Quadratic optimization with quantum computing

Biweekly Presentation IV

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Mathematical background

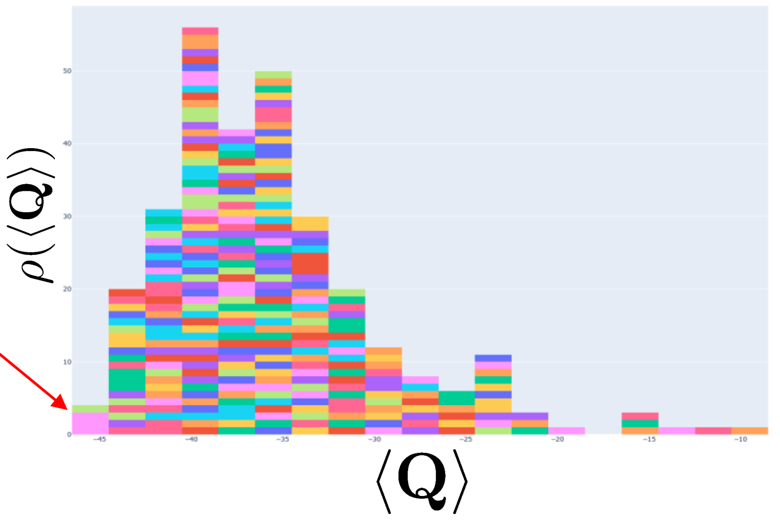
- Expected value of a matrix: $\langle \mathbf{b} | \mathbf{Q} | \mathbf{b} \rangle$

$$\langle 10010 | \mathbf{Q} | 10010 \rangle = \text{scalar}$$

- Find $|\mathbf{b}\rangle$ such that $\min(\langle \mathbf{b} | \mathbf{Q} | \mathbf{b} \rangle)$
  - Explicitly (calculate **all** $\langle \mathbf{Q} \rangle$)
  - ★ By **sampling** $\rho(\langle \mathbf{Q} \rangle)$

- Better than random sampling?
  - Optimizing is hard, because $|\mathbf{b}\rangle$ is discrete
  - Need to find **continous parameters** to repr $|\mathbf{b}\rangle$

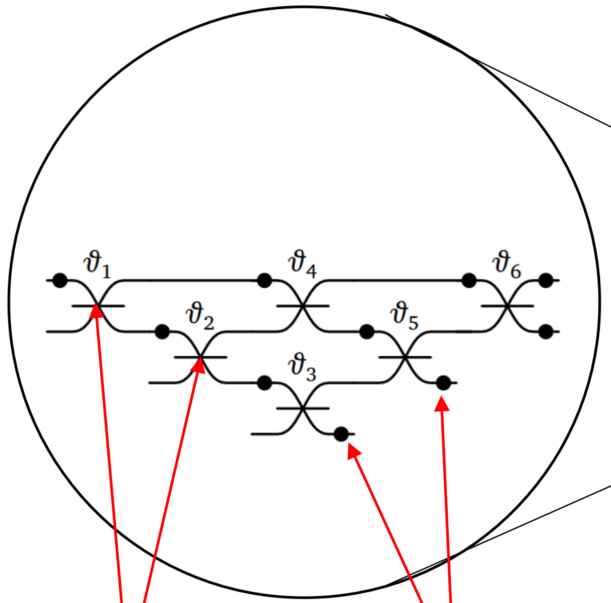
- ***Q*** is a symmetric matrix (n x n)
- ***b*** is a **binary** vector (n)



