Quadratic optimization with quantum computing

Biweekly Presentation III

Bálint Hantos

Supervisor: Péter Rakyta

Combinatorial optimization

- Combinatorial optimisation problems:
  - Discrete variables (instead of continous)
  - Optimizing over **discrete** >> **continous** variables
  - Constraints?
    - Continous: add to the objective function! (eg. Lagrangian multiplier)
    - Discrete is hard, if there are constraints (check every case)
  - It is even harder for **binary** (0,1) variables: brute force method

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ \cdots \\ 1 \end{bmatrix}$$

Quadratic unconstrained binary optimization - QUBO

$\langle\, \boldsymbol{b} \mid \boldsymbol{Q} \mid \boldsymbol{b} \,\rangle$

$\langle\, 10010 \mid \boldsymbol{Q} \mid 10010 \,\rangle$ = scalar

- $\boldsymbol{Q}$ is a symmetric matrix (n x n)
- $\boldsymbol{b}$ is a **binary** vector (n)

Optimization problem:
We want to find $\boldsymbol{b}$ such that

$$\min(\langle\, \boldsymbol{b} \mid \boldsymbol{Q} \mid \boldsymbol{b} \,\rangle)$$

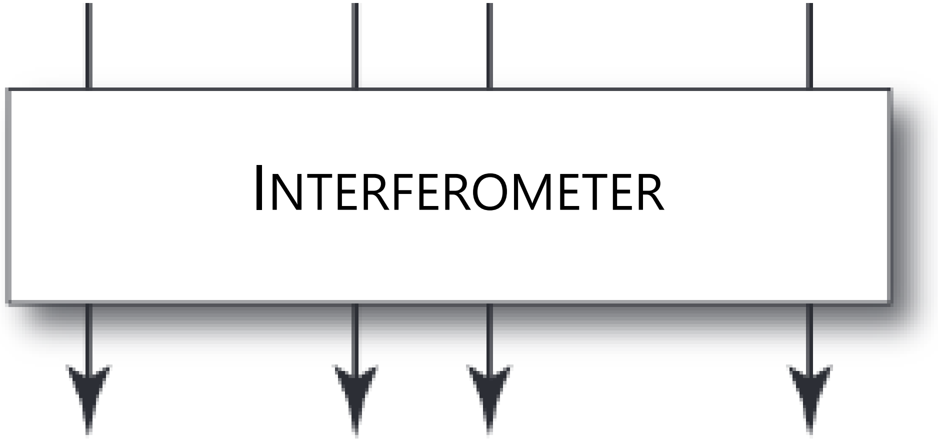
$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ \dots \\ 1 \end{bmatrix}$$

Size of problem $n \rightarrow 2^n$ possible solutions

Binary optimization is expensive.

Boson sampling

Single photon quantum states



Multiphoton quantum state

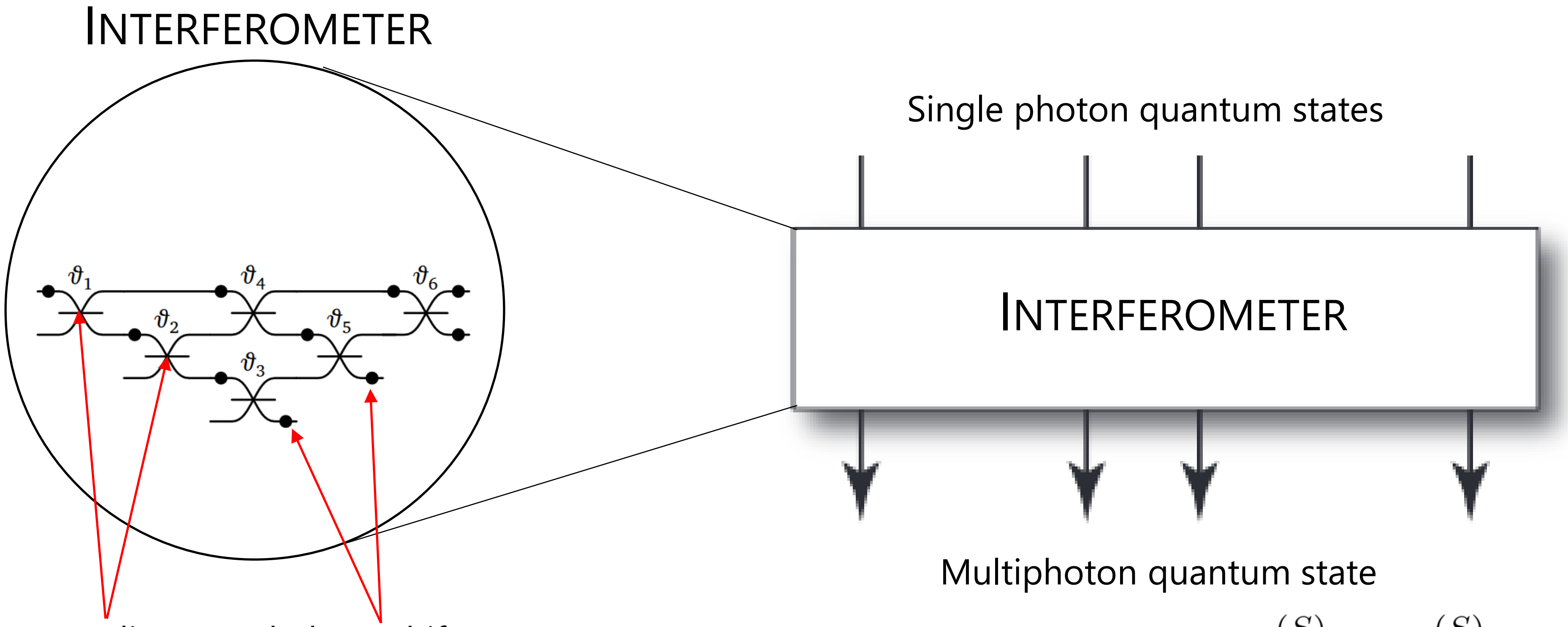
$$\sum_S \alpha_S |n_1^{(S)}, \ldots, n_m^{(S)}\rangle$$

