Multiplication Rule

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Events are not independent

$$P(A \cap B) = P(A).P(B/A)$$

This rule says that the probability of the intersection of the events A and B equals the product of the probability of A and the probability of B given that A has happened or known to you. This is symbolized in the second term of the above expression as P (B/A). P(B/A) is called the conditional probability of B given the fact that A has happened.

We can also write if B has already happened.

$$P(A \cap B) = P(B).P(A/B)$$

Multiplication Rule

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Events are not independent-Example

From a pack of cards, 2 cards are drawn in succession one after the other. After every draw, the selected card is not replaced. What is the probability that in both the draws you will get Spades?

Solution:

Let A = getting Spade in the first draw

Let B = getting spade in the second draw.

The cards are not replaced. This situation requires the use of conditional probability.

P(A) = 13/52 (There are 13 Spades and 52 cards in a pack)

P(B/A)=12/51(There are 12 Spades and 51 cards because the first card selected is not replaced after the first draw)

= (13/52).(12/51)=156/2652= 1/17.

$$P(A \cap B) = P(A).P(B/A)$$

Marginal Probability

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Contingency table consists of rows and columns of two attributes at different levels with frequencies or numbers in each of the cells. It is a matrix of frequencies assigned to rows and columns.

The term marginal is used to indicate that the probabilities are calculated using a contingency table (also called joint probability table).

Problem for Discussion

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- Of the cars on a used car lot, 70% have air conditioning (AC) and 40% have a CD player (CD). 20% of the cars have both.

- What is the probability that a car has a CD player, given that it has AC ?

  i.e., we want to find $P(CD \mid AC)$

Solution for the Problem

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Attributes	CD	NO CD	Marginal Total
AC	20	50	70
NO AC	20	10	30
Marginal Total	40	60	100

From the table it is easy to see that there are 70 cars with AC out of which 20 have CD.
Hence P(CD | AC) = 20/70

Bayes' Theorem

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- Bayes' Theorem is used to revise previously calculated probabilities based on new information.
- Developed by Thomas Bayes in the 18th Century.
- It is an extension of conditional probability.

Bayes' Theorem

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$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \cdots + P(A \mid B_k)P(B_k)}$$

- where:

  B_i = i^{th} event of k mutually exclusive and collectively exhaustive events

  A = new event that might impact $P(B_i)$

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Bayes' Theorem Discussion Problem

- A drilling company has estimated a 40% chance of striking oil for their new well.
- A detailed test has been scheduled for more information. Historically, 60% of successful wells have had detailed tests, and 20% of unsuccessful wells have had detailed tests.
- Given that this well has been scheduled for a detailed test, what is the probability that the well will be successful?