Boson sampling

- Need to find **continous parameters** to repr $|\mathbf{b}\rangle$

INTERFEROMETER

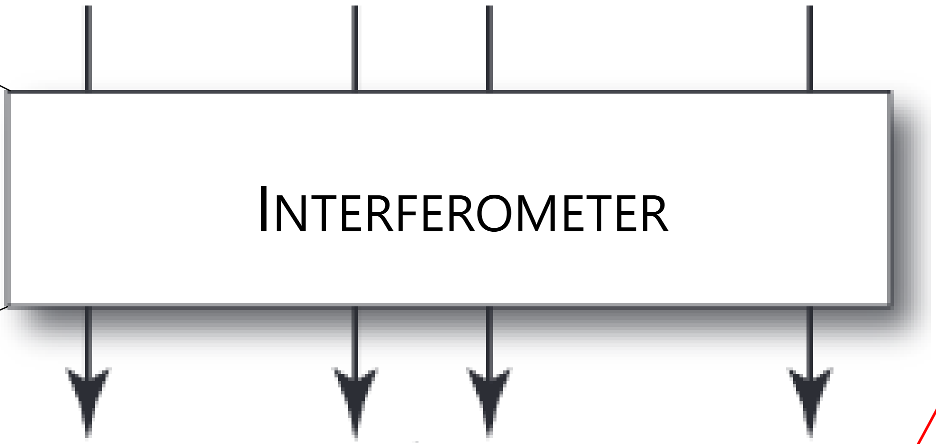


Beam splitters and phase shifters

ϑ_i and ψ_j

continous

Single photon quantum states

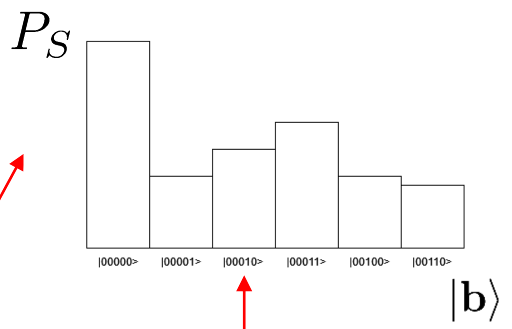


Multiphoton quantum state

$$\Psi(\vartheta_i, \psi_j) = \textstyle\sum_S \alpha_S \underbrace{|n_1^{(S)}, \dots, n_m^{(S)}\rangle}_{\approx |\mathbf{b}\rangle}$$

