Numerical Analysis of Blockage and Optimization of Heat Transfer Performance of Fractal-like Microchannel Nets

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The conjugate fluid flow and heat transfer characteristics of fractal-like microchannel nets embedded in a disk-shape heat sink are investigated using a three-dimensional computational fluid dynamics (CFD) approach. A constant heat flux is applied to the top wall of the heat sink. The intrinsic advantages of fractal-like microchannel nets such as low flow resistance, temperature uniformity, and reduced danger of blockage compared with the traditional parallel channel nets are demonstrated. In addition, various optimized designs with parameters such as the number of branches, number of branching levels, and number of channels that reach the center of the disk are addressed in this context. [DOI: 10.1115/1.2159007]

1 Introduction

The performance of microelectronic devices has improved significantly over the past three decades. Associated with these improvements, effective thermal management is key to ensure reliable and efficient performance of such electronic devices. Various innovative methods have been investigated to meet this demand. Among them, microchannel heat sink with high heat flux is possibly the most effective cooling technique. Naturally it has attracted many researchers' attention since the idea was first introduced for cooling integrated circuits by Tuckerman and Pease [1]. The review article by Sohban and Garimella [2] notes that numerous investigations of single-phase flow and heat transfer in straight microchannels on heat sinks have been conducted during the past decade. However, results from different researchers are often conflicting. Several authors suggest that fluid flow and thermal transport in microchannels with characteristic lengths between 50 μm and 1 mm are different from macroscale phenomena. By contrast, Obot [3] reviewed numerous papers related to friction as well as heat and mass transfer in microchannels. Based on a careful re-examination of previous work, he concluded that there is no supporting evidence to suggest that fluid flow in microchannels behaves differently from that in normal-scale channels. For fluid flow, Kleinstreuer [4] presented a global Knudsen number, $\mathrm{Kn}=\lambda_{\mathrm{IM}}/l_{\mathrm{system}}$, where l_{system} is the system dimension scale and λ_{IM} is the intermolecular length (3 Å for water). Hence, for scales of characteristic length larger than 10 μm, the conventional Navier-Stokes equations are appropriate to describe the transport phenomena in terms of the flow and heat transfer.

Use of microchannels is an attractive method for electronic device cooling due to its compactness and high surface-to-volume ratio. However, this is achieved at the expense of increased pumping power. Another disadvantage of microchannel heat sinks is the uneven temperature distribution along the wall of the heat sink, which is especially undesirable for electronic cooling applications since nonuniformity of temperature can affect the performance and lifetime of chips.

The efficient transport characteristics of natural systems can provide useful hints for optimal solutions of many problems. The structure of the living organism may provide inspiration for the design of an effective microchannel cooling systems. Bejan and Errera [5] first discussed the tree network for cooling applications. They proposed the architecture of the volume-to-point path such that the flow resistance is minimum. They found that a tree network has the minimal resistance path. Bejan [6] showed that the total heat flux convected by a double tree is proportional to the total volume raised to power 3/4. By adapting the constructal theory of Bejan [7,8], Pence [9] proposed a fractal-like bifurcating flow network in a two-dimensional heat sink. Wechsatol et al. [10] analyzed the optimization of networks in a disk-shaped body considering the minimum overall resistance between one point (O) and many points situated equidistantly on the circle centered at O. Their results emphasize the robustness of optimized fractal-like nets for the fluid flow.

Pence [11] developed a one-dimensional model to predict both the pressure distribution and wall surface temperature along and in a fractal-like branching channel network. Results were compared with an array of straight channels having the same channel length and convective surface area as the branching network. For identical pressure drop, flow rate and total pumping power through both networks, a lower maximum wall temperature along the fractal-like network was found. Chen and Cheng [12] designed a new fractal branching channel net for cooling of chips. Their results show that a fractal-like channel network has a stronger heat transfer capability and requires lower pumping power. Alharbi et al. [13] investigated three-dimensional fractal-like branching networks. They found that the local pressure recovery at each bifurcation results in a lower total pressure drop than that with conventional parallel straight channel networks.

Enfield et al. [14] developed a two-dimensional model to predict concentration profiles and degree of mixing resulting from diffusion in laminar flow within a fractal-like merging network. It is found that the geometric variations yield a threshold value to provide a mechanism for predicting the optimum number of branching levels with respect to the degree of mixing (DoM). Senn and Poulikakos [15] conducted a three-dimensional simulation for a treelike net. Compared with a serpentine flow pattern, results show that the tree net requires only almost half the pressure drop and a larger heat transfer capability than the corresponding serpentine flow pattern having the same surface area and inlet Reynolds number.

The recent paper by Ghodossi [16] demonstrated analytically that the fractal microchannel networks have no advantage com-

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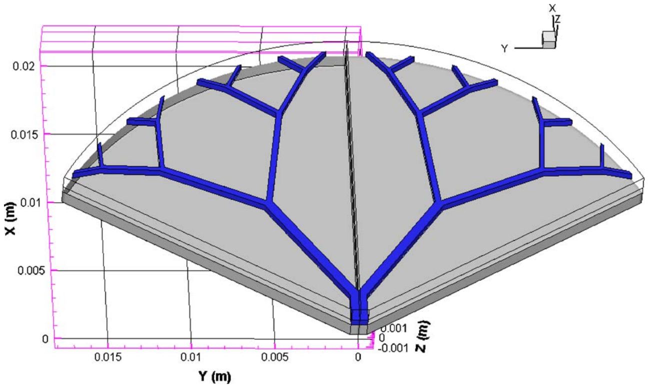

Fig. 1 One typical physical model of fractal-like branching channels embedded in a heat sink attached to a chip; 1/n part of the disk is shown (n=4 here). Radius of the disk, R=20 mm.

