Individual Spread Footing

Manpreet Kaur, Amritpal Singh, Monisha
Guru Nanak Dev Engineering College, Ludhiana

February 16, 2016

Contents

List of Figures

List of Charts

List of Tables

Chapter 12

Individual Spread Footing

12.1 Introduction

Clause 34 of the *IS456:2000* gives the provisions governing the design of reinforced concrete footings. These provisions are similar to those given by the ACI Committee 318. The design of footings in accordance with *IS456:2000* differs from that by the *IS456:1964* in the following aspects.

I Perimeter shear stress must not exceed the allowable value. This aspect was not given in the *IS456:1964*. But it is similar to the familiar concept of punching shear stress.

II Bond-stress-criterion was given in the *IS456:1964*, but it is omitted in the *IS456:2000*. Instead, development length of footing bars is required to be checked at the sections where bending moment is critical.

III 25% excess pressure on edge of footing was allowed by the *IS456:1964* when a footing is eccentrically loaded. This concession is withdrawn by the *IS456:2000*, thereby adding to the cost of eccentrically loaded footings.

The *IS456:2000* requires footings to be designed for the following limit states.

(a) Perimeter shear

(b) Bending moment

(c) Beam shear

(d) Development length of footing bars

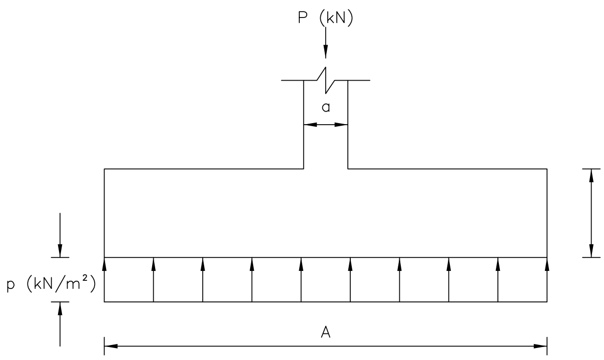
(e) Development length of column bars

12.2 Types of Individual Footings

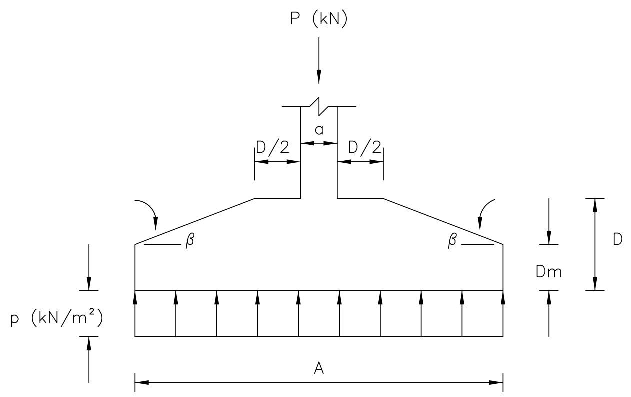
Individual spread footings can be either square or rectangular in plan, the area of a rectangular footing with sides A and B being given by,

$$A \times B = \frac{P}{p} \tag{12.1}$$

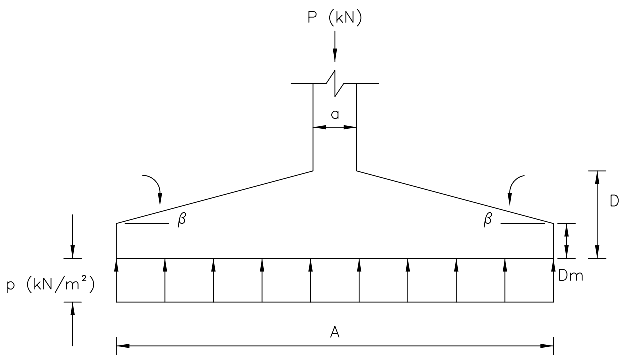
where P is the column load in kN and p denotes the net allowable soil pressure in kN/m^2. In this development, self-weight of footing may not be considered. This involves only a small error in that, the weight of the concrete of footing is assumed here to be approximately equal to the weight of the earth displaced by it. Further, it is assumed here that the soil pressure under the footing is uniform. This is a reasonable assumption as discussed elsewhere. Various types of individual spread footings are shown in Fig.12.1. With the area of footing known from Equation12.1, dimensions A and B are easily finalise. The only dimension of footing which remains to be known is the depth (D) of the footing. A common way of design of footing is to assume D, rather generously, with a view to reduce steel area as well as to help provide fixity to the column base, in order to be close to the assumptions made in the frame analysis of superstructure.



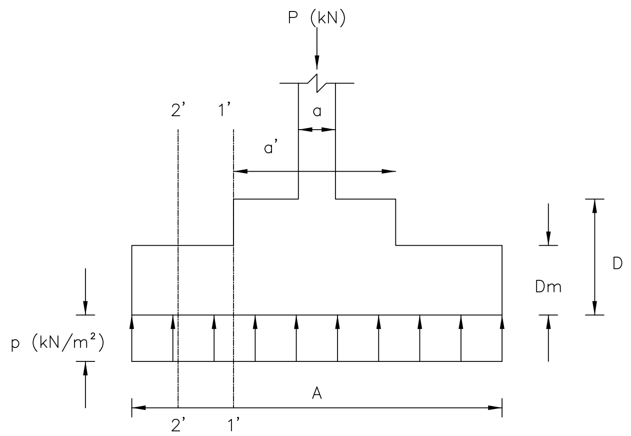
(a) Uniformly deep footing



(b) Sloped footing (slope starting from $D/2$ away from the edge of column)

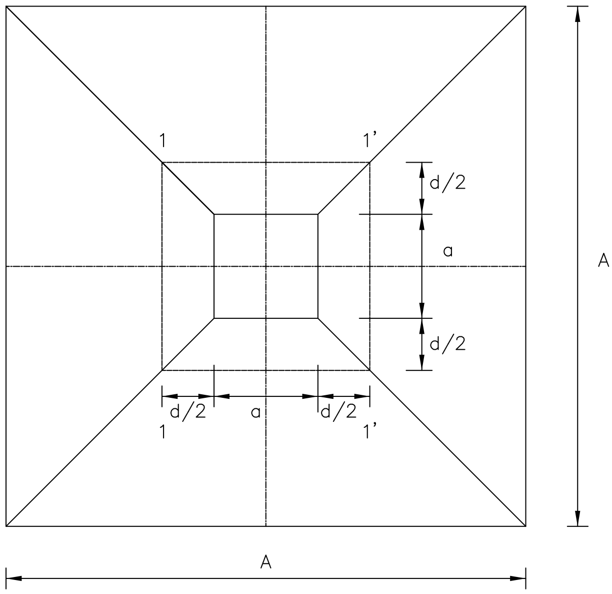


(c) Sloped footing (slope starting from the edge of column)

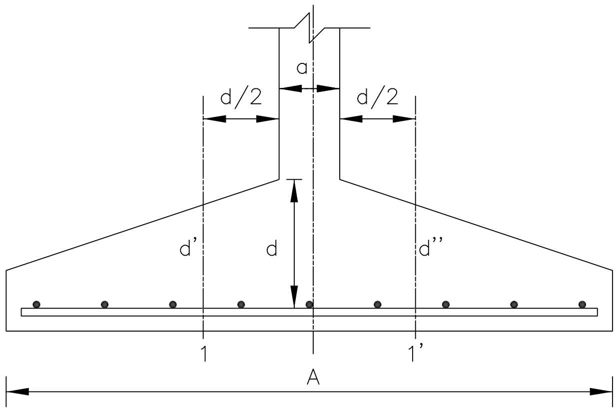


(d) Stepped footing

Figure 12.1: Types of individual spread footing.



(a) Critical perimeter 1-1-1-1 in plan for perimeter shear



(b) Section of sloped footing showing reduced depth d' for perimeter shear

Figure 12.2: Perimeter shear for square footing.

12.3 Design for Perimeter Shear

Depth of footing is fixed from the consideration of perimeter shear stress which depends on concrete quality, being independent of types of reinforce steel. For a square footing of uniform depth with a square column of side a (Fig.12.2a), perimeter shear stress τ_v is given by

$$\tau_v = \frac{V_u}{b_0 \times d} = \frac{1.5 \times S_p}{4(a+d) \times d} \tag{12.2}$$

where

$$S_p = P - p \times (a+d)^2 \tag{12.3}$$

and b_0 = Perimeter of critical closed section.

