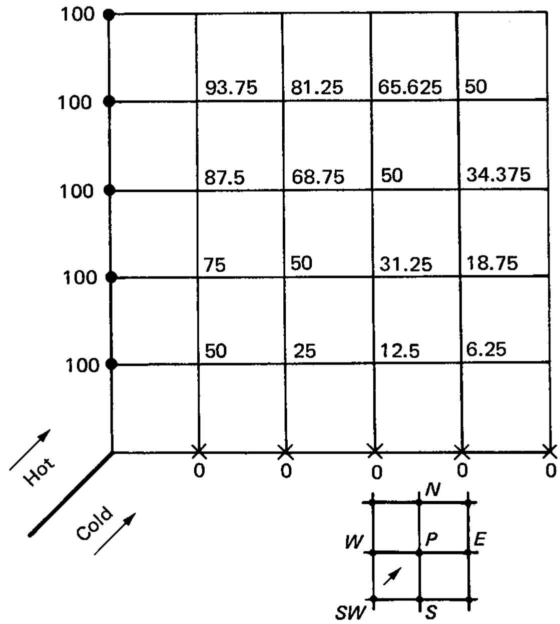

Figure 5.15 Situation with flow at 45° to the grid lines.

be necessary to involve more neighbors in the discretization equation. Although a few such schemes have been worked out [for example, Raithby (1976b)] and have shown an impressive reduction in false diffusion, they are significantly more complicated and so far insufficiently tested. For these reasons, we shall not discuss them here. (8) A more detailed discussion of false diffusion has been given by Raithby (1976a).

5.7 CLOSURE

In this chapter, we have completed the construction of the general discretization equation for the dependent variable ϕ. The convection term was the only addition that we made here, but it led to a number of interesting considerations. Our formulation ensures physically realistic behavior and thus holds the key to successful computation in the presence of fluid flow. The flow field itself, of course, must also be calculated in most cases. It is to this matter that we turn our attention in the next chapter.

PROBLEMS

5.1 In a steady two-dimensional situation, the variable ϕ is governed by

$$\operatorname{div}(\rho \mathbf{u} \phi) = \operatorname{div}(\Gamma \operatorname{grad} \phi) + a - b\phi ,$$

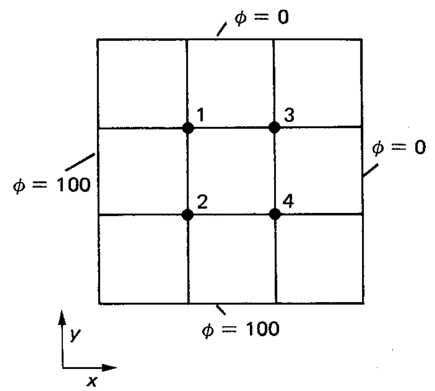

Figure 5.16 Boundary conditions for Problem 5.1.

where $\rho = 1$, $\Gamma = 1$, $a = 10$, and $b = 2$. The flow field is such that $u = 1$ and $v = 4$ everywhere. For the uniform grid shown in Fig. 5.16, $\Delta x = \Delta y = 1$. The values of ϕ are given for the four boundaries. Adopting the control-volume design according to Practice A in Section 4.6-1, calculate the values of ϕ_1, ϕ_2, ϕ_3, and ϕ_4 by use of:
(*a*) The central-difference scheme
(*b*) The upwind scheme
(*c*) The hybrid scheme
(*d*) The power-law scheme

5.2 Obtain the exact solution of the equation

$$\frac{d}{dx}\left(\rho u \phi - \Gamma \frac{d\phi}{dx}\right) = S ,$$

where ρu, Γ, and S are all constant; the boundary conditions are $\phi = \phi_0$ at $x = 0$, and $\phi = \phi_L$ at $x = L$. Use the exponential scheme to obtain a numerical solution of the problem for various values of $\rho u L/\Gamma$ and $(SL^2/\Gamma)/(\phi_L - \phi_0)$. Do you get perfect agreement with the exact solution? Why?

5.3 A parallel-flow heat exchanger is governed by

$$m_h c_h \frac{dT_h}{dx} = \frac{UA}{L}(T_c - T_h) \qquad \text{and} \qquad m_c c_c \frac{dT_c}{dx} = \frac{UA}{L}(T_h - T_c) ,$$

where m, c, and T stand for the mass flow rate, the specific heat, and the temperature, respectively; the subscripts h and c denote the hot and cold fluids, respectively; U is the overall heat transfer coefficient between the two fluids; A is the total heat transfer area; and L is the length of the heat exchanger. The inlet temperatures $T_{h,\text{in}}$ and $T_{c,\text{in}}$ are given. Obtain a numerical solution for the dimensionless temperatures $(T_h - T_{c,\text{in}})/\Delta T$ and $(T_c - T_{c,\text{in}})/\Delta T$ as functions of x/L for the conditions $m_h c_h = m_c c_c$ and $UA/m_h c_h = 1$. The temperature difference ΔT equals $T_{h,\text{in}} - T_{c,\text{in}}$. Compare the numerical results with the exact solution. (Although the two coupled equations can be handled iteratively by sequential solution for T_h and T_c, a direct simultaneous solution is often advantageous for such a case. This can be achieved by use of the algorithm for two coupled variables, which was outlined in Problem 4.17.)

5.4 Consider the one-dimensional distribution of a variable ϕ governed by convection and diffusion. The flow field is created by the flow in a porous-walled duct; m_x denotes the x-direction mass flow rate along the duct at any location x, and m_L is the rate of mass