



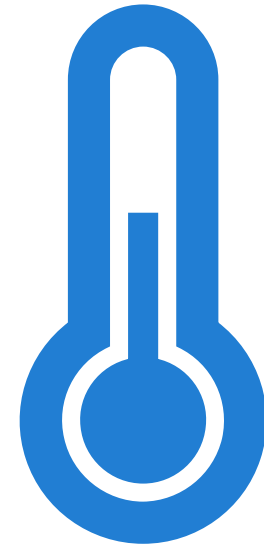
FORECASTING MONTHLY AVERAGE TEMPERATURE IN ROME

SARIMA AND SARIMAX MODELS WITH CLIMATE COVARIATES

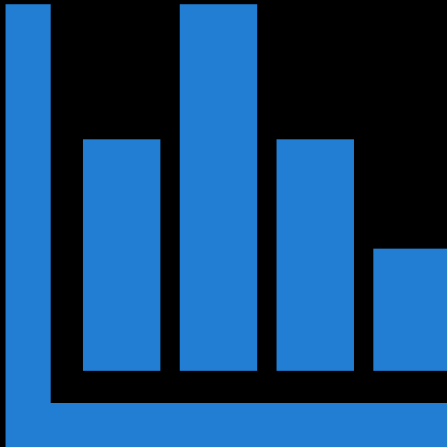
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Motivation and Research Questions

- Context: Monthly average temperature (TAVG) for Roma Ciampino station, Jan 1951 – Sep 2022.
- Why temperature?
Affects energy demand, health risks, agriculture, tourism.
Mediterranean cities face rising temperatures and more frequent heat extremes.
- Methodological goal:
Compare simple benchmarks, SARIMA, and SARIMAX models.
Assess whether climate covariates improve forecasting performance.
- Research questions:
How accurate can 1-step-ahead forecasts of TAVG be?
Do precipitation, heat days, ENSO, and seasonal dummies add predictive power beyond SARIMA?



Data and Variables



- Sample: 861 monthly observations (1951–2022), Roma Ciampino station (ID IT000016239).
- Target variable:
TAVGt : monthly average temperature in °C (used in levels).
- Covariates:
PRCPt : total monthly precipitation.
DX90t : number of days with maximum temperature above a high threshold (heat days).
ENSOt : Niño 3.4 anomaly index (global climate driver).
- Calendar features:
 $\sin(2\pi\text{month}/12)$ and $\cos(2\pi\text{month}/12)$ to capture cyclical seasonality.
- Lags and split:
Regressors enter mostly with one-month lags (e.g. *PRCPt-1*, *DX90t-1*, *ENSOt-1*).
Training sample: 1951–2008; test sample: 2008–2022 for genuine out-of-sample evaluation.

Exploratory Analysis

- Main features of TAVG:

Very regular annual cycle with strong seasonality.

Gradual warming over time: recent decades warmer than early sample.

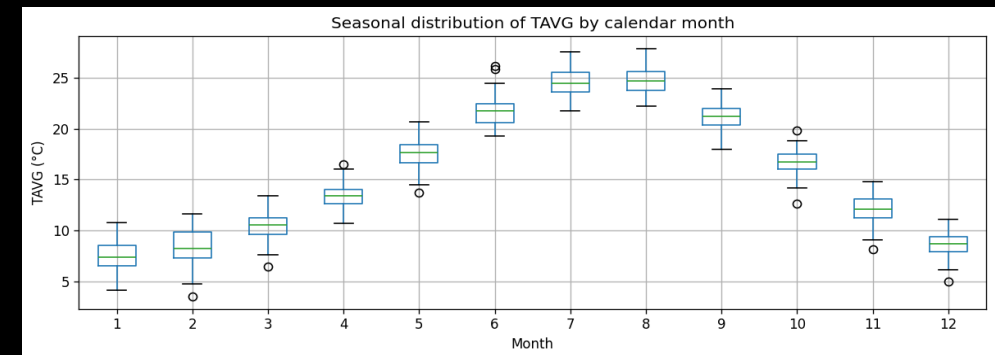
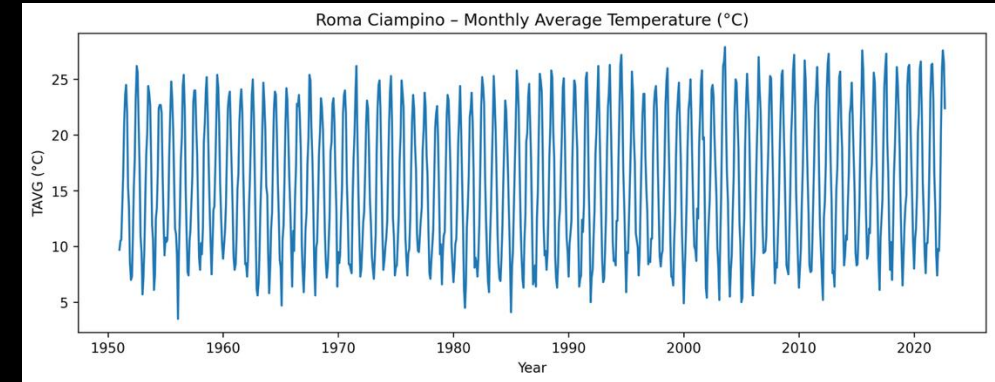
Winter months around 7–9°C; summer months around 25°C.

