📘 Section 5.4 – Properties of Discretisation Schemes

When solving convection-diffusion problems using numerical methods like the **finite volume method**, it's important that the **discretisation scheme** behaves correctly. There are three main properties that a good scheme must follow:

🔷 5.4.1 **Conservativeness**

What it means:
The scheme must **conserve the quantity** (like mass, energy, or momentum). That means:

> **The amount of the quantity leaving one control volume must be equal to the amount entering the next one.**

Why it's important:
It keeps the **total amount of the property consistent** across the domain, with **no artificial losses or gains**.

In Example 5.1:
The **central differencing scheme** used in the finite volume method **does satisfy conservativeness**, because:

- The flux at shared faces between two cells is computed the **same way** from both sides.

✅ **Conclusion:**
The scheme is conservative in both **low and high velocity** cases.

🔷 5.4.2 **Boundedness**

What it means:
The solution ϕ should stay within **realistic limits**.
In Example 5.1, this means:

- $\phi = 1$ at the inlet (left boundary)
- $\phi = 0$ at the outlet (right boundary)
- So, ϕ should stay **between 0 and 1** in the whole domain.

When is a scheme bounded?

- When all the coefficients in the discretised equations are **positive**.
- And source terms don't push the solution outside physical range.

In Example 5.1:

- When velocity is **low** (e.g. $u = 0.1$), the solution is **smooth** and stays within [0, 1].
- But when velocity is **high** (e.g. $u = 2.5$), central differencing creates **negative coefficients**.
  This can cause:
  - $\phi < 0$
  - $\phi > 1$
  - **Oscillations** or **non-physical results**

❌ **Conclusion:**
Boundedness is **only satisfied** when **diffusion dominates**. It is **not satisfied** in **convection-dominated** problems.

🔷 5.4.3 **Transportiveness**

What it means:
The scheme should respect the **flow direction**. That means:

> **When fluid moves from left to right, the value of ϕ in a cell should be influenced more by the upstream (left-side) value.**

Why it matters:
In convection-dominant flows, the property (like heat or species) is mostly carried by the fluid. The scheme must handle this by **prioritizing upstream values**.

In Example 5.1:

- Central differencing **does not do this**. It averages values **from both sides equally**, so it **doesn't follow the direction of flow**.
- For small u, this is not a big problem.
- For large u, this leads to **wrong transport behavior** — the solution does not follow the flow.

❌ **Conclusion:**
Central differencing is **not transportive**, especially in **high velocity** (convection-dominant) cases.

📘 Section 5.5 – Assessment of Central Differencing Scheme

This section reviews how **central differencing** performs when solving real convection-diffusion problems, like in Example 5.1.

🧠 Summary:

1. **Good for Diffusion-Dominated Problems:**
   - When **Péclet number** $Pe < 2$
   - The solution is **accurate and stable**
   - Works well in **smooth, low-velocity** cases
2. **Fails for Convection-Dominated Problems:**
   - When $Pe > 2$, problems begin:
     - **Negative coefficients**
     - **Unbounded results** (values outside expected range)
     - **Unrealistic oscillations**
3. **Should Not Be Used for Strong Convection:**
   - Instead, use other schemes:
     - **Upwind differencing** (more stable)
     - **Hybrid scheme** (combines upwind and central)
     - **QUICK scheme** (more accurate for smooth flows)

✅ Final Summary for Example 5.1

Property	Low Velocity (u = 0.1)	High Velocity (u = 2.5)	Notes
Conservativeness	✅ Satisfied	✅ Satisfied	Total flux is conserved
Boundedness	✅ Satisfied	❌ Violated	Solution may go beyond 0–1
Transportiveness	⚠️ Weak issue	❌ Violated	Scheme doesn't follow flow direction

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