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# Statistics Formulas

Statistics is  
the grammar of science.  
*Karl Pearson*

# Descriptive Statistics

## Population

Size  $N$

Mean  $\mu = \frac{\sum_{i=1}^N x_i}{N}$

Variance  $\sigma^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N}$

Standard deviation  $\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \mu)^2}{N}}$

## Sample

Sample size  $n$

Mean  $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

Variance  $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$

Standard deviation  $S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$

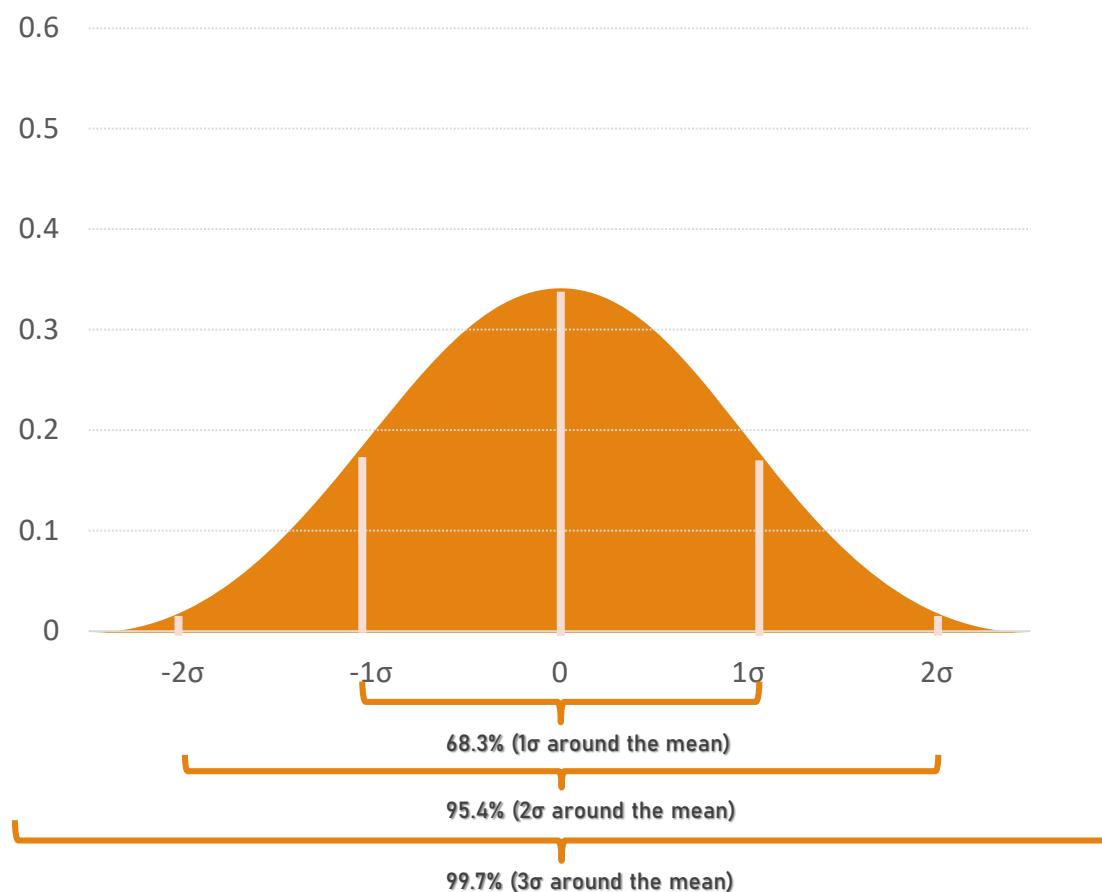
## Mode

The mode is defined by the most common value of the sample or the population.  
If there are two most frequent values we may say the dataset is bimodal.

## Median

The median is the value exactly in the middle of a sample or a population.

# Normal distribution



## Empirical rule

For normal distributions, about

- 68% of data points fall within 1 standard deviation of the mean.
- 95% of data points fall within 2 standard deviations of the mean.
- 99.7% of data points fall within 3 standard deviations of the mean.

