

TC

No. of operations

$\propto$  input size

def hello (n) :

n=21

for i in range(n) :

print ("Hello Student")

$O(n)$

$O(1)$

n=1

21

~~n=1M~~

21

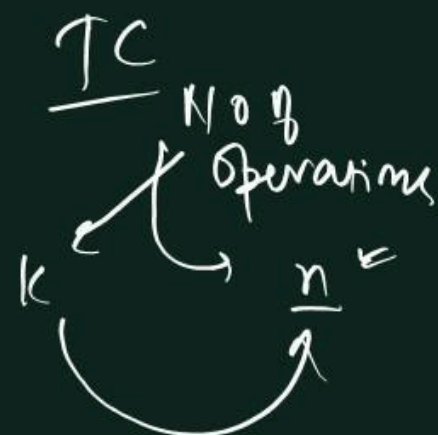
n=1B

21

(TC)

```
def fn(n)
    i = s = 1
    while (s <= n)
        i = i + 1
        s = s + i
    print('Hello')
```

$$s_{(k+1)} > n$$



$$\begin{aligned} s_1 &= 1 \\ s_2 &= 1 + 2 \\ s_3 &= 1 + 2 + 3 \\ &\vdots \\ s_k &= 1 + 2 + 3 + \dots + k \end{aligned}$$

$$\begin{aligned} (s_{k+1}) &= 1 + 2 + 3 + \dots + (k+1) \\ &= \frac{(k+1)(k+2)}{2} \end{aligned}$$

$$\hookrightarrow \frac{(k+1)(k+2)}{2} > n$$

$$(k+1)(k+2) \simeq n$$

$$\rightarrow \left( \cancel{k^2} + \underbrace{3k+2} \right) \simeq n$$

$$k^2 \simeq n$$

$$\Rightarrow \left( \cancel{k} \right) \simeq \sqrt{n}$$

$$T_C = O(\sqrt{n})$$

TC = ?



```
for i in range(n)
```

```
    for j in range(i+1)
```

```
        print('Hello')
```

Non op

1 + 2 + 3

$n(n+1)$

$\Rightarrow \frac{n^2 + n}{2}$

$O(n^2)$   
 $O(n \cdot \sqrt{n})$

$$-4/5 \rightarrow \textcircled{1}$$

$$\text{Divisor} \times \text{Quotient} + \text{Remainder} = \text{No}$$

$$\Rightarrow 5 \times -1 + R = -4$$

$$\Rightarrow -5 + R = -4$$

$$\Rightarrow \textcircled{R = 1}$$

$$\begin{array}{r} -4/5 \\ \overline{) 08} \\ \underline{-5} \\ 3 \end{array}$$

Quotient/Remainder?

$$-123/5$$

$$\begin{array}{r} -24 \cdot 6 \\ \hline \boxed{-25} \end{array}$$

$$\begin{aligned} -25 \times 5 + R &= -123 \\ \Rightarrow -125 + R &= -123 \\ \Rightarrow \boxed{R=2} \end{aligned}$$

$$\begin{array}{r} 123 \\ \hline \end{array} / 5$$

$$\boxed{3}$$



(TC)

↓ No of operation

↳ inputs

def fn(n):

$(n/2)$

$O(n \log n)$  ✓

$\overline{O(n)}$  ✓

$\overline{O(n^2)}$

→ for i in range( $n/2$ , n):

→ for j in range( ~~$n/2$~~ , n)

print('Hello')

$j = j \times 2$



```

def fn(n):
    i = n
    while (i > 1):
        i = i/3
        for j in range(i):
            print('Hello')

