ENG331 - ENG722
CONTROL SYSTEMS 1

Task 4 – Simulation

Assesses ILO 1: "Predict characteristics of uncompensated third order and higher linear time-invariant systems." And ILO 4: "Communicate the design process of a control system using system block diagrams and signal flow graphs."

Laboratory Information Sheet

This Laboratory is separated into four tasks:

1. Development of plant model and investigation of system responses in simulation
2. Investigation of physical plant responses
3. Evaluation and tuning of a closed-loop controller applied to a physical plant
4. Evaluation of plant behaviour in simulation

Each task will be assessed separately. There is no formal page limit but you should be able to answer each question in the space provided. Total answer text should be kept to < 2 pages.

You do not need to submit a formal structured lab report. Simply address each **highlighted question** and provide the requested measurements and tables.

Task 4 Instructions

In this task you will now be working with the Coupled tanks system entirely in simulation, and exploring in further detail the dynamic behaviour of the system and the operation of a cascaded controller.

Part 1: Modelling [10 marks, ILO 1]

Nonlinear model

1. We will start by adapting the nonlinear Simulink model created in Lab Task 1 for use in simulating the full closed-loop system in configuration #3. In Simulink create a model that represents the system in Figure 1.
    - You should have a subdiagram for each block in Figure 1.
    - Label each signal as shown in Figure 1, and label each block. You should also set all named signals to be logged, these can then be viewed using the Data Visualiser in Simulink or you can add a scope to each signal.
    - At this stage the controller block should just pass the input signal ($e(t)$) to its output ($V_P(t)$).
    - The Tank 1 block should output both the height and the flow through the orifice. The output flow from Tank 1 should be used as the input to Tank 2.
    - You may wish to adjust the example nonlinear Simulink model for the Tanks given in the Solution to Task 1 Q7 so that it uses an integration rather than a differentiation as this is more reliable and performant when using a numerical solver.

    Use your system identification from Lab Task 2 to set the pump gain, orifice diameters and discharge coefficients for each tank.

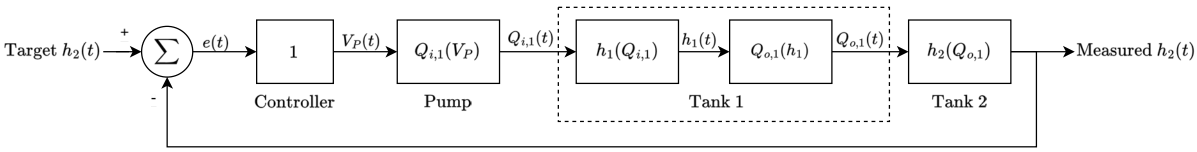

Figure 1 Nonlinear time-domain model of closed-loop system in configuration #3

2. Adjust the nonlinear model to better represent the physical system by adding limits to represent the physical limits of the pump and tank subsystems. Add saturation limits to limit the pump voltage output by the controller to $V_p \epsilon [0,12]$ V, and to limit the height of each tank

to $h_i \epsilon [0,25]$ cm. *Note: you should ensure this limit is applied to the height of each tank before it is fed back into the calculation differential equation, not just on the output of the block.*

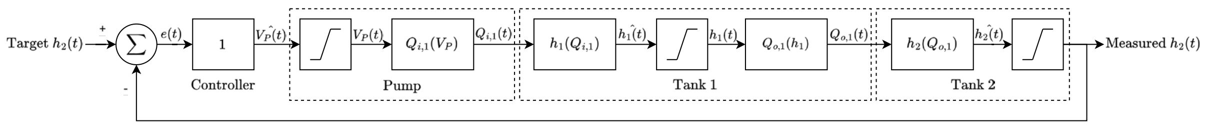

Figure 2 Nonlinear time-domain model of closed-loop system in configuration #3 with actuator and plant saturation

3. As you will have observed in Lab Tasks 2 & 3, the measurement of the Tank height is quite noisy. This is in part due to the turbulent flow in the tanks, but is also impacted by the inherent noise in the analogue measurement of the transducer voltage.

   We can represent this by adding a noise signal at the output of Tank 2. We can create the noise signal using a Random number block, which will give a normally distributed noise about a mean value and with some variance, you can play around with the parameters of this block until you get a noise signal that is similar to that observed in Lab Task 2/3 (suggested approximate numbers when working in SI units are mean=0, variance=1e-5).

