MAT 344 | Introduction to Combinatorics

https://q.utoronto.ca/courses/99258/
Text: Applied Combinatorics - Keller & Trotter

What is Combinatorics?

Combinatorics is a field of mathematics concerned with counting and finite structures. Combinatorics is a very diverse subject that has many applications to other fields of mathematics and computer science. The goal of this course is to introduce you to a variety of techniques and ideas that will help you solve a wide range of problems. For example, you probably know the algebraic identity $1 + 2 + \ldots + n = \frac{n(n-1)}{2}$. A combinatorial proof of this identity can be given by simply counting a set of objects in two different ways, and proofs of this sort are very transparent and enlightening. The topics we will cover include graphs, recurrences, induction, generating functions, inclusion-exclusion and probability.

Texbook

We will use *Applied Combinatorics* by M.T. Keller and W.T. Trotter, 2017 edition as our main textbook. Available for free at `http://www.rellek.net/appcomb/` in many formats. You can also purchase a printed copy from the UofT Bookstore.

Contact information

Instructor	Lectures	Office	Office Hours	Email
Balázs Elek	Tu. 13:00-14:00, BA 1130 Th. 13:00-15:00, BA 1130	BA6402	We. 13:00-15:00 Th. 15:00-16:00	`balazse@math.utoronto.ca`

TA	Tutorials	Email
Keegan Dasilva Barbosa	Tu 14:00-15:00, BA1210 Tu 15:00-16:00, BA1210	`keegan.dasilvabarbosa@mail.utoronto.ca`
Reila Zheng	Tu 14:00-15:00, BA2139 Fr. 12:00-13:00, BA1240	`reila@math.utoronto.ca`

E-mail policy: Please **do not** ask us math questions by email. Emails are a poor medium for explanations, and these questions are much better asked and answered elsewhere:

If you have a question about the class material, check the syllabus and the Quercus announcements first. Next, use the Discussion section on the course Quercus – many of your fellow classmates may have the same question, and others may know the answer to your question. Of course, you are always welcome to ask questions during lecture, tutorials or office hours.

If you have a question or concern about administrative aspects of the course, illness verification forms and other requests for special accommodation; send an email to the instructor.

E-mails must include "MAT344" in the subject. Responses will come within 1-2 business days.

Important dates

- May 12, *Last day to enroll*
- June 13, *Last class before break*
- June 15, *Break begins*
- June 17, *Midterm*
- July 2, *Classes resume*
- July 15, *Last day to drop*
- August 8, *Last day of class*

Learning outcomes

Students will demonstrate the ability to:
- Select and justify appropriate tools (induction, graphs, recurrences, complexity theory, generating functions, probability) to analyze a counting problem.
- Analyze a counting problem by proving an exact or approximate enumeration, or a method to compute one efficiently.
- Describe solutions to iterated processes by relating recurrences to induction, generating functions, or combinatorial identities.
- Identify when an exact solution is intractable, and use estimates to describe its approximate size.
- Prove combinatorial identities by counting a set of objects in two ways.
- Construct counting problems which show the usefulness or limitations of combinatorial tools.

Assessment

When you submit work for assessment, you are usually interested in the mark you receive. Graded assessments are *summative* - we are evaluating how well you demonstrate understanding, and adding up all of these to give a final grade. Ideally, you will also be interested in improving your work on later assessments. Assessments which give you feedback to improve are *formative* - we are hoping to guide you toward improvement.

The final exam is strictly summative. Work that you do in class, tutorials, or show us during office hours is strictly formative. The weekly assignments and midterm are both summative and formative. Every formative assessment is an opportunity to gain confidence or improve your abilities, so you should take full advantage of them, even when they are not graded.

Crowdmark

To help you effectively get feedback on your problem sets and midterm exam, we are using Crowdmark for all graded assessments, so that we can give more detailed feedback and you can easily access it. In particular, **all problem sets must be turned in via Crowdmark**. Uploads to Crowdmark must be legible - if a grader cannot read your file, it will not be marked and you will not be allowed to resubmit it.

