9.9.5.10

EE24BTECH11010 - Balaji B

Question:
Find the area of the region included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.
Answer:

Variable	Description
$\mathbf{u}$	Negative of centre
f	Constant of circle
$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$	Points of intersection of parabola and circle
A	Area between the conics

TABLE I: Variables used

The general equation of Circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The parameters of the given circle is

$$\mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, f = 0 \tag{2}$$

Substituting the parameters of the circle in general equation, we get

$$\|\mathbf{x}\|^2 + 2\begin{pmatrix} -4 & 0 \end{pmatrix}\mathbf{x} + 0 = 0 \tag{3}$$

To find the point of intersection of parabola and circle, substitute parabola in circle.

$$\because \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

The parabola can be represented as

$$y = \pm 2\sqrt{x} \tag{5}$$

Substituting this in above $\mathbf{x}$, we get

$$\mathbf{x} = \begin{pmatrix} x \\ \pm 2\sqrt{x} \end{pmatrix} \tag{6}$$

Substituting this in circle, we get

$$\left\|\begin{pmatrix} x \\ \pm 2\sqrt{x} \end{pmatrix}\right\|^2 + 2\begin{pmatrix} -4 & 0 \end{pmatrix}\begin{pmatrix} x \\ \pm 2\sqrt{x} \end{pmatrix} + 0 = 0 \tag{7}$$

$$\therefore x(x-4) = 0 \tag{8}$$

$$\implies x = 0, 4 \tag{9}$$

The point of intersection of circle and parabola can be given by

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{10}$$

$$\mathbf{x}_2 = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{11}$$

$$\mathbf{x}_3 = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{12}$$

Thus, the desired Area is given as

$$A = 2\left[\int_0^4 \sqrt{4x}dx + \int_4^8 \sqrt{8x - x^2}\right] \tag{13}$$

$$A = 2\left[\left[\frac{4}{3}x^{\frac{3}{2}}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{8x - x^2} + 8\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8\right] \tag{14}$$

$$A = \frac{64}{3} + 8\pi \tag{15}$$

$\therefore$ The Area A is $\frac{64}{3} + 8\pi$

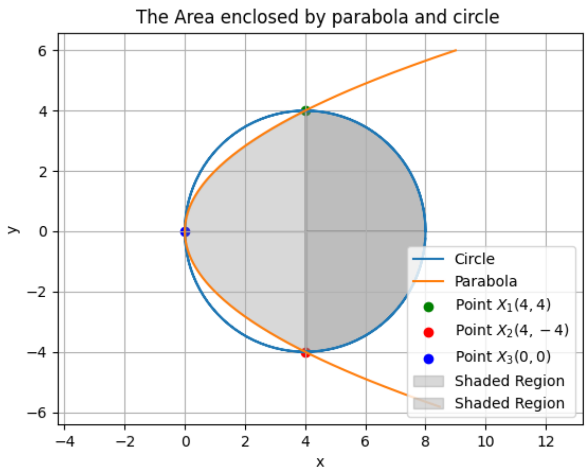

Fig. 1: The Area enclosed by parabola and circle