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# Finite Volume Method for Analysis of Convective Longitudinal Fin with Temperature-Dependent Thermal Conductivity and Internal Heat Generation

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**Keywords**: Heat transfer, Longitudinal fin, Finite Volume Method, Temperature-dependent thermal conductivity and Internal heat generation

**Abstract.** In this study, heat transfer in a longitudinal rectangular fin with temperature-dependent thermal properties and internal heat generation has been analyzed using finite volume method. The numerical solution was validated with the exact solution for the linear problem. The developed heat transfer models were used to investigate the effects of thermo-geometric parameters, coefficient of heat transfer and thermal conductivity (non-linear) parameters on the temperature distribution, heat transfer and thermal performance of the longitudinal rectangular fin. From the results, it shows that the fin temperature distribution, the total heat transfer, and the fin efficiency are significantly affected by the thermo-geometric of the fin. Therefore, the results obtained in this analysis serve as basis for comparison of any other method of analysis of the problem and they also provide platform for improvement in the design of fin in heat transfer equipment.

## 1. Introduction

Fins are used to implement the flow of heat between a source and a sink in various types of heat-transfer equipment and components.In practice, various types of fins with different geometries are used, but the simplicity of its design, ease of construction and manufacturing process, has made rectangular fins to be widely applied in heat-transfer equipment. Also, for ordinary fins problem, the thermal properties of the fin and the surrounding medium (thermal conductivity and heat transfer coefficient) are assumed to be constant, but if large temperature difference exists within the fin, typically, between tip and the base of the fin, the thermal conductivity and the heat transfer coefficient are not constant but temperature-dependent. Therefore, while analyzing the fin, effects of the temperature-dependent thermal properties must be taken into consideration. In carrying out such analysis, the thermal conductivity may be modelled for such and other many engineering applications by power law and by linear dependency on temperature while the heat transfer coefficient can be expressed as power law for which the exponents represent different phenomena as reported by Khani and Aziz [1], Ndlovu and Moitsheki [2]. Such dependency of thermal conductivity and heat transfer coefficient on temperature renders the problem highly non-linear and difficult to solve analytically. It is also very realistic to consider the temperature-dependent internal heat generation in the fins as applied in electric-current carrying conductor, nuclear rods or any other heat generating components of thermal systems.

Over the past few decades, the solutions of the highly non-linear differential equations have been constructed using different techniques. Aziz and Enamul-Huq [3] and Aziz [4] applied regular perturbation expansion to study a pure convection fin with temperature dependent thermal conductivity. Few years later, Campo and Spaulding [5] predicted the thermal behaviour of uniform circumferential fins using method of successive approximation. Chiu and Chen [6] and Arslanturk [7] adopted the Adomian Decomposition Method (ADM) to obtain the temperature distribution in a pure convective fin with variable thermal conductivity. The same problem was solved by Ganji [8] with the aid of the homotopy perturbation method originally proposed by He [9]. In the same year, Chowdhury and Hashim [10] applied Adomian decomposition method to evaluate the temperature distribution of straight rectangular fin with temperature dependent surface flux for all possible types of heat transfer while in the following year, Rajabi [11] applied Homotopy perturbation method (HPM) to calculate the efficiency of straight fins with temperature-dependent thermal conductivity. Also, a year later, Mustapha [12] adopted Homotopy analysis method (HAM) to find the efficiency of straight fins with temperature-dependent thermal conductivity. Meanwhile, Coskun and Atay [13] utilized variational iteration method (VIM) for the analysis of convective straight and radial fins with temperature-dependent thermal conductivity. Also, Languri *et al.* [14] applied both variation iteration and Homotopy perturbation methods for the evaluation of efficiency of straight fins with temperature-dependent thermal conductivity while Coskun and Atay [15] applied variational iteration method to analyse the efficiency of convective straight fins with temperature-dependent thermal conductivity. Besides, Atay and Coskum [16] employed variation iteration and finite element methods to carry out comparative analysis of power-law-fin type problems. Domairry and Fazeli [17] used Homotopy analysis method to determine the efficiency of straight fins with temperature-dependent thermal conductivity. Chowdhury *et al.* [18] investigated a rectangular fin with power law surface heat flux and made a comparative assessment of results predicted by HAM, HPM, and ADM. Khani *et al*. [19] used Adomian decomposition method (ADM) to provide series solution to fin problem with a temperature-dependent thermal conductivity while Moitsheki *et al.* [20] applied the Lie symmetry analysis to provide exact solutions of the fin problem with a power-law temperature-dependent thermal conductivity while Hosseini *et al.* [21] applied homotopy analysis method to generate approximate but accurate solution of heat transfer in fin with temperature-dependent internal heat generation and thermal conductivity. The application of differential transform method (DTM) to solve differential equations without linearization, discretization or no approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives and computation of derivatives symbolically. This method was applied by Joneidi *et al.* [22], Moradi and Ahmadikia [23, 24] and the method was also used by Mosayebidorcheh *et al.* [25], Ghasemi *et al.* [26], Ganji and Dogonchi [27] to solve the fin problem. However, the search for the arbitrary value that will satisfy the second boundary condition necessitated the use of Maple or Mathematica software and such could result in additional computational cost in the generation of solution to the problem. This drawback is not only peculiar to DTM, other approximate analytical methods such as HPM, HAM, ADM and VIM also required additional computational cost and time for the determination of such auxiliary parameters in their procedures of implementation. Also, most of the approximate analytical methods give accurate predictions only when the nonlinearities are weak, they fail to predict accurately for strong nonlinear models. Also, the methods often involved complex mathematical analysis leading to analytic expression involving a large number terms and when such methods as HPM, HAM, ADM and VIM are routinely implemented, they can sometimes lead to erroneous results as observed by Fernandez [28], Aziz and Bouaziz [29]. In practice, approximate analytical solutions with large number of terms are not convenient for use by designers and engineers. Although, some numerical methods such as finite difference method (FDM) and finite element methods (FEM) have been adopted to analyze heat transfer in fins, the FDM and FEM do not enforce the conservation principle in its original form as does the finite volume method (FVM). However, there are limited studies in literatures on the applications of finite volume methods for heat transfer analysis in fin. The inherent advantages, wide range of applications and high level of accuracy of the method justify the consideration of the method for the problem under consideration. From industrial point of view, finite volume method is known as robust and cheap method of discretization of conservation laws. It is preferable to other numerical methods because it enforces conservation on each cell, and thus ensures that both local and global conservation are guaranteed no matter how coarse the mesh. Also, its preference to other numerical methods is as a result of the fact that boundary conditions can be applied non-invasively. This is true because the values of the conserved variables are located within the volume element and not at the nodes or surfaces. FVM is geometrically flexible like finite element method, it enjoys an advantage in memory use and speed for large problems, higher speed flows and source term dominated flows. It is especially powerful on coarse non-uniform grids and in calculations where the mesh moves to track interfaces or shocks. It can handle Neumann boundary condition as readily as the Dirichlet boundary condition. Hence, in this work, finite volume method was applied to analyze heat transfer in a longitudinal rectangular fin with temperature-dependent thermal properties and internal heat generation.

