

Self-Organising Systems

Rudolf Mayer



Institut für Softwaretechnik & Interaktive Systeme
(mayer@ifs.tuwien.ac.at)

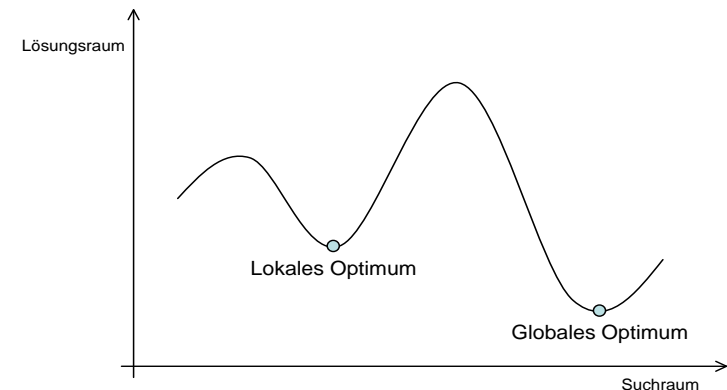
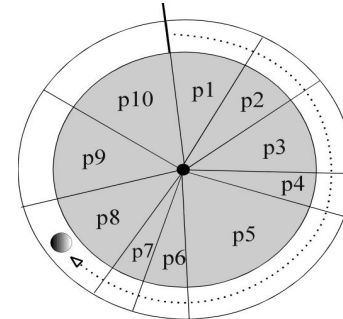


SBA Research
(www.sba-research.org)

- Brief Recap & Genetic algorithms, continued
- Genetic Programming
- Ant Colony Optimisation
- Agents

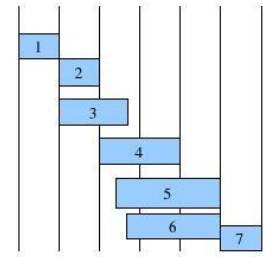
Recap: Genetic Algorithms

- Genetic Algorithms
 - Coding – binary, gray code, ...
 - Starting population – initialisation methods
 - Genetic operators: reproduction / crossover / mutation
 - Fitness function
 - Selection (Roulette wheel)
 - *Population diversity*
 - *Tournament selection*
 - *Crowding, Fitness sharing*
 - *Adaptation of mutation & crossover*



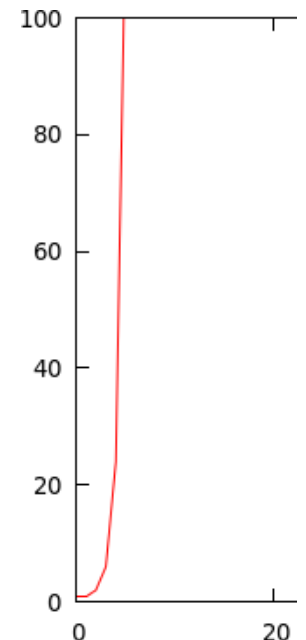
Recap: Genetic Algorithms

- Genetic Algorithms Applications
 - Generally heuristic optimisation / search tasks
 - Often NP (hard) problems
 - Scheduling (job scheduling) – ideal assignments of jobs to resources at particular times
 - Timetabling
 - Travelling Salesman – finding the optimal route
 - Economics (e.g. supply/demand model)



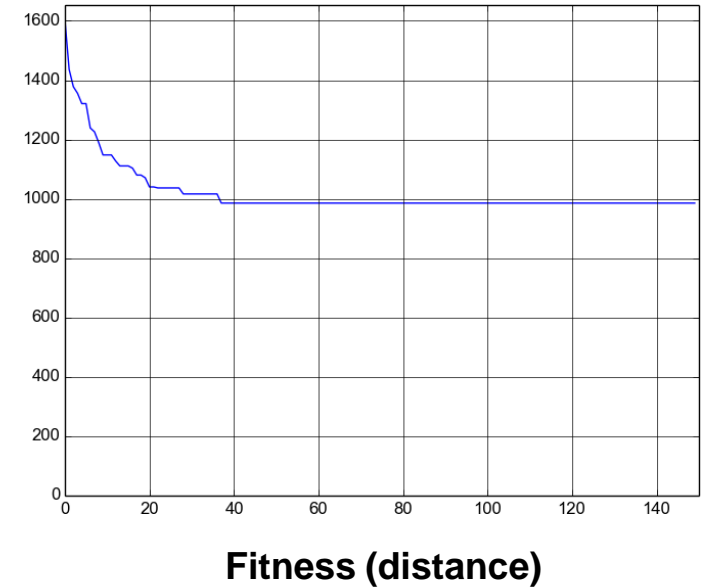
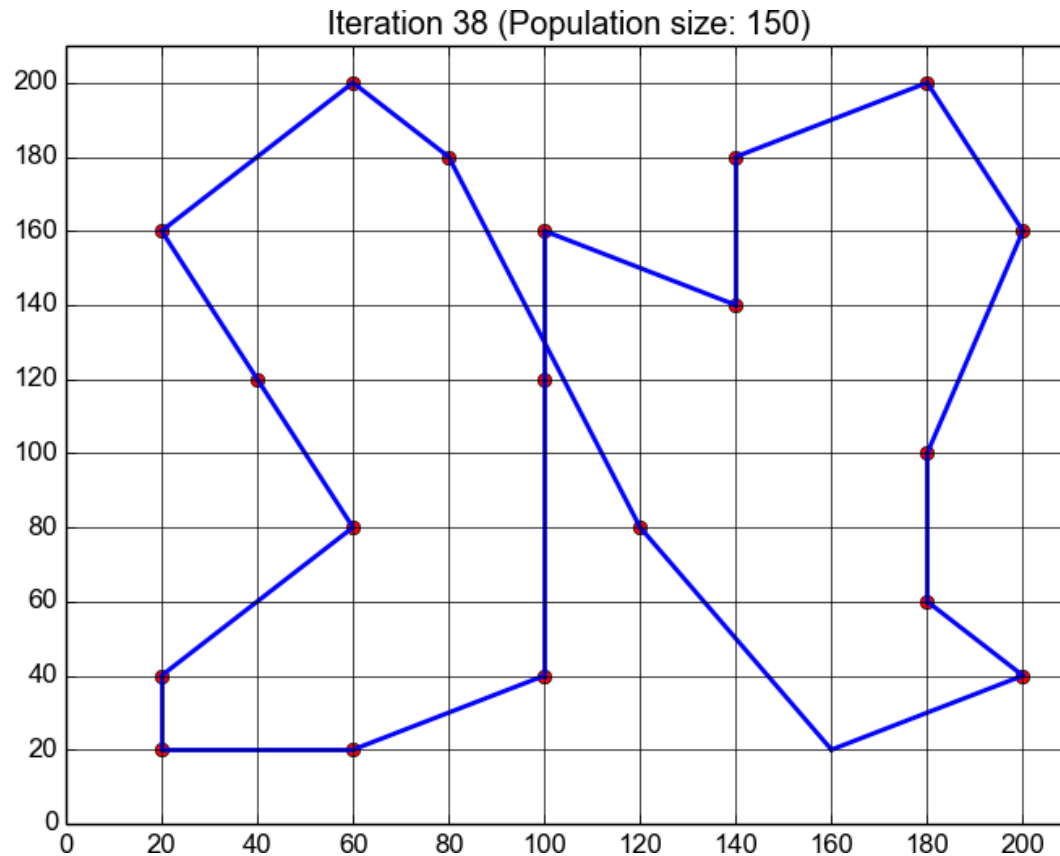
Recap: GA: Travelling salesman

- Given a set of locations, visit all locations in the optimal route
- Simple example: 3 cities
 - E.g. Vienna, Graz, Linz
- *How many possible routes?*
 - 6 different routes (be less if starting & end city are fixed)
- *General number of routes?*
 - Factorial: $n! = n \times (n-1)!$
 - $3! = 3 \times 2 \times 1 = 6$



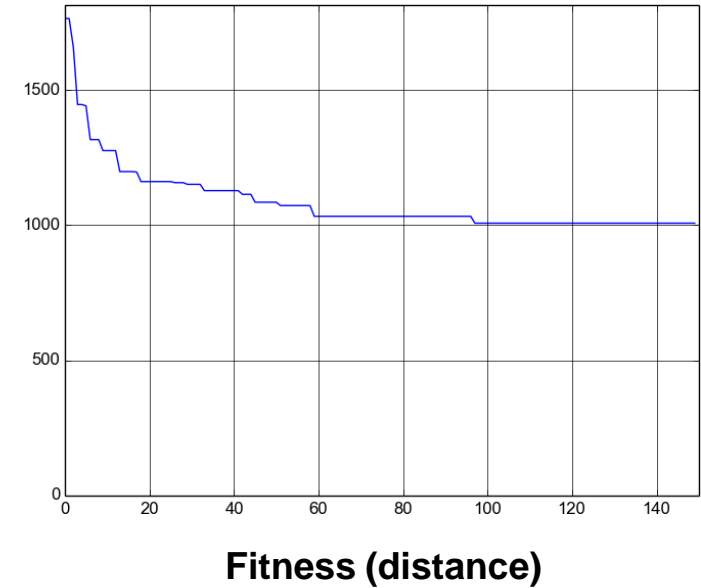
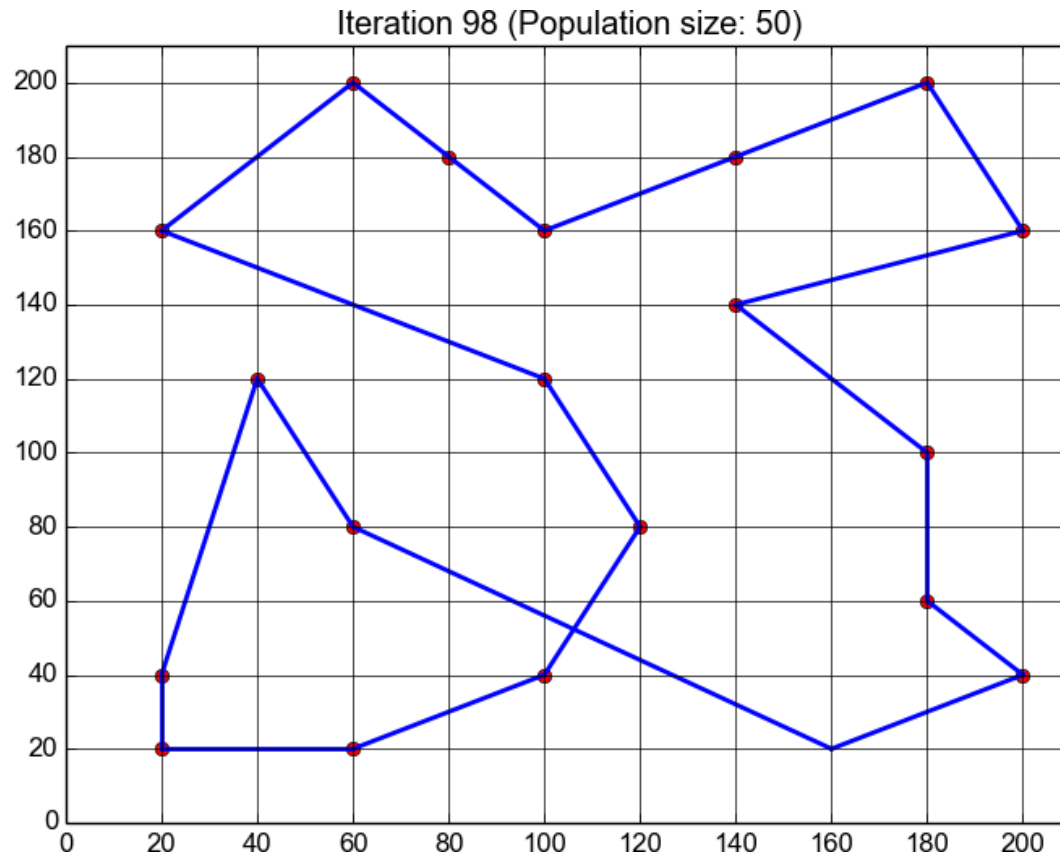
Recap: GA: Travelling salesman

- Run 1
 - Population size: 150, Iterations: 150
 - Final solution in Iteration 38



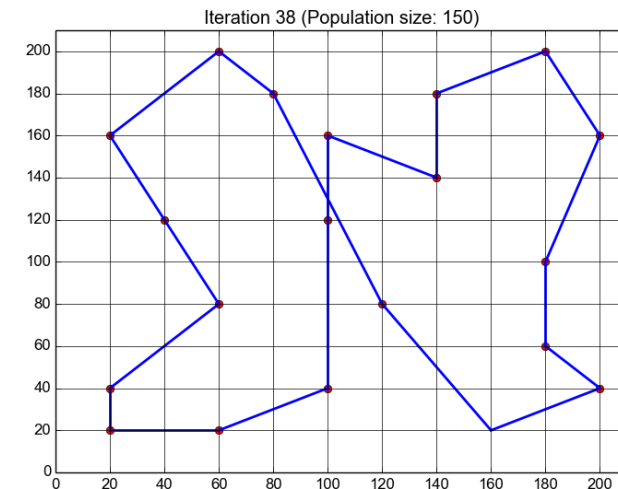
Recap: GA: Travelling salesman

- Run 2
 - Population size: 50, Iterations: 150
 - Final solution in Iteration 98



Recap: GA: Travelling salesman

- Mind: population size vs. phenotype (solution) size
- In the previous example
 - 20 cities
 - Phenotype / solution contains 20 indices
 - Population size 150
 - At each iteration (generation): create 150 ***candidate*** solutions
 - In the example: always shown the best solution in that iteration



Recap: GA: Travelling salesman

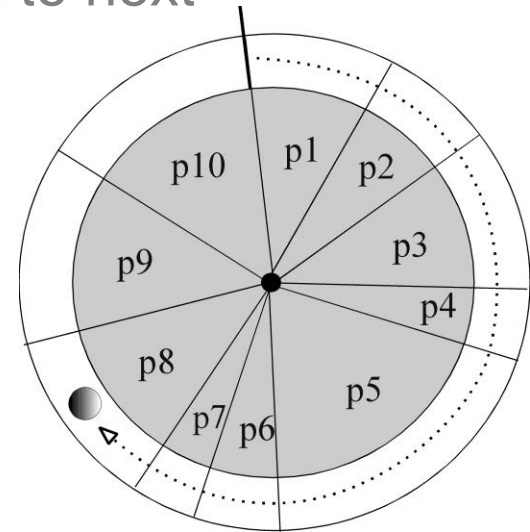
- Encoding
- Adaptations of mutation & crossover operators
 - To ensure valid solutions
- Relatively quick convergence
- *Optimal solution found?*

- Models the principle of „surviving the fittest“ observed in nature
- Simple approach: only select the ***n*** fittest
 - Favours the fitter individuals, suppresses the weak ones
 - Each individual should have some chance to pass to next generation

