ENG204 - Signals and Linear Systems – Assignment 1.2

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1.1 Part a

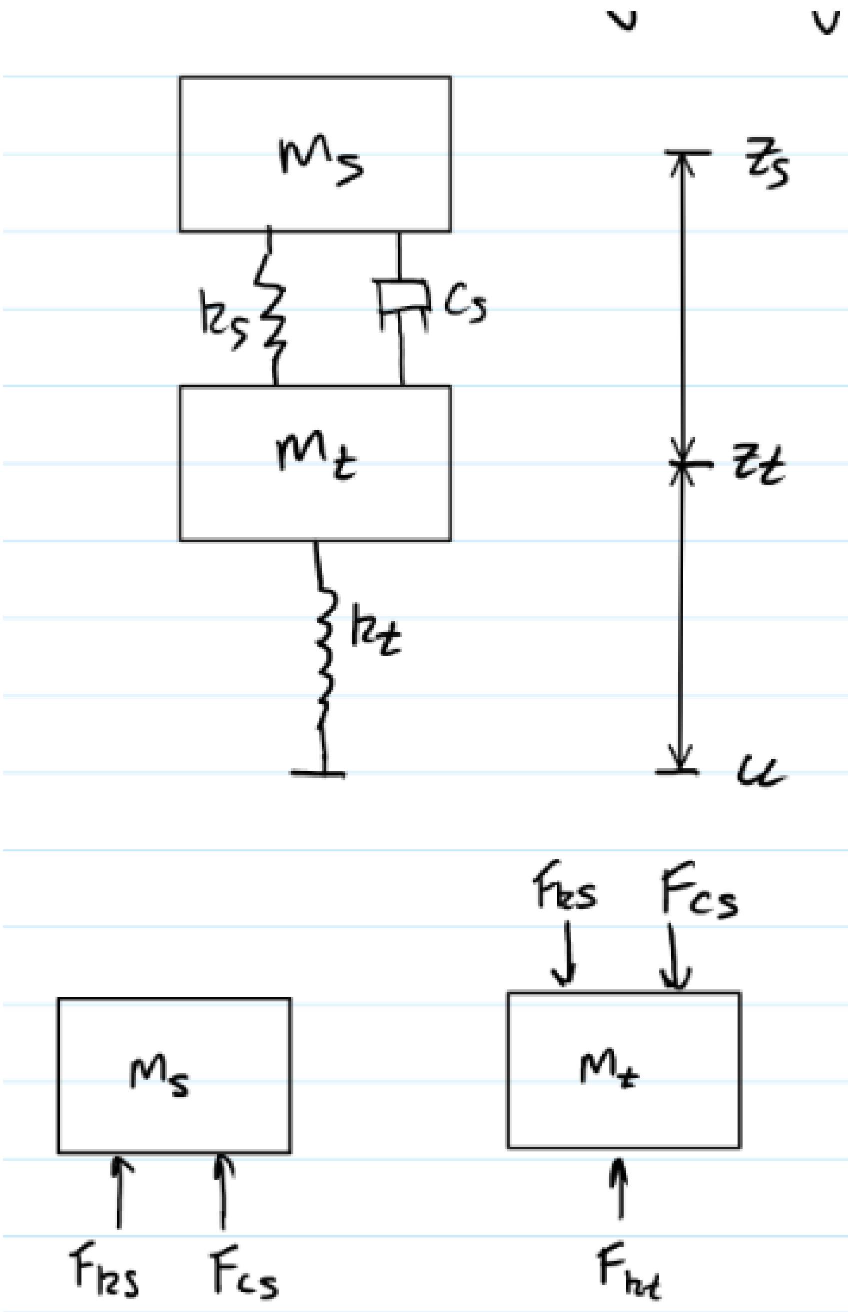

Figure 1: Free Body Diagram

The gravational forces can be ignored because they will cause a constant downward force. Where we will only be looking at the differences in forces between the systems. In other words, the system will respond in the same way whether or not the gravitational forces are included. This is because the system that is being looked at only cares about the differences in forces, and not their total value.

1.2 Part b

1.2.1 Differential Equation

```
clc
clear
pkg load symbolic
syms mt at ms as  ...
    Fkt Fks Fcs Fks
% sum F_y = ma
eq1 = mt*at == Fkt - Fks - Fcs;
eq2 = ms*as == Fks + Fcs;


% Sub in force equations
syms kt u zt zs dzs dzt ks cs
FktEqu = kt*(u-zt);
FksEqu = ks*(zt-zs);
FcsEqu = cs*(dzt-dzs);
eq1=subs(subs(subs(eq1,Fkt,FktEqu),Fks,FksEqu),Fcs,FcsEqu);
eq2=subs(subs(subs(eq2,Fkt,FktEqu),Fks,FksEqu),Fcs,FcsEqu);
```

Using these equations from the free body diagram:

- $a_t m_t = -F_{cs} - F_{ks} + F_{kt}$
- $a_s m_s = F_{cs} + F_{ks}$

And:

- $F_{kt} = k_t\left(u - z_t\right)$
- $F_{ks} = k_s\left(-z_s + z_t\right)$
- $F_{cs} = c_s\left(-\dot{z}_s + \dot{z}_t\right)$

Will result in the differential equations for the system:

- $a_t m_t = \dot{v}_t m_t = \ddot{z}_t m_t = -c_s\left(-\dot{z}_s + \dot{z}_t\right) - k_s\left(-z_s + z_t\right) + k_t\left(u - z_t\right)$
- $a_s m_s = \dot{v}_s m_s = \ddot{z}_s m_s = c_s\left(-\dot{z}_s + \dot{z}_t\right) + k_s\left(-z_s + z_t\right)$

1.2.2 Difference Equations

Next we can create the difference equations by subsituting in the forward differences for the velocities and accelerations:

- $\dot{z}_s = \frac{-z_s[n]+z_s[n+1]}{T_s}$
- $\dot{z}_t = \frac{-z_t[n]+z_t[n+1]}{T_s}$
- $a_s = \dot{v}_s = \frac{-v_s[n]+v_s[n+1]}{T_s}$
- $a_t = \dot{v}_t = \frac{-v_t[n]+v_t[n+1]}{T_s}$

```
% Sub in forward difference
syms zs zs1 zs2 zt zt1 zt2 Ts vs1 vt1 vs vt
% First forward difference of displacement
dzsEqu = (zs1-zs)/(Ts);
```

```
dztEqu = (zt1-zt)/(Ts);
% First forward differences of acceleration
dvsEqu = (vs1-vs)/(Ts);
dvtEqu = (vt1-vt)/(Ts);
eq1=subs(eq1,at,dvtEqu);
eq1=subs(eq1,dzs,vs);
eq1=subs(eq1,dzt,vt);
eq1=expand(simplify(eq1));
eq2=subs(eq2,as,dvsEqu);
eq2=subs(eq2,dzs,vs);
eq2=subs(eq2,dzt,vt);
eq2=expand(simplify(eq2));
```

