29-07-2022- shift-1

EE24BTECH11010 - Balaji B

1) Let R be a relation from the set $\{1, 2, 3, ..., 60\}$ to itself such that $R = \{(a, b) : b = pq,$ where $p, q \geq 3$ are prime numbers $\}$. Then, the number of elements in R is : (July-2022)

a) 600
b) 660
c) 540
d) 720

2) $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to: (July-2022)

a) 244
b) 224
c) 245
d) 265

3) Let A and B be two 3×3 non-zero real matrices such that AB is a zero matrix. Then (July-2022)

a) the system of linear equation $AX = 0$ has a unique solution
b) the system of linear equation $AX = 0$ has infinitely many solutions
c) B is an invertible matrix
d) $adj(A)$ is an invertible matrix

4) If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + ... + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is: (July-2022)

a) 198
b) 202
c) 212
d) 218

5) If $\lim_{x\to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$, where $\alpha, \beta, \gamma \in \mathbb{R}$, then which of the following is NOT correct? (July-2022)

a) $\alpha^2 + \beta^2 + \gamma^2 = 6$
b) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$
c) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$
d) $\alpha^2 - \beta^2 + \gamma^2 = 4$

6) The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\sin x+\cos x} dx$ is equal to: (July-2022)

a) $\tan^{-1}(2)$
b) $\tan^{-1}(2) - \frac{\pi}{4}$
c) $\frac{1}{2}\tan^{-1}(2) - \frac{\pi}{8}$
d) $\frac{1}{2}$

7) Let the solution curve $y = y(x)$ of the differential equation $\left(1 + e^{2x}\right)\left(\frac{dy}{dx} + y\right) = 1$ pass through the point $\left(0, \frac{\pi}{2}\right)$. Then, $\lim_{x\to\infty} e^x y(x)$ is equal to: (July-2022)

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$

8) Let a line L pass through the point of intersection of the lines $bx + 10y - 8 = 0$ and $2x - 3y = 0, b \in \mathbb{R} - \left\{\frac{4}{3}\right\}$. If the line L also passes through the point $(1, 1)$ and touches the circle $17\left(x^2 + y^2\right) = 16$, then the eccentricity of the ellipse $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$ is : (July-2022)

a) $\frac{2}{\sqrt{5}}$ b) $\sqrt{\frac{3}{5}}$ c) $\frac{1}{\sqrt{5}}$ d) $\sqrt{\frac{2}{5}}$

9) If the foot of perpendicular from the point $A(-1, 4, 3)$ on the plane $P : 2x+my+nz = 4$ is $\left(-2, \frac{7}{2}, \frac{3}{2}\right)$, then the distance of the point A from the plane P, measured parallel to a line with direction ratios $3, -1, -4$, is equal to:

a) 1
b) $\sqrt{26}$
c) $2\sqrt{2}$
d) $\sqrt{14}$

10) Let $\mathbf{a} = 3\hat{i} + \hat{j}$ and $\mathbf{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\mathbf{c}$ be a vector satisfying $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b} + \lambda\mathbf{c}$. If $\mathbf{b}$ and $\mathbf{c}$ are non-parallel, then the value of λ is: (July-2022)

a) -5
b) 5
c) 1
d) -1

11) The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot\alpha$ is equal to : (July-2022)

a) $\frac{6}{5}$ b) $\frac{9}{5}$ c) $\frac{4}{3}$ d) $\frac{7}{3}$

12) The statement $(p \wedge q) \implies (p \wedge r)$ is equivalent to:

a) $q \implies (p \wedge r)$
b) $p \implies (p \wedge r)$
c) $(p \wedge r) \implies (p \wedge q)$
d) $(p \wedge q) \implies r$

13) Let the circumcentre of a triangle with vertices $A(a, 3), B(b, 5)$ and $C(a, b), ab > 0$ be $P(1, 1)$. If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to : (July-2022)

a) 2
b) $\frac{4}{7}$
c) $\frac{2}{7}$
d) 4

14) Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $\left(\hat{a} + \hat{b}\right)$ and $\left(\hat{a} + 2\hat{b} + 2\left(\hat{a} \times \hat{b}\right)\right)$, then the value of $164\cos^2\theta$ is equal to : (July-2022)

a) $90 + 27\sqrt{2}$
b) $45 + 18\sqrt{2}$
c) $90 + 3\sqrt{2}$
d) $54 + 90\sqrt{2}$

15) $f(\alpha) = \int_1^{\alpha} \frac{\log_{10} t}{1+t} dt, \alpha > 0$, then $f\left(e^3\right) + f\left(e^{-3}\right)$ is equal to: (July-2022)

a) 9
b) $\frac{9}{2}$
c) $\frac{9}{\log_e(10)}$
d) $\frac{9}{2\log_e(10)}$