If you would like to request a regrade on an assessment, you will need to make a written submission on Crowdmark explaining what you believe was marked incorrectly. TAs will not discuss grading in tutorials. If an assessment is regraded, it will be carefully scrutinized, and your mark may go down.

Assessments

Problem Sets
Weekly, due Tuesdays at 4am — *10%*
10 problem sets covering the previous week's material, due each Monday.
Best 8 out of 10 will count.

Tutorial participation
Weekly — *10%*
11 Tutorial sessions, if you attend and participate, you will receive full marks for the tutorial.
Best 9 out of 11 will count

Midterm
Monday, June, 17, 13:10-15:00 — *30%*
Material from the first 6 weeks, up to and including chapter 5

Final Exam
To be Scheduled — *50%*
Cumulative

Calculator policy

Calculators are not necessary for the problem sets and the exams; for that reason, they are not allowed during exams. We would much prefer that you write your solutions in the form of 17^6 rather than 24137569 (for example).

Late/Missed Assessments

Best 8 of 10 problem sets count, so missing at most two does not require a reason. Late problem sets will not be accepted. Missing more than two assignments will require an illness verification form.

Best 9 of 11 tutorials will count, so missing at most two does not require a reason. Since the tutorial marks are for participation, you should attend unless you have a very good reason. If you have a serious long term illness that prevents you from attending multiple sessions, email the instructor.

Most midterm conflicts can be resolved with an early sitting, on June 17, 10:10-12:00. If you have an academic conflict, email the instructor with a photo of your schedule. Missing the midterm for medical reasons requires a verification of illness form, and the weight will be put on the final exam. The form can be found at `http://www.illnessverification.utoronto.ca`, and must be filled out by a doctor. Submitting a forged medical note is an academic offence.

Tutorials

Tutorials start the second week of classes, May 13. Tutorials give you the opportunity to work in-depth on problems in small groups with TA guidance. The problems will require you to apply course concepts and justify your solutions to others. You also get participation credit, which contributes to your final grade. Tutorials will be your best opportunity to practice solving novel questions under time constraints, like you would on a test, and get immediate feedback on your solutions from peers and TAs. Solutions will be posted on Quercus after all tutorials have finished.

Role in your program

Prerequisites: MAT 223. We will expect you to have a solid understanding of algebraic manipulations, solving linear systems and set notation, as well as some familiarity with writing proofs.

Programs recommending MAT344: Applied Math Specialist, Focus in Theory of Computation, Math Applications in Economics and Finance, Math & Applications Teaching Specialist, Math Major.

Role in your program: We hope to explain the connection between enumeration and algorithm complexity, and motivate pedagogical questions which can be solved by combinatorial methods, while maintaining the standards of 300-level mathematics courses. These standards include clear communication in written proofs.

Calendar

Strings and Sets **Week 1**
Chapter 2.1-2.3 *May 6–May 10*
Introduction to enumeration of strings of letters or numbers with restrictions, as well as permutations and combinations

Binomial Coefficients **Week 2**
*Chapter 2.4-2.6, **PS 1 due May 13*** *May 13–May 17*
Combinatorial proofs and the binomial theorem

Well-ordering, Recurrence and Induction **Week 3**
*Chapter 3.1-3.8, **PS 2 due May 20*** *May 20–May 24*
Our first look at recurrence relations, motivating the formal proof system of induction.

Strong Induction, Pigeonhole Principle, and Complexity **Week 4**
*Chapters 3.9, 4, **PS 3 due May 27*** *May 27–May 31*
A variant of induction suitable for recurrences; a famous existance theorem; and an introduction to growth rates of functions, motivated by how difficult certain enumeration problems are.

Graph basics **Week 5**
*Chapter 5.1-5.4, **PS 4 due Jun 3*** *June 3–June 7*
Notation and basic properties of graphs.

Counting graphs **Week 6**
*Chapter 5.5-5.7, **PS 5 due Jun 10*** *June 10–June 14*
Both exact and asymptotic enumerations for certain graphs of given size.

