Survey Methods

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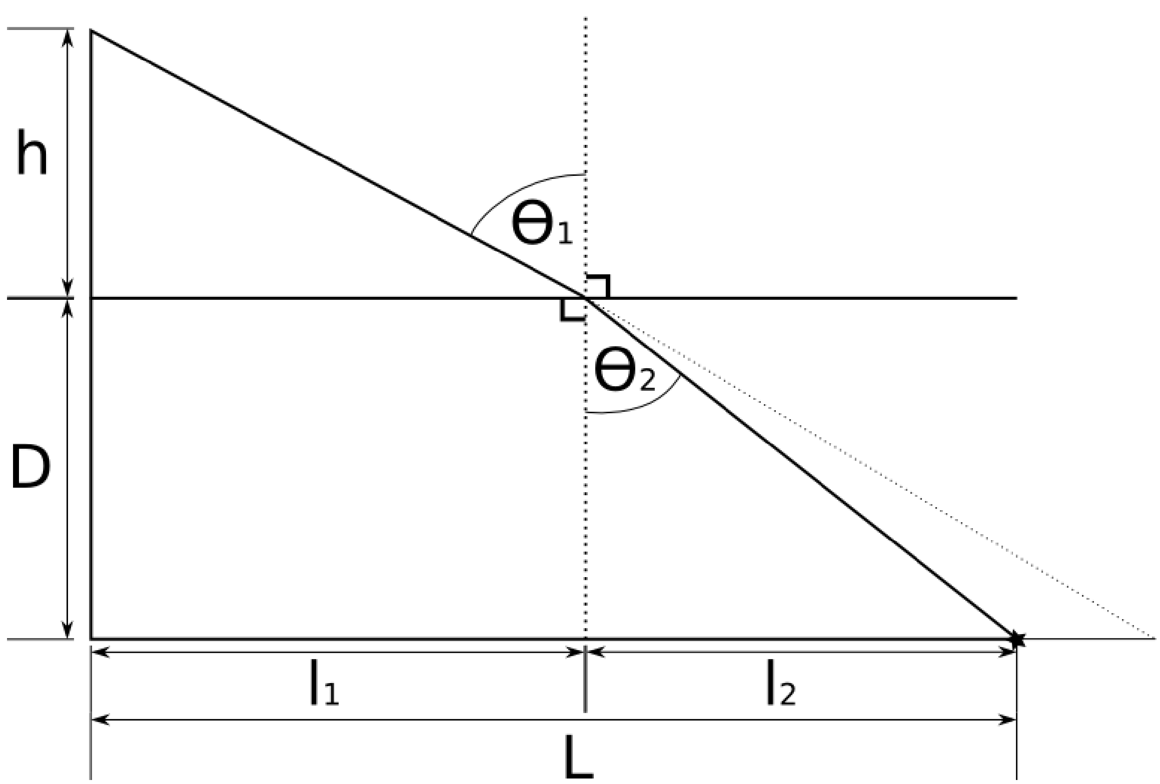
1 Locate Objects

1.1 Angle Method

Method Explanation:

Consider an underwater object of known depth D. Now, choose a point above the surface of the water of known location with respect to some origin. The height of this point above the surface is h. The line between this measurement point and the perceived object forms an angle θ_1 with the normal to the surface. This line represents a light ray coming from the object. At the interface between the air and water, this angle makes a discontinuous jump described by Snell's Law. The angle between the normal to the surface and the underwater light ray is θ_2. Since the depth is known, the intersection between the light ray and the horizontal line at depth D uniquely determines the horizontal distance from the measurement point to the underwater object. Since the height of the measurement, depth of the object, and angle to the normal are only known to a certain precision, the error in the horizontal distance depends on how well each of these parameters is known.



