## EE24BTECH11010 - Balaji B

#### **Question:**

Find out whether the lines 6x - 3y + 10 = 0 and 2x - y + 9 = 0 intersect at a point, parallel, or coincident.

#### **Theoretical Solution:**

Let  $a_1,b_1$ , and  $c_1$  and  $a_2,b_2$ , and  $c_2$  be the coefficients of x,y, and 1 in lines 1 and 2 respectively.

We get:

$$\frac{a_1}{a_2} = \frac{6}{2} \tag{0.1}$$

$$\frac{b_1}{b_2} = \frac{3}{1} \tag{0.2}$$

$$\frac{c_1}{c_2} = \frac{10}{9} \tag{0.3}$$

$$m_1 = \frac{-a_1}{b_1} = \frac{6}{3} = \frac{2}{1} \tag{0.4}$$

$$m_2 = \frac{-a_2}{h_2} = \frac{2}{1} \tag{0.5}$$

As all the ratios are equal to each other  $m_1$  and  $m_2$  equal

... The lines doesn't intersect at a point

## **Computational Solution:**

We represent the system in matrix form:

$$A = \begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -10 \\ -9 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{0.6}$$

LU factorization using update equaitons

Given a matrix **A** of size  $n \times n$ , LU decomposition is performed row by row and column by column. The update equations are as follows:

# **Step-by-Step Procedure:**

- 1. Initialization: Start by initializing L as the identity matrix L = I and U as a copy of A.
- 2. Iterative Update: For each pivot k = 1, 2, ..., n: Compute the entries of U using the first update equation. Compute the entries of L using the second update equation.
- 3. Result: After completing the iterations, the matrix A is decomposed into  $L \cdot U$ , where L is a lower triangular matrix with ones on the diagonal, and U is an upper triangular matrix.

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### 1. Update for $U_{k,j}$ (Entries of U)

For each column  $j \ge k$ , the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \text{ for } j \ge k.$$

This equation computes the elements of the upper triangular matrix U by eliminating the lower triangular portion of the matrix.

#### 2. Update for $L_{i,k}$ (Entries of L)

For each row i > k, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left( A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \text{ for } i > k.$$

This equation computes the elements of the lower triangular matrix **L**, where each entry in the column is determined by the values in the rows above it. Using a code we get L,U as

$$L = \begin{pmatrix} 1 & 0 \\ 0.33 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 6 & -3 \\ 0 & 0 \end{pmatrix}. \tag{0.7}$$

Solving Ax = b

Forward Substitution: Solve Ly = b:

$$\begin{pmatrix} 1 & 0 \\ 0.33 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -10 \\ -9 \end{pmatrix}. \tag{0.8}$$

From the first row:

$$y_1 = -10. (0.9)$$

From the second row:

$$0.33y_1 + y_2 = -9 \tag{0.10}$$

$$3(-10) + y_2 = 16 \tag{0.11}$$

$$y_2 = 46 (0.12)$$

Thus:

$$y = \begin{pmatrix} -10\\46 \end{pmatrix}. \tag{0.13}$$

Back Substitution: Solve Ux = y:

$$\begin{pmatrix} 6 & -3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ 46 \end{pmatrix}. \tag{0.14}$$

From the first row:

$$6x - 3y = -10. (0.15)$$

From the second row:

0 = 46 (contradiction). (0.1)	.16	)
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The system of equations is inconsistent and has no solution. The matrix A is singular (non-invertible), as indicated by the zero  $u_{22}$  in the U-matrix.

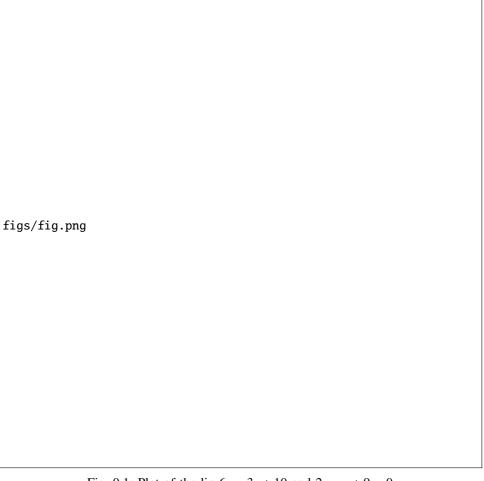


Fig. 0.1: Plot of the  $\lim 6x - 3y + 10$  and 2x - y + 9 = 0