31-01-2023- shift-2

EE24BTECH11010 - Balaji B

1) Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha\,(>0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of $100+n$ students are respectively 50 and 350, then the sum of variances of classes A and B is : (January-2023)

   a) 450
   b) 900
   c) 650
   d) 500

2) Let $\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\mathbf{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be three vectors. If $\mathbf{r}$ is a vector such that, $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$, then $25\,|\mathbf{r}|^2$ is equal to : (January-2023)

   a) 336
   b) 449
   c) 339
   d) 560

3) Let H be the hyperbola, whose foci are $\left(1 \pm \sqrt{2}, 0\right)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is : (January-2023)

   a) $\frac{5}{2}$
   b) 3
   c) 2
   d) $\frac{3}{2}$

4) Let $\alpha > 0$. If $\int_0^\alpha \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$, then α is equal to: (January-2023)

   a) 4
   b) 2
   c) $2\sqrt{2}$
   d) $\sqrt{2}$

5) The complex number $z = \frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to: (January-2023)

   a) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
   b) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$
   c) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
   d) $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

6) The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is (January-2023)

7) Let the area of the region $\left\{(x,y): |2x-1| \le y \le \left|x^2 - x\right|, 0 \le x \le 1\right\}$ be A. Then $(6A + 11)^2$ is equal to (January-2023)

8) If $^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to:

9) If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^l} \right)^9$ is -84 and the coefficient of x^{-3l} is $2^\alpha \beta$, where $\beta < 0$ is an odd number, then $|\alpha l - \beta|$ is equal to

10) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that $|\mathbf{a}| = \sqrt{31}$, $4|\mathbf{b}| = |\mathbf{c}| = 2$ and $2(\mathbf{a} \times \mathbf{b}) = 3(\mathbf{c} \times \mathbf{a})$. If the angle between $\mathbf{b}$ and $\mathbf{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\mathbf{a} \times \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}} \right)^2$ is equal to

11) Let S be the set of all $a \in \mathbb{N}$ such that the area of the triangle formed by the tangent at the point $P(b, c)$, $b, c \in \mathbb{N}$, on the parabola $y^2 = 2ax$ and the lines $x = b, y = 0$ is 16 unit^2, then $\sum_{a \in S} a$ is equal to

12) The sum $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \text{.......} + 15.29^2$ is

13) Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space $[0, 60]$ is less than or equal to a. If $P(A) = \frac{11}{36}$, then a is equal to

14) Let $A = \left[a_{ij} \right]$, $a_{ij} \in \mathbb{Z} \cap [0, 4]$, $1 \le i, j \le 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is

15) Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix $\text{Adj}\left(2. \text{Adj}\left(2A^{-1} \right) \right)$ is 2^{84}, then n is equal to