Which produces the two following difference equations:

- $-\frac{m_t v_t[n]}{T_s} + \frac{m_t v_t[n+1]}{T_s} = c_s v_s[n] - c_s v_t[n] + k_s z_s[n] - k_s z_t[n] + k_t u[n] - k_t z_t[n]$
- $-\frac{m_s v_s[n]}{T_s} + \frac{m_s v_s[n+1]}{T_s} = -c_s v_s[n] + c_s v_t[n] - k_s z_s[n] + k_s z_t[n]$

1.3 Part c

We need to find the matricies of the two following equations:

- $\underline{q}[n+1] = \underline{A}q[n] + \underline{b}x[n]$
- $y[n] = \underline{C}q[n] + dx[n]$

We chose the following state varibles:

- $q_1[n] = z_t[n]$
- $q_2[n] = v_t[n] = \dot{z}_t[n]$
- $q_3[n] = z_s[n]$
- $q_4[n] = v_s[n] = \dot{z}_s[n]$
- $q_1[n+1] = z_t[n+1]$
- $q_2[n+1] = v_t[n+1] = \dot{z}_t[n+1]$
- $q_3[n+1] = z_s[n+1]$
- $q_4[n+1] = v_s[n+1] = \dot{z}_s[n+1]$
- $x[n] = u[n]$
- $y[n] = \begin{bmatrix} z_t[n] \\ z_s[n] \end{bmatrix}$

1.3.1 State Equation

Need to solve for the matricies in $\underline{q}[n+1] = \underline{A}q[n] + \underline{b}x[n]$. To do this we will subsitute in the state varibles into the difference equations.

```
syms q1n q1n1 q2n q2n1 q3n q3n1 q4n q4n1

eq1 = subs(eq1,zt,q1n);
eq1 = subs(eq1,vt,q2n);
eq1 = subs(eq1,zs,q3n);
eq1 = subs(eq1,vs,q4n);
eq1 = subs(eq1,zt1,q1n1);
eq1 = subs(eq1,vt1,q2n1);
eq1 = subs(eq1,zs1,q3n1);
eq1 = subs(eq1,vs1,q4n1);

eq2 = subs(eq2,zt,q1n);
eq2 = subs(eq2,vt,q2n);
eq2 = subs(eq2,zs,q3n);
eq2 = subs(eq2,vs,q4n);
eq2 = subs(eq2,zt1,q1n1);
eq2 = subs(eq2,vt1,q2n1);
eq2 = subs(eq2,zs1,q3n1);
eq2 = subs(eq2,vs1,q4n1);

equq1n1 = q1n+Ts*q2n;
equq3n1 = q3n+Ts*q4n;

eq1 = subs(eq1, q1n1, equq1n1);
eq1 = subs(eq1, q3n1, equq3n1);
eq2 = subs(eq2, q1n1, equq1n1);
eq2 = subs(eq2, q3n1, equq3n1);

eq1 = expand(simplify(solve(eq1,q2n1)));
eq2 = expand(simplify(solve(eq2,q4n1)));
```

Which gives us the following equations:

- $q_1[n+1] = q_1[n] + T_s \cdot q_2[n]$
- $q_2[n+1] = -\frac{T_s c_s q_2[n]}{m_t} + \frac{T_s c_s q_4[n]}{m_t} - \frac{T_s k_s q_1[n]}{m_t} + \frac{T_s k_s q_3[n]}{m_t} - \frac{T_s k_t q_1[n]}{m_t} + \frac{T_s k_t u[n]}{m_t} + q_2[n]$
- $q_3[n+1] = q_3[n] + T_s \cdot q_4[n]$
- $q_4[n+1] = \frac{T_s c_s q_2[n]}{m_s} - \frac{T_s c_s q_4[n]}{m_s} + \frac{T_s k_s q_1[n]}{m_s} - \frac{T_s k_s q_3[n]}{m_a s} + q_4[n]$

These four equations can be used to fill in the matrix A and B:

$$\underline{A} = \begin{bmatrix} 1 & T_s & 0 & 0 \\ -\frac{T_s k_s}{m_t} - \frac{T_s k_t}{m_t} & 1 - \frac{T_s c_s}{m_t} & \frac{T_s k_s}{m_t} & \frac{T_s c_s}{m_t} \\ 0 & 0 & 1 & T_s \\ \frac{T_s k_s}{m_s} & \frac{T_s c_s}{m_s} & -\frac{T_s k_s}{m_s} & 1 - \frac{T_s c_s}{m_s} \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 \\ \frac{T_s k_t}{m_t} \\ 0 \\ 0 \end{bmatrix}$$

Therefore the entire state equation is:

$$\begin{bmatrix} q_1[n+1] \\ q_2[n+1] \\ q_3[n+1] \\ q_4[n+1] \end{bmatrix} = \begin{bmatrix} 1 & T_s & 0 & 0 \\ -\frac{T_s k_s}{m_t} - \frac{T_s k_t}{m_t} & 1 - \frac{T_s c_s}{m_t} & \frac{T_s k_s}{m_t} & \frac{T_s c_s}{m_t} \\ 0 & 0 & 1 & T_s \\ \frac{T_s k_s}{m_s} & \frac{T_s c_s}{m_s} & -\frac{T_s k_s}{m_a s} & 1 - \frac{T_s c_s}{m_s} \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \\ q_3[n] \\ q_4[n] \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{T_s k_t}{m_t} \\ 0 \\ 0 \end{bmatrix} [u[n]]$$

1.3.2 Output Equation

Need to solve for the matricies in $y[n] = \underline{C}q[n] + dx[n]$ using:

- $q_1[n] = z_t[n]$
- $q_3[n] = z_s[n]$

Therefore:

$$\underline{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Therefore the output equation is:

$$\begin{bmatrix} z_t[n] \\ z_s[n] \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \\ q_3[n] \\ q_4[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [u[n]]$$

