2024-MA-14-26

EE24BTECH11010 - BALAJI B

1) Consider the following limit
$$\lim_{\epsilon\to 0} \frac{1}{\epsilon} \int_0^\infty e^{-x/\epsilon} \left(\cos(3x) + x^2 \sqrt{x+4}\right) dx$$
Which one of the following is correct? [2024 ME]
   a) The limit does not exist
   b) The limit exists and is equal to 0
   c) The limit exists and is equal to 3
   d) The limit exists and is equal to π
2) Let $\mathbb{R}\left[X^2, X^3\right]$ be a substring of $\mathbb{R}[X]$ generated by X^2 and X^3. Consider the following statements.
I. The ring $\mathbb{R}\left[X^2, X^3\right]$ is a unique factorization domain.
II. The ring $\mathbb{R}\left[X^2, X^3\right]$ is a principle ideal domain.
Which one of the following is correct? [2024 MA]
   a) Both I and II are TRUE
   b) I is TRUE and II is FALSE
   c) I is FALSE and II is TRUE
   d) Both I and II are FALSE
3) Given a prime number p, let $n_p(G)$ denote the number of p-Sylow subgroups of a finite group G. Which one of the following is TRUE for every group G of order 2024? [2024 MA]
   a) $n_{11}(G) = 1$ and $n_{23}(G) = 11$
   b) $n_{11}(G) \in \{1, 23\}$ and $n_{23}(G) = 1$
   c) $n_{11}(G) = 23$ and $n_{23} \in \{1, 88\}$
   d) $n_{11}(G) = 23$ and $n_{23}(G) = 11$
4) Consider the following statements.
I. Every compact Hausdroff space is normal.
II. Every metric space is normal. Which one of the following is correct?
[2024 MA]
   a) Both I and II are TRUE
   b) I is TRUE and II is FALSE
   c) I is FALSE and II is TRUE
   d) Both I and II are FALSE
5) Consider the topology on $\mathbb{Z}$ with basis $\{S(a, b) : a, b \in \mathbb{Z} \text{ and } a \neq 0\}$, where
$$S(a, b) = \{an + b : n \in \mathbb{Z}\}$$
Consider the following statements.
I. $S(a, b)$ is both open and closed for each $a, b \in \mathbb{Z}$ with $a \neq 0$
II. The only connected set containing $x \in \mathbb{Z}$ is $\{x\}$
Which one of the following is correct? [2024 MA]

a) Both I and II are TRUE
b) I is TRUE and II is FALSE
c) I is FALSE and I is TRUE
d) Both I and II are FALSE

6) Let $A \in M_2(\mathbb{C})$ be a normal matrix. Consider the following statements.
I. If all the eigenvalues of A are real, then A is Hermitian.
II. If all the eigenvalues of A have absolute value 1, then A is unitary.
Which one of the following is correct? [2024 MA]

a) Both I and II are TRUE
b) I is TRUE and II is FALSE
c) I is FALSE and II is TRUE
d) Both I and II are FALSE

7) Let $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ and $T : M_2(\mathbb{C}) \to M_2(\mathbb{C})$ be a linear transformation given by $T(B) = AB$. The characteristic polynomial of T is [2024 MA]

a) $X^2 - 8X + 16$
b) $X^2 - 4$
c) $X^2 - 2$
d) $X^2 - 16$

8) Let

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ -1 & 1 & 2 \end{pmatrix}$$

and $\mathbf{b}$ be a 3×1 real column vector. Consider the statements.
I. The Jacobi iteration method for the system $(A + \epsilon I_3)\mathbf{x} = \mathbf{b}$ onverges for any initial approximation and $\epsilon > 0$
II. The Gauss-Seidel iteration method for the system $(A + \epsilon I_3)\mathbf{x} = \mathbf{b}$ converges for any initial approximation and $\epsilon > 0$
Which of the following is correct? [2024 MA]

a) Both I and I are TRUE
b) I is TRUE and II is FALSE
c) I is FALSE and II is TRUE
d) Both I and I are FALSE