Overall mean $\approx 15.6^\circ\text{C}$, standard deviation $\approx 6.3^\circ\text{C}$.

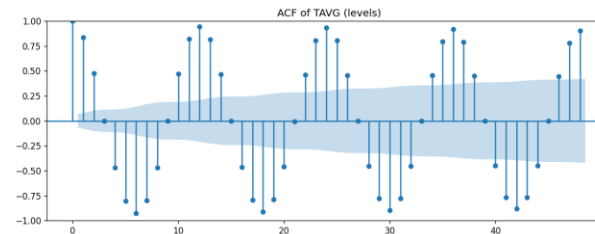
- Monthly boxplots confirm:

Classical Mediterranean pattern: mild winters and hot summers.

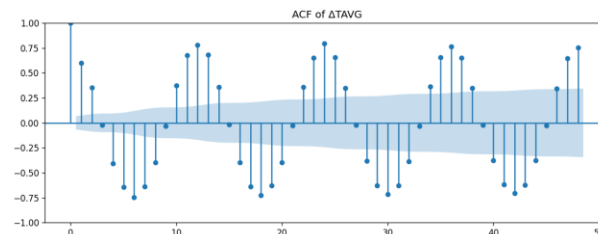
High within-month stability and large across-season differences.



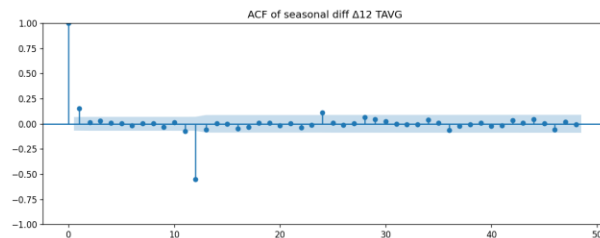
Stationarity and Autocorrelation



ACF of levels of TAVG



ACF first differences of TAVG



ACF seasonal differences of TAVG

- Unit root testing:
ADF test on TAVG_t: test statistic around -3.82 , p -value ≈ 0.003 .
Reject simple unit root, but clear seasonality and low-frequency movements remain.
- Differencing strategy:
Apply one non-seasonal difference $(1 - L)$.
Apply one seasonal difference at lag 12 $(1 - L_{12})$.
Transformed series: $(1 - L)(1 - L_{12})TAVG_t$.

Models Considered

Benchmark models:

- Constant mean: $y_t = \mu + \varepsilon_t$.
- Random walk with drift: $y_t = \mu + y_{t-1} + \varepsilon_t$.
- Seasonal naive: $\hat{y}_{t+h|t} = y_{t+h-12}$.