1.4 Part d

This code creates an array of systems with varing k_s and c_s.

```
clear
clc
pkg load symbolic
pkg load control
% Student ID 1 = 651790
% Student ID 2 = 652137
ms=2296;
mt=236;
kt=250000;
ksMin = 10000;
csMin = 500;
ksMax = 250000;
csMax = 2000;
Ts = 0.0001;
idx = 0;
numOfSys = 10;
for i =0:numOfSys;
    for j =0:numOfSys;
        ks = ksMin + (i/numOfSys)*(ksMax-ksMin);
        cs = csMin + (j/numOfSys)*(csMax-csMin);
        % Setup State Space Matricies
        A = [1-(Ts*cs)/mt,(Ts*cs)/mt,-1*(Ts*ks+Ts*kt)/mt,(Ts*ks)/mt;
             (Ts*cs)/ms,1-(Ts*cs)/ms,(Ts*ks)/ms,-1*(Ts*ks)/ms;
             Ts,0,1,0;
             0,Ts,0,1];
        B = [(Ts*kt)/mt;
             0;
             0;
             0];
        C = [0,0,1,0;
             0,0,0,1];
        D = [0;
             0];
```

```
        idx = idx +1;
        sysArray(idx).A = A;
        sysArray(idx).B = B;
        sysArray(idx).C = C;
        sysArray(idx).D = D;
        sysArray(idx).ks=ks;
        sysArray(idx).cs=cs;
    end
end
% Create the state-space system
sys = cell(length(sysArray), 1);
for i = 1:length(sysArray)
    sys{i} = ss(sysArray(i).A, sysArray(i).B, sysArray(i).C, sysArray(i).D,Ts);
end
```

1.4.1 Impulse Response

```
for i = 1:length(sys)
    figure;
    hold on;
    impulse(sys{i});
    titleStr = sprintf('Impulse Response with ks = %i and cs = %i', sysArray(i).ks,
    ↪  sysArray(i).cs);
    title(titleStr, 'FontSize', 15);
    hold off;
end
```

In the following graphs y1 is the wheel and y2 is the vehicle. So we want y2 to stabalise as fast as possible, with the least amount of change in displacement.