The allowable perimeter shear stress τ_a (clause 31.6.3 of the *IS456:2000*) is given by,

$$\tau_a = k_s.\tau_c = k_s \times 0.25\sqrt{f_{ck}} \tag{12.4}$$

where, f_{ck} is to be put in N/mm^2.

$k_s = 1.0$ for square columns and also for rectangular with aspect ratio $\left(\dfrac{b}{a}\right) \leq 2.0$. For the condition $\tau_v = \tau_a$, Equation(12.2) and (12.4) give,

$$-\frac{\frac{a^2}{A^2}+\frac{2\,ad}{A^2}+\frac{d^2}{A^2}-1}{\frac{ad}{A^2}+\frac{d^2}{A^2}}=\frac{0.0670\,\alpha\sqrt{fck}}{p}=k(say) \tag{12.5}$$

For a square sloped footing with a square column of side a (Fig.12.2b),

$$\tau_v=\frac{V_u}{b_0\times d''}=\frac{1.5\times S_p}{4(a+d)\times d''} \tag{12.6}$$

Assuming $d''=\alpha.d$, the condition $\tau_v=\tau_a$ gives,

$$-\frac{\frac{a^2}{A^2}+\frac{2\,ad}{A^2}+\frac{d^2}{A^2}-1}{\frac{ad}{A^2}+\frac{d^2}{A^2}}=\frac{0.0670\,\alpha\sqrt{fck}}{p}=k \tag{12.7}$$

$\alpha=1.0$ for footings of Types (a), (b) and (d) (Fig.12.1), while $\alpha<1.0$ for sloped footings of Type (c). The overall depth of footing is given by,

$$D=c+d+\phi \tag{12.8}$$

Here, d is regarded as an average value for either steel layer. For sloped footings (Fig.12.2b), simple geometry gives,

$$\alpha=\frac{d''}{d}=\frac{D_m}{d}+\frac{D-D_m}{d}.\frac{\left(1-\frac{a}{A}-\frac{d}{A}\right)}{\left(1-\frac{a}{A}\right)}=\frac{(c+\phi)}{d} \tag{12.9}$$

Chart12.1 is developed on the basis of Equation(12.5) and (12.7) and it applies to both uniformly deep and sloped square footings. It can also be used for rectangular footings with rectangular columns by using average values of a and A, provided an equal overhang is left on all sides of column, which requires,

$$-a+b=-A+B \tag{12.10}$$

Solution of numerical examples gives an idea that it is possible to develop thumb rules for fixing depth of footings. It may be noted that there is no dire need of exactness in fixing the value of depth of footing, only it should be more than adequate for the actions imposed on a footing. Table12.1, based on Equation(12.5), is developed for footings of Types (a) and (b) (Fig.12.1).It gives values of D/A for various practicable values of p. It is seen that, for safety in beam shear, these values are to be increased by 10% in case of steel types Fe 415and Fe 500. For sloped and stepped footings (Types c and d), the depth of footing given by Table12.1 may be increased by 20%. The depth at the free end of a footing may be restricted to 150 mm, which is the minimum prescribed by the *IS456:2000* for spread footings.

12.4 Design for Moment and Beam Shear

Section 1-1 in Fig.12.3 is the critical section for bending moment. The bending moment for full width B is given by,

$$M_{1-1}=\frac{1}{8}\,(A-a)Bp \tag{12.11}$$

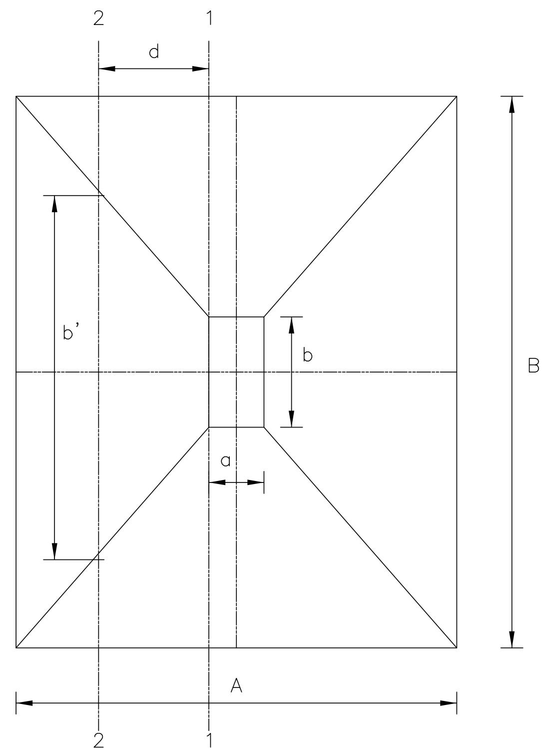
For footings of uniform depth and also stepped footings, the concrete compression zone is rectangular and charts of Chapter 2 are used to calculate the required area of steel. But for slopped footings, the concrete compression zone is of a trapezoidal shape and Chart4.1 of Chapter 4 is to be used for finding steel area. Chart4.1 can be used for both uniformly deep ($\gamma=0$) and sloped footings. The calculated steel area should not be less than the specied minimum steel area (Table11.4 of Chapter 11) for spread footings which may be regarded as slabs for this purpose.

Section 2-2 in Fig.12.3 is the critical section for beam shear. The shear force and moment at section $2'-2'$ for the full width of footing is given by,

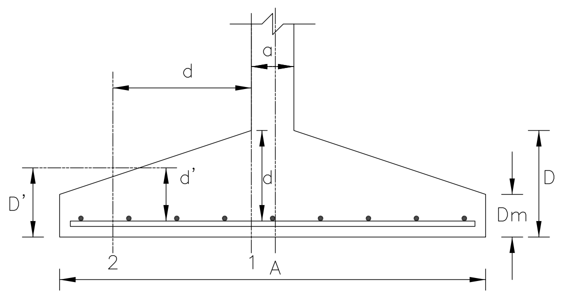
$$S_{2-2}=\frac{1}{2}\,(A-a-2\,d)Bp \tag{12.12}$$

$$M_{2-2}=\frac{1}{8}\,(A-a-2\,d)^2Bp \tag{12.13}$$

Beam shear stress τ_v for footings of uniform depth and stepped footings is given by,



(a) Plan of sloped footing



(b) Section of sloped footing

Figure 12.3: Critical sections for bending moment and beam shear in footings.

$$\tau_v = \frac{V_u}{bd} = \frac{1.5 \times S_{2-2}}{bd} \tag{12.14}$$

where,

$$b = \text{B for uniformaly deep footings,}$$
$$= \text{a for stepped footing}(Fig.12.1d)$$

For sloped footings, clause 40.1.1 of the *IS456:2000* gives, (-ve sign applies here),

$$\tau_v = \frac{V_u}{b'd'} - \frac{M_u}{b'd'^2}.tan\beta \tag{12.15}$$

where b', d' are shown in Fig.12.3a and Fig.12.3b and M_u is the ultimate moment at section 2-2.

$$D' = D_m + \frac{(D - D_m)\left(\frac{a}{A} + \frac{2\,d}{A} - 1\right)}{\frac{a}{A} - 1} \tag{12.16}$$

$$d' = D' - (c + \phi) \tag{12.17}$$

$$b' = b + \frac{2\,(B - b)d}{A - a}\beta \tag{12.18}$$

For Type (c), Fig.12.1c gives,

$$\tan(\beta) = \frac{2\,(D - D_m)}{A - a} \tag{12.19}$$

For Type (b), Fig.12.1b gives,

$$\tan(\beta) = \frac{2\,(D - D_m)}{A - D - a} \tag{12.20}$$

The calculated value of τ_v must not exceed the allowable stress τ_a, as shear reinforcement is just not provided in individual footings for reasons of economy, the same as in solid slabs. The allowable stress in concrete solid slabs τ_a is given by,

$$\tau_a = k\tau_c \tag{12.21}$$

τ_c is given by Table19 of the *IS456:2000* depending on the steel area provided for moment at the critical section 2-2 (the minimum value of τ_c is assumed for $p_t \leq 0.15$ in Table19 and

$$\begin{aligned} k &= 1.0 \text{ for } D \geq 300mm \\ &= 1.1 \text{ for } D = 250mm \\ &= 1.2 \text{ for } D = 200mm \\ &= 1.25 \text{ for } D = 175mm \\ &= 1.30 \text{ for } D \leq 150mm \end{aligned}$$