$$P_S = |\alpha_S|^2$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ \cdots \\ 1 \end{bmatrix}$$

Gard et al. "An introduction to boson-sampling.„ 2016

Boson sampling: interferometer



Beam splitters and phase shifters

$$\vartheta_i \quad \text{and} \quad \psi_j$$

Multiphoton quantum state

$$\Psi(\vartheta_i, \psi_j) = \sum_S \alpha_S |n_1^{(S)}, \ldots, n_m^{(S)}\rangle$$

$$P_S = |\alpha_S|^2$$

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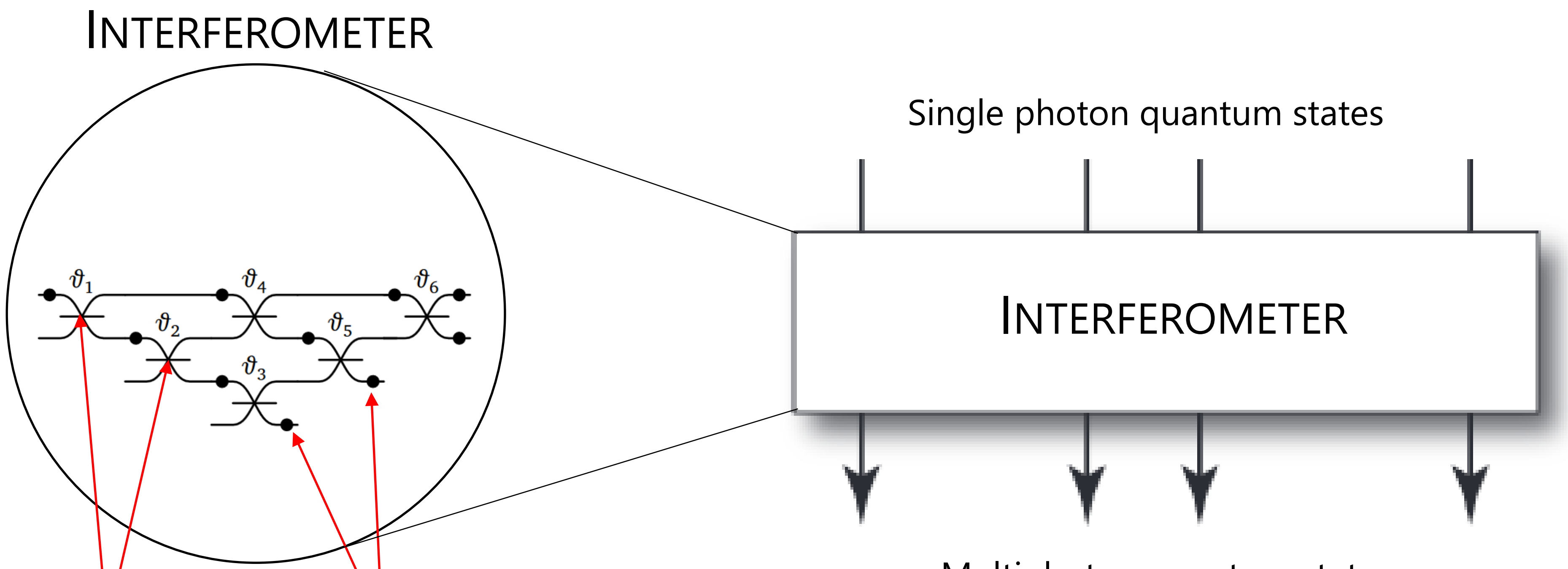
Boson sampling: interferometer