## Z-score

$$z = \frac{x-\mu}{\sigma}$$

# Z-Score Table (positive)

<b>Z</b>	<b>0</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
-3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
-3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
-3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
-3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
-3	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
-2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
-2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
-2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
-2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
-2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
-2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
-2	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
-1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

# Z-Score Table (negative)

<b>Z</b>	<b>0</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
<b>0</b>	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
<b>0.1</b>	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
<b>0.2</b>	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
<b>0.3</b>	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
<b>0.4</b>	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
<b>0.5</b>	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
<b>0.6</b>	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
<b>0.7</b>	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
<b>0.8</b>	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
<b>0.9</b>	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
<b>1</b>	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
<b>1.1</b>	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
<b>1.2</b>	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
<b>1.3</b>	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
<b>1.4</b>	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
<b>1.5</b>	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
<b>1.6</b>	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
<b>1.7</b>	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
<b>1.8</b>	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
<b>1.9</b>	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
<b>2</b>	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
<b>2.1</b>	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
<b>2.2</b>	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
<b>2.3</b>	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
<b>2.4</b>	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
<b>2.5</b>	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
<b>2.6</b>	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
<b>2.7</b>	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
<b>2.8</b>	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
<b>2.9</b>	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
<b>3</b>	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
<b>3.1</b>	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
<b>3.2</b>	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
<b>3.3</b>	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
<b>3.4</b>	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976

## Bayes' theorem

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

## At least one success or failure

$$P(\text{at least 1 success}) = 1 - P(\text{all failure})$$

$$P(\text{at least 1 failure}) = 1 - P(\text{all successes})$$

## Probability of an event

$$P(\text{event}) = \frac{\text{number of outcome that meets our criteria (success)}}{\text{number of all possible outcomes}}$$

For events where every outcome has the same probability.

## Addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

## Multiplication rule

$$\text{For independent events: } P(A \text{ and } B) = P(A) \cdot P(B)$$

$$\text{For dependent events: } P(A \text{ and } B) = P(A) \cdot P(B|A)$$

## Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

## Binomial density function

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

"Probability of k successes in n attempts"

$$E(X) = \mu = np$$

$$\sigma^2 = np(1-p)$$

## Poisson density function

$$P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

$$E(X) = \mu = \lambda$$

$$\sigma^2 = \lambda$$

## Poisson cumulative density function (c.d.f.)

$$P(X \leq n) = e^{-\lambda} \sum_{k=0}^n \frac{\lambda^k}{k!}$$

## Sampling distribution of the sample mean

Mean:  $\mu_{\bar{x}} = \mu$

Standard deviation:  $SE_{\bar{x}} = \sigma_{\bar{x}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

## Sampling distribution of the sample proportion

Mean:  $\mu_{\hat{p}} = p$

Standard deviation:  $SE_{\hat{p}} = \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

## Confidence interval

$$\begin{aligned}\hat{p} &\pm z^* SE_{\hat{p}} \\ \hat{p} &\pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\end{aligned}$$

## Margin of error

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

# *t*-table

## Significance level

One-sided	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
Two-sided	0.5	0.40	0.3	0.20	0.10	0.05	0.02	0.01	0.002	0.001
1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \quad \text{t-score} \quad \text{where } s \text{ is the sample standard deviation}$$

## Regression line

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{(\sum y) - m(\sum x)}{n}$$

## Correlation coefficient

$$r_{xy} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}$$

## Residual

residual = actual value – predicted value

$$e_i = a_i - p_i$$

## MSE (Mean-Square-Error)

$$MSE = \frac{\sum e_i^2}{n}$$

## MAE (Mean-Absolute-Error)

$$MAE = \frac{\sum |e_i|}{n}$$

## RMSE (Root-Mean-Square-Error)

$$RMSE = \sqrt{\frac{\sum e_i^2}{n}}$$

# F Table for $\alpha=0.10$

(see next table below for 12 to  $\infty$ )

