Control Systems Assignment-1

ES18BTECH11011

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1 Question 58

Control systems Engineering by Norman S. Nise. Chapter 2 question 58.
Q: In a magnetic levitation experiment a metallic object is held up in the air suspended under an electromagnet. The vertical displacement of the object can be described by the following nonlinear differential equation:

$$m\frac{d^2H}{dt^2} = mg - k\frac{I^2}{H^2}$$

here:
m = mass of object, g = gravity acceleration constant
k = positive constant
H = distance between electromagnet and object (output signal)
I = electromagnet current(input signal)

a. Show that a system's equilibrium will be achieved when: $H_o = I_o\sqrt{\frac{k}{mg}}$
Solution:
When the system is at equilibrium the second derivative of $H(x)$ is zero. That is: $\frac{d^2H}{dt^2} = 0$
It implies that the given equation and substituting the value of $\frac{d^2H}{dt^2} = 0$ in it will give:

$$mg - k\frac{I^2}{H^2} = 0$$

$$mg = k\frac{I^2}{H^2}$$

$$H^2 = k\frac{I^2}{mg}$$

$$H = I\sqrt{\frac{k}{mg}}$$

which intern gives H as H_o and I as I_o which are the equilibrium values of the system.So,

$$H_o = I_o\sqrt{\frac{k}{mg}}$$

b. Linearize the equation about the equilibrium point found in Part a and show that the resulting transfer function obtained from the linearized differential equation can be expressed as:

$$\frac{\delta H(s)}{\delta I(s)} = -\frac{a}{s^2 - b^2}$$

Solution:
Performing the linearization by defining the following terms as $\delta H = H(t) - H_o$ and $\delta I = I(t) - I_o$. Substituting the above values will give in the original equation will give us:

$$m\frac{d^2(H_o + \delta H}{dt^2} = mg - k\frac{(I_o + \delta I)^2}{(H_o + \delta H)^2}$$

let $\gamma = mg - k\frac{(I_o+\delta I)^2}{(H_o+\delta H)^2}$

Getting a first order taylor series approximation, which is calculate the below expression:

$$m\frac{d^2\delta H}{dt^2} = \left.\frac{\partial\gamma}{\partial\delta H}\right|_{\delta H=0,\delta l=0}\delta H + \left.\frac{\partial\gamma}{\partial\delta I}\right|_{\delta H=0,\delta I=0}\delta I$$

substituting γ in the above equation

$$m\frac{d^2\delta H}{dt^2} = \left.\frac{\partial}{\partial\delta H}(mg - k\frac{(I_o + \delta I)^2}{(H_o + \delta H)^2})\right|_{\delta H=0,\delta l=0}\delta H + \left.\frac{\partial}{\partial\delta I}(mg - k\frac{(I_o + \delta I)^2}{(H_o + \delta H)^2})\right|_{\delta H=0,\delta I=0}\delta I$$

After taking partial derivative on right hand side of the equation respectively:

$$m\frac{d^2\delta H}{dt^2} = k\left.\frac{2\,(I_0 + \delta I)^2\,(H_0 + \delta H)}{(H_0 + \delta H)^4}\right|_{\delta H=0,\delta I=0}\delta H - k\left.\frac{2\,(I_0 + \delta I)\,(H_0 + \delta H)^2}{(H_0 + \delta H)^4}\right|_{\delta H=0,\delta I=0}\delta I$$

Partial derivative of

$$\frac{\partial}{\partial\delta H}(mg) = 0$$

$$\frac{\partial}{\partial\delta I}(mg) = 0$$

$$\frac{\partial}{\partial \delta H}(-k\frac{(I_o+\delta I)^2}{(H_o+\delta H)^2}) = k\frac{2\left(I_0+\delta I\right)^2\left(H_0+\delta H\right)}{\left(H_0+\delta H\right)^4}$$

$$\frac{\partial}{\partial \delta I}(-k\frac{(I_o+\delta I)^2}{(H_o+\delta H)^2}) = -k\frac{2\left(I_0+\delta I\right)\left(H_0+\delta H\right)^2}{\left(H_0+\delta H\right)^4}$$

Similar to partial derivative of $\frac{u}{v}$ which is $\frac{vdu-udv}{v^2}$

After substituting $\delta H = 0$ and $\delta I = 0$ in the above differential we get:

$$\frac{d^2\delta H}{dt^2} = \frac{2kI_0^2}{mH_0^3}\delta H - \frac{2kI_0}{mH_0^2}\delta I$$

Laplace transform on the above equation transfer function can be obtained:

$$\delta H(s)s^2 = \frac{2kI_o}{mH_o^3}\delta H(s) - \frac{2kI_o}{mH_o^2}\delta I(s)$$

$$\delta H(s)(s^2 - \frac{2kI_o}{mH_o^3}) = -\frac{2kI_o}{mH_o^2}\delta I(s)$$

$$\frac{\delta H(s)}{\delta I(s)} = -\frac{\frac{2kI_o}{mH_o^2}}{s^2 - \frac{2kI_o}{mH_o^3}}$$

The given equation can be expressed in the form of $\frac{\delta H(s)}{\delta I(s)} = -\frac{a}{s^2-b^2}$ at equibilrium point byleariztion.
Here $a = \frac{2kI_o}{mH_o^2}$ and $b = \sqrt{\frac{2kI_o}{mH_o^3}}$