```

$\rightarrow O(n)$   
 $\rightarrow O(\log n)$

$T(n) = T(n/3) + n/3 + O(1)$

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \text{---}$$

$$\log_2 n \quad n/3 + n/9 + n/27 \quad \text{---}$$

$$= n \left( \frac{1}{3} + \frac{1}{9} \right) \text{---}$$

$$O(n \times \log_3 n)$$

$$1 + 3 + 9$$

①  $i = n$   
 while ( $i > 1$ ):  
 $i = i/3$

$$\frac{n}{3^k} \approx 1$$

$$\Rightarrow 3^k \approx n$$

$$\Rightarrow \log_3 3^k \approx \log_3 n$$

$$\Rightarrow k \approx \log_3 n$$

①  $i = n$

②  $i = n/3$

③  $i = n/3^2$

$(k+1)^{th} = \left(\frac{n}{3^k}\right)$

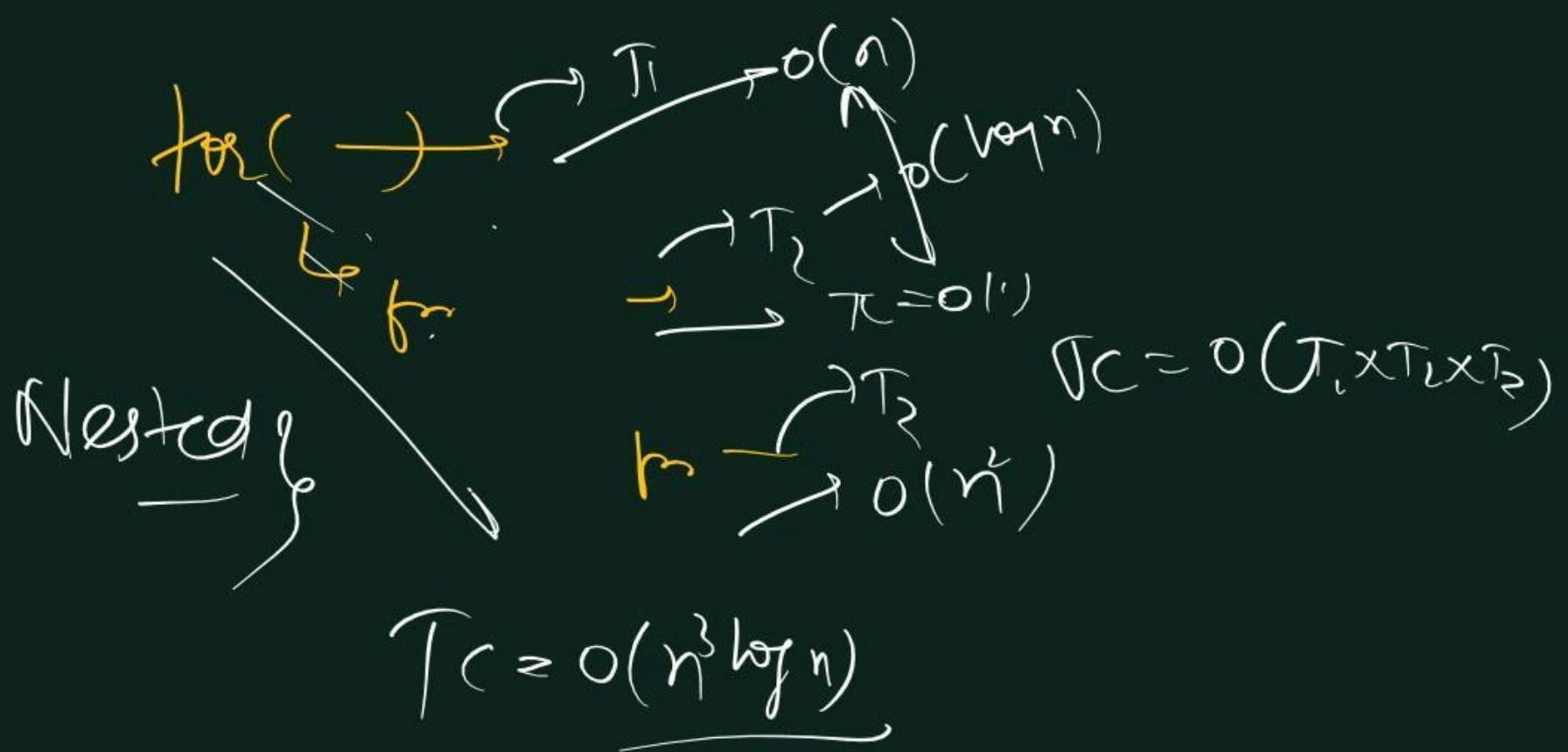
$i(k) = \frac{n}{3^{k-1}}$

$$\begin{array}{l}
 \textcircled{1} \quad 1 \\
 \textcircled{2} \quad 3 \\
 \textcircled{3} \quad 3^2 \\
 \vdots \\
 \textcircled{k} \quad 3^{k-1}
 \end{array}
 \left. \vphantom{\begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \vdots \\ \textcircled{k} \end{array}} \right\} (T+1)^{\frac{n}{3}} = \textcircled{3^k}$$

$$\begin{aligned}
 & \left. \vphantom{\begin{array}{l} 3^k \approx n \\ \log_3 \end{array}} \right\} 3^k \approx n \\
 & \Rightarrow \underline{\log_3} \\
 & \Rightarrow \log_3^k \approx \log_3 n \\
 & \Rightarrow \boxed{k \approx \log_3 n}
 \end{aligned}$$

$$\left\{ \begin{array}{l} \text{for } 1 \rightarrow TC_1 \rightarrow O(n) \\ \text{for } 2 \rightarrow TC_2 \rightarrow O(\log n) \\ \text{for } 3 \rightarrow TC_3 \rightarrow O(n^2) \end{array} \right\} \rightarrow \text{Max}(TC_1, TC_2, TC_3)$$