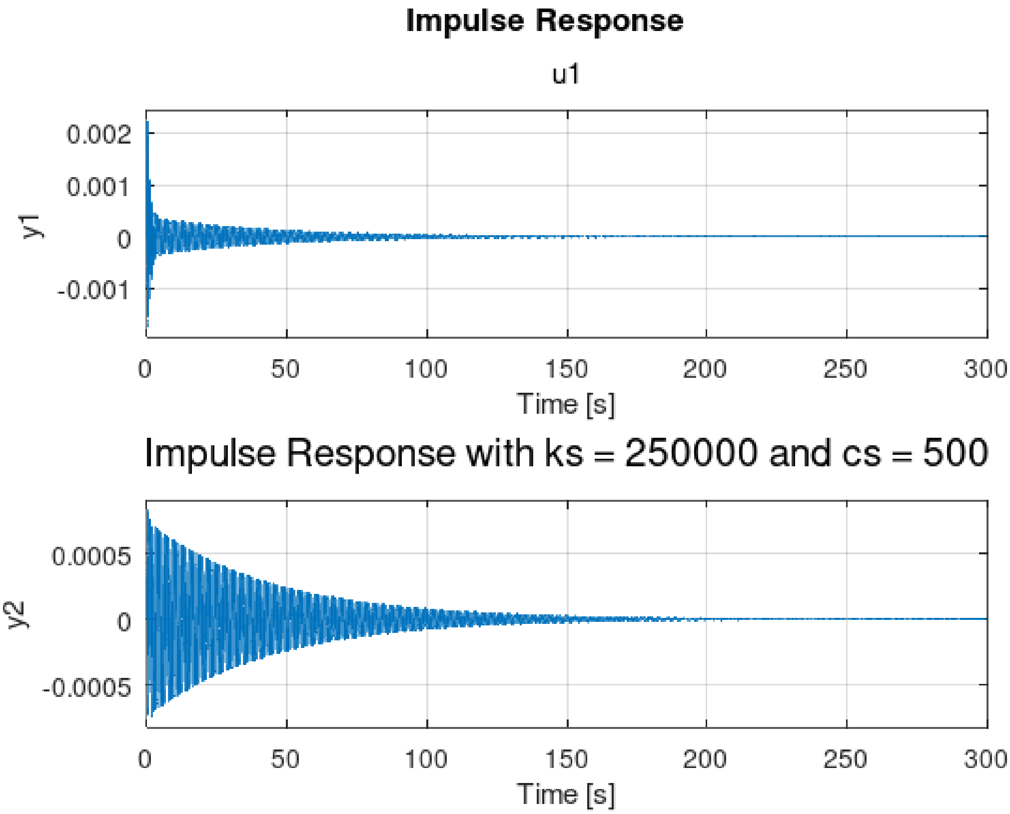

Figure 2: 111th system

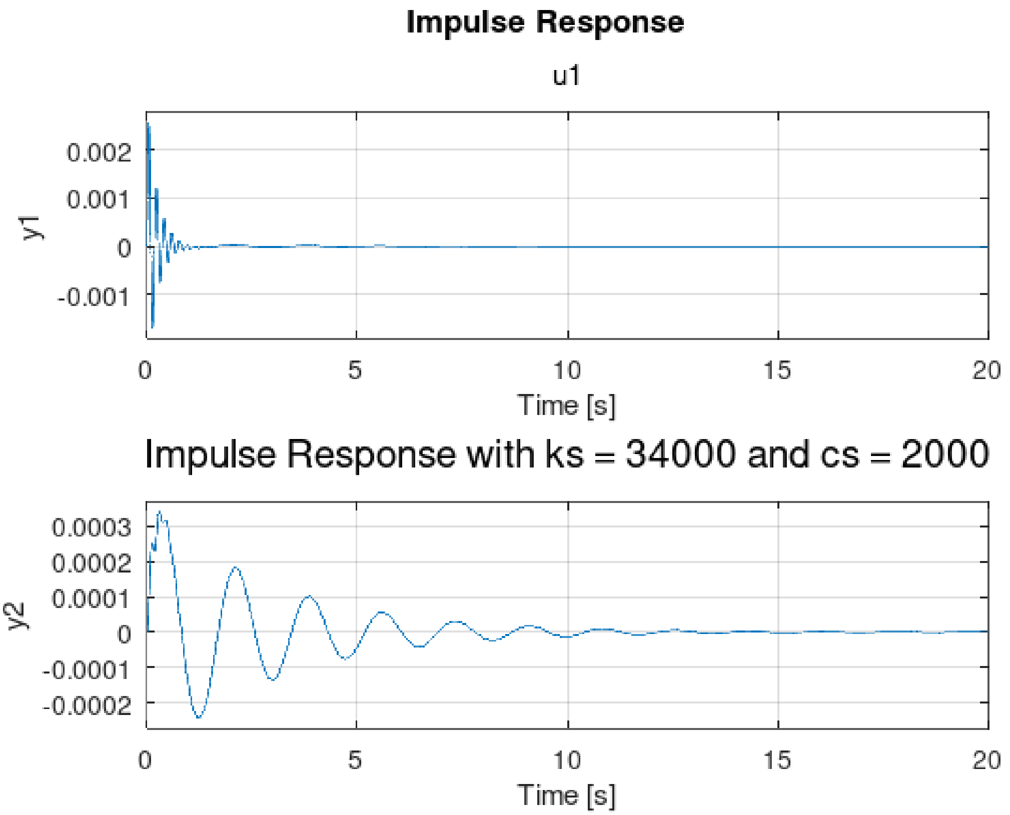

Figure 3: 22nd system

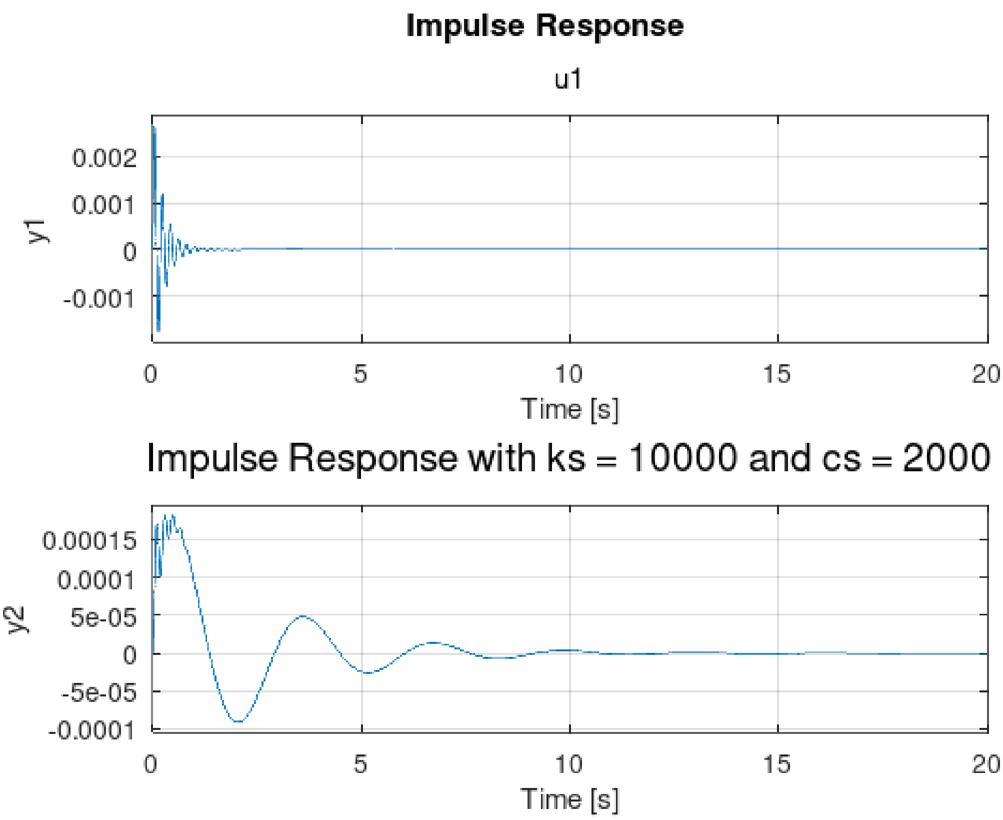

Figure 4: 11th system

From these graphs we can see that having a high k_s and a low c_s resuts in a slow convergence and very high frequencies. Where as having a low k_s and high c_s results in a faster convergence at low frequency. However there is a sweet spot between both where the best response is gathered, which can be seen in the 11th response.

1.4.2 Stability

Check if the system is stable using the eigenvalues of A.

```
for i = 1:length(sysArray)
    eigen=abs(eig(sysArray(i).A));
    if all(eigen < 1)
        fprintf("The %i th system is stable\n", i)
        maxEig = max(eigen);
        fprintf("The max eigen value is %f\n", maxEig)
    else
        fprintf("The %i th system is unstable\n", i)
    end
end
```

This output shows that the system is stable for all $10,000 \leq k_s \leq 250,000 N/m$ and $500 \leq c_s \leq 2000 Ns/m$. We would also expect to the the most stable system to occur when the largest of the eigenvalues is the smallest, which we can see occurs on the 11th system. The full output can be seen in Appendix A

1.4.3 Damping

```
ms=2296;
ksMin = 10000;
```

```
csMin = 500;
ksMax = 250000;
csMax = 2000;
idx = 0;
numOfSys = 10;
for i =0:numOfSys;
    for j =0:numOfSys;
        idx = idx +1;
        ks = ksMin + (i/numOfSys)*(ksMax-ksMin);
        cs = csMin + (j/numOfSys)*(csMax-csMin);
        damp = cs / (2 * sqrt(ms * ks));
        fprintf('For the %i th system the damping factor is %f\n', idx, damp)
    end
end
```

We want a damping factor of 1, this is when the system will be critically damped (contributors, 2024). The output shows that the closest to 1 is 0.208696 which occurs on the 11 th system, this aligns with the graphs and eigenvalues. The full output can be seen in Appendix B.