Normally, depth of footing given for perimeter shear (Chart12.1) is more than adequate to satisfy the requirements of beam shear. But when steel type Fe 415 and Fe 500 are used as reinforcement beam shear may govern the depth of footing. For stepped footings, additional checks for moment and beam shear are required to be made for the portion of the footing of depth D_m (Fig.12.1d). When section $1' - 1'$ is the critical section for moment and section $2' - 2'$ is that for beam shear (Fig.12.1d), expressions for moment and shear are given as,

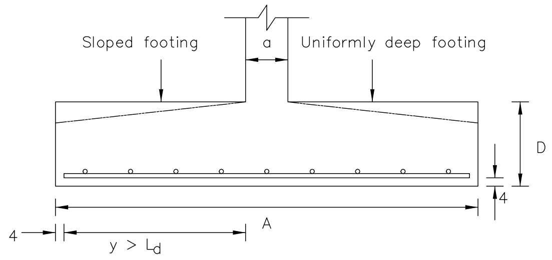
$$M_{1'-1'} = p.B.\frac{(A - a')^2}{B} \tag{12.22}$$

$$S_{2'-2'} = p.B\left[\frac{(A - a')^2}{B} - d_m\right] \tag{12.23}$$

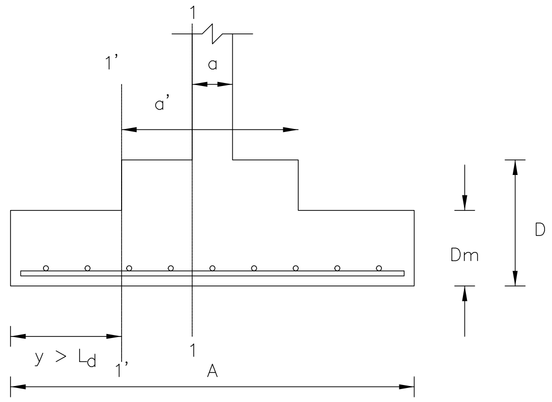
For finding steel area, Chart4.1 (with $\gamma = 0$), may be used, as the concrete compression zone is of a rectangular shape of width equal to B. Shear stress τ_v is given by,

$$\tau_v = \frac{V_v}{bd} = \frac{1.5S_{2-2}}{B.d_m} \tag{12.24}$$

and it must not exceed τ_a given by Equation(12.21), failing which depth D_m should be suitably increased. Normally $D_m = 0.30\ D$ to $0.50\ D$ is kept in stepped footings and perimeter shear stress can be checked to be safe by using first principles.



(a) Section of uniformaly and sloped footing



(b) Section of stepped footing

Figure 12.4: Development length of footing bars.

12.5 Development Length of Bars

Column dowel bars should extend into footings for a distance equal to the development length (Table11.3 of Chapter 11) of column bars in compression (or in tension when moment in column is large). With the clause 26.2.2.2 of the *IS456:2000*, column bars can always be adequately anchored in the footing, whatever be the depth of footing.

For development length of footing bars, there should be adequate bar length available (y), either straight or bent-up or both measured from the face of column. Referring to Fig.12.4a, for sloped or uniformly deep footings,

$$y = \frac{1}{2}A - \frac{1}{2}a - 4 > L_d(tension) \tag{12.25}$$

where 40 mm is taken as clear cover over ends of bars in footings. If Equation12.25 is not satisfied, there are two ways to tackle this problem :

1. bend bars up, as shown dotted in Fig.12.4a

2. choose smaller diameter for bars.

For stepped footings Fig.12.4b, the available straight length of bars beyond the critical section $1' - 1'$ is,

$$y' = \frac{(A - a')}{2} - 4 > L_d(tension) \tag{12.26}$$

Normally, full steel area required at section 11 is provided throughout and the steel strength σ_s at section 1 - 1 may be less than its maximum value of 0.87 f_y. The value of L_d (tension) should be calculated for the appropriate value of σ_s.

12.6 Selection of Type of Footings

Footings of uniform depth (Type a), though commonly used in practice for reasons of ease in design and construction, are the costliest. These consume more concrete quantity (about 25% to 45%) than that by sloped footings. This type is suitable only for small footings with overall depth being restricted to, say, 300 mm.

For footings of intermediate size, sloped footings with slope starting from $\frac{D}{2}$ away from the edge of column (Type b), are quite suitable. This type is quite economical giving concrete and steel quantities quite reasonable in comparison with other types. This type is easy to design as well as to execute. This type is recommended for most individual footings encountered in buildings with overall depth greater than 300 mm. The depth at the free end of footing may be kept at 150 mm, the specified minimum given by the *IS456:2000*. The depth (D) of this type of footing is kept the same as that for footings of uniform depth.

For large-sized footings, sloped footings with the slope starting from the edge of column (Type c) or stepped footings (Type d) are preferred to other types, as these give the least quantities for concrete and steel consumption. The stepped footings give the least steel quantity, while the sloped footings (Type c), give the least concrete quantity. The depth for these types of footings works out to be about 20% more than that for footings of uniform depth. Stepped footings are a little cumbersome in construction, while the sloped footings are easier in execution, albeit a little more labour-intensive than the footings of uniform depth.

12.7 Examples

Example 12.1. Square footing of Uniform Depth (Type a).
Given:

$$P = 1000kN$$
$$p = 200.kN/m^2$$
$$a = 400mm$$
$$f_{ck} = 15N/mm^2$$
$$f_y = 415N/mm^2$$

Required: Design the footing
Solution:

(i) **Dimensions of footing**

Equation12.1 gives,

$$A^2 = \frac{P}{p} = 5.0053 \times 10^6 mm^2$$
$$A = 2236.mm$$
$$A = 2250mm$$

Provided 2250×2250 base and

$$p = 197.5kN/m^2$$

Table12.1 gives for $p = 0.00020$, and steel Fe 415,

$$D = 0.222\,A = 500.mm$$

(ii) **Check for perimeter shear**

Chart12.1 gives for,

$$k = \frac{670.\,\alpha\sqrt{f_{ck}}}{p} = 13.15$$
$$\frac{a}{A} = 0.178$$

$$\frac{A}{d} = 5.5$$

$$d = 410.$$

Equation12.8 gives with $c = 40.mm, \phi = 12.$,

$$D = c + d + \phi = 464.0mm$$

$$D = 500.mm \text{ is safe}$$

(iii) **Design for moment** Equation12.11 gives,

$$M_{1-1} = \frac{1}{8}\,(A-a)^2 Bp = 190.1kNm$$

With rectangular compression zone, Chart(2.2) gives for,

$$d = D - c - \phi = 423.0mm$$

$$k = 0.04723$$

$$\mu = 0.047$$

$$A_{st} = \frac{1.15\,Adf_{ck}\mu}{f_y} = 1878.mm^2$$

$$\frac{A_{st}}{A} = 834.7mm^2/m$$

Table11.4 gives the minimum tension steel area in footings taken as slab,

$$A_{st}(min) = 1.2\,D = 600.mm^2/m$$

$\phi 12/135$ c/c both ways provided giving an area= $837.3mm^2/m$
which is exceeded by that provided.

(iv) **Design for beam shear**
Equation12.12 gives,

$$S_{2-2} = \frac{1}{2}\,(A - a - 2\,d)Ap = 223.1kN$$

Equation12.14 gives

$$\tau_v = \frac{1.5\,S_{22}}{Ad} = 0.35N/mm^2$$

$$\frac{100\,A_s}{bd} = 0.1980$$

Table 19 of the *IS456:2000* gives,

$$\tau_c = 0.32N/mm^2$$

With

$$k = 1.0$$

Equation12.21 gives,

$$\tau_a = \tau_c = 0.32N/mm^2$$

With

$$\tau_v = \tau_a, D = 50mm \text{ is safe}$$

(v) **Check on development length of footing bars**
Table(11.13) gives, for footings bars of ϕ 12 Fe 415),

$$L_d(tension) = 55\,\phi = 660.mm$$

Equation12.25 gives,

$$y = \frac{1}{2}\,A - \frac{1}{2}\,a - 4 = 921.mm$$

with $y > L_d$ (tension), footing bars will develop full strength at the critical section.