- **Roulette wheel**

- Probability P_i of selecting i -th individual is proportional to its fitness f_i
 - (*Fitness proportionate selection*)

$$P_i = \frac{f_i}{\sum_{j=1}^{PopSize} f_j}$$

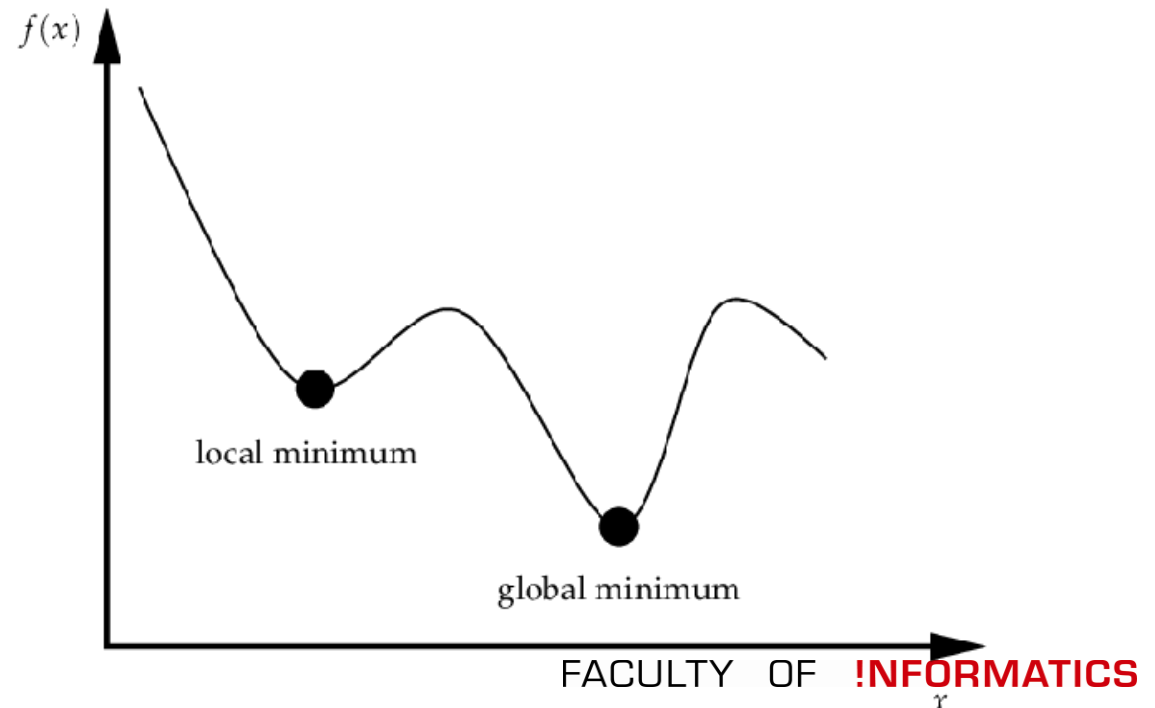


- **Other selection schemes**

- Stochastic universal sampling, ***Tournament selection***, Reminder stochastic sampling, Rank-based selection

Population diversity

- Diversity is crucial
 - Early homogenization of genetic material causes premature convergence
 - ➔ Disables valuable exploration of the search space
 - ➔ May lead to local optimum



Population diversity

- Crowding, Fitness sharing
 - The more dense a region of the search space (more individuals of the current population) → the less fit they become
- Modified **selection**, e.g. restricted tournament selection
 - Select individual from various subsets
 - Subsets contain only similar individuals, achieved e.g. by clustering
 - More diversity, as also individuals from sub-optimal groups get chosen
- Adaptation of **mutation & crossover** genetic operators
 - When the population converges: change (increase) mutation and/or crossover parameters and/or recombination operators
 - Exploration is increased

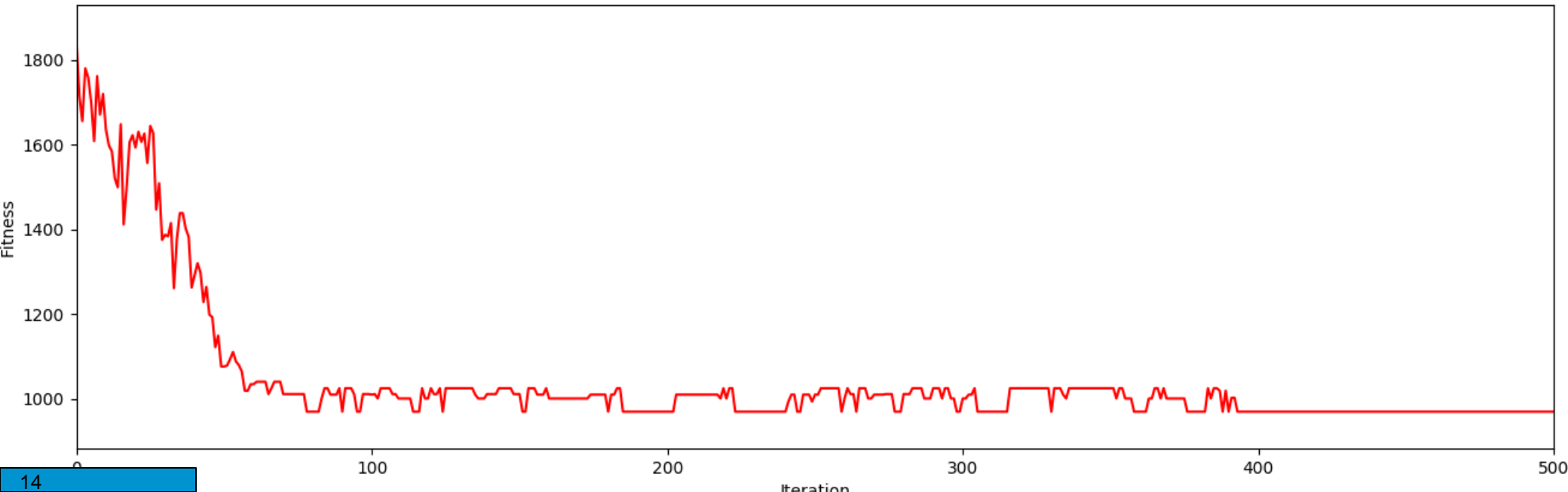
GAs: Recap general ideas

- Ideas are:
 - Adaption
 - Bring new „ideas“ into a system (Mutation)
 - Select among competitors (Selection)
 - Bring together successful individuals (Mating)

„It's Alive“: The Coming Convergence of Information, Biology, and Business“, Christoph Meyer, Stan Davis

GA: elitist algorithm

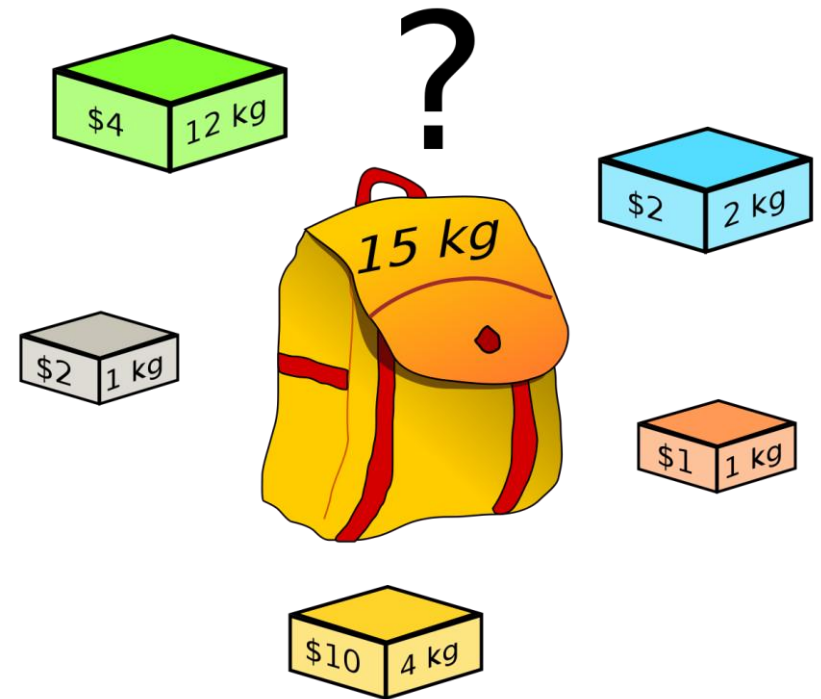
- Elitist algorithm variation:
 - During selection: always keep best-known solution so far
 - *Why?*
 - Avoid regress in optimal solution found so far
 - If population big enough: not influencing population diversity too much
 - Alternative: Pocket algorithm – keep best known solution as *result*



GA: Other example

- Given a set of items, each with a weight and a value
- Determine which items to include in a collection so
 - That the total weight is less than or equal to a given limit
 - And the total value is maximised

- How to represent as GA?*



- Recap
- Genetic Programming
- Ant Colony Optimisation
- Agents

Genetic Programming (GP)

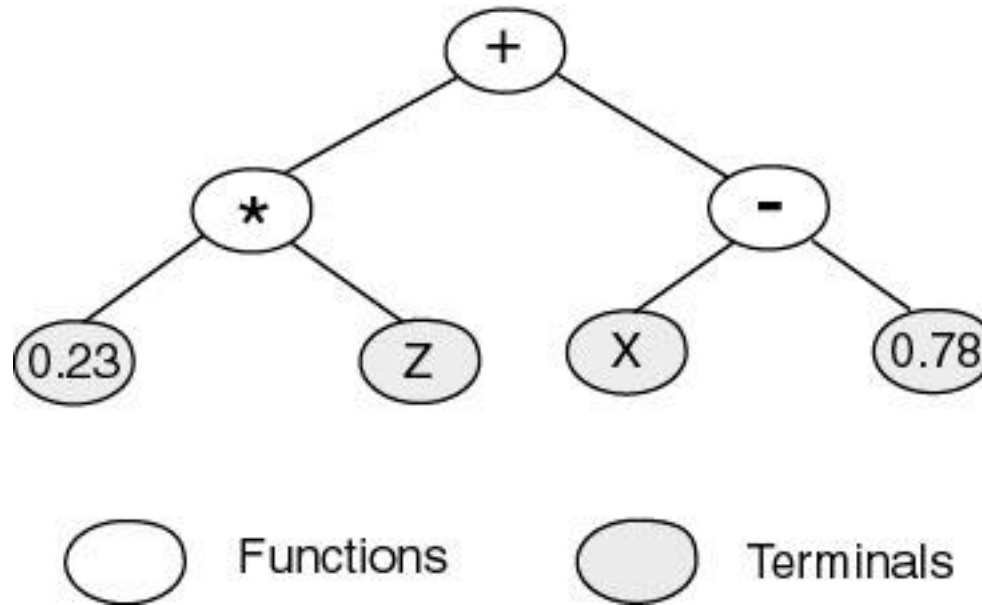
- Evolutionary Algorithm, similar to Genetic Algorithms
- Structures that undergo adaptation are **trees**
 - Variable size and shape
- Trees represent programs / *instructions*
 - Composed of functions (inner nodes) and terminals (leaf nodes) chosen for the problem at hand:
 - Terminals T – operands: input variables of the program
 - Functions F – operations on the data
- Tree evaluated in a recursive manner

Genetic Programming (GP)

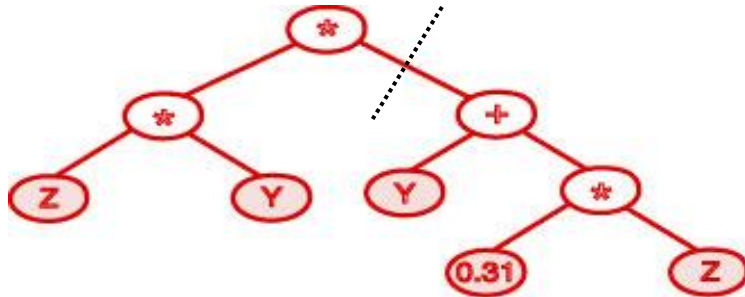
- Trees composed of functions and terminals
 - Terminals T – input variables
 - real-valued, integer or logical constants, functions w/o argument
 - Functions F
 - arithmetic functions (+, -, *, /)
 - algebraic functions (sin, cos, exp, log)
 - logic functions (AND, OR, NOT)
 - conditional operators (If-Then-Else, cond?true:false)
 - other problem specific operations
- Closure condition
 - It is necessary that the output of an arbitrary function or terminal can be an input of any other function
 - Favours languages like LISP & generally functional languages

GP: Representation

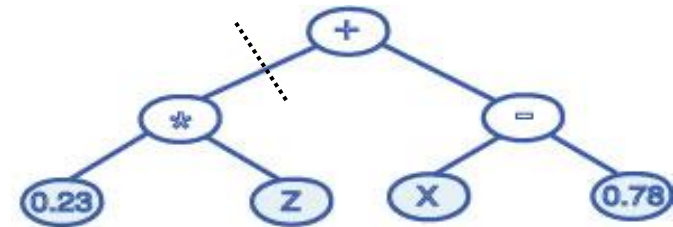
Ex.: Tree representation of expression **$0.23 * Z + X - 0.78$**



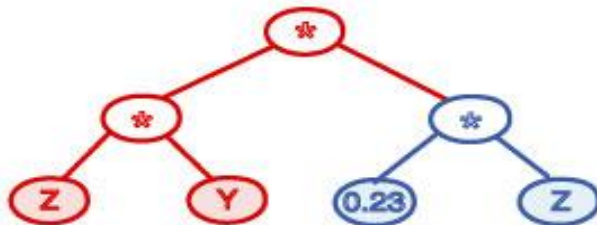
GP: Crossover



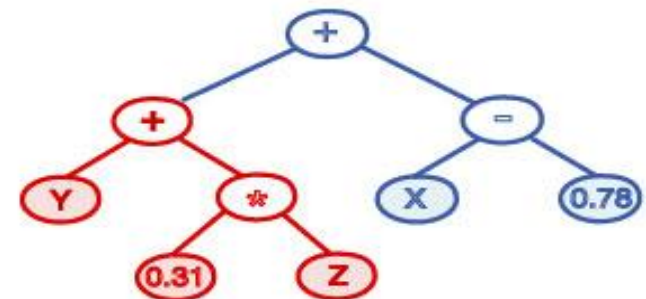
Parent 1: $Z * Y * (Y + 0.31 * Z)$



Parent 2: $0.23 * Z + X - 0.78$



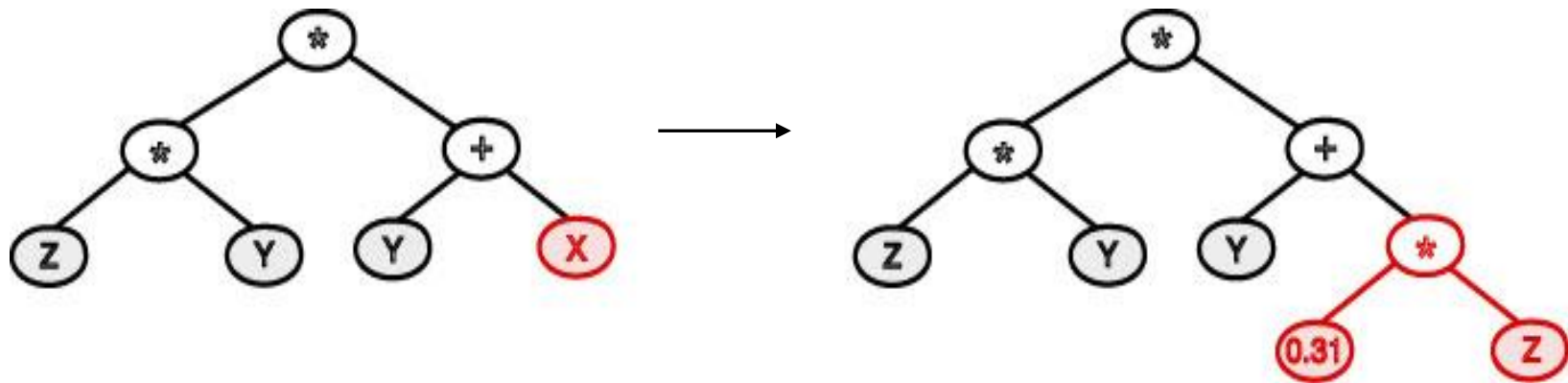
Child 1: $0.23 * Y * Z^2$



Child 2: $Y + 0.31 * Z + X - 0.78$

■ Mutation

- replaces selected sub tree with a randomly generated new one



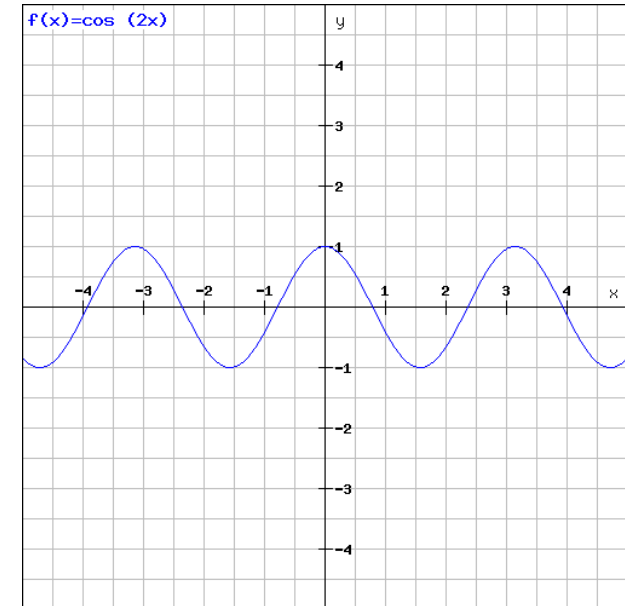
- Other operators proposed, e.g. permutation