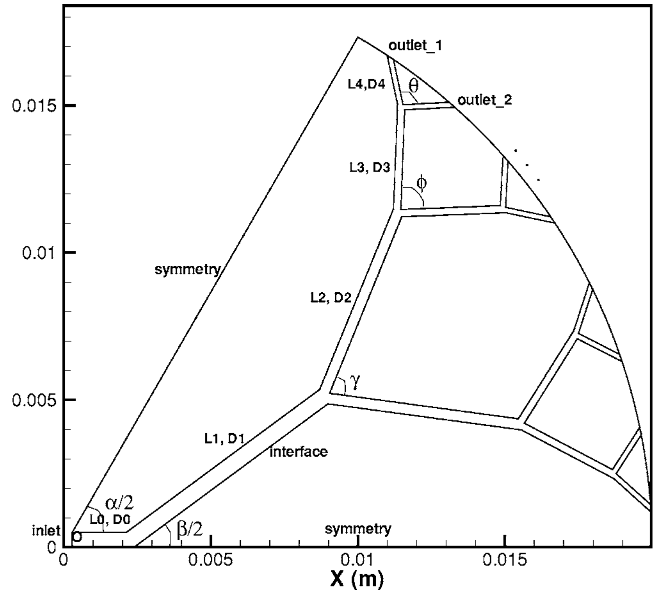

Fig. 2 The 2D profile of the computational domain

pared with conventional straight channels considering the effect of Reynolds number. The analytical results conflict with those discussed earlier. However, it is noted in [16] that the important effect of pressure recovery at each bifurcating location has been neglected; this is probably why the conclusion defers from the one reached by [12] using a complete model for the hydrodynamics.

In this paper, we study numerically both heat transfer and fluid flow in fractal-like microchannel networks which are conjugated with the heat sinks for the cooling of the microelectronic chips of circular shape. Parametric studies for effects of blockage, number of branches, levels, and tubes are carried out to identify possible optimum designs.

2 Method and Analysis

Beginning with Murray's study on blood vessels [17], it has been found that there is an optimal size step (change in hydraulic diameter) at each paring node of the fractal-like networks such that the global flow resistance is minimized. It is given by

$$D_{l+1}/D_l = n^{-1/3} \tag{1}$$

where D is the hydraulic diameter, and n is the number of branches into which each channel splits. For the present analysis, $n=2,3$. Subscript l denotes a low-order branching level and $l+1$ denotes a higher-order branching level at a bifurcation. The first channel emanating from the center of the disk is the zeroth-order branch, i.e., $l=0$. A typical schematic physical model is shown in Fig. 1. This model includes three parts: heat sink (in silicon) at the bottom (gray shaded part), fractal-like channel networks embedded in the sink (black shaded part) and the chip at the top attached to the heat sink (transparent part). The objective of this system is to remove the heat generated by the chip using the fractal-like microchannel network shown. Figure 1 is just one typical case. In the present analysis, for the fixed heat sink and chip with a radius of 20 mm, the structures of the channel networks such as number of bifurcating branches, number of channels at level 0 and number of branching levels are changed for comparison. Note that the region marked with coordinate system (x-, y-, z-) is the computational domain.

It is the objective of the present work to analyze the pressure and temperature distributions in fractal-like microchannel networks using a conjugated three-dimensional computational fluid dynamics analysis. The CFD package, Fluent 6.2 [18], was employed for this purpose. The computations were performed using water with temperature-dependent properties as the working fluid. A steady, incompressible and laminar flow is considered. The three-dimensional continuity, momentum and energy equations governing laminar transport are as follows:

Continuity:

$$\frac{\partial V_i}{\partial x_i} = 0 \tag{2}$$

Momentum:

$$\rho\left(\frac{\partial(V_iV_j)}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left(\mu\frac{\partial V_j}{\partial x_i}\right) - \frac{\partial p}{\partial x_i} \tag{3}$$

Energy:

$$\rho\left(\frac{\partial(V_ic_pT)}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left(\lambda\frac{\partial T}{\partial x_i}\right) \tag{4}$$

As shown in Fig. 2, in this study, the flow emanates from the center of the disk to the point at the perimeter of the disk. The related boundary conditions are described as follows:

Inlet: mass flow inlet ($\dot{m}$), the total mass flow rate is 3.6 g/s, which is distributed evenly into every channel at zeroth-order level. For example, if the disk has four channels at the center, then $\dot{m}$=0.9 g/s is the mass flow rate as inlet boundary condition of the computational domain. Note that the corresponding Reynolds number is around 200 at inlet and the typical Kn is about 0.0015.

Outlet: pressure outlet. the reference pressure (atmosphere pressure) is used here.