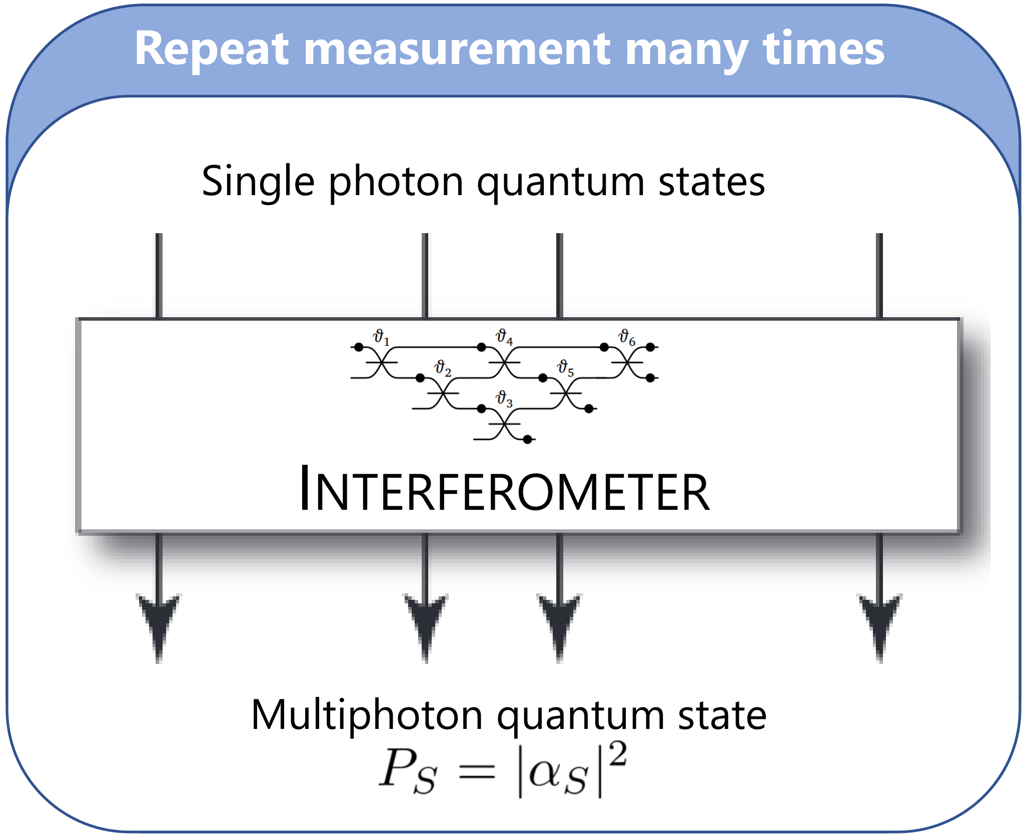
$$P_S = |\alpha_S|^2$$

Distribution of $|\mathbf{b}\rangle$



Boson sampling

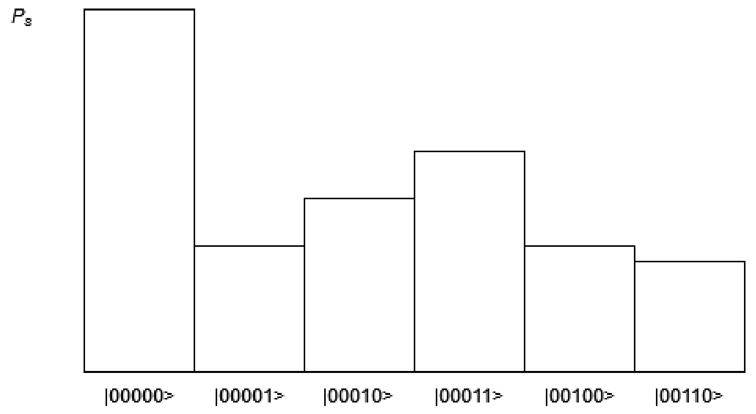
- Need to find **continous parameters** to repr



