

# WS5

# Dynamic Simulation of Assistive Robotics and Human Motion

## Day 1: Inverse Modelling

- Intro to simulation, engines and approaches (forward/inverse). (15 m)
- Neuromechanics basics. (15 m)
- Introduction to OpenSim, musculoskeletal modelling and inverse modelling (1 h)
- Q&A + IT help (30 m)

## Day 2: Forward Sim + Control

- MuJoCo and forward modelling basics. (25 m)
- Control basics and introducing programming challenge (15 m)
- Arm prototyping exercise (1 h)
- Q&A (10 m)

## Day 3: Advanced use-cases

- Reinforcement learning & motion synthesis (10 m)
- Simulations for vision-based robotics (15 m)
- Virtual prostheses, sim2real concerns and methods. (20 m)
- Ankle exo challenge (1h)
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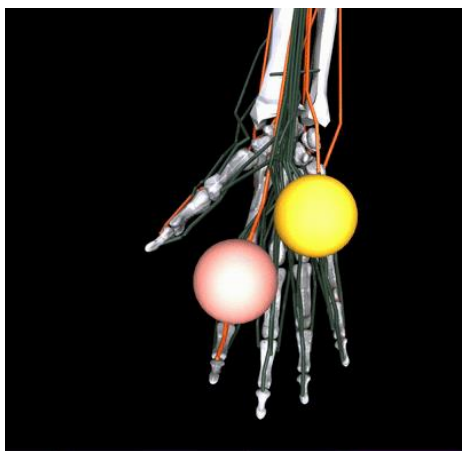
Why go through  
the trouble of  
physics-based  
animation?



Every interaction needs to be set  
up to not violate our expectations.

Can we enforce them implicitly?

# Quick history of MuJoCo

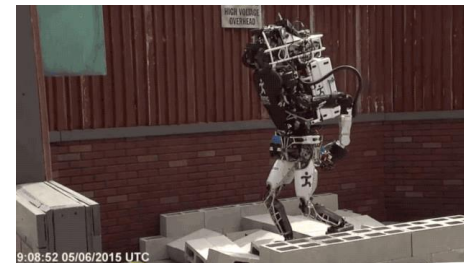


Optimal  
control in  
biomechanics

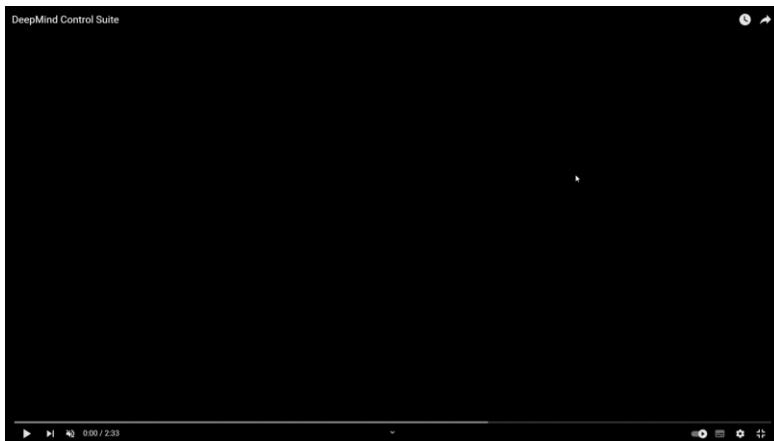
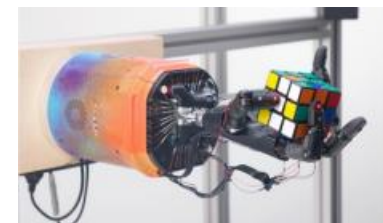
DeepMind  
Acquisition +  
Open Sourcing

