2020-ST-53-65

EE24BTECH11010 - BALAJI B

1) Let (X, Y) be a random vector such that, for any $y > 0$, the conditional probability density fuction of X given $Y = y$ is
$$f_{X|Y=y}(x) = ye^{-yx},\ x > 0.$$
If the marginal probability density function of Y is
$$g(y) = ye^{-y},\ y > 0$$
then $E(Y|X = 1) =$ __________ (correct up to one decimal place)

[2020 ST]

2) Let (X, Y) be a random vector with the joint moment generating function
$$M_{X,Y}(s, t) = e^{2s^2+t},\ -\infty < s, t < \infty.$$
Let $\Phi(.)$ denotes the distribution function normal distribution and $p = P(X + 2Y < 1)$. If $\Phi(0) = 0.5$, $\Phi(0.5) = 0.6915$, $\Phi(1) = 0.8413$ and $\Phi(1.5) = 0.9332$ then the value of $2p + 1$ (round off to two decimal places) equals __________

[2020 ST]

3) Consider a homogeneous Markov chain $\{X_n\}_{n\geq 0}$ with state $\{0, 1, 2, 3\}$ and one-step transition probability matrix
$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
Assume that $P(X_0 = 1) = 1$. Let p be the probability that state 0 will be visited before 3. Then $6 \times p =$ __________

[2020 ST]

4) Let (X, Y) be a random vector with joint probability mass function
$$f_{X,Y}(x, y) = \begin{cases} {}^{x}C_y \left(\frac{1}{4}\right)^x & y = 0, 1, 2 \ldots x\ ;\ x = 1, 2 \ldots \\ 0 & \text{otherwise} \end{cases}$$
where ${}^{x}C_y = \frac{x!}{y!(x-y)!}$. Then variance of Y equals __________

[2020 ST]

5) Let X be a discrete random variable with probability mass function $f \in \{f_0, f_1\}$ where

	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$
$f_0(x)$	0.10	0.10	0.10	0.10	0.60
$f_1(x)$	0.05	0.06	0.08	0.09	0.72

The power of the most powerful level $\alpha = 0.1$ test for testing $H_0 : X \sim f_0$ against $H_1 : X \sim f_1$, based on X, equals __________ (correct up to two decimal places).

[2020 ST]

6) $\mathbf{X} = (X_1, X_2, X_3)$ be a random vector following $N_3(\mathbf{0}, \sum)$ distribution, where $\sum = \begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 1 \end{pmatrix}$. Then the partial correlation coefficient between X_2 and X_3, wuth fixed X_1, equals __________ (correct up to two decimal places)

[2020 ST]

7) Let X_1, X_2, X_3 and X_4 be a random sample from a populaion having probability density function $f_\theta(x) = f(x-\theta)$, $-\infty < x < \infty$, where $\theta \in (-\infty, \infty)$ and $f(-x) = f(x)$, for all $x \in (-\infty, \infty)$. For testing $H_0 : \theta = 0$ against $H_1; \theta = 0$ against $H_1 : \theta < 0$ let T^+ denotes the Wilcoxon Signed-rank statistic. Then under H_0,

$$32 \times P(T^+ \leq 5) = \_\_\_\_\_\_\_\_\_\_$$

[2020 ST]

8) A simple linear regression model with unknown intercept and unknown slope is fitted to the following data

x	-2	-1	0	1	2
y	3	5	8	9	10

using the method of ordinary least squares. Then the predicted value of y corresponding to $x = 5$ is __________

[2020 ST]

9) Let $D = \{(x, y, z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} : 0 \leq x, y, z \leq 1, x + y + z \leq 1\}$ where $\mathbb{R}$ denotes the set of all real numbers to $x5$. If

$$I = \int\int_D\int (x+y)\, dxdydz,$$

then $84 \times I =$ __________

[2020 ST]

10) Let the random vector (X, Y) have the joint distribution function

$$F(x, y) = \begin{cases} 0 & x < 0 \text{ or } y < 0 \\ \frac{1-e^{-x}}{4} & x \geq 0, 0 \leq y < 1 \\ 1 - e^{-x} & x \geq 0, y \geq 1 \end{cases}$$

Let Var(X) and Var(Y) denote the variance X and Y, then respectively. Then

$$16 \text{ Var}(X) + 32 \text{ Var}(Y) = \_\_\_\_\_\_\_\_\_\_$$

[2020 ST]

11) Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables with $E(X_1) = 0$, $E\left(X_1^2\right)$ and $E\left(X_1^4\right) = 3$. Further, let

$$Y_n = \frac{X_1^2 + X_2^2 + \cdots + X_n^2}{n}$$

If

$$\lim_{n\to\infty} P\left(Y_n + \frac{\sqrt{n}(Y_n - 1)}{\sqrt{3}} \leq 2\right) = \Phi(c),$$

where $\Phi(.)$ denotes the cumulative distribution function of the standard normal distribution, then $c^2 =$ __________(correct up to one decimal place)

[2020 ST]

12) Let the random vector $\mathbf{X} = (X_1, X_2, X_3)$ have the joint probability density function

$$f_{\mathbf{X}}(x_1, x_2, x_3) = \begin{cases} \frac{81}{4} x_1^2 x_2^2 x_3^2, & -1 \le x_1 \le x_2 \le x_3 \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then the variable of the random variable $X_1 + X_2 + X_3$ equals ____________ (correct up to one decimal place) [2020 ST]

13) Let $X_1, \ldots, X_5$ be a random sample from a distribution with the probability density function

$$f(x; \theta) = \frac{1}{2} e^{-|x-\theta|}, x \in (-\infty, \infty),$$

where $\theta \in (-\infty, \infty)$. For testing $H_0 : \theta = 0$ against $H_1 : \theta < 0$, let $\sum_{i=1}^{5} Y_i$ be the sign test statistic, where

$$Y_i = \begin{cases} 1, & X > 0 \\ 0, & \text{otherwise} \end{cases}$$

Then the size of the test, which rejects H_0 if and only if $\sum_{i=1}^{5} Y_i \le 2$, equals ____________ (correct up to one decimal place). [2020 ST]