


Section 5.4 – Properties of Discretisation Schemes

In computational fluid dynamics (CFD), a good discretisation scheme must ensure **physical reliability** of the solution. Three essential properties are:


5.4.1 Conservativeness

- **Definition:** A scheme is conservative if the net flux entering a control volume equals the net flux leaving adjacent volumes.
- **Importance:** Ensures **global and local conservation** of quantities like mass, momentum, or energy.
- **Achieved When:** The flux at a shared face between two control volumes is **identically calculated** from both sides (e.g., same interpolation).

 **Central differencing** is conservative under structured grids and steady conditions.


5.4.2 Boundedness

- **Definition:** A discretisation scheme is bounded if the computed solution stays **within physical limits** (e.g., $0 \leq \phi \leq 1$).
- **Why It Matters:** Prevents unphysical oscillations and negative values.
- **Mathematically:** For boundedness, all coefficients in the discretised equation should be **positive**, and source terms non-negative or handled correctly.

 Central differencing **can violate boundedness** for high convection (large Peclet number), leading to overshoots or undershoots.

5.4.3 Transportiveness

- **Definition:** A scheme is transportive if it respects the **direction of flow** and accurately propagates transported quantities.
- **Concept:** The scheme must **bias the interpolation** in the direction of the flow (upstream influence).
- **Failure Consequences:** If a scheme lacks transportiveness, the **solution smears** and doesn't represent sharp gradients.

 Central differencing is **not transportive** — it doesn't prioritize upstream information.

Section 5.5 – Assessment of the Central Differencing Scheme

This section evaluates how central differencing performs when applied to convection-diffusion problems.

Main Points:

1. **Second-order Accurate:** It is mathematically more accurate in regions where the solution varies smoothly.
2. **Fails for High Convection (Peclet Number $Pe > 2$):**

- Leads to **non-physical oscillations**.
 - Cannot maintain **boundedness**.
 - Becomes **numerically unstable**.
3. **Ideal Only for Diffusion-dominant Flows:**
- Central differencing works best when **convection is weak** and **diffusion is strong**.
 - For **pure convection**, it's inadequate.
4. **Alternatives Preferred for Convection-Dominant Problems:**
- Upwind scheme (first-order accurate but bounded).
 - Hybrid or QUICK schemes for better balance.

✅ Conclusion of Section 5.5

While **central differencing** is attractive for its accuracy in smooth flows, it **fails the key physical properties**—boundedness and transportiveness—under high-convection conditions. Therefore, it's **not suitable** for general-purpose CFD in convection-dominated problems.

Would you like a visual mind map of these concepts or a comparison table between central, upwind, and other schemes?



I am Not getting your intention please explain and give the related info to the example Example 5.1

Great — let's connect these **theoretical properties** directly to **Example 5.1** so you can understand how they apply practically. I'll explain each concept **in the context of the example**.

📘 Quick Recap: **Example 5.1 – 1D Convection-Diffusion Problem**

You're solving:

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx} \left(\Gamma \frac{d\phi}{dx} \right)$$

with:

- Domain: $x \in [0, 1]$
- $\phi(0) = 1, \phi(1) = 0$
- Uniform grid
- Using **central differencing**
- Cases:
 - $u = 0.1 \rightarrow$ **diffusion-dominant**
 - $u = 2.5 \rightarrow$ **convection-dominant**

Now let's analyze the **discretisation properties** in context of this example:

◆ 1. Conservativeness in Example 5.1

✓ What It Means:

The flux leaving one control volume face must equal the flux entering the adjacent cell at that face.

✓ In This Example:

- You're using **finite volume method**.
- Central differencing ensures the **same interpolation is used at shared faces**.
- Hence, **Example 5.1 is conservative**.
- It respects the **local conservation of ϕ** (whatever goes out of one cell enters the next).

✓ **Conservativeness is satisfied** in both low and high velocity cases.

◆ 2. Boundedness in Example 5.1

⚠ What It Means:

The solution ϕ should stay **within physical bounds**.

For this example:

$$0 \leq \phi \leq 1$$

⚠ In This Example:

- When $u = 0.1 \rightarrow$ solution is smooth and stays within $[0, 1]$.
- When $u = 2.5 \rightarrow$ central differencing leads to:
 - **Negative coefficients** in the discretised matrix.
 - **Overshoot/undershoot** (you saw values like $\phi > 1$, even **negative** or > 1).
 - Violates **boundedness**.

● **Central differencing is NOT bounded** when convection dominates.

◆ 3. Transportiveness in Example 5.1

⚠ What It Means:

The scheme should reflect the **direction of the flow** (from left to right) by giving **more weight to upstream values**.

⚠ In This Example:

- Central differencing treats upstream and downstream faces **symmetrically**.
- Doesn't account for the **direction of the flow**.
- When $u = 2.5$, the solution does **not transport information** properly from left to right.
- Instead, the scheme becomes **oscillatory or unstable**.

● **Transportiveness fails for high velocity** in central differencing.

📊 **In Summary (Applied to Example 5.1):**

Property	Low Velocity ($u = 0.1$)	High Velocity ($u = 2.5$)	Status
Conservativeness	✅ Yes (FVM ensures it)	✅ Yes	✅ OK
Boundedness	✅ Solution is smooth	❌ Overshoots	⚠️ Violated
Transportiveness	⚠️ Symmetric but OK	❌ Not directional	🔴 Violated

💡 So What Does This Tell You?

- In Example 5.1:
 - **Central differencing is fine for low velocities** (diffusion-dominated problems).
 - **It fails for convection-dominated flows** — producing unstable, unphysical solutions.

➡ That's why the book **criticizes central differencing** in Section 5.5.