Multiphoton quantum state

Beam splitters and phase shifters

ϑ_i and ψ_j

continous

$$\Psi(\vartheta_i, \psi_j) = \sum_S \alpha_S |n_1^{(S)}, \ldots, n_m^{(S)}\rangle$$

$$P_S = |\alpha_S|^2$$

discrete

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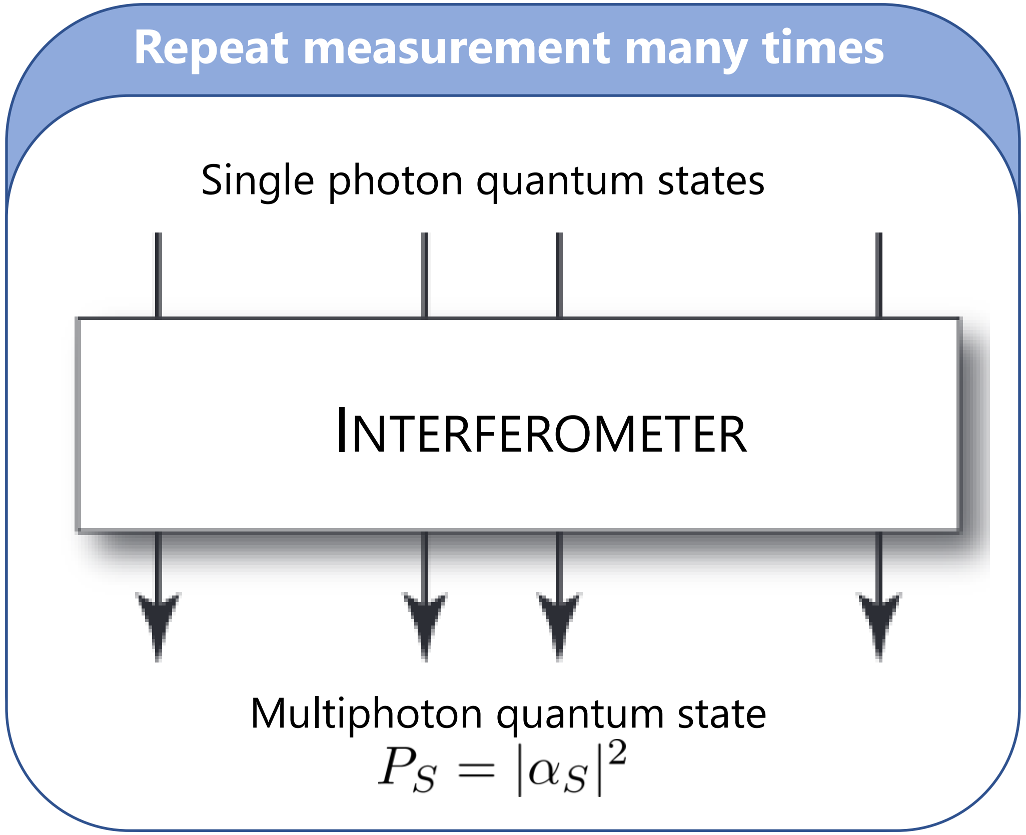
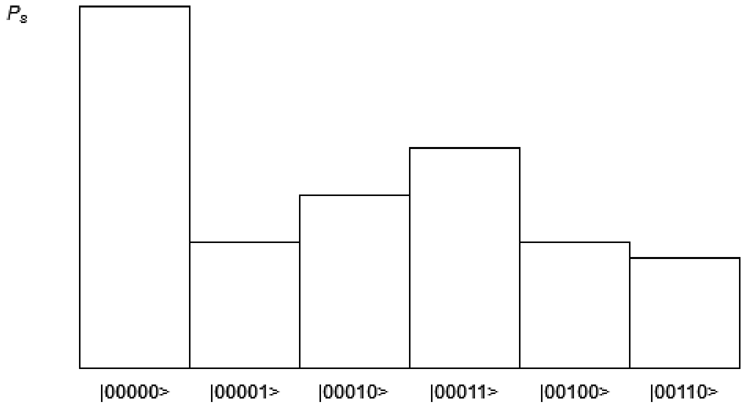
Boson sampling: distribution and energy