Interface: As shown in Figs. 1 and 2, compared with the outer heat sink and chip, channels are of smaller scales. To ensure accuracy of simulation in the channels and control the total number of grids, we need to use two-level grids with different size steps for the channels and the outer part. To connect the two parts, a conjugate interface between channels and outer walls was chosen.

Symmetrical walls: Since the fractal-like channel network is

Table 1 Nonoptimized channel dimensions for fractal-like flow network, n=3, L_{tot}=20 mm

l	H_l (mm)	w_l (mm)	D_l (mm)	L_l (mm)
0	0.250	0.539	0.342	5.66
1	0.250	0.366	0.297	7.38
2	0.250	0.249	0.249	4.39
3	0.250	0.169	0.202	2.30

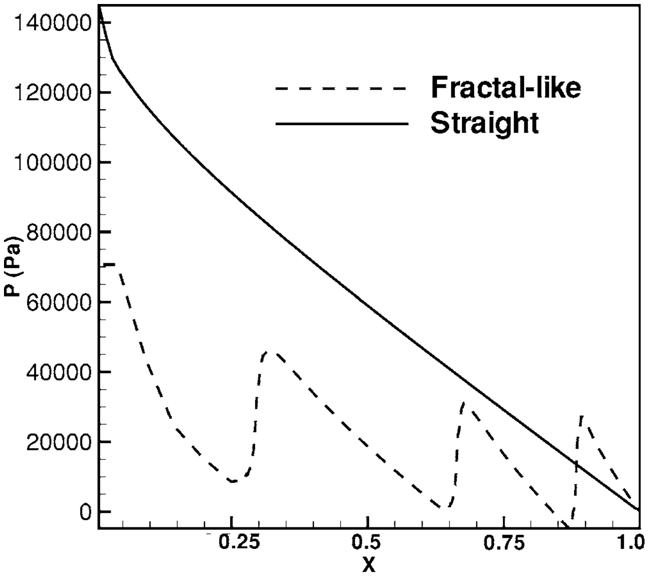

Fig. 3 Pressure distribution through fractal-like and straight microchannel networks

arranged regularly in the system, we use symmetric boundary conditions to reduce the computational cost.

Along the top wall of the chip: a constant heat flux (q'') is applied ($q''=10\ \mathrm{W/cm^2}$ in present analysis); Other walls are specified to be adiabatic.

Grid-independence of the final results was checked for all the computations. The final grid system used here is about $36\times 16 \times \mathrm{mod}(L_i/0.1\ \mathrm{mm})$ for each channel. Here L_i represents the channel length at level i. And for the outer domain, we use a size step of 0.2 mm for the x- and y-directions and 40 grids in the z-direction (refer to Fig. 1). For the current grid system, the maximum aspect ratio of the grids is about 5. To ensure computational accuracy, double-precision computation is employed.

