Survey Methods

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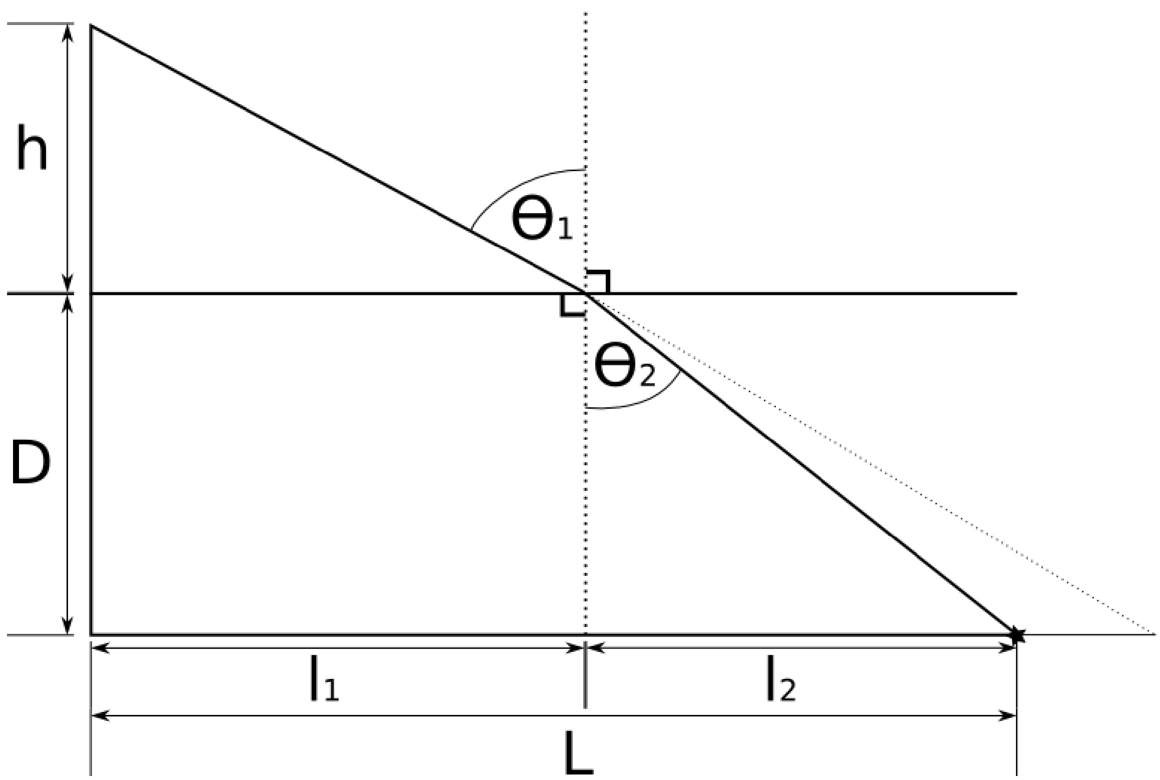
1 Locate Measurement Positions

2 Locate Objects

2.1 Angle Method

Method Explanation:

Consider an underwater object of known depth D. Now, choose a point above the surface of the water of known location with respect to some origin. The height of this point above the surface is h. The line between this measurement point and the perceived object forms an angle θ_1 with the normal to the surface. This line represents a light ray coming from the object. At the interface between the air and water, this angle makes a discontinuous jump described by Snell's Law. The angle between the normal to the surface and the underwater light ray is θ_2. Since the depth is known, the intersection between the light ray and the horizontal line at depth D uniquely determines the horizontal distance from the measurement point to the underwater object. Since the height of the measurement, depth of the object, and angle to the normal are only known to a certain precision, the error in the horizontal distance depends on how well each of these parameters is known.



