① (12) Balkrishna

Ans 2: Since the $max^m$ degree of polynomial $T \Rightarrow 2$
so $\dim(P_2) \Rightarrow 3$

Kernel

So a subset of Kernel T is $T(A) = 0$

$$(a-b) + (b-c)x + (c-a)x^2 \Rightarrow 0$$

$$a = b = c = t \text{ (let)}$$

new matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

$\dim^n$ of kernel is 1, b/c there's only one independent parameter as 't'

A/c to rank nullity Theorem :-

$$\text{rank}(T) + \text{nullity}(T) \rightarrow \dim(W)$$

$$\text{rank}(T) + 1 = 4$$

So, rank of T is 3 & nullity is 1

Ans 3

$$A - \lambda I \Rightarrow 0$$

$$\det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$2-\lambda \Rightarrow \pm 1$$

$$\lambda \Rightarrow 1, 3$$

for $\lambda = 1$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x - y = 0$ let t

$x = y$ $y = t$

$$t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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(1) ~~Rnk Rand~~

Find rank?

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_4 \to R_4 - R_3 = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

∴ Rank is 3 Ans

(4) (12) Dalwriting

Ques 10 Brief description of linear transformation for computer vision for rotating 2-D image

⇒ Linear Transformation for rotate 2D - image involves applying a rotat$^n$ matrix to each pixel coordinate This matrix rotates points counter clockwise by an angle θ around the origin. it ~~po~~ preserves geometric properties like parallelism and distance Rotate$^n$ is essential in tasks like image allignment and object detection in computer vision.

Q.8 using Jacobi's method (perform 3 iterates)

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$
$$x_0 = 1, \quad y = 1, \quad z_0 = 1$$

⇒ first eq$^n$ $x = \frac{1}{3}(23 + 6y - 2z)$

second$^n$ eq$^n$ $y = (-15 + 4x + z)$

third eq$^n$ $z = \frac{1}{7}(16 - x + 3y)$

$$x(0) = 1, \quad y(0) = 1, \quad z(0) = 1$$

**Iterate$^n$ - 1** :-

$$x(1) \Rightarrow (23 + 6 - 2)/3 \Rightarrow 9$$
$$y(1) = (-15 + 4 + 1) = -10$$
$$z(1) = (16 - 1 + 3)/7 = 18/7$$

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→ and eigen vectors are same as of A
$v_1 = t\begin{bmatrix}1\\1\end{bmatrix}$ $v_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

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Ans 4 Eqⁿ → $x^{k+1} \Rightarrow \frac{7.85 + 0.1y^{k} + 0.2z^{k}}{3}$

$y^{k+1} \Rightarrow \frac{-19.3 - 0.1x^{k+1} - 0.3z^{k}}{7}$

$z^{k+1} \Rightarrow \frac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$

we know $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

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continue 3 for $\lambda = 3$

$$\begin{bmatrix}-1 & -1\\-1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$-x - y = 0$ let $x = t$
$x = -y$ $y = -t$

so Eigen value $v_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

Now, find for $A^{-1}$
eigen values of $A^{-1}$ will be $\frac{1}{\lambda_1}, \frac{1}{\lambda_2} \rightarrow 1, \frac{1}{3}$

→ and eigen vector are same as of A
$v_1 = t\begin{bmatrix}1\\1\end{bmatrix}$ $v_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

Now, for $A + 4I$
→ eigen values for $A + 4I$ will be $\lambda_1 + 4, \lambda_2 + 4$
⇒ 5, 7

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5 Define consistent or inconsistent system of equation:-

Ans

consistent (atleast one soln)

Dependent (infinite soln)

Independent (unique soln)

$$A = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_1$
$R_3 \to R_3 - 2R_2$
$R_4 \to R_4 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$
$\rho(A:B) = 2$
$n = 3$
$\rho(A) = \rho(A:B) \neq n$

consistent, but infinite soln.

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M2 Baltram

1) Translatⁿ to origin:

translate the image, so that it centre alligns with origin.

2) Rotatⁿ :- Apply rotatⁿ matrix

3) Translatⁿ Back :- translate it back with its original position by adding coordinate of centre.

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Q7) Determine whether set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case $S$ is not a basis determine the dimⁿ of basis of subspace spanned by $S$.

Ans ~~of~~ $a(1,2,3) + b(3,1,0) + c(-2,1,3) = (0,0,0)$

$$a + 3b - 2c = 0$$
$$2a + b + c = 0$$
$$3a + 3c = 0$$
$$c \rightarrow a \quad b = -9$$

only one solⁿ is possible is $a = b = c = 0$ so linearly independent

Since dimⁿ of $V_3(R)$ is 3 and S also contain 3 vector and $S \rightarrow L$ then it spans $V_3(R)$ making it a basis for $V_3(R)$.

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Iteratⁿ - 2

$x(2) \rightarrow \frac{1}{3}(23 + 6(-10) - 36/7)$

$= 10.62$

$y(2) : (-15 + 4(10.62) + 18/7)$

$= 30.04$

$z(3) = \frac{1}{7}(16 - 10.62 + 3(30.4))$

$= 13.8$

Iteratⁿ - 3

$x(3) = (23 + 6(30.4) - 2(13.8))/3$

$= 59.27$

$y(3) \Rightarrow (-15 + 4(59.27) + 13.8) \Rightarrow 235.25$

$z(3) \Rightarrow (16 - 59.27 + 3(235.88))/7 = 67.4$

So, after three iteratⁿ $x(3) \Rightarrow 59.27$, $y(3) \Rightarrow 235.88$, $z(3) \Rightarrow 67.4$

Q.9 Affine transformation Rotation

Ans Suppose we have a 2-D image represented as grid or pixels. we can use AT matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ rotation of image by θ to rotate it around centre.

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(6) Determine whether fxn $T: P_2 \Rightarrow P_2$ is linear transformation or not:-

Ans:- $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

1) Additive :-

$T(u+v) \Rightarrow T(u)+T(v)$

$u = a_1+b_1x+c_1x^2$

$v = a_1+b_2x+C_2x^2$

$T(v+u) = T((a_1+a_2)+(b_1+b_2)x+(C_1+C_2)x^2))$

$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$\Rightarrow (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+$
$(c_2+1)x^2$

$\Rightarrow T(u)+T(v)$ hence proved

2) Homogenity:-

$T(Ku) = kT(u)$

$T(K(a+bx+cx^2))$

$T(Ka+Kbx+Kcx^2)$

$\Rightarrow (Ka+Kb+Kc+1)+(Ka+Kb+Kc+1)x+$
$(Ka+Kb+Kc+1)x^2$

$\Rightarrow K(a+1)+K(b+1)x+K(c+1)x^2$

$\Rightarrow KT(u)$

Hence proved.

Hence, it's a linear Transformatn.