$\rightarrow \underline{O(n^2)}$





$(n/2) \rightarrow \log n$   
 $(n/2 + 1) \rightarrow \log n$   
 $j = n/2 + 1$ ?  
 $T = O(n \log n)$   
 $n/2$

```

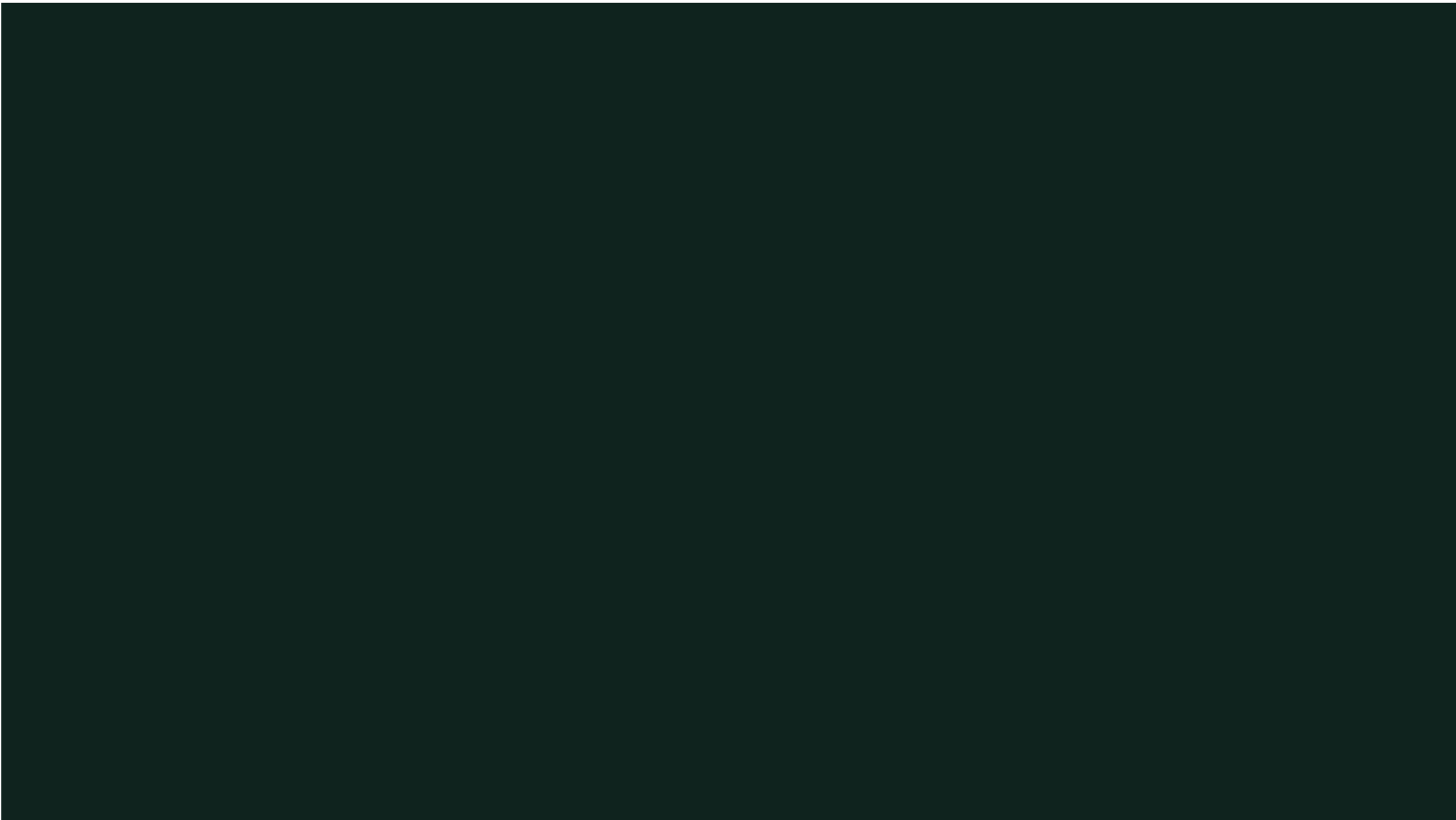
def hello(n):
    for i in range(n/2, n):
        j = 1
        while (j < n):
            j = j * 2
        for k in range(n/2, n):
            break;
            print("hello")

```

$(n/2) \rightarrow O(n)$   
 $j = 1$   
 $\text{while}(j < n) \rightarrow O(\log n)$   
 $j = j * 2$   
 $\rightarrow O(1)$   
 $\text{for } k \text{ in range}(n/2, n)$   
 $\text{break;}$   
 $\text{print("hello")}$



def



## Type of Tc

Best Case

Average case

Worst case