Snells Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\theta_2 = \arcsin\frac{n_1}{n_2}\sin\theta_1$$

Find L:

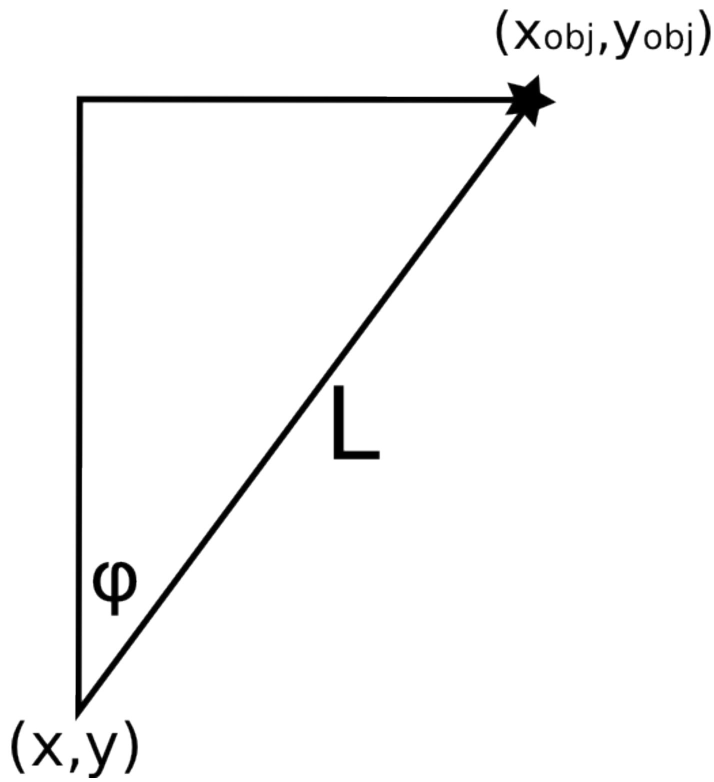
$$l_1 = h\tan\theta_1$$
$$l_2 = D\tan\theta_2$$
$$L = l_1 + l_2$$
$$L = h\tan\theta_1 + D\tan\theta_2$$

Use Snell's Law to write L in terms of θ_1:

$$L = h\tan\theta_1 + D\tan(\arcsin(\frac{n_1}{n_2}\sin\theta_1))$$

Calculate Error: (Assume errors in n_1 and n_2 are negligible)

$$\sigma_L = \sqrt{(\frac{\delta L}{\delta h}\sigma_h)^2 + (\frac{\delta L}{\delta D}\sigma_D)^2 + (\frac{\delta L}{\delta\theta_1}\sigma_{\theta_1)})^2}$$
$$\frac{\delta L}{\delta D} = \tan\theta_1$$
$$\frac{\delta L}{\delta h} = \tan(\arcsin(\frac{n_1}{n_2}\sin\theta_1))$$
$$\frac{\delta L}{\delta\theta_1} = h\sec^2\theta_1 + \frac{(\frac{n_1}{n_2})\cos\theta_1}{(1-(\frac{n_1}{n_2})^2\sin^2\theta_1)^{3/2}}$$



Calculate x_{obj} and y_{obj} from x, y, L, and ϕ:

$$x_{obj} = x + L\sin\phi$$

$$y_{obj} = y + L\sin\phi$$

Calculate error in x_{obj} and y_{obj}:

$$\sigma_{x_{obj}} = \sqrt{(\frac{\delta x_{obj}}{\delta x}\sigma_x)^2 + (\frac{\delta x_{obj}}{\delta L}\sigma_L)^2 + (\frac{\delta x_{obj}}{\delta \phi}\sigma_\phi)^2}$$

$$\sigma_{y_{obj}} = \sqrt{(\frac{\delta y_{obj}}{\delta y}\sigma_y)^2 + (\frac{\delta y_{obj}}{\delta L}\sigma_L)^2 + (\frac{\delta y_{obj}}{\delta \phi}\sigma_\phi)^2}$$

Calculate Partials:

$$\frac{\delta x_{obj}}{\delta x} = 1$$

$$\frac{\delta x_{obj}}{\delta x} = \cos\phi$$

$$\frac{\delta x_{obj}}{\delta x} = -L\sin\phi$$

$$\frac{\delta x_{obj}}{\delta x} = 1$$

$$\frac{\delta x_{obj}}{\delta x} = \sin\phi$$

$$\frac{\delta x_{obj}}{\delta x} = L\cos\phi$$

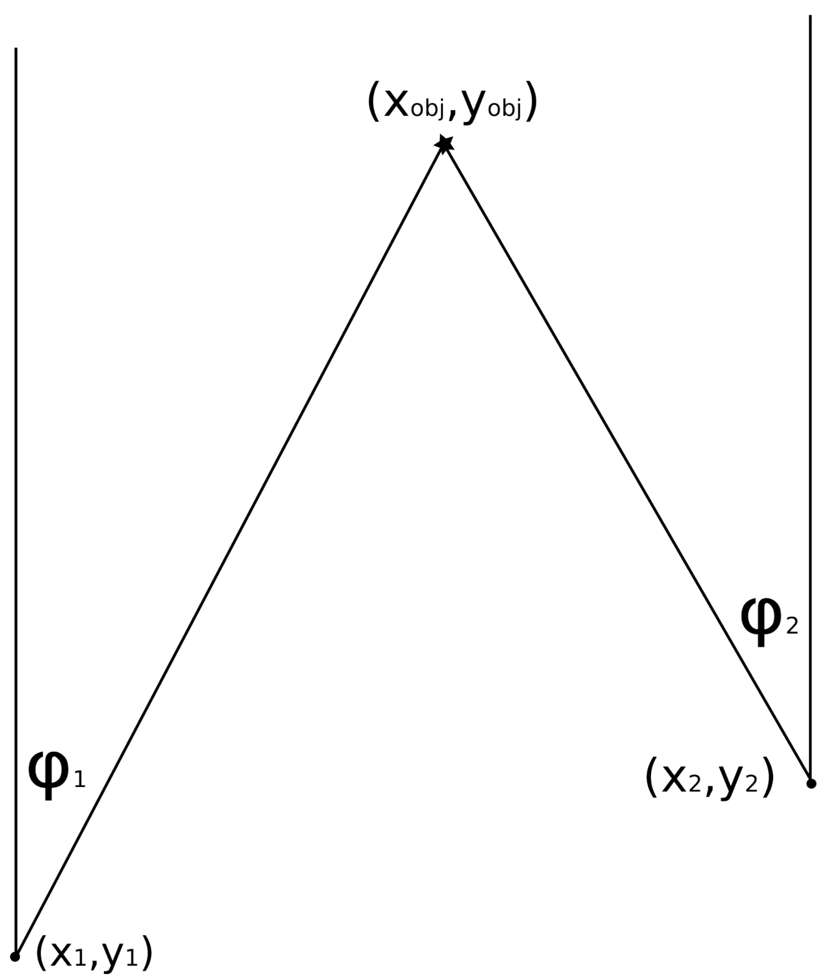
Procedure:

1. Locate the measurement point with respect to the origin.
2. Measure the height from the water to the measurement point.
3. Measure θ_1. (Angle between measurement vector and object)
4. Measure ϕ. (Angle between measurement vector and magnetic North)
5. Calculate x_{obj} and y_{obj}.

2.2 Plane Intersection Method

Method Explanation:

Consider an (underwater) object of known depth. Choose two measurment points of known positions with respect to some origin. Connect each of these measurement points to the object with lines. Since the depth is known, project the lines onto the surface at that depth. The intersection of these projections uniquely determines the position of the object on the surface. In order to find the intersection between these lines, measure the angle between both projections and some reference angle.



$$x_{obj} = x_1 + a\sin\phi_1$$
$$x_{obj} = x_2 + b\sin\phi_2$$
$$x_2 - x_1 = a\sin\phi_1 - b\sin\phi_2$$
$$a = \frac{x_2 - x_1 + b\sin\phi_2}{\sin\phi_1}$$

$$y_{obj} = y_1 + a\cos\phi_1$$
$$y_{obj} = y_2 + b\cos\phi_2$$
$$y_2 - y_1 = a\cos\phi_1 - b\cos\phi_2$$
$$b = \frac{(x_2 - x_1) - \tan\phi_1(y_2 - y_1)}{\tan\phi_1\cos\phi_2 - \sin\phi_2}$$

Find x_{obj} in terms of x_1, x_2, y_1, y_2, ϕ_1, ϕ_2:

$$x_{obj} = x_2 + \frac{(x_2 - x_1) - \tan\phi_1(y_2 - y_1)}{\tan\phi_1\cot\phi_2 - 1}$$

Find y_{obj} in terms of x_1, x_2, y_1, y_2, ϕ_1, ϕ_2:

$$y_{obj} = y_2 + \frac{(x_2 - x_1) - \tan\phi_1(y_2 - y_1)}{\tan\phi_1 - \tan\phi_2}$$