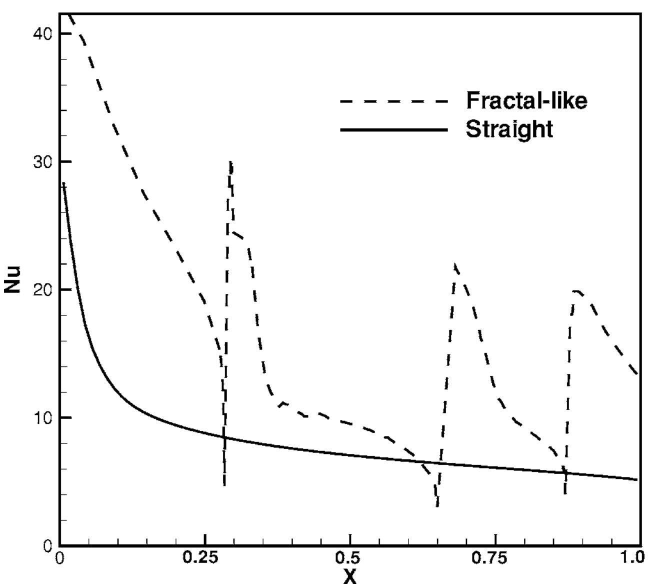

Fig. 4 Local heat flux through fractal-like and straight microchannel networks

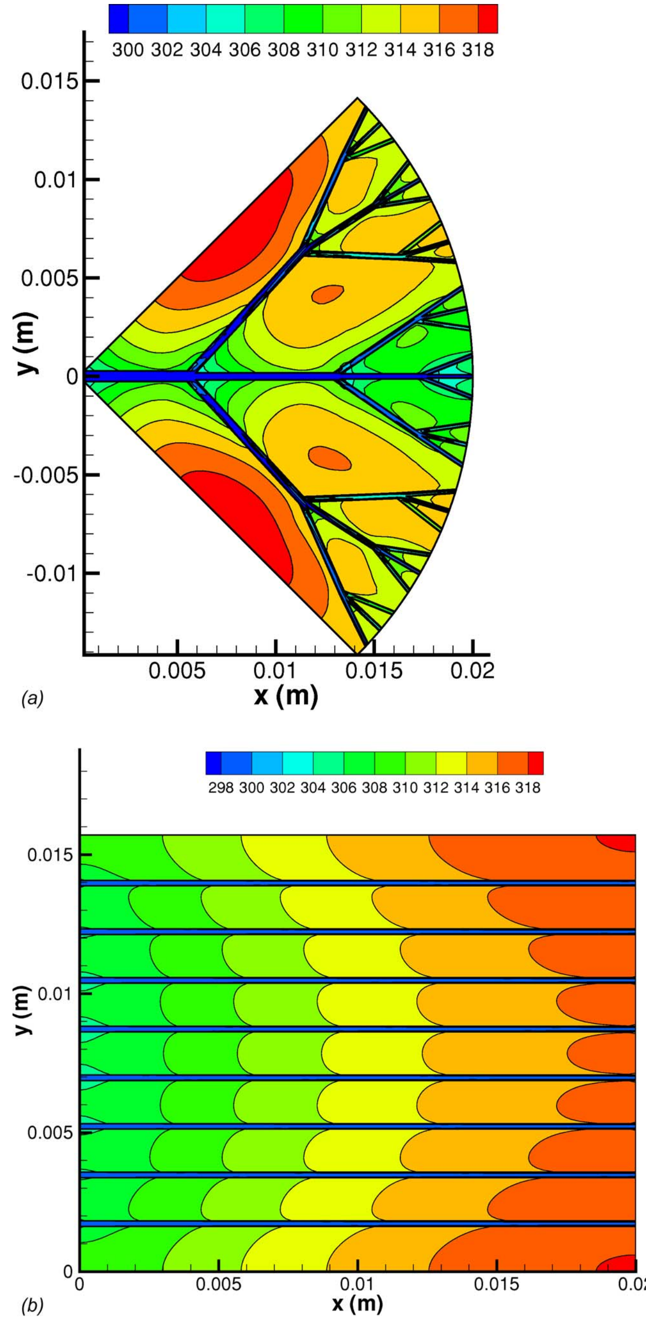

Fig. 5 Temperature distribution over mid-depth plane for (*a*) fractal-like and (*b*) straight parallel nets

3 Results and Discussion

The numerical results are validated with analytical and numerical solutions from literature, as suggested by Senn and Poulikakos [19]. The numerical value of $f\,\mathrm{Re}=56.94$ of the parallel straight nets is in good agreement with the analytical solution given by Moody (or Darcy) [20] as

$$f\,\mathrm{Re} = 24\left[1 - 192\pi^{-5}\sum_{n=1,3,\ldots}^{\infty} n^{-5}\tanh\left(\frac{n\pi}{2}\right)\right]^{-1} \cong 56.908 \quad (5)$$

The model was assessed first by comparing the fractal-like networks with parallel straight channels. Three-dimensional conjugated computational results were carried out to investigate the fluid flow and thermal transport phenomena in the fractal-like microchannel networks. The channel dimensions for the fractal-like

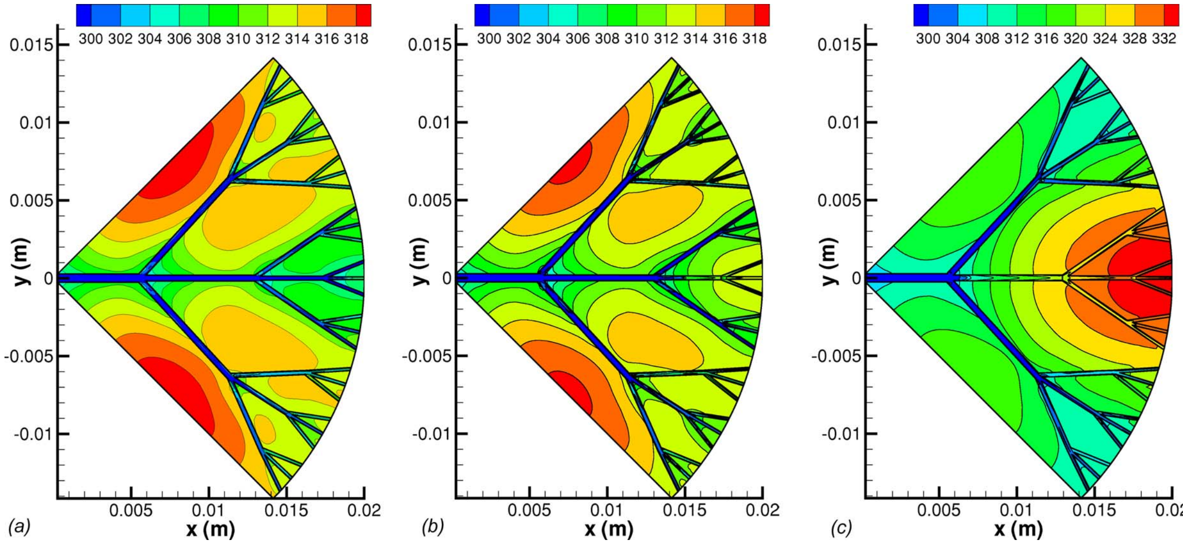

Fig. 6 Temperature distribution over mid-depth plane for blockage of the fractal-like nets at the outlet side of channel: (*a*) 3*n*, (*b*) 2*e*, and (*c*) 1*b*

network are shown in Table 1. To compare its performance with flow through straight channel, a straight channel net is composed of a series of parallel channels of the same hydraulic diameter (8 channels with $D=0.2$ mm and $L=20$ mm) as the terminal channel in the fractal-like flow net. Also, the total channel length, total heat convective surface area, heat flux between fractal-like and straight channels are identical.

