KME272 - Assesment 1.2

Baley Eccles - 652137

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Contents

1 KME272 - Assesment 1.2

1.1 1

1.1.1 (i)

$$g(x) = 3x^3 + x - 5$$

$$g'(x) = 9x^2 + 1$$

The minimum occurs at 1.1

$$g'(1.1) = 9 \cdot (1.1)^2 + 1 = 11.89 > 1$$

So the root will be found

1.1.2 (ii)

$$g(x) = \frac{-1}{k}(3x^3 - kx - 5)$$

$$g'(x) = \frac{-1}{k}(9x^2 - k)$$

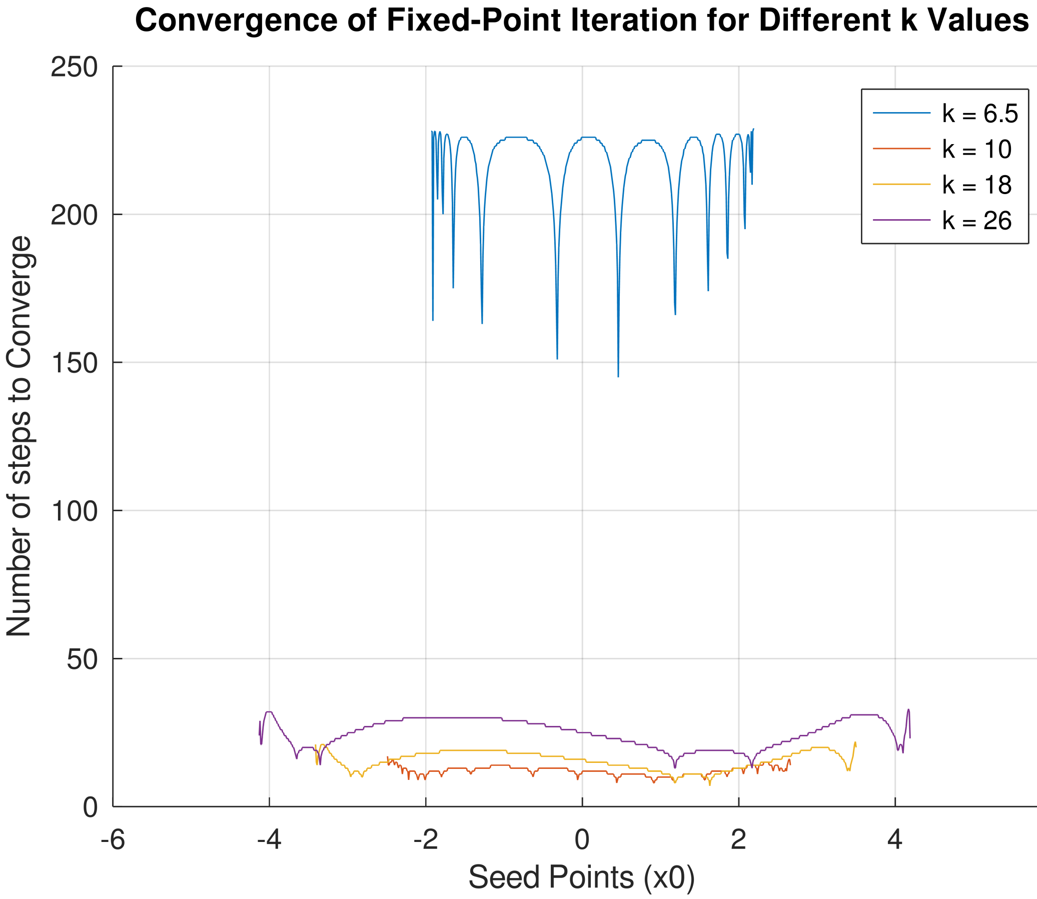
$$|g'(x)| > 1$$

$$k < 9x^2 - k$$

$$x > \sqrt{\frac{2k}{9}} \text{ or } x < -\sqrt{\frac{2k}{9}}$$

1.1.3 (iii)

```
kValues = [6.5, 10, 18, 26];
tol = 10 (-6);
seeds = -5:0.01:5; % For octave 0.001 doesnt work it is too slow
stepsCount = zeros(length(seeds), length(kValues));
maxSteps = 250;
f = @(x, k) -1/k * (3*x 3 - k*x - 5);
for j = 1:length(kValues)
    k = kValues(j);
    for i = 1:length(seeds)
        x0 = seeds(i);
        x_n = x0;
        steps = 0;

        while true
            x_n1 = f(x_n, k);
            steps = steps + 1;
            if abs(x_n1 - x_n) < tol
                break;
            end
            if steps > maxSteps
                steps = NaN;
                break;
            end
            x_n = x_n1;
        end

        stepsCount(i, j) = steps;
    end
end

fig = figure ();
hold on;

for j = 1:length(kValues)
    plot(seeds, stepsCount(:, j), 'DisplayName', ['k = '    num2str(kValues(j))]);
end

hold off;

xlabel('Seed Points (x0)');
ylabel('Number of steps to Converge');
title('Convergence of Fixed-Point Iteration for Different k Values');
legend show;
grid on;
print('KME272-Assesment-1-2-Part1.png','-dpng','-r1000');
```



1.2 2

```
% Spliting the interval
R=1;
L=10;
SubInterval = 0:0.1:R;
% Using the intermediate value theorem
for i = 1:length(SubInterval)-1
    % f(SubInterval(i),R,L)
    % f(SubInterval(i+1),R,L)
    if f(SubInterval(i),R,L) > 0 && f(SubInterval(i+1),R,L) < 0
        % the interval contains the root
        Interval = [SubInterval(i), SubInterval(i+1)];
    end
        if f(SubInterval(i),R,L) < 0 && f(SubInterval(i+1),R,L) > 0
        % the interval contains the root
        Interval = [SubInterval(i), SubInterval(i+1)];
    end
end
a=Interval(1);
b=Interval(2);
fprintf("Both root finding methods will begin on the interval [%f,%f]\n",a,b)