   It is helpful to be able to control when this noise is present in the simulation. We can do this by multiplying the noise signal by a unit step before it is added to the output of Tank 2. By adjusting the Step Time parameter of the unit step, we can control the time at which the noise will be enabled. i.e. $n(t) = \eta(t) \times u(t - T_{noise})$ where $\eta(t)$ is a normally distributed random process, and T_{noise} is the time at which to enable the measurement noise.

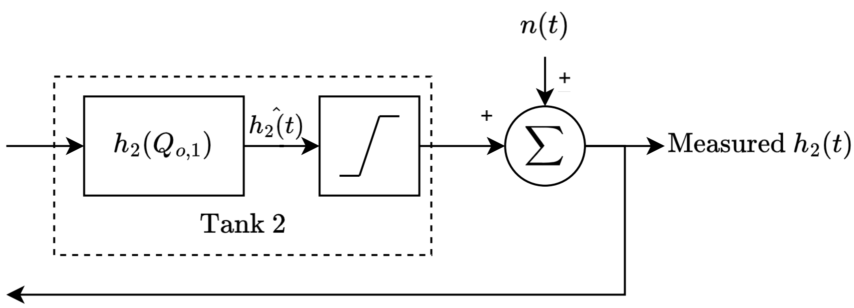

Figure 3 System with additive measurement noise.

4. You will have noticed that the apparatus is fitted with a controllable bypass valve that will open an additional outflow path from Tank 1. We can model this in the same way as we model the main orifice, but with will use a larger diameter. Modify your Simulink subsystem for Tank

1 so that there is an additional outflow that does not feed into Tank 2 but instead exits the system through an orifice with a diameter of 10mm with Cd = 0.8.

We can control when this outflow is present by multiplying the diameter of the bypass disturbance by a step function before it is used in the model calculation. We can then configure the step time of this signal so that the disturbance acts once the system has reached steady state.

5. ***Attach your completed nonlinear Simulink model to your submission as a separate file. [2 marks]***

6. ***Use your nonlinear model to generate a plot of the step response of the closed-loop system for an input signal of $\boldsymbol{r(t) = 0.01u(t) + 0.1\,m}$, and disturbance $\boldsymbol{d(t) = 0.1u(t-50)}$, and noise $\boldsymbol{n(t) = \eta(t)u(t-100)}$, where $\boldsymbol{\eta(t)}$ is the output of the random number block.*** *Ensure your plot shows $h_2(t)$ as well as the controller output and the actual pump voltage $V_P(t)$* ***[3 marks]***

The system we have developed here will act as a theoretical prediction of the real coupled tanks systems response. The model you have developed here will be critical for the Design Project. However, in order to design controllers, we need an LTI model. You could build an LTI model representation in the Laplace domain in Simulink (and you can explore this in the Design Report if you wish), but it is just as convenient to use the Control Systems Toolbox in MATLAB.

Linearised model

In Lab Tanks 1 we developed a linearised model relating the height of Tank 1 to the voltage of the pump. We need to do a little more work to adapt this to separate linearised models for the Pump and both Tanks.

We will create a MATLAB script with tf or zpk variables to represent each block in the closed loop system (see Figure 1), and use MATLAB to investigate the behaviour of the system.

Create a MATLAB script that initialises all the system and controller parameters, and calculates the transfer functions as tf or zpk objects for all blocks in the closed-loop system, and for the overall closed-loop transfer function, as below.

1. Controller:

   Create an appropriately named variable to hold the transfer function of the PIDF controller given in Lab Task 3. You will need variables for K_P, K_i, K_d, and T_f.

2. Pump:

   We could use a numeric variable for the pump gain, but in case we want to use a more complex pump model, we should instead use a zpk object with no poles or zeros. You will also need a variable for the pump gain K_{pump}.

3. Tank 1&2:

   We previously linearised this tank to give an equation for $\frac{\Delta H_1(s)}{\Delta V_P(s)}$. Because we have separated the Pump to its own system, and we need to calculate the output flow of Tank 2 rather than its height, we need to do a little more work. The linearised equation from Task 1 was:

$$A_1 \frac{d}{dt}\delta h_1(t) \approx K_{pump}\delta V_p(t) - \frac{A_{o1}C_{d1}\sqrt{2g}}{2\sqrt{h_{1,0}}}\delta h_1(t)$$

   Because $K_{pump}V_P(t) = q_i(t)$, we can write:

$$A_1 \frac{d}{dt}\delta h_1(t) \approx \delta q_i(t) - \frac{A_{o1}C_{d1}\sqrt{2g}}{2\sqrt{h_{1,0}}}\delta h_1(t)$$