It may be noted that with $D = 500.\ mm$, this type of footing of uniform depth is not economical. Footing of Types (b) and (c) with sloping depth would be more economical than the present design.

Example 12.2. Rectangular sloped footing of Type (c).
Given:
Same as in Ex. 12.1 with

$$a = 400mm$$
$$b = 600mm$$

and the footing is to have equal overhangs on all sides of column.
Required: Design the footing
Solution:

(i) **Dimensions of footing**
Equation12.1 gives,

$$A \times B = \frac{P}{p} = 5.005 \times 10^6$$

$$A \times B = \frac{1000}{0.02} = 50000mm^2$$

The equal overhang condition, Equation12.9 gives,

$$-a + b = -A + B = 200$$

The solution of these two equations gives,

$$A = 2140mm$$
$$B = 2340mm$$

Practical designer may choose $A = 2140mm$ and $B = 2340mm$

$$p = \frac{P}{AB} = 200.kN/m^2$$

Table12.1 gives for $p = .02$ and steel Fe 415,

$$\frac{D}{A} = \frac{1}{4.5} \times 1.20$$
$$D = \frac{2}{15}A + \frac{2}{15}B = 600.mm$$
$$D = 600.mm \text{ and } D_m = 150.mm$$

(ii) **Check for perimeter shear**

Equation12.8 gives, with $c = 40mm$, $\phi = 12.mm$,

$$d = D - c - \phi = 545.mm$$

Equation12.9 gives,

$$\alpha = \frac{D_m}{d} - \frac{c+\phi}{d} + \frac{(D - D_m)\left(\frac{a}{A} + \frac{d}{A} - 1\right)}{d\left(\frac{a}{A} - 1\right)} = 0.74$$

Chart12.1 gives for,

$$k = \frac{670.\,\alpha\sqrt{f_{ck}}}{p} = 9.64$$

$$\frac{a}{A}(\text{average value}) = \frac{a+b}{A+B} = 0.22$$

$$\frac{A}{d} = 4.98 \text{ or d } = 0.201\,A = 430.$$

$$D = c + d + \phi = 482.mm$$
$$D = 600.mm \text{ is safe in perimeter shear.}$$

(iii) **Design for moment**

Equation12.11 gives,

$$M_{1-1} = \frac{1}{8}\,(A-a)^2 Bp = 176.9kNm$$

with trapezoidal compression zone, Chart 4.1 gives for,

$$b = 600mm, d = 545.mm$$

$$\tan\beta = \frac{2\,(D-D_m)}{B-b} = 0.5156$$

$$\gamma = \frac{d}{b\times\tan\beta} = 1.8$$

$$k = \frac{200000\,M_{1-1}\gamma_f}{3\,bd^2} = 0.12$$

$$\mu = 0.11$$

$$A_{st} = \frac{100\,bdf_{ck}\mu}{87\,f_y} = 1496.mm^2$$

$$\frac{A_{st}}{B} = 639.3mm^2/m$$

Minimum steel for average value of $D = \frac{1}{2}\,D + \frac{1}{2}\,D_m = 373.mm$ regarding footings as slabs,

$$A_{st}(min) = 1.2\times 373. = 448.mm^2/m$$

Provide ϕ 12/176 c/c bothways, $area = 642.3mm^2/m$

(iv) **Design for beam shear**

Equation(12.12)gives

$$S_{2-2} = \frac{1}{2}\,(A-a-2\,d_m)Bp = 152.kN$$

Equation(12.16) gives,

$$D' = D_m + \frac{(D-D_m)\left(\frac{a}{A}+\frac{2\,d}{A}-1\right)}{\frac{a}{A}-1} = 316.mm$$

Equation(12.17) gives,

$$d' = D' - c - \phi = 264.mm$$

Equation(12.18) gives,

$$b' = b + \frac{2\,(B-b)d}{A-a} = 1691.mm$$

Equation(12.13) gives

$$M_{2-2} = \frac{1}{8}\,(A-a-2\,d)^2 Bp = 24.631kNm$$

Equation12.15 gives,

$$\tau_v = \frac{228.}{1691.\times 264.} - \frac{1.5\times 24.631\times 0.5156}{1691.\times (264.)^2}$$

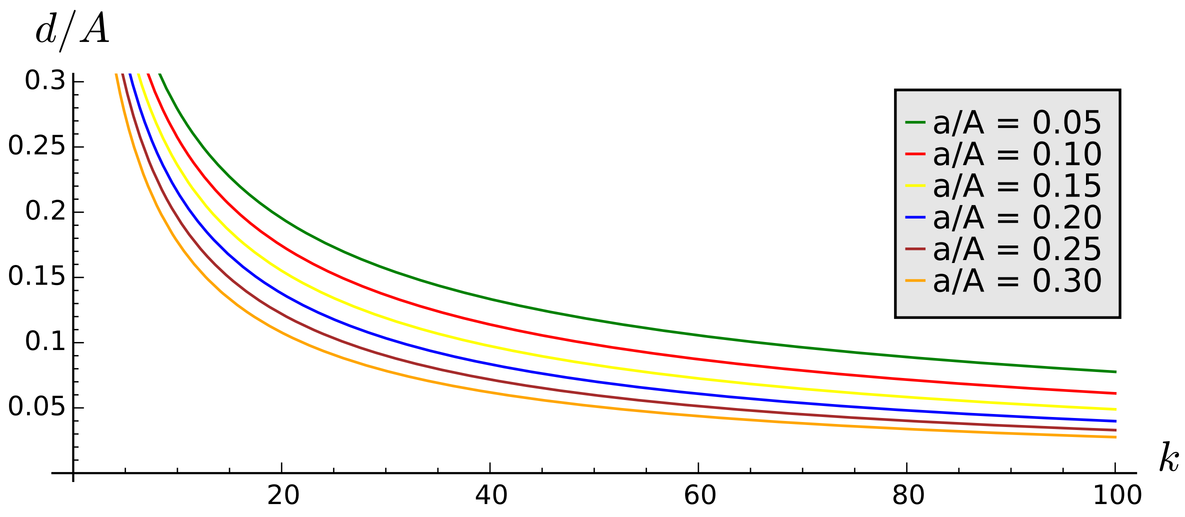


Chart 12.1: Effective Depth (d) of Square Individual Footings for Safety of perimeter shear

$$\tau_v = 0.510 N/mm^2$$

For

$$\frac{100\,A_s}{b'\,d'} = 0.243$$

Table19 gives $\tau_c = 0.35\ N/mm^2$

$$k = 1.0, \tau_a = \tau_c = 0.0350 N/mm^2 > \tau_u = 0.34 N/mm^2,$$

$$D = 600.mm \text{ is safe in beam shear}$$

(v) **Check on development length of footing bars**
Table(11.3)gives for ϕ 12 bars

$$L_d(tension) = 55\,\phi = 660mm$$

Chart12.1 Effective Depth (d) of Square Individual Footings for Safety in perimeter shear.

$$k = \frac{\left[1 - \left(\frac{a}{A}\right)^2 - 2\left(\frac{a}{A}\right)^2\left(\frac{d}{A}\right) - \left(\frac{d}{A}\right)^2\right]}{\left[\left(\frac{a}{A}\right)\left(\frac{d}{A}\right) + \left(\frac{d}{A}\right)^2\right]}$$

$$k = 0.067\frac{\sqrt{f_{ck}}}{p}$$

Equation12.25 gives,

$$y = \frac{1}{2}A - \frac{1}{2}a - 4 = 866.mm > 660mm \text{ OK},$$

Notes:

1. fck in N/mm^2
2. p in kN/mm^2b
3. For rectangular column $a \times b$ Assume $a = \frac{(a+b)}{2}$ as in approximation provided $a/b \leq 0.50$.

