BASIC PROBABILITY

The likelihood of an event occurring

$$P(X) = \frac{\text{preferred outcomes}}{\text{sample space}}$$

$$P(A, B) = P(A) + P(B)$$

EXPECTED VALUES

Trial	Experiment	Experimental Probability	Expected Value
Observing an event and record the outcome	Collection of one or multiple trials	Probability of an event based on the experiment	Specific outcome we expect to occur when we run an experiment

Categorical: $E(X) = n \cdot p$

Numeric: $E(X) = \sum_{i=1}^{n} n_i \cdot p_i$

FREQUENCY

Probability frequency distribution	Collection of the probabilities for each possible outcome of an event.
Why?	To try and predict future events when the expected value is unattainable.
Frequency	Number of times a given value or outcome appears in the sample space.
Frequency distribution table	Table that matches each distinct outcome in the sample space to its associated frequency.
How?	By dividing every frequency by the size of the sample space.

COMPLEMENTS

Everything an event is **not**

A’ = Not A

P(A’) = 1-P(A)

A + A’ = Sample space

COMBINATORICS

The likelihood of an event occurring

$$P(X) = \frac{\text{preferred outcomes}}{\text{sample space}}$$

$$P(A, B) = P(A) + P(B)$$

Without repetition | With repetition

PERMUTATIONS (arrange)

$$P_n = P_n^n = n!$$

How many ways are to arrange 3 letters a, b, c?

How many ways are to arrange 2 letters a and 2 letters b?

$$P_{n_1,\ldots,n_k} = \frac{(\sum n_i)!}{\prod n_i!}$$

VARIATIONS (pick and arrange)

$$V_p^n = P_p^n = \frac{n!}{(n-p)!}$$

Hoy many words of 2 different letters can you make with 4 letters a, b, c, d?

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$$\bar{V}_p^n = n^p$$

COMBINATIONS (pick)

$$C_p^n = \frac{n!}{p!\,(n-p)!}$$

Hoy many ways are there to pick 2 different letters out of 4 letters a, b, c, d?

Hoy many ways are there to pick 2 letters out of 4 letters a, b, c, d?

$$\bar{C}_p^n = \mathrm{C}_{\mathrm{p}}^{\mathrm{n+p-1}}$$

FACTORIALS

0! = 1
If n < 0, n! doesn't exist

If n > 0, n > k

$$(n+k)! = n! \cdot (n+1) \cdot \ldots \cdot (n+k)$$

$$(n-k)! = \frac{n!}{(n-k+1) \cdot \ldots \cdot n}$$

$$\frac{n!}{k!} = (k+1) \cdot \ldots \cdot n$$

BAYESIAN INFERENCE

$x \in A$, $x \notin A$: element x is a part of set A (x NOT in A)

$A \ni x$: set A contains element x

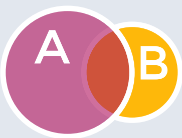
$\forall x$: for all/any x

$A \subseteq B$: A is a subset of B

Mutually exclusive

$A \cap B = \emptyset$

All complements are mutually exclusive, but not all mutually exclusive sets are complements



Intersect

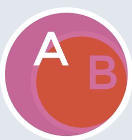
$A \cap B$

Satisfies all the events simultaneously

Union

$A \cup B = A + B - A \cap B$

Satisfies at least one of the events



Completely overlap

Independent Events

$P(A|B) = P(A)$

Theoretically probability remains unaffected by other events

Dependent Events

$P(A|B) \neq P(A)$

Probabilities of dependent events vary as conditions change

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- B has occurred
- Only elements of the intersection can satisfy A
- P(A|B) not the same meaning as P(B|A)

Law of Total Probability

$A = B_1 + \dots + B_n$

$P(A) = P(A|B_1) \cdot P(B_1) + \dots + P(A|B_n) \cdot P(B_n)$

Additive Law

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Rule

$P(A \cap B) = P(A|B) \cdot P(B)$

Bayes' Law

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

DISTRIBUTIONS

Show the possible values a random variable can take and how frequently they occur.