Robotics  
Community

RL Community



DARPA Robotics challenge

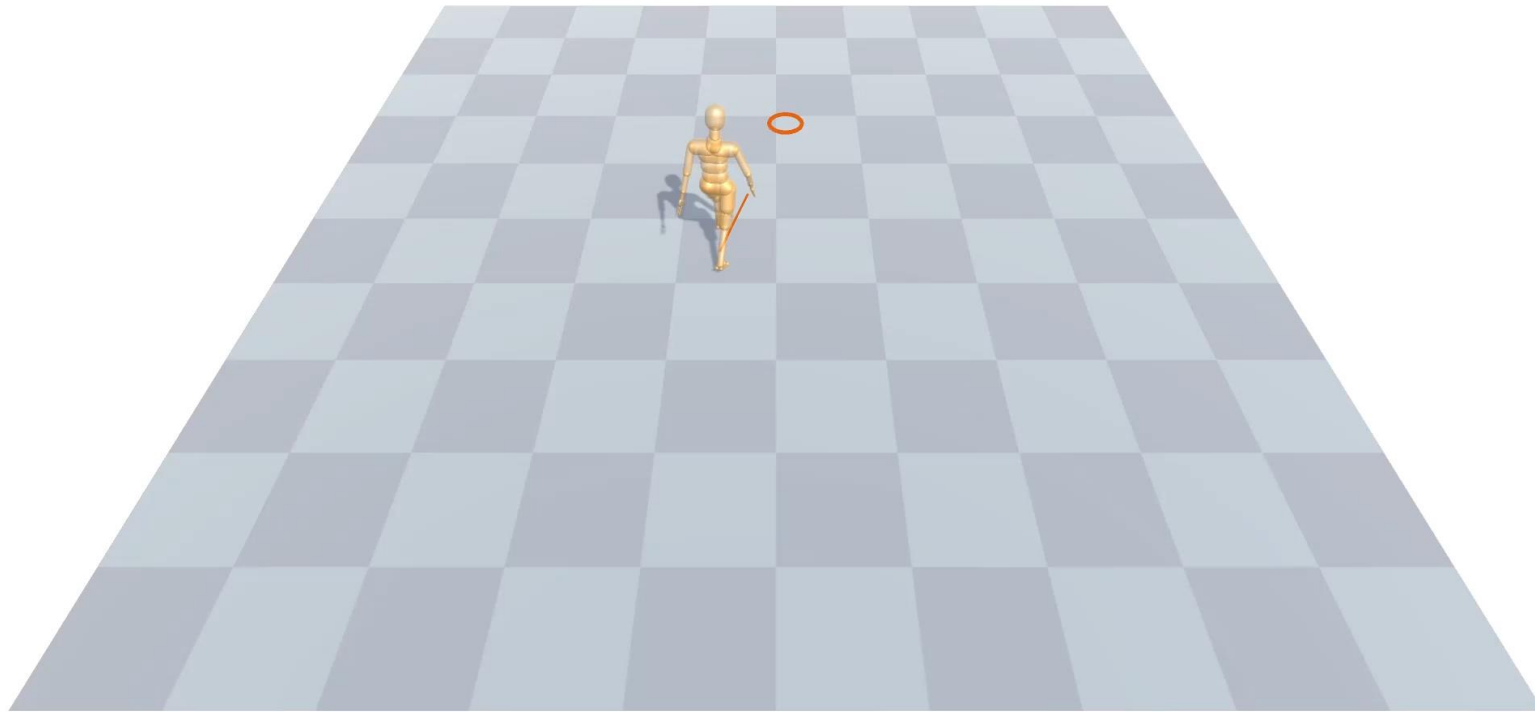


# Example use case: Virtual Prototyping

- Same motor decode and motion capture
- Contact on virtual hand's middle finger activates force-graded stimulation of subject's phantom middle finger
- Higher force on middle finger causes a higher frequency of stimulation

D. T. Kluger et al., "Virtual Reality Provides an Effective Platform for Functional Evaluations of Closed-Loop Neuromyoelectric Control," in IEEE TNSRE 2019

# Example use case: Motion Synthesis



# Example use case: Digital twin



# Example use case: Shared autonomy control with vision based control



Automatically controlled:

- Hand Preshape
- Hand Aperture
- Wrist Rotation

Myoelectrically controlled:

