

# clipper

The PyPHS\* development team<sup>1</sup>

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## 1 System netlist

line	label	dictionary.component	nodes	parameters
$\ell_1$	input	electronics.source	('','#', 'A')	{ type voltage
$\ell_2$	d1	electronics.diode	('B', '#')	{ Is ('Is', 2e-09)
$\ell_3$	C	electronics.capacitor	('A', 'B')	{ R ('Rd', 0.5)
$\ell_4$	output	electronics.source	('B', '#')	{ v0 ('v0', 0.026)
$\ell_5$	d2	electronics.diode	('','#', 'B')	{ mu ('mu', 1.7)

”

## 2 Core dimensions

The system

$$\dim(\mathbf{l}) = n_{\mathbf{l}} = 1$$

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\*<https://pyphs.github.io/pyphs/>

<sup>†</sup><https://www.ircam.fr/recherche/equipes-recherche/systemes-et-signaux-sonores-audioacoustique-instruments-s>

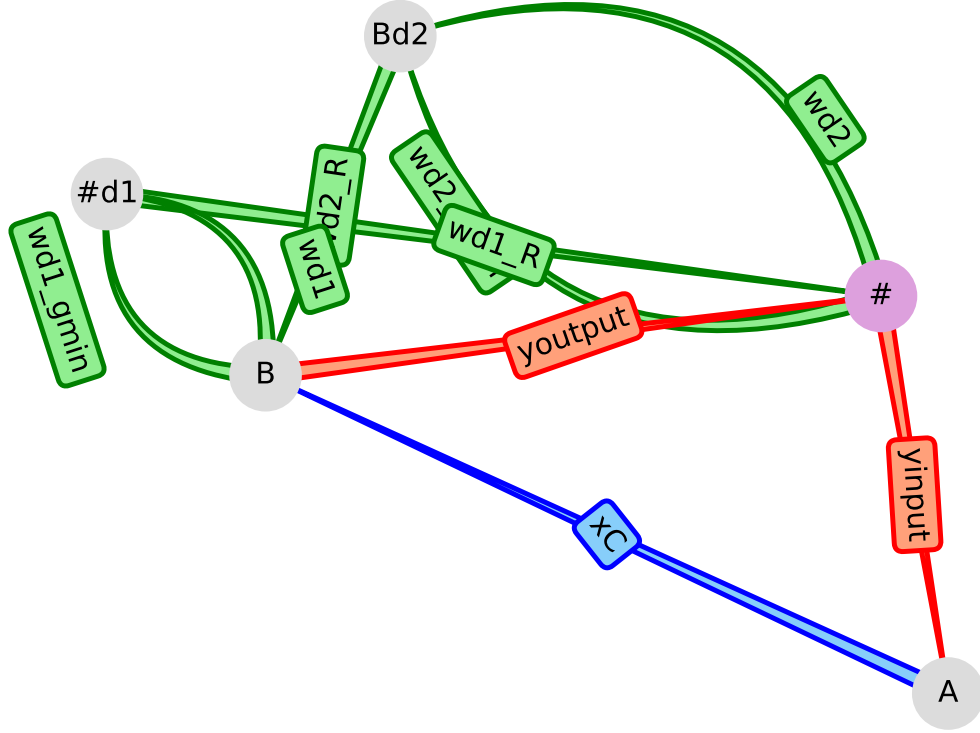


Figure 1: Graph of system clipper.

$$\dim(\mathbf{n}_l) = n_{n_l} = 2$$

$$\dim(\mathbf{x}) = n_{\mathbf{x}} = 1$$

$$\dim(\mathbf{x}_l) = n_{x_l} = 1$$

$$\dim(\mathbf{x}_{n_l}) = n_{x_{n_l}} = 0$$

$$\dim(\mathbf{w}) = n_{\mathbf{w}} = 2$$

$$\dim(\mathbf{w}_l) = n_{w_l} = 0$$

$$\dim(\mathbf{w}_{n_l}) = n_{w_{n_l}} = 2$$

$$\dim(\mathbf{y}) = n_{\mathbf{y}} = 2$$

$$\dim(\mathbf{p}) = n_{\mathbf{p}} = 0$$

$$\dim(\mathbf{o}) = n_{\mathbf{o}} = 0$$

$$\dim(\mathbf{cy}) = n_{\mathbf{cy}} = 0$$

### 3 Core quantities

#### 3.1 Core constants

parameter	value (SI)
$I_s$	2e-09
$R_d$	0.5
$v_0$	0.026
$m_u$	1.7
$g_{\min}$	1e-12
$C_{\text{symbol}}$	1e-09