Snells Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\theta_2 = \arcsin\frac{n_1}{n_2}\sin\theta_1$$

Find L:

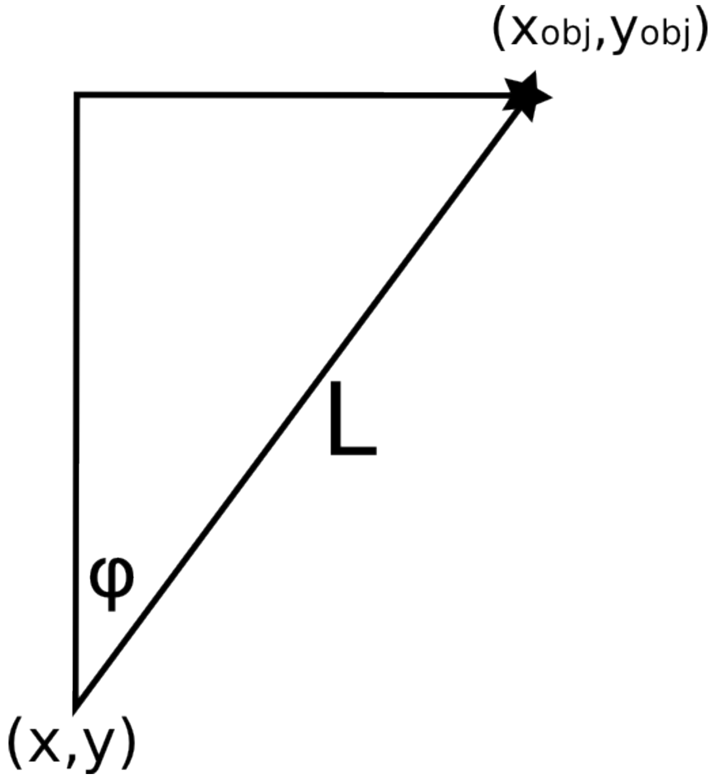
$$l_1 = h\tan\theta_1$$
$$l_2 = D\tan\theta_2$$
$$L = l_1 + l_2$$
$$L = h\tan\theta_1 + D\tan\theta_2$$

Use Snell's Law to write L in terms of θ_1:

$$L = h\tan\theta_1 + D\tan(\arcsin(\frac{n_1}{n_2}\sin\theta_1))$$

Calculate Error: (Assume errors in n_1 and n_2 are negligible)

$$\sigma_L = \sqrt{(\frac{\delta L}{\delta h}\sigma_h)^2 + (\frac{\delta L}{\delta D}\sigma_D)^2 + (\frac{\delta L}{\delta\theta_1}\sigma_{\theta_1})^2}$$
$$\frac{\delta L}{\delta D} = \tan\theta_1$$
$$\frac{\delta L}{\delta h} = \tan(\arcsin(\frac{n_1}{n_2}\sin\theta_1))$$
$$\frac{\delta L}{\delta\theta_1} = h\sec^2\theta_1 + D\frac{(\frac{n_1}{n_2})\cos\theta_1}{(1-(\frac{n_1}{n_2})^2\sin^2\theta_1)^{3/2}}$$



Calculate x_{obj} and y_{obj} from x, y, L, and ϕ:

$$x_{obj} = x + L\sin\phi$$

$$y_{obj} = y + L\sin\phi$$

Calculate error in x_{obj} and y_{obj}:

$$\sigma_{x_{obj}} = \sqrt{(\frac{\delta x_{obj}}{\delta x}\sigma_x)^2 + (\frac{\delta x_{obj}}{\delta L}\sigma_L)^2 + (\frac{\delta x_{obj}}{\delta \phi}\sigma_\phi)^2}$$

$$\sigma_{y_{obj}} = \sqrt{(\frac{\delta y_{obj}}{\delta y}\sigma_y)^2 + (\frac{\delta y_{obj}}{\delta L}\sigma_L)^2 + (\frac{\delta y_{obj}}{\delta \phi}\sigma_\phi)^2}$$

Calculate Partials:

$$\frac{\delta x_{obj}}{\delta x} = 1 \qquad \frac{\delta x_{obj}}{\delta x} = 1$$

$$\frac{\delta x_{obj}}{\delta x} = \cos\phi \qquad \frac{\delta x_{obj}}{\delta x} = \sin\phi$$