GP: problem definition

- Goal: find a tree that represents an (optimal) program
- Challenges
 - How to evaluate optimality?
 - For which types of programs is this doable?
 - Size of search space
- Example: finding identical mathematical functions

GP Example: Finding a trigonometric identity

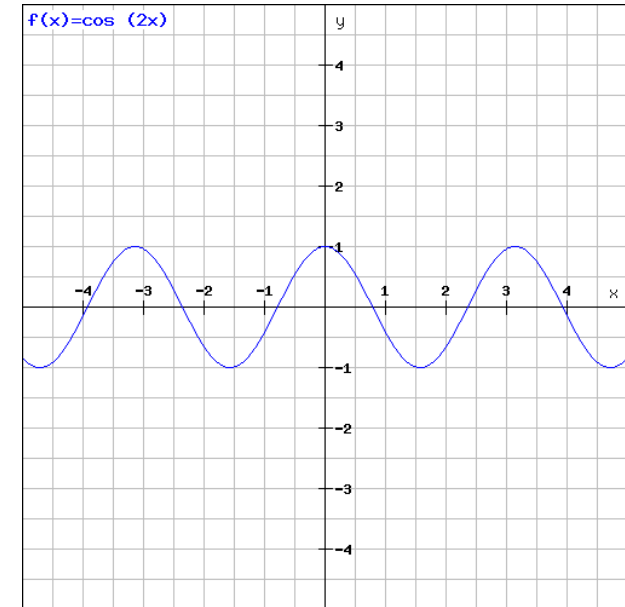
- Goal is to find a solution for the right side of the equation $\cos 2x \equiv ?$



- Terminals: $T = \{X, \text{constant } 1.0\}$
- Functions: $F = \{+, -, *, \%, \text{SIN}\}$
- Fitness: Sum of absolute difference between y_i and the value generated by the tested expression for given x_i

GP Example: Finding a trigonometric identity

- Goal is to find a solution for the right side of the equation $\cos 2x \equiv ?$



- Stopping criterion:
solution with fitness (error) $< 0,01$
- Test cases: 20 pairs (x_i, y_i) (population size)
 - x_i randomly chosen from the interval $\langle 0, 2\pi \rangle$
 - $y_i = \cos(2 x_i)$ (groundtruth)

GP Example: Results

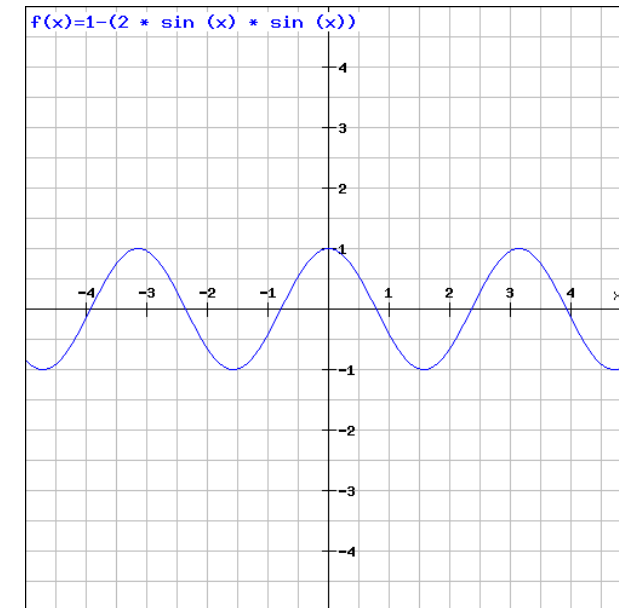
■ 1st test run

- 13th generation: following solution found (prefix notation)

$$(- (- 1 (* (\sin X) (\sin X)))) (* (\sin X) (\sin X))$$

- After reformulating: **$1 - 2 \sin^2 (x)$**

- $1 - 2 \sin^2 (x)$ is a *known identity* of $\cos(2x)$:
- $\cos(2x) = \cos^2 (x) - \sin^2 (x) = 2 \cos^2 (x) - 1$
 $= 1 - 2 \sin^2 (x)$



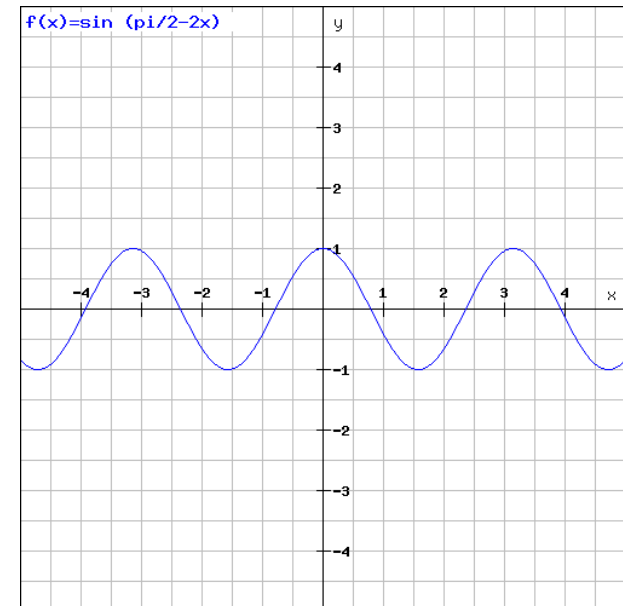
■ 2nd test run

- in 34th generation: solution $(- 1 (* (* (\sin X) (\sin X)) 2))$
 - the same expression as in the first run

$$(\sin (-(-2(*X2)))$$

))))))))

))

$$\left. \vphantom{\frac{\pi}{2}} \right\} \approx \frac{\pi}{2}$$
$$\cos 2x = \sin (\pi/2 - 2x)$$


GP: properties

- Learns an equation/program
- Most often represented as tree
 - Closure condition
- Bloating effect – individuals would be reducible to shorter (more readable) form (cf. “cos 2x” example earlier)
 - Difficult to interpret
 - Approaches:
 - Limit allowed depth of individuals
 - Punish individuals with large size
 - Combination of both
- Requires the goodness of the program to be easily **evaluated** by fitness function

GP: Further readings

- <http://www.human-competitive.org/awards>
 - Papers awarded in the “human-competitive results” challenge
 - (Results were produced by any form of genetic and evolutionary computation, regularly contains GP results)

- Further material:
 - <http://www.genetic-programming.com/>
(outdated)

 - <http://geneticprogramming.com> (updated more recently)

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Ant Colony Optimisation

heuristic approach

- Representative of swarm intelligence algorithm
 - *More algorithms to follow in upcoming lectures*
- Basic algorithm of artificial ant colonies
- Extensions to these algorithms
- Initial Application areas

Why ants?

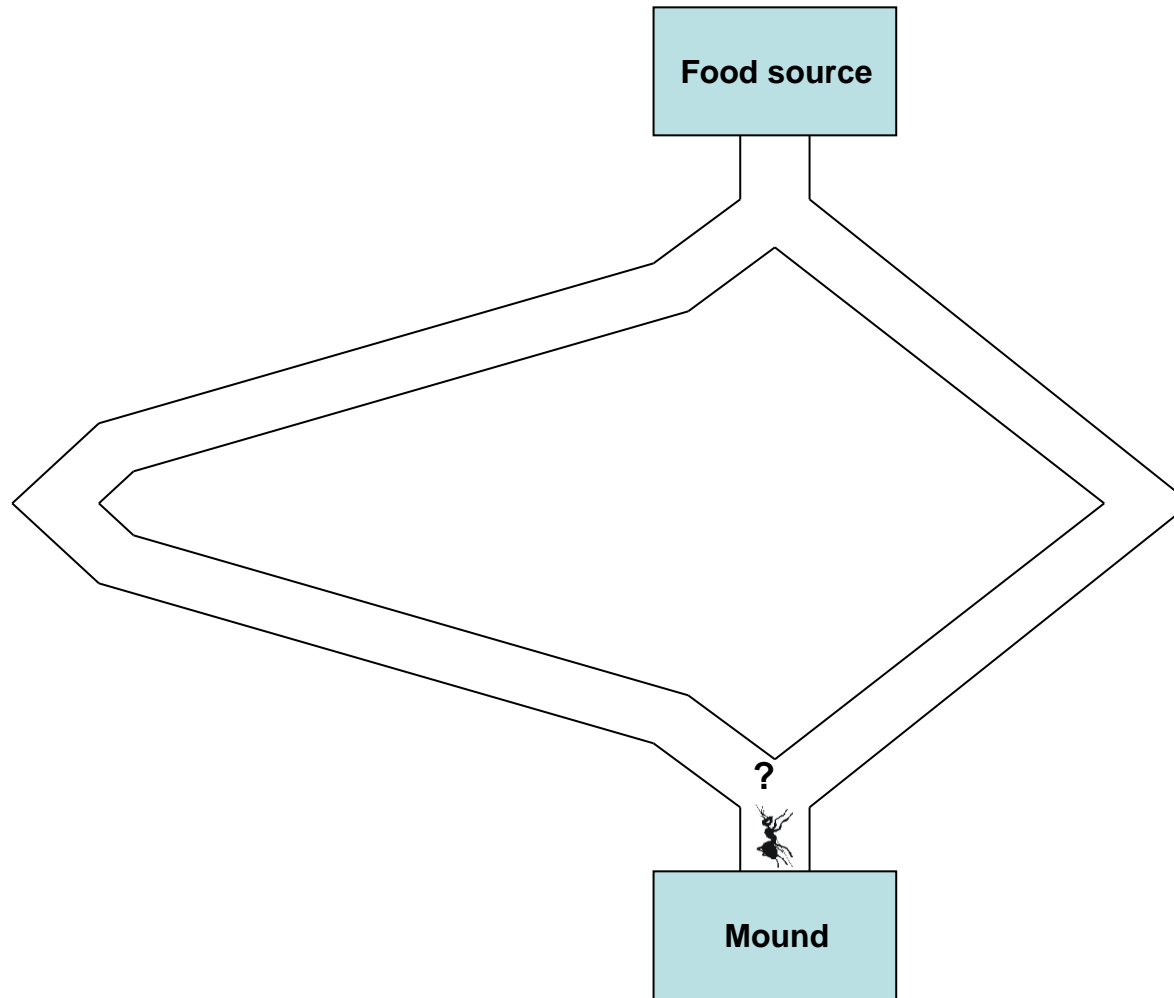
- Ants are relatively simple individuals
- Characteristics
 - De-centralised decision making
 - Feedback, random behaviour, ...
 - Communication between ants, via pheromones (trail-laying / trail-following behaviour)
- ➔ Capabilities to solve complex problems
 - Building of geometrically complex mounds
 - **Finding of shortest paths to food sources**
- Self-organising system



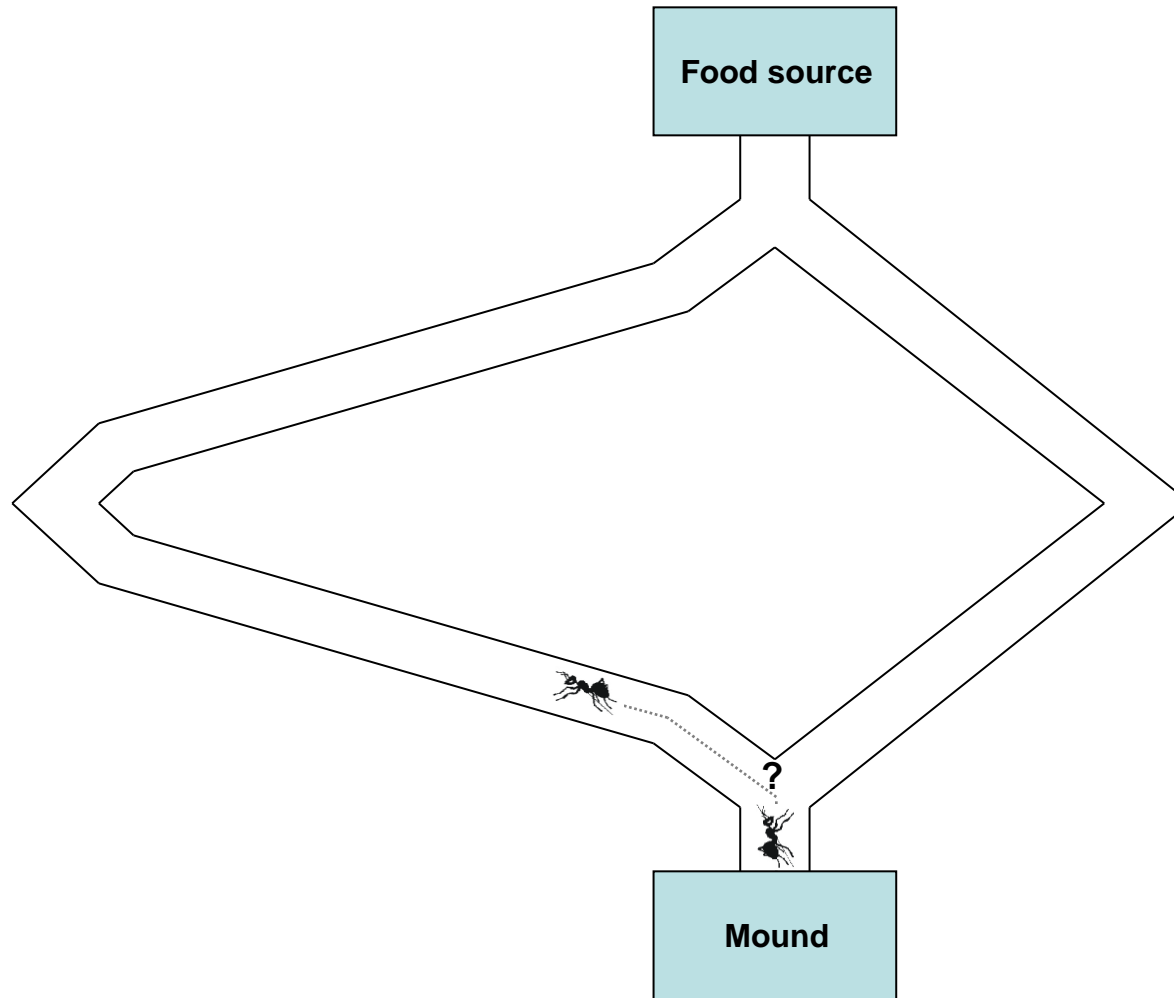
Finding the shortest path

- Ants wander (randomly) to find food
- Once found, return to the mound
 - Leaving pheromone trail
- Subsequent ants will likely follow that trail
- If they also find food, they will reinforce that trail (“solution”)

