📘 What Does First-Order Accuracy Mean?

A **first-order accurate scheme** means the **error reduces linearly** with grid spacing Δx:

$$\text{Error} \propto \Delta x$$

So if you halve the mesh size (make grid finer), the error is roughly halved.

🧪 Example 5.2 Setup Recap

We solve a 1D **steady-state convection-diffusion** problem using UDS with:

- Domain: $x \in [0, 1]$
- $\Gamma = 0.1$, $\rho = 1$, $\Delta x = 0.25$
- 5 nodes → coarse grid
- Boundary Conditions:

$$\phi(0) = 1, \quad \phi(1) = 0$$

- Two cases:
  - (i) $u = 0.1$ m/s → $Pe = 0.25$
  - (ii) $u = 2.5$ m/s → $Pe = 6.25$

📉 Why UDS Is First-Order

In UDS, the face value is approximated as:

$$\phi_e = \phi_P, \quad \phi_w = \phi_W$$

This approximation causes a **truncation error** (Taylor expansion):

$$\left(\frac{d\phi}{dx}\right)_e = \frac{\phi_E - \phi_P}{\Delta x} = \left(\frac{d\phi}{dx}\right)_{\text{exact}} + \frac{\Delta x}{2}\left(\frac{d^2\phi}{dx^2}\right) + O(\Delta x^2)$$

The **leading error term is proportional to** Δx → hence **first-order accuracy**.

🧾 In Example 5.2: Observe This in Action

Case	Grid	Error Behavior
(i) $u = 0.1$	Coarse (5 nodes)	Numerical and exact solution are close. Diffusion dominates, so error is small even with low order scheme.
(ii) $u = 2.5$	Same grid	UDS performs poorly. Strong convection → high numerical diffusion → φ profile becomes almost linear instead of exponential.

If we **refined the grid** (e.g., use 10 or 20 points), UDS would give **better results** — but the improvement would be **linear with respect to grid refinement**, confirming first-order accuracy.

📌 Summary

✅ **First-order accuracy in UDS** means:

- Error ∝ Δx
- Accurate only with **very fine grids**
- In Example 5.2:
  - Works okay for low Pe
  - Performs **poorly for high Pe**, showing visible **numerical diffusion**
  - Better results with grid refinement, but slowly