## 2. Problem Formulation

Consider a straight fin of length $L$ that is exposed on both faces to a convective environment at temperature, $T_\infty$ and with heat transfer co-efficient, $h$ and internal heat generation, $q$ shown in Fig.1. Assuming that the heat flow in the fin and its temperatures remain constant with time, the temperature of the medium surrounding the fin is uniform, the fin base temperature is uniform., there is no contact resistance where the base of the fin joins the prime surface, the fin thickness is small compared with its height and length, so that temperature gradients across the fin thickness and heat transfer from the edges of the fin may be neglected. The dimension $x$ pertains to the *height coordinate* which has its origin at the fin baseand has a positive orientation from fin base to fin tip, it could therefore be stated that the

Rate of heat conduction into the element at $x$ = Rate of heat conduction from the element at $x+dx$
+ Rate of heat convection from the element
+ Rate of heat internal generation in the element (1)

Mathematically, the thermal energy balance could be expressed as shown in eq. (2)

$$q_x = q_{x+dx} + q_{conv.} + q_{\text{int.}} \tag{2}$$

i.e.

$$q_x - q_{x+dx} = q_{conv.} + q_{\text{int.}} \tag{3}$$

$$q_x - \left( q_x + \frac{\delta q}{\delta x} dx \right) = hP(T - T_c)dx + q_{\text{int.}}(T)dx \tag{4}$$

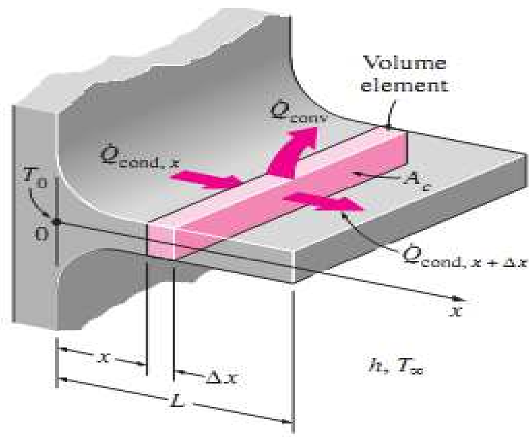



Fig.1 The geometry of straight rectangular convecting fin

As $dx \to 0$, eq. (4) reduces to Eq. (5)

$$-\frac{dq}{dx} = hP(T - T_c) + q_{\text{int.}}(T) \tag{5}$$

From Fourier's law of heat conduction

$$q = -k(T)A_{cr}\frac{dT}{dx} \tag{6}$$

Substituting eq. (6) into eq. (5), eq. (7) was obtained

$$\frac{d}{dx}\left(k(T)A_{cr}\frac{dT}{dx}\right)=hP(T-T_c)+q(T) \tag{7}$$

Further simplification of eq. (7) gives the governing differential equation for the fin as given by

