Week 1: Newton's Laws

Summary

- Physics is a *language* – in particular the language of a certain kind of *worldview*. For philosophically inclined students who wish to read more deeply on this, I include links to terms that provide background for this point of view.

  - Wikipedia: http://www.wikipedia.org/wiki/Worldview
  - Wikipedia: http://www.wikipedia.org/wiki/Semantics
  - Wikipedia: http://www.wikipedia.org/wiki/Ontology

  Mathematics is the natural language and logical language of physics, not for any particularly deep reason but because it *works*. The components of the semantic language of physics are thus generally expressed as mathematical or logical *coordinates*, and the semantic expressions themselves are generally mathematical/algebraic *laws*.

- **Coordinates** are the *fundamental adjectival modifiers* corresponding to the differentiating properties of "things" (nouns) in the real Universe, where the term fundamental can also be thought of as meaning *irreducible* – adjectival properties that cannot be readily be expressed in terms of or derived from other adjectival properties of a given object/thing. See:

  - Wikipedia: http://www.wikipedia.org/wiki/Coordinate System

- **Units.** Physical coordinates are basically mathematical numbers with units (or can be so considered even when they are discrete non-ordinal sets). In this class we will consistently and universally use *Standard International* (SI) units unless otherwise noted. Initially, the irreducible units we will need are:

  a) *meters* – the SI units of *length*
  b) *seconds* – the SI units of *time*
  c) *kilograms* – the SI units of *mass*

  All other units for at least a short while will be expressed in terms of these three, for example units of *velocity* will be meters per second, units of *force* will be kilogram-meters per second squared. We will often give names to some of these combinations, such as the SI units of force:

$$1 \text{ Newton} = \frac{\text{kg-m}}{\text{sec}^2}$$

Later you will learn of other irreducible coordinates that describe elementary particles or extended macroscopic objects in physics such as electrical charge, as well as additional derivative quantities such as energy, momentum, angular momentum, electrical current, and more.

- **Laws of Nature** are essentially *mathematical postulates* that permit us to understand natural phenomena and *make predictions* concerning the time evolution or static relations between the coordinates associated with objects in nature that are consistent mathematical theorems of the postulates. These predictions can then be compared to *experimental observation* and, if they are consistent (uniformly successful) we *increase our degree of belief in them*. If they are inconsistent with observations, we *decrease* our degree of belief in them, and seek alternative or modified postulates that work better[23].

  The entire body of human scientific knowledge is the more or less successful outcome of this process. This body is not fixed – indeed it is *constantly changing* because it is an *adaptive* process, one that self-corrects whenever observation and prediction fail to correspond or when new evidence spurs insight (sometimes revolutionary insight!)

  Newton's Laws built on top of the analytic geometry of Descartes (as the basis for at least the abstract spatial coordinates of things) are the *dynamical principle* that proved *successful* at predicting the outcome of many, many everyday experiences and experiments as well as cosmological observations in the late 1600's and early 1700's all the way up to the mid-19th century[24]. When combined with associated empirical *force laws* they form the basis of the physics you will learn in this course.

- **Newton's Laws**:

  a) **Law of Inertia:** Objects at rest or in uniform motion (at a constant velocity) in an *inertial reference frame* remain so unless acted upon by an unbalanced (net) force. We can write this algebraically[25] as

$$\sum_i \vec{F}_i = 0 = m\vec{a} = m\frac{d\vec{v}}{dt} \quad \Rightarrow \quad \vec{v} = \text{constant vector} \tag{1.1}$$

[23] Students of philosophy or science who *really* want to read something cool and learn about the fundamental basis of our knowledge of reality are encouraged to read e.g. Richard Cox's *The Algebra of Probable Reason* or E. T. Jaynes' book *Probability Theory: The Logic of Science*. These two related works *quantify* how science is not (as some might think) absolute truth or certain knowledge, but rather *the best set of things to believe* based on our overall experience of the world, that is to say, "the evidence".