4. Chart can be used for squarish rectangular footings provided an equal overhang is left beyond faces of column with $A = \frac{A+B}{2}$
For equal overhang $(b-a) = (B-A)$
5. $\alpha = 1.0$ for types (a),(b) and (d)(Fig.12.1)
6. $\alpha < 1.0$ for type (c). Use Equation12.9
7. $\alpha = \frac{D_m}{d} + \frac{D - D_m}{d} . \frac{\left(1 - \frac{a}{A} - \frac{d}{A}\right)}{\left(1 - \frac{a}{A}\right)} - \frac{(c+\phi)}{d}$

12.8 Conclusion

Provisions of the *IS456:2000* on footings have been applied to individual spread footings, square, or rectangular in plan with depth uniform, varying or stepped. Design aids are given for fixing depth of footings and checking it in respect of requirements of safety in perimeter shear. Procedure for design of footings for bending moment, shear and development length of tension steel bars is given in detail and examples are given to illustrate it. For a large-sized rectangular footing, a footing beam in the long direction will be more economical than the traditional isolated footing. Also, for large square footings, two cross footing beams with a uniform base slab will make for economy.

p(kN/m^2)	A/D	Fe415,Fe500
50	8	7
100	6	5.5
150	5.5	5
200	5	4.5
250	4.5	4
300	4	3.5

Table 12.1: Depth of Footing for Safe Bearing Capacity

Note:

1. **'A' is the average of sides of rectangular footing.**
2. **For sloped (type c) and stepped (type d) footings, increased depth given by the above table by 20%.**

Chapter 13

Limit State Design-Introduction

13.1 Reinforced Concrete Codes

Indian Standard Code of Practice for Plain and Reinforced Concrete was first published in 1953 and its second revision *IS456:2000* is referred here in after as the old code, While its third revision was made in 1978 and is fourth and the latest revision *IS456:2000* is called here in after as the new code or simply the Code. *IS456:1964* mainly deals with the working stress method of design of reinforced concrete and gives the ultimate load method of design in its Appendix B. But the position of these two methods of design is reversed in the new code. The working stress method is relegated to Appendix B of the Code, while the limit state method, which is a rationalisation of the ultimate load method, forms the main body of the Code. In general, working stress method leads to a more conservative design than that by the limit state method. This manual concerns itself with the limit state method as given in Section 5 of the Code.

13.2 Requirements of Design

All structures must be designed to be safe, serviceable and economical. Safety means that structures must not fail under loads unless exceeded by a given margin called overload safety factors. Serviceability means that structures must perform well throughout their service life in both appearance and comfort to their users. This requirement includes cracking and deection under working loads. Economy means that structures must be designed in such a way as to minimise the quantities of materials used in them. Although safety and serviceability are the basic requirements, the test of an acceptable structural design is economy.

In design of reinforced concrete structures, all critical sections are checked for the effect of forces acting on them. So, the economy of design achieved here refers to the critical sections. his approach in itself may make a marginal difference, say, within 5% of cost of structure, which in itself, is about 40% to 50% of the total cost of building. More substantial economies in cost of structures are achieved by the correct choice of structural systems like load-bearing walls, frames, shear walls and their several combinations. Also, sizing of columns and beams and spacing of frames in framed buildings will affect the economy. These decisions will be crucial for economy of structures, but these are beyond the scope of this manual.

13.3 Limit State Method

Limit state method includes consideration of structures at both the working and the ultimate load levels with a view to satisfy the requirements of safety and serviceability. It offers an integrated approach to design of structures. A reinforced concrete section needs to be checked for forces acting on it, together with relevant serviceability requirements. Each force or a serviceability requirement is called a limit state. The aim of design is to ensure that a reinforced concrete section does not reach any of the limit states to which it may be subjected. The usual approach is to design a section for a limit state which is likely to govern it and then check it for the remaining limit states. The Code gives principles for design of reinforced concrete sections under the following limit states :

(i) Flexure

(ii) Compression

(iii) Axial load and uniaxial bending

(iv) Axial load and biaxial bending

(v) Shear

(vi) Torsion

(vii) Deflection

(viii) Cracking

Limit states (i) to (vi) refer to ultimate limit states or limit states of collapse, while the last two limit states refer to serviceability requirements. Limit state of tension, as may occur in design of concrete hangers or suspenders, is not given by the Code. The other relevant limit states for concrete structures can be impact, vibration, fatigue, durability, re-resistance, lightning etc. These limit states are satisfied by good quality concrete, adequate size of members, adequate cover to reinforcement, provision of minimum steel area and other special details. Limit state of cracking is satisfied by adhering to the requirements of minimum steel area and spacing of bars in members. Thus, analysis of reinforced concrete sections under the various ultimate limit states (i) to (vi) is required. Further, limit state of deection, which is crucial for bending membersslabs and beamsneeds consideration in detail.

13.4 Safety Factors

The system of partial safety factors is followed in the *IS456:2000* as against the system of unied safety factors followed in the old code. The values of partial safety factors Yf for loads are given in Table18 of the Code, which are valid for various load-combinations mentioned therein. Table18 of the *IS456:2000* gives ultimate loads for the two common load-combinations as follows:

$$UL = 1.5(DL + LL)UL = 1.2(DL + LL + WLorEL) \tag{13.1}$$

The values of partial safety factors for loads are to be supplemented with partial safety factors

$$\gamma_m(concrete) = 1.50 \tag{13.2}$$
$$\gamma_m(steel) = 1.15 \tag{13.3}$$

The *IS456:1964* gave values of 1.85 and 1.4 as factors of safety for total loads for the two load-combinations considered in Equation1.1 and Equation1.2 respectively and practical examples of design of slabs and singly reinforced beams show, that these compare well with the systems of partial safety factors incorporated in the *IS456:2000*.

13.5 Stress-Strain Diagrams for Materials

Fig.21 of the Code gives the stress-strain diagram for concrete which is rectangular-parabolic in shape. The design strength at the extreme bre in compression is given by,

$$f_c = 0.67\frac{f_{ck}}{\gamma_m} = 0.67\frac{f_{ck}}{1.5} = 0.45f_{ck} \tag{13.4}$$

Where, f_{ck} is the characteristic cube strength of concrete in compression at 28days. The factor 0.67 in Equation1.5 accounts for variation in strength of concrete in actual structure as against that in the cube test of concrete. Equationfor concrete stress diagram of Fig.21 of the Code is given with reference to Equation1.1. For the range x = O to g (A to B),

$$y = -\frac{h}{g}^2 x^2 + \frac{2h}{g}x \tag{13.5}$$

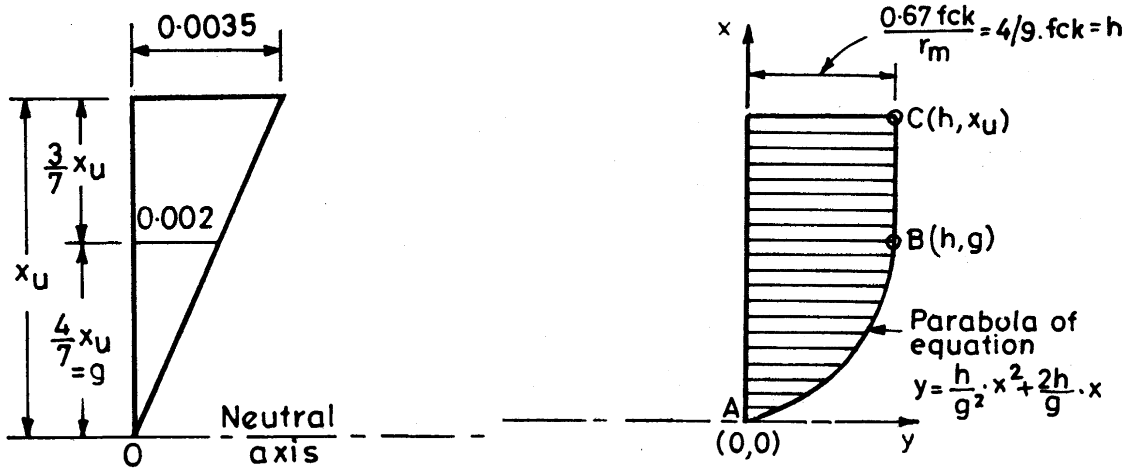
and for the range x = g to x_u (B to C),

$$y = h \tag{13.6}$$

$$h = \frac{4}{9}f_{ck}$$

$$g = \frac{4}{7}x_u$$

For a given shape of concrete compression zone, the amount of concrete compressive force (C) and its location from the neutral axis ($\bar{x}_1$) can be obtained by integration (Fig.1.2).