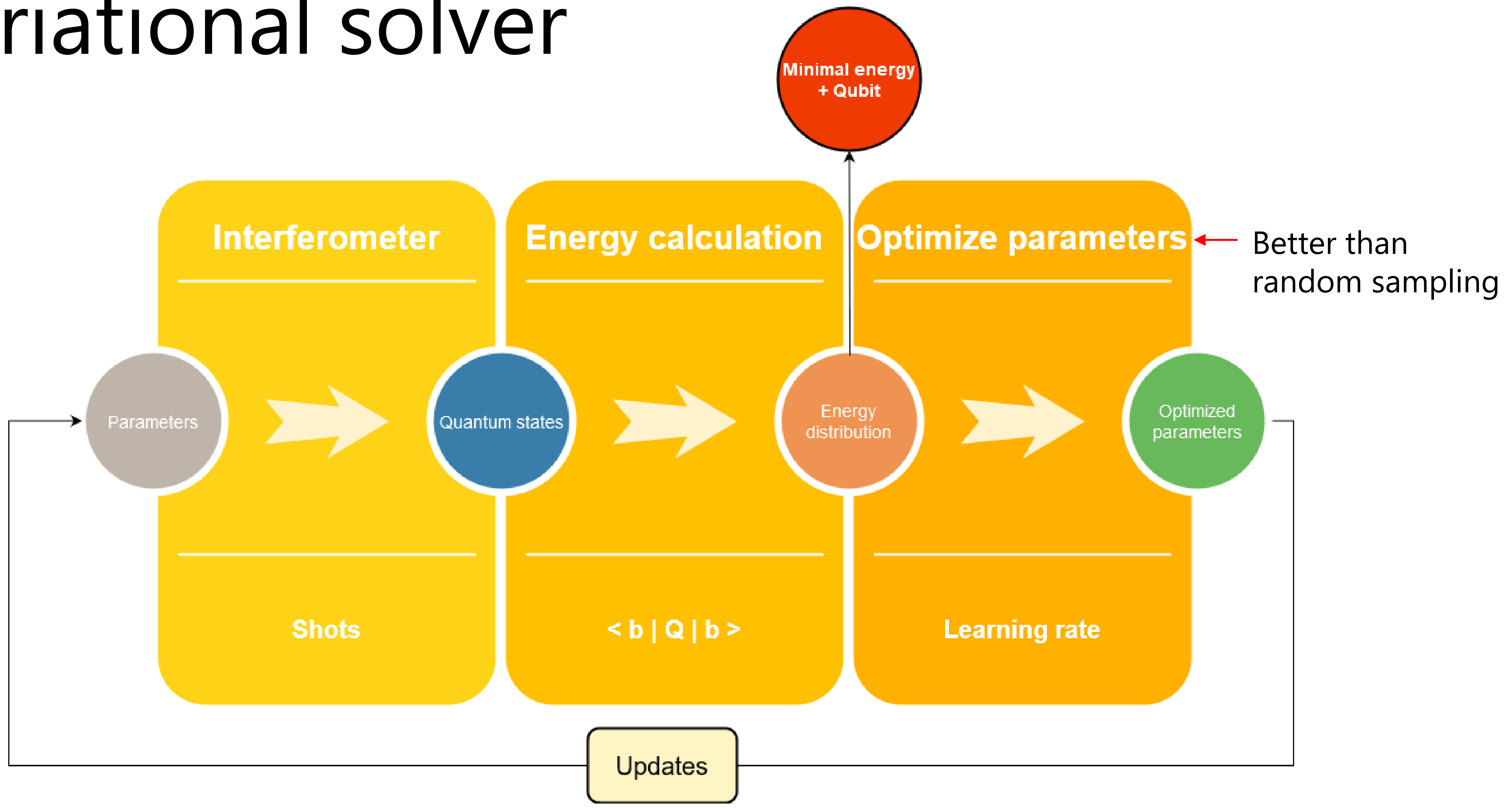
Continous variables ϑ_i and ψ_j

Distribution of quantum states:

$$P_S(\vartheta, \psi) = |\alpha_S|^2$$



Variational solver



Setup

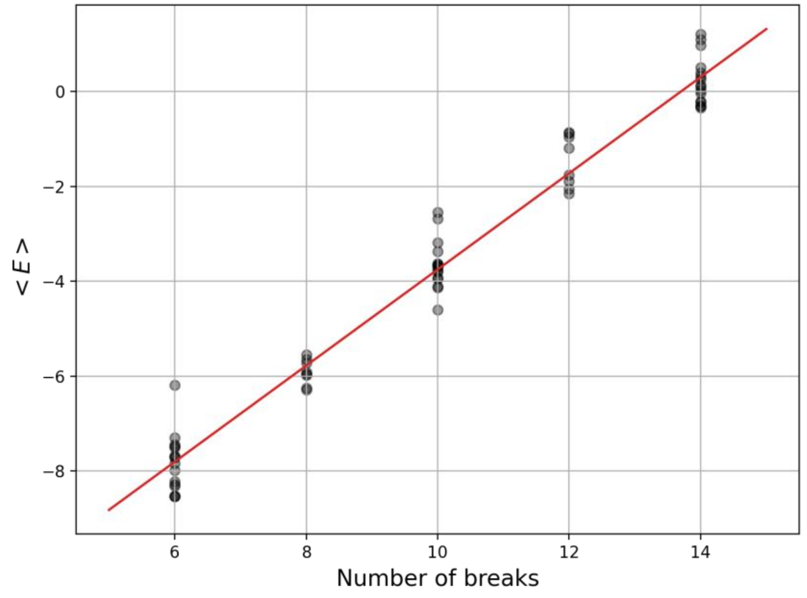
- Piquasso – simulator for photonic quantum computations
- Piquassoboost – for performance improvement

- Personal use:
  - Ubuntu on Docker for a separated application environment

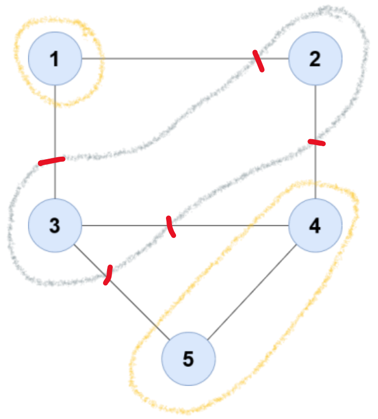
- Budapest Quantum Computing Group server:
  - 64 Core CPU
  - FPGA server