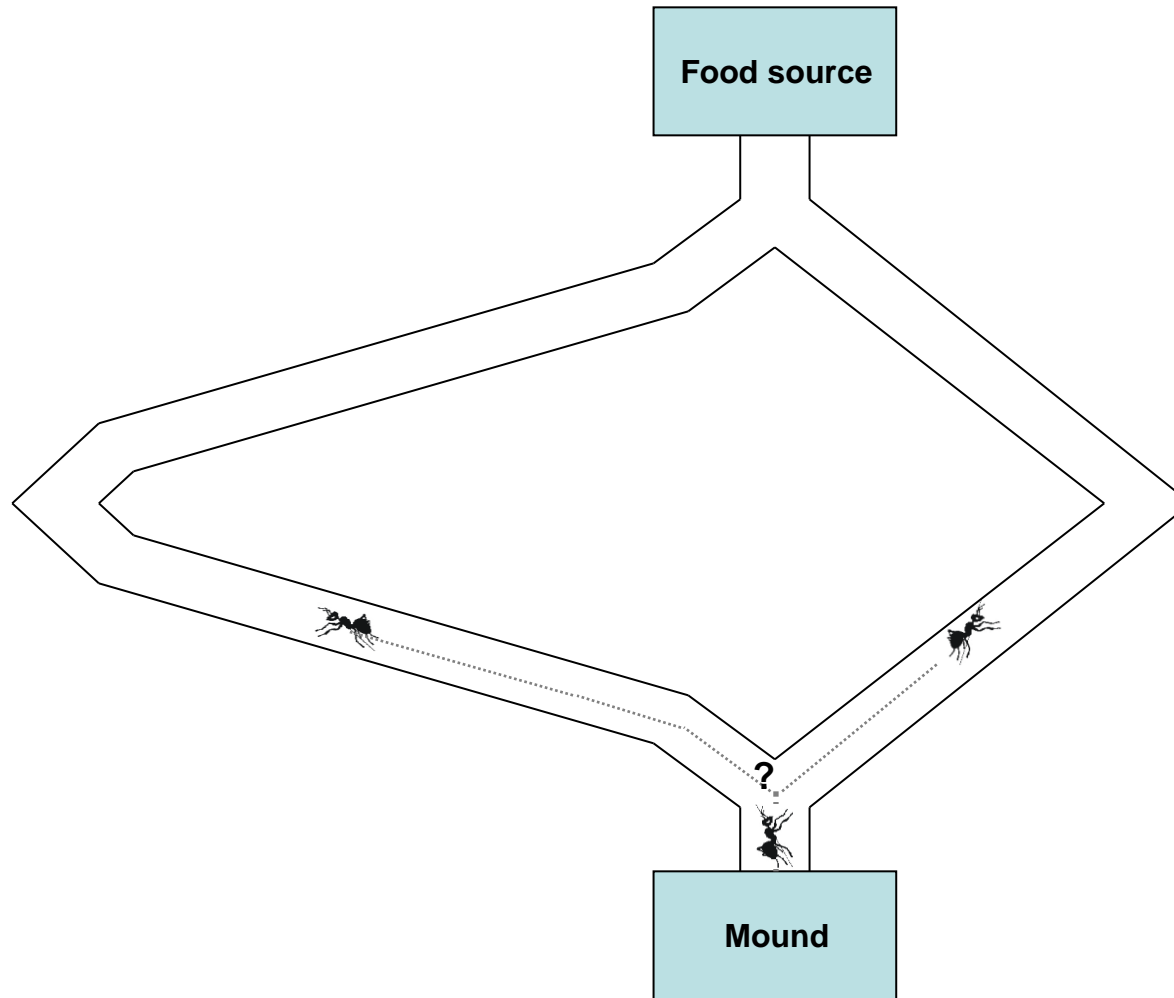
Finding the shortest path



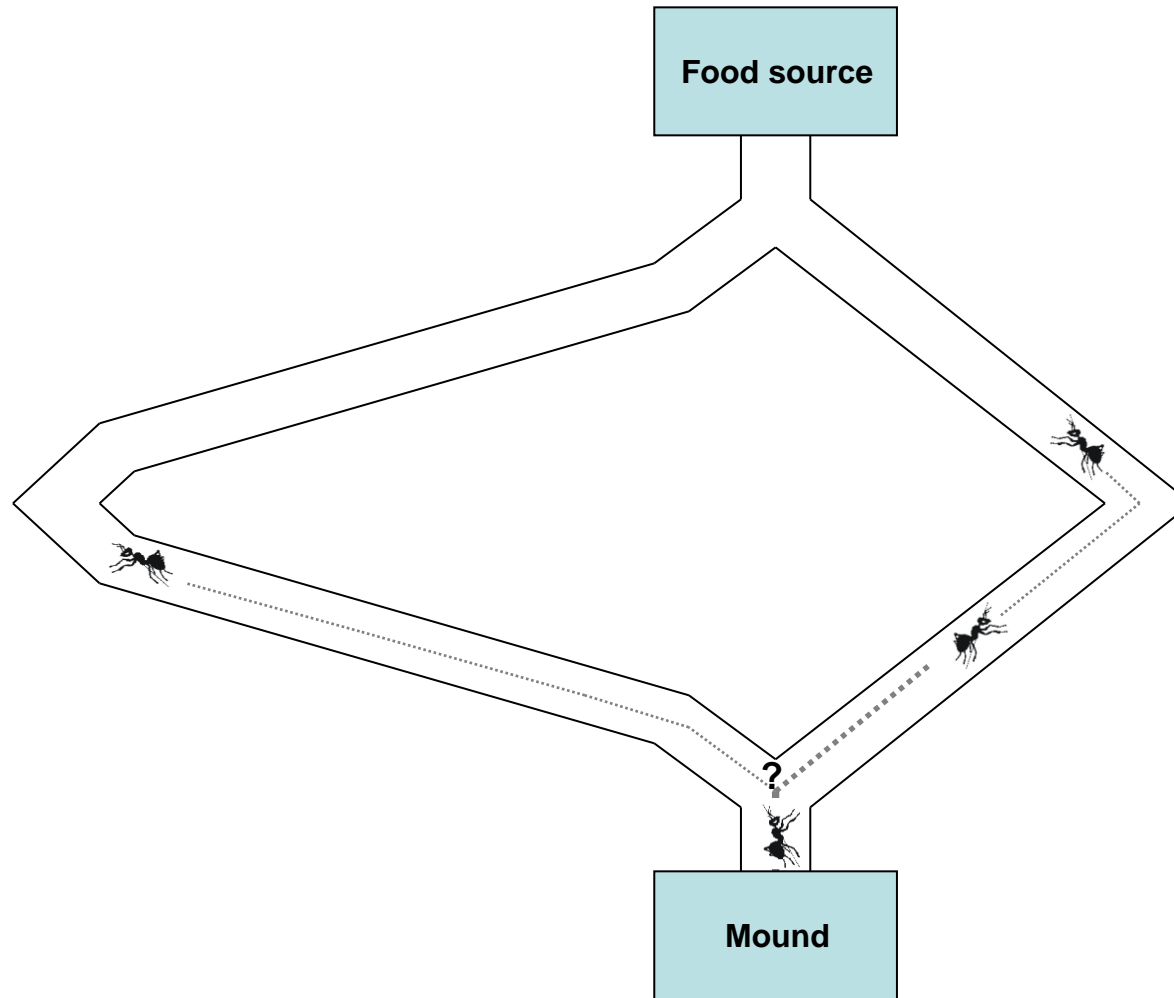
Finding the shortest path



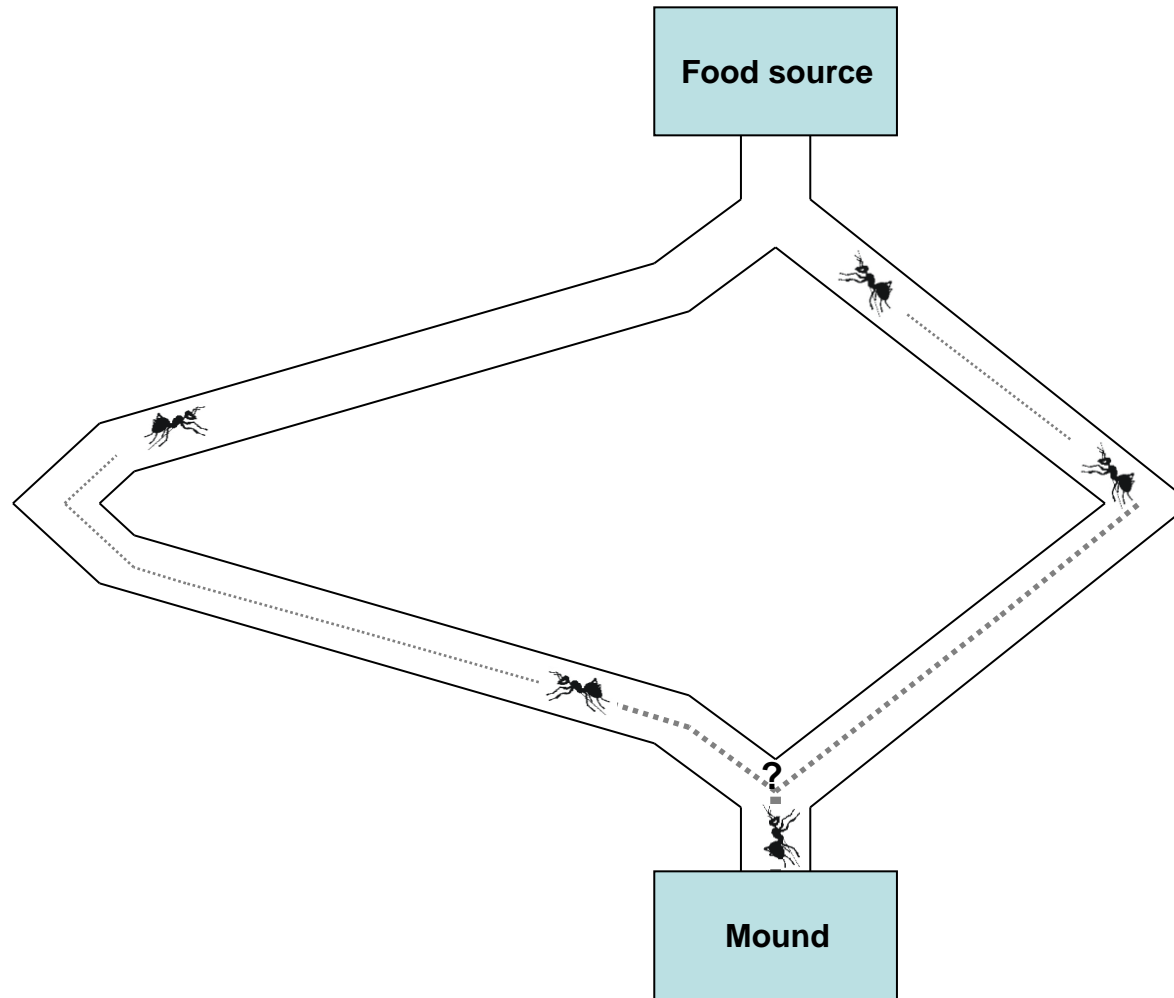
Finding the shortest path



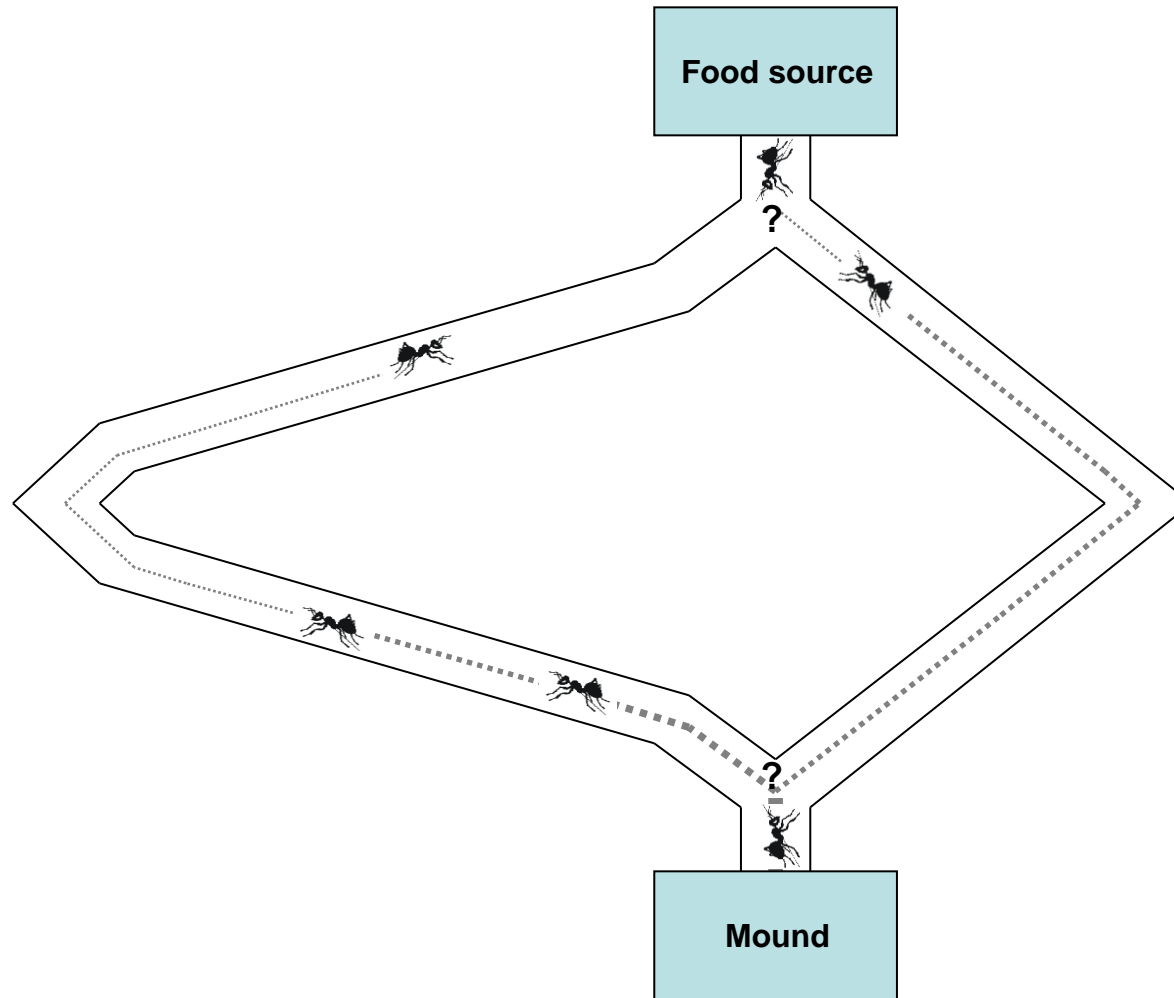
Finding the shortest path



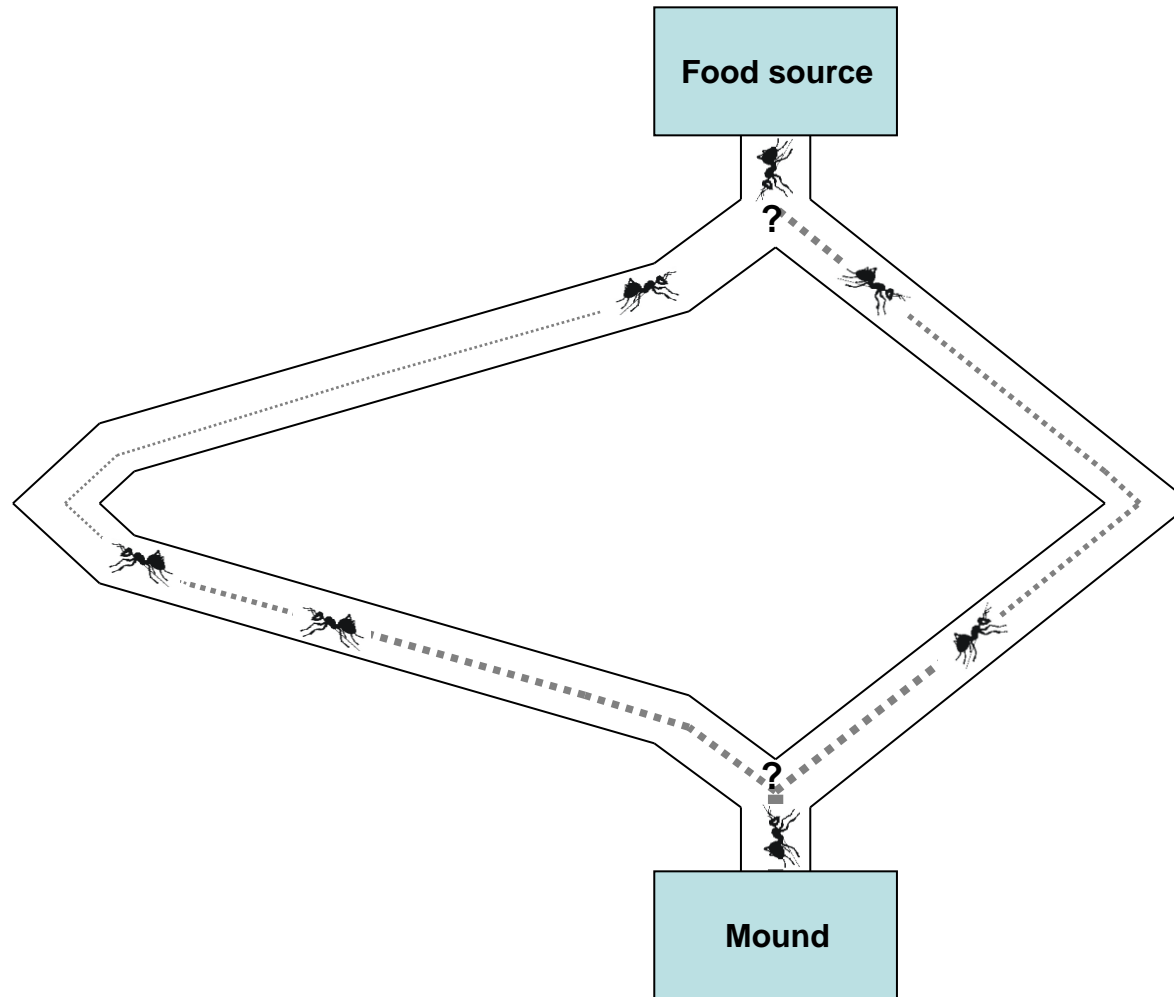
Finding the shortest path



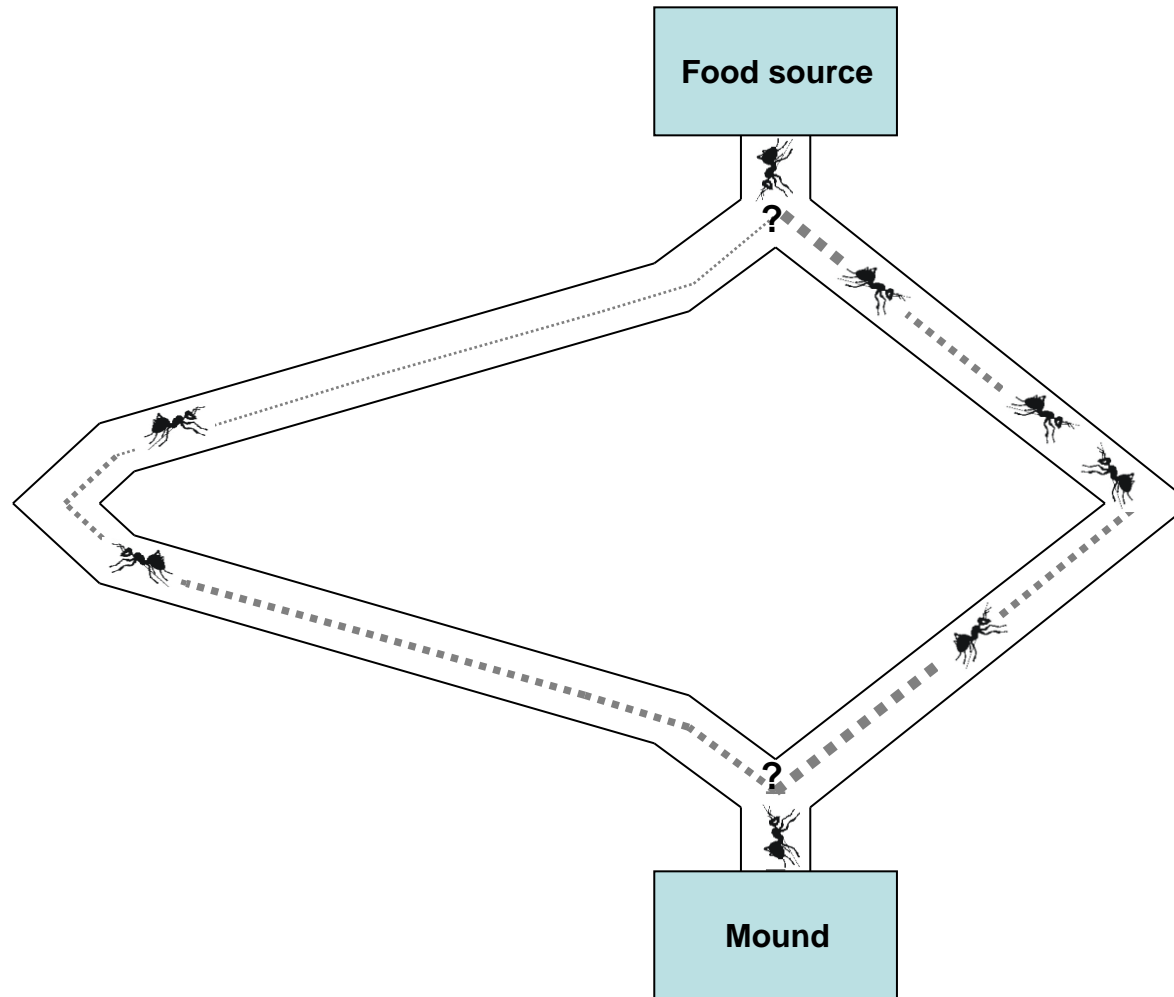
Finding the shortest path



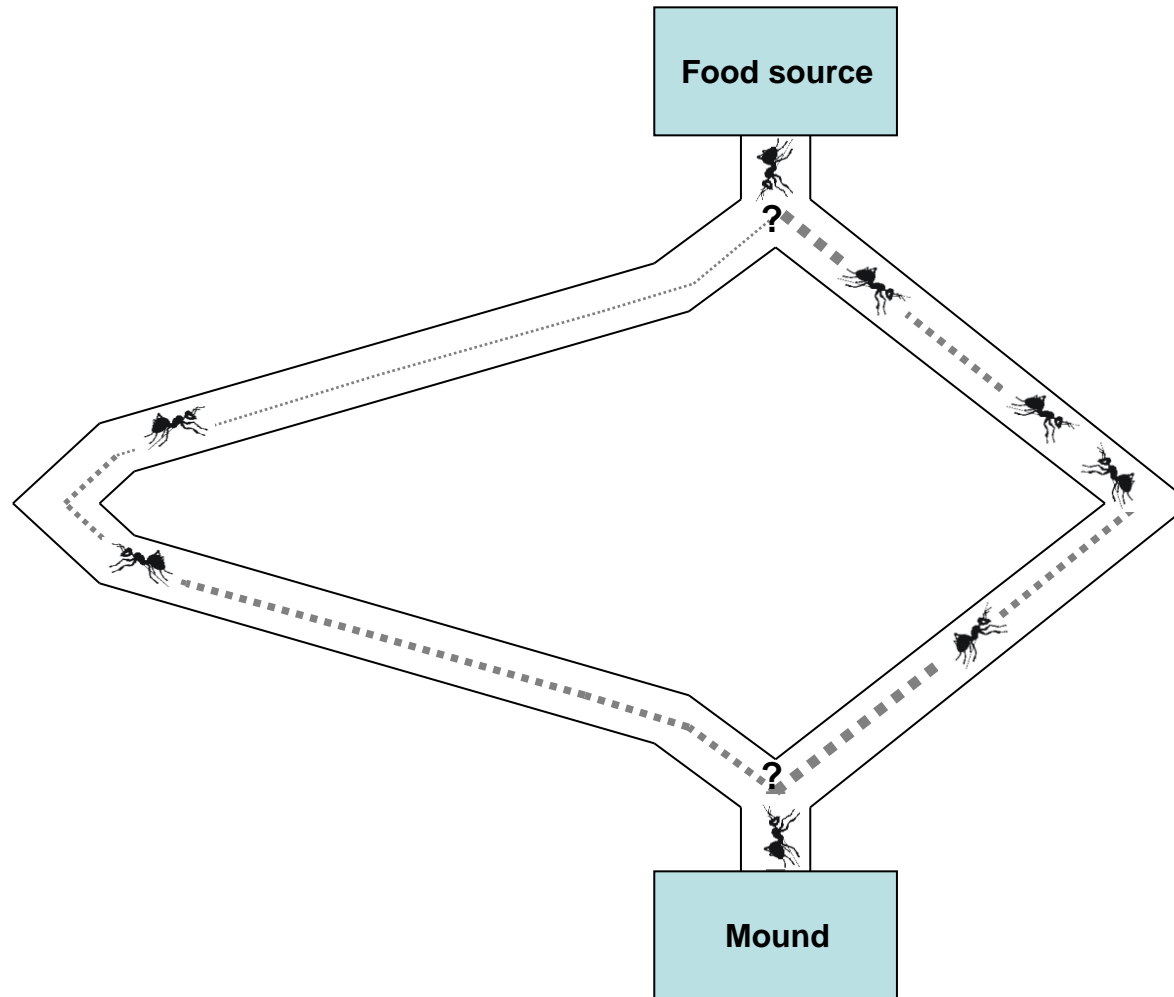
Finding the shortest path