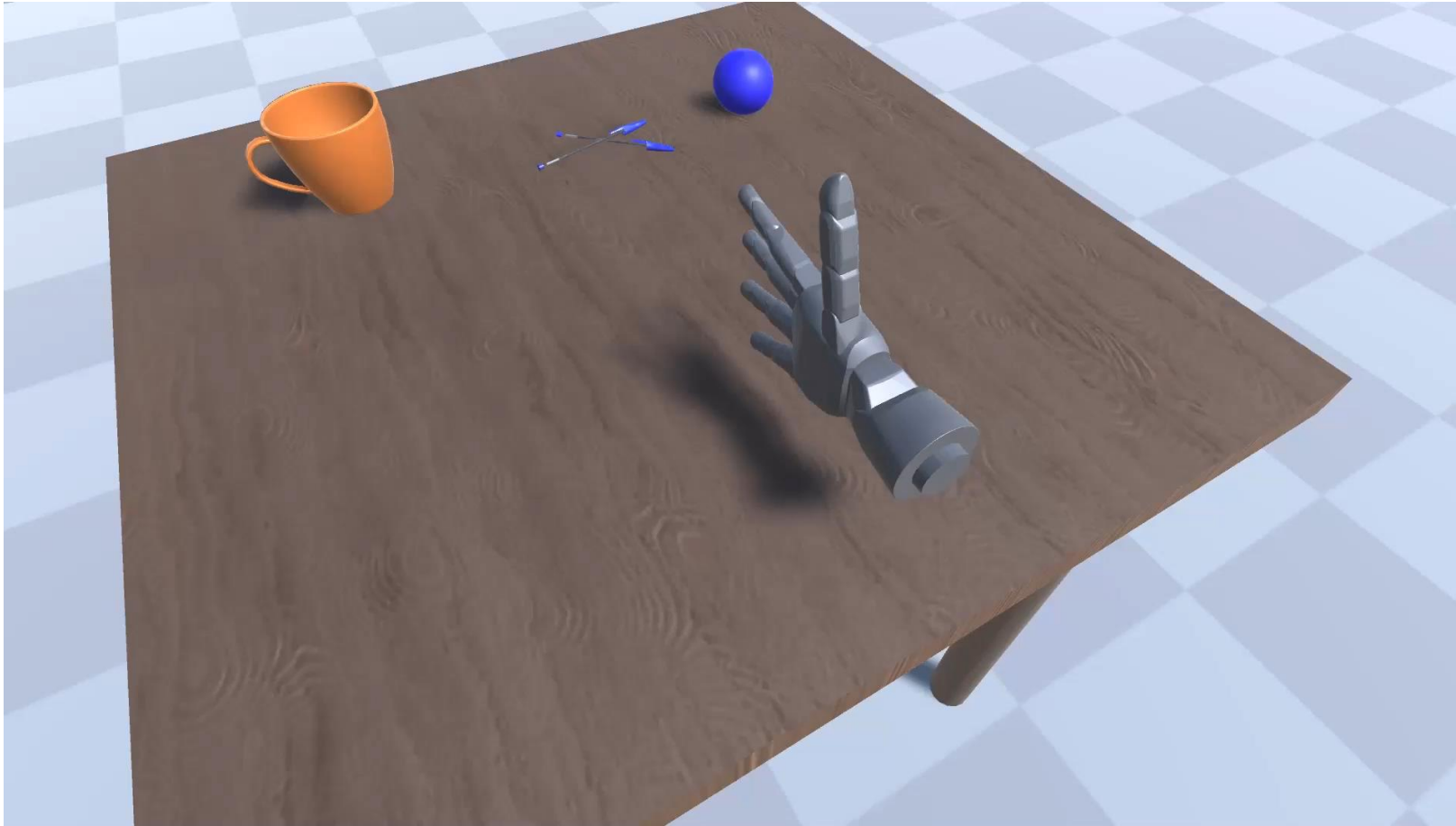
- Grasping/releasing

Reduce:

- Compensatory movement
- Muscle activation



Generate datasets or evaluate performance in simulation





# Differences from OpenSim

OpenSim has a dedicated GUI  
(Wrapping the Simbody physics engine)



MuJoCo needs to be combined with a development environment for a proper UI (e.g., Coppelia, Unity)

Elastic Foundations contact model



Optimisation based constraint resolution with force impulses (constraints in velocity vs acceleration)

Muscle-tendon equilibrium resolved dynamically



Tendon assumed to be unstretchable, other parameters adapted to match behaviour

Well explored and understood MSk model library



System-identified robotic model library (but MSk models are being translated from OpenSim)

