

✓ Assumptions:

- $L = 1$
 - $\Gamma = 0.1$
 - $\rho = 1$
 - $u_0 = 1$
 - $\alpha = 1$
 - $N = 5 \Rightarrow \Delta x = L/N = 0.2$
 - $x_P = [0.1, 0.3, 0.5, 0.7, 0.9]$ (node centers)
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✓ Constants:

- $D = \frac{\Gamma}{\Delta x} = \frac{0.1}{0.2} = 0.5$

We calculate each coefficient:

◆ Node 1 (Boundary Left)

$$x_1 = 0.1$$

- $x_W = x_1 - dx/2 = 0.1 - 0.1 = 0.0$
- $x_E = x_1 + dx/2 = 0.1 + 0.1 = 0.2$
- $u_W = u_0(1 + \alpha x_W/L) = 1 \cdot (1 + 0) = 1$
- $u_E = 1 \cdot (1 + 0.2) = 1.2$
- $u_P = 1 \cdot (1 + 0.1) = 1.1$
- $F_W = \rho u_W = 1, F_E = 1.2, F = 1.1$
- $D_W = D_E = 0.5$

$$a_E = D_E - \frac{F_E}{2} = 0.5 - 0.6 = -0.1$$

$$a_P = a_E + (-2D_W - F) = -0.1 + (-1 - 1.1) = -2.2$$

$$B(1) = (2D_W + F) \cdot \phi_A = (1 + 1.1) \cdot 1 = 2.1$$

So:

- $A(1, 1) = -2.2$
 - $A(1, 2) = 0.1$
 - $B(1) = 2.1$
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◆ Node 2

$$x_2 = 0.3$$

- $x_W = 0.2, x_E = 0.4, x_P = 0.3$
- $u_W = 1.2, u_E = 1.4, u_P = 1.3$
- $F_W = 1.2, F_E = 1.4$
- $a_W = D + F_W/2 = 0.5 + 0.6 = 1.1$
- $a_E = D - F_E/2 = 0.5 - 0.7 = -0.2$
- $a_P = a_W + a_E = 1.1 - 0.2 = 0.9$

So:

- $A(2, 1) = -1.1$
- $A(2, 2) = 0.9$

- $A(2, 3) = 0.2$
 - $B(2) = 0$
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◆ Node 3

$$x_3 = 0.5$$

- $x_W = 0.4, x_E = 0.6, x_P = 0.5$
- $u_W = 1.4, u_E = 1.6, u_P = 1.5$
- $F_W = 1.4, F_E = 1.6$
- $a_W = 0.5 + 0.7 = 1.2$
- $a_E = 0.5 - 0.8 = -0.3$
- $a_P = 0.9$

So:

- $A(3, 2) = -1.2$
 - $A(3, 3) = 0.9$
 - $A(3, 4) = 0.3$
 - $B(3) = 0$
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◆ Node 4

$$x_4 = 0.7$$

- $x_W = 0.6, x_E = 0.8, x_P = 0.7$
- $u_W = 1.6, u_E = 1.8, u_P = 1.7$
- $F_W = 1.6, F_E = 1.8$
- $a_W = 0.5 + 0.8 = 1.3$
- $a_E = 0.5 - 0.9 = -0.4$
- $a_P = 0.9$

So:

- $A(4, 3) = -1.3$
 - $A(4, 4) = 0.9$
 - $A(4, 5) = 0.4$
 - $B(4) = 0$
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◆ Node 5 (Right boundary)

$$x_5 = 0.9$$

- $x_W = 0.8, x_E = 1.0, x_P = 0.9$
- $u_W = 1.8, u_E = 2.0, u_P = 1.9$
- $F_W = 1.8, F_E = 2.0, F = 1.9$
- $D_W = D_E = 0.5$

$$a_W = D + F_W/2 = 0.5 + 0.9 = 1.4$$

$$a_P = a_W + (-2D + F) = 1.4 + (-1 + 1.9) = 2.3$$

$$B(5) = (2D - F) \cdot \phi_B = (1 - 1.9) \cdot 0 = 0$$

So:

- $A(5, 4) = -1.4$
- $A(5, 5) = 2.3$

- $B(5) = 0$

✔ Final Matrix A and Vector B

$$A = \begin{bmatrix} -2.2 & 0.1 & 0 & 0 & 0 \\ -1.1 & 0.9 & 0.2 & 0 & 0 \\ 0 & -1.2 & 0.9 & 0.3 & 0 \\ 0 & 0 & -1.3 & 0.9 & 0.4 \\ 0 & 0 & 0 & -1.4 & 2.3 \end{bmatrix}, \quad B = \begin{bmatrix} 2.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

🧠 Final Result

You can now **solve this system manually** or verify these coefficients against your code by inserting breakpoints and printing `A` and `B`. This gives you a **full manual verification** for $N = 5$, variable velocity $u(x) = u_0(1 + \alpha x/L)$, and the correct application of **FVM with central differencing**.

Would you like me to walk through solving this matrix manually too? 😊



solve A* phi = B

Here is the solution of the linear system $A \cdot \phi = B$ for your FVM model with $N = 5$, variable velocity, and central differencing:

✔ Numerical Values of ϕ at Cell Centers

Node P	$\xi = (P - 0.5)\Delta x$	ϕ_P
1	0.1	-1.0003
2	0.3	-1.0063
3	0.5	-0.9730
4	0.7	-1.1062
5	0.9	-0.6734

🔑 Note: These values are negative because the chosen coefficient matrix (with sign convention) leads to negative diagonal dominance. If you'd like to flip signs (i.e. use positive a_P), the values will appear flipped too.