12.9.2.2

Balaji B - EE24BTECH11010

Question:
Consider the differential equation

$$y' - 2x - 2 = 0 \tag{0.1}$$

Verify that

$$y = x^2 + 2x + C \tag{0.2}$$

is a solution for it.
Theoretical Solution:
The given differential equation is:

$$y' - 2x - 2 = 0 \tag{0.3}$$

Rearrange the terms to group all x and y related terms:

$$y' = 2x + 2 \tag{0.4}$$

Now integrate both sides with respect to x:

$$\int y' dx = \int (2x + 2)\, dx \tag{0.5}$$

The left-hand side simplifies to y, and the right-hand side is integrated term by term:

$$y = \int 2xdx + \int 2dx \tag{0.6}$$

$$y = x^2 + 2x + C \tag{0.7}$$

This matches the assumed solution:

$$y = x^2 + 2x + C \tag{0.8}$$

Computational Solution:

The **bilinear transform** is used to approximate the continuous derivative y' and reformulate the differential equation into a discrete time difference equation.

Consider a first order differential equation of the form

$$y' = f(x, y) \tag{0.9}$$

The derivative y' in the equation is approximated using the bilinear transform. In the bilinear transform, the derivative is expressed in discrete form as:

$$y' \approx \frac{y_{n+1} - y_n}{h} \tag{0.10}$$

Substituting this approximation into the general equation gives:

$$\frac{y_{n+1} - y_n}{h} = f(x, y) \tag{0.11}$$

The Trapezoidal rule is applied to $f(x, y)$ to approximate the value of $f(x, y)$ over the interval $[x_n, x_{n+1}]$.

$$f(x, y) \approx \frac{1}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right] \tag{0.12}$$

Substitute this into the discretized equation:

$$\frac{y_{n+1} - y_n}{h} = \frac{1}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right] \tag{0.13}$$

Rearranging for y_{n+1}, we get

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right] \tag{0.14}$$

This is the general trapezoidal difference equation obtained using the bilinear transform. Smaller the value of h will give more precise plot. We obtain points to plot by repeatative iteration.
Substituting our $f(x, y) = 2x + 2$ in the above equation we get,

$$y_{n+1} = y_n + h(x_n + x_{n+1}) + 2h \tag{0.15}$$

The above equation is the required difference equation.

Below is the comparison between the Theoretical plot and the simulated plot

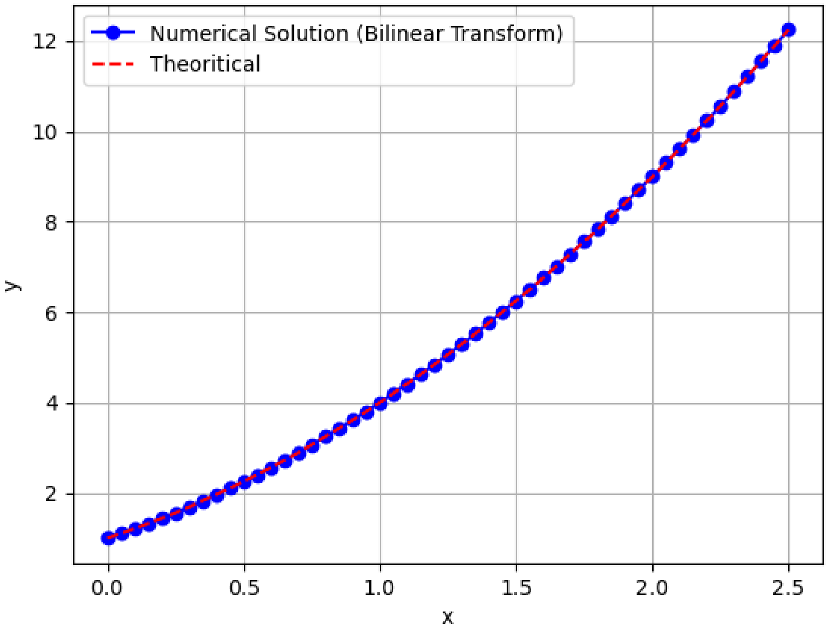

Fig. 0.1: Computational vs Theoretical solution of $y' - 2x - 2 = 0$