OPTIMIZATION OF HEAT SINK FIN GEOMETRIES FOR HEAT SINKS IN NATURAL CONVECTION

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Abstract

Computer analysis of heat sink thermal performance for a specified configuration is commonly applied, typically requiring iterative application of the analysis program for various possible configurations to discern performance trends and optimum design conditions. Incorporation of minimization routines into the analysis eliminates the manual, iterative design analysis and identifies the optimum design. Optimization of fin geometry for heat sinks in natural convection with rectangular cross section fins at a constant fin spacing was implemented for steady state and intermittent duty cycle operation. The method of implementation and results are discussed.

Nomenclature

- a -fin/channel aspect ratio, L/S
- Gr_x -Grashof number evaluated for dimension x
- H -height of fin channel
- L -fin length
- Nu_x -Nusselt number
- r -fin channel characteristic length, 2LS/(2L+S)
- Pr -Prandtl number
- Ra^* -modified Rayleigh number, $(r/H)Gr_rPr$
- S -space between adjacent fins
- V -a constant, -11.8 ($inch^{-1}$)
- ψ -fin channel configuration factor

Introduction

Heat sink design goals may vary, but the most frequent concern is minimization of the heat sink temperature. Electronic circuit flow considerations may determine the location of heat sources and many dimensions may be constrained by product size requirements. The fin array geometry is frequently one of the few parameters completely at the thermal system engineer's discretion. Usually the fins take the form of rectangular cross section plates extending from a heat spreader, or backplane, at regular intervals.

Studies by Elenbass reported an expression for the optimum spacing for natural convection between parallel plates [1] as:

$$S = (50HGr_sPr)^{.25} \qquad (1)$$

However the U shaped fin channel is only roughly approximated by infinite, parallel, vertical walls. Additionally, radiant heat transfer can typically account for 20 to 30% of the heat being transferred [2]. Mathematical correlations are available for the U shaped fin channel, but when incorporating the temperature dependent air properties and considering the nonlinearity of thermal radiation the heat transfer is no longer a simple, differentiable mathematical expression.

A heat sink analysis program uses numerical methods to predict temperature distributions on the heat sink for the user supplied heat sink geometry, heat sink material, and heat sources. The heat sink design process requires the iterative application of the program to determine performance trends and the optimum design.

Incorporation of minimization routines into the analysis potentially eliminates iterative design analysis, pinpointing the optimum design. A "down hill simplex method in multidimensions" due to Nelder and Mead [3] for function minimization was selected as an optimization scheme to find the fin array geometry which minimizes the average heat sink temperature for steady state and intermittent duty cycle operation.

Heat Sink Thermal Analysis

Typically finite difference or finite element analysis approaches are used to predict heat sink temperature distributions. Initially a more simplified approach was taken, to assume that the whole heat sink is at a uniform temperature and optimize the heat sink fin configuration based on the predicted, average heat sink temperature. It was anticipated that when the approach was made functional, a more sophisticated numerical analysis could be incorporated, however good correlation was found when compared with a finite difference model. The Biot number, the ratio of the convection thermal resistance to the conduction thermal resistance of the body, is very small for practical heat sink configurations, typically less than .01. For Biot numbers less than .1 the error in assuming a uniform temperature is less than 5% [4], so this lumped parameter model simulates the heat convection and radiation quite well for highly conductive, metal heat sinks.

Convection heat transfer from the fin array channel is sometimes modeled as convection from infinite, parallel, isothermal, vertical walls. This may be sufficient for extremely long fins but for shorter fins the effects of the heat sink backplane surface on the fluid flow and heat transfer become more significant. A better correlation was proposed by Van de Pol and Tierny [5], which models the U shaped cross section of a pair of fins mounted to a common backplane as:

$$Nu_r = \frac{Ra^*}{\psi}\{1-\exp[-\psi(2Ra^*)^{-.75}]\} \qquad (2)$$

where:

$$\psi = \frac{24[1-0.483e^{-0.17/a}]}{\{(1+a/2)[1+(1-e^{-0.83/a})(9.14\ a^{.5}\ e^{V}-0.61)]\}^3} \qquad (3)$$

which is valid for the limiting case of infinitely long fins and converges to the solution for a vertical plate for fins of zero length. Although the correlation was developed for isothermal surfaces, with fin efficiencies for most cast or extruded aluminum heat sinks approaching 99% efficient, there is negligible error for most applications. For non-isothermal fins the thermal conductance from the fin was reduced by the fin efficiency as a simple approximation.

Temperature dependent physical properties of air can be approximated by least squares fit polynomial functions. A linearized approximation of a radiation heat transfer using gray body analysis provided an effective radiation heat transfer coefficient.

Optimization Approach

An optimum fin configuration can be anticipated by considering the warmed air rising along a hot vertical wall. A layer of warmed air is built up along the surface, tending to insulate the hot surface from the cool ambient air. For a given wall height, a decrease in the wall spacing would cause these boundary layers to interfere. Intuitively one would expect greater interference to eventually become detrimental to fluid flow and heat transfer within the fin channel. On the other hand, for a given heat sink width increasing the fin spacing one can fit fewer fins on the heat sink. So a trade off is expected between increasing the number of fins by making them thinner and more closely spaced versus decreased natural convection between fins too closely spaced.

