Solving quadratic optimization problems with bosonic quantum computer simulator

Report I

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Motivation

The field of quantum computing is a promising subject because of the powerful applications in solving hard optimization problems.

Introduction and theoretical background

The idea of the boson sampler stemmed initally from the KLM-protocol. Knill, Laflamme and Milburn showed, that linear optics with photo detection and single photon sources is sufficient for universal quantum computing. However, it is not feasible to implement at scale, because we would need billions of linear optical elements and millions of single photon sources.

But by taking it apart and looking at the elements, we can see, that it is constructed of gates made up by linear optic network and some additional parts. The linear optic part consists of beam splitters and phase shifter, characterized by angles ϑ_i and ψ_i for each piece of equipment accordingly.

Boson sampling[1]

The boson sampler is an M-mode linear interferometer, where the inputs are single photons, and the outputs are multi photon states. This output is measured so we get a single multi photon $|\mathbf{n}\rangle = |n_1, n_2, ..., n_M\rangle$ Fock-vector, where n_i are the number of photons in the i state.

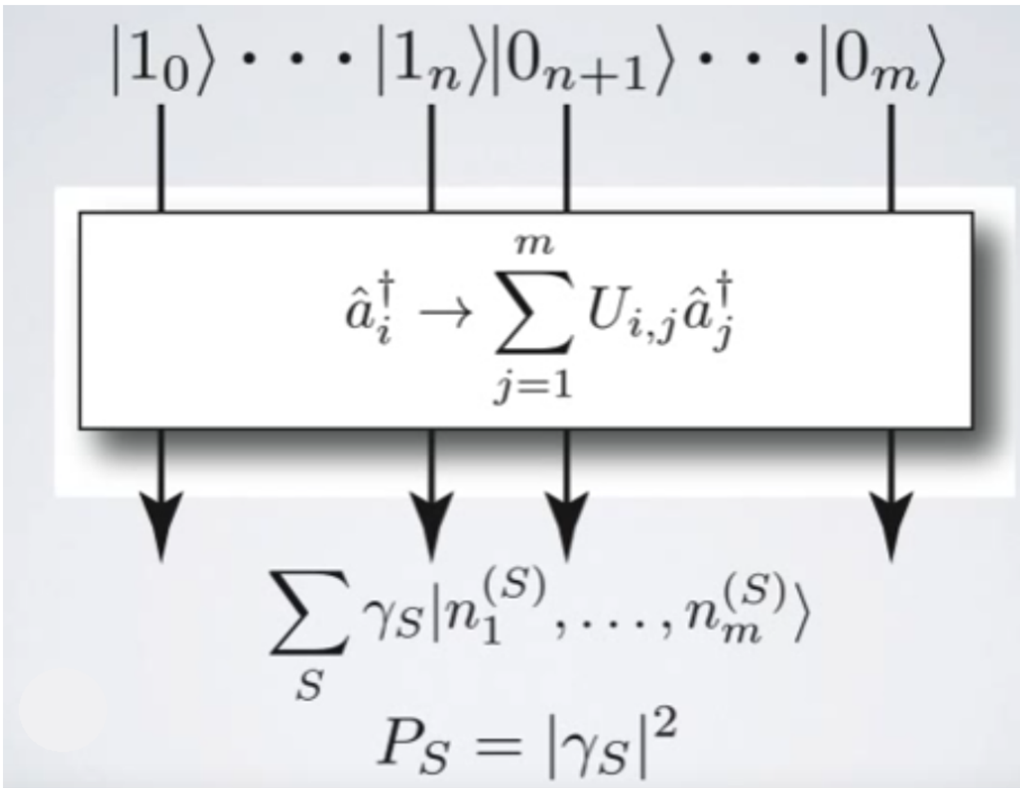

Figure 1: Schema of a boson sampler.
On top, there are single photon states and vacuum states, that are propagated through a linear optics network. At the bottom there are the superpositions of the multiphoton optical states. Image credits to [1].

In order to interpret the measurement in a qubit basis the Fock-vector is mapped onto a M component $|\mathbf{b^{(j)}}\rangle$ binary vector with the $\wp_j$ parity function. Note that the different optical states can be mapped to the same bit strings. So if a $|\mathbf{n}\rangle$ and $|\mathbf{n}'\rangle$ with amplitudes α and β are mapped to the same binary vector $|\mathbf{b^{(j)}}\rangle$ then the associated amplitude is $\sqrt{\alpha^2+\beta^2}$.

Simulating such an equipment on a classical computer, that samples bozons in this way is pretty hard because the probabilities (and amplitudes) associated with the optical states involves calculating the permanent of a complex-valued matrix, which is #P-complete [2]:

$$P(|\mathbf{n}\rangle) \propto |\mathrm{Per}(A_{|\mathbf{n}\rangle})|^2,$$

where $A_{|\mathbf{n}\rangle}$ is a unitary matrix describing the linear optics network configuration.

Variational bosonic solver[3]

Most problems in physics and other fields of study have no exact solutions. The variational approach is a way of obtaining a numerical solution to such problems. It approaches the exact solution (if there is any) as close as it is needed. We generally think of the E energy as an objective function, and the goal is to minimize it by finding the optimal values for the free parameters.

Continuing from the point where we left at the parity function. The $|\mathbf{b^{(j)}}\rangle$ bitstring has a corresponding amplitude $\beta_{|\mathbf{b^{(j)}}\rangle}$. Then the expected value of the energy is calculated in the following way:

$$E_{|\mathbf{b^{(j)}}\rangle} = \sum_{|\mathbf{b^{(j)}}\rangle} \beta_{|\mathbf{b^{(j)}}\rangle} \langle \mathbf{b^{(j)}} | \, H \, | \mathbf{b^{(j)}} \rangle$$

.

The variational bosonic solver algorithm tries to find the E_{min} minimal energy and the corresponding configuration $|\mathbf{b}_{min}\rangle$. In each epoch the gradient descent step is made in the direction to the minimal energy value by adjusting ϑ_i and ψ_i angles of the beam splitters and phase-shifters.

Quadratic uncostrained binary optimization (QUBO)[4]

It is a combinatorial optimization problem, that means that the goal is to find an extremum over a discrete set. Finding the optimal solution to a QUBO is equivalent to minimizing H of the Ising-model. It has a wide range of applications apart from physics, for example in binary portfolio optimization, or various other problems in finance and mathematics.

QUBO is a NP hard problem, which means that is computationally expensive.

Let $f_Q(x)$ be a quadratic binary mapping $f_Q(x) : \mathbb{B}^m \to \mathbb{R}$ in the form of

$$f_Q(x) = \langle x | \, Q \, | x \rangle \, .$$

The goal is to find $x^* = \text{argmin}(f(x))$ for the given $Q \,\epsilon\, \mathbb{R}^{m\times m}$ symmetric matrix.

References

[1] Introduction to boson-sampling - Youtube.com, 2022. [online] Available at: https://www.youtube.com/watch?v=tlgYp-I5dvs [Accessed 18 February 2022]

[2] Aaronson, Scott, and Alex Arkhipov. "The computational complexity of linear optics." Proceedings of the forty-third annual ACM symposium on Theory of computing. 2011. https://arxiv.org/abs/1011.3245

[3] Bradler, Kamil, and Hugo Wallner. "Certain properties and applications of shallow bosonic circuits." arXiv preprint arXiv:2112.09766 (2021). https://arxiv.org/abs/2112.09766

[4] Quadratic unconstrained binary optimization - Wikipedia, 2022 [Accessed 18 February 2022]