

4.4.2.14

EE24BTECH11010 - Balaji B

Question:

Show that two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where $b_1b_2 \neq 0$ are Perpendicular if $a_1a_2 + b_1b_2 = 0$.

Answer:

Variable	Description
\mathbf{m}_1	Direction vector of line 1
\mathbf{m}_2	Direction vector of line 2
\mathbf{h}_1	$\begin{pmatrix} 0 \\ c_1 \end{pmatrix}$
\mathbf{h}_2	$\begin{pmatrix} 0 \\ c_2 \end{pmatrix}$

TABLE I: Variables Used

The equation of line is given by,

$$y = mx + c \quad (1)$$

$$x = x \quad (2)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = x \begin{pmatrix} 1 \\ m \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \quad (3)$$

$$\mathbf{x} = k\mathbf{m} + \mathbf{h} \quad (4)$$

Writing the line 1 in the form of the above equation, we get

$$a_1x + b_1y + c_1 = 0 \quad (5)$$

$$y = -\frac{a_1}{b_1}x - \frac{c_1}{b_1} \quad (6)$$

$$\mathbf{x} = x \begin{pmatrix} 1 \\ -\frac{a_1}{b_1} \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{c_1}{b_1} \end{pmatrix} \quad (7)$$

$$\therefore \mathbf{m}_1 = \begin{pmatrix} 1 \\ -\frac{a_1}{b_1} \end{pmatrix} \quad (8)$$

Writing the line **2** in the form of the above equation, we get

$$a_2x + b_2y + c_2 = 0 \quad (9)$$

$$y = -\frac{a_2}{b_2}x - \frac{c_2}{b_2} \quad (10)$$

$$\mathbf{x} = x \begin{pmatrix} 1 \\ -\frac{a_2}{b_2} \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{c_2}{b_2} \end{pmatrix} \quad (11)$$

$$\therefore \mathbf{m}_2 = \begin{pmatrix} 1 \\ -\frac{a_2}{b_2} \end{pmatrix} \quad (12)$$

For the lines to be perpendicular $m_1^\top m_2 = 0$.

$$\begin{pmatrix} 1 & -\frac{a_1}{b_1} \end{pmatrix} \begin{pmatrix} 1 \\ -\frac{a_2}{b_2} \end{pmatrix} = 0 \quad (13)$$

$$1 + \frac{a_1 a_2}{b_1 b_2} = 0 \quad (14)$$

$$a_1 a_2 + b_1 b_2 = 0 \quad (15)$$

Let's consider $a_1 = 1, b_1 = 1, c_1 = 0$ and $a_2 = 1, b_2 = -1, c_2 = 0$.

The equation of line **1** will be

$$\mathbf{x} = x \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (16)$$

The equation of line **2** will be

$$\mathbf{x} = x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (17)$$

From above we have $m_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $m_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

For lines to be perpendicular:

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0 \quad (18)$$

\therefore The lines are perpendicular.

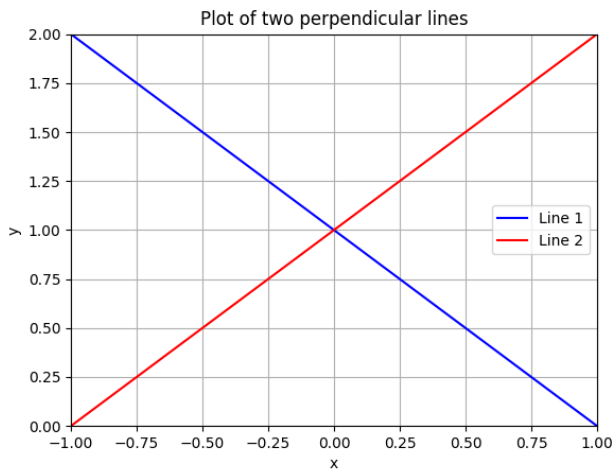


Fig. 1: Plot of two perpendicular lines