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-\frac{h}{A_{cr}}P(T-T_\infty)+q_{\mathrm{int.}}(T)=0 \tag{8}$$

where the boundary conditions are

$$\begin{aligned}&x=0,\quad T=T_o\\&x=L,\quad \frac{dT}{dx}=0\end{aligned} \tag{9}$$

For many engineering applications, the thermal conductivity and the coefficient of heat transfer are temperature-dependent. Therefore, the temperature-dependent thermal properties and internal heat generation are given respectively by

$$k\left(T\right)=k_\infty[1+\lambda(T-T_\infty)] \tag{10}$$

and

$$q_{\mathrm{int}}\left(T\right)=q_\infty[1+\psi(T-T_\infty)] \tag{11}$$

Substituting eqs. (10) and (11) into equation (8), we arrived at

$$\frac{d}{dx}\left[k_\infty[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right]-\frac{hP(T-T_\infty)}{A_c}+q_\infty[1+\psi(T-T_\infty)]=0 \tag{12}$$

Introducing the following dimensionless parameters into eq. (12) viz;

$$\begin{aligned}&X=\frac{x}{L},\quad \theta=\frac{T-T_\infty}{T_o-T_\infty},\quad K=\frac{k}{k_\infty},\quad M^2=\frac{Ph_oL^2}{A_ck_\infty},\\&Q=\frac{q_\infty A_c}{h_oP(T_o-T_\infty)},\quad \gamma=\psi(T_o-T_\infty),\quad \beta=\lambda(T_o-T_\infty)\end{aligned} \tag{13}$$

The dimensionless governing differential eq. (14) and the boundary conditions were arrived at

$$\frac{d}{dX}\left[(K(\theta)]\frac{d\theta}{dX}\right]-M^2\theta+M^2Q(1+\gamma\theta)=0 \tag{14}$$

The boundary conditions are

$$\begin{aligned}&X=0,\quad \theta=1\\&X=1,\quad \frac{d\theta}{dX}=0\end{aligned} \tag{15}$$

where

$$K(\theta)=1+\beta\theta$$

## 3. Method of Solution

The above non-linear Eq. (14) does permit the generation of any closed form solution. Therefore, recourse has to be made to either approximation analytical method, semi-numerical method or numerical method of solution. In this work, finite volume method is used.The method divides the domain into a finite number of non-overlapping cells or control volumes (Fig. 2) over which conservation of function (the dependent variable) is enforced in a discrete sense. It is possible to start the discretization process with a direct statement of conservation on the control volume. Integrate the governing equation over the control volume. The finite volume formulation of the fin equation is

$$\int_w^e \left[ \frac{d}{dX}[K(\theta)] - M^2\theta + M^2 Q + M^2 Q\gamma\theta \right] dV = 0 \tag{16}$$

Since $V=AdX$, e q. (16) could be expressed as

$$\int_\omega^e \frac{d}{dX}\left[ K(\theta)\frac{d\theta}{dX}\right] AdX - M^2(1-QY)\int_\omega^e \theta AdX + M^2 Q \int_\omega^e AdX = 0 \tag{17}$$

Since the area, A of the fin is constant, eq. (17) reduces to

$$\int_\omega^e \frac{d}{dX}\left[ K(\theta)\frac{d\theta}{dX}\right] dX - M^2(1-QY)\int_\omega^e \theta dX + M^2 Q \int_\omega^e dX = 0 \tag{18}$$

In order to derive the discretization equation, the grid point cluster in Fig. 2 is used. Point P represents the main point, where the temperature is to be determined. The east and the west neighbours of the main grid point are represented by E and W, respectively. The narrow line depicts the face of the control volume while letter e and w denote these faces. The thickness in the y and z direction is assumed as unity for the one-dimensional problem under consideration. This implies that the temperature variation is in x-direction only and grid points are uniformly distributed in the direction only.

