## ❓ Why does central differencing fail for convection-dominated flow when $N = 5$, but seems to work when $N \geq 20$?

The answer lies in the **Péclet number** and how it relates to **grid size**.

---

### 🔷 What is the **Cell Peclet Number**?

The **Péclet number** (Pe) for a control volume is:

$$\text{Pe} = \frac{F}{D} = \frac{\rho u \Delta x}{\Gamma}$$

Where:

- $\rho u$ = convective flux (convection strength)
- $\Gamma$ = diffusion coefficient
- $\Delta x = \frac{L}{N}$ = grid size (cell width)

#### 🔍 So: **Pe is directly proportional to $\Delta x$**

This means:

- Coarse grid (small $N$, large $\Delta x$) → **Large Pe**
- Fine grid (large $N$, small $\Delta x$) → **Small Pe**

---

### 📉 Central Differencing Becomes Unstable When:

$$\boxed{\text{Pe} > 2}$$

Because:

- The scheme produces **negative coefficients**
- Leads to **oscillations**, **overshoot**, and **unbounded behavior**

---

### 🧠 So What's Happening in Your Case?

### 🔷 Case 1: $N = 5$

- $\Delta x = 1/5 = 0.2$
- For example, if $u = 2.5$, then:

$$\text{Pe} = \frac{1 \cdot 2.5 \cdot 0.2}{0.1} = 5$$

This is **well above 2**, so the central differencing scheme is **unstable and inaccurate**.

---

### 🔷 Case 2: $N = 20$

- $\Delta x = 1/20 = 0.05$
- Same velocity $u = 2.5$:

$$\text{Pe} = \frac{1 \cdot 2.5 \cdot 0.05}{0.1} = 1.25$$

Now **Pe < 2** → the scheme is **stable**, **bounded**, and behaves well!

---

## ✅ Final Answer:

The **central differencing scheme** assumes the solution varies **smoothly** between nodes.
When the **grid is coarse**, this assumption fails in convection-dominated flows, and the scheme gives **unphysical results**.

But when the **grid is refined** (higher $N$, smaller $\Delta x$), the **local Peclet number** becomes smaller, and the **solution appears smooth** over short distances — so the **central differencing works again**.

---

## 🧪 Practical Rule of Thumb:

> 🔺 **Use central differencing only when:**
>
> $$\text{Local Peclet Number (Pe)} < 2$$

Otherwise:

- Use **upwind**, **hybrid**, or **higher-order schemes** like QUICK.