Finding the shortest path

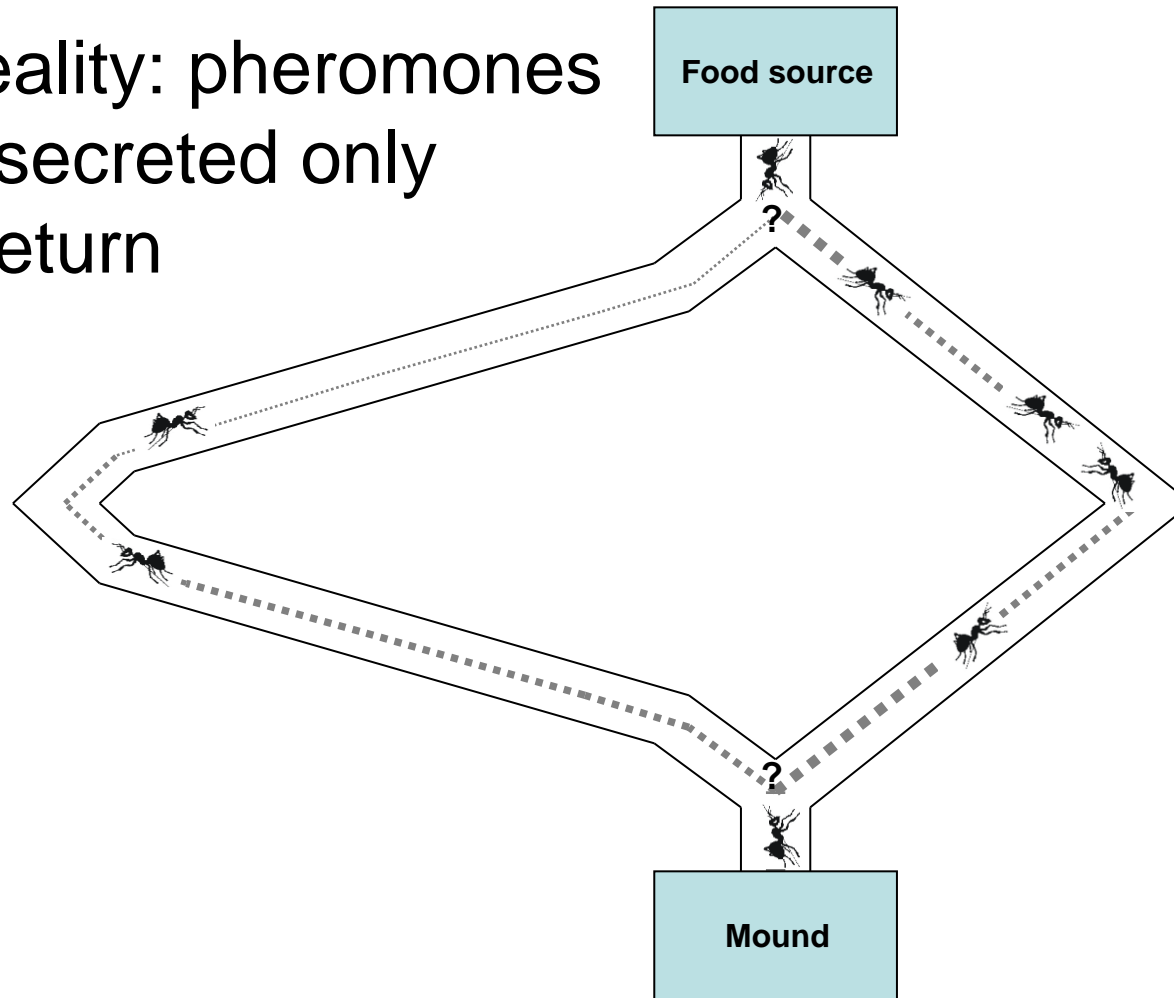


Finding the shortest path



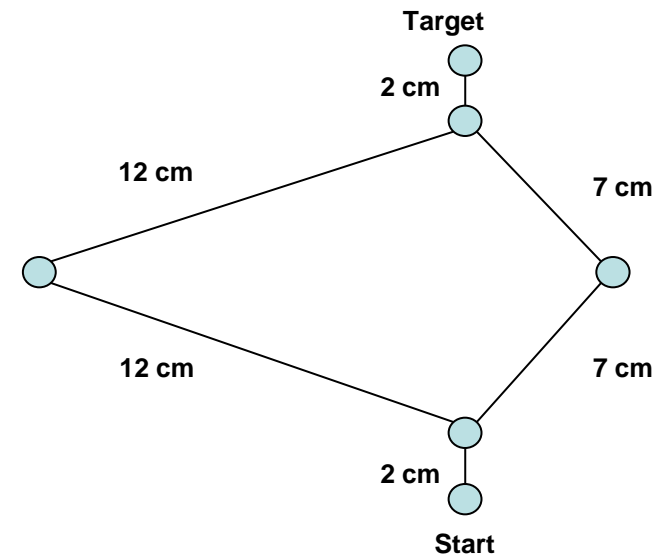
Finding the shortest path

- In reality: pheromones are secreted only on return



- Initial Algorithm: Ant System
Coloni, Dorigo and Maniezzo, 1991

- Differences zu natural ants:
 - Artificial ants move only in one direction
 - Rule for decision
 - Pheromones are secreted after end of one iteration
 - *Natural ants don't optimise only by shortest path*