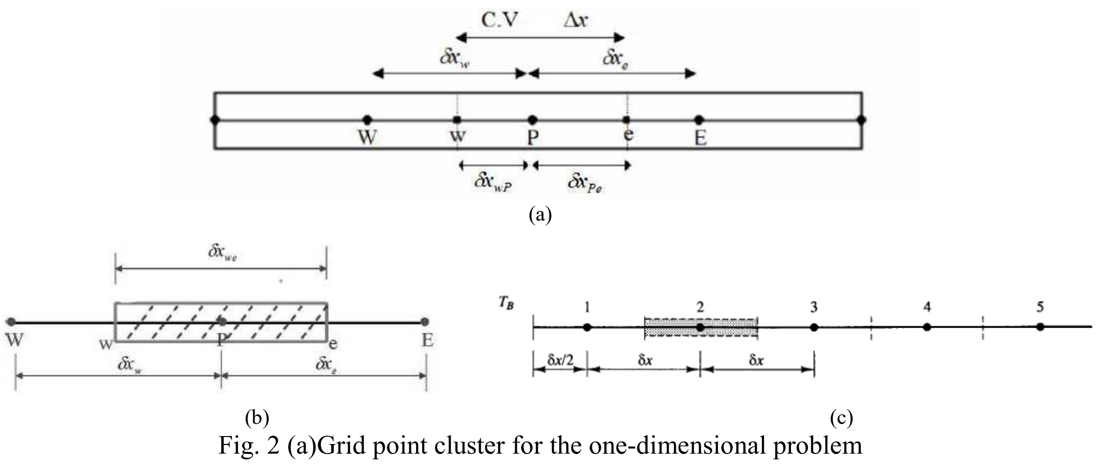



Fig. 2 (a)Grid point cluster for the one-dimensional problem
(b) Arrangement of control volumes (c) The grid used for the one-dimensional problem

Hence, the integration of eq. (18) gives eq. (19)

$$\left[K(\theta)\frac{d\theta}{dX}\right]_e - \left[K(\theta)\frac{d\theta}{dX}\right]_\omega - M^2(1-QY)\theta dX + M^2 Q dX = 0 \tag{19}$$

The finite volume discretization of eq. (19) for the first nodal point in Fig. 2 is expressed as

$$[K(\theta)]_e\left(\frac{\theta_E-\theta_P}{\delta X}\right)-[K(\theta)]_W\left(\frac{\theta_P-\theta_B}{\frac{\delta X}{2}}\right)-M^2(1-QY)\theta_P\delta X+M^2Q\delta X=0 \tag{20}$$

Collecting the like terms, we arrived at

$$\left[\frac{[K(\theta)]_e}{\delta X}+\frac{2[K(\theta)]_B}{\delta X}+M^2(1-Q\gamma)dX\right]\theta_P=\frac{[K(\theta)]_e}{\delta X}\theta_E+\frac{2[K(\theta)]_B}{\delta X}\theta_B+M^2Q\delta X \tag{21}$$

where

$$[K(\theta)]_e=1+\beta\theta_E$$

$$[K(\theta)]_B=1+\beta\theta_B$$

$$[K(\theta)]_w=1+\beta\theta_W$$

Thus, eq. (21) becomes

$$\left[\frac{(1+\beta\theta_E)}{\delta X}+\frac{2(1+\beta\theta_W)}{\delta X}+M^2(1-QY)(\delta X)^2\right]\theta_P=\frac{(1+\beta\theta_E)}{\delta X}\theta_E+\frac{2(1+\beta\theta_W)}{\delta X}\theta_B+M^2QdX \tag{22}$$

Using the nodal number shown in Fig. 2

$$\left[(1+\beta\theta_2)+2(1+\beta\theta_B)+M^2(1-QY)(\delta X)^2\right]\theta_1-(1+\beta\theta_2)\theta_2=2(1+\beta\theta_2)\theta_B+M^2Q(\delta X)^2 \tag{23}$$

For the middle nodal points

$$\int_\omega^e\frac{d}{dX}\left[K(\theta)\frac{d\theta}{dX}\right]dV-M^2(1-QY)\int_\omega^e\theta dX+M^2Q\int_\omega^e\theta dX=0 \tag{24}$$

Similarly;

$$\left[K(\theta)\frac{d\theta}{dX}\right]_e-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2(1-QY)\theta_PdX+M^2QdX=0 \tag{25}$$

The finite volume discretization of eq. (18) for the middle nodal points is Fig. 2 is expressed as

$$[K(\theta)]_e\left[\frac{\theta_E-\theta_P}{\delta X}\right]-[K(\theta)]_w\left[\frac{\theta_P-\theta_W}{\delta X}\right]-M^2(1-QY)\theta_P\delta X+M^2Q\delta x=0 \tag{26}$$

Eq. (26) could be further expressed as

$$\frac{(1+\beta\theta_E)(\theta_E-\theta_P)}{\delta X}-\frac{(1+\beta\theta_W)(\theta_P-\theta_W)}{\delta X}-M^2(1-QY)\theta_P\delta X+M^2Q\delta X=0 \tag{27}$$

Collection of like terms in eq. (26) leads to

$$\left[(1+\beta\theta_E)+(1+\beta\theta_W)+M^2(1-QY)(\delta X)^2\right]\theta_P-(1+\beta\theta_E)\theta_E-(1+\beta\theta_W)\theta_W=M^2Q(\delta X)^2 \tag{28}$$