#### 3.2 Core variables

The system variables are:

- the *state*  $\mathbf{x} : t \mapsto \mathbf{x}(t) \in \mathbb{R}^1$  associated with the system's energy storage:

$$\mathbf{x} = \begin{pmatrix} x_C \end{pmatrix}$$

- the *state increment*  $\mathbf{d}_{\mathbf{x}} : t \mapsto \mathbf{d}_{\mathbf{x}}(t) \in \mathbb{R}^1$  that represents the numerical increment during a single simulation time-step:

$$\mathbf{d}_{\mathbf{x}} = \begin{pmatrix} d_{xC} \end{pmatrix}$$

- the *dissipation variable*  $\mathbf{w} : t \mapsto \mathbf{w}(t) \in \mathbb{R}^2$  associated with the system's energy dissipation:

$$\mathbf{w} = \begin{pmatrix} w_{d1} \\ w_{d2} \end{pmatrix}$$

### 3.3 Core inputs

The input (*i.e.* controlled quantities) are:

- the *input variable*  $\mathbf{u} : t \mapsto \mathbf{u}(t) \in \mathbb{R}^2$  associated with the system's energy supply (sources):

$$\mathbf{u} = \begin{pmatrix} u_{\text{output}} \\ u_{\text{input}} \end{pmatrix}$$

- the *parameters*  $\mathbf{p} : t \mapsto \mathbf{p}(t) \in \mathbb{R}^0$  associated with variable system parameters:

$$\mathbf{p} = \text{Empty}$$

### 3.4 Core outputs

The output (*i.e.* observed quantities) are:

- the *output variable*  $\mathbf{y} : t \mapsto \mathbf{y}(t) \in \mathbb{R}^2$  associated with the system's energy supply (sources):

$$\mathbf{y} = \begin{pmatrix} y_{\text{output}} \\ y_{\text{input}} \end{pmatrix}$$

- the *observer*  $\mathbf{o} : t \mapsto \mathbf{o}(t) \in \mathbb{R}^0$  associated with functions of the above quantities:

$$\mathbf{o} = \text{Empty}$$

### 3.5 Core connectors

The connected quantities are:

- the *connected inputs*  $\mathbf{u}_c : t \mapsto \mathbf{u}_c(t) \in \mathbb{R}^0$

$$\mathbf{u}_c = \text{Empty}$$

- the *connected outputs*  $\mathbf{y}_c : t \mapsto \mathbf{y}_c(t) \in \mathbb{R}^0$

$$\mathbf{y}_c = \text{Empty}$$

## 4 Core constitutive relations

### 4.1 Core storage function

The system's storage function is:

$$H(\mathbf{x}) = \frac{0.5}{C_{\text{symbol}}} \cdot x_C^2$$

The gradient of the system's storage function is:

$$\nabla H(\mathbf{x}) = \left( g_{xC} \right)$$

The elements of the storage function's gradient are given below:

$$g_{xC} = \frac{1.0}{C_{\text{symbol}}} \cdot x_C$$

The Hessian matrix of the storage function is:

$$\Delta H(\mathbf{x}) = \left( \frac{1.0}{C_{\text{symbol}}} \right)$$

The Hessian matrix of the linear part of the storage function is:

$$\mathbf{Q} = \left( \frac{1.0}{C_{\text{symbol}}} \right)$$

### 4.2 Core dissipation function

The dissipative function is:

$$\mathbf{z}(\mathbf{w}) = \begin{pmatrix} z_{d1} \\ z_{d2} \end{pmatrix}$$

The elements of the dissipation function are given below:

$$z_{d1} = I_s \cdot \left( e^{\frac{w_{d1}}{m_u \cdot v_0}} - 1 \right)$$

$$z_{d2} = I_s \cdot \left( e^{\frac{w_{d2}}{m_u \cdot v_0}} - 1 \right)$$

The jacobian matrix of the dissipation function is:

$$\mathcal{J}_z(\mathbf{w}) = \begin{pmatrix} \frac{I_s \cdot e^{\frac{w_{d1}}{m_u \cdot v_0}}}{m_u \cdot v_0} & 0 \\ 0 & \frac{I_s \cdot e^{\frac{w_{d2}}{m_u \cdot v_0}}}{m_u \cdot v_0} \end{pmatrix}$$