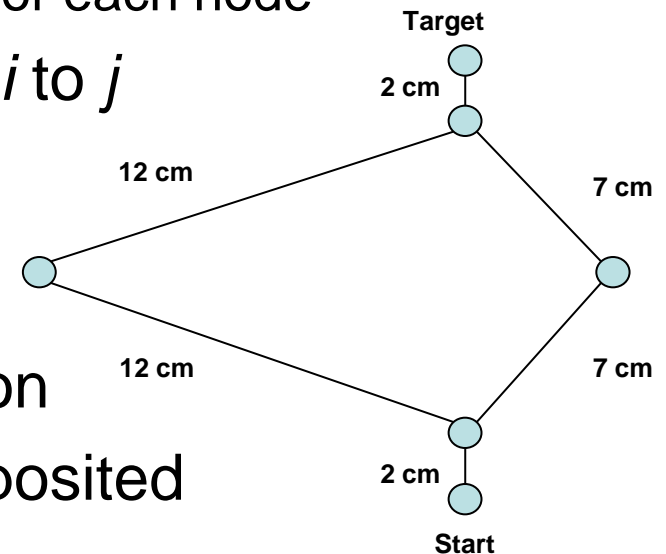
State transition rules

- Decide to which node an ant will move
- Prevent cyclic paths
 - keeping a list of potential transition nodes for each node

➔ Probability p of k -th ant to move from i to j

Information available

- **Local** ($\eta_{i,j}$): desirability of state transition
- **Global** ($\tau_{i,j}$): amount of pheromone deposited



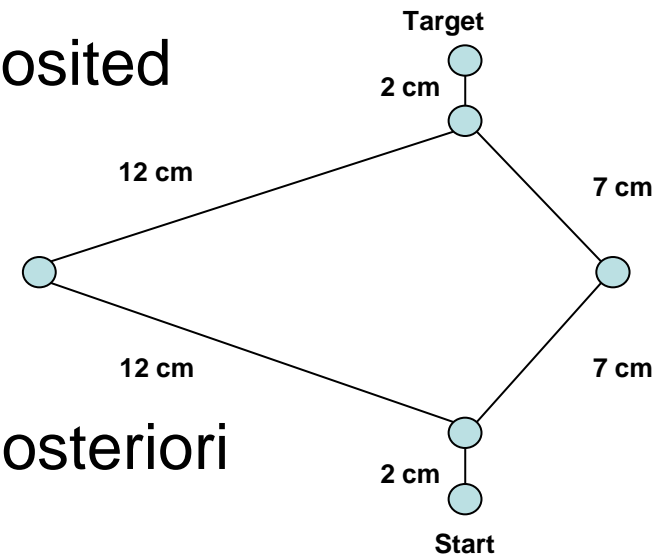
state transition rules.
based on local and global information.

local = apriory, jsut next section.
global , all toether

$$p_{ij}^k = \frac{\eta_{ij}^\alpha \cdot \tau_{ij}^\beta}{\sum_{l \in J_i^k} \eta_{il}^\alpha \cdot \tau_{il}^\beta}$$

Information available

- Local ($\eta_{i,j}$): desirability of state transition normally static over time
 - A priori knowledge about the problem domain
 - Typically: length of path, i.e. $\eta_{ij} = \frac{1}{d_{ij}}$
- Global ($\tau_{i,j}$): amount of pheromone deposited changes over time
 - A posteriori knowledge



Other parameters

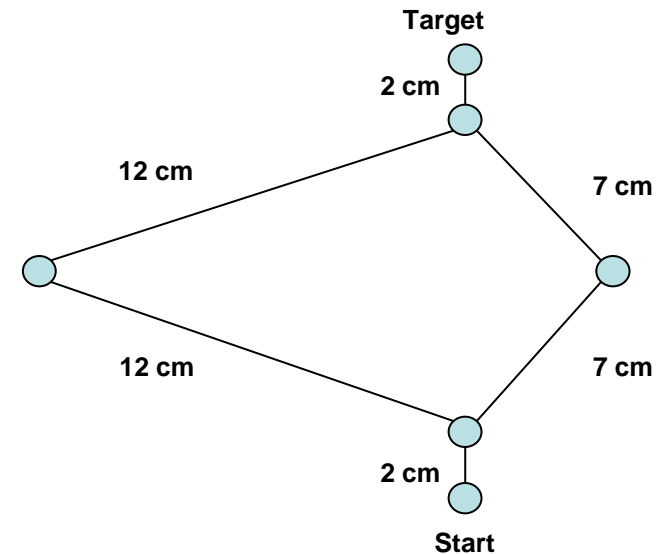
- α, β : control influence of a-priori vs. a-posteriori

$$p_{ij}^k = \frac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{l \in J_i^k} \tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}}$$

$$p_{ij}^k = \frac{\tau_{ij}^\alpha \cdot \eta_{ij}^\beta}{\sum_{l \in J_i^k} \tau_{il}^\alpha \cdot \eta_{il}^\beta}$$

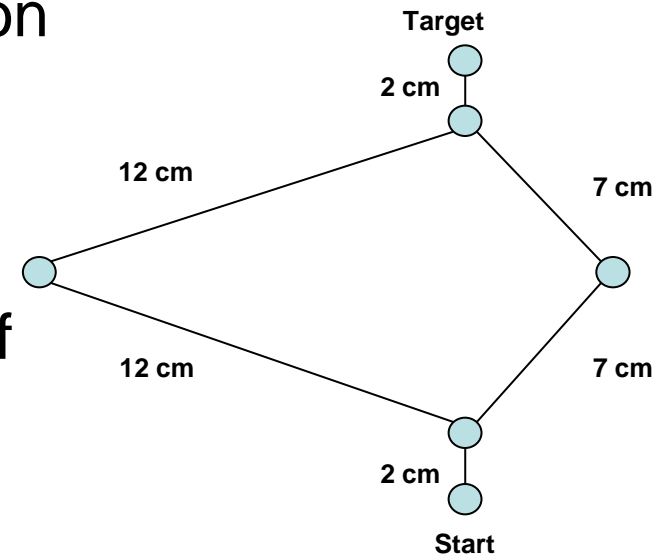
1. Compute for every possible transition
2. Select according to probabilities

or do highest/roulet/tournament
"diversity vs convergence"



■ Update of pheromones

- When all ants reached the target: pheromone trails are updated
- Pheromones deposited on each edge used by an ant
- Amount depending on quality of solution
 - Typically on length of path (L)
 - *Compare to fitness function in GA*
- After each iteration a certain amount of pheromones vaporises: ρ



$$\tau_{ij,t+1} = (1 - \rho) \cdot \tau_{ij,t} + \Delta \tau_{ij}$$

$$\Delta \tau_{ij} = \sum_{k=1}^m \Delta \tau_{ij}^k$$

$$\Delta \tau_{ij}^k = Q / L^k$$

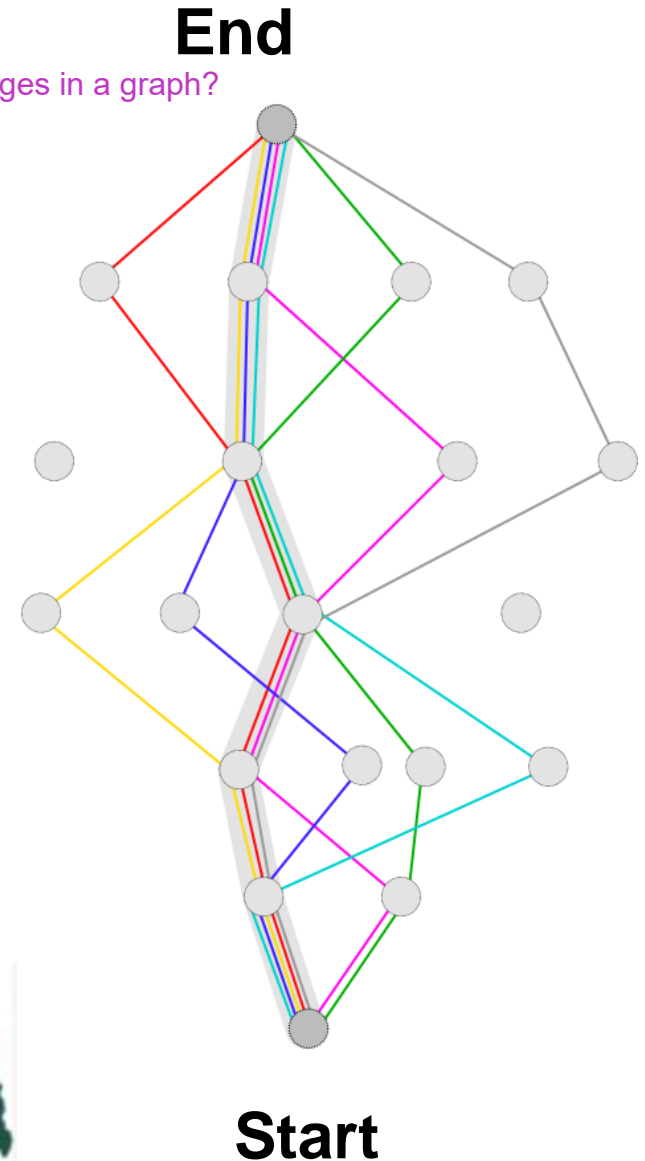
(Sum of all newly deposited pheromones by all ants 1..m)

Q: Constant

- Initialise all edges with desirability according to segment length
- For $t = 1$ to t_{\max} iterations *(iterations)*
 - For $k = 1$ to m number of ants *(ants)*
 - Node i = Start node
 - While Node i not = Target node
 - Apply transition rule \rightarrow move to next node
 - End While
 - End For
 - Compute path length of all ants, store shortest path
 - Apply pheromone update rule on edges
- End For

what are analogies. here the path is very obvious. but paths might be edges in a graph?
so maybe even clustering? distance measures?

- Often used to solve shortest-path problems
- Popular example: Travelling sales man
 - Ants don't travel from pre-defined start to end node
 - Goal reached when all nodes visited



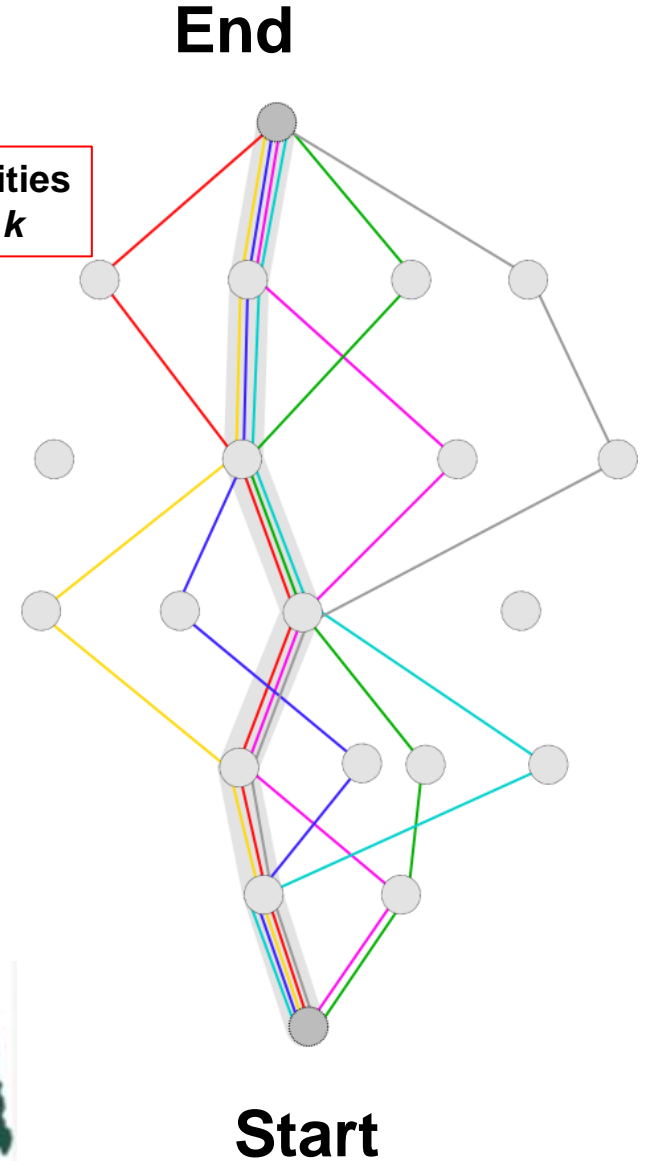
■ Travelling sales man

- Adaption of transition probability

$$p_{ij}^k = \begin{cases} \frac{\tau_{ij}^\alpha \cdot \eta_{ij}^\beta}{\sum_{l \in J_i^k} \tau_{il}^\alpha \cdot \eta_{il}^\beta} & \text{if } j \in C_k(i) \\ 0 & \text{otherwise} \end{cases}$$