Using the interior nodal numbers

$$\left[(1+\beta\theta_{n+1})+(1+\beta\theta_{n-1})+M^2(1-QY)(\delta X)^2\right]\theta_n-(1+\beta\theta_{n+1})\theta_{n+1}-(1+\beta\theta_{n-1})\theta_{n-1}=M^2Q(\delta X)^2 \tag{29}$$

where *n = 2, 3, 4,…, N-1, N is the number of number points on the one-dimensional mesh*

for the last node (N), it is analyzed as follows

$$\int_\omega^e \frac{d}{dX}\left[K(\theta)\frac{d\theta}{dX}\right]-M^2(1-QY)\int_\omega^e \theta dX+M^2Q\int_\omega^e dX=0 \tag{30}$$

The integration eq. (30) as before, gives

$$\left[K(\theta)\frac{d\theta}{dX}\right]_e-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2(1-QY)\,\theta_P dX+M^2Qdx=0 \tag{31}$$

Using the second boundary condition $x=1,\quad \frac{d\theta}{dX}=0$, we have;

$$\left[K(\theta)\frac{d\theta}{dX}\right]_e-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2(1-QY)\,\theta_P dX+M^2Qdx=0 \tag{32a}$$

From the boundary condition, $\left[K(\theta)\frac{d\theta}{dX}\right]_e=0$

Therefore, eq. (32a) becomes

$$0-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2(1-QY)\,\theta_P dX+M^2Qdx=0 \tag{32b}$$

Applying the second boundary condition to eq. (31) as shown in eq. (32) and after finite volume discretization, eq. (31) reduces to

$$\frac{-(1+\beta\theta_w)(\theta_P-\theta_W)}{(\delta X/2)}-M^2(1-QY)\theta_P\delta X+M^2Q\delta x=0 \tag{33}$$

Rearranging the terms in eq. (33) leads to eq. (34)

$$\left[2(1+\beta\theta_w)+M^2(1-QY)(\delta X)^2\right]\theta_P-2(1+\beta\theta_w)\theta_w=M^2Q(\delta X)^2 \tag{34}$$

Thus, for the last node, we have

$$\left[2(1+\beta\theta_{N-1})+M^2(1-QY)(\delta X)^2\right]\theta_N-2(1+\beta\theta_{N-1})\theta_{N-1}=M^2Q(\delta X)^2 \tag{35}$$

The non-linear systems of eq. (23), all the equations in eq. (29) and eq. (35) are solved with the aid of MATLAB using ***fsolve***.

## 4. Fin Parameter for Thermal Performance Indication

The performance indication parameter for the fin such as the efficiency of the fin is analyzed.

### 4.1 Fin efficiency

The amount of heat dissipated from the entire fin is found by using Newton's law of cooling as

$$Q_f = \int_0^1 Ph(T - T_\infty)dX \tag{36}$$

Also, the maximum heat dissipated is obtained if the fin base temperature is kept constant throughout the fin i.e.

$$Q_{\max} = Ph_b L(T_b - T_\infty) \tag{37}$$

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature and is given by

$$\eta = \frac{Q_f}{Q_{\max}} = \frac{\int_0^L Ph(T - T_\infty)dx}{Ph_b L(T_b - T_\infty)} \tag{38}$$

Therefore, the fin efficiency in dimensionless variables is given by

$$\eta = \int_0^1 \theta \, dX \tag{39}$$

It is very important to point out that the thermo-geometric parameter or the fin performance factor, M could be written in terms of Biot number, Bi and the aspect ratio, $a_r$ as shown in eq. (40).

$$M^2 = \frac{Ph_b L^2}{A_c k_a} = \frac{(2L)h_b L^2}{(L\delta)k_a} = \frac{2h_b \delta L^2}{\delta^2 k_a} = \frac{2h_b \delta}{k_a}\left(\frac{L}{\delta}\right)^2 = 2Bia_r^2$$

$$\text{Where} \quad Bi = \frac{h_b \delta}{k_a}, \; a_r = \frac{L}{\delta} \tag{40}$$

From equation (40), it implies that $M = a_r \sqrt{2Bi}$

## 5. Results and Discussion

Figs. 3-8 show that the dimensionless temperature distribution falls monotonically along fin length for all various thermogeometric, thermal conductivity and convective heat transfer parameters. For larger values of the thermogeometric parameter *M*, the more the heat convected from the fin through its length and the more thermal energy is efficiently transferred into environment through the fin length. In the situation of negligible heat loss from the fin tip (insulated tip) to the environment, the fin temperature decreases along the fin length also, and the temperature decreasing rate is the same around fin base area.