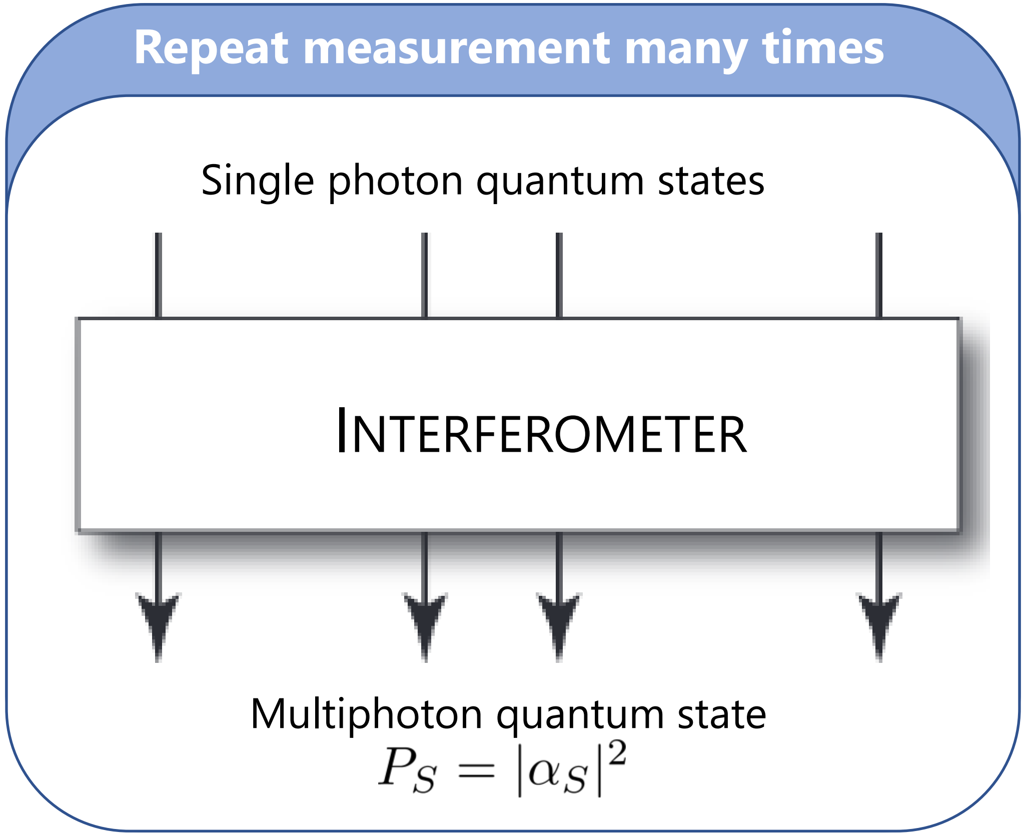


Distribution of quantum states:

$$P_S(\vartheta, \psi) = |\alpha_S|^2$$



Boson sampling: distribution and energy



Distribution of quantum states: $P_S = |\alpha_S|^2$

Mapping to qubit basis:
$|\mathbf{n}\rangle \rightarrow |\mathbf{b}\rangle$ **binary** vectors
$\sum_{s \subset S} P_s \rightarrow \beta_{|\mathbf{b}\rangle} \,\epsilon\, [0,1]$

$$E(\psi, \vartheta) = \sum_{|\mathbf{b}\rangle} \beta_{|\mathbf{b}\rangle} \langle \mathbf{b}| \, Q \, |\mathbf{b}\rangle = \langle Q \rangle$$

Depends on ϑ_i and ψ_j

Fixed

<u>Goal</u>: find $|\mathbf{b}^*\rangle = \mathrm{argmin}(\langle \mathbf{b}^*| \, Q \, |\mathbf{b}^*\rangle)$

Variational solver: gradient descent

Apply gradient descent:

$$\vartheta_i' \to \vartheta_i - \eta \frac{\partial E}{\partial \vartheta_i}$$

Need to compute the gradient

Approximating the gradient: $\frac{\partial E}{\partial \vartheta_i} \approx \frac{E(\vartheta_i+\varepsilon)-E(\vartheta_i)}{\varepsilon}$