DF2	DF1									
	1	2	3	4	5	6	7	8	9	10
1	39.86346	49.5	53.59324	55.83296	57.24008	58.20442	58.90595	59.43898	59.85759	60.19498
2	8.52632	9	9.16179	9.24342	9.29263	9.32553	9.34908	9.36677	9.38054	9.39157
3	5.53832	5.46238	5.39077	5.34264	5.30916	5.28473	5.26619	5.25167	5.24	5.23041
4	4.54477	4.32456	4.19086	4.10725	4.05058	4.00975	3.97897	3.95494	3.93567	3.91988
5	4.06042	3.77972	3.61948	3.5202	3.45298	3.40451	3.3679	3.33928	3.31628	3.2974
6	3.77595	3.4633	3.28876	3.18076	3.10751	3.05455	3.01446	2.98304	2.95774	2.93693
7	3.58943	3.25744	3.07407	2.96053	2.88334	2.82739	2.78493	2.75158	2.72468	2.70251
8	3.45792	3.11312	2.9238	2.80643	2.72645	2.66833	2.62413	2.58935	2.56124	2.53804
9	3.3603	3.00645	2.81286	2.69268	2.61061	2.55086	2.50531	2.46941	2.44034	2.41632
10	3.28502	2.92447	2.72767	2.60534	2.52164	2.46058	2.41397	2.37715	2.34731	2.3226
11	3.2252	2.85951	2.66023	2.53619	2.45118	2.38907	2.34157	2.304	2.2735	2.24823
12	3.17655	2.8068	2.60552	2.4801	2.39402	2.33102	2.28278	2.24457	2.21352	2.18776
13	3.13621	2.76317	2.56027	2.43371	2.34672	2.28298	2.2341	2.19535	2.16382	2.13763
14	3.10221	2.72647	2.52222	2.39469	2.30694	2.24256	2.19313	2.1539	2.12195	2.0954
15	3.07319	2.69517	2.48979	2.36143	2.27302	2.20808	2.15818	2.11853	2.08621	2.05932
16	3.04811	2.66817	2.46181	2.33274	2.24376	2.17833	2.128	2.08798	2.05533	2.02815
17	3.02623	2.64464	2.43743	2.30775	2.21825	2.15239	2.10169	2.06134	2.02839	2.00094
18	3.00698	2.62395	2.41601	2.28577	2.19583	2.12958	2.07854	2.03789	2.00467	1.97698
19	2.9899	2.60561	2.39702	2.2663	2.17596	2.10936	2.05802	2.0171	1.98364	1.95573
20	2.97465	2.58925	2.38009	2.24893	2.15823	2.09132	2.0397	1.99853	1.96485	1.93674
21	2.96096	2.57457	2.36489	2.23334	2.14231	2.07512	2.02325	1.98186	1.94797	1.91967
22	2.94858	2.56131	2.35117	2.21927	2.12794	2.0605	2.0084	1.9668	1.93273	1.90425
23	2.93736	2.54929	2.33873	2.20651	2.11491	2.04723	1.99492	1.95312	1.91888	1.89025
24	2.92712	2.53833	2.32739	2.19488	2.10303	2.03513	1.98263	1.94066	1.90625	1.87748
25	2.91774	2.52831	2.31702	2.18424	2.09216	2.02406	1.97138	1.92925	1.89469	1.86578
26	2.90913	2.5191	2.30749	2.17447	2.08218	2.01389	1.96104	1.91876	1.88407	1.85503
27	2.90119	2.51061	2.29871	2.16546	2.07298	2.00452	1.95151	1.90909	1.87427	1.84511
28	2.89385	2.50276	2.2906	2.15714	2.06447	1.99585	1.9427	1.90014	1.8652	1.83593
29	2.88703	2.49548	2.28307	2.14941	2.05658	1.98781	1.93452	1.89184	1.85679	1.82741
30	2.88069	2.48872	2.27607	2.14223	2.04925	1.98033	1.92692	1.88412	1.84896	1.81949
40	2.83535	2.44037	2.22609	2.09095	1.99682	1.92688	1.87252	1.82886	1.7929	1.76269
60	2.79107	2.39325	2.17741	2.04099	1.94571	1.87472	1.81939	1.77483	1.73802	1.70701
120	2.74781	2.34734	2.12999	1.9923	1.89587	1.82381	1.76748	1.72196	1.68425	1.65238
$\infty$	2.70554	2.30259	2.0838	1.94486	1.84727	1.77411	1.71672	1.6702	1.63152	1.59872

# F Table for $\alpha=0.10$

(table for 12 to  $\infty$ )