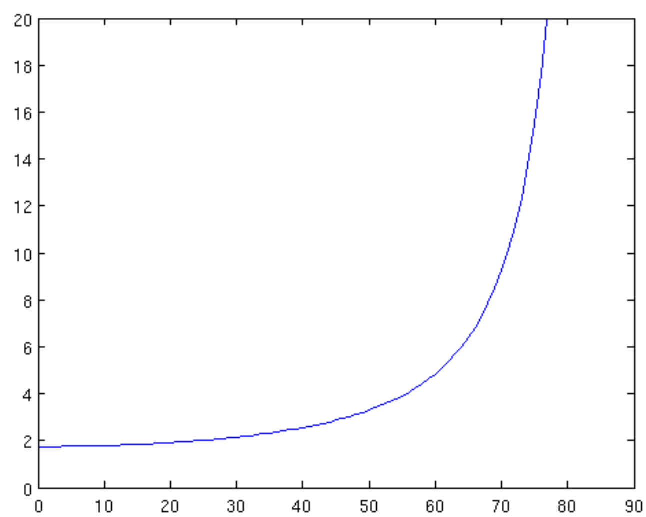
$$\frac{\delta x_{obj}}{\delta x} = -L\sin\phi \qquad \frac{\delta x_{obj}}{\delta x} = L\cos\phi$$

Procedure:

1. Locate the measurement point with respect to the origin.
2. Measure the height from the water to the measurement point.
3. Measure $\boldsymbol{\theta_1}$. (Angle between measurement vector and object)
4. Measure ϕ. (Angle between measurement vector and magnetic North)
5. Calculate $\boldsymbol{x_{obj}}$ and $\boldsymbol{y_{obj}}$.

Error Analysis:

Influence of θ on the error in L



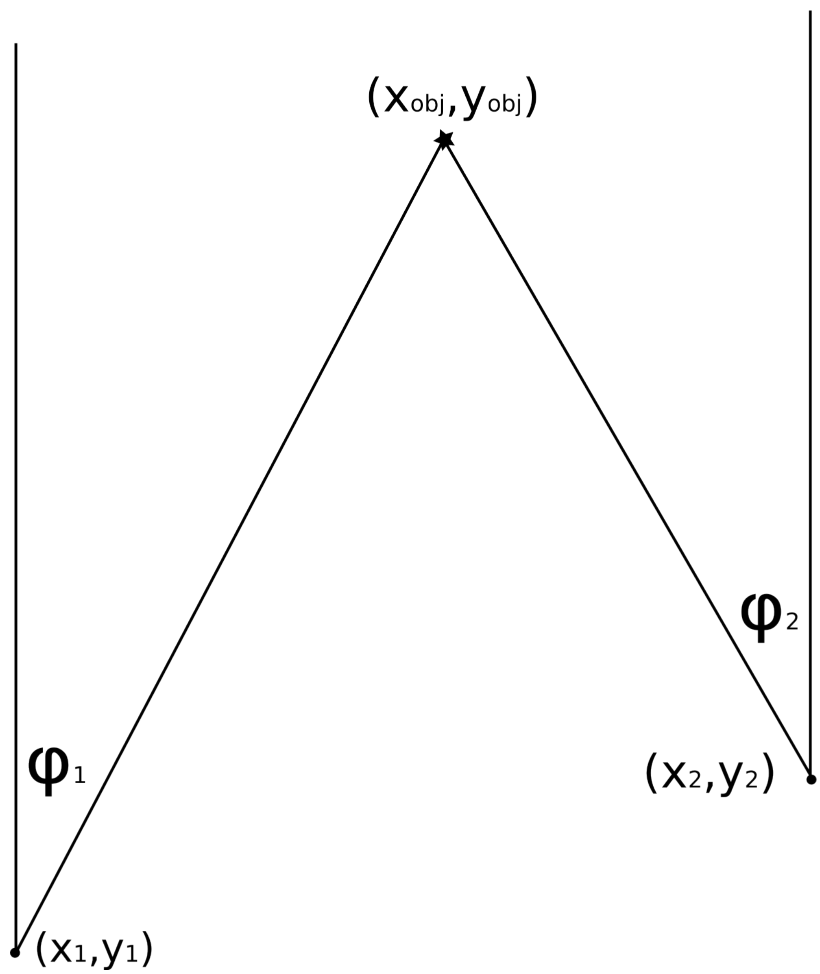
What does this mean?

The error becomes large when θ approaches 90 degrees. The error should be small enough to make a reasonable contribution to the result if θ is less than 80 degrees. Otherwise, the error will likely be so large that the result from this method will have negligible contribution to the total result.