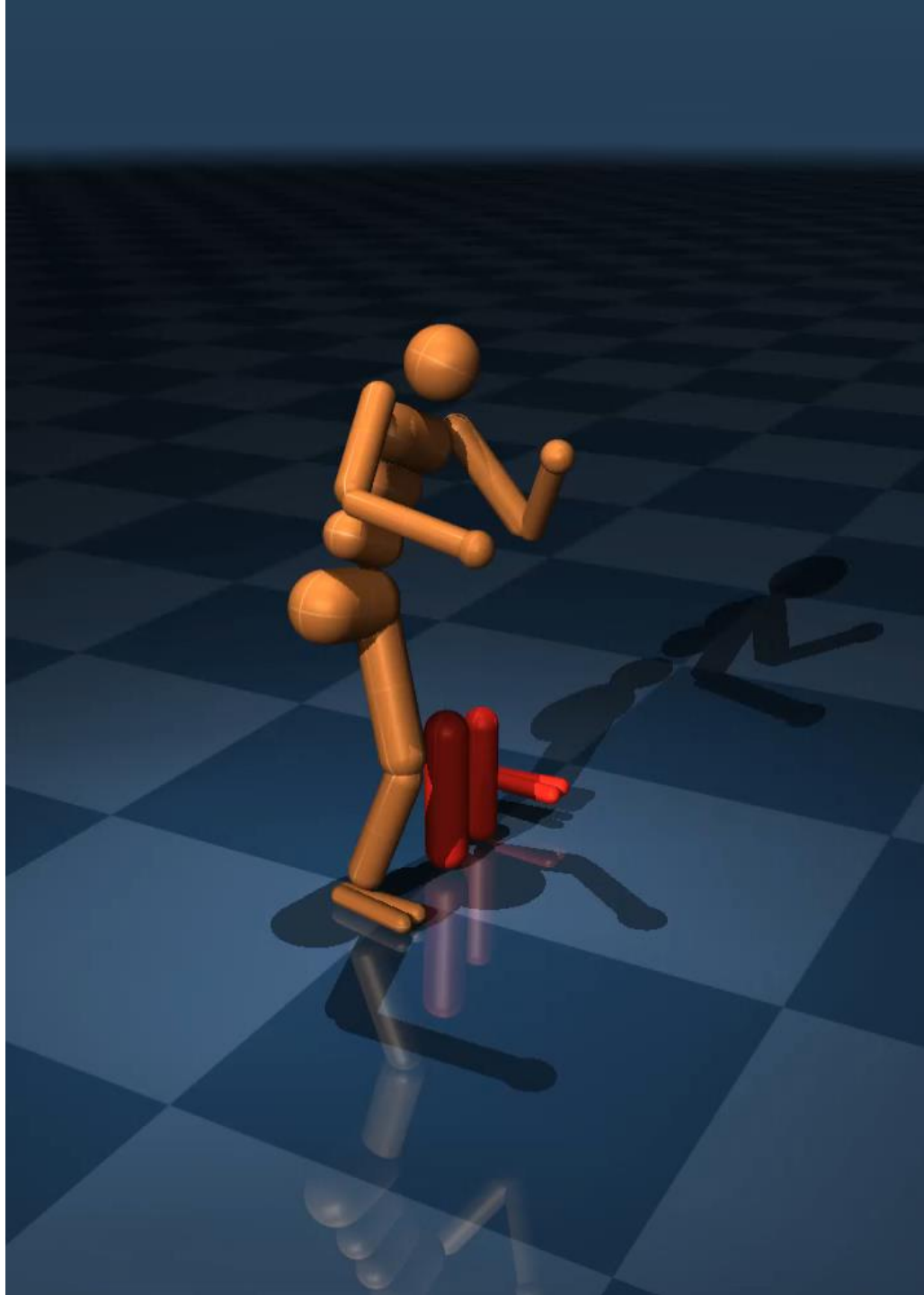
MuJoCo is usually\* 60-4000 times faster

# Developing with MuJoCo

- Python, Simulink, C++, C# API / wrappers
- Visual GUI in Coppelia Sim or in Unity
- Custom GUIs
- GPU accelerated learning, engine rewritten in Jax (MJX)