DF2	DF1									
	12	15	20	24	30	40	60	120	$\infty$	
1	60.70521	61.22034	61.74029	62.00205	62.26497	62.52905	62.79428	63.06064	63.32812	
2	9.40813	9.42471	9.44131	9.44962	9.45793	9.46624	9.47456	9.48289	9.49122	
3	5.21562	5.20031	5.18448	5.17636	5.16811	5.15972	5.15119	5.14251	5.1337	
4	3.89553	3.87036	3.84434	3.83099	3.81742	3.80361	3.78957	3.77527	3.76073	
5	3.26824	3.23801	3.20665	3.19052	3.17408	3.15732	3.14023	3.12279	3.105	
6	2.90472	2.87122	2.83634	2.81834	2.79996	2.78117	2.76195	2.74229	2.72216	
7	2.66811	2.63223	2.59473	2.57533	2.55546	2.5351	2.51422	2.49279	2.47079	
8	2.50196	2.46422	2.42464	2.4041	2.38302	2.36136	2.3391	2.31618	2.29257	
9	2.37888	2.33962	2.29832	2.27683	2.25472	2.23196	2.20849	2.18427	2.15923	
10	2.28405	2.24351	2.20074	2.17843	2.15543	2.13169	2.10716	2.08176	2.05542	
11	2.20873	2.16709	2.12305	2.10001	2.07621	2.05161	2.02612	1.99965	1.97211	
12	2.14744	2.10485	2.05968	2.03599	2.01149	1.9861	1.95973	1.93228	1.90361	
13	2.09659	2.05316	2.00698	1.98272	1.95757	1.93147	1.90429	1.87591	1.8462	
14	2.05371	2.00953	1.96245	1.93766	1.91193	1.88516	1.85723	1.828	1.79728	
15	2.01707	1.97222	1.92431	1.89904	1.87277	1.84539	1.81676	1.78672	1.75505	
16	1.98539	1.93992	1.89127	1.86556	1.83879	1.81084	1.78156	1.75075	1.71817	
17	1.95772	1.91169	1.86236	1.83624	1.80901	1.78053	1.75063	1.71909	1.68564	
18	1.93334	1.88681	1.83685	1.81035	1.78269	1.75371	1.72322	1.69099	1.65671	
19	1.9117	1.86471	1.81416	1.78731	1.75924	1.72979	1.69876	1.66587	1.63077	
20	1.89236	1.84494	1.79384	1.76667	1.73822	1.70833	1.67678	1.64326	1.60738	
21	1.87497	1.82715	1.77555	1.74807	1.71927	1.68896	1.65691	1.62278	1.58615	
22	1.85925	1.81106	1.75899	1.73122	1.70208	1.67138	1.63885	1.60415	1.56678	
23	1.84497	1.79643	1.74392	1.71588	1.68643	1.65535	1.62237	1.58711	1.54903	
24	1.83194	1.78308	1.73015	1.70185	1.6721	1.64067	1.60726	1.57146	1.5327	
25	1.82	1.77083	1.71752	1.68898	1.65895	1.62718	1.59335	1.55703	1.5176	
26	1.80902	1.75957	1.70589	1.67712	1.64682	1.61472	1.5805	1.54368	1.5036	
27	1.79889	1.74917	1.69514	1.66616	1.6356	1.6032	1.56859	1.53129	1.49057	
28	1.78951	1.73954	1.68519	1.656	1.62519	1.5925	1.55753	1.51976	1.47841	
29	1.78081	1.7306	1.67593	1.64655	1.61551	1.58253	1.54721	1.50899	1.46704	
30	1.7727	1.72227	1.66731	1.63774	1.60648	1.57323	1.53757	1.49891	1.45636	
40	1.71456	1.66241	1.60515	1.57411	1.54108	1.50562	1.46716	1.42476	1.37691	
60	1.65743	1.60337	1.54349	1.51072	1.47554	1.43734	1.3952	1.34757	1.29146	
120	1.6012	1.545	1.48207	1.44723	1.40938	1.3676	1.32034	1.26457	1.19256	
$\infty$	1.54578	1.48714	1.4206	1.38318	1.34187	1.29513	1.23995	1.1686	1	

# F Table for $\alpha=0.05$

(see next table below for 12 to  $\infty$ )