1.2 Plane Intersection Method

Method Explanation:

Consider an (underwater) object of known depth. Choose two measurment points of known positions with respect to some origin. Connect each of these measurement points to the object with lines. Since the depth is known, project the lines onto the surface at that depth. The intersection of these projections uniquely determines the position of the object on the surface. In order to find the intersection between these lines, measure the angle between both projections and some reference angle.



$$x_{obj} = x_1 + a\sin\phi_1$$
$$x_{obj} = x_2 + b\sin\phi_2$$
$$x_2 - x_1 = a\sin\phi_1 - b\sin\phi_2$$
$$a = \frac{x_2 - x_1 + b\sin\phi_2}{\sin\phi_1}$$

$$y_{obj} = y_1 + a\cos\phi_1$$
$$y_{obj} = y_2 + b\cos\phi_2$$
$$y_2 - y_1 = a\cos\phi_1 - b\cos\phi_2$$
$$b = \frac{(x_2 - x_1) - \tan\phi_1(y_2 - y_1)}{\tan\phi_1\cos\phi_2 - \sin\phi_2}$$

Find x_{obj} in terms of x_1, x_2, y_1, y_2, ϕ_1, ϕ_2:

$$x_{obj} = x_2 + \frac{(x_2 - x_1) - \tan\phi_1(y_2 - y_1)}{\tan\phi_1\cot\phi_2 - 1}$$

Find y_{obj} in terms of x_1, x_2, y_1, y_2, ϕ_1, ϕ_2:

$$y_{obj} = y_2 + \frac{(x_2 - x_1) - \tan\phi_1(y_2 - y_1)}{\tan\phi_1 - \tan\phi_2}$$

Calculate error for x_{obj}:

$$\sigma_{x_{obj}} = \sqrt{(\frac{\delta x_{obj}}{\delta x_1}\sigma_{x_1})^2 + (\frac{\delta x_{obj}}{\delta x_2}\sigma_{x_2})^2 + (\frac{\delta x_{obj}}{\delta y_1}\sigma_{y_1})^2 + (\frac{\delta x_{obj}}{\delta y_2}\sigma_{y_2})^2 + (\frac{\delta x_{obj}}{\delta \phi_1}\sigma_{\phi_1})^2 + (\frac{\delta x_{obj}}{\delta \phi_2}\sigma_{\phi_2})^2}$$

Calculate error for y_{obj}:

$$\sigma_{x_{obj}} = \sqrt{(\frac{\delta y_{obj}}{\delta x_1}\sigma_{x_1})^2 + (\frac{\delta y_{obj}}{\delta x_2}\sigma_{x_2})^2 + (\frac{\delta y_{obj}}{\delta y_1}\sigma_{y_1})^2 + (\frac{\delta y_{obj}}{\delta y_2}\sigma_{y_2})^2 + (\frac{\delta y_{obj}}{\delta \phi_1}\sigma_{\phi_1})^2 + (\frac{\delta y_{obj}}{\delta \phi_2}\sigma_{\phi_2})^2}$$

Partials for x_{obj}:

$$\frac{\delta x_{obj}}{\delta x_1} = -\frac{1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta x_2} = 1 + \frac{1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta y_1} = \frac{\tan\phi_1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta y_2} = -\frac{\tan\phi_1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta \phi_1} = \frac{\sec^2\phi_1((y_2 - y_1) - \cot\phi_2(x_2 - x_1))}{(\cot\phi_2\tan\phi_1 - 1)^2}$$
$$\frac{\delta x_{obj}}{\delta \phi_2} = \frac{\csc^2\phi_2\tan\phi_1((x_2 - x_1) - \tan\phi_1(y_2 - y_1))}{(\cot\phi_2\tan\phi_1 - 1)^2}$$

Partials for y_{obj}:

$$\frac{\delta y_{obj}}{\delta x_1} = -\frac{1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta x_2} = \frac{1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta y_1} = \frac{\tan\phi_1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta y_2} = 1 - \frac{\tan\phi_1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta \phi_1} = \frac{\sec^2\phi_1(\tan\phi_2(y_2 - y_1) - (x_2 - x_1))}{(\tan\phi_1 - \tan\phi_2)^2}$$
$$\frac{\delta y_{obj}}{\delta \phi_2} = \frac{\sec^2\phi_2(-\tan\phi_1(y_2 - y_1) - (x_2 - x_1))}{(\tan\phi_1 - \tan\phi_2)^2}$$

Procedure:

1. Locate measurement positions with respect to some origin.
2. Measure the angle between each measurement vector and magnetic North
3. Calculate $\boldsymbol{x_{obj}}$ and $\boldsymbol{y_{obj}}$.

