kme272 - Assesment 1.5

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1.1 Q1

```
clear
clc
pkg load symbolic
x_0=-4;
x_1=0;
x_2=4;
y_0=3;
y_1=5;
y_2=1;
syms x L_1 L_2 L_3


L_0 = ((x - x_1) * (x - x_2)) / ((x_0 - x_1) * (x_0 - x_2));
L_1 = ((x - x_0) * (x - x_2)) / ((x_1 - x_0) * (x_1 - x_2));
L_2 = ((x - x_0) * (x - x_1)) / ((x_2 - x_0) * (x_2 - x_1));
% Display the LaTeX representations
latex(L_0)
latex(L_1)
latex(L_2)
P_2=y_0*L_0+y_1*L_1+y_2*L_2;
latex(P_2)
```

$$L_0(x) = \frac{(x - x_1) \cdot (x - x_2)}{(x_0 - x_1) \cdot (x_0 - x_2)} = \frac{x\,(x-4)}{32}$$

$$L_1(x) = \frac{(x - x_0) \cdot (x - x_2)}{(x_1 - x_0) \cdot (x_1 - x_2)} = -\frac{(x-4)\,(x+4)}{16}$$

$$L_2(x) = \frac{(x - x_0) \cdot (x - x_1)}{(x_2 - x_0) \cdot (x_2 - x_1)} = \frac{x\,(x+4)}{32}$$

$$P_2(x) = y_0 \cdot L_0(x) + y_1 \cdot L_1(x) + y_2 \cdot L_2(x) = \frac{3x\,(x-4)}{32} + \frac{x\,(x+4)}{32} - \frac{5\,(x-4)\,(x+4)}{16}$$

1.2 Q2

- $$f[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0}$$
- $$f[x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1}$$
- $$f[x_2, x_3] = \frac{y_3 - y_2}{x_3 - x_2}$$
- $$f[x_0, x_1, x_2] = \frac{y_1 - y_0}{x_1 - x_0} - \frac{y_2 - y_1}{x_2 - x_1}$$
- $$f[x_1, x_2, x_3] = \frac{y_2 - y_1}{x_2 - x_1} - \frac{y_3 - y_2}{x_3 - x_2}$$
- $$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$$

Using these equations we can fill in the table

x	f[x]	f[x$_0, x_1$]	[f[x$_0, x_1, x_2$]	f[x$_0, x_1, x_2, x_3$]
-4	3			
0	5	0.5		
4	1	-1	-0.1875	
6	-1	-1	0	0.01875

$$P_3(x) = f[x_0\ ] + f[x_0\ , x_1\ ](x - x_0\ ) + f[x_0\ , x_1\ , x_2\ ](x - x_0\ )(x - x_1\ ) + f[x_0\ , x_1\ , x_2\ , x_3\ ](x - x_0\ )(x - x_1\ )(x - x_2\ )$$

$$P_3(x) = 3 + 0.5(x+4) - 1(x+4)(x) - 0.01875(x+4)(x)(x-4)$$

1.3 Q3

```
clear
clc
pkg load symbolic
syms Mn hn xn1 Mn1 hn yn yn1 xn a hn1
sn=(Mn1/(6*hn1))*(xn-xn)^3 +(Mn/(6*hn1))*(xn-xn1)^3 +
↪  ((yn1/hn1)-(1/6)*hn1*Mn1)*(xn-xn)+((yn/hn1)-(1/6)*hn1*Mn)*(xn-xn1);
snDiff=(-Mn1/(2*hn1))*(xn-xn)^2+(Mn/(2*hn1))*(xn-xn1)^2+(hn1/6)*(Mn1-Mn)+((yn-yn1)/hn1);
eq= sn+snDiff == a;
latex(eq)
```

$$\frac{M_n\,(x_n - x_{n-1})^3}{6h_{n-1}} + \frac{M_n\,(x_n - x_{n-1})^2}{2h_{n-1}} + \frac{h_{n-1}\,(-M_n + M_{n-1})}{6} + (x_n - x_{n-1})\left(-\frac{M_n h_{n-1}}{6} + \frac{y_n}{h_{n-1}}\right) + \frac{y_n - y_{n-1}}{h_{n-1}} = \alpha$$

1.4 Q4

```
clear
clc
pkg load symbolic
syms M0 M1 M2 h0 h1 h2 x0 x1 x2 y0 y1 y2 a x
s0=(M0/(6*h0))*(x1-x1)^3 +(M1/(6*h0))*(x1-x)^3 +
↪  ((y0/h0)-(0/6)*h0*M0)*(x1-x1)+((y1/h0)-(0/6)*h0*M1)*(x1-x);
eq1=subs(diff(diff(s0,x),x),x,x0);
latex(expand(simplify(eq1)))


s1=(M1/(6*h1))*(x2-x2)^3 +(M2/(6*h1))*(x2-x1)^3 +
↪  ((y1/h1)-(1/6)*h1*M1)*(x2-x2)+((y2/h1)-(1/6)*h1*M2)*(x2-x1);
eq2= s1+diff(s1,x2)==a;
latex(expand(simplify(eq2)))
```