SARIMA model:

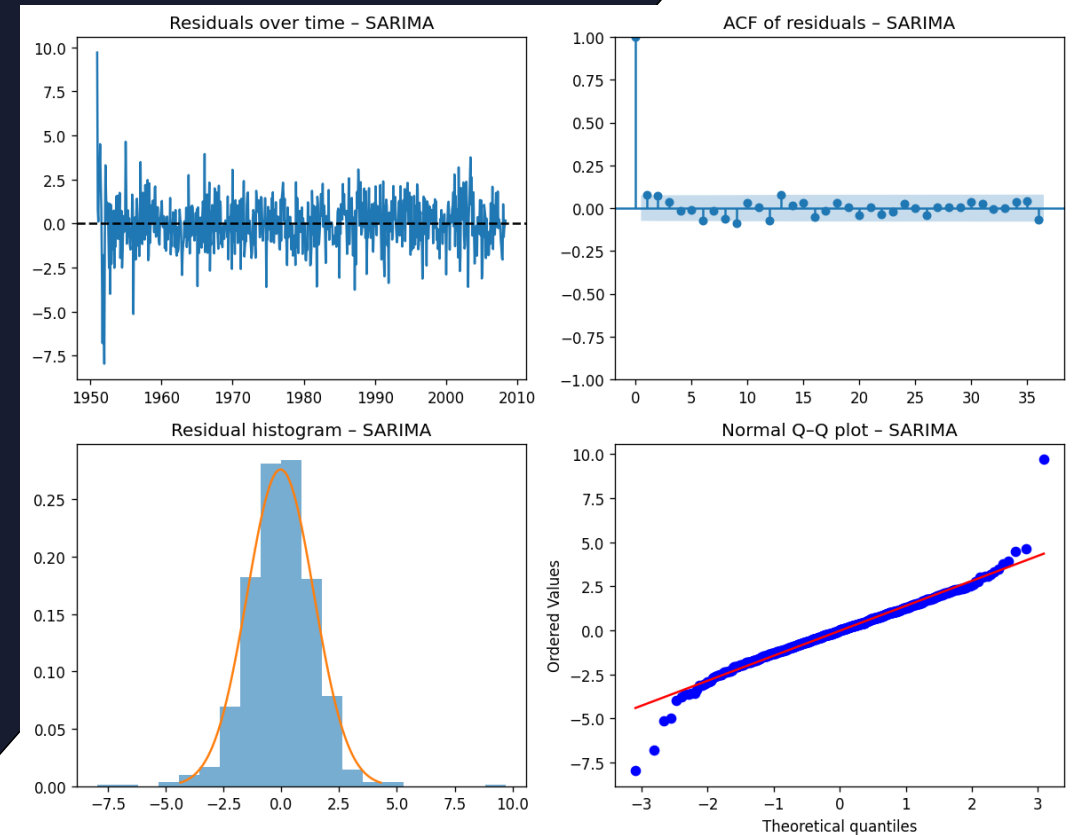
- Seasonal ARIMA with both non-seasonal and seasonal differencing.
- Orders chosen via AIC/BIC over a grid of candidates.
- Selected specification: $\text{SARIMA}(0, 1, 2) \times (0, 1, 1)_{12}$.