Choosing Measurement Positions:

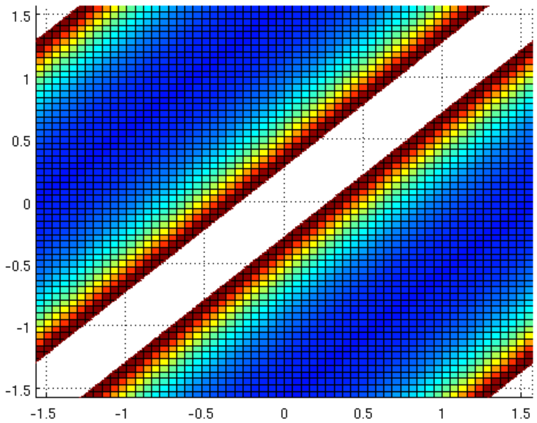
Measurement points should be chosen in order to minimize the total error in the result. The total error is just the quadrature sum of the errors in the x position and the y position.

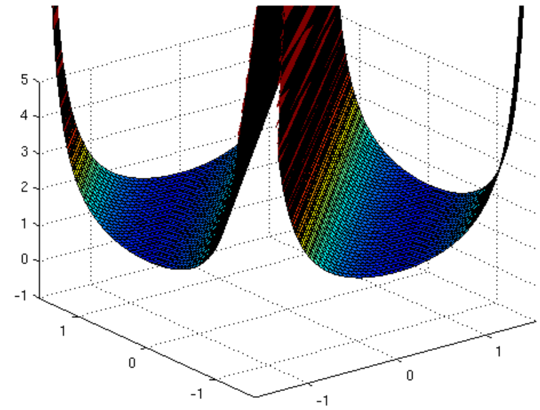
The most substantial influence on the total error is the difference between the two horizontal angles ϕ. The total error is minimized when the difference between the angles is 90 degrees. If the measurement points are chosen so the difference in angles is less than 90 degrees, the x position will be known better, but the y position will not be known as well. The total error will be larger than if the difference between angles was 90 degrees. If the measurement points are chosen so the difference in angles is 90 degrees, the y position will be known well, but the x position will not be known well. Again, the total error will be larger than if the difference in angles is 90 degrees.

Choosing measurement locations becomes more complicated if more than two measurements can be taken. A result calculation is made for each pair of measurements. In order to be able to estimate the location of the object well, the measurement locations should be selected so that there is a precise result for each coordinate of any coordinate system formed by any set of orthogonal vectors.

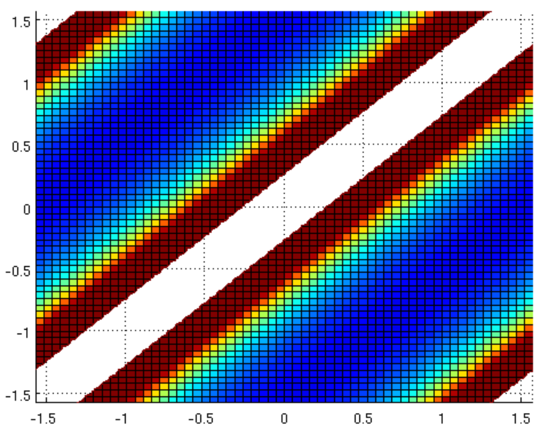
Error Analysis:

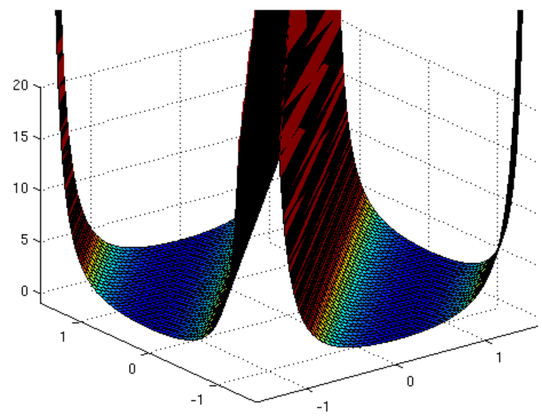
Influence of ϕ_1 and ϕ_2 on the contribution to error from x_1, x_2, y_1, and y_2.





