

✅ Code Functionality Summary

- ✅ Modular FVM Solver using `TriDiagonalCoeffMatrix` with name-value inputs.
- ✅ Compares numerical solution with the analytical solution:

$$\phi(x) = \phi_A + (\phi_B - \phi_A) \cdot \frac{e^{\frac{\rho U x}{\Gamma}} - 1}{e^{\frac{\rho U L}{\Gamma}} - 1}$$

- ✅ Looping through combinations of:
 - $U = 0.1, U = 2.5$
 - $N = 5, N = 20$
- ✅ Prints a well-formatted **comparison table** with:
 - Node index
 - Distance
 - FVM and analytical values
 - Absolute difference
 - % error

✅ Results Interpretation

For each (U, N) combination:

- You'll get a **graph** comparing FVM and analytical solutions.
- A **table** showing error at each node.

The error should:

- Be **small** for low U (diffusion-dominant).
- Increase for high U and low N , due to **numerical diffusion** (central differencing becomes unstable for high Peclet numbers).
- Decrease as **grid is refined** (increased N).

✅ Possible Enhancements (Optional)

If you'd like to go further:

- 🔄 Add **upwind differencing** as an option and compare.
- 📈 Compute **maximum error** and plot error convergence vs. N .
- 💬 Add LaTeX-style legends in plots for publication-quality graphics.

✅ What You've Built

You've created a **clean, educational, and practical** FVM code with:

- Central differencing
- Boundary condition handling
- Accuracy verification
- Multiple test cases

👏 **Well done!** If you'd like me to convert this into a PDF report format, export plots automatically, or integrate symbolic math for derivation — just ask!

Perfect! Let's walk through **manual finite volume calculations** for the **central differencing scheme** on the convection-diffusion equation for **each case and each node**.

PROBLEM SETUP

We are solving:

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx}(\Gamma \frac{d\phi}{dx})$$

Using **central differencing** with:

- $\rho = 1$
- $\Gamma = 0.1$
- $L = 1.0 \text{ m}$
- $\phi(0) = 1, \phi(1) = 0$
- Uniform grid: $N = 5 \Rightarrow \Delta x = 0.2$
- Cell centers: $x_P = 0.1, 0.3, 0.5, 0.7, 0.9$

We'll walk through two cases:

CASE 1: $U = 0.1 \text{ m/s}$

Constants:

- $F = \rho U = 1 \cdot 0.1 = 0.1$
- $D = \Gamma / \Delta x = 0.1 / 0.2 = 0.5$
- $a_W = D + F/2 = 0.5 + 0.05 = 0.55$
- $a_E = D - F/2 = 0.5 - 0.05 = 0.45$

Node 1 (Left Boundary)

- Boundary condition: $\phi_A = 1$
- Modified coefficients:

$$a_P = 3D + F/2 = 1.5 + 0.05 = 1.55$$

$$a_E = -a_E = -0.45$$

$$B = (2D + F) \cdot \phi_A = (1 + 0.1) \cdot 1 = 1.1$$

Equation:

$$1.55 \cdot \phi_1 - 0.45 \cdot \phi_2 = 1.1$$

Node 2 (Internal)

$$a_W = 0.55, \quad a_E = 0.45, \quad a_P = 1.0$$

Equation:

$$-0.55 \cdot \phi_1 + 1.0 \cdot \phi_2 - 0.45 \cdot \phi_3 = 0$$

◆ Node 3 (Internal)

Same as node 2:

$$-0.55 \cdot \phi_2 + 1.0 \cdot \phi_3 - 0.45 \cdot \phi_4 = 0$$

◆ Node 4 (Internal)

Same:

$$-0.55 \cdot \phi_3 + 1.0 \cdot \phi_4 - 0.45 \cdot \phi_5 = 0$$

◆ Node 5 (Right Boundary)

- $\phi_B = 0$
- Coefficients:

$$a_P = 3D - F/2 = 1.5 - 0.05 = 1.45$$

$$a_W = -a_E = -0.55$$

$$B = (2D - F) \cdot \phi_B = (1 - 0.1) \cdot 0 = 0$$

➡ Equation:

$$-0.55 \cdot \phi_4 + 1.45 \cdot \phi_5 = 0$$

✓ Final System (CASE 1, $U = 0.1$):

$$\begin{bmatrix} 1.55 & -0.45 & 0 & 0 & 0 \\ -0.55 & 1.00 & -0.45 & 0 & 0 \\ 0 & -0.55 & 1.00 & -0.45 & 0 \\ 0 & 0 & -0.55 & 1.00 & -0.45 \\ 0 & 0 & 0 & -0.55 & 1.45 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

📖 CASE 2: $U = 2.5$ m/s

✳ Constants:

- $F = \rho U = 2.5$
- $D = 0.5$
- $a_W = D + F/2 = 0.5 + 1.25 = 1.75$
- $a_E = D - F/2 = 0.5 - 1.25 = -0.75$

◆ Node 1:

$$a_P = 3D + F/2 = 1.5 + 1.25 = 2.75$$

$$a_E = -a_W = -0.75$$

$$B = (2D + F) \cdot \phi_A = (1 + 2.5) \cdot 1 = 3.5$$

➡ Equation:

$$2.75 \cdot \phi_1 - 0.75 \cdot \phi_2 = 3.5$$

◆ Node 2:

$$a_W = 1.75, \quad a_E = -0.75, \quad a_P = a_W + a_E = 1.0$$
$$-1.75 \cdot \phi_1 + 1.0 \cdot \phi_2 + 0.75 \cdot \phi_3 = 0$$

◆ Node 3:

$$-1.75 \cdot \phi_2 + 1.0 \cdot \phi_3 + 0.75 \cdot \phi_4 = 0$$

◆ Node 4:

$$-1.75 \cdot \phi_3 + 1.0 \cdot \phi_4 + 0.75 \cdot \phi_5 = 0$$

◆ Node 5 (Right boundary):

$$a_P = 3D - F/2 = 1.5 - 1.25 = 0.25$$
$$a_W = -a_P = -1.75$$
$$B = (2D - F) \cdot \phi_B = (1 - 2.5) \cdot 0 = 0$$

➡ Equation:

$$-1.75 \cdot \phi_4 + 0.25 \cdot \phi_5 = 0$$

✅ Final System (CASE 2, $U = 2.5$):

$$\begin{bmatrix} 2.75 & -0.75 & 0 & 0 & 0 \\ -1.75 & 1.00 & 0.75 & 0 & 0 \\ 0 & -1.75 & 1.00 & 0.75 & 0 \\ 0 & 0 & -1.75 & 1.00 & 0.75 \\ 0 & 0 & 0 & -1.75 & 0.25 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Would you like me to solve these systems step-by-step (e.g. with Gauss elimination or TDMA)? Or want to verify your code against these coefficients?



solve for phi

Here are the **manual solutions** for ϕ at each node for both cases:

📘 **CASE 1: $U = 0.1$ m/s (Diffusion-dominant)**

Node P	x_P	ϕ_P
1	0.1	0.9421
2	0.3	0.8006
3	0.5	0.6276
4	0.7	0.4163
5	0.9	0.1579

CASE 2: $U = 2.5$ m/s (Convection-dominant)

Node P	x_P	ϕ_P
1	0.1	1.6506
2	0.3	1.3856
3	0.5	2.0040
4	0.7	0.5611
5	0.9	3.9278

Observations:

- Case 1 gives a smooth and expected decay of ϕ from 1 to 0.
- Case 2 shows non-physical behavior: overshoots, large positive values → due to central differencing instability for high Peclet numbers (you need upwind differencing here).