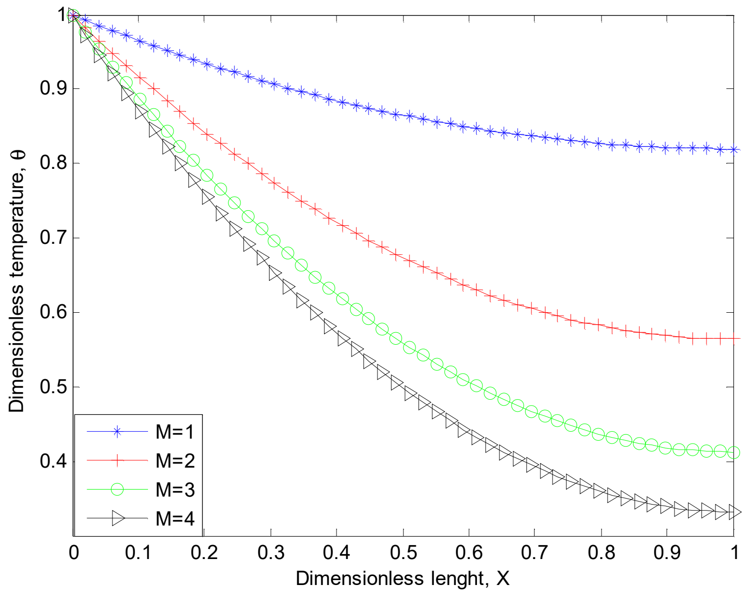



Fig.3 Dimensionless temperature distribution in the fin when β=0.5, Q=0.4, γ=0.6

Figs. 3 show the variation of dimensionless temperature with dimensionless length and also the effect of the thermogeometric parameter on the straight fin with an insulated tip. From the figure, as the thermogeometric parameter increases, the rate of heat transfer (the convective heat transfer) through the fin increases as the temperature in the fin drops faster (becomes steeper which reflects high base heat flow rates) as depicted in the figure. It can be inferred from the results that the ratio of convective heat transfer to conductive heat transfer at the base of the fin ($h_b/k_b$) has much effect on the temperature distribution, rate of heat transfer at the base of the fin, efficiency and effectiveness of the fin. As $h_b$ increases (or $k_b$ decreases), the ratio $h_b/k_b$ increases at the base of the fin and consequently the temperature along the fin, especially at the tip of the fin decreases i.e. the tip end temperature decreases as M increases. The profile has steepest temperature gradient at $M=1.0$, but its much higher value gotten from the lower value of thermal conductivity than the other values of $M$ in the profiles produces a lower heat-transfer rate. This shows that the thermal performance or efficiency of the fin is favoured at low values of thermogeometric parameter since the aim (high effective use of the fin) is to minimize the temperature decrease along the fin length, where the best possible scenario is when $T=T_b$ everywhere.

Fig.4 shows the variation of dimensionless temperature with dimensionless length in longitudinal convecting fin with insulated tip. The effects of thermogeometric and thermal conductivity parameters on the dimensionless temperature distribution and consequently, on the rate of heat transfer are shown. From the figure, it is obvious that as the thermogeometric parameter increases, the rate of heat transfer through the fin increases. This is because as the fin convective heat transfer increases, more heat is transferred by conduction through the fin thereby increases the temperature distribution in the fin and consequently the rate of heat transfer.

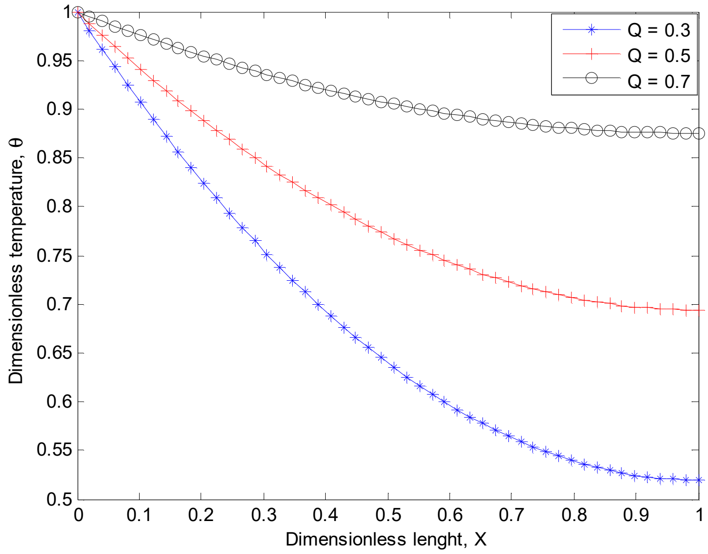



Fig.5 Dimensionless temperature distribution in the fin parameters when β=0.2, M=2, γ=0.2

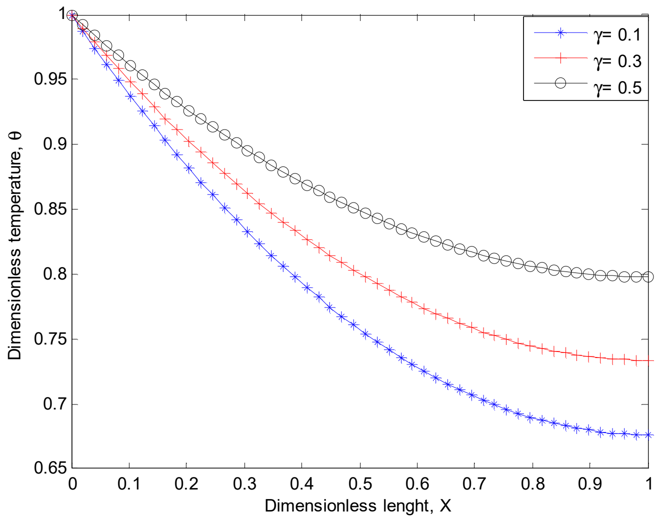



Fig.6 Dimensionless temperature distribution in the fin parameters when M=2, β=0.3, Q=0..3