Which gives the two following equations:

$$-\frac{M_1 x_0}{h_0} + \frac{M_1 x_1}{h_0} = 0$$

$$a = \frac{M_2 h_1 x_1}{6} - \frac{M_2 h_1 x_2}{6} - \frac{M_2 h_1}{6} - \frac{M_2 x_1^3}{6h_1} + \frac{M_2 x_1^2 x_2}{2h_1} + \frac{M_2 x_1^2}{2h_1} - \frac{M_2 x_1 x_2^2}{2h_1} - \frac{M_2 x_1 x_2}{h_1} + \frac{M_2 x_2^3}{6h_1} + \frac{M_2 x_2^2}{2h_1} - \frac{x_1 y_2}{h_1} + \frac{x_2 y_2}{h_1} + \frac{y_2}{h_1}$$

And in matrix form:

$$\begin{bmatrix} -\frac{x_0}{h_0} + \frac{x_1}{h_0} & 0 \\ 0 & \frac{M_2 h_1 x_1}{6} - \frac{M_2 h_1 x_2}{6} + \frac{M_2 h_1}{6} - \frac{M_2 x_1^3}{6h_1} + \frac{M_2 x_1^2 x_2}{2h_1} - \frac{M_2 x_1^2}{2h_1} - \frac{M_2 x_1 x_2^2}{2h_1} + \frac{M_2 x_1 x_2}{h_1} + \frac{M_2 x_2^3}{6h_1} - \frac{M_2 x_2^2}{2h_1} \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha + \frac{x_1 y_2}{h_1} - \frac{x_2 y_2}{h_1} + \frac{y_2}{h_1} \end{bmatrix}$$

\]

1.5 Q5

```
clear
clc
pkg load symbolic
syms M0 M1 M2 h0 h1 h2 x0 x1 x2 y0 y1 y2 a x x3 y3
s0=(M0/(6*h0))*(x1-x1)^3 +(M1/(6*h0))*(x1-x)^3 +
↪  ((y0/h0)-(0/6)*h0*M0)*(x1-x1)+((y1/h0)-(0/6)*h0*M1)*(x1-x);
eq1=subs(diff(diff(s0,x),x),x,x0);
s1=(M1/(6*h1))*(x2-x2)^3 +(M2/(6*h1))*(x2-x1)^3 +
↪  ((y1/h1)-(1/6)*h1*M1)*(x2-x2)+((y2/h1)-(1/6)*h1*M2)*(x2-x1);
eq2= s1+diff(s1,x2)==a;

x0value=-4;
x1value=0;
x2value=4;
x3value=6;
y0value=3;
```

```
y1value=5;
y2value=1;
y3value=-1;
h0value=x1value-x0value;
h1value=x2value-x1value;
h2value=x3value-x2value;
avalue=1/2;

eq1=subs(eq1,x0,x0value);
eq1=subs(eq1,x1,x1value);
eq1=subs(eq1,x2,x2value);
eq1=subs(eq1,x3,x3value);
eq1=subs(eq1,y0,y0value);
eq1=subs(eq1,y1,y1value);
eq1=subs(eq1,y2,y2value);
eq1=subs(eq1,y3,y3value);
eq1=subs(eq1,h0,h0value);
eq1=subs(eq1,h1,h1value);
eq1=subs(eq1,h2,h2value);
eq1=subs(eq1,a,avalue);

eq2=subs(eq2,x0,x0value);
eq2=subs(eq2,x1,x1value);
eq2=subs(eq2,x2,x2value);
eq2=subs(eq2,x3,x3value);
eq2=subs(eq2,y0,y0value);
eq2=subs(eq2,y1,y1value);
eq2=subs(eq2,y2,y2value);
eq2=subs(eq2,y3,y3value);
eq2=subs(eq2,h0,h0value);
eq2=subs(eq2,h1,h1value);
eq2=subs(eq2,h2,h2value);
eq2=subs(eq2,a,avalue);

latex(expand(simplify(eq1)))
latex(expand(simplify(eq2)))
```

Subsituting the values into the two equations gives:

$$M_1 = 0$$

$$M_2 = -\frac{9}{16}$$

Which gives the matrix equation:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{9}{16} \end{bmatrix}$$

1.6 Q6

```
% Given data points
x = [-4, 0, 4, 6];
y = [3, 5, 1, -1];
alpha = -1/2;

% Construct the matrix for the spline
```