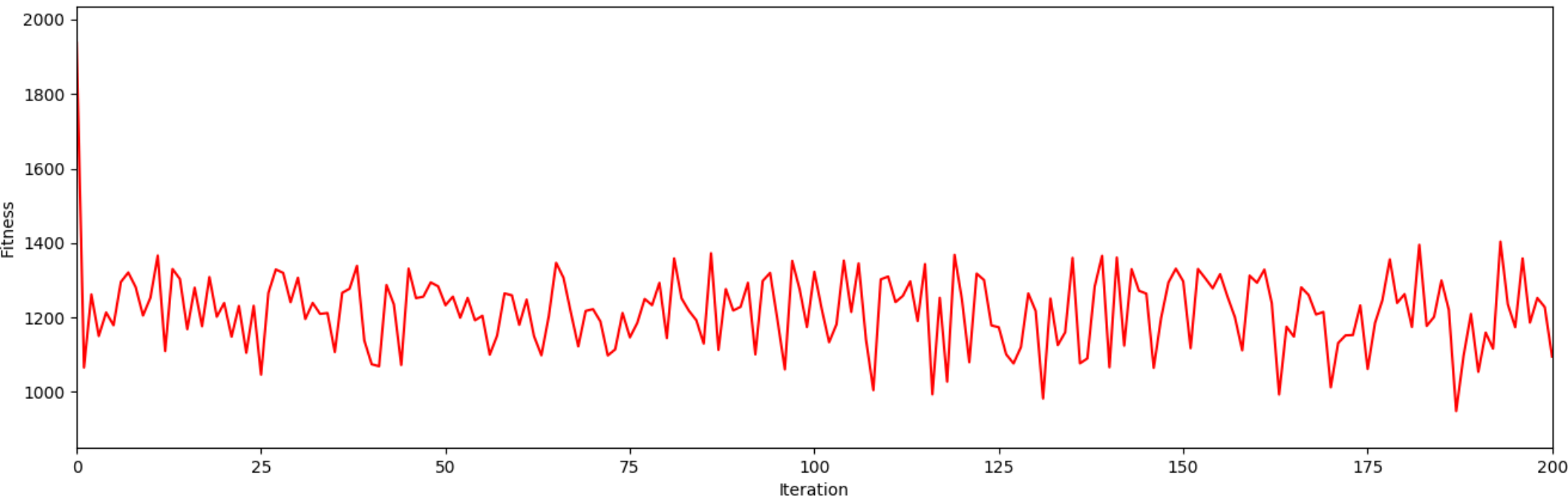
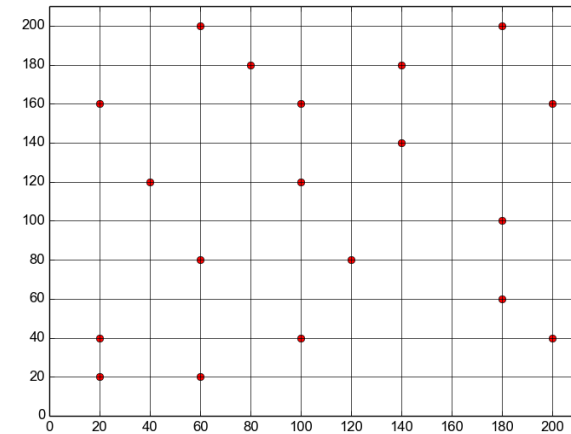
Set of remaining cities (nodes) for ant k

- Goal reached when all nodes visited



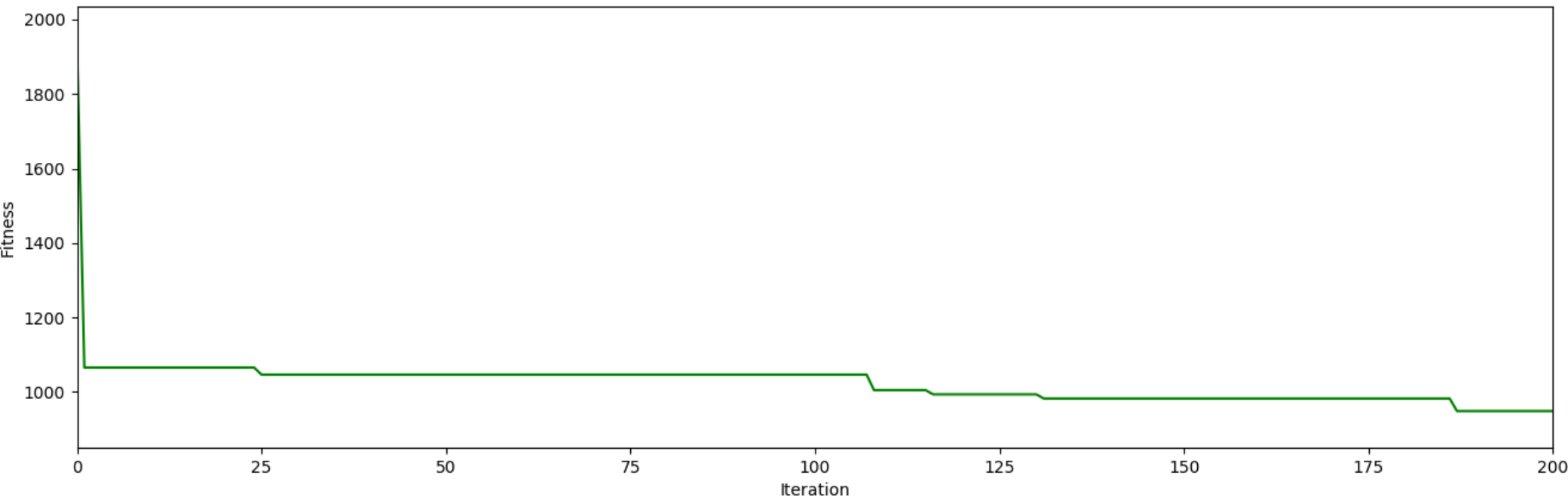
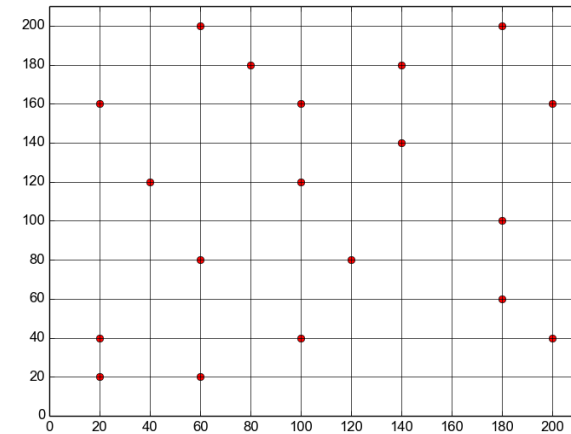
Ants on TSP problem

- Same optimisation example as with GA
 - Finds **good** solution very **fast**
 - Best solution fluctuates a lot over iterations
 - Relatively **slow improvements** from initial good solution



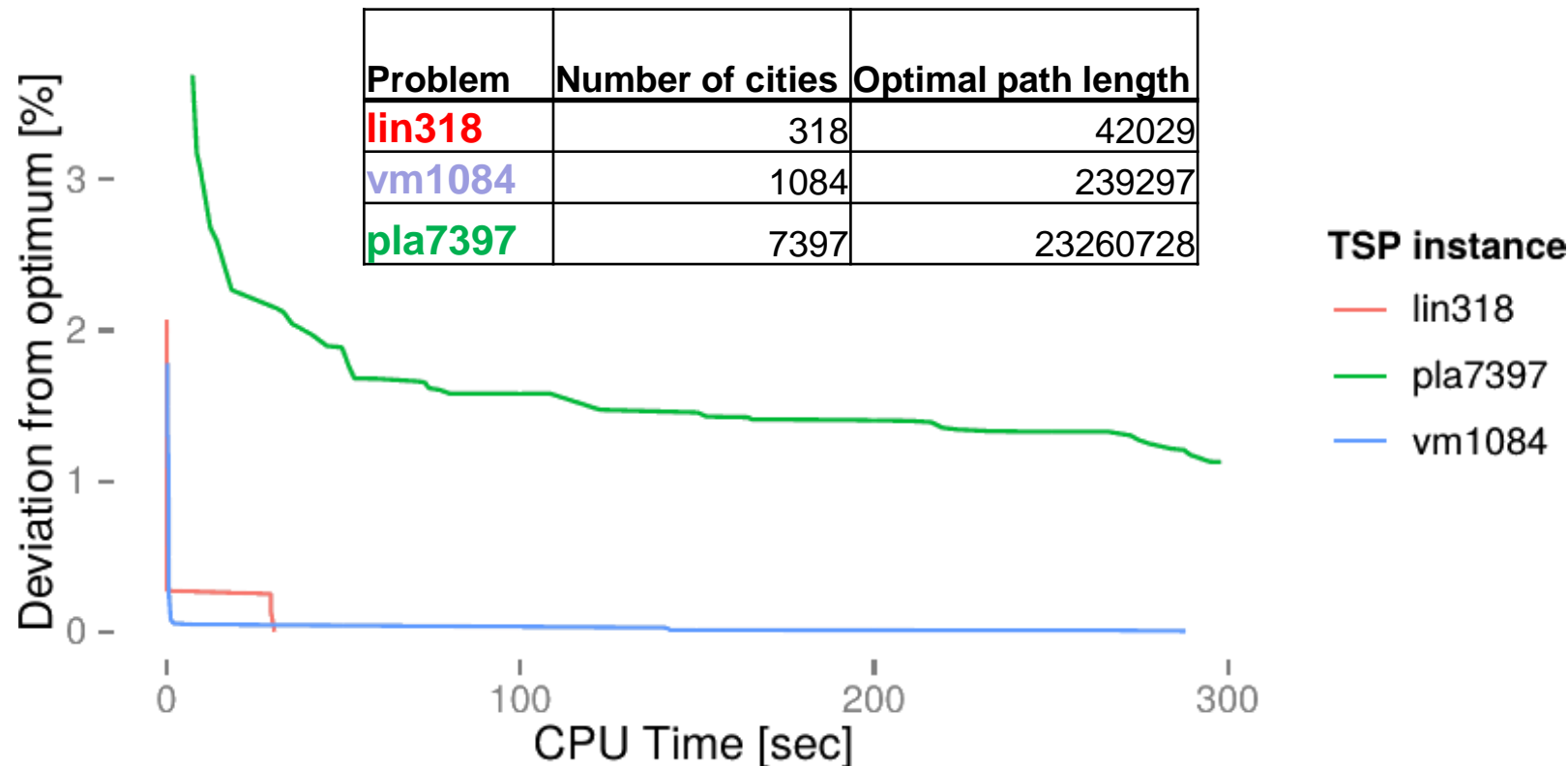
Ants on TSP problem

- Same optimisation example as with GA
 - Finds **good** solution very **fast**
 - Best solution fluctuates a lot over iterations
 - Relatively **slow improvements** from initial good solution
 - (only showing best solution)



■ Problem:

Works well for smaller tasks, not so good for larger problems with many nodes and edges



- Problem:

Works well for smaller tasks, not so good for larger problems with many nodes and edges

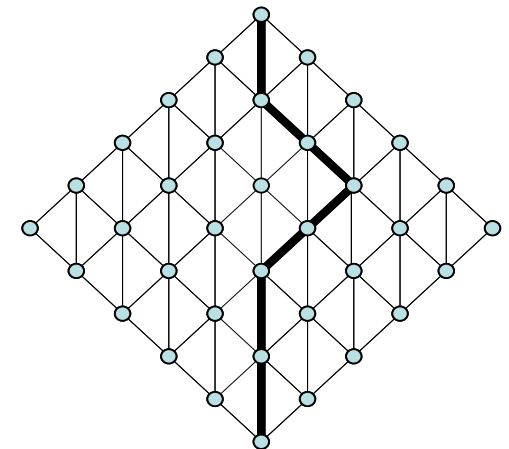
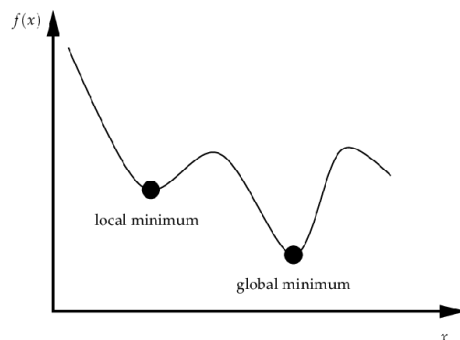
→ Several extensions to the original algorithm:

- AS with Elitist Strategy
- Ant Colony System
- Hybrid Ant System
- MAX-MIN Ant System

- Proposed by Coloni, Dorigo and Maniezzo
- Idea:
 - Currently best path contains edges of eventual optimal path
- Extension:
 - e ants additionally support the current best path

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} + \Delta \tau_{ij} + e \cdot \Delta \tau_{ij}^e$$

- Problem:
 - Too high value of e : \rightarrow local optimum



- Proposed by Dorigo & Gambardella
- Idea:
 - Reduce effort for exploring new ways
 - But do that more efficiently
- Changes:next slides detail
 - New transition rule, favouring current best solution
 - New pheromone update rule
 - Additionally: local pheromone update rule
 - Using candidate lists for each node check neighboring edges for optimum

- Pheromone update rule
 - In each iteration only the currently best solution is reinforced
 - Search focussed more on this neighbourhood

- Local pheromon update rule
 - After each state transition: pheromone trail of this edge is reduced
 - Currently used edge becomes less attractive, ants search elsewhere
 - Search space is expanded

- Transition rule:
 - Explore new paths
 - Utilise existing results

$$p_{ij}^k = \frac{\tau_{ij}^\alpha \cdot \eta_{ij}^\beta}{\sum_{l \in J_i^k} \tau_{il}^\alpha \cdot \eta_{il}^\beta}$$

$$\arg \max_{u \in J_i^k} \tau_{iu}^\alpha \cdot \eta_{iu}^\beta$$

$$j = \begin{cases} \arg \max_{l \in J_i^k} \tau_{il}^\alpha \cdot \eta_{il}^\beta & \text{for } q \leq q_0 \\ S & \text{for } q > q_0 \end{cases}$$

- q: random number 0..1
- q₀: parameter (0..1): guide amount of exploration
 - close to 0? → little exploration, likely stuck in local optimum
 - close to 1? → very similar to original ant system

- Local update rule: applied after each state transition

k : parameter, $0 < k < 1$

$$\tau_{ij} \leftarrow (1 - \kappa) \cdot \tau_{ij} + \kappa \Delta \tau_{ij}$$

- Global update rule: after all ants have completed route
- ρ : pheromone decay

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} + \rho \Delta \tau_{ij}$$

$$\Delta \tau_{ij} = \begin{cases} (L_{gb})^{-1} & \text{if } (i, j) \in \text{global-best-tour} \\ 0 & \text{otherwise} \end{cases}$$

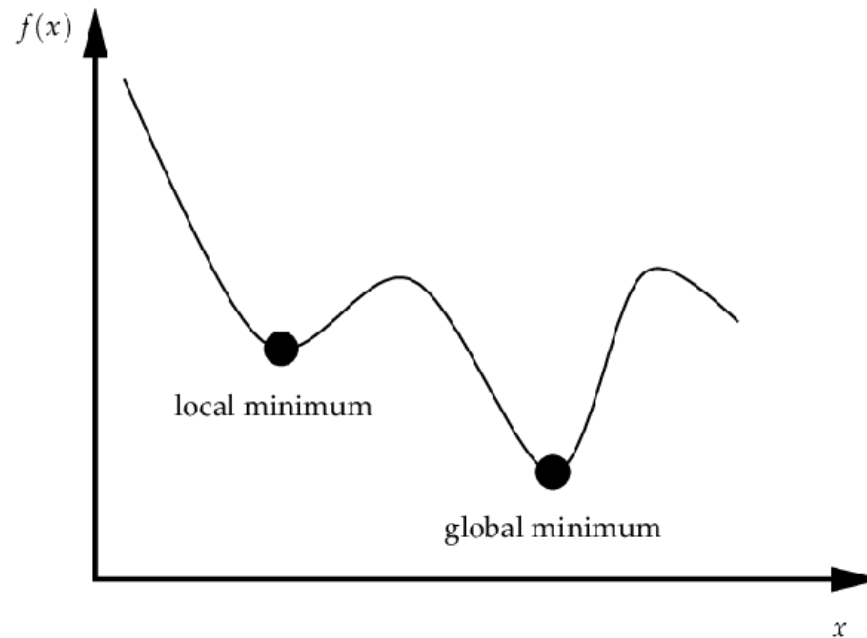
L_{gb} : length of global-best-tour (from trail beginning)