Calculate error for x_{obj}:

$$\sigma_{x_{obj}} = \sqrt{(\frac{\delta x_{obj}}{\delta x_1}\sigma_{x_1})^2 + (\frac{\delta x_{obj}}{\delta x_2}\sigma_{x_2})^2 + (\frac{\delta x_{obj}}{\delta y_1}\sigma_{y_1})^2 + (\frac{\delta x_{obj}}{\delta y_2}\sigma_{y_2})^2 + (\frac{\delta x_{obj}}{\delta \phi_1}\sigma_{\phi_1})^2 + (\frac{\delta x_{obj}}{\delta \phi_2}\sigma_{\phi_2})^2}$$

Calculate error for y_{obj}:

$$\sigma_{x_{obj}} = \sqrt{(\frac{\delta y_{obj}}{\delta x_1}\sigma_{x_1})^2 + (\frac{\delta y_{obj}}{\delta x_2}\sigma_{x_2})^2 + (\frac{\delta y_{obj}}{\delta y_1}\sigma_{y_1})^2 + (\frac{\delta y_{obj}}{\delta y_2}\sigma_{y_2})^2 + (\frac{\delta y_{obj}}{\delta \phi_1}\sigma_{\phi_1})^2 + (\frac{\delta y_{obj}}{\delta \phi_2}\sigma_{\phi_2})^2}$$

Partials for x_{obj}:

$$\frac{\delta x_{obj}}{\delta x_1} = -\frac{1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta x_2} = 1 + \frac{1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta y_1} = \frac{\tan\phi_1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta y_2} = -\frac{\tan\phi_1}{\tan\phi_1\cot\phi_2 - 1}$$
$$\frac{\delta x_{obj}}{\delta \phi_1} = \frac{\sec^2\phi_1((y_2 - y_1) - \cot\phi_2(x_2 - x_1))}{(\cot\phi_2\tan\phi_1 - 1)^2}$$
$$\frac{\delta x_{obj}}{\delta \phi_2} = \frac{\csc^2\phi_2\tan\phi_1((x_2 - x_1) - \tan\phi_1(y_2 - y_1))}{(\cot\phi_2\tan\phi_1 - 1)^2}$$

Partials for y_{obj}:

$$\frac{\delta y_{obj}}{\delta x_1} = -\frac{1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta x_2} = \frac{1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta y_1} = \frac{\tan\phi_1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta y_2} = 1 - \frac{\tan\phi_1}{\tan\phi_1 - \tan\phi_2}$$
$$\frac{\delta y_{obj}}{\delta \phi_1} = \frac{\sec^2\phi_1(\tan\phi_2(y_2 - y_1) - (x_2 - x_1))}{(\tan\phi_1 - \tan\phi_2)^2}$$
$$\frac{\delta y_{obj}}{\delta \phi_2} = \frac{\sec^2\phi_2(-\tan\phi_1(y_2 - y_1) - (x_2 - x_1))}{(\tan\phi_1 - \tan\phi_2)^2}$$

Procedure:

1. Locate measurement positions with respect to some origin.
2. Measure the angle between each measurement vector and magnetic North
3. Calculate x_{obj} and y_{obj}.

Notes:

In order to minimize the total error with two measurements, the difference between the angles should be 90 degrees. That is, the measurement vectors from the measurement points to the object should be orthogonal.

Choosing measurement points that are closer to each other (the difference between the angles will be smaller than 90 degrees) will decerase the error in the direction formed by the midpoint of the measurement points and the object point

For Example, if the measurement points were located symmetrically in the x direction and had the same y coordinate. If the difference between the angles is smaller than 90 degrees, there is more certainty in the x coordinate and less in the y coordinate. The sum of these errors in quadrature is going to be greater than that if the difference in the angles is 90 degrees. If the angle between the measurement points is larger than 90 degrees, there is going to be more certainty in the y coordinate and less in the x coordinate. The sum of these errors in quadrature is also going to be greater than that if the difference in the angle is 90 degrees.

This example can be expanded to any two measurement points because they are symmetric around their midpoint and have the same component in the direction that is orthogonal to the line formed by their midpoint and the object.

This must be because there can be no preferred coordinate system. When these errors are projected onto our x,y coordinate system, the errors in x and y might be the same, but their sum in quadrature will be greater than if the difference in angles were to be 90 degrees.