fprintf("------------------------------------------------------------\n",a,b)
fprintf("The bisection method\n")
step = 1;
tol = 10 -6;
while true
```

```
    c = (a+b)/2;
    if f(a,R,L) * f(c,R,L) < 0
        b=c;
    else
        a=c;
    end
    epsilon = abs(f(c,R,L));
    if step > 2 && epsilon < tol
        break;
    end
    step = step +1;
    lastc = c;
    fprintf("steps = %i, epsilon = %f, c = %f\n",step,epsilon,c)
end
fprintf("------------------------------------------------------------\n",a,b)
fprintf("The regula-falsi method\n")
a=Interval(1);
b=Interval(2);
step = 1;
while true
    c=a-f(a,R,L)*((b-a)/(f(b,R,L)-f(a,R,L)));
    epsilon = abs(f(c,R,L));
    fprintf("steps = %i, epsilon = %f, c = %f\n",step,epsilon,c)
    if epsilon <= tol
        break
    else
        % Check which interval the root is in
        if f(b,R,L)*f(c,R,L) < 0
            a=c;
        else
            b=c;
        end
    end
    step = step + 1;
end
fprintf("------------------------------------------------------------\n",a,b)

function        = f(     )
    Vmax=L*(R 2*(pi/2 - asin(0/R)) - 0*sqrt(R 2-0 2)); % Max occurs when h = 0
    retval= L*(R 2*(pi/2 - asin(h/R)) - h*sqrt(R 2-h 2)) - 0.9*Vmax;
end
```

```
Both root finding methods will begin on the interval [0.000000,0.100000]
------------------------------------------------------------
The bisection method
steps = 2, epsilon = 0.571213, c = 0.050000
steps = 3, epsilon = 0.072204, c = 0.075000
steps = 4, epsilon = 0.176968, c = 0.087500
steps = 5, epsilon = 0.052414, c = 0.081250
steps = 6, epsilon = 0.009887, c = 0.078125
steps = 7, epsilon = 0.021265, c = 0.079688
steps = 8, epsilon = 0.005690, c = 0.078906
steps = 9, epsilon = 0.002099, c = 0.078516
steps = 10, epsilon = 0.001795, c = 0.078711
steps = 11, epsilon = 0.000152, c = 0.078613
steps = 12, epsilon = 0.000822, c = 0.078662
steps = 13, epsilon = 0.000335, c = 0.078638
steps = 14, epsilon = 0.000092, c = 0.078625
steps = 15, epsilon = 0.000030, c = 0.078619
steps = 16, epsilon = 0.000031, c = 0.078622
------------------------------------------------------------
The regula-falsi method
steps = 1, epsilon = 0.001002, c = 0.078671
steps = 2, epsilon = 0.000002, c = 0.078621
steps = 3, epsilon = 0.000000, c = 0.078621
------------------------------------------------------------
```