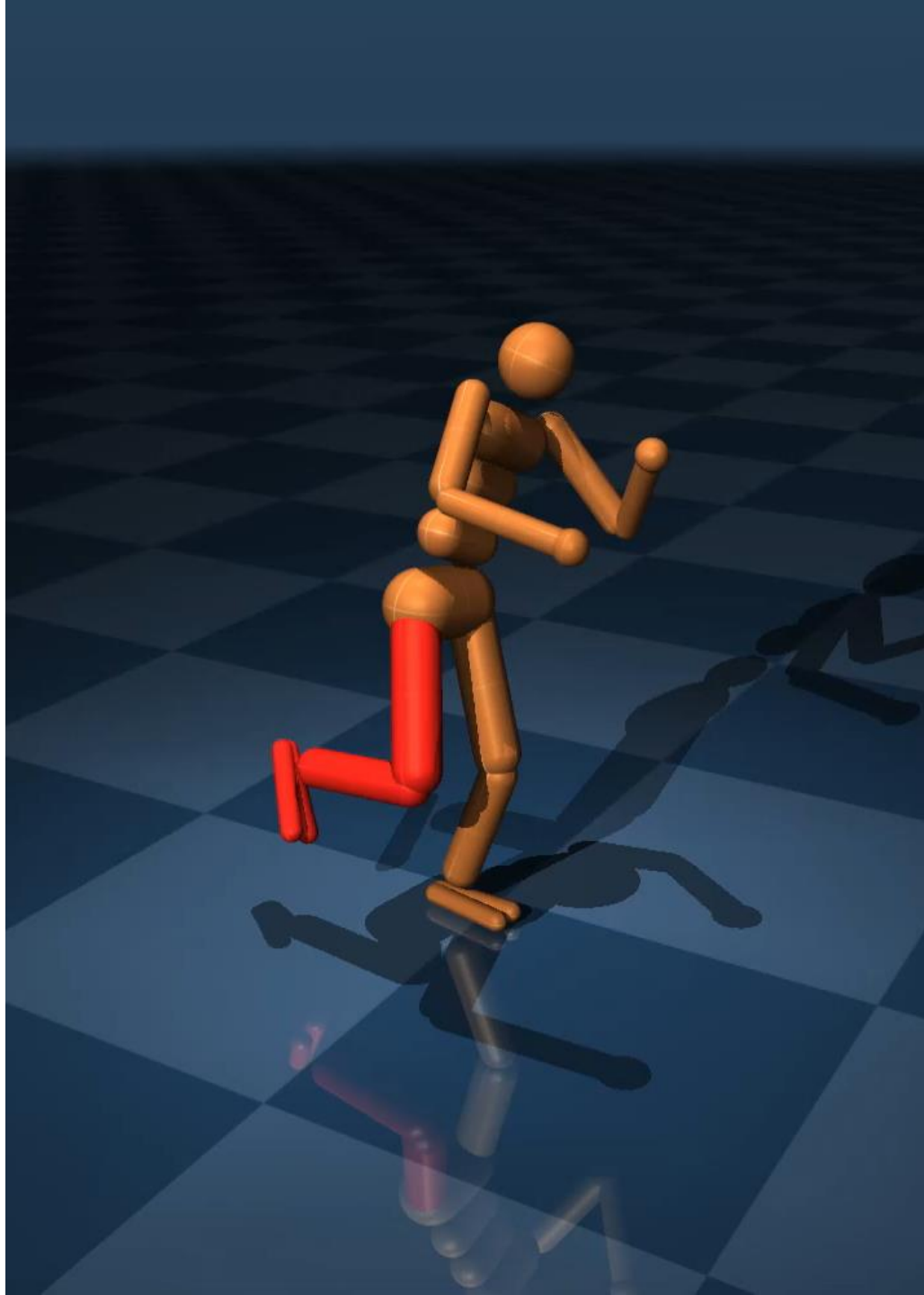
# Simulation State

Global Coordinates



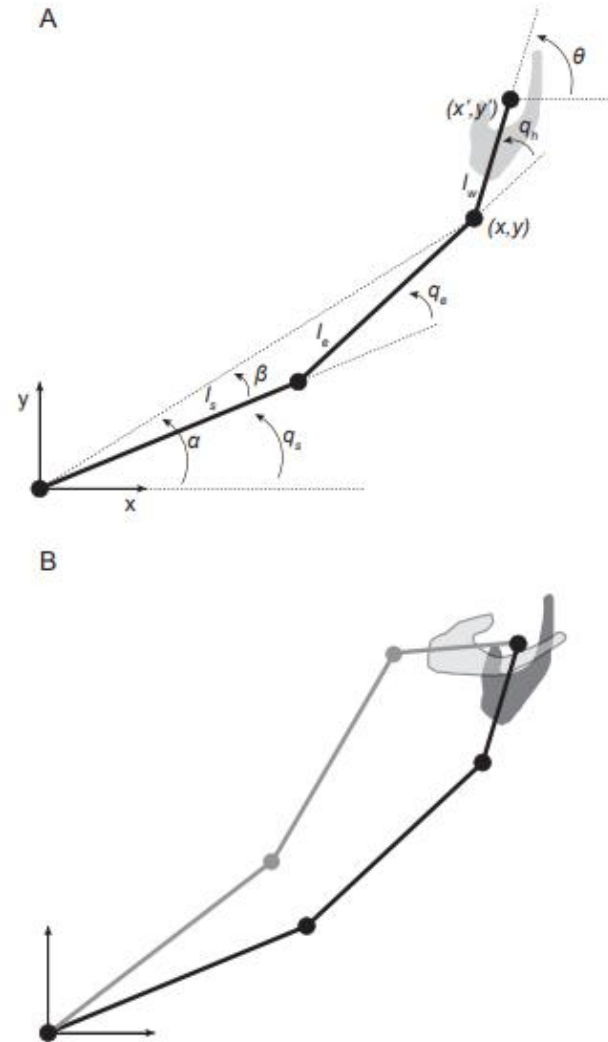
# Simulation State

Generalised Coordinates



# Simulation State

Not always 1:1 mapping between different state representations



# Simulation State

Convert between state representations

$$\mathbf{J}(\mathbf{q}) = \left( \frac{\partial x_i}{\partial q_j} \right) \longrightarrow \partial x = \mathbf{J} \partial q$$

Hand		Joint		Muscle	
$\mathbf{x}$	←	$\mathbf{q}$	→	$\lambda$	position
$\dot{\mathbf{x}}$	$\mathbf{J}$ ←	$\dot{\mathbf{q}}$	$\mathbf{J}_\mu$ →	$\dot{\lambda}$	velocity
$\mathbf{F}$	$\mathbf{J}^T$ →	$\boldsymbol{\tau}$	$\mathbf{J}_\mu^T$ ←	$\mu$	force
$\mathbf{K}_x$	$\mathbf{J}^T \mathbf{K}_x \mathbf{J} + \frac{d\mathbf{J}^T}{dq} \mathbf{F}$ →	$\mathbf{K}$	$\mathbf{J}_\mu^T \mathbf{K}_\mu \mathbf{J}_\mu + \frac{d\mathbf{J}_\mu^T}{dq} \mu$ ←	$\mathbf{K}_\mu$	stiffness
$\mathbf{D}_x$	$\mathbf{J}^T \mathbf{D}_x \mathbf{J}$ →	$\mathbf{D}$	$\mathbf{J}_\mu^T \mathbf{D}_\mu \mathbf{J}_\mu$ ←	$\mathbf{D}_\mu$	damping