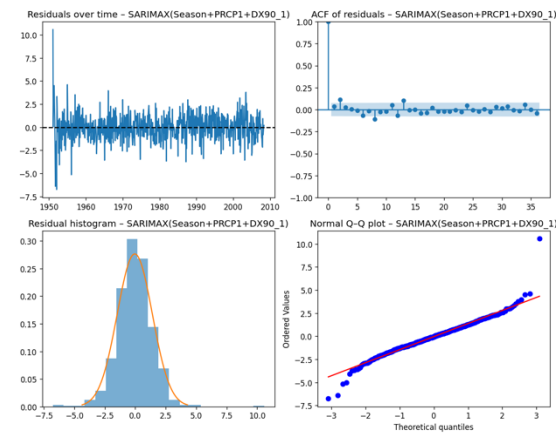
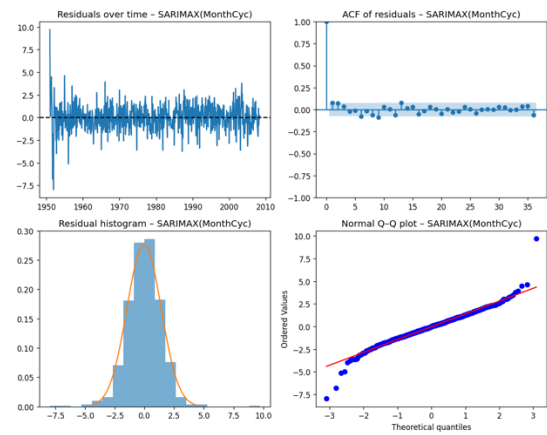
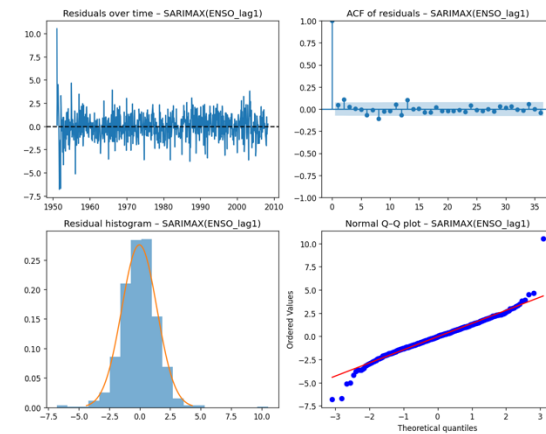
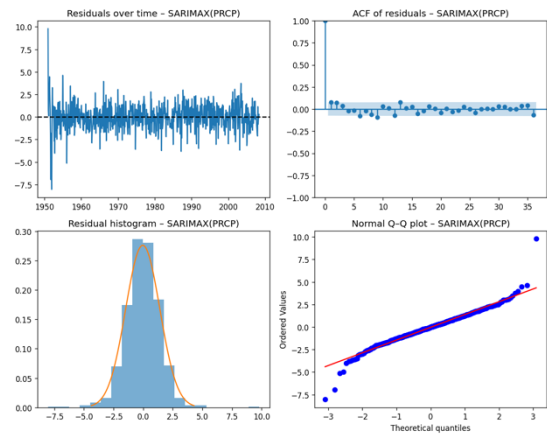
SARIMAX models:

- Same differencing structure plus exogenous regressors.
- Variants:
 - Lagged precipitation PRCP_{t-1} .
 - Month sin/cos seasonality.
 - Four-exogenous set: PRCP_{t-1} , DX_{90t-1} , ENSOT , ENSOT_{-1} .
 - ENSO-only specifications (e.g., ENSOT_{-1}).

Estimation Strategy and Diagnostics

- Train–test setup:
Models estimated on 1951–2008.
Genuine 1-step-ahead forecasts produced for 2008–2022.
- Model selection:
SARIMA orders chosen by AIC/BIC.
SARIMAX variants compared on test RMSFE and MAFE.
- Residual diagnostics (SARIMA):
Residual ACF shows no strong remaining autocorrelation.
Residuals approximately symmetric; mild tail deviations only.





I-Step-Ahead Forecast Results

- Metrics:

Root Mean Squared Forecast Error (RMSFE).

Mean Absolute Forecast Error (MAFE).

- Benchmarks:

Constant mean and RW with drift: RMSFE around 6.6–7.2.

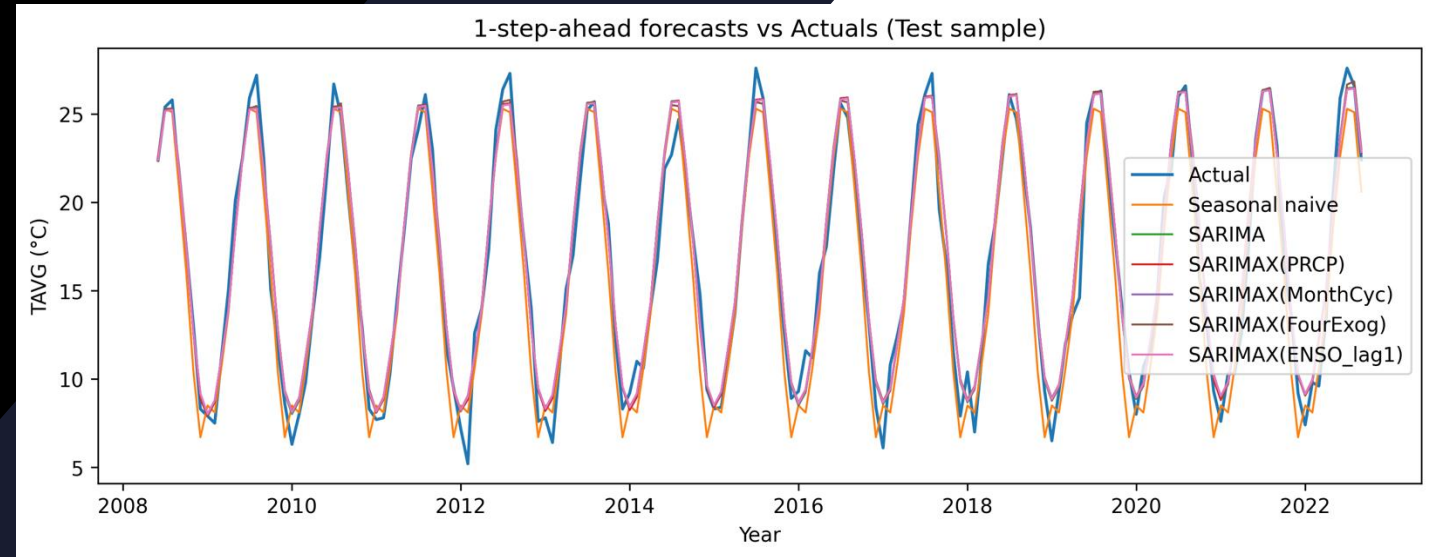
Seasonal naive: RMSFE ≈ 1.65 .