DF2	DF1									
	1	2	3	4	5	6	8	9	10	
1	161.4476	199.5	215.7073	224.5832	230.1619	233.986	238.8827	240.5433	241.8817	
2	18.5128	19	19.1643	19.2468	19.2964	19.3295	19.371	19.3848	19.3959	
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8452	8.8123	8.7855	
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.041	5.9988	5.9644	
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8183	4.7725	4.7351	
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.1468	4.099	4.06	
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.866	3.7257	3.6767	3.6365	
8	5.3177	4.459	4.0662	3.8379	3.6875	3.5806	3.4381	3.3881	3.3472	
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2296	3.1789	3.1373	
10	4.9646	4.1028	3.7083	3.478	3.3258	3.2172	3.0717	3.0204	2.9782	
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	2.948	2.8962	2.8536	
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.8486	2.7964	2.7534	
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.7669	2.7144	2.671	
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.6987	2.6458	2.6022	
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.6408	2.5876	2.5437	
16	4.494	3.6337	3.2389	3.0069	2.8524	2.7413	2.5911	2.5377	2.4935	
17	4.4513	3.5915	3.1968	2.9647	2.81	2.6987	2.548	2.4943	2.4499	
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5102	2.4563	2.4117	
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.4768	2.4227	2.3779	
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.599	2.4471	2.3928	2.3479	
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4205	2.366	2.321	
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.3965	2.3419	2.2967	
23	4.2793	3.4221	3.028	2.7955	2.64	2.5277	2.3748	2.3201	2.2747	
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.3551	2.3002	2.2547	
25	4.2417	3.3852	2.9912	2.7587	2.603	2.4904	2.3371	2.2821	2.2365	
26	4.2252	3.369	2.9752	2.7426	2.5868	2.4741	2.3205	2.2655	2.2197	
27	4.21	3.3541	2.9604	2.7278	2.5719	2.4591	2.3053	1.7306	1.6717	
28	4.196	3.3404	2.9467	2.7141	2.5581	2.4453	2.2913	2.236	2.19	
29	4.183	3.3277	2.934	2.7014	2.5454	2.4324	2.2783	2.2229	2.1768	
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.2662	2.2107	2.1646	
40	4.0847	3.2317	2.8387	2.606	2.4495	2.3359	2.1802	2.124	2.0772	
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.097	2.0401	1.9926	
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.175	2.0164	1.9588	1.9105	
$\infty$	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	1.9384	1.8799	1.8307	

# F Table for $\alpha=0.05$

(table for 12 to  $\infty$ )

DF2	DF1								
	12	15	20	24	30	40	120	$\infty$	
1	243.906	245.9499	248.0131	249.0518	250.0951	251.1432	253.2529	254.3144	
2	19.4125	19.4291	19.4458	19.4541	19.4624	19.4707	19.4874	19.4957	
3	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5494	8.5264	
4	5.9117	5.8578	5.8025	5.7744	5.7459	5.717	5.6581	5.6281	
5	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.3985	4.365	
6	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7047	3.6689	
7	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.2674	3.2298	
8	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	2.9669	2.9276	
9	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7475	2.7067	
10	2.913	2.845	2.774	2.7372	2.6996	2.6609	2.5801	2.5379	
11	2.7876	2.7186	2.6464	2.609	2.5705	2.5309	2.448	2.4045	
12	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.341	2.2962	
13	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2524	2.2064	
14	2.5342	2.463	2.3879	2.3487	2.3082	2.2664	2.1778	2.1307	
15	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1141	2.0658	
16	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.0589	2.0096	
17	2.3807	2.3077	2.2304	2.1898	2.1477	2.104	2.0107	1.9604	
18	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	1.9681	1.9168	
19	2.308	2.2341	2.1555	2.1141	2.0712	2.0264	1.9302	1.878	
20	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.8963	1.8432	
21	2.2504	2.1757	2.096	2.054	2.0102	1.9645	1.8657	1.8117	
22	2.2258	2.1508	2.0707	2.0283	1.9842	1.938	1.838	1.7831	
23	2.2036	2.1282	2.0476	2.005	1.9605	1.9139	1.8128	1.757	
24	2.1834	2.1077	2.0267	1.9838	1.939	1.892	1.7896	1.733	
25	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.7684	1.711	
26	2.1479	2.0716	1.9898	1.9464	1.901	1.8533	1.7488	1.6906	
27	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7306	1.6717	
28	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7138	1.6541	
29	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.6981	1.6376	
30	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.6835	1.6223	
40	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.5766	1.5089	
60	1.9174	1.8364	1.748	1.7001	1.6491	1.5943	1.4673	1.3893	
120	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.3519	1.2539	
$\infty$	1.7522	1.6664	1.5705	1.5173	1.4591	1.394	1.2214	1	

# F Table for $\alpha=0.025$

(see next table below for 12 to  $\infty$ )