1.5 Part e

1.5.1 Varing Frequency

The following code is going to use the 11th system, as it has been shown to be the best.

```
fMin=1000;
fStep=fMin;
fMax=10*fMin;
for i = fMin:fStep:fMax
    um = 1;
    f = i; % Frequency
    w0 = 2 * pi * f;
    t = 0:Ts:10;
    u = um * sin(w0 * t);
    figure;
    hold on;
    y = lsim(sys{11}, u, t);
    plot(t, y(:, 2));
    titleStr = sprintf('Response of Quarter-Car Suspension System to Sinusoidal Input at
    ↪  %i Hz', f);
    title(titleStr, 'FontSize', 10);
    xlabel('Time (s)');
    ylabel('Displacement');
    legendEntry = sprintf('System %d: ks = %.2f, cs = %.2f', 11, sysArray(11).ks,
    ↪  sysArray(11).cs);
    legend(legendEntry);
    hold off;
    filename = sprintf('ENG204-Assignment-2-Sinusoidal-f-%i.png', f);
    print(filename,'-dpng','-r100');
end
```

Frequencies from 0.1 Hz to 10k Hz were analised. Some samples can be seen here:

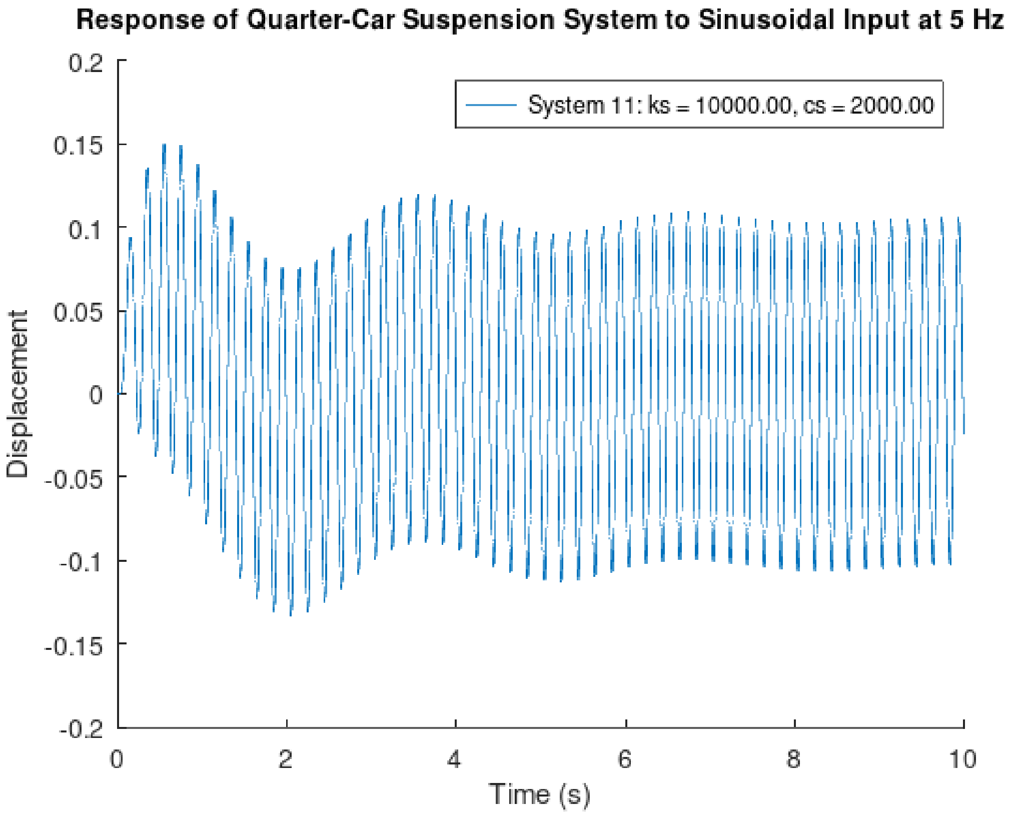

Figure 5: 5Hz

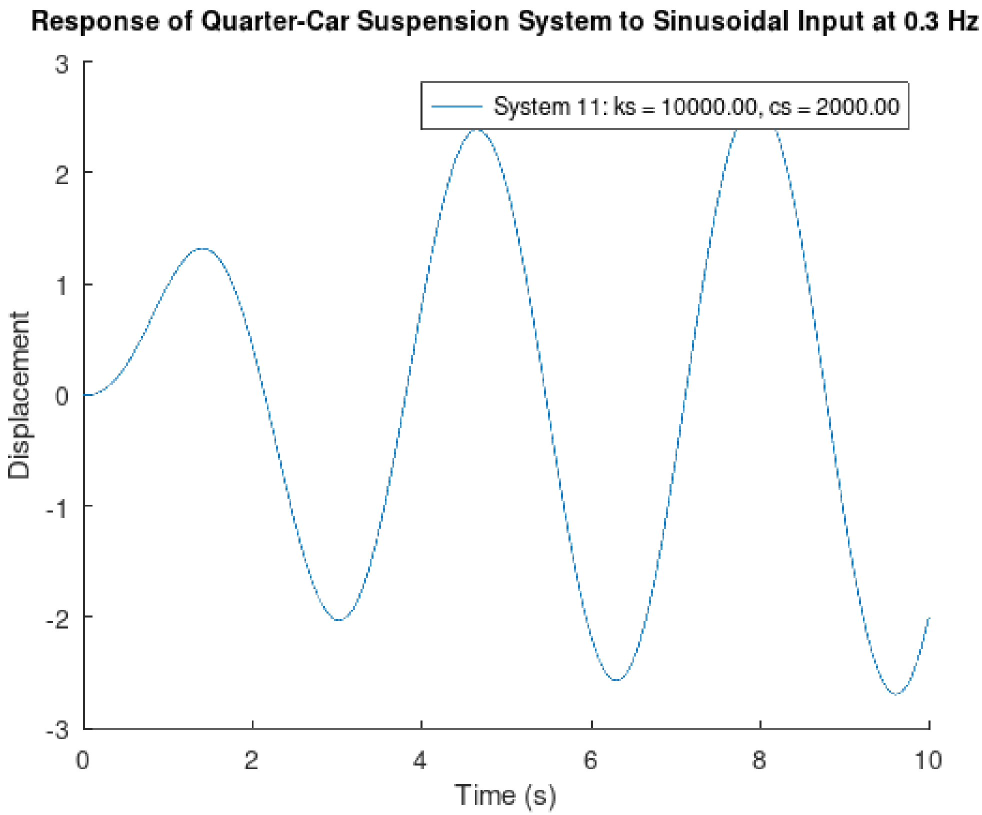

Figure 6: 0.3Hz

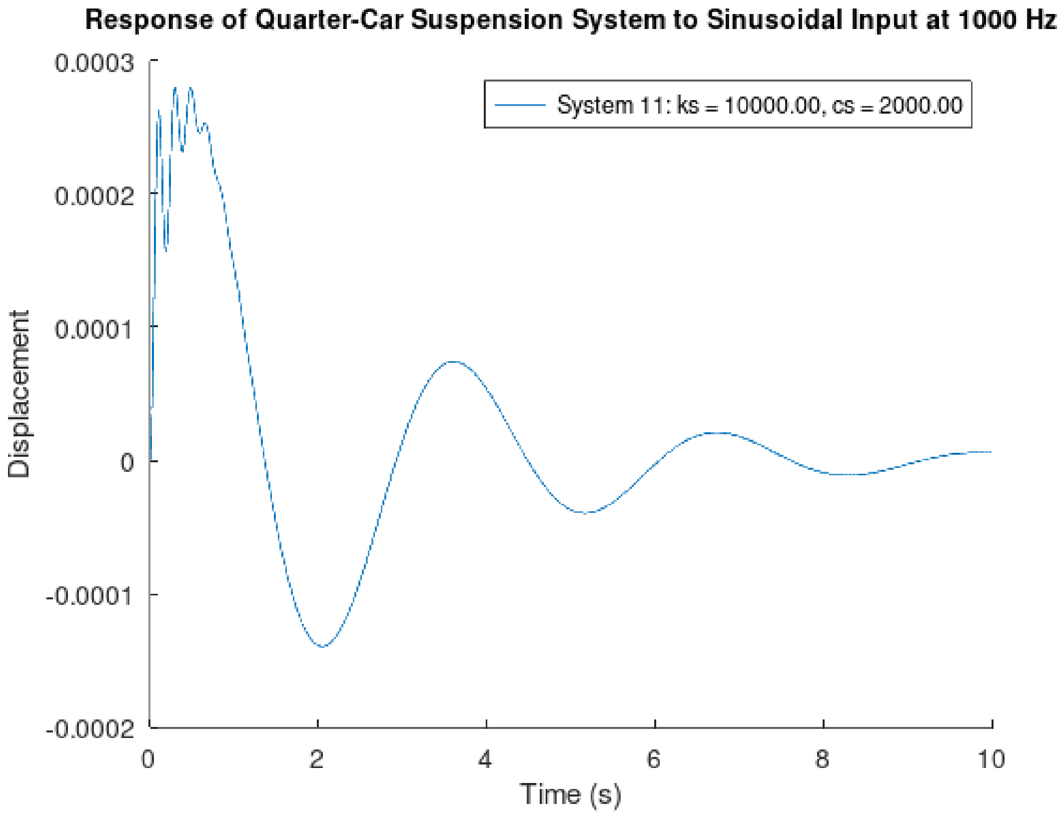

Figure 7: 1000Hz