The jacobian matrix of the linear part of the dissipation function is:

$$\mathbf{Z}_l = \begin{pmatrix} g_{\min} & 0 & 0 & 0 \\ 0 & g_{\min} & 0 & 0 \\ 0 & 0 & R_d & 0 \\ 0 & 0 & 0 & R_d \end{pmatrix}$$

## 5 Core structure

$$\begin{pmatrix} \frac{d\mathbf{x}}{dt} \\ \mathbf{w} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{M}_{xx} & \mathbf{M}_{xw} & \mathbf{M}_{xy} \\ \mathbf{M}_{wx} & \mathbf{M}_{ww} & \mathbf{M}_{wy} \\ \mathbf{M}_{yx} & \mathbf{M}_{yw} & \mathbf{M}_{yy} \end{pmatrix} \cdot \begin{pmatrix} \nabla H \\ \mathbf{z} \\ \mathbf{u} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} \mathbf{M}_{xx} & \mathbf{M}_{xw} & \mathbf{M}_{xy} \\ \mathbf{M}_{wx} & \mathbf{M}_{ww} & \mathbf{M}_{wy} \\ \mathbf{M}_{yx} & \mathbf{M}_{yw} & \mathbf{M}_{yy} \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{J}_{xx} & \mathbf{J}_{xw} & \mathbf{J}_{xy} \\ -^\top \mathbf{J}_{xw} & \mathbf{J}_{ww} & \mathbf{J}_{wy} \\ -^\top \mathbf{J}_{xy} & -^\top \mathbf{J}_{wy} & \mathbf{J}_{yy} \end{pmatrix}}_{\mathbf{J}} - \underbrace{\begin{pmatrix} \mathbf{R}_{xx} & \mathbf{R}_{xw} & \mathbf{R}_{xy} \\ ^\top \mathbf{R}_{xw} & \mathbf{R}_{ww} & \mathbf{R}_{wy} \\ ^\top \mathbf{R}_{xy} & ^\top \mathbf{R}_{wy} & \mathbf{R}_{yy} \end{pmatrix}}_{\mathbf{R}}$$

### 5.1 Core M-structure

$$\mathbf{M} = \begin{pmatrix} -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} & \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} & 1.0 & -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \\ -\frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} & 0 & 0 & -\frac{1.0}{R_d \cdot g_{\min} + 1} \\ \frac{1.0}{R_d \cdot g_{\min} + 1} & 0 & -\frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} & 0 & \frac{1.0}{R_d \cdot g_{\min} + 1} \\ -1.0 & 0 & 0 & 0 & -1.0 \\ -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} & \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} & 1.0 & -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix}$$

$$\mathbf{M}_{xx} = \begin{pmatrix} -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix}$$

$$\mathbf{M}_{xw} = \begin{pmatrix} \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix}$$

$$\begin{aligned}
\mathbf{M}_{xy} &= \begin{pmatrix} 1.0 & -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{M}_{xcy} &= \text{Empty} \\
\mathbf{M}_{wx} &= \begin{pmatrix} -\frac{1.0}{R_d \cdot g_{\min} + 1} \\ \frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{M}_{ww} &= \begin{pmatrix} -\frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} & 0 \\ 0 & -\frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{M}_{wy} &= \begin{pmatrix} 0 & -\frac{1.0}{R_d \cdot g_{\min} + 1} \\ 0 & \frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{M}_{wcy} &= \text{Empty} \\
\mathbf{M}_{yx} &= \begin{pmatrix} -1.0 \\ -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{M}_{yw} &= \begin{pmatrix} 0 & 0 \\ \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{M}_{yy} &= \begin{pmatrix} 0 & -1.0 \\ 1.0 & -\frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{M}_{ycy} &= \text{Empty} \\
\mathbf{M}_{cyx} &= \text{Empty} \\
\mathbf{M}_{cyw} &= \text{Empty} \\
\mathbf{M}_{cyy} &= \text{Empty} \\
\mathbf{M}_{cy cy} &= \text{Empty}
\end{aligned}$$