DF2	DF1									
	1	2	3	4	5	6	8	9	10	
1	647.789	799.5	864.163	899.5833	921.8479	937.1111	956.6562	963.2846	968.6274	
2	38.5063	39	39.1655	39.2484	39.2982	39.3315	39.373	39.3869	39.398	
3	17.4434	16.0441	15.4392	15.101	14.8848	14.7347	14.5399	14.4731	14.4189	
4	12.2179	10.6491	9.9792	9.6045	9.3645	9.1973	8.9796	8.9047	8.8439	
5	10.007	8.4336	7.7636	7.3879	7.1464	6.9777	6.7572	6.6811	6.6192	
6	8.8131	7.2599	6.5988	6.2272	5.9876	5.8198	5.5996	5.5234	5.4613	
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.8993	4.8232	4.7611	
8	7.5709	6.0595	5.416	5.0526	4.8173	4.6517	4.4333	4.3572	4.2951	
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.102	4.026	3.9639	
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.8549	3.779	3.7168	
11	6.7241	5.2559	4.63	4.2751	4.044	3.8807	3.6638	3.5879	3.5257	
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.5118	3.4358	3.3736	
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.388	3.312	3.2497	
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.2853	3.2093	3.1469	
15	6.1995	4.765	4.1528	3.8043	3.5764	3.4147	3.1987	3.1227	3.0602	
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.1248	3.0488	2.9862	
17	6.042	4.6189	4.0112	3.6648	3.4379	3.2767	3.061	2.9849	2.9222	
18	5.9781	4.5597	3.9539	3.6083	3.382	3.2209	3.0053	2.9291	2.8664	
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	2.9563	2.8801	2.8172	
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	2.9128	2.8365	2.7737	
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.874	2.7977	2.7348	
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.8392	2.7628	2.6998	
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.8077	2.7313	2.6682	
24	5.7166	4.3187	3.7211	3.3794	3.1548	2.9946	2.7791	2.7027	2.6396	
25	5.6864	4.2909	3.6943	3.353	3.1287	2.9685	2.7531	2.6766	2.6135	
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.7293	2.6528	1.878	
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.7074	2.6309	2.5676	
28	5.6096	4.2205	3.6264	3.2863	3.0626	2.9027	2.6872	2.6106	2.5473	
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.884	2.6686	2.5919	2.5286	
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.6513	2.5746	2.5112	
40	5.4239	4.051	3.4633	3.1261	2.9037	2.7444	2.5289	2.4519	2.3882	
60	5.2856	3.9253	3.3425	3.0077	2.7863	2.6274	2.4117	2.3344	2.2702	
120	5.1523	3.8046	3.2269	2.8943	2.674	2.5154	2.2994	2.2217	2.157	
INF	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.1918	2.1136	2.0483	

# F Table for $\alpha=0.025$

(table for 12 to  $\infty$ )