For low frequencies the vehicle experiences the worst movement, at high frequencies the vehicle experiences very low amount of movement. The worst amplitude occurs at 0.3 Hz, it has a maximum magnitude ≈ 3.

1.5.2 Varing Suspension

The following code is going to use many systems at 0.3Hz, as it was shown to be the worst for the best case.

```
um = 1;
f = 0.3; % Frequency
w0 = 2 * pi * f;
t = 0:Ts:10;
u = um * sin(w0 * t);

for i = 1:length(sys)
    figure;
    hold on;
    y = lsim(sys{i}, u, t);
    plot(t, y(:, 2));

    titleStr = sprintf('%i th System to Sinusoidal Input at %i Hz', i , f);
    title(titleStr, 'FontSize', 10);
    xlabel('Time (s)');
    ylabel('Displacement');
    legendEntry = sprintf('System %d: ks = %.2f, cs = %.2f', i, sysArray(i).ks,
    ↪  sysArray(i).cs);
    legend(legendEntry);
    hold off;
```

```
    filename = sprintf('ENG204-Assignment-2-Sinusoidal-f-0.3-%i.png', i);
    print(filename,'-dpng','-r100');
end
```

The best preforming system was the last system, this system has the highest k_s and c_s, the magnitude of the output is ≈ 1.5. Decreasing the value of k_s and c_s tends to increase the magnitdue of the output. Where the worst peforming system was the first one with the lowest k_s and c_s, it has a magnitude of ≈ 6.