# Evolving state

Integration based  
simulation

## Equation of motion

$$M\dot{v} = \sum \tau$$

$$M\dot{v} + c = \tau + J^T f$$

Forward modelling:

$\tau$  known, what is  $\dot{v}$ ?

$$\dot{v} = M^{-1}(\tau + J^T f - c)$$

Inverse modelling:

$\dot{v}$  known, what is  $\tau$ ?

$$\tau = M\dot{v} + c - J^T f$$

Symbol	Size	Description
$n_Q$		number of position coordinates
$n_V$		number of degrees of freedom
$n_C$		number of active constraints
$q$	$n_Q$	joint position
$v$	$n_V$	joint velocity
$\tau$	$n_V$	applied force: passive, actuation, external
$c(q, v)$	$n_V$	bias force: Coriolis, centrifugal, gravitational
$M(q)$	$n_V \times n_V$	inertia in joint space
$J(q)$	$n_C \times n_V$	constraint Jacobian
$r(q)$	$n_C$	constraint residual
$f(q, v, \tau)$	$n_C$	constraint force

# Evolving state

Integration based  
simulation

Timestep of  $h$  seconds (e.g. 0.005 s)

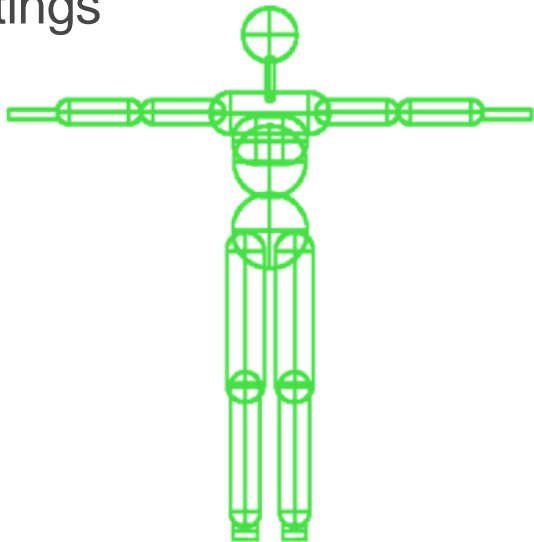
velocity:  $v_{t+h} = v_t + h\dot{v}_t$

position:  $q_{t+h} = q_t + hv_t$

# System representation in MuJoCo

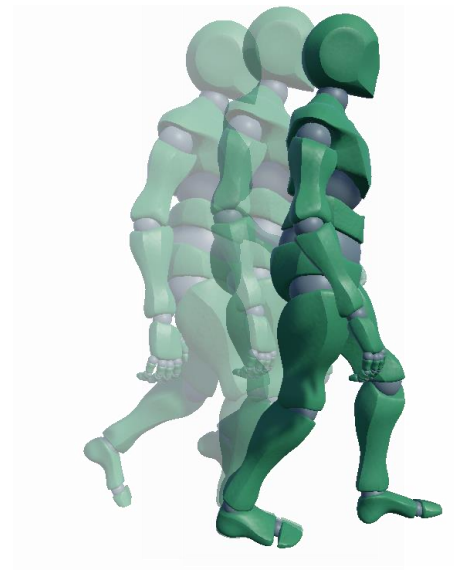
## Model

- Static information about model
- Reference frames, DoFs
- Inertia, geometry, colliders, constraints
- Passive properties, actuation setup
- Sim settings