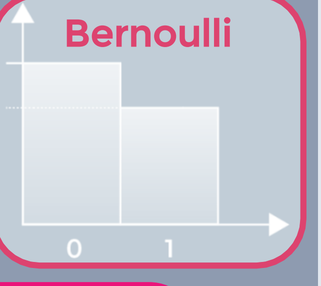
- Y actual outcome
- Y one of the possible outcomes
- $P(Y = y) = p(y)$
- Probability function: function that assigns a probability to each distinct outcome in the sample space

	Population	Sample
Mean	μ	$\bar{x}$
Variance	σ^2	s^2
Standard Deviation	σ	s

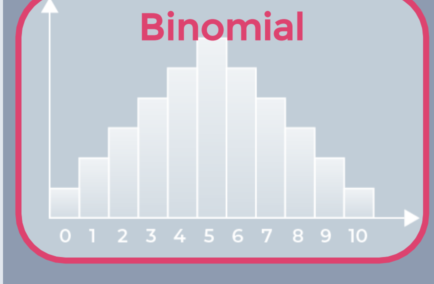
DISCRETE

- Finite number of outcomes
- Can add up individual value to determine the probability of an interval
- Expressed with table, graph or piecewise function
- Expected values might be unattainable

- $Y \sim U(a, b)$
- $Y \sim U(a)$ for categorical
- Outcomes are equally likely
- No predictive power



- $Y \sim Bern(p)$
- 1 trial, 2 possible outcomes
- $E(Y) = p$
- $\sigma^2 (Y) = p \cdot (1-p)$



- $Y \sim B(n, p)$
- Measures the p 1 of the possible outcomes over n trials
- $P(Y=y) = p(y) = C(y, n) \cdot p^y \cdot (1-p)^{n-y}$
- $E(Y) = n \cdot p$
- $\sigma^2 (Y) = n \cdot p \cdot (1-p)$

- $Y \sim Po(\lambda)$
- Measures the frequency over an interval of time or distance ($\lambda \geq 0$)
- $P(Y=y) = p(y) = \dfrac{\lambda^y}{y!\, e^{-\lambda}}$
- $E(Y) = \lambda$
- $\sigma^2 (Y) = \lambda$

DISTRIBUTIONS

- PDF: Probability Density Function
- CDF: Cummulative Density Function

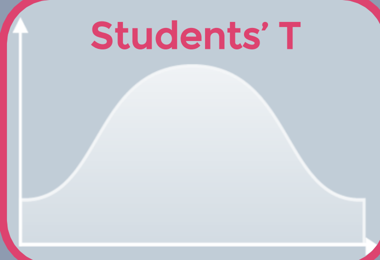
CONTINUOUS

- Infinitely many consecutive possible values
- Cannot add up individual value to determine the probability of an interval
- Expressed with graph or continuous function
- $P(Y=y) = p(y) = 0$ for any individual value y ($P(Y< y) = P(Y \leq y)$)

Normal

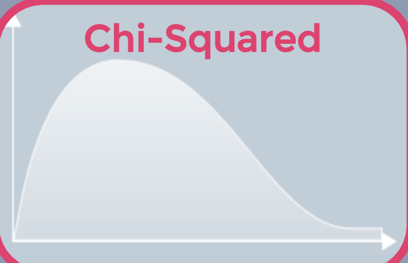
- $Y \sim N(\mu, \sigma^2)$
- $E(Y) = \mu$
- $\sigma^2(Y) = \sigma^2$
- 68% of all ist values fall in the interval $(\mu - \sigma, \ \mu + \sigma)$

Students' T



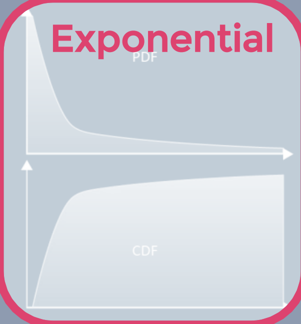
- $Y \sim t(k)$
- Small sample size approximation of a Normal (accounts for extreme values better)
- If k>1: $E(Y) = \mu$ and $\sigma^2 (Y) = s2 \cdot k / (k-2)$

Chi-Squared



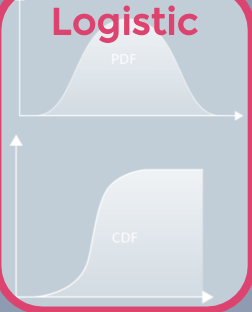
- $Y \sim \chi 2(\lambda)$
- Square of the t-distribution
- $E(Y) = k$
- $\sigma^2 (Y) = 2k$

Exponential



- $Y \sim Exp(\lambda)$
- $E(Y) = 1/\lambda$
- $\sigma^2 (Y) = 1/\lambda^2$

Logistic



- $Y \sim Logistic(\mu, s)$
- Continous variable inputs and binary outcome
- CDF ↑ when values near the mean
- ↓ *s*, the quicker it reaches values close to 1
- $E(Y) = \mu$
- $\sigma^2 (Y) = s^2 \cdot n^2 / 3$