Practically infeasible

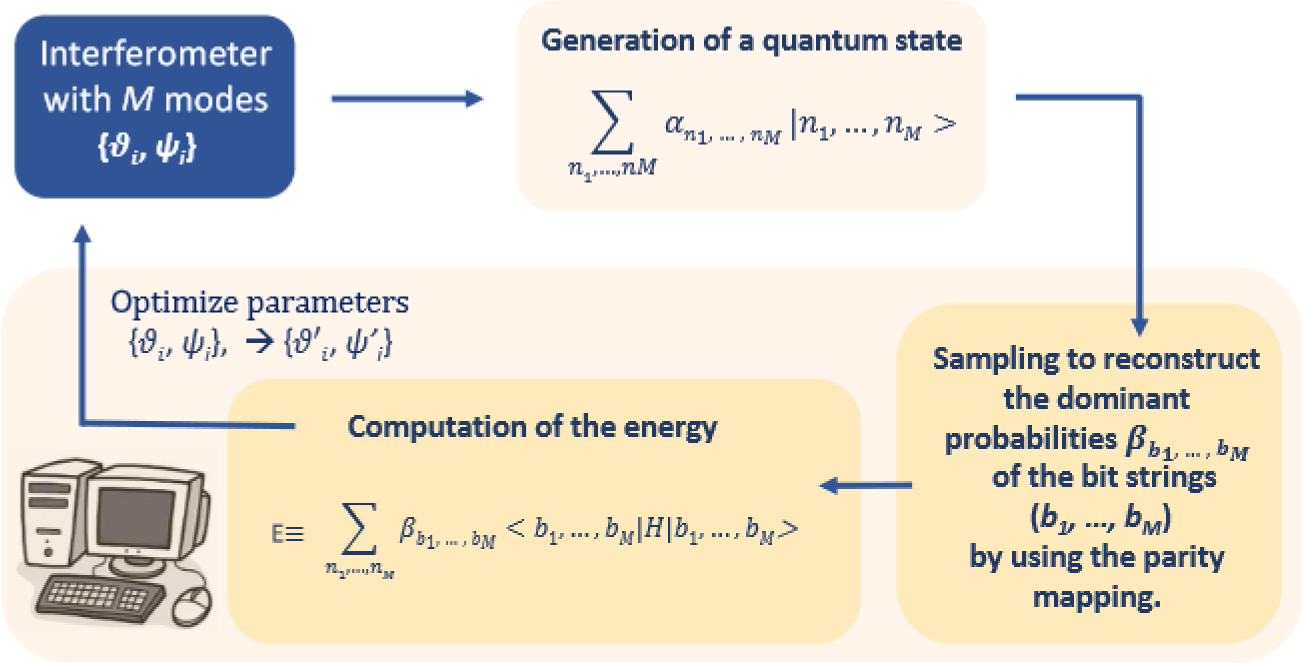
- Very high accuracy in tuning optical components (ε is small)
- Need many samples to capture difference bw distributions ($E(\vartheta_i + \varepsilon)$ and $E(\vartheta_i)$)

Use the *parameter shift rule* instead:

$$2\frac{\partial E(\vartheta_i)}{\vartheta_i} = E(\vartheta_i + \pi/2) - E(\vartheta_i - \pi/2)$$

- Hint: trigonometric functions backing it up

Model overview



Break minimization: formulation

Take a timetable

Slot	1	2	3	4	5	6
team 1	2	3	4	2	3	4
team 2	1	4	3	1	4	3
team 3	4	1	2	4	1	2
team 4	3	2	1	3	2	1

Create an assignment schema

Slot	1	2	3	4	5	6
team 1	z_1	z_2	z_3	$1-z_1$	$1-z_2$	$1-z_3$
team 2	$1-z_1$	z_5	z_4	z_1	$1-z_5$	$1-z_4$
team 3	z_6	$1-z_2$	$1-z_4$	$1-z_6$	z_2	z_4
team 4	$1-z_6$	$1-z_5$	$1-z_3$	z_6	z_5	z_3