Figure 3 shows the centerline pressure distributions for both flow nets. For comparison, nondimensionalized length X ($X=x/L$) is used to plot the curve. The pressure distribution presented for fractal-like channel nets is that for the middle channels, i.e., that along the path by the following branches: $l=0$, $l=1b$, $l=2e$, $l=3n$. It is noted that, for convenience, each channel is identified by "$l=2e$," where "2" represents the branch level and "e" represents the number of channels along the clockwise direction (denoted from "a"). Results show that the pressure drop through the straight channel network is about 70 kPa higher than that through the fractal-like network, which is consistent with that from other researchers [11]. One possible reason for this is that there is pressure recovery at each bifurcation, which results in reduction of the total pressure drop.

Figure 4 shows the Nusselt number distribution along the top wall of the middle channel for both fractal-like and straight channel nets. Here, $\mathrm{Nu}_l=hD_l/k$, where D_l is defined by the local hydraulic diameter of each branch for fractal-like nets since such nets has no identical characteristic length. It is found from the figure that fractal-like nets have a higher average Nusselt number than the straight nets, which show again the advantage of fractal-like channels nets over straight channel nets.

Figures 5(*a*) and 5(*b*) show the temperature distribution along the mid-depth plane ($z'=0.5$) including the sink and channel for fractal-like and straight nets. It is observed that the temperature difference between the maximum and minimum values, $\Delta T_{\max}$, on the plane for fractal-like nets is about 20 K. For parallel channels of identical hydraulic diameter as the terminal channel of the microscale fractal-like channel networks, the temperature difference is also about 20 K. Although the overall $\Delta T_{\max}$ is nearly the same, the distribution is very different for the two cases. For traditional parallel channels, the temperature gradually increases from the inlet side to the outlet side. Such a gradient could affect the chip's performance. What is more desirable is a more evenly distributed temperature field which may be achieved using fractal-like channel networks. As shown in Fig. 5(*a*), the temperature distribution is more uniform, except for two regions with high temperature, which is attributed to being the farthest in distance from the channels and hence the retardation of thermal diffusion.

3.1 Effects of Blockage.

One main problem in fractal-like microchannel networks which should be considered is the possible blockage of fluid flow in the channels. It is noted that the phenomenon of blockage, which could be caused by particulate or static-electricity, can be dangerous for the cooled system. For parallel straight microchannel networks, a blocked channel results in the break-up of the system due to the increased temperature of the stagnated fluid; it would exceed the rating of the chip and sink materials. Hence, it is necessary to evaluate the effects of possible blockage in fractal-like channel networks with their more complicated structured geometry as compared with straight channels.

Here the following three cases are investigated to examine harmful effects of total flow blockage:

Case 1: blockage at outlet of channel $3n$;
Case 2: blockage at outlet of channel $2e$;
Case 3: blockage at outlet of channel $1b$.

The predicted temperature distributions over the mid-depth plane ($z'=0.5$) are shown in Figs. 6(*a*)–6(*c*). One interesting finding is that, when the outlet of channel $3n$ is blocked, the maximum temperature difference, $\Delta T_{\max}$, has not increased much as compared with the case without blockage. The difference between the two cases is that there exist more regions (two for the case without blockage and one more for this case) with the highest temperature. This indicates that for fractal-like channel networks, the effects of the blockage of fluid flow in the channel, especially for the channel at high-order branching level, are smaller than those in straight case. This means that such a system could have greater stability, which is important for a microelectronic device cooling system. Even when channel $2e$ is blocked, though it is at a lower branch level than channel $3n$, the maximum temperature difference is still nearly the same, as shown in Fig. 6(*b*); this demonstrates the strong reliability of such fractal-like channel nets as compared with straight ones.

Figure 6(*c*) shows the result of blockage at the outlet of channel $1b$ which is close to the main channel from the center of the disk. Although $\Delta T_{\max}$ is increased from 20 K for the case without blockage to 34 K, the system also stays safe considering the condition of break-up. From the discussion above, it is concluded that the advantages of fractal-like microchannel networks are obvious, e.g., the temperature distribution is more uniform and the adverse effects of blockage of flow are minimized. Hence such systems

Table 2 Dimensions of cases studied for analysis of system optimization

Case	1	2	3	4	5		6
b	2	2	2	2	2		3
l	2	2	2	3	4		1
n	3	4	5	3	3		3
L_0 (mm)	3.14	6.74	9.16	3.06	2.18		7.2
L_1 (mm)	10.28	9.28	8.16	8.32	8.40		12.8
L_2 (mm)	8.64	6.1	4.7	7.36	6.74	L'_1	15.2
L_3 (mm)	–	–	–	3.88	3.6		–
L_4 (mm)	–	–	–	–	1.72		–
α (deg)	120	90	72	120	120		120
β (deg)	38.85	38.64	38.31	40.6	37.5		57.7
γ (deg)	36.77	39.78	41.65	36.79	38.1	D_1/D_0	0.68
ϕ (deg)	–	–	–	42.39	43.1	D'_1/D_0	0.71
θ (deg)	–	–	–	–	49.8		–

have potential for application in the field of MEMS cooling. Of course it should be noted that the manufacture of such networks is complicated.