   Which gives a perturbation transfer function

$$\frac{\Delta H_1(s)}{\Delta Q_i(s)} \approx \frac{\frac{1}{A_1}}{s + \frac{A_{o1}C_{d1}\sqrt{2g}}{A_1 2\sqrt{h_{1,0}}}}$$

   And because $q_{o1}(t) = A_{o1}C_{d1}\sqrt{2gh_1(t)}$, and therefore $\delta q_{o1}(t) \approx \frac{A_{o1}C_{d1}\sqrt{2g}}{2\sqrt{h_{1,0}}}\delta h_1(t)$

$$\therefore \frac{\Delta Q_{o1}(s)}{\Delta Q_i(s)} \approx \frac{\frac{A_{o1}C_{d1}\sqrt{2g}}{A_1 2\sqrt{h_{1,0}}}}{s + \frac{A_{o1}C_{d1}\sqrt{2g}}{A_1 2\sqrt{h_{1,0}}}}$$

   We can do the same for Tank 2 to get:

$$\frac{\Delta H_2(s)}{\Delta Q_{o1}(s)} \approx \frac{\frac{1}{A_2}}{s + \frac{A_{o2}C_{d2}\sqrt{2g}}{A_2 2\sqrt{h_{2,0}}}}$$

   Note that these are both linearised about the operating height of each tank.

Create tf objects for the Tank 1 and Tank 2 transfer functions. You will need variables for the tank parameters.

4. Feedback:

   We can now specify the complete closed-loop system structure using the ‘feedback’ command, with the controller, pump, Tank 1, and Tank 2 transfer functions specified in cascade, and ‘1’ as the second argument to specify negative unity feedback.

5. ***Calculate the steady-state height of Tank 1 if Tank 2 is at 10cm. [1 mark]***

   Use this to configure the transfer functions for Tank 1 and Tank 2. Configure the controller such that $K_p = 1, K_i = 0, K_d = 0, T_f = 0$. Use the same parameters for pump gain and the tank parameters as for the nonlinear Simulink model.

6. ***Plot the step response of the closed loop system for a step change in the target Tank 2 height of 0.01u(t) m.*** *Note that this is really the perturbation in Tank height 2 about the operating point, not the absolute value of the height of Tank 2. Ensure your plot shows $\delta h_2(t)$ as well as the controller output $\delta V_P(t)$* ***[2 marks]***

7. ***Include your MATLAB script with appropriate comments for this section. [2 marks]***

Part 2: Analysis [30 marks, ILO 4]

Use theory as well as your developed nonlinear Simulink model, and your now complete linearised model to answer the following questions.

1. If we model the disturbance as an additive step flow signal at the input to Tank 2 (with a negative magnitude) in the linearised model
    a. ***Use theory to predict what the steady state error in the closed loop would be using your tuned PIDF controller (from Lab Task 3) due to the disturbance alone. [2 marks]***
    b. ***Use your linearised MATLAB model (not Simulink) to plot the response to a step disturbance [2 marks]***

       Hint: think about how we used the feedback function previously to generate the closed-loop transfer function and how we would need to modify this function call if the input to the closed-loop system was the disturbance.
    c. ***Is this method of modelling the disturbance a good approximation of the actual behaviour of the system?** (compare to the simulated results from your nonlinear model)* ***[2 marks]***

2. Robustness of the controller you tuned in Lab Task 3
    a. ***Plot the root-locus of the system with your tuned controller at the tuned operating point [2 marks]***
        i. ***Comment on the sensitivity of the dominant closed-loop poles at the tuned operating point to the loop gain.** Note: you can observe this directly from the root-locus, you are not required to calculate from the sensitivity equation.* ***[2 marks]***
    b. ***Generate the Nyquist diagram of the system with your tuned controller at the tuned operating point [2 marks]***
        i. ***What are the gain and phase margins? [2 marks]***
    c. ***Generate the Nyquist diagram of the system with your tuned controller at a significantly different operating point [2 marks]***
        i. ***What are the gain and phase margins? [2 marks]***
    d. ***Use the Nyquist diagrams to comment on the robustness of the controller [2 marks]***

3. Sensitivity
    a. ***Generate the gang of four transfer functions for the tuned system [4 marks]***
    b. ***Plot the step responses and frequency responses of the sensitivity functions as Nyquist diagrams or Bode plots. [4 marks]***
    c. ***Comment on what you can observe about the behaviour of the closed-loop system from these plots. [2 marks]***