The downhill simplex method was selected for determining the optimum fin geometry. This method requires only function evaluations, not derivatives and can be applied to optimization of multiple dimensions simultaneously. An added advantage is that it may be used to find the local optimum value, i.e. a relative minima, of a function. Although this algorithm is not considered fast or efficient, it is easily implemented.

Three problems occur in applying this optimization scheme to a heat sink. One is that the heat sink temperature becomes a discontinuous function of the fin geometry. Take for example the case of increasing the fin spacing while holding the fin thickness and length constant. As one increases the fin spacing, the heat transfer might improve and lower the heat sink temperature until the heat sink backplane is full of fins. Any increase in fin spacing at this point will force a fin to drop of the heat sink and produce a discontinuity as shown in Figure 1. This presents a problem for the downhill simplex minimization scheme as the simplex may reflect over the valleys of discontinuities that represent the true optimum case.

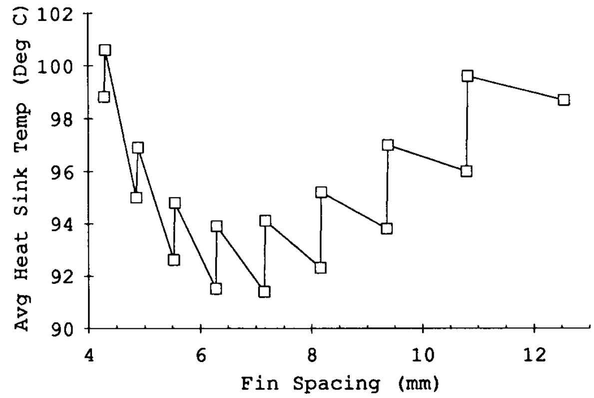

Figure 1 Average Heat Sink Temperature vs Fin Spacing with 5 mm Thick Fin for Steady State Performance

To deal with the known discontinuities, the predicted temperature is checked for an integer number of fins on either side of the proposed solution. If a lower temperature is discovered, this new point is used as a new vertex for the simplex and the optimization is continued.

Because of the high conductivity of the fin materials used, the fin efficiency of typical fins is quite large, typically over 90%. The resulting optimum fin thicknesses are very thin as shown in Figure 2 for a typical steady state heat sink. As interesting as this may be from a theoretical view point, it may not be a practical design. Therefore it was desired to bound the fin geometries by practical ranges a priori and determine the optimum configuration within this specified range. Reflection of the simplex out of the bounded range was prevented by allowing simplex motion in the proposed vectorial direction but the magnitude of motion was limited to prevent stepping outside the specified range.

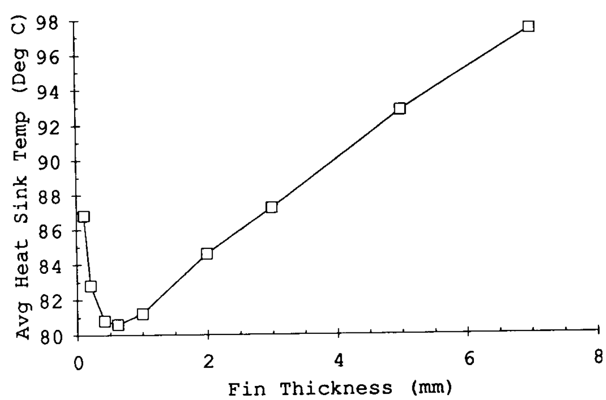

Figure 2 Average Heat Sink Temperature vs Fin Thickness for Optimal Fin Spacing and Steady State Performance (discontinuities removed for clarity)

Finally, Nelder and Mead observed that the simplex could find a relative minima and settle on this solution before finding the true minimum value at some other region of the available fin configurations. To prevent this, the minimization process is restarted from other points in the range of potential configurations to force the simplex to traverse the whole range. This process is sped up by using the minimum point found from one minimization as one of the simplex vertices for the subsequent minimizations.

A final solution is usually identified with five or six minimizations for a total of about 200 trials in a real time of only a second or two. The optimization procedure consistently identified the same optimum point as was found by manually iterating a heat sink analysis program.

DISCUSSION OF RESULTS

Figure 3 depicts the average heat sink temperature as a function of fin thickness and spacing for a typical steady state application. This data was produced by iteratively analyzing the heat sink for the various configurations using a conventional heat sink analysis program, much as the designer might. As previously noted, the discontinuities of the graph are due to the discrete changes of heat sink convection area that occur as another fin is added or deleted from the heat sink.