[24] Although they failed in the late 19th and early 20th centuries, to be superceded by relativistic quantum mechanics. Basically, everything we learn in this course is *wrong*, but it nevertheless *works damn well* to describe the world of macroscopic, slowly moving objects of our everyday experience.

[25] For students who are still feeling very shaky about their algebra and notation, let me remind you that $\sum_i \vec{F}_i$ stands for "The sum over i of all force $\vec{F}_i$", or $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$ We will often use $\sum$ as shorthand for summing over a list of similar objects or components or parts of a whole.

b) **Law of Dynamics:** The net force applied to an object is directly proportional to its acceleration in an *inertial reference frame*. The constant of proportionality is called the **mass** of the object. We write this algebraically as:

$$\vec{F} = \sum_i \vec{F}_i = m\vec{a} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt} \tag{1.2}$$

where we introduce the *momentum* of a particle, $\vec{p} = m\vec{v}$, in the last way of writing it.

c) **Law of Reaction:** If object B exerts a force $\vec{F}_{AB}$ on object A *along a line connecting the two objects*, then object A exerts an equal and opposite reaction force of $\vec{F}_{BA} = -\vec{F}_{AB}$ on object B. We write this algebraically as:

$$\vec{F}_{ij} = -\vec{F}_{ji} \tag{1.3}$$

$$\text{(or)} \quad \sum_{i,j} \vec{F}_{ij} = 0 \tag{1.4}$$

(the latter form means that the sum of all *internal* forces between particles in any closed system of particles cancel).

- **An inertial reference frame** is a ***spatial coordinate system plus an independent time coordinate*** that is either at rest or moving at a constant speed, a *non-accelerating* set of coordinates that can be used to describe the locations of real objects as a function of time. However, this definition is inadequate, because acceleration itself is defined only *relative to a frame*. This leaves us with a problem: non-accelerating relative to *what frame*? We have to identify at least one inertial reference frame before we can talk about other frames that do not accelerate relative to *it*.

The way we can identify *any* inertial reference frame is as follows. As you will see, ***all actual physical forces in the laws of nature*** are ***interaction*** laws. For any observed force pushing an object, there exists *another object* somewhere that is doing the pushing, a "Newton's Third Law partner". No force exists in isolation, and no object can exert a force on itself. A signature of a *non*-inertial reference frame is that within it, there exists an observed "pseudoforce" that arises in Newton's Second Law due to the acceleration of the frame. This pseudoforce ***has no Newton's Third Law partner!*** A *consistent* definition of an inertial reference frame is therefore:

> **Inertial Reference Frame**: ***Any frame where all observed forces that occur in all statements of Newton's Second Law for all particles are pairwise interactions between particles.*** In other words, there are no forces that act on any particle in complete isolation from and independent of the other particles in the system

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In physics one has considerable leeway when it comes to choosing the (inertial) coordinate frame to be used to solve a problem – some lead to much simpler solutions than others!

- **Forces of Nature** (weakest to strongest):

  a) Gravity
  b) Weak Nuclear
  c) Electromagnetic
  d) Strong Nuclear

  It is possible that there are more forces of nature waiting to be discovered. Because physics is not a *dogma*, this presents no real problem. If they are, we'll simply give the discoverer a Nobel Prize and add their name to the "pantheon" of great physicists, add the force to the list above, and move on. Science, as noted, is self-correcting.

- **Force is a vector**. For each force rule below we therefore need *both* a formula or rule for the magnitude of the force (which we may have to compute in the solution to a problem – in the case of forces of constraint such as the normal force (see below) we will *usually* have to do so) and a way of determining or specifying the *direction* of the force. Often this direction will be obvious and in corresponence with experience and mere common sense – strings pull, solid surfaces push, gravity points down and not up. Other times it will be more complicated or geometric and (along with the magnitude) may vary with position and time.