- SARIMA vs SARIMAX:

SARIMA(0, 1, 2) \times (0, 1, 1)₁₂: RMSFE ≈ 1.25 , MAFE ≈ 0.97 .

Best SARIMAX with PRCP -1 : RMSFE ≈ 1.2490 , MAFE ≈ 0.9643 .

Exogenous variables give only marginal gains over SARIMA.



10-Year Ahead Forecast Scenario

- Forecast horizon: Oct 2022 – Sep 2032.

- Models used:

Selected SARIMA model.

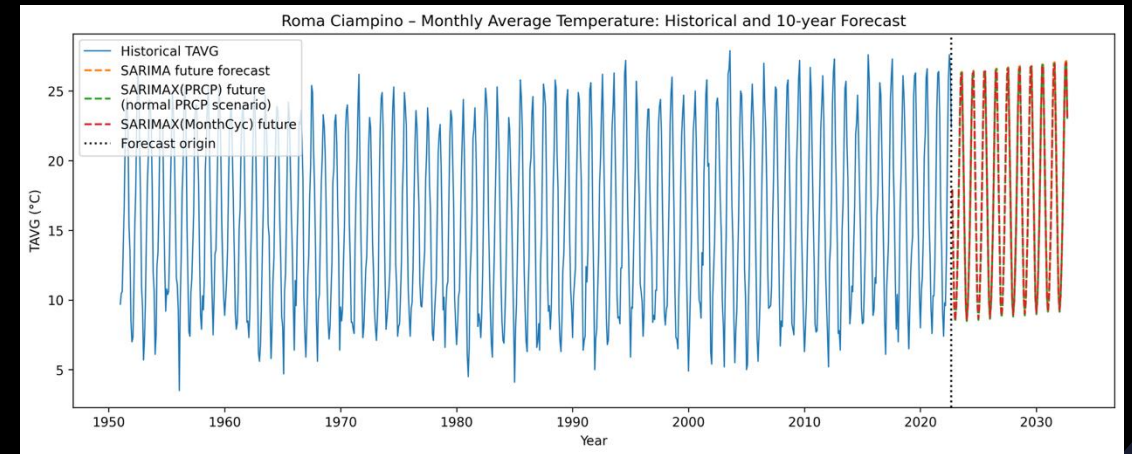
Best-performing SARIMAX specifications.

- Qualitative behaviour:

Very persistent seasonal pattern in all models.

Winter forecasts around 8–10°C; summer peaks around 26–27°C.

Average forecasted TAVG over the horizon $\approx 17.2^\circ\text{C}$



Conclusions and Extensions

- Main findings:

Strong seasonality: seasonal naive is already a strong benchmark.

SARIMA(0, 1, 2) × (0, 1, 1)₁₂ provides accurate and robust 1-step-ahead forecasts.

SARIMAX with lagged precipitation is slightly best in RMSFE/MAFE, but gains over SARIMA are minimal. Most predictive power is contained in the temperature series itself.

- Possible extensions:

Use richer climate covariates (radiative forcing, large-scale indices, circulation patterns).

Explore non-linear or regime-switching models and volatility (e.g., GARCH).

Produce density forecasts and prediction intervals for risk and policy analysis.