## 5.2 Core J-structure

$$\begin{aligned}
\mathbf{J} &= \begin{pmatrix} 0 & \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} & 1.0 & 0 \\ -\frac{1.0}{R_d \cdot g_{\min} + 1} & 0 & 0 & 0 & -\frac{1.0}{R_d \cdot g_{\min} + 1} \\ \frac{1.0}{R_d \cdot g_{\min} + 1} & 0 & 0 & 0 & \frac{1.0}{R_d \cdot g_{\min} + 1} \\ -1.0 & 0 & 0 & 0 & -1.0 \\ 0 & \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} & 1.0 & 0 \end{pmatrix} \\
\mathbf{J}_{xx} &= \text{Zeros} \\
\mathbf{J}_{xw} &= \begin{pmatrix} \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{J}_{xy} &= \begin{pmatrix} 1.0 & 0 \end{pmatrix} \\
\mathbf{J}_{xcy} &= \text{Empty} \\
\mathbf{J}_{wx} &= \begin{pmatrix} -\frac{1.0}{R_d \cdot g_{\min} + 1} \\ \frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{J}_{ww} &= \text{Zeros} \\
\mathbf{J}_{wy} &= \begin{pmatrix} 0 & -\frac{1.0}{R_d \cdot g_{\min} + 1} \\ 0 & \frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{J}_{wcy} &= \text{Empty} \\
\mathbf{J}_{yx} &= \begin{pmatrix} -1.0 \\ 0 \end{pmatrix} \\
\mathbf{J}_{yw} &= \begin{pmatrix} 0 & 0 \\ \frac{1.0}{R_d \cdot g_{\min} + 1} & -\frac{1.0}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{J}_{yy} &= \begin{pmatrix} 0 & -1.0 \\ 1.0 & 0 \end{pmatrix} \\
\mathbf{J}_{ycy} &= \text{Empty} \\
\mathbf{J}_{cyx} &= \text{Empty} \\
\mathbf{J}_{cyw} &= \text{Empty} \\
\mathbf{J}_{cyy} &= \text{Empty} \\
\mathbf{J}_{cyey} &= \text{Empty}
\end{aligned}$$

### 5.3 Core R-structure

$$\begin{aligned}
\mathbf{R} &= \begin{pmatrix} \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} & 0 & 0 & 0 & \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \\ 0 & \frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} & 0 & 0 & 0 \\ 0 & 0 & \frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} & 0 & 0 & 0 & \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{R}_{xx} &= \begin{pmatrix} \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{R}_{xw} &= \text{Zeros} \\
\mathbf{R}_{xy} &= \begin{pmatrix} 0 & \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
\mathbf{R}_{xcy} &= \text{Empty}
\end{aligned}$$



$$\begin{aligned}
& \mathbf{R}_{\mathbf{w}\mathbf{x}} = \text{Zeros} \\
\mathbf{R}_{\mathbf{w}\mathbf{w}} &= \begin{pmatrix} \frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} & 0 \\ 0 & \frac{1.0 \cdot R_d}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
& \mathbf{R}_{\mathbf{w}\mathbf{y}} = \text{Zeros} \\
& \mathbf{R}_{\mathbf{w}\mathbf{c}\mathbf{y}} = \text{Empty} \\
\mathbf{R}_{\mathbf{y}\mathbf{x}} &= \begin{pmatrix} 0 \\ \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
& \mathbf{R}_{\mathbf{y}\mathbf{w}} = \text{Zeros} \\
\mathbf{R}_{\mathbf{y}\mathbf{y}} &= \begin{pmatrix} 0 & 0 \\ 0 & \frac{2.0 \cdot g_{\min}}{R_d \cdot g_{\min} + 1} \end{pmatrix} \\
& \mathbf{R}_{\mathbf{y}\mathbf{c}\mathbf{y}} = \text{Empty} \\
& \mathbf{R}_{\mathbf{c}\mathbf{y}\mathbf{x}} = \text{Empty} \\
& \mathbf{R}_{\mathbf{c}\mathbf{y}\mathbf{w}} = \text{Empty} \\
& \mathbf{R}_{\mathbf{c}\mathbf{y}\mathbf{y}} = \text{Empty} \\
& \mathbf{R}_{\mathbf{c}\mathbf{y}\mathbf{c}\mathbf{y}} = \text{Empty}
\end{aligned}$$