4.4.2.14

EE24BTECH11010 - Balaji B

Question:
Show that two lines $a_1x+b_1y+c_1 = 0$ and $a_2+b_2+c_2 = 0$ where $b_1b_2 \neq 0$ are Perpendicular if $a_1a_2 + b_1b_2 = 0$.
Answer:

Variable	**Description**
$\mathbf{m_1}$	Direction vector of line 1
$\mathbf{m_2}$	Direction vector of line 2
$\mathbf{h_1}$	$\begin{pmatrix} 0 \\ c_1 \end{pmatrix}$
$\mathbf{h_2}$	$\begin{pmatrix} 0 \\ c_2 \end{pmatrix}$

TABLE I: Variables Used

The equation of line is given by,

$$y = mx + c \tag{1}$$
$$x = x \tag{2}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = x\begin{pmatrix} 1 \\ m \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \tag{3}$$
$$\mathbf{x} = k\mathbf{m} + \mathbf{h} \tag{4}$$

Writing the line **1** in the form of the above equation, we get

$$a_1x + b_1y + c_1 = 0 \tag{5}$$
$$y = -\frac{a_1}{b_1}x - \frac{c_1}{b_1} \tag{6}$$
$$\mathbf{x} = x\begin{pmatrix} 1 \\ -\frac{a_1}{b_1} \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{c_1}{b_1} \end{pmatrix} \tag{7}$$
$$\therefore \mathbf{m_1} = \begin{pmatrix} 1 \\ -\frac{a_1}{b_1} \end{pmatrix} \tag{8}$$

Writing the line **2** in the form of the above equation, we get

$$a_2x + b_2y + c_2 = 0 \tag{9}$$

$$y = -\frac{a_2}{b_2}x - \frac{c_2}{b_2} \tag{10}$$

$$\mathbf{x} = x\begin{pmatrix} 1 \\ -\frac{a_2}{b_2} \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{c_2}{b_2} \end{pmatrix} \tag{11}$$

$$\therefore \mathbf{m_2} = \begin{pmatrix} 1 \\ -\frac{a_2}{b_2} \end{pmatrix} \tag{12}$$

For the lines to be perpendicular $m_1^\top m_2 = 0$.

$$\begin{pmatrix} 1 & -\frac{a_1}{b_1} \end{pmatrix}\begin{pmatrix} 1 \\ -\frac{a_2}{b_2} \end{pmatrix} = 0 \tag{13}$$

$$1 + \frac{a_1}{b_1}\frac{a_2}{b_2} = 0 \tag{14}$$

$$a_1a_2 + b_1b_2 = 0 \tag{15}$$

Let's consider $a_1 = 1, b_1 = 1, c_1 = 0$ and $a_2 = 1, b_2 = -1, c_2 = 0$.

The equation of line **1** will be

$$\mathbf{x} = x\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{16}$$

The equation of line **2** will be

$$\mathbf{x} = x\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{17}$$

From above we have $m_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $m_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
For lines to be perpendicular:

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0 \tag{18}$$

$\therefore$ The lines are perpendicular.

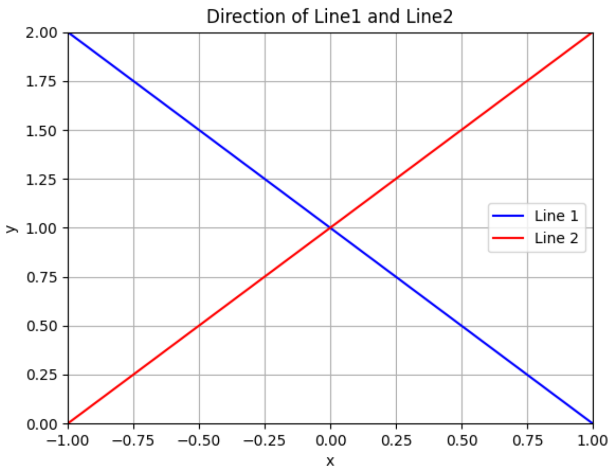

Fig. 1: Plot of the line