📘 5.6.1 – Assessment of the Upwind Differencing Scheme (UDS)

🔷 Key Points

1. ✅ **Stability**
   - UDS is **unconditionally stable**.
   - Works even when convection dominates diffusion (high Péclet number).
   - No non-physical oscillations in the solution.
2. ⚠️ **Accuracy**
   - UDS is **first-order accurate in space**.
   - It introduces a **truncation error** of:

$$\text{Error} = \left(\frac{\partial \phi}{\partial x}\right)_e - \frac{\phi_P - \phi_W}{\Delta x} = \frac{\Delta x}{2}\left(\frac{\partial^2 \phi}{\partial x^2}\right) + O(\Delta x^2)$$

   - This shows the **leading error term is proportional to Δx**, i.e., **first-order**.
3. 🚫 **Numerical Diffusion**
   - UDS adds **artificial (numerical) diffusion** especially at high **Péclet numbers**.
   - It causes **smearing** of the solution: sharp gradients get flattened.
   - This error increases with convection strength and coarser grids.
4. 📉 **Physical Effect**
   - In convection-dominated problems, UDS may give **non-physical results**.
   - Gradients are under-predicted → poor resolution near boundaries.

📊 Comparison with Example 5.2

In **Example 5.2**, the problem is solved using UDS for two flow velocities:

Case	(i) $u = 0.1$ m/s (Low $Pe = 0.25$)	(ii) $u = 2.5$ m/s (High $Pe = 6.25$)
Type	Diffusion-dominated	Convection-dominated
Accuracy	Good agreement with exact	Large error due to smearing
φ profile	Smooth, close to analytical solution	Flattened, smeared profile

Case	(i) $u = 0.1$ m/s (Low $Pe = 0.25$)	(ii) $u = 2.5$ m/s (High $Pe = 6.25$)
Effect of UDS	Small numerical diffusion	Large artificial diffusion
First-order error	Minimal due to small Δx	Dominant due to low spatial accuracy

✅ Conclusion

- UDS is **conservative** and **stable**, but only **first-order accurate**.
- Suitable for **preliminary analysis**, not for **accurate prediction** in convection-dominated problems.
- For better results, use **higher-order schemes** like **QUICK** or **Hybrid**.