Apply $f(\boldsymbol{z})$ objective function

$$f(\mathbf{z}) \sim \sum_k [z_k z_{k'} + (1 - z_k)(1 - z_{k'})]$$

Group the coefficients

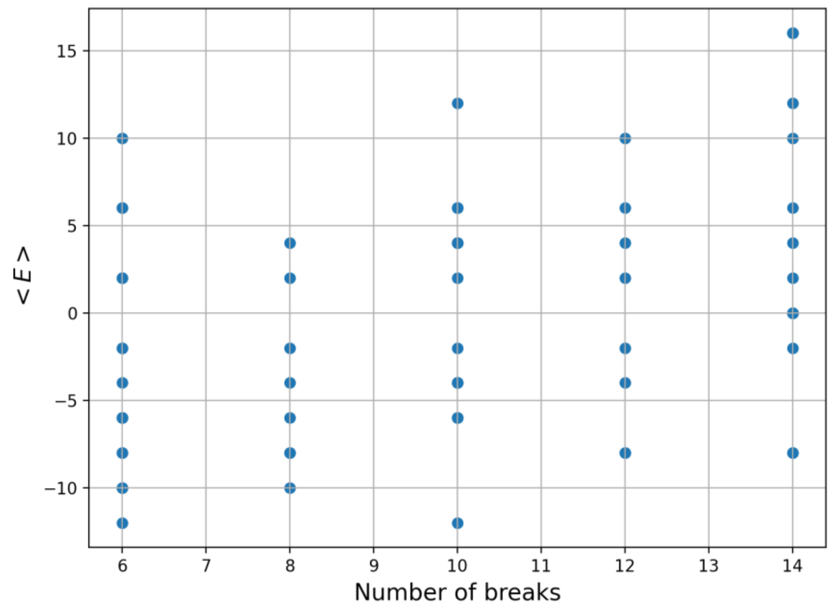
$$\begin{bmatrix} 0 & 4 & -2 & 2 & -4 & 0 \\ 4 & -4 & 4 & 4 & 0 & -4 \\ -2 & 4 & -2 & 0 & 4 & -2 \\ 2 & 4 & 0 & -6 & 4 & 2 \\ -4 & 0 & 4 & 4 & -4 & 4 \\ 0 & -4 & -2 & 2 & 4 & 0 \end{bmatrix}$$

Find **z** vector(s) /w minimal <**Q**>

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ \dots \\ 1 \end{bmatrix}$$

Progress

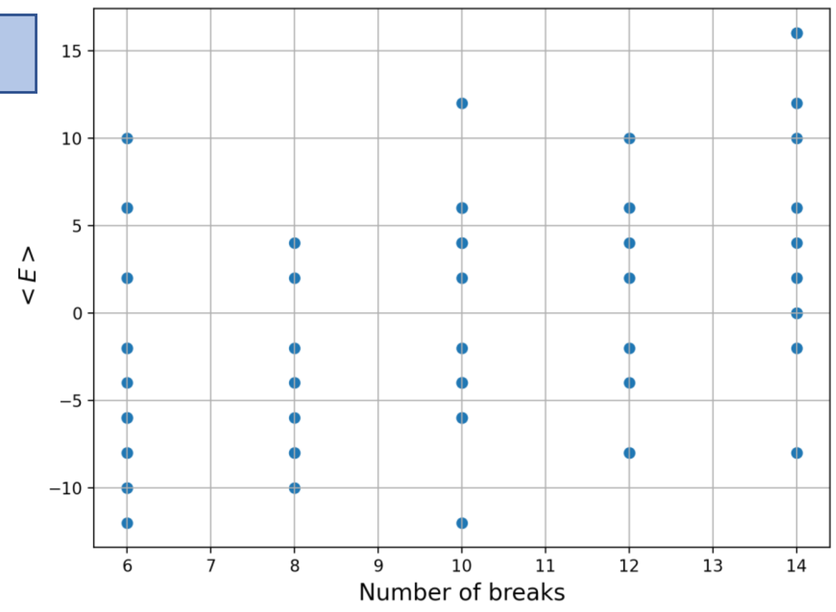
- Calculated the <**z**|**Q**|**z**> for all **z** in a tournament
- Counted breaks for every **z**
- Compared **energy** and **number of breaks**
- Expected a linear connection between breaks and energy (<**Q**> = <**E**>)
- Found no clear correspondence bw breaks and expected value
- Perhaps **Q** calculated wrongly



Progress

- Calculated the <**z**|**Q**|**z**> for all **z** in a tournament
- Counted breaks for every **z**
- Compared **energy** and **number of breaks**
- Expected a linear connection between breaks and energy (<**Q**> = <**E**>)
- **Q was** calculated wrongly (change up $z_4 - z_5$)
- Overlapping data points – noise
- Correspondence bw energies and breaks
- **The lowest energy** configuration paired with a min-break → **quantum annealing** is viable

previous



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