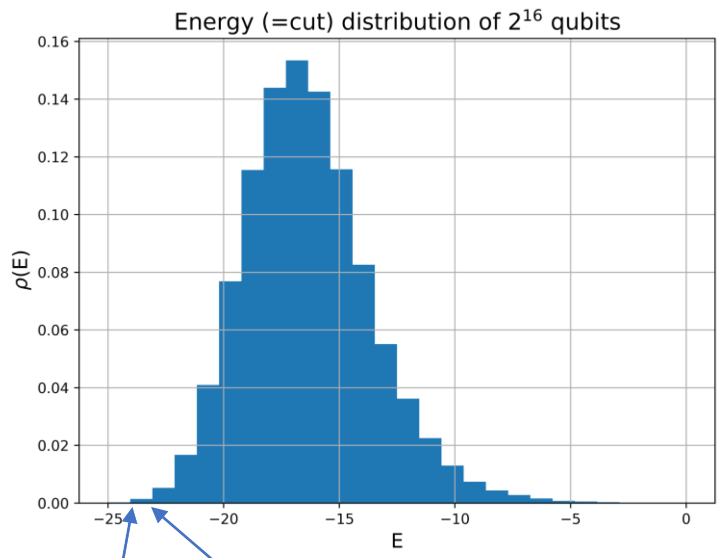
Results: break minimization

- **Break:** team plays at the same place two times in a row
- Compared **energy** and **number of breaks**
- Expected a linear connection between breaks and energy (<**Q**> = <**E**>)
- **The lowest energy** configuration paired with a min-break → **quantum annealing** is viable
- The global minimum (6) was succesfully **found** with the **Piquasso** model