DF2	DF1								
	12	15	20	24	30	40	120	$\infty$	
1	976.7079	984.8668	993.1028	997.2492	1001.414	1005.598	1014.02	1018.258	
2	39.4146	39.4313	39.4479	39.4562	39.465	39.473	39.49	39.498	
3	14.3366	14.2527	14.1674	14.1241	14.081	14.037	13.947	13.902	
4	8.7512	8.6565	8.5599	8.5109	8.461	8.411	8.309	8.257	
5	6.5245	6.4277	6.3286	6.278	6.227	6.175	6.069	6.015	
6	5.3662	5.2687	5.1684	5.1172	5.065	5.012	4.904	4.849	
7	4.6658	4.5678	4.4667	4.415	4.362	4.309	4.199	4.142	
8	4.1997	4.1012	3.9995	3.9472	3.894	3.84	3.728	3.67	
9	3.8682	3.7694	3.6669	3.6142	3.56	3.505	3.392	3.333	
10	3.6209	3.5217	3.4185	3.3654	3.311	3.255	3.14	3.08	
11	3.4296	3.3299	3.2261	3.1725	3.118	3.061	2.944	2.883	
12	3.2773	3.1772	3.0728	3.0187	2.963	2.906	2.787	2.725	
13	3.1532	3.0527	2.9477	2.8932	2.837	2.78	2.659	2.595	
14	3.0502	2.9493	2.8437	2.7888	2.732	2.674	2.552	2.487	
15	2.9633	2.8621	2.7559	2.7006	2.644	2.585	2.461	2.395	
16	2.889	2.7875	2.6808	2.6252	2.568	2.509	2.383	2.316	
17	2.8249	2.723	2.6158	2.5598	2.502	2.442	2.315	2.247	
18	2.7689	2.6667	2.559	2.5027	2.445	2.384	2.256	2.187	
19	2.7196	2.6171	2.5089	2.4523	2.394	2.333	2.203	2.133	
20	2.6758	2.5731	2.4645	2.4076	2.349	2.287	2.156	2.085	
21	2.6368	2.5338	2.4247	2.3675	2.308	2.246	2.114	2.042	
22	2.6017	2.4984	2.389	2.3315	2.272	2.21	2.076	2.003	
23	2.5699	2.4665	2.3567	2.2989	2.239	2.176	2.041	1.968	
24	2.5411	2.4374	2.3273	2.2693	2.209	2.146	2.01	1.935	
25	2.5149	2.411	2.3005	2.2422	2.182	2.118	1.981	1.906	
26	2.4908	2.3867	2.2759	2.2174	2.157	2.093	1.954	1.878	
27	2.4688	2.3644	2.2533	2.1946	2.133	2.069	1.93	1.853	
28	2.4484	2.3438	2.2324	2.1735	2.112	2.048	1.907	1.829	
29	2.4295	2.3248	2.2131	2.154	2.092	2.028	1.886	1.807	
30	2.412	2.3072	2.1952	2.1359	2.074	2.009	1.866	1.787	
40	2.2882	2.1819	2.0677	2.0069	1.943	1.875	1.724	1.637	
60	2.1692	2.0613	1.9445	1.8817	1.815	1.744	1.581	1.482	
120	2.0548	1.945	1.8249	1.7597	1.69	1.614	1.433	1.31	
INF	1.9447	1.8326	1.7085	1.6402	1.566	1.484	1.268	1	

# F Table for $\alpha=0.01$

(see next table below for 12 to  $\infty$ )

DF2	DF1									
	1	2	3	4	5	6	8	9	10	
1	4052.181	4999.5	5403.352	5624.583	5763.65	5858.986	5981.07	6022.473	6055.847	
2	98.503	99	99.166	99.249	99.299	99.333	99.374	99.388	99.399	
3	34.116	30.817	29.457	28.71	28.237	27.911	27.489	27.345	27.229	
4	21.198	18	16.694	15.977	15.522	15.207	14.799	14.659	14.546	
5	16.258	13.274	12.06	11.392	10.967	10.672	10.289	10.158	10.051	
6	13.745	10.925	9.78	9.148	8.746	8.466	8.102	7.976	7.874	
7	12.246	9.547	8.451	7.847	7.46	7.191	6.84	6.719	6.62	
8	11.259	8.649	7.591	7.006	6.632	6.371	6.029	5.911	5.814	
9	10.561	8.022	6.992	6.422	6.057	5.802	5.467	5.351	5.257	
10	10.044	7.559	6.552	5.994	5.636	5.386	5.057	4.942	4.849	
11	9.646	7.206	6.217	5.668	5.316	5.069	4.744	4.632	4.539	
12	9.33	6.927	5.953	5.412	5.064	4.821	4.499	4.388	4.296	
13	9.074	6.701	5.739	5.205	4.862	4.62	4.302	4.191	4.1	
14	8.862	6.515	5.564	5.035	4.695	4.456	4.14	4.03	3.939	
15	8.683	6.359	5.417	4.893	4.556	4.318	4.004	3.895	3.805	
16	8.531	6.226	5.292	4.773	4.437	4.202	3.89	3.78	3.691	
17	8.4	6.112	5.185	4.669	4.336	4.102	3.791	3.682	3.593	
18	8.285	6.013	5.092	4.579	4.248	4.015	3.705	3.597	3.508	
19	8.185	5.926	5.01	4.5	4.171	3.939	3.631	3.523	3.434	
20	8.096	5.849	4.938	4.431	4.103	3.871	3.564	3.457	3.368	
21	8.017	5.78	4.874	4.369	4.042	3.812	3.506	3.398	3.31	
22	7.945	5.719	4.817	4.313	3.988	3.758	3.453	3.346	3.258	
23	7.881	5.664	4.765	4.264	3.939	3.71	3.406	3.299	3.211	
24	7.823	5.614	4.718	4.218	3.895	3.667	3.363	3.256	3.168	
25	7.77	5.568	4.675	4.177	3.855	3.627	3.324	3.217	3.129	
26	7.721	5.526	4.637	4.14	3.818	3.591	3.288	3.182	3.094	
27	7.677	5.488	4.601	4.106	3.785	3.558	3.256	3.149	3.062	
28	7.636	5.453	4.568	4.074	3.754	3.528	3.226	3.12	3.032	
29	7.598	5.42	4.538	4.045	3.725	3.499	3.198	3.092	3.005	
30	7.562	5.39	4.51	4.018	3.699	3.473	3.173	3.067	2.979	
40	7.314	5.179	4.313	3.828	3.514	3.291	2.993	2.888	2.801	
60	7.077	4.977	4.126	3.649	3.339	3.119	2.823	2.718	2.632	
120	6.851	4.787	3.949	3.48	3.174	2.956	2.663	2.559	2.472	
$\infty$	6.635	4.605	3.782	3.319	3.017	2.802	2.511	2.407	2.321	