## Data

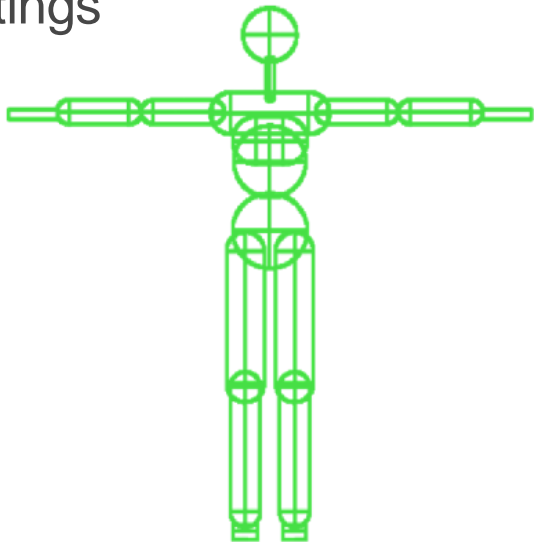
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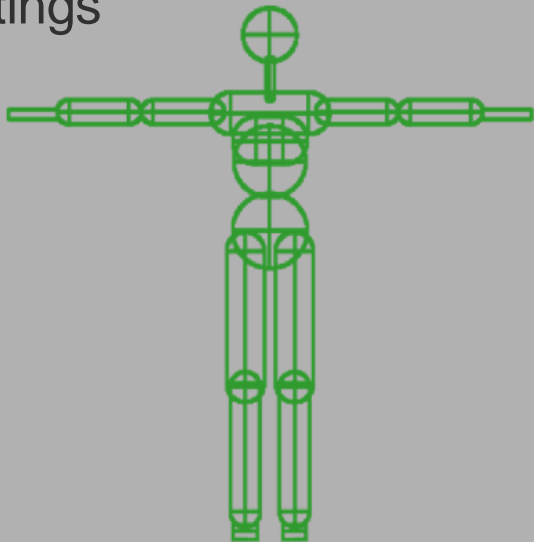
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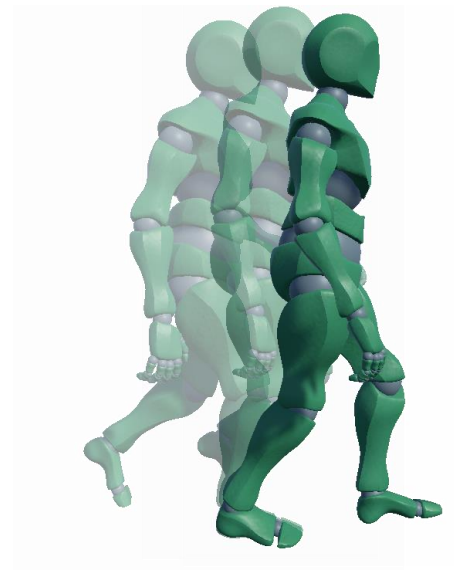
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# Simulation Loop

```
void mj_step1(const mjModel* m, mjData* d)
{
    mj_checkPos(m, d);
    mj_checkVel(m, d);
    mj_fwdPosition(m, d);
    mj_sensorPos(m, d);
    mj_energyPos(m, d);
    mj_fwdVelocity(m, d);
    mj_sensorVel(m, d);
    mj_energyVel(m, d);

    // if we had a callback we would be using mj_step, but call it anyway
    if( mjcb_control )
        mjcb_control(m, d);
}

void mj_step2(const mjModel* m, mjData* d)
{
    mj_fwdActuation(m, d);
    mj_fwdAcceleration(m, d);
    mj_fwdConstraint(m, d);
    mj_sensorAcc(m, d);
    mj_checkAcc(m, d);

    // compare forward and inverse solutions if enabled
    if( mjENABLED(mjENBL_FWDINV) )
        mj_compareFwdInv(m, d);

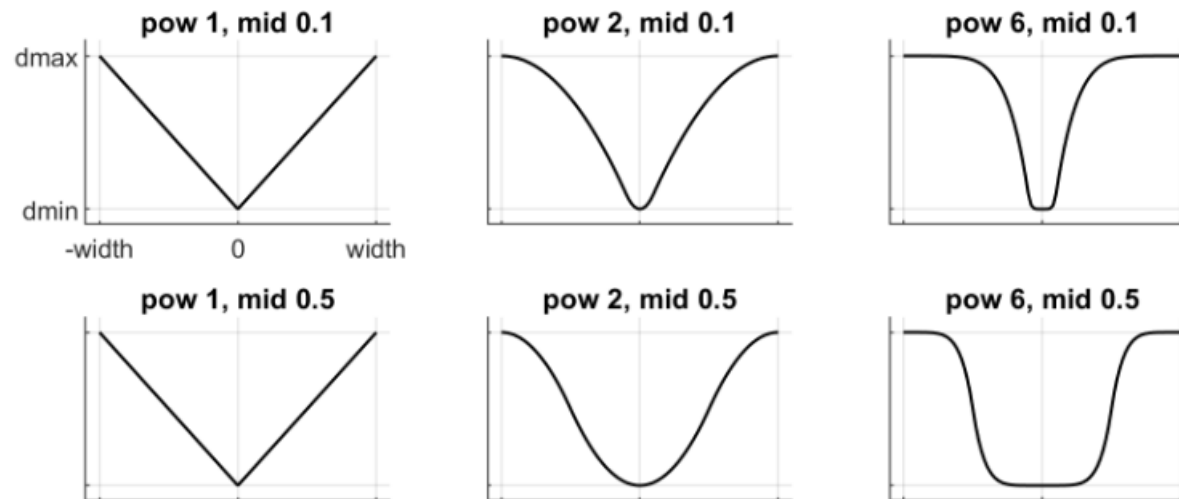
    // integrate with Euler; ignore integrator option
    mj_Euler(m, d);
}
```

# Resolving constraints

- Contacts assumed soft, but not purely spring-damper-like

Relaxed complementarity → Allow penetration of rigid bodies

- Can solve as convex optimization, much faster
- “It’s a feature, not a bug.” Model compressibility, recover contact forces from kinematics.
- Choose parameter set to model the interaction you are looking for.