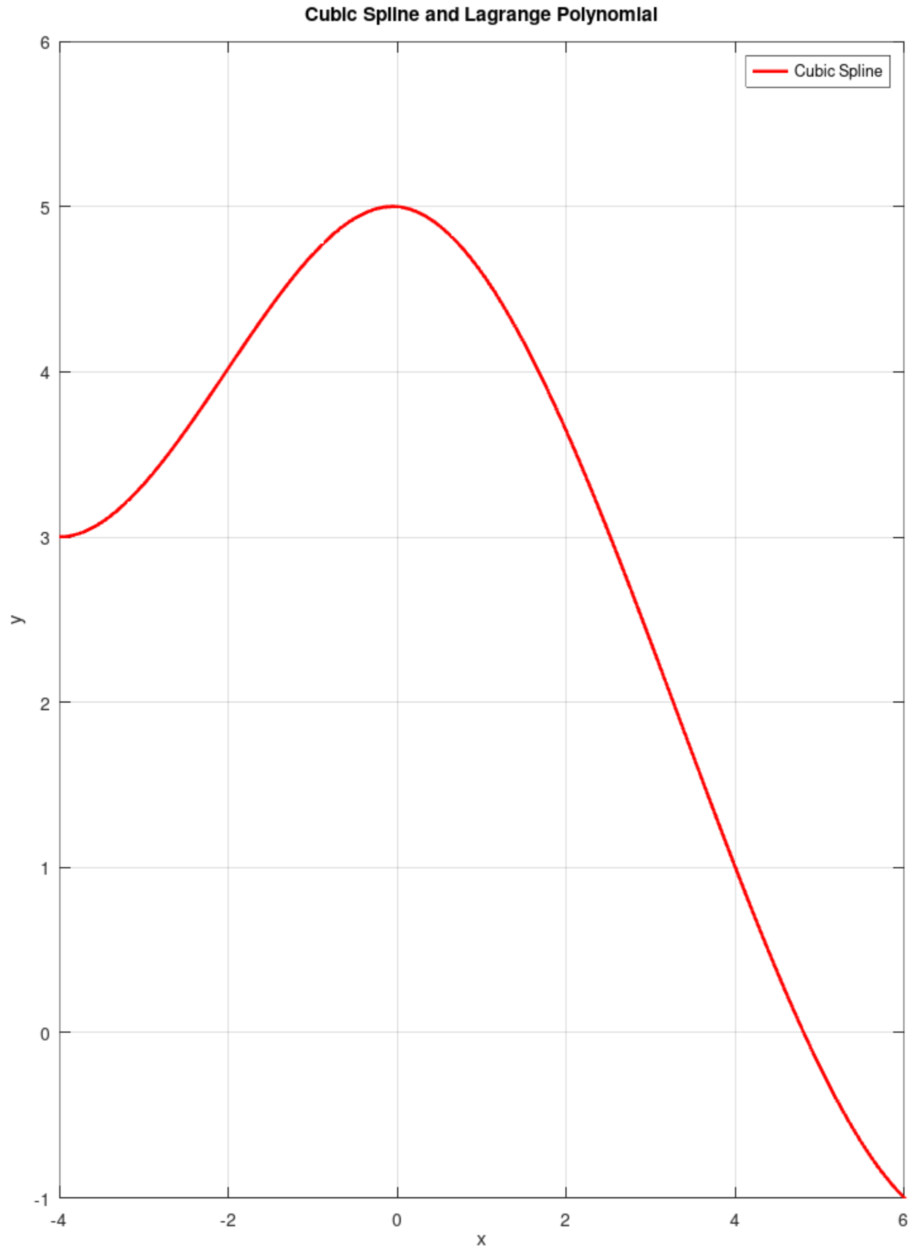
```
A = [1, 0; 0, 1];
b = [0; alpha];

% Solve for M1 and M2
M = A\b;

% Generate spline
xx = linspace(-4, 6, 100);
yy = spline(x, [M(1), y, M(2)], xx);

% Plotting
figure;
plot(xx, yy, 'r-', 'LineWidth', 2);
hold on;

% Overlay the Lagrange polynomial
P2 = @(x) ... % Define P2 based on your calculations
plot(xx, P2(xx), 'b--', 'LineWidth', 2);
legend('Cubic Spline', 'Lagrange Polynomial');
title('Cubic Spline and Lagrange Polynomial');
xlabel('x');
ylabel('y');
grid on;
hold off;
filename = sprintf('KME272-Assignment-5.png');
print(filename,'-dpng','-r100');
```


Cubic Spline and Lagrange Polynomial
Cubic Spline
y
x
6
5
4
3
2
1
0
-1
-4
-2
0
2
4
6