30-01-2024- shift-2

EE24BTECH11010 - Balaji B

1) If z is a complex number, then the number of common roots of the equations $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to (January-2024)

   a) 0
   b) 2
   c) 1
   d) 3

2) Suppose $2-p, p, 2-\alpha, \alpha$ are the coefficients of four consecutive terms in the expansion of $(1+x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals (January-2024)

   a) 8
   b) 4
   c) 6
   d) 10

3) If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to

   a) 9
   b) 12
   c) 11
   d) 10

4) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \frac{x}{\left(1+x^4\right)^{\frac{1}{4}}}$ and $g(x) = f(f(f(f(x))))$. Then, $18\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)\, dx$ is equal to (January-2024)

   a) 36
   b) 33
   c) 39
   d) 42

5) Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3×3 matrix, where $x \sin\theta = y \sin\left(\theta + \frac{2\pi}{3}\right) = z \sin\left(\theta + \frac{4\pi}{3}\right) \neq 0, \theta \in (0, 2\pi)$. For a square matrix M, let trace (M) denote the sum of all diagonal entries of M. Then among the statements:

   (I) $\text{Trace}(R) = 0$
   (II) If $\text{Trace}(\text{adj}(\text{adj}(\text{R}))) = 0$, then R has exactly one non-zero entry. (January-2024)

   a) Only (I) is true
   b) Only (II) is true
   c) Both (I) and (II) are true
   d) Neither (I) nor (II) is true

6) Let $Y = Y(X)$ be a curve lying in the first quadrant such that the area enclosed by the line $Y - y = Y'(X - x)$ and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)} + 1, Y'(x) \neq 0$. If $Y(1) = 1$, then $12Y(2)$ equals

(January-2024)

7) Let a line passing through the point $(-1,2,3)$ intersect the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha,\beta,\gamma)$ and $L_2 : \frac{x+2}{-3} + \frac{y-2}{-2} + \frac{z-1}{4}$ at $N(a,b,c)$. Then, the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals

(January-2024)

8) Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x-\alpha) + y^2 = 16$, where $\alpha \in (5,9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β , then the value of $(\alpha\beta)^2$ equals

(January-2024)

9) Let $\alpha = \sum_{k=0}^{n}\left(\frac{({}^nC_k)^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1}\left(\frac{{}^nC_k\ {}^nC_{k+1}}{k+2}\right)$. If $5\alpha = 6\beta$, then n equals

(January-2024)

10) Let S_n be the sum to n-terms of an arithmetic progression $3, 7, 11, \dots$. If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals

(January-2024)

11) In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C . A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is

(January-2024)

12) The number of symmetric relations defined on the set $\{1,2,3,4\}$ which are not reflexive is

(January-2024)

13) The number of real solutions of the equation $x\left(x^2 + 3|x| + 5|x-1| + 6|x-2|\right) = 0$ is

(January-2024)

14) The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x-2y+4 = 0$ and the positive coordinate axes is

(January-2024)

15) The variance σ^2 of the data

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

Is

(January-2024)