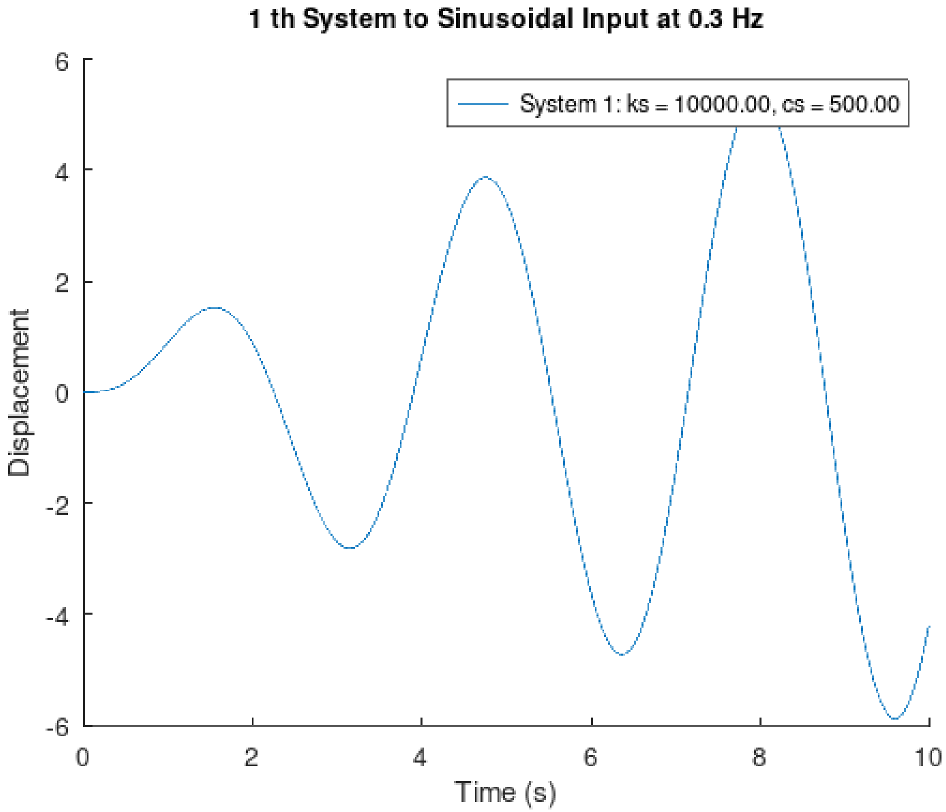

Figure 8: 1st system at 0.3Hz

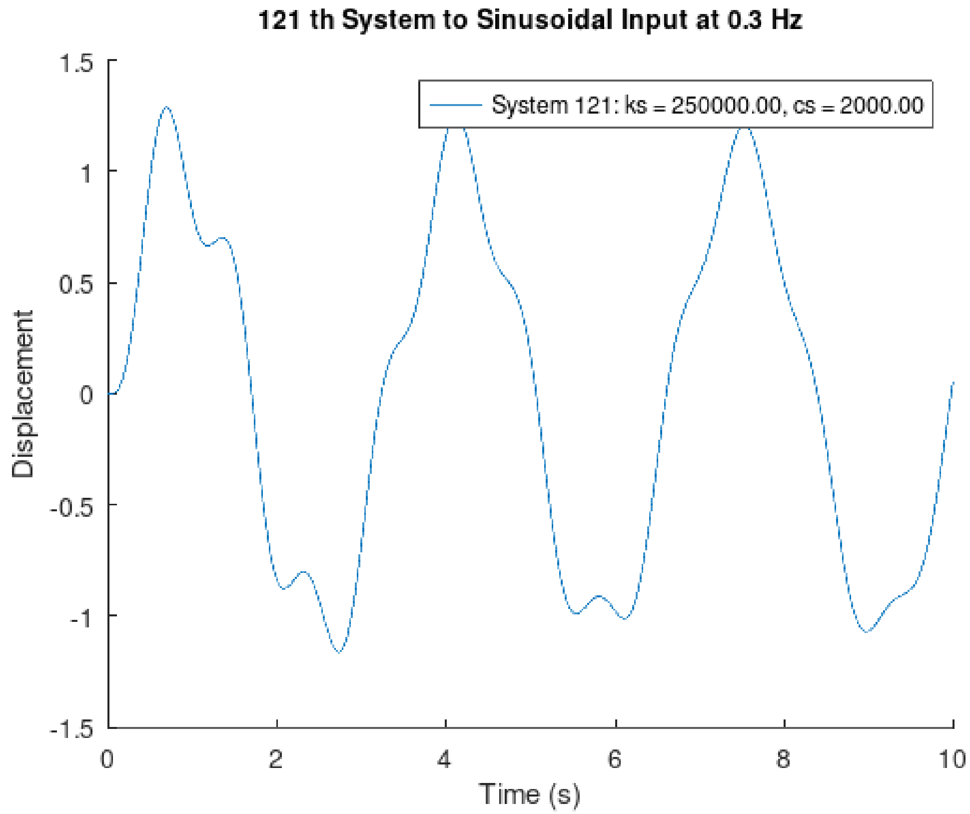

Figure 9: 121st system at 0.3Hz

1.6 Part f

The metrics involved in these calculations depend heavily on the use-case of the suspension system being designed. We will assume that the system is designed for general consumer usage, with typical speeds varying from 50km/h to 110km/h(13.8889 to 30.5556 m/s). These speeds impact the frequency of the oscillations as well as the peak suspension displacements. This will lead to varying vehicle smoothness depending on the speed as well as the suspension configuration(ks and cs values). The graphical demonstration developed through MATLAB shows clearly that as the speed increases the amplitude decreases and the frequency increases. This is due to the relationship between frequency and velocity, $\omega = 2\pi v/\lambda$. Where λ is the wavelength of the road bumps. Many documentations have been made on this topic, with varying approaches based on the specific vehicle and usage scenarios. (AI, 2024) and (Goga and KĐúþik, 2012)

References

AI, C. (2024). *Car suspension design* [Accessed: 2024-30-08]. https://www.collimator.ai/tutorials/car-suspension-design

contributors, W. (2024). Damping [Accessed: 2024-30-08]. https://en.wikipedia.org/wiki/Damping

Goga, V., & KĐúþik, M. (2012). Optimization of vehicle suspension parameters with use of evolutionary computation [Accessed: 2024-30-08]. https://www.sciencedirect.com/science/article/pii/S1877705812045651

1.7 Appendix A

The 1 th system is stable
The max eigen value is 0.999990
The 2 th system is stable
The max eigen value is 0.999987
The 3 th system is stable
The max eigen value is 0.999984
The 4 th system is stable
The max eigen value is 0.999981
The 5 th system is stable
The max eigen value is 0.999978
The 6 th system is stable
The max eigen value is 0.999975
The 7 th system is stable
The max eigen value is 0.999972
The 8 th system is stable
The max eigen value is 0.999969
The 9 th system is stable
The max eigen value is 0.999966
The 10 th system is stable
The max eigen value is 0.999963
The 11 th system is stable
The max eigen value is 0.999960
The 12 th system is stable
The max eigen value is 0.999992
The 13 th system is stable
The max eigen value is 0.999989
The 14 th system is stable
The max eigen value is 0.999987
The 15 th system is stable
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The 16 th system is stable
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The 18 th system is stable
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The 19 th system is stable
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The 20 th system is stable
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The max eigen value is 0.999988
The 75 th system is stable
The max eigen value is 0.999987
The 76 th system is stable
The max eigen value is 0.999985

The 77 th system is stable
The max eigen value is 0.999984
The 78 th system is stable
The max eigen value is 0.999997
The 79 th system is stable
The max eigen value is 0.999996
The 80 th system is stable
The max eigen value is 0.999995
The 81 th system is stable
The max eigen value is 0.999994
The 82 th system is stable
The max eigen value is 0.999992
The 83 th system is stable
The max eigen value is 0.999991
The 84 th system is stable
The max eigen value is 0.999990
The 85 th system is stable
The max eigen value is 0.999989
The 86 th system is stable
The max eigen value is 0.999988
The 87 th system is stable
The max eigen value is 0.999987
The 88 th system is stable
The max eigen value is 0.999986
The 89 th system is stable
The max eigen value is 0.999997
The 90 th system is stable
The max eigen value is 0.999996
The 91 th system is stable
The max eigen value is 0.999995
The 92 th system is stable
The max eigen value is 0.999994
The 93 th system is stable
The max eigen value is 0.999993
The 94 th system is stable
The max eigen value is 0.999992
The 95 th system is stable
The max eigen value is 0.999991
The 96 th system is stable
The max eigen value is 0.999991
The 97 th system is stable
The max eigen value is 0.999990
The 98 th system is stable
The max eigen value is 0.999989
The 99 th system is stable
The max eigen value is 0.999988
The 100 th system is stable
The max eigen value is 0.999997
The 101 th system is stable
The max eigen value is 0.999997
The 102 th system is stable