(a) Linear concrete strain diagram

(b) Rectangular parabolic concrete stress diagram

Figure 13.1: Equations of the concrete stress diagram

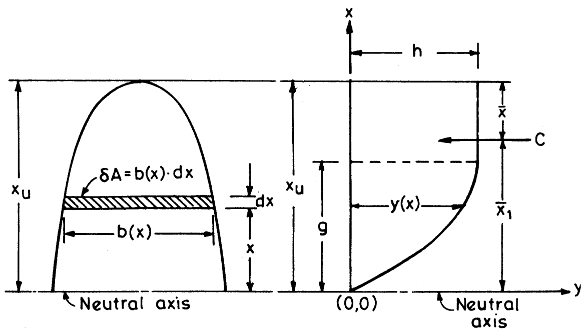
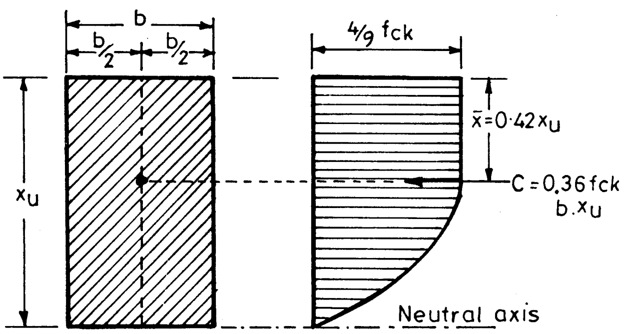
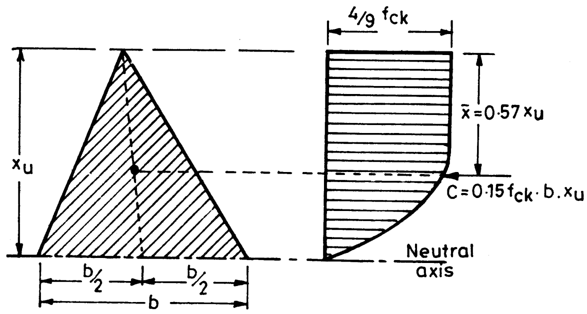


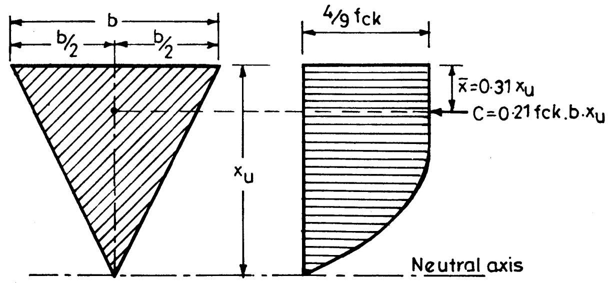
Figure 13.2: Concrete compression zone of any given shape under the stress diagram of the Code



(a) Rectangular concrete compression zone.



(b) Triangular concrete compression zone.



(c) Inverted triangular concrete comparison zone.

Figure 13.3: Concrete stress block parameters for three basic shapes of concrete compression zone.

$$C = \int_{x=g}^{x=0} b(x).y(x).\,dx + \int_{x=x_u}^{x=g} b(x).h.\,dx \tag{13.7}$$

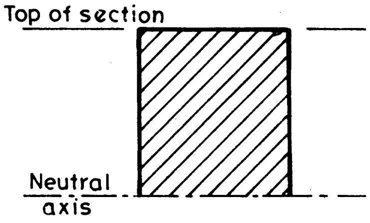
$$C.(\bar{x}_1) = \int_{x=g}^{x=0} b(x).y(x).x.\,dx + \int_{x=x_u}^{x=g} b(x).h.x.\,dx \tag{13.8}$$

The location of the concrete compressive force from the top ber of section is given by (Fig.1.2),

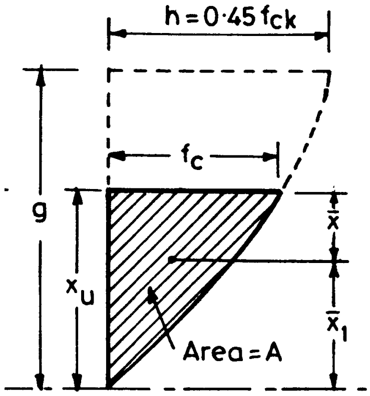
$$\bar{x} = x_u - \bar{x}_1 \tag{13.9}$$

Equation(1.8) to Equation(1.10) are applied to rectangular and triangular shapes of concrete compression zone and the results are given in Fig.1.3.

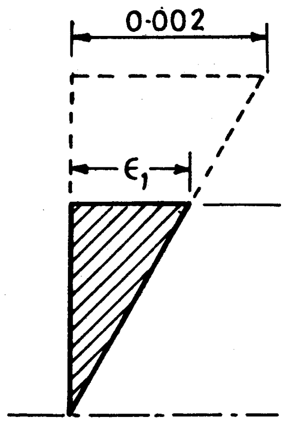
When the concrete stress diagram is only a part-parabola (Fig.1.4), the expressions for the area (A) of the stress diagram and the location of its centre of gravity ($\bar{x}$) from the top bre of section are given below.



(a) Concrete compression zone



(b) Concrete stress diagram part-parabola



(c) Linear strain diagram

Figure 13.4: Part-parabolic concrete stress diagram

$$\frac{x_u}{g} = \frac{\epsilon}{0.002} \tag{13.10}$$

$$A = -\frac{1}{3}.\frac{h^2}{g}.x_u^2 + \frac{h}{g}.x_u^2 \tag{13.11}$$

$$A.\bar{x} = -\frac{1}{4}.\frac{h^2}{g}.x_u^4 + \frac{2}{3}.\frac{h}{g}x_u^3 \tag{13.12}$$

$$\bar{x} = -x_u - \bar{x_1} \tag{13.13}$$

$$f_c = -\frac{h^2}{g}.x_u^2 + 2.\frac{h}{g}.x_u \tag{13.14}$$

When the neutral axis falls below the section, the expressions for the area (A) of the concrete stress diagram and the location of its centre of gravity ($\bar{x}$) are given below (Fig.1.5). Fig.23 of the *IS456:2000* gives stress-strain curves for steel types (ordinary mild steel plain bars) and Fe 415 and Fe 500 (high strength steel deformed bars) with the modulus ofelasticity of steel being $E_s = 2 * 10^5 N/mm^2$, which is given to be the same for all types of reinforcing steel.The design strength of steel, equal in both tension and compression,is given by,

$$f_s = \frac{f_y}{\gamma_m} = \frac{f_y}{1.15} = 0.87 f_y \tag{13.15}$$

where, fy is the characteristic yield strength of steel, as shown in Fig.23 of the *IS456:2000.* Table1.1 gives values of design stresses in steel type Fe 415 for various strain values. This table is used extensively for developing design aids for steel type Fe 415. In this manual, no separate design aids are given for steel type Fe 500. As the stresss-strain diagram for Fe 500 is similar to that for Fe 415 and as the difference in the yield stresses of the two steel types is small, it is seen that. steel area for Fe 500 in the design of reinforced concrete sections can always be found with reasonable accuracy by using design aids for steel.

type Fe 415 and using a $multiplier = \frac{415}{500} = 0.83$ to the steel area obtained for Fe 415. Thus, for all reinforced concrete members, design aids are givenonly for the two steel types Fe 250 and Fe 415.

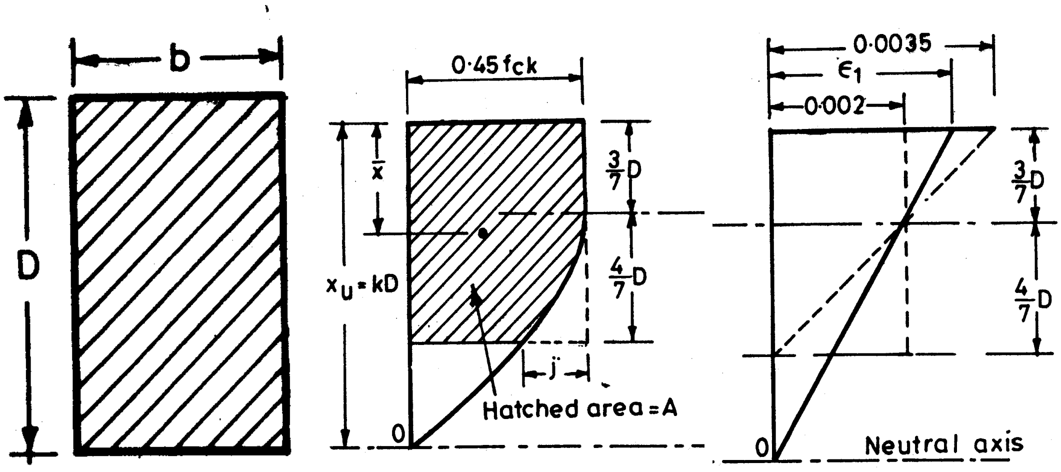
13.6 Behaviour of Reinforced Concrete Members at Failure

A member under bending with or without axial load of either comperssive or tensile nature may fail under either of the following three modes of failure 4.

