Comparison of Differencing Schemes for Convection-Diffusion

1. Central Differencing Scheme

- **Accuracy**: Second-order accurate for diffusion-dominated flows (low Peclet numbers, $|Pe| < 2$)[1][2].
- **Limitations**:
  - Produces unphysical oscillations (numerical instability) for $|Pe| > 2$ due to negative coefficients in discretized equations[1][2].
  - High false diffusion in convection-dominated flows[1].
- **Best For**: Problems with $|Pe| < 2$, where diffusion dominates.

2. Upwind Differencing Scheme

- **Accuracy**: First-order accurate. Stable for all Pe but introduces **false diffusion** (smearing of sharp gradients)[1][2].
- **Advantages**:
  - Guarantees bounded solutions (no oscillations) for high $|Pe|$ [1][3].
  - Reflects flow direction by using upstream node values[2].
- **Best For**: Convection-dominated flows ($|Pe| > 2$) where stability is critical.

3. Hybrid Scheme

- **Design**: Combines central differencing ($|Pe| < 2)and upwind differencing(|Pe| \geq 2$)[2][4].
- **Advantages**:
  - Stable for all Pe.
  - Reduces false diffusion compared to pure upwind schemes[1][2].
- **Limitations**:
  - First-order accuracy for $|Pe| \geq 2$ [2].
  - Discontinuous transition between schemes at $|Pe| = 2$ [5].
- **Best For**: General-purpose simulations with varying Pe.

4. Power-Law Scheme

- **Design**: Uses an exponential interpolation based on the exact solution of the 1D convection-diffusion equation [Initial Context].
- **Advantages**:
  - Smoothly adapts between central (linear) and upwind (constant) approximations based on Pe.
  - No abrupt transitions, reducing numerical errors [Initial Context].
  - Handles both diffusion- and convection-dominated flows without oscillations [Initial Context].
- **Limitations**:
  - Computationally more intensive than hybrid schemes.
  - Rarely used in modern CFD codes due to the prevalence of high-resolution schemes like QUICK[1].

Key Comparison Table

Scheme	Accuracy Order	Stability	False Diffusion	Adaptivity to Pe	Ideal Use Case
Central Differencing	2nd	Unstable ($Pe > 2$)	High	No	Low Pe flows
Upwind Differencing	1st	Stable	Moderate	No	High Pe flows
Hybrid	1st/2nd	Stable	Low	Yes (Pe -based)	General-purpose simulations
Power-Law	Variable	Stable	Very Low	Yes (smooth)	All Pe ranges

Practical Insights

1. **Central Scheme**: Best for laminar flows or low-velocity scenarios (e.g., heat conduction) but fails in convection-dominated cases[1][2].
2. **Upwind Scheme**: Preferred for high-speed flows (e.g., aerodynamics) but introduces smearing near shocks[1][3].
3. **Hybrid Scheme**: Balances stability and accuracy but sacrifices higher-order precision for Pe \geq 2 [5][2].

4. **Power-Law**: Outperforms hybrid schemes in accuracy for intermediate Pe but is less common in modern implementations [Initial Context].

For most engineering applications, hybrid and power-law schemes are preferred due to their robustness across a wide range of Pe. The choice depends on computational resources and the need for precision vs. stability[1][2].

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1. https://www.isca.in/MATH_SCI/Archive/v4/i2/2.ISCA-RJMSS-2016-009.pdf
2. https://en.wikipedia.org/wiki/Hybrid_difference_scheme
3. https://www.youtube.com/watch?v=sQwRp57cEYg
4. https://www.kns.org/files/pre_paper/26/61김윤일pdf
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