4-qubit Grover Search Algorithm

Master Degree in Artificial Intelligence for Science and Technology

Course: Quantum Simulation

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Exam Project - Report

Quantum computing is a promising revolution in the field of computing. It should offer quantum advantages over its classical counterparts owing to the development of quantum hardware and exploitation of entanglement characteristics. The objective is to demonstrate speedup for problems that are difficult to implement in classical computers.
The two main algorithms that exploit the power of quantum computers are the Shor Algorithm, which faces the problem of factorisation in prime numbers of a given integer at a time which is the power of O(logN).
The second main algorithm is Grover's search algorithm, which is one of the most popular quantum computing algorithms that can search for an entity with high probability in an unstructured list using only O($\sqrt{N}$).
In this report we will focus on the Grover's Search Algorithm.

Grover's Search Algorithm was developed by Grover in 1996, and has the objective of searching for an object in an unstructured data array of N items.
The algorithm was divided into four main stages:

- Initialization: All the qubits are set in superpositions, all the states have an amplitude equal to $\frac{1}{\sqrt{N}}$
- Oracle: the oracle is like a black box, takes an input states and inverts the selected coefficient of the target base state. The oracle works by performing a phase flip on the marked state. In case of the 3-qubit and of the 4-qubit Grover the implementation of the oracle is done by using an Hadamard Gate on the last qubit, followed by a multi control X. A better visualization of the entire circuit will be provided successively.

- Amplification: The amplification is applied to the state of our interest, this corresponds to a HZH gate.
- Measurement: In the end we apply a measurement on all the qubits.

The last element we need to implement the circuit is the number of iterations we need to perform to maximise the probability of finding the right state.
For a dataset of dimension $N = 2^n$, where n is the number of qubits, the optimal number of iterations, t, is given by:

$$t = \frac{\pi\sqrt{N}}{4} \tag{1}$$

Then, for 4-qubits, the optimal number of iterations is equal to 3.14, whereas for 3-qubit the optimal number is equal to 2.
In the result part we will study the performance of the algorithm on both a simulator and on a real quantum computer.

The second part of this project was related to the application of a measurement-error mitigation technique. The objective is to reduce measurement errors by applying measurement error mitigation to the Grover Algorithm itself.
The algorithm we applied consists of two steps:
First, to calibrate the measurement errors, we built a full calibration matrix, M.
The second step involves using a filter to reduce the error on the measurement computing the inverse of the calibration matrix M^{-1}.

Results

First, the Grover Algorithm was implemented using three qubits and two iterations.
The Grover algorithm was implemented by adding Control Z to the Oracle, which allowed us to search for state 011 instead of state 111.
A visual representation of the circuit is shown in Fig. 1.

We ran the circuit first on the simulator using the IBMQ portal [1]. The circuit was tested on a QASM Simulator in QISKIT, where QASM represents the Quantum Assembly Language Simulator. The simulator was run for 1000 shots, and the results were obtained in the form of a count histogram, where each bin corresponded to a different state.
The results in 2 shows that the simulator is able to correctly identify the state we are searching for the 94.7% of the time.

[1] quantum-computing.ibm.com/

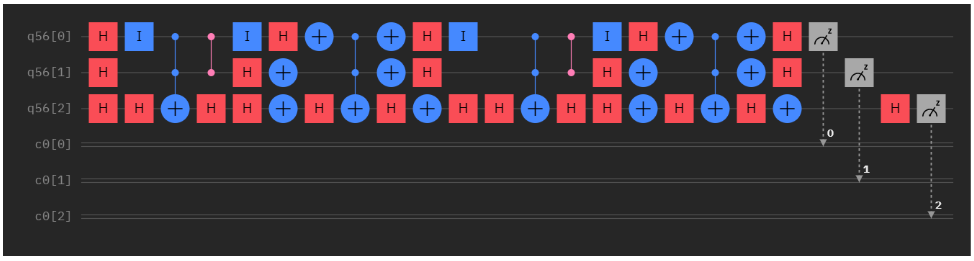

Figure 1: Circuit for the Grover Algorithm on 3 qubit

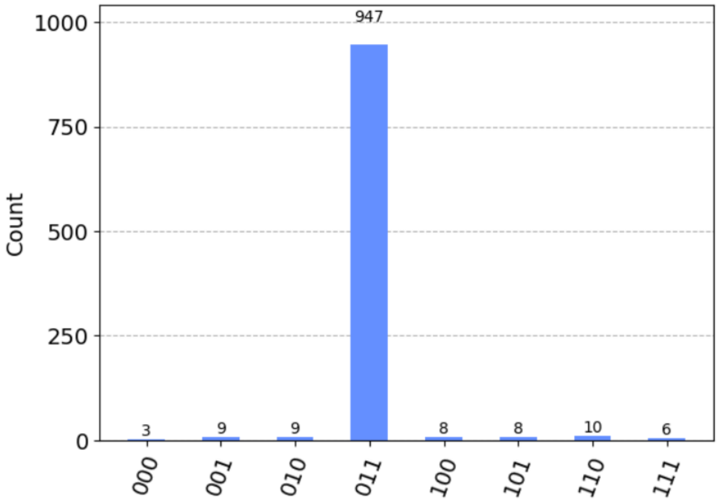

Figure 2: Counts for the 3 qubit on a simulator

The second test was performed using a real quantum computer, instead of a simulator. In particular, given the free resources available on the IBMQ, I tried two different architectures: Lima and Nairobi.
In this case, the output is calculated using the Sampler function of QISKIT, which returns a quasi-probability distribution, as can be seen in Fig 3. The test was performed using 10000