(i) Tension failure.Concrete crushed or not crushed,but tension steel well in yield.

(ii) Balanced failure.Concrete crushed and tension steel just in yield.

(iii) Compression failure.Concrete crushed and tension steel not in yield.



(a) Section fully under compression

(b) Concrete stress diagram

(c) Concrete strain diagram

Figure 13.5: Concrete stress block parameters when netural Axis is below the section

With a very low amount of tension steel, failure will be initiated by yielding of reinforcement and collapse will take place with snapping of reinforcement at the ultimate elongation , of steel (83m) as given by the breaking tests of different types of steel specied by relevant codes5 6. At this stage, strain in concrete at the extreme bre of section (81) may be less than the maximum crushing strain of concrete (0.0035). When the section is not excessively under-reinforced, failure is initiated by the yielding of tension steel. Consequently, the neutral axis shifts upwards and the nal collapse is also associated with the crushing of concrete. These two modes of failure fall under the category of tension failure which takes place gradually and gives ample warning before collapse.

When the amount of tension steel is large, the section is over-reinforced. Then the concrete reaches its ultimate crushing strain (ϵ_{stu}) before the tension steel starts yielding. This is called compression failure and it leads to a sudden collapse without warning due to the brittle nature of concrete. For this reason, it needs to be avoided or provided for, in design based on limit state of collapse. Balanced failure is just a transition between the tension and the compression failures. In general, members should be so designed that tension failure is ensured at the limit state of collapse. In order to achieve this, the necessary criterion is that the tension steel must be well in yield. The Code, in its clause 38.1 (f) has, therefore, stipulated that the minimum strain in the tension steel shall exceed the linear elastic yield strain by 0.002. This ensures that the tension steel of a type with xed yield point like Fe 250 will denitely be in yield, thereby, giving ductility to the number. In cold worked steels like Fe 415 and Fe 500, the yield stress is associated with steel strain equal to $\epsilon_{std} = \frac{f_y}{1.15E_s} + 0.002$, thereby, satisfying the above requirement of the Code. Members under axial compression with or without moment fail under compression failure.This can not be avoided. But in order to reduce the suddenness and explosion of this type of failure, concrete crushing strain is reduced from 0.0035 to 0.002 and lateral ties are provided in members to improve ductility.

13.7 Unified Approach

It would be ideal to consider a member under bending and axial load (P_u, M_u) as a general case, of which a beam $(P_u = 0, M_u)$, a column $(+veP_u, M_u)$ and a tension member (veP_u, M_u) form only particular cases. This may be called a unied approach.The Code,in its Section 5 on the limit state method of structural design, has giVen a limited unied approach for members under bending combined with axial compression but has made no mention of tension members. SP-16 $DesignAids$[8] have however,given charts for design of tension members with equal steel on opposite faces or equal steel on all four faces, without making clear the basis of their development.The old code had given separate treatments for beams,columns and ties giving no unied approach for their design. There is no doubt that these different members behave under load in special patterns but it is quite fascinating to include all these special patterns into a unied approach so that design aids can be developed which can be used with equal ease for design of beams, columns as well as tension members. The compartmentalised approach for beams, columns and ties has led to proliferation of design aids and loss of continuity, clarity and conciseness in design. Further, the unied approach can be extremely useful for design of reinforced concrete sections subject to biaxial bending combined with axial load of either nature.

For the limit state of collapse for bending and axial compression, the Code lists the relevant assumptions in clauses 38.1 and 39.1. For the limit state of collapse for bending combined with axial tension, the following additional assumptions are made.

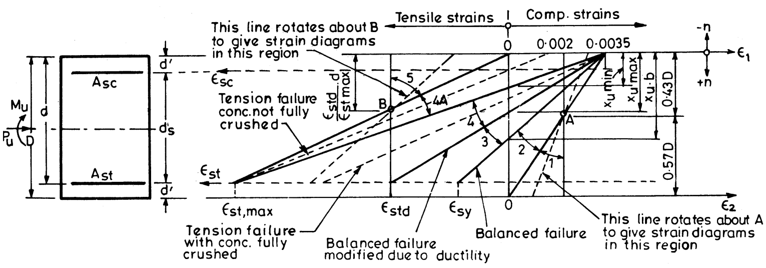


Figure 13.6: Various strain diagrams applicable for a section under bending and axial load.

(i) The minimum tensile strain in axial tension is taken equal to ϵ_{std},

$$where, \epsilon_{std} = \frac{f_y}{1.15\epsilon_s} + 0.002 \tag{13.16}$$

(ii) For bending combined with axial tension, the maximum tensile strain in the most highly stretched steel layer shall be equal to 1/10th the ultimate elongation (ϵ_{stu}) which is given by the tensile tests of different types of steelas stipulated in relevant *codes*[5,6].This gives,

$$\epsilon_{st}, max = \frac{1}{10}.\epsilon_{stu} \tag{13.17}$$

The maximum tensile strain in steel is not given by the Code. The obvious value for the maximum tension steel strain is its snapping strain 83m. But the values of snapping strains for steel types Fe 250,Fe 415 and Fe 500 are quite high.Therefore,in order toput a limit on excessive cracking and spalling of concrete of tension zone, it is assumed that the maximum tensile strain may be limited to 1/10th the snapping strain of steel.For steel types Fe 250, Fe 415 and Fe 500, this assumption gives values of 0.020, 0.0145 and 0.012 respectively for the maximum tensile strains. These values compare well with 0.005 and 0.01 assumedfor all types of reinforcing steel by DIN 1045[9] and CEB Model *Code*[10] respectively. Further, the requirement for ensuring ductility in sections,

$$\epsilon_{st} = \epsilon_{std} = \frac{f_y}{1.15E_s} + 0.002$$

has been given by the Code for tension failure of beams only. In the unied approach, this requirement of ductility is extended to tension failure of sections under bending with or without axial load of either compressive 0r tensile nature. As it is known that a ductile tension failure is a gradual one and gives ample warning before collapse, it is advantageous to ensure ductility for all members (not only for beams) under tension failure.

With these additional assumptions, a section under bending combined with axial load of either compressive or tensile nature can be analysed for its full range of variation of axial load from $+P_u(compression)to-P_u(tension)$ with pure bending case ($P_u = 0$) following as a transition. The design assumptions for a section under bending and axial load are portrayed in Fig. 1.6, by the various strain diagrams applicable for the full range of variation of Pu.