The effects of internal heat generation parameter on the temperature distribution are depicted in Fig. 5 while Fig. 6 shows the effects of internal heat generation on the fin thermal performance at different thermogeometric parameters. From the figures, as the internal heat generation parameter increases the temperature gradient of the fins decreases. This is because, as the rate of internal heat generation within fin increase, the thermal performance of the fin decreases. However, the figures show that the dimensionless temperature gradient of the fin length increases as the thermogeometric parameter increases.

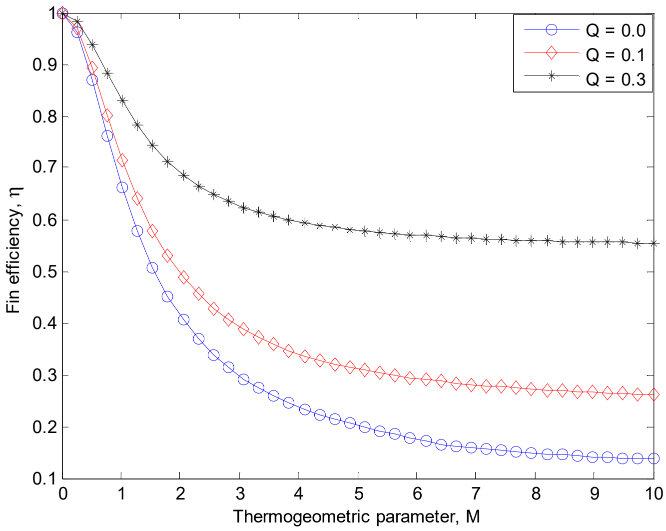
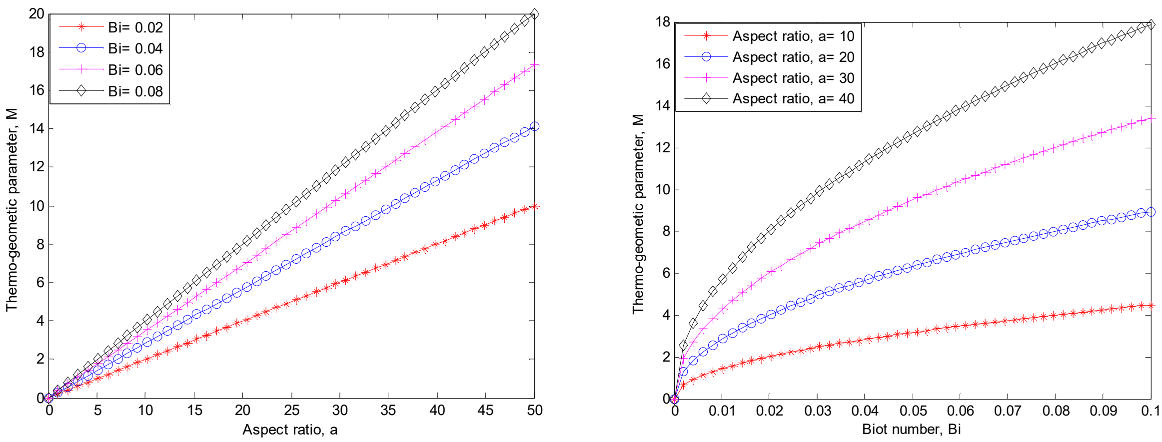



Fig.7 Dimensionless temperature distribution in the fin parameters when β=0.1, γ=0.8

Also, Fig.7 illustrates the effects of internal heat generation and aspect ratio on the effectiveness of the fin for the temperature dependent thermal conductivity and heat transfer coefficient. From the figures, it could be seen that as the rate of internal heat generation increases and aspect ratio increases (in case of effectiveness of the fin), higher local temperature is produced in the fin, thereby increases the efficiency and the effectiveness of the fin.Also, from the results, it shows that high efficiency and effectiveness of fin could be achieved by using small values of thermogeometric parameter and this could be realized using a fin of small length or by using a material of better thermal conductivity.