3.2 Effects of Number of Branches, Levels, and Tubes. As discussed earlier, the simulation results show the advantages of the micro-scale fractal-like channel networks as compared with the straight parallel channels. It is noted, however, that the design tested is not optimized. Considering the complicated structure of such systems, many factors need to be included for the optimization of the system. These factors include the number of the branches, levels, and channels. The dimensional scales used here comply with the optimized design of tree-shaped networks for fluid flow in a disk given by Wechsatol et al. [10]. They recently investigated the effects on flow resistance using analytical methods. The cases analyzed in the present study are listed in Table 2.

Figure 7 shows the effects of the number of channels that reach the central point on the maximum temperature value at the mid-depth plane. From this figure we find that with the increased number of ducts in the center, the maximum temperature is clearly decreased: about 3.6 K from $n=3$ to $n=4$ and 2.5 K from $n=4$ to $n=5$. It is understandable that more channels emanating from the center would make the temperature distribution on the chip more uniform, although the total mass flow rate of the channel networks has not changed. This means that using more channels can increase the cooling performance. However, it can also be noted from Fig. 7 that with the increased number of ducts attached to the center, its effects on the maximum temperature is decreased. Thus, there should be an optimal value as a compromise between the number of ducts at the center and the complexity of the networks. The latter factor would make the process of manufacture more difficult; fabrication costs should be considered in any optimization process.

The influence of the number of branching levels on the maximum temperature is shown in Fig. 8. From it, similar trends can be seen as discussed above. The maximum temperature, $T_{\max}$, decreases sharply by about 7.7 K from $l=2$ to $l=3$. However, in the transition from two levels of complexity to the next, third level, the maximum temperature only increases by 2.5 K, which is much smaller as compared with the former value. Such trends are also found from the change in the number of branches used. In general, although the use of more complicated structures (more numbers of ducts at the center, branches and branching levels) could reduce the maximum temperature distribution, the use of complicated structures is not always a better option considering the drawbacks of complex structures e.g., difficulty in manufacture and the potential for blockage of fluid flow in the channels.

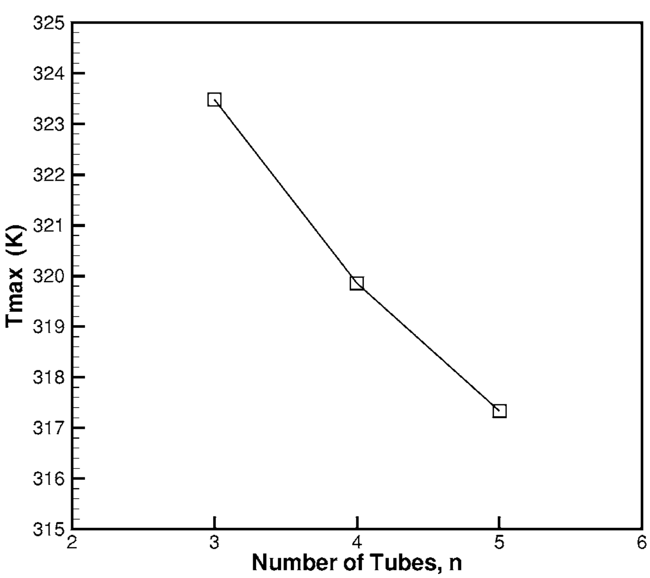

Fig. 7 Maximum temperature over mid-depth plane, $z'=0.5$, with changed number of tubes

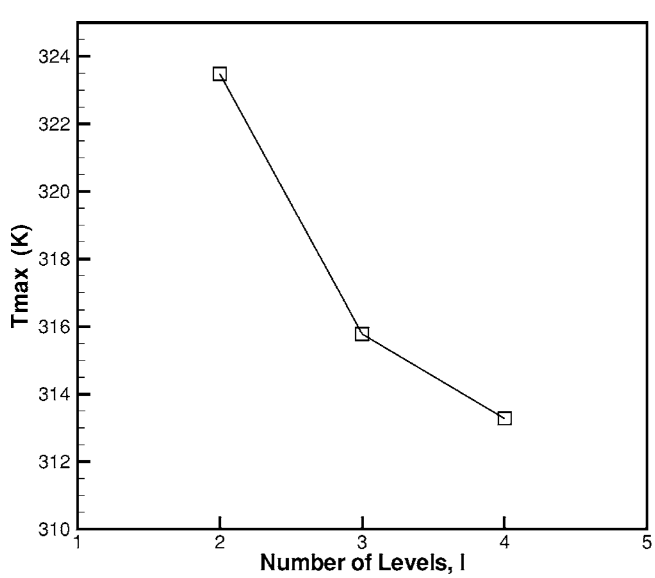

Fig. 8 Maximum temperature over mid-depth plane, $z'=0.5$, with changed number of levels