For a section under bending combined with axial compression, clause 39.1 (b) of the Code gives the equation,

$$\epsilon_1 = 0.0035 - 0.75\epsilon_2 \tag{13.18}$$

which represents various strain diagrams given in the line rotating about the point A (Fig.1.6). On a similar analbgy, the line rotating about the point B (Fig.1.6) gives different strain diagrams applicable for a section under bending combined with axial tension, which can be got from the following relations for different types of reinforcing steel.

$$ForFe250, \epsilon_{st} = 0.0020 - 5.45\epsilon_1 \tag{13.19}$$
$$Fe415, \epsilon_{st} = 0.0145 - 2.82\epsilon_1 \tag{13.20}$$
$$Fe500, \epsilon_{st} = 0.012 - 1.86\epsilon_1 \tag{13.21}$$

The above relationships have been derived by using the principles of similar triangles along with the values of $\epsilon_{st}, max, \epsilon_{std}$ for various types of steel given in Table1.2. Figure 1.6 gives by similar triangles,

$$n = \frac{\epsilon_1}{\epsilon_1 + \epsilon_{st}} \tag{13.22}$$

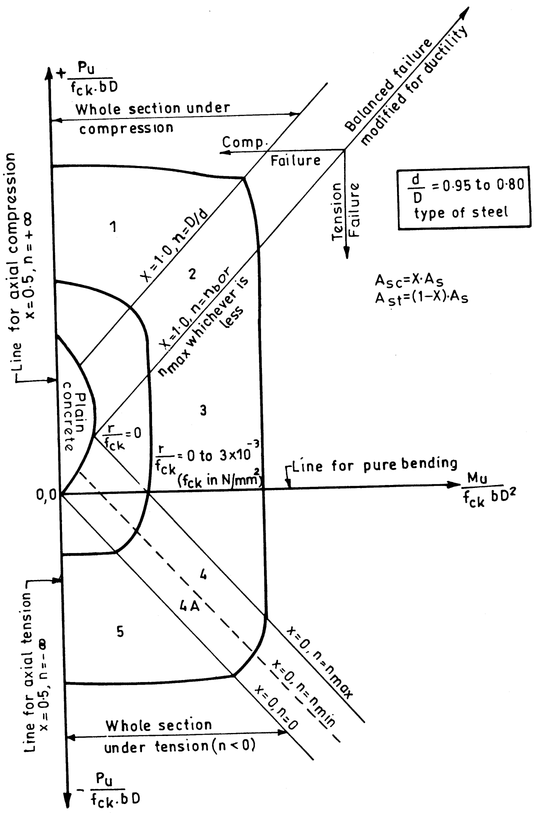


Figure 13.7: Interaction curves for a section under bending and axial load.

$n_b(balanced), n_max, n_min$ are calculated by the Equation1.26 and are given in Table1.2 for ready reference. Balanced failure is given by $\epsilon_1 = 0.0035, \epsilon_{st} = \epsilon_{sy}$ (ϵ_{sy} corresponds to a steel strain when $f_{st} = 0.87f_y$, to be read from stress-strain curves for steel given in Fig.23 of the *IS456:2000*, balanced failure modied for ductility is given for $\epsilon_1 = 0.0035, \epsilon_{st} = \epsilon - std$ and tension failure with concrete not fully crushed is given for $\epsilon_1 < 0.0035, \epsilon_{st} > \epsilon_{std}$. Tension failure with concrete not fully crushed is given for $n < n_min, When \epsilon_{st} = \epsilon_{st} max and \epsilon_1 < 0.0035. When n. <= 0$, section is fully under tension and concrete strength is to be entirely neglected and only steel areas are to be utilised to resist external forces. When $n > n_b$, compression failure is indicated where concrete is crushed ($\epsilon_1 = 0.002 to 0.0035$), but tension steel is not yield, it may rather be in compression as well. Fig.1.7 shows typical interaction curves for a section under bending and axial load with different Regions 1 to 5 marked therein, which are also shown in fig 1.6. The region-wise load moment combinations with corresponding strain values are summarised below.

Region 1. Whole section under compression, design governed by compression failure with $\epsilon_1 = 0.002 to 0.0035, \epsilon_2 = 0 to 0.002 and n = \infty to D/d$. pplicable for axial compression with or without small moment.

Region 2. Section partly under compression and partly under tension, design governed by compression failure with $\epsilon_1 = 0.0035(constant) and \epsilon_{st} < \epsilon_{sy} and n = D/d to n_b$. Applicable for axial compression with large moment.

Region 3. Section partly under compression and partly under tension, design governed , by tension failure modied for ductility with

$$\epsilon_1 = 0.0035(constant)$$

$$\epsilon_{st} = \epsilon_{sy} to \epsilon_{std}(theoretical)$$

$$= \epsilon_{std}(assumed constant for ductility).$$

$$n <= n_b$$

$$> n_{max}(theoretical)$$

But

$$n = n_{max}(assumed constant for ductility).$$

This is a region when the value of X is not xed.For a xed arrangement of steelin section,this region reduces to a straight line.

Region 4. Section partly under compression and partly under tension,design governed by tension failure with concrete fully crushed with,

$$\epsilon_1 = 0.0035(constant)$$

$$\epsilon_{st} = \epsilon_{std} to \epsilon_{st}, max$$

$$and n = n_{max} t0 n_{min}$$

Applicable for moment and axial load of either nature. Region (4A) Section partly under compression and partly under tension, design governed by tension failure with concrete not fully crushed with

$\epsilon_1 = 0 to 0.0035, \epsilon_{st}, \epsilon_{st}, max(constant) and n = 0 to n_{min}$

Applicable for moment and axial load mostly tensile in nature.

Region 5. Whole section under tension, design governed by tension failure with $\epsilon_1 = 0 to \epsilon_{std}(tensile strains), \epsilon_{st} = \epsilon_{std} to \epsilon_{st}, max and n = 0 to \infty$. Applicable for axial tension with or without small moment.

13.8 Analysis and Design of Structures

In practice, structures are analysed by the well-known principles and methods ofTheory of Structures under vertical and horizontal loads at the working load level.There are distinct advantages in using working loads during analysis of structures. Firstly, these give a realistic assessment of loads on members and secondly, column loads are then directly usable for design of footings, with the soil bearing capacity known at the working load level.The choice of a common load factor of 1.5 for both dead and live loads makes it easy to get ultimate forces which are used for design of all critical sections in structural members. design of structural membersslabs, beams, columns and footingsbased on the limit state method in accordance with *IS456:2000*. For each type of structural member, only one type of design aid is given, which is chosen on the basis of its effectiveness and convenience in practice. All design aids have been based on the governing formulae which have been fully derived and given in the subsequentchapters. For a complete understanding of the subject, this step. is quite important. Further, design aids are based strictly on the theoretical development and do not incorporate in themselves other design restrictions of the Code like minimum steel area in members, minimum eccentricity and slenderness effects on columns etc. These effects are dealt with separately outside the design aids. Design aids are developed in such a way that these are effective for both design and review of reinforced concrete sections. A unied approach for design of sections under bending (both uniaxial and biaxial) and axial load of either compressive or tensile nature is followed by which the same charts are capable of being used for beams, columns and ties. Design aids are prepared for two steel types Fe 250 and Fe 415 only. Most design aids are made independent of concrete quality.

Numerical examples have been given to illustrate the use of design aids. A consistent use of units for various quantities is important in analysis and design of structures. With SI units, it is convenient to work out loads in kN/m^2, moments and torques in or kNm, shears and column loads in kN and soil pressures in N/mm^2 or kN/cm^2. The Code gives values of all stresses f_{ck}, f_y etc., in N/mm^2.'For working out non-dimensional parameters like $\frac{P_u}{f_{ck}bD}, \frac{M_u}{f_{ck}bD^2}$ etc.

It is a general practice to provide one type of reinforcing steel in a building. This makes it easy for procurement of steel and supervision at the site. However, clause 26.1 of the Code permits use of two types of reinforcing steel in a building, one for main reinforcement (say, Fe 415 or Fe 500) and the other for stirrups in members (say Fe 250). As the percentage increase in strength of the two types steel Fe 415(or Fe 500) and Fe 250 is much greater than the percentage increase in their costs, it is economical to use deformed steel bars Fe 415 (or Fe 500) for both main and secondary reinforcement in all members of buildings with the following two exceptions.

(i) Reinforcement in Slabs

Mild steel plain bars (Fe 250) should be used in slabs in order to reduce slab thickness for control of deflection. For a general economy of the building as a whole, thin slabs are preferable (Chapter 6). In practice, Fe 415 steel is used in slabs with more steel provided at the midspan in order to get less slab thickness.

(ii) Transverse Reinforcement in Columns

The pitch and diameter of lateral ties in columns are given in clause 26.5.3.2 (c) of the Code and these both are independent of steel quality. Fe 250, Fe 250 can, therefore, be used for lateral ties in column for reasons of economy. Diameter 6 mm bars are rolled in Fe 250 steel and these are used for column ties and stirrups in lintels.

Provision of more than one type of reinforcing steel in structural members of a buildings calls for a careful planning and supervision at the site.

13.9 Conclusion

The general principles and the basic assumptions of the limit state method of design of reinforced concrete structures are given in this chapter, which are to be read in conjunction with the relevant clauses of the Code.

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