10.3.2.2.3

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Question

Find out whether the lines $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ intersect at a point, parallel, or coincident.

Parameters

The parameters for the problem are given as follows:

Variable	**Description**
A	Matrix consisting of coefficients in the linear equation
L	Lower triangular matrix
U	Upper triangular matrix
$\mathbf{x}$	Solution to the linear equation

Theoretical Solution

Let a_1,b_1, and c_1 and a_2,b_2, and c_2 be the coefficents of x,y, and 1 in lines 1 and 2 respectively.
We get:

$$\frac{a_1}{a_2} = \frac{6}{2} \tag{3.1}$$

$$\frac{b_1}{b_2} = \frac{3}{1} \tag{3.2}$$

$$\frac{c_1}{c_2} = \frac{10}{9} \tag{3.3}$$

$$m_1 = \frac{-a_1}{b_1} = \frac{6}{3} = \frac{2}{1} \tag{3.4}$$

$$m_2 = \frac{-a_2}{b_2} = \frac{2}{1} \tag{3.5}$$

Conclusion: Since $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$ but $m_1 = m_2$, the lines are parallel.

Computational Solution

We represent the system in matrix form:

$$A = \begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -10 \\ -9 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{3.6}$$

LU factorization using update equaitons

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Step-by-Step Procedure:

1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.

2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.

3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U) For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix **L**, where each entry in the column is determined by the values in the rows above it.

Using a code we get L,U as

$$L = \begin{pmatrix} 1 & 0 \\ 0.33 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 6 & -3 \\ 0 & 0 \end{pmatrix}. \tag{3.7}$$

Solving $Ax = b$
Forward Substitution: Solve $Ly = b$:

$$\begin{pmatrix} 1 & 0 \\ 0.33 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -10 \\ -9 \end{pmatrix}. \tag{3.8}$$

From the first row:

$$y_1 = -10. \tag{3.9}$$

From the second row:

$$0.33y_1 + y_2 = -9 \tag{3.10}$$
$$3(-10) + y_2 = 16 \tag{3.11}$$
$$y_2 = 46 \tag{3.12}$$

Back Substitution: Solve $Ux = y$

$$\begin{pmatrix} 6 & -3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ 46 \end{pmatrix}. \tag{3.13}$$

From the first row:

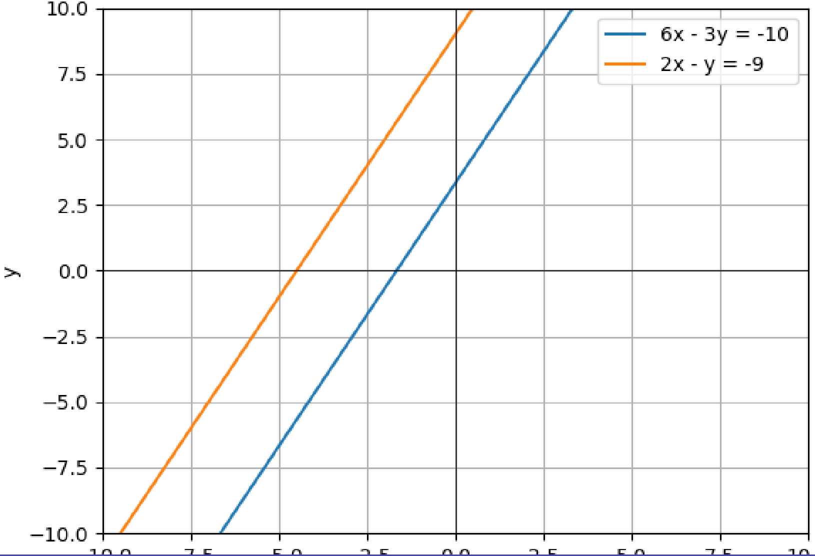
$$6x - 3y = -10. \tag{3.14}$$

From the second row:

$$0 = 46 \quad \text{(contradiction)}. \tag{3.15}$$

The system of equations is inconsistent and has no solution. The matrix A is singular (non-invertible), as indicated by the zero u_{22} in the U-matrix.

Plot



Codes

Code:
https://github.com/Balaji29-code/EE1003/tree/main/Problem-6/codes