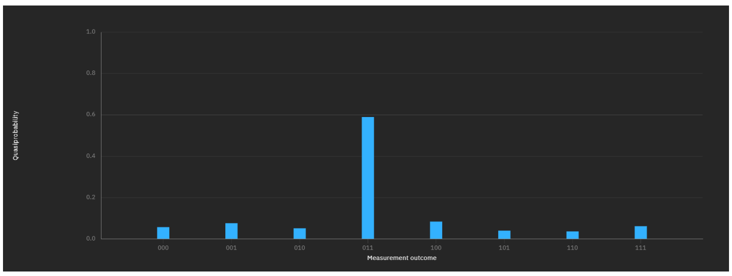

Figure 3: Quasiprobability distribution on LIMA for 3 qubit

shots and the highest probability, corresponding to state 011 (the one we are searching for), is equal to 0.59.
Studying the same circuit with the same conditions on Nairobi I obtained the same results in the order of ± 0.01 in terms of probability, for this reason I will not upload the corresponding plot. Is worth noticing, however, that the queue time to access the Nairobi Quantum Computer was of three days, much more respect to LIMA that required just a couple of hours.

The second part is related to the implementation of the Grover Algorithm on a 4-qubit circuit.
The structure of the resulting circuit is shown (Fig. 4). In this case, the focus was on founding

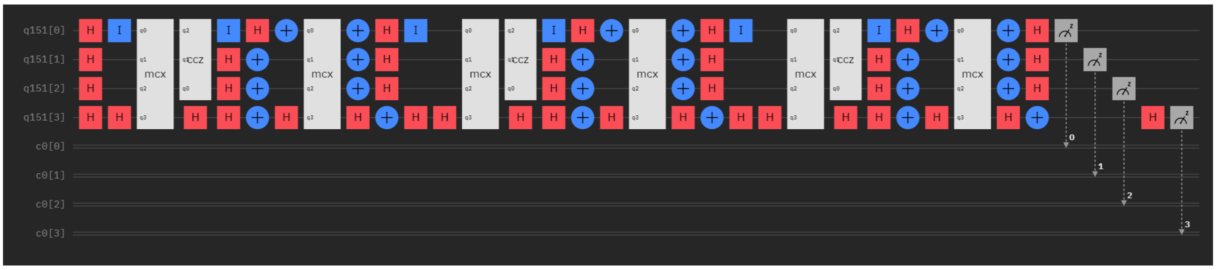

Figure 4: Circuit for the Grover Algorithm on 4 qubit

state 0111. The choice of the state is arbitrary.
For the case of three qubits, I tried to run the circuit first on a simulator, finding a total count for the right state equal to 951 over 1000 shots, as shown in Fig. 5. The results of the simulator

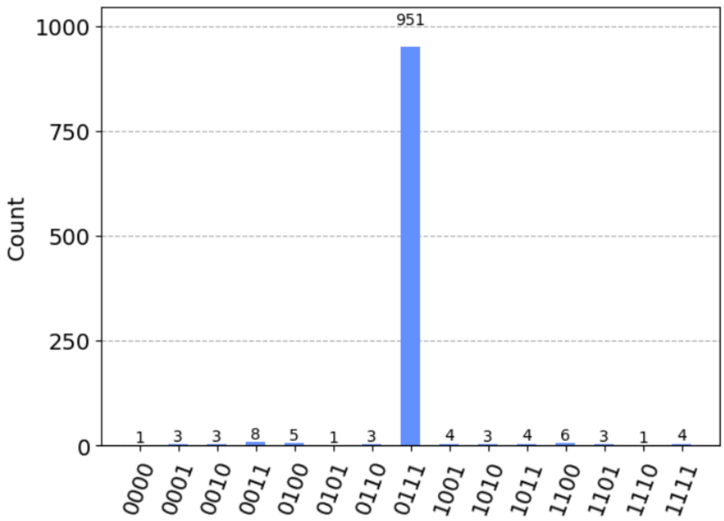

Figure 5: Counts for the 4 qubit on a simulator

are promising; they show high performance for the algorithm, even with a 4 qubit circuit.

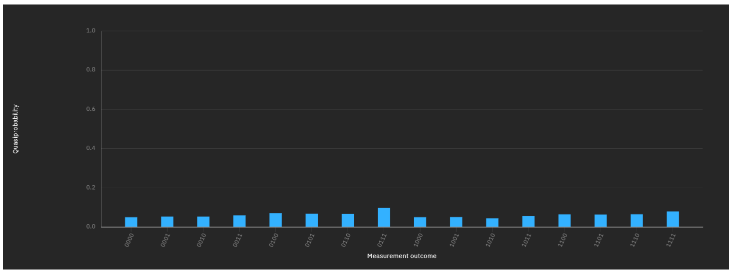

Figure 6: Quasiprobability distribution on LIMA for 4 qubit

Therefore, I tested the circuit on the quantum computer LIMA and found the distribution in Fig. 6.

We can see that in this case, the quasi-probability is much worse than that in the case with three qubits. Still the highest probability is found in the state that we are searching but with a quasiprobability of 0.098.

At this point, the task was changed. The new objective is to perform measurement error mitigation to improve the results found in the case of the 4 qubit implementation.
First, the calibration matrix for the LIMA was computed. Depending on the time of the test, the result could change slightly because the calibration was performed repeatedly at different times of the day. I tested different scenarios, but the difference was so small that it was not significant for my task.
The resulting calibration matrix is in Fig: 7 From the calibration matrix, it is also possible