(a) (b)

Fig. 8 Effects of Biot number on the thermos-geometric parameter of the fin

The effects of Biot number and aspect ratio on the thermo-geometric parameter (the fin performance factor) are shown in Figs. 8. From the results, the fin performance factor increases as the aspect ratio and Biot number increase. However, the thermal performance or efficiency of the fin is favoured at low values of thermogeometric parameter since the aim (high effective use of the fin) is to minimize the temperature decrease along the fin length, where the best possible scenario is when $T=T_b$ everywhere. It must be pointed out that equation (40) shows the direct relationship between the thermo-geometric parameter, M and the Biot number, Bi which directly depends on the fin length. A small value of M corresponds to a relatively short and thick fin of poor thermal conductivity and a high value of M implies a long fin or fin with low value of thermal conductivity. Since, the thermal performance or efficiency of the fin is favoured at low values of thermo-geometric fin parameter, very long fins are to be avoided in practice. A compromise is reached for one-dimensional analysis of fins *0 < Bi <0.1*. When the Biot number is greater than 0.1, two dimensional analysis of the fin is recommended as one-dimensional analysis predicts unreliable results for such limit.

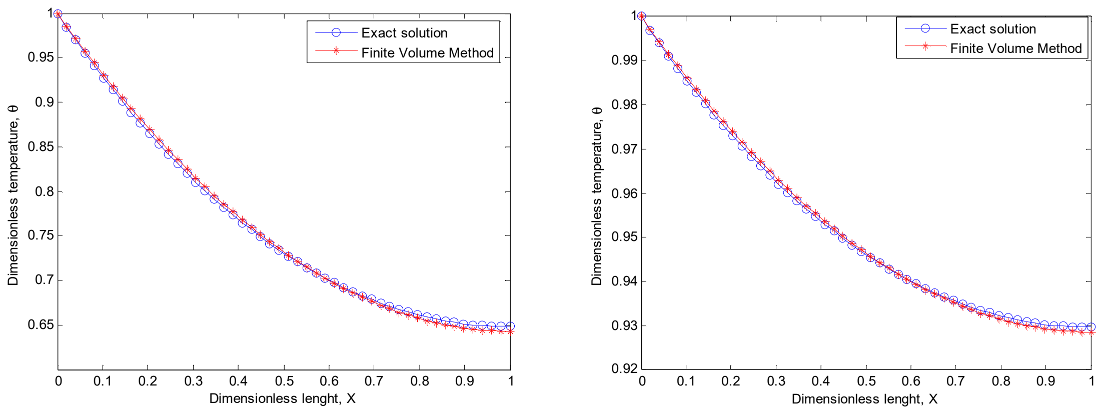



Fig.9 Dimensionless temperature distribution in the fin parameters when (a) M=1, β=0, Q=0 (b) M=1.5, Q=0.8

The finite volume method of solution was validated by the exact solution in Figs. 9a and 9b for the linear thermal model of the fin. This gives the confidence in the predicted results by the finite volume method for the non-linear problems in which no closed-form solution is difficult or impossible to obtain.

## Conclusion

In this work, steady state heat transfer in a longitudinal rectangular fin with temperature-dependent thermal properties and internal heat generation has been analyzed by using finite volume method. The solution was validated by the exact solution for the linear problem. The developed heat transfer models were used to investigate the effects of thermo-geometric, coefficient of heat transfer and thermal conductivity (non-linear) parameters on the temperature distribution, heat transfer and thermal performance of the longitudinal rectangular fin. These results serve as basis for comparison of any other method of analysis of the problem and they also provide platform for improvement in the design of fin in heat transfer equipments such as air–land–space vehicles and their power sources, in chemical, refrigeration, and cryogenic processes, in electrical and electronic equipment, in conventional furnaces and gas turbines, in the design of firebox for the generation of steam power from fossil fuels. In process heat dissipators and waste heat boilers, and in nuclear-fuel modules, steam power plants, automobiles radiators etc.

## Nomenclature

$a_r$ aspect ratio
A cross sectional area of the fins, $m^2$
Bi Biot number
h heat transfer coefficient, $Wm^{-2}k^{-1}$
$h_b$ heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
H dimensionless heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
j geometric parameter
k thermal conductivity of the fin material, $Wm^{-1}k^{-1}$
$k_b$ thermal conductivity of the fin material at the base of the fin, $Wm^{-1}k^{-1}$
K dimensionless thermal conductivity of the fin material, $Wm^{-1}k^{-1}$
L Length of the fin, m
M dimensionless thermo-geometric fin parameter
$m^2$ thermo-geometric fin parameter $m^{-1}$

n convective heat transfer power
P perimeter of the fin, m
T Temperature, K
$T_\infty$ ambient temperature, K
$T_b$ Temperature at the base of the fin, K
x fin axial distance, m
X dimensionless length of the fin
Q dimensionless heat transfer
$q_i$ the uniform internal heat generation in W/m$^3$

**Greek Symbols**
β thermal conductivity parameter or non-linear parameter
δ thickness of the fin, m
$\delta_b$ fin thickness at its base.
γ dimensionless internal heat generation parameter
θ dimensionless temperature
$\theta_b$ dimensionless temperature at the base of the fin
η efficiency of the fin
ε effectiveness of the fin

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