The max eigen value is 0.999996
The 103 th system is stable
The max eigen value is 0.999995
The 104 th system is stable
The max eigen value is 0.999994
The 105 th system is stable
The max eigen value is 0.999993
The 106 th system is stable
The max eigen value is 0.999992
The 107 th system is stable
The max eigen value is 0.999992
The 108 th system is stable
The max eigen value is 0.999991
The 109 th system is stable
The max eigen value is 0.999990
The 110 th system is stable
The max eigen value is 0.999989
The 111 th system is stable
The max eigen value is 0.999998
The 112 th system is stable
The max eigen value is 0.999997
The 113 th system is stable
The max eigen value is 0.999996
The 114 th system is stable
The max eigen value is 0.999995
The 115 th system is stable
The max eigen value is 0.999995
The 116 th system is stable
The max eigen value is 0.999994
The 117 th system is stable
The max eigen value is 0.999993
The 118 th system is stable
The max eigen value is 0.999992
The 119 th system is stable
The max eigen value is 0.999992
The 120 th system is stable
The max eigen value is 0.999991
The 121 th system is stable
The max eigen value is 0.999990

1.8 Appendix B

For the 1 th system the damping factor is 0.052174
For the 2 th system the damping factor is 0.067826
For the 3 th system the damping factor is 0.083478
For the 4 th system the damping factor is 0.099131
For the 5 th system the damping factor is 0.114783
For the 6 th system the damping factor is 0.130435
For the 7 th system the damping factor is 0.146087
For the 8 th system the damping factor is 0.161739
For the 9 th system the damping factor is 0.177392
For the 10 th system the damping factor is 0.193044
For the 11 th system the damping factor is 0.208696
For the 12 th system the damping factor is 0.028295
For the 13 th system the damping factor is 0.036784
For the 14 th system the damping factor is 0.045273
For the 15 th system the damping factor is 0.053761
For the 16 th system the damping factor is 0.062250
For the 17 th system the damping factor is 0.070738
For the 18 th system the damping factor is 0.079227
For the 19 th system the damping factor is 0.087715
For the 20 th system the damping factor is 0.096204
For the 21 th system the damping factor is 0.104693
For the 22 th system the damping factor is 0.113181
For the 23 th system the damping factor is 0.021664
For the 24 th system the damping factor is 0.028163
For the 25 th system the damping factor is 0.034663
For the 26 th system the damping factor is 0.041162
For the 27 th system the damping factor is 0.047661
For the 28 th system the damping factor is 0.054160
For the 29 th system the damping factor is 0.060659
For the 30 th system the damping factor is 0.067159
For the 31 th system the damping factor is 0.073658
For the 32 th system the damping factor is 0.080157
For the 33 th system the damping factor is 0.086656
For the 34 th system the damping factor is 0.018220
For the 35 th system the damping factor is 0.023686
For the 36 th system the damping factor is 0.029152
For the 37 th system the damping factor is 0.034618
For the 38 th system the damping factor is 0.040084
For the 39 th system the damping factor is 0.045550
For the 40 th system the damping factor is 0.051016
For the 41 th system the damping factor is 0.056482
For the 42 th system the damping factor is 0.061948
For the 43 th system the damping factor is 0.067414
For the 44 th system the damping factor is 0.072880
For the 45 th system the damping factor is 0.016025
For the 46 th system the damping factor is 0.020833
For the 47 th system the damping factor is 0.025640
For the 48 th system the damping factor is 0.030448
For the 49 th system the damping factor is 0.035255
For the 50 th system the damping factor is 0.040063

For the 51 th system the damping factor is 0.044870
For the 52 th system the damping factor is 0.049678
For the 53 th system the damping factor is 0.054485
For the 54 th system the damping factor is 0.059293
For the 55 th system the damping factor is 0.064100
For the 56 th system the damping factor is 0.014470
For the 57 th system the damping factor is 0.018812
For the 58 th system the damping factor is 0.023153
For the 59 th system the damping factor is 0.027494
For the 60 th system the damping factor is 0.031835
For the 61 th system the damping factor is 0.036176
For the 62 th system the damping factor is 0.040517
For the 63 th system the damping factor is 0.044858
For the 64 th system the damping factor is 0.049200
For the 65 th system the damping factor is 0.053541
For the 66 th system the damping factor is 0.057882
For the 67 th system the damping factor is 0.013295
For the 68 th system the damping factor is 0.017284
For the 69 th system the damping factor is 0.021272
For the 70 th system the damping factor is 0.025261
For the 71 th system the damping factor is 0.029249
For the 72 th system the damping factor is 0.033238
For the 73 th system the damping factor is 0.037226
For the 74 th system the damping factor is 0.041215
For the 75 th system the damping factor is 0.045204
For the 76 th system the damping factor is 0.049192
For the 77 th system the damping factor is 0.053181
For the 78 th system the damping factor is 0.012366
For the 79 th system the damping factor is 0.016076
For the 80 th system the damping factor is 0.019786
For the 81 th system the damping factor is 0.023496
For the 82 th system the damping factor is 0.027206
For the 83 th system the damping factor is 0.030916
For the 84 th system the damping factor is 0.034626
For the 85 th system the damping factor is 0.038336
For the 86 th system the damping factor is 0.042046
For the 87 th system the damping factor is 0.045756
For the 88 th system the damping factor is 0.049466
For the 89 th system the damping factor is 0.011609
For the 90 th system the damping factor is 0.015091
For the 91 th system the damping factor is 0.018574
For the 92 th system the damping factor is 0.022056
For the 93 th system the damping factor is 0.025539
For the 94 th system the damping factor is 0.029021
For the 95 th system the damping factor is 0.032504
For the 96 th system the damping factor is 0.035987
For the 97 th system the damping factor is 0.039469
For the 98 th system the damping factor is 0.042952
For the 99 th system the damping factor is 0.046434
For the 100 th system the damping factor is 0.010975
For the 101 th system the damping factor is 0.014267

For the 102 th system the damping factor is 0.017560
For the 103 th system the damping factor is 0.020852
For the 104 th system the damping factor is 0.024145
For the 105 th system the damping factor is 0.027437
For the 106 th system the damping factor is 0.030730
For the 107 th system the damping factor is 0.034022
For the 108 th system the damping factor is 0.037315
For the 109 th system the damping factor is 0.040607
For the 110 th system the damping factor is 0.043900
For the 111 th system the damping factor is 0.010435
For the 112 th system the damping factor is 0.013565
For the 113 th system the damping factor is 0.016696
For the 114 th system the damping factor is 0.019826
For the 115 th system the damping factor is 0.022957
For the 116 th system the damping factor is 0.026087
For the 117 th system the damping factor is 0.029217
For the 118 th system the damping factor is 0.032348
For the 119 th system the damping factor is 0.035478
For the 120 th system the damping factor is 0.038609
For the 121 th system the damping factor is 0.041739