# F Table for $\alpha=0.01$

(table for 12 to  $\infty$ )

DF2	DF1								
	12	15	20	24	30	40	120	$\infty$	
1	6106.321	6157.285	6208.73	6234.631	6260.649	6286.782	6339.391	6365.864	
2	99.416	99.433	99.449	99.458	99.466	99.474	99.491	99.499	
3	27.052	26.872	26.69	26.598	26.505	26.411	26.221	26.125	
4	14.374	14.198	14.02	13.929	13.838	13.745	13.558	13.463	
5	9.888	9.722	9.553	9.466	9.379	9.291	9.112	9.02	
6	7.718	7.559	7.396	7.313	7.229	7.143	6.969	6.88	
7	6.469	6.314	6.155	6.074	5.992	5.908	5.737	5.65	
8	5.667	5.515	5.359	5.279	5.198	5.116	4.946	4.859	
9	5.111	4.962	4.808	4.729	4.649	4.567	4.398	4.311	
10	4.706	4.558	4.405	4.327	4.247	4.165	3.996	3.909	
11	4.397	4.251	4.099	4.021	3.941	3.86	3.69	3.602	
12	4.155	4.01	3.858	3.78	3.701	3.619	3.449	3.361	
13	3.815	3.665	3.587	3.507	3.425	3.341	3.165	3.1748	
14	3.8	3.656	3.505	3.427	3.348	3.266	3.094	3.004	
15	3.666	3.522	3.372	3.294	3.214	3.132	2.959	2.868	
16	3.553	3.409	3.259	3.181	3.101	3.018	2.845	2.753	
17	3.455	3.312	3.162	3.084	3.003	2.92	2.746	2.653	
18	3.371	3.227	3.077	2.999	2.919	2.835	2.66	2.566	
19	3.297	3.153	3.003	2.925	2.844	2.761	2.584	2.489	
20	3.231	3.088	2.938	2.859	2.778	2.695	2.517	2.421	
21	3.173	3.03	2.88	2.801	2.72	2.636	2.457	2.36	
22	3.121	2.978	2.827	2.749	2.667	2.583	2.403	2.305	
23	3.074	2.931	2.781	2.702	2.62	2.535	2.354	2.256	
24	3.032	2.889	2.738	2.659	2.577	2.492	2.31	2.211	
25	2.993	2.85	2.699	2.62	2.538	2.453	2.27	2.169	
26	2.958	2.815	2.664	2.585	2.503	2.417	2.233	2.131	
27	2.926	2.783	2.632	2.552	2.47	2.384	2.198	2.097	
28	2.896	2.753	2.602	2.522	2.44	2.354	2.167	2.064	
29	2.868	2.726	2.574	2.495	2.412	2.325	2.138	2.034	
30	2.843	2.7	2.549	2.469	2.386	2.299	2.111	2.006	
40	2.665	2.522	2.369	2.288	2.203	2.114	1.917	1.805	
60	2.496	2.352	2.198	2.115	2.028	1.936	1.726	1.601	
120	2.336	2.192	2.035	1.95	1.86	1.763	1.533	1.381	
$\infty$	2.185	2.039	1.878	1.791	1.696	1.592	1.325	1	

**Statistical thinking  
will one day be  
as necessary a qualification  
for efficient citizenship  
as the ability  
to read and write.**

*Herbert George "H.G." Wells  
(1866 – 1946, English writer)*