- **Force Rules** The following set of force rules will be used both in this chapter and throughout this course. All of these rules can be derived or understood (with some effort) from the forces of nature, that is to say from "elementary" natural laws.

  a) **Gravity** (near the surface of the earth):

  $$F_g = mg \tag{1.5}$$

  The direction of this force is *down*, so one could write this in vector form as $\vec{\boldsymbol{F}}_g = -mg\hat{\boldsymbol{y}}$ in a coordinate system such that up is the $+y$ direction. This rule follows from Newton's Law of Gravitation, the elementary law of nature in the list above, evaluated "near" the surface of the earth where it is approximately constant.

  b) **The Spring** (Hooke's Law) in one dimension:

  $$F_x = -k\Delta x \tag{1.6}$$

  This force is directed back to the equilibrium point of unstretched spring, in the *opposite direction* to Δx, the displacement of the mass on the spring from equilibrium. This rule arises from the primarily electrostatic forces holding the atoms or molecules of the spring material together, which tend to linearly oppose *small* forces that pull them apart or push them together (for reasons we will understand in some detail later).

  c) The **Normal Force**:

  $$F_\perp = N \tag{1.7}$$

  This points perpendicular and away from solid surface, magnitude sufficient to oppose the force of contact *whatever it might be!* This is an example of a *force* *of constraint* – a force whose magnitude is determined by the *constraint* that one solid object cannot generally interpenetrate another solid object, so that the solid surfaces exert whatever force is needed to prevent it (up to the point where the "solid" property itself fails). The physical basis is once again the electrostatic molecular forces holding the solid object together, and *microscopically* the surface deforms, however slightly, more or less like a spring.

d) **Tension** in an Acme (massless, unstretchable, unbreakable) string:

$$F_s = T \tag{1.8}$$

This force simply transmits an *attractive* force between two objects on opposite ends of the string, in the directions of the taut string at the points of contact. It is another constraint force with no fixed value. Physically, the string is like a spring once again – it microscopically is made of bound atoms or molecules that pull ever so slightly apart when the string is stretched until the restoring force balances the applied force.

e) **Static Friction**

$$f_s \leq \mu_s N \tag{1.9}$$

(directed opposite towards net force parallel to surface to contact). This is another force of constraint, as large as it needs to be to keep the object in question travelling at the same speed as the surface it is in contact with, up to the *maximum* value static friction can exert before the object starts to slide. This force arises from mechanical interlocking at the microscopic level plus the electrostatic molecular forces that hold the surfaces themselves together.

f) **Kinetic Friction**

$$f_k = \mu_k N \tag{1.10}$$

(opposite to direction of relative sliding motion of surfaces and parallel to surface of contact). This force *does* have a fixed value when the right conditions (sliding) hold. This force arises from the forming and breaking of microscopic adhesive bonds between atoms on the surfaces plus some mechanical linkage between the small irregularities on the surfaces.

g) **Fluid Forces, Pressure**: A fluid in contact with a solid surface (or anything else) in general exerts a force on that surface that is related to the *pressure* of the fluid:

$$F_P = PA \tag{1.11}$$

which you should read as "the force exerted by the fluid on the surface is the pressure in the fluid times the area of the surface". If the pressure varies or the surface is curved one may have to use calculus to add up a total force. In general the direction of the force exerted is *perpendicular* to the surface. An object at rest in a fluid often has balanced forces due to pressure. The force arises from the molecules in the fluid literally bouncing off of the surface of the object, transferring momentum (and exerting an average force) as they do so. We will study this in some detail and will even derive a kinetic model for a gas that is in good agreement with real gases.

h) **Drag Forces**:

$$F_d = -bv^n \tag{1.12}$$

(directed opposite to relative velocity of motion through fluid, n usually between 1 (low velocity) and 2 (high velocity). This force also has a determined value, although one that depends in detail on the motion of the object. It arises first because the surface of an object moving through a fluid is literally bouncing fluid particles off in the leading direction while moving away from particles in the trailing direction so that there is a differential pressure on the two surfaces, second from "friction" between the fluid particles and the surface.