Results: Max-cut





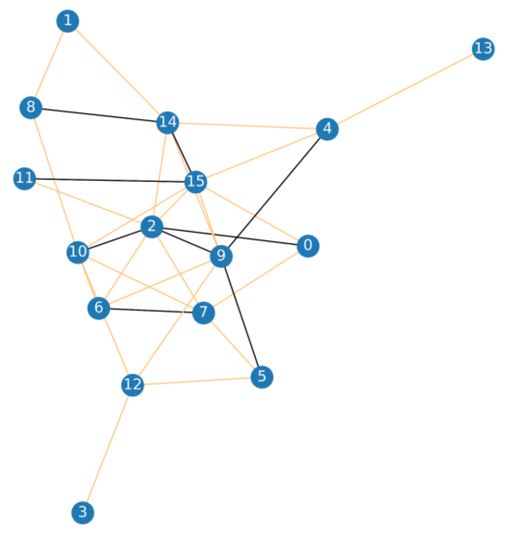
Variational (best): - 24

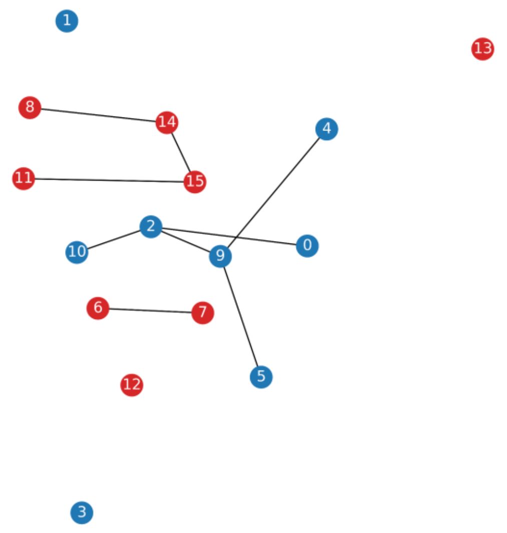
exact: min(E) = -24

Shots: 20
Updates: 30

Network without
the cut
N = 16

Network with
Max-cut
N = 16

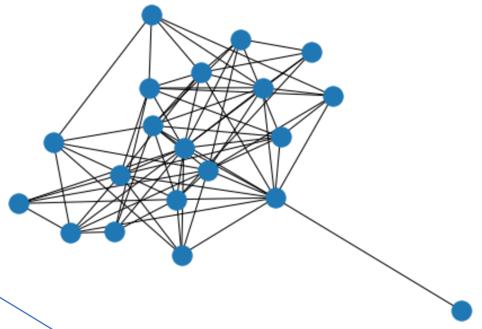


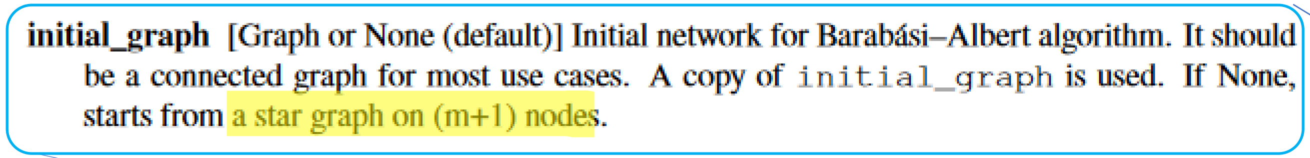


Side note: Barabási-Albert graph

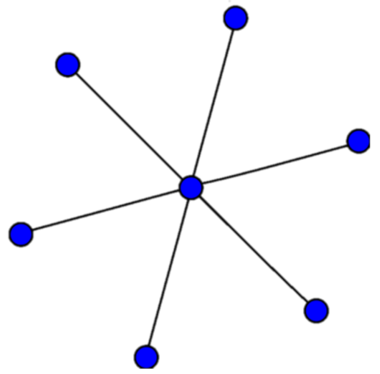
- Last report: How can there be a such a lonely node?
- Parameters: n – final num of nodes

  m – edges from new nodes

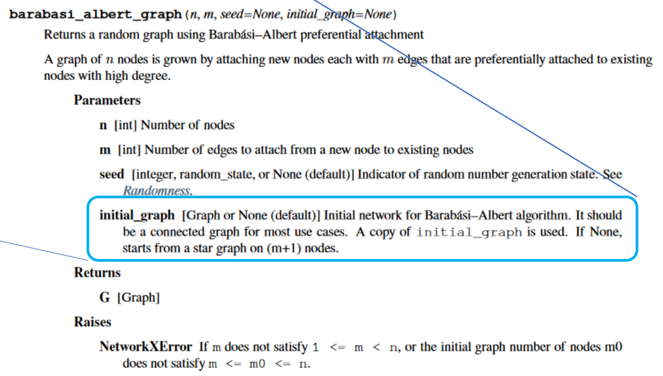




initial_graph [Graph or None (default)] Initial network for Barabási–Albert algorithm. It should be a connected graph for most use cases. A copy of `initial_graph` is used. If None, starts from a star graph on (m+1) nodes.



$m = 5$



barabasi_albert_graph (*n*, *m*, *seed=None*, *initial_graph=None*)

Returns a random graph using Barabási–Albert preferential attachment

A graph of n nodes is grown by attaching new nodes each with m edges that are preferentially attached to existing nodes with high degree.

Parameters

n [int] Number of nodes

m [int] Number of edges to attach from a new node to existing nodes

seed [integer, random_state, or None (default)] Indicator of random number generation state. See *Randomness*.

initial_graph [Graph or None (default)] Initial network for Barabási–Albert algorithm. It should be a connected graph for most use cases. A copy of `initial_graph` is used. If None, starts from a star graph on (m+1) nodes.

Returns

G [Graph]

Raises

NetworkXError If `m` does not satisfy `1 <= m < n`, or the initial graph number of nodes m0 does not satisfy `m <= m0 <= n`.

NetworkX reference - networkx — NetworkX documentation. (n.d.). Retrieved April 26, 2022, from https://networkx.org/documentation/stable/_downloads/networkx_reference.pdf