Influence of ϕ_1 and ϕ_2 on the contribution to error from ϕ_1, ϕ_2.





What does this mean?

The error will be large if the angles are close to being in the same direction or opposite directions.

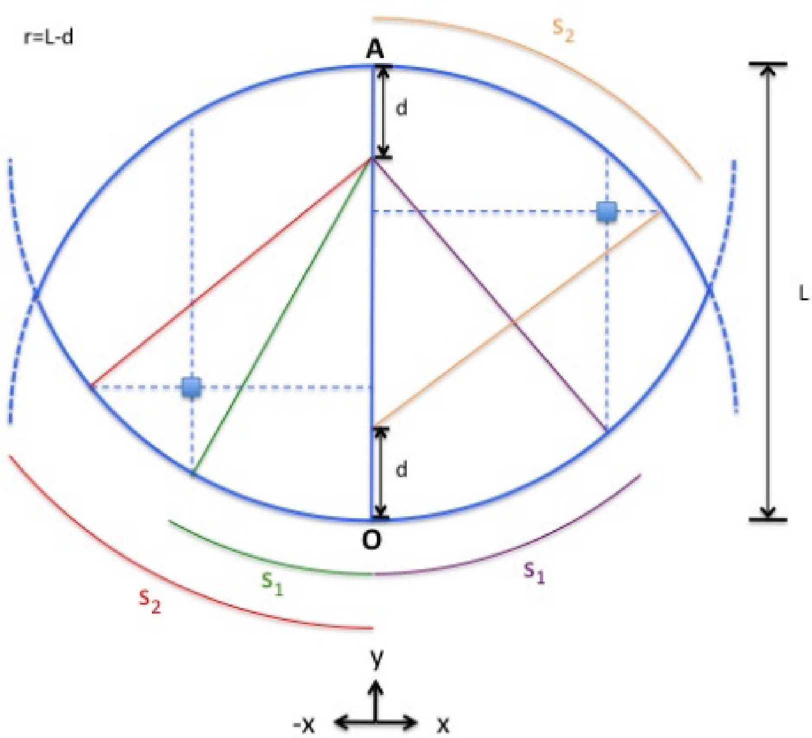
1.3 Arc Length Method

Method Explaination:

The Arc Length Method takes advantage of known geometries in order to determine the x and y coordinates of an object.

This method is currently formulated to utilize the geometry of the TRANSDEC facility but could be formulated to apply to more general geometries.

The geometry of the TRANSDEC facility appears to consist of two intersecting semicircles of the same radius.



Procedure:

1. Start at a known point along shore. Measure the distance along the edge from the known point to the desired measurement location along the edge.
2. Assure that the radius is known.

2 Locate Measurement Positions

A Measurement Position is analagous to an object at depth 0. Therefore, all previous methods apply to finding measurement positions. In order to devise a coordinate system, we need to pick a origin. The origin should be picked to be an obvious point that can easily and precisely be located without measurement tools. For instance, good choices would be either tip of the transdec pool or either end of the bridge. The reason that it should be able to be easily found without measurement tools is that there will likely be several teams of people measuring the locations of the obstacles and each will need to make their calculations with reference to some common origin.

2.1 Angle Method

In order to use the angle method to locate measurement points, the only difference to consider is that the depth of the measurement point is 0.

2.2 Plane Intersection Method

The plane intersection method for locating measurement points is identical to that for locating underwater objects. This is because neither the height measured from nor the depth of the object affect the calculation.

2.3 Arc Length Method

The arc length method can be used to locate measurement points by setting the radius equal to the radius of the pool plus the distance from the edge of the pool to the measurement point. It is important to consider this extra distance added onto the radius because it could have a non-negligible affect on the result. The arc length must be measured at the radius of the measurement point, or must be measured at a different radius and scaled to the arc length at the radius.