# Thank you for the attention!

Feel free to reach out to me if you have questions!  
[bkh16@ic.ac.uk](mailto:bkh16@ic.ac.uk)

Both OpenSim and MuJoCo has easy to follow tutorials and documentation to getting started, as well as communities that can answer questions and solve problems:

<https://github.com/opensim-org/opensim-core>

<https://github.com/google-deepmind/mujoco>

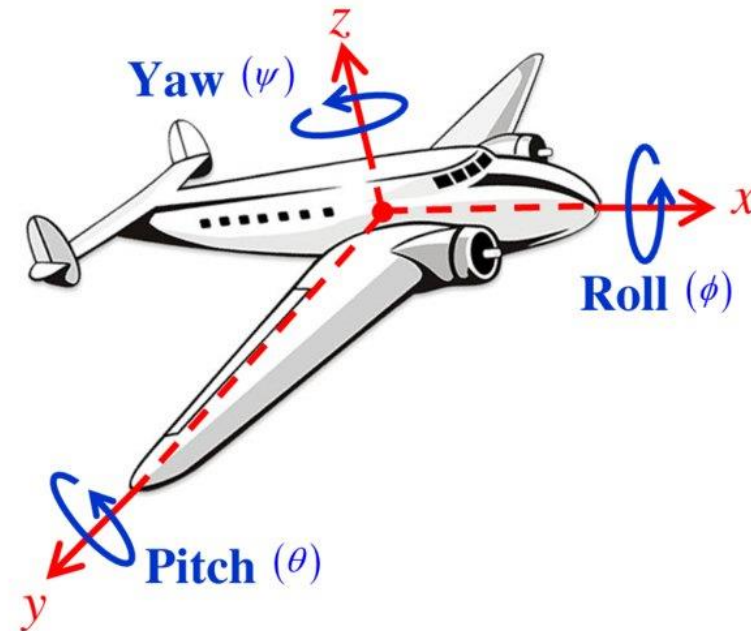
Recommended reading:

De Groote, F. and Falisse, A., 2021. Perspective on musculoskeletal modelling and predictive simulations of human movement to assess the neuromechanics of gait. *Proceedings of the Royal Society B*, 288(1946), p.20202432.



# Representing 3 DoF rotations

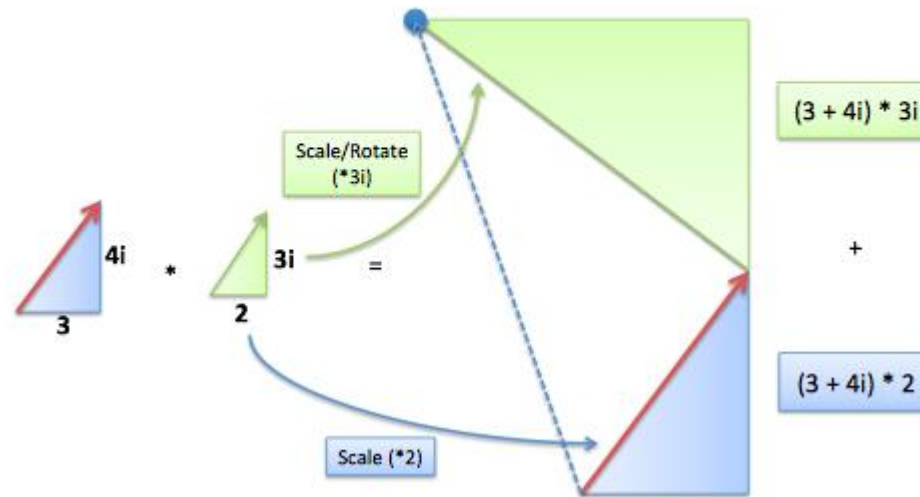
Euler angles are a simple and intuitive way, but suffer from a few key issues



# Representing 3 DoF rotations

Unintuitively, you need more than three numbers to accurately represent 3 DoFs with shared center of rotation (e.g., ball joints or free joints).

## Complex Multiplication



# Representing 3 DoF rotations

- 4 numbers for position, 3 for velocity

Quaternions extend complex numbers to represent 3D rotations continuously, with familiar, multiplication-based notation.

## Complex Multiplication