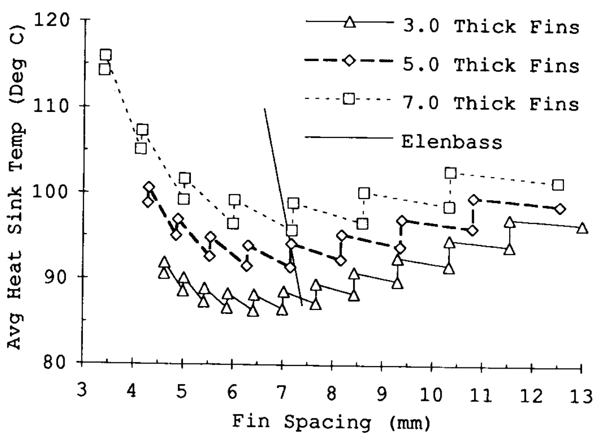

Figure 3 Average Heat Sink Temperature vs Fin Thickness and Spacing for Steady State Performance

An analysis of fin length is consistent with the intuitive answer, that the longer the fin the better, but there are diminishing returns for successively longer and longer fins. Since increasing fins beyond some nominal length, decreases manufacturability and thereby increases cost, it should not be permitted to increase without bound.

For a typical EIA (Electronics Industry Association) intermittent duty cycle [6] analysis, Figure 4 shows how fin spacing and thickness vary for the specified configuration. It appears that similar trends govern the intermittent duty cycle heat sink as the steady state heat sink. That is, that thinner fins appear better and an optimum spacing does exist. However further investigation reveals more significant trends.

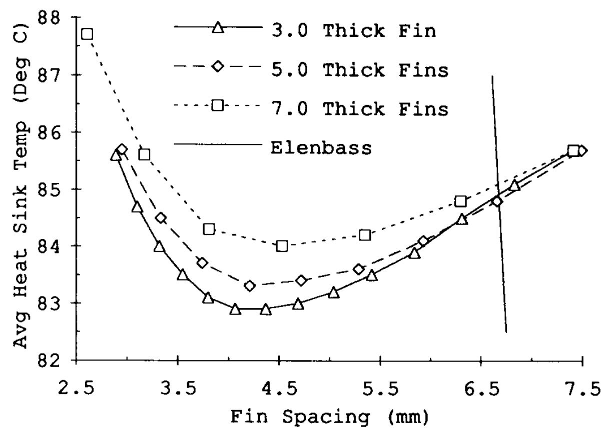

Figure 4 Average Heat Sink Temperature vs Fin Thickness and Fin Spacing for EIA Duty Cycle Performance (discontinuities removed for clarity)

Figure 5 shows the optimum fin dimensions as a function of the heat sink backplane thickness and reveals that the optimum fin spacing tended to remain fairly constant, about 4.3 mm. As the optimum fin spacing is related to the interference of the boundary layers, it is not surprising that it is independent of backplane thickness. However the heat sink backplane thickness has a definite impact on the fin thickness. For an intermittent duty cycle, the thermal inertia, or a thermal "flywheel" effect, of the heat sink can play an important role in cooling the heat sink. Heat stored in the heat sink during high dissipation conditions can be dissipated by this thermal "flywheel" during low dissipation portions of the duty cycle for a lower peak temperature. When the heat sink backplane is thin, a thicker fin may be better as it increases the thermal inertia of the heat sink at the expense of increased fin area.

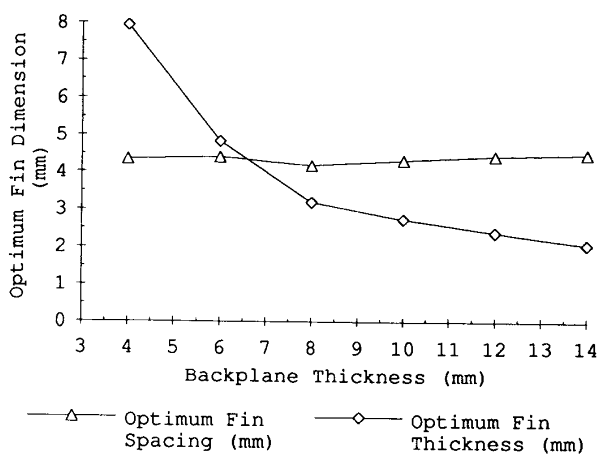

Figure 5 Optimum Fin Spacing and Thickness vs Heat Sink Backplane Thickness for EIA Duty Cycle Performance

While the optimum configuration tends to favor a larger backplane thickness, analytical modeling reveals that it has a diminishing return as well as being increasingly more difficult to fabricate.

CONCLUSIONS

An approach has been developed for determining the optimum fin configurations for a given heat sink in natural convection. The proposed algorithm allows the computer to hunt for the lowest temperature performance of a user specified range of fin geometries. The heat transfer analysis is based on methods used in conventional heat sink analysis programs, so the results correlate well.

There is opportunity for much future work. Since backplane thickness plays a significant role for intermittent duty heat sinks, it would be appropriate to also optimize the heat sink backplane thickness. The addition of finite difference schemes to predict temperatures at specific points on the heat sink could permit optimizing the heat sink for cooling at specific points. Variable fin spacing, heat sink backplane thickness, and fin thickness are all feasible and could be handled by this method, however their application may be limited. Perhaps the most potential lies in the opportunity to incorporate fabrication capability rules as the bounding limits of the optimization range. The resulting heat sink proposal would thus be the heat sink that performs at the lowest temperature within the practical range of fabrication process capability.

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