9) For the initial value problem

$$y' = f(x, y),\ y(x_0) = y_0$$

generate approximations y_n to $y(x_n)$, $x_n = x_0 + nh$, for a fixed $h > 0$ and $n = 1, 2, 3, \ldots$ using the recursion formula

$$y_n = y_{n-1} + ak_1 + bk_2, \text{ where}$$
$$k_1 = hf(x_{n-1}, y_{n-1}) \text{ and } k_2 = hf(x_{n-1} + \alpha h, y_{n-1} + \beta k_1)$$

Which one of the following choices of a, b, α, β for the above recursion formula gives the Runge-Kuta method of order 2? [2024 MA]

a) $a = 1, b = 1, \alpha = 0.5, \beta = 0.5$
b) $a = 0.5, b = 0.5, \alpha = 2, \beta = 2$
c) $a = 0.25, b = 0.75, \alpha = \frac{2}{3}, \beta = \frac{2}{3}$

d) $a = 0.5, b = 0.5, \alpha = 1, \beta = 2$

10) Let $u = u(x,t)$ be the solution of

$$\frac{\partial u}{\partial t} - 4\frac{\partial^2 u}{\partial x^2} = 0\ ,\ 0 < x < 1\ ,\ t > 0,$$
$$u(0,t) = u(1,t) = 0\ ,\ t \geq 0,$$
$$u(x,0) = \sin(\pi x)\ ,\ 0 \leq x \leq 1$$

Define $g(t) = \int_0^1 \left(u(x,t)^2\right) dx$, for $t > 0$. Which one of the following is correct?

[2024 MA]

a) g is decreasing on $(0,\infty)$ and $\lim_{t\to\infty} g(t) = 0$
b) g is decreasing on $(0,\infty)$ and $\lim_{t\to\infty} g(t) = \frac{1}{4}$
c) g is increasing on $(0,\infty)$ and $\lim_{t\to\infty} g(t)$ does not exist
d) g is increasing on $(0,\infty)$ and $\lim_{t\to\infty} g(t) = 3$

11) y_1 and y_2 are two different solution of the ordinary differential equation

$$y' + \sin(e^x)\, y = \cos\left(e^{x+1}\right)\ ,\ 0 \leq x \leq 1,$$

then which of the following is the general solution on $[0,1]$? [2024 MA]

a) $c_1 y_1 + c_2 + y_2\ ,\ c_1, c_2 \in \mathbb{R}$
b) $y_1 = c_1(y_1 - y_2)\ ,\ c \in \mathbb{R}$
c) $c y_1 + (y_1 - y_2)\ ,\ c \in \mathbb{R}$
d) $c_1(y_1 + y_2) + c_2(y_1 - y_2)\ ,\ c_1, c_2 \in \mathbb{R}$

12) Consider the following Linear Programming Problem **P**

$$\begin{array}{ll} \text{minimize} & 5x_1 + 2x_2 \\ \text{subject to} & 2x_1 + x_2 \leq 2, \\ & x_1 + x_2 \geq 1 \\ & x_1, x_2 \geq 0. \end{array}$$

The optimal value of the problem **P** is equal to [2024 MA]

a) 5
b) 0
c) 4
d) 2

13) Let $p = \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right) \in \mathbb{R}^4$ and $f : \mathbb{R}^4 \to \mathbb{R}$ br a differential function such that $f(p) = 6$ and $f(\lambda x) = \lambda^3 f(x)$, for every $\lambda \in (0,\infty)$ and $x \in \mathbb{R}^4$. The value of

$$12\frac{\partial f}{\partial x_1}(p) + 6\frac{\partial f}{\partial x_2}(p) + 4\frac{\partial f}{\partial x_3}(p) + 3\frac{\partial f}{\partial x_4}(p)$$

is equal to ____________ (answer in integer) [2024 MA]