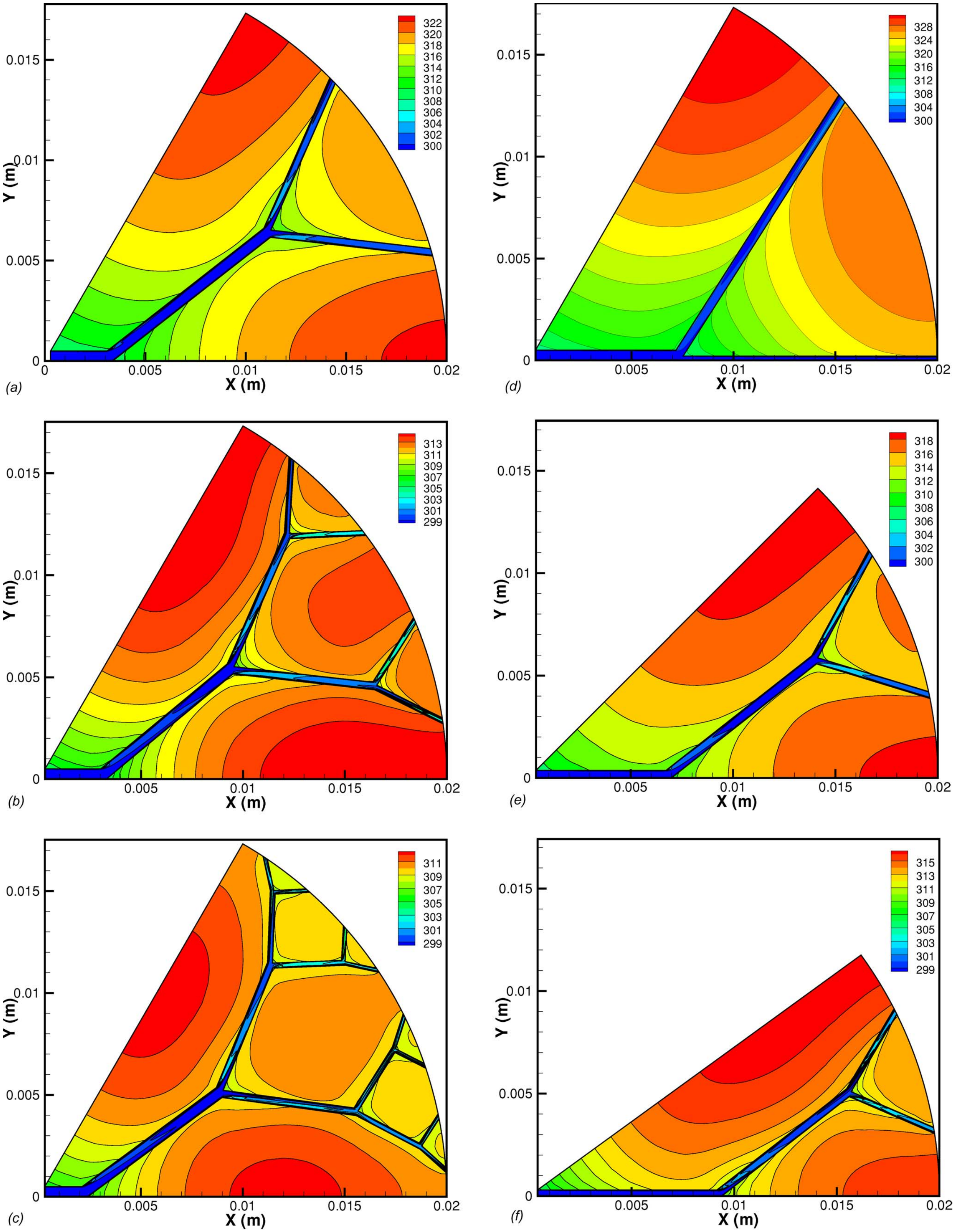

Fig. 9 Temperature distributions at mid-depth plane, $z'=0.5$ for (*a*) $b=2$, $l=2$, $n=3$; (*b*) $b=2$, $l=3$, $n=3$; (*c*) $b=2$, $l=4$, $n=3$; (*d*) $b=3$, $l=2$, $n=3$; (*e*) $b=2$, $l=2$, $n=4$; (*f*) $b=2$, $l=2$, $n=5$

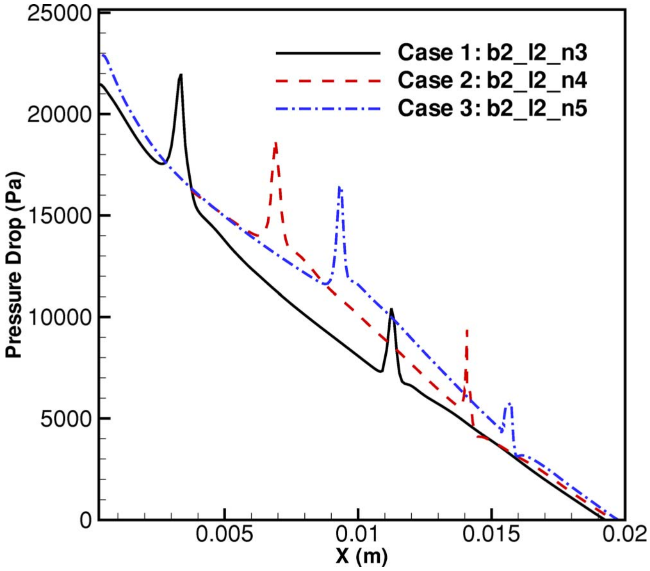

Fig. 10 Pressure distribution through fractal-like channel networks for cases with different number of tubes

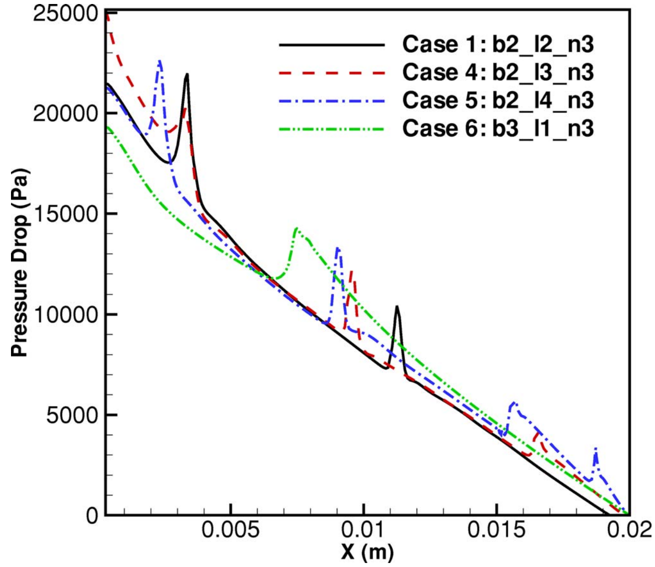

Fig. 11 Pressure distribution through fractal-like channel networks for cases with different number of levels

Figures 9(*a*)–9(*c*) show the detailed temperature distributions over the mid-depth plane for the cases with different number of branching levels. From these figures it is observed that increased levels affect the maximum-temperature regions. With gradually increased l, the highest-temperature point shifts from the perimeter to the center of disk. Also, the area of such regions shrinks with increased l.

Figures 9(*a*), 9(*e*), and 9(*f*) display the influence of the number of tubes emanating from the center on the temperature distribution at the mid-depth plane, $z'=0.5$. Different from the effects of level number, the increased number of tubes does not affect the highest-temperature region. The effect is just to decrease the temperature evenly. This happens because the increased level number does not change the shape of the channel system but just squeezes the channels using smaller angles between bifurcating channels. However, the increased number of branching levels change the distribution of channels which is more concentrated near the perimeter.

Finally, it is interesting to discuss the effects of the numbers, n, l, b, on the pressure distribution through the fractal-like microchannel networks which is shown in Figs. 10 and 11. Figure 10 indicates that fewer ducts produce smaller pressure drop. It confirms the discussion above that use of fewer tubes is better if the maximum temperature is not out of range. It is noted that the fluctuation of the pressure at the conjunction of channels is due to the highly localized pressure recovery at each bifurcation, which tends to lower the total pressure drop as compared with conventional straight channel networks.

The influence of branching level number on the pressure drop distribution is shown in Fig. 11. It is found that the three-bifurcating channels could reduce the pressure drop obviously since the case of tripling just has one level. For the cases with paired branches, with increased branching levels, the change of pressure drop is not obvious. Figure 11 indicates that the case of 3 levels has the highest pressure drop. One possible reason for this is that its angle at the first bifurcation has the highest value as shown in Table 2, this results in the high pressure drop at the first main channel.

4 Conclusions

In the present research, flow and heat transfer through fractal-like branching channel networks were investigated using a conjugated three-dimensional CFD approach. Results of comparison with straight tube networks, temperature, and pressure distributions are presented and compared. Also, the optimization of networks is discussed by considering the effects of different factors.

Temperature distributions for the fractal-like channel networks show that the damage of blockage of the fluid flow could be reduced to at a large extent, which is impossible in straight channel networks. These results emphasize the robustness of fractal-like channel networks for electronic cooling which need high reliability.

Comparison of the number of ducts that reach the center of a disk configuration, the branching levels and the branches provide a useful way to optimize the fractal-like branching channel networks in account for the influence on temperature and pressure drop distribution. Furthermore, in application of such systems, balance between structural optimization and complication of manufacture should be considered.

Acknowledgment

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Nomenclature

b = number of bifurcations
c_p = specific heat capacity (J/kg K)
D = hydraulic diameter (mm)
f = friction factor
H = channel height (mm)
h = heat transfer coefficient (W/m^2 K)
Kn = Knudsen number, Kn=λ_{IM}/l_{system}, l_{system} is system dimension scale, λ_{IM} is intermolecular length
L = channel length (mm)
l = number of branching levels
$\dot{m}$ = mass flow rate (kg m s^{-1})
n = number of channels at the zeroth level
n = sequence number, in Eq. (5)
Nu = Nusselt number
q'' = heat flux (W/cm^2)
R = disk radius (mm)
Re = Reynolds number
ΔT = temperature difference (K)
T = temperature (K)
v = velocity (m/s)

w = channel width (mm)
X = dimensionless transverse direction (x/L)
x = coordinate (m)
x,y,z = coordinate (m)
z' = dimensionless transverse direction (z/h)

Greek Symbols

$\alpha, \beta, \ldots$ = angles between two near channels at level 0,1,… (rad)
λ_{IM} = intermolecular length (Å)
μ = dynamic viscosity (Pa s)
ρ = density (kg/m^3)
0 = channel touching the center
1,2,… = channels positioned progressively closer to the perimeter

Subscripts

i,j = indices in Einstein summation convention
l = number of branching levels
max = maximum
tot = total

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