Break **No classes**
***Midterm Jun 17**, up to and including Chapter 5* *June 15–July 1*

Inclusion-Exclusion **Week 7**
Chapter 7 *July 1–July 4*
A counting principle that applies to collections of intersecting sets. Applications of Inclusion-exclusion

Generating functions **Week 8**
*Chapter 8, **PS 6 due Jul 8*** *July 8–July 12*
A bookkeeping method to store information about sequences in a useful way.

Recurrences **Week 9**
*Chapter 9, **PS 7 due Jul 15*** *July 15–July 17*
A bookkeeping method to store information about sequences in a useful way.

Probability **Week 10**
*Chapter 10.1-10.3, **PS 8 due Jul 22*** *July 22–July 26*
Basic concepts and their relation to enumeration.

Discrete random variables and Graph algorithms **Week 11**
*Chapters 10.4-10.6, 12.1-12.2, **PS 9 due Jul 29*** *July 29–August 2*
Using probability to compute expectation. Finding minimal spanning trees.

Graph algorithms **Week 12**
*Chapters 12.3-12.6, 13, **PS 10 due Jul Aug 5*** *August 5–August 9*
Minimal spanning trees of directed graphs. Networks and the Max-flow min-cut theorem.

Collaboration Policy, and Academic Integrity

Collaboration and discussion are an important part of mathematics. A great part of having a large group learning combinatorics is that you can help each other by discussing concepts and difficulties you have outside of class. Naturally, this will extend to working on problem sets, so we need to establish boundaries to let us fairly evaluate everyone. You are welcome to discuss problem set questions together, but all work that you submit must be your own. Remember that cheating is always possible and may increase your homework grade a bit. But it will hurt your appreciation of yourself, your knowledge and your exam grades a lot more.

Here are some tips to avoid committing an academic offence:
- Do not photograph or copy anyone's problem set solutions.
- Write your final draft alone, using only your own notes.
- Do not share your drafts with any other students.
- Do not ask for solutions to problem set questions online, post solutions, or look up posted solutions.

For example, if you type up your solutions using notes that anyone else wrote, or if you write your solutions with another student looking over your shoulder, you are both committing an academic offence.

For more information, see `https://www.academicintegrity.utoronto.ca/`.

Resources

We recommend LaTeXfor typesetting assignments. It is the standard for mathematical writing, and produces great looking documents (like this syllabus). If you are new to LaTeX, try Overleaf for an online editor or LyX (with TexLive or MacTex) for an introductory desktop editor.

You can find Sage Math cells at `https://sagecell.sagemath.org/` for quick computations. Sage is based on Python, and many helpful articles are available. The online version of the textbook also has embedded Sage Math cells with some code already written to demonstrate the computations (for example, in `http://www.rellek.net/book/s_intro_number.html`).

There are office hours on Wednesdays 13:00-15:00 every week so that students can drop in to get advice about course material, assessments, or you academic goals. Stop by and say hi!

Other books

You may want to learn about other combinatorial topics that we don't have time to discuss, like Latin squares or block designs, or more about specific topics. Some other books you may find relevant are:

- *Combinatorics Through Guided Discovery* by K. P. Bogart is an inquiry-based learning book that covers many of the topics of the course, and is a great source for insightful problems. Available for free at `http://bogart.openmathbooks.org/`.
- *Combinatorics* by J. Morris is a more traditional theorem-proof style textbook that includes topics like Latin squares and designs. Available for free at `http://www.cs.uleth.ca/~morris/Combinatorics/Combinatorics.html`.
- *Combinatorics and Graph Theory* by D. Guichard is another more traditional style textbook with an emphasis on Graph theory. Available for free at `https://www.whitman.edu/mathematics/cgt_online/book/`.
- *generatingfunctionology* by H. S. Wilf is a book focused on generating functions and their applications. Available for free at `https://www.math.upenn.edu/~wilf/DownldGF.html`