- Candidate lists *faster but not optimal?*
 - For large-scale problems
 - Each node has a list of candidate solutions
 - E.g. list of preferred cities to visit
 - List contains the c / closes nodes, ordered by distance
 - E.g. 15-20 nodes
 - Transition rule applied only for the nodes in the candidate list
 - Unless?
 - No node from the candidate list can be visited

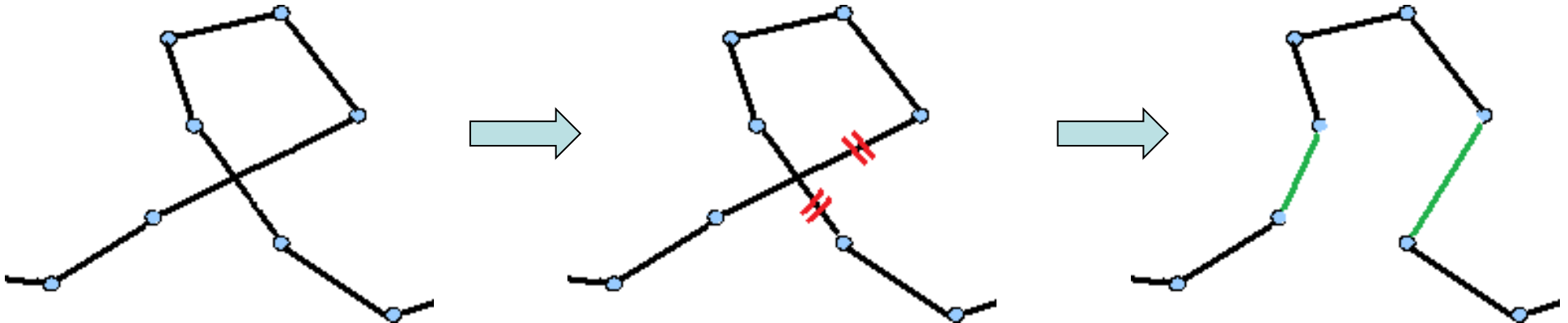
- Propose by Gambardella, Taillard & Dorigo
- Idea
 - Ant algorithms arrive relatively quickly at a good solution, but take long to find the optimum
 - Local search methods take relatively long for a good solution, but then find the optimum relatively quickly
 - Combine both methods
- Extensions:
 - Integrate a local search method to ant system algorithm
- Shows strong performance improvements

Local search

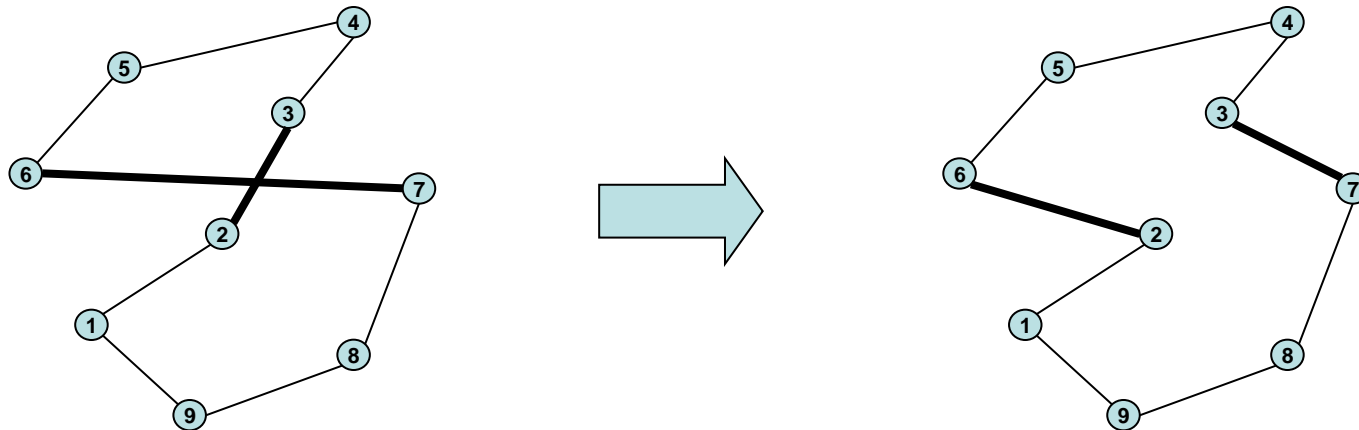
- Existing solution improved by iterative, small changes
- If no improvement possible – method stops
- Normally only discovers local optima



- Methods typically used with ant colony algorithms
 - 2-opt-Methode: reorder route so it doesn't cross itself
 - Delete two edges, and re-add them crossed-over

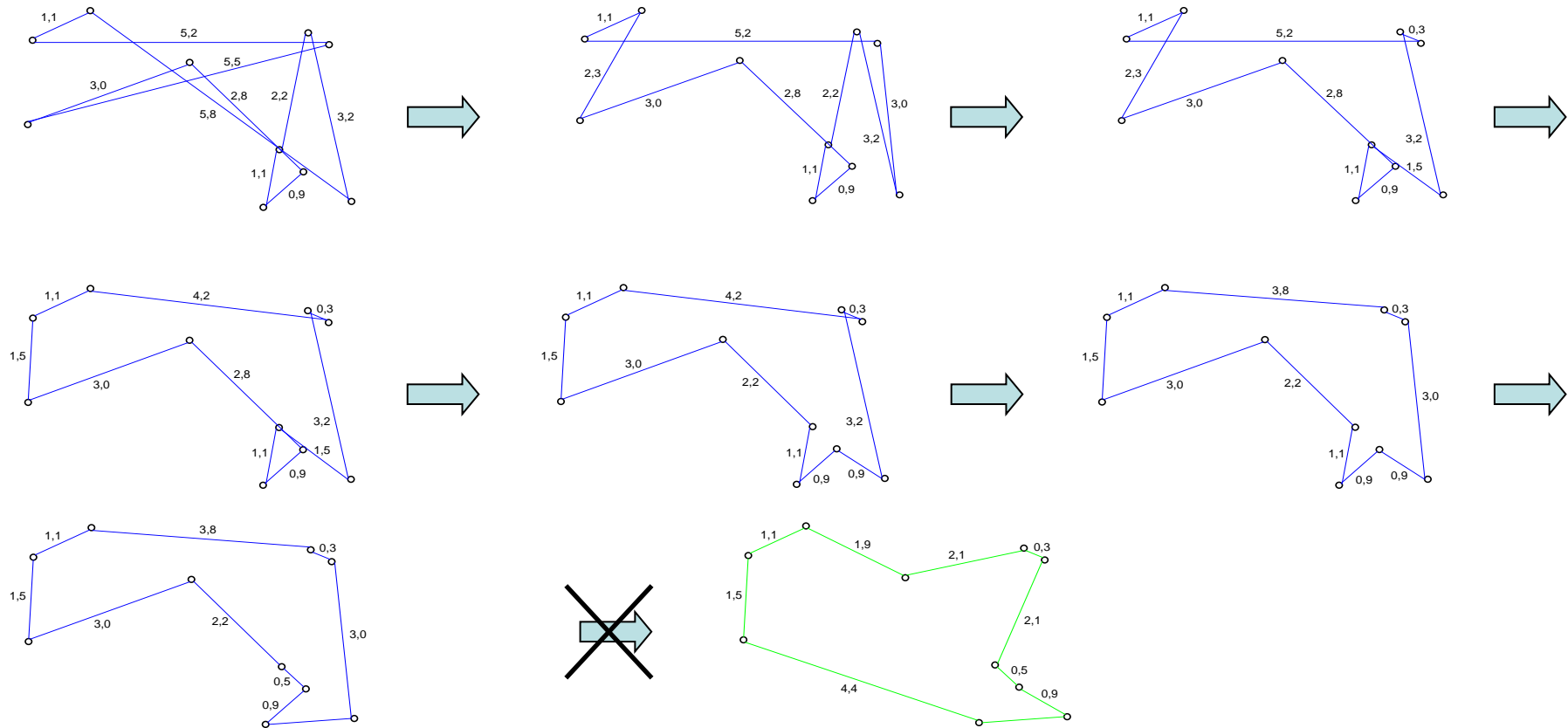


- Methods typically used with ant colony algorithms:
 - 2-opt-Methode: reorder route so it doesn't cross itself
 - Delete two edges, and re-add them crossed-over



- Methods typically used with ant colony algorithms:
 - 2-opt method
 - Delete two edges, and re-add them crossed-over
 - 3-opt method
 - Delete three edges at a time
 - Reconnect in all possible ways
 - Chose best solution
 - Lin-Kernighan method
 - Decide number of replacements at each step

■ Example: TSP



Length: 17,7
(found with local search)

Length: 16,2
(optimum)

- Application of local search variants: at various points and to various degrees possible
 - After each ant found a solution
 - Only for a specific percentage of solutions
 - Only on the best solution of a iteration

MAX-MIN Ant System (MMAS)

- Proposed by Stützle & Hoos
- Idea
 - Allowing only the best ant to update the trail can enhance solution-finding performance
 - However, can lead to early stagnation
- ➔ Prevent too large or too small amounts of pheromones on the edges
 - Too large amount ➔ local optimum
 - Too small amount ➔ edge is never used in solution space
- ➔ No edge that is never / always chosen

- Extension:
 - Introduce explicit maximum and minimum edge strength
 - Parameters τ_{\min} and τ_{\max}
 - Depending on the average edge length
 - Initialise all edges with τ_{\max}
 - Evaporation/decay after each iteration
 - Only edges along the best path are reinforced
 - Can still lead to a local optimum
 - ➔ Integration of local search

- Can still lead to a local optimum
 - ➔ Integration of local search
 - When trail branching is low (limited exploration)
 - ➔ Adjust trail (edge) intensities (“smoothing”)
 - Proportional to the difference between τ_{ij} and τ_{\max}
- Empiric evaluation: results compared to ACS
 - MMAS doesn’t always produce **best** result
 - But in **average**: results are better

Problem	ACS best	ACS avg	MMAS best	MMAS avg
eil51.tsp	426	428	426	426.7
kroA100.tsp	21282	21420	21282	21302.8
D198.tsp	15888	16054	15963	16048
ftv170.atsp	2774	2826.5	2787	2807.75
kro124.atsp	36241	36857	36416	36572.9

- Ant Colony Optimization
 - „Ant Colony Optimization“ introduced as term by Dorigo and Di Caro
 - **Umbrella** term for all ant systems, meta-heuristics
- „Most successful“ ACO algorithms:
 - ACS
 - MMAS
- Extensions can overcome some limitations, but –
introduce new parameters that need to be carefully set

- Combinatorial optimization problems
 - Scheduling
 - Vehicle routing problems: minimum cost set of routes
 - Serving all customers
 - Demand of each customer $<$ vehicle capacity
 - Visited exactly once
 - Multi-depot vehicle routing problem (mutli depots to start from)
 - Periodic vehicle routing problem
 - Split delivery (demand can be $>$ capacity)

Application areas for ACO algorithms

- *Generally: heuristic optimisation for NP-hard problems*
 - *Complexity rises exponentially with number of nodes*
 - *Solution can not be found with exhaustive search*
 - *Solution goodness can be validated*
- ACO: problem needs to be formulated as ***graph***

- Structure changes during learning phase
 - Nodes and/or edges get deleted
 - Node and/or edges get added
 - Local information changed

- Examples:
 - Dynamic TSP
 - Construction sites, traffic conditions → edge value/costs **change**
 - Update of targets: **new** target nodes (new customers), **removed** targets (customer cancelled order)

- After change of problem structure:
one time update of pheromone amounts

- Different strategies for update
 - Proximity to changed node/edges not considered
 - Proximity to changed node/edges considered according to local information
 - Proximity to changed node/edges considered according to global information

- Amount of update
 - Too little: stay in local optimum
 - Too much: loss of performance

Example System: AntNet

- Di Caro & Dorigo, 1997
- Optimal routing of data packets in a network, considering the current load
 - → distributed, dynamic problem
- Information for each node i :
 - Routing table, containing state transition probabilities $P_{j,z}^i(t)$
 - Matrix Γ_i , containing
 - estimated average time $\mu_{i \rightarrow z}$
 - variance $\sigma_{i \rightarrow z}^2$

Example System: AntNet

- Initialisation phase
 - Ants → initialise routing table and matrix
- Main phase
 - Ants → update routing table and matrix
 - Data packets routed according to probabilities
- Types of ants:
 - Forward ants
 - Backward ants (have priority)

■ Algorithm

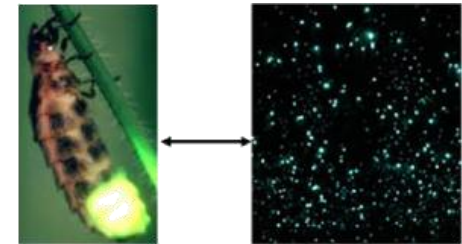
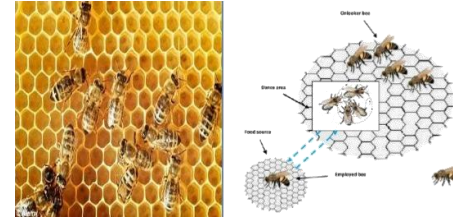
- Periodically on each node a “forward ant” is placed, with a random target node
 - Ants decide upon routing table and local information on load of the next node
 - Each ant has table $S^k_{s \rightarrow z}$, which records after each transition the node number and the time of leaving starting node
 - Once at the target node \rightarrow a “backward ant” with table $S^k_{s \rightarrow z}$ is generated and sent to the starting node
 - Ant updates routing table and matrix on each node, using information from $S^k_{s \rightarrow z}$
- \rightarrow Updates information on current load on the network

More details in upcoming lecture

- Advantages of ACO algorithms
 - Can be adapted to various tasks
 - Can solve dynamic problems
 - Decentralised, with global state
 - Can find good solutions relatively quickly

- Disadvantages
 - Takes relatively long to find best solution
 - combination with local search, if applicable

- Particle swarm optimisation (~1995)
 - Particles moved around in search space
 - According to local knowledge and best global solution
- Artificial bee colony (~2000)
 - Example: Dynamic Server Allocation
- Glow worm swarm optimisation (~2005)
 - Glow worms light according to their fitness; attract others
- Intelligent water drops (2007)
- Multi-swarm optimisation (multiple particle swarms)



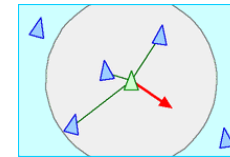
More in depth: upcoming lectures

■ Applications

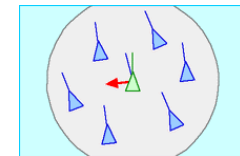
- Problem solving
 - Heuristic optimisation
- Simulate (natural) behaviour
 - E.g. “Boids” (1986): simulation of bird flock behaviour

– “Operations”

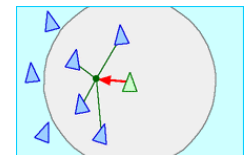
» Steer to avoid local overcrowding



» Steer towards direction of birds in proximity



» Steer towards average position of birds in proximity



- Algorithm used e.g. in games & CGI film sequences
- <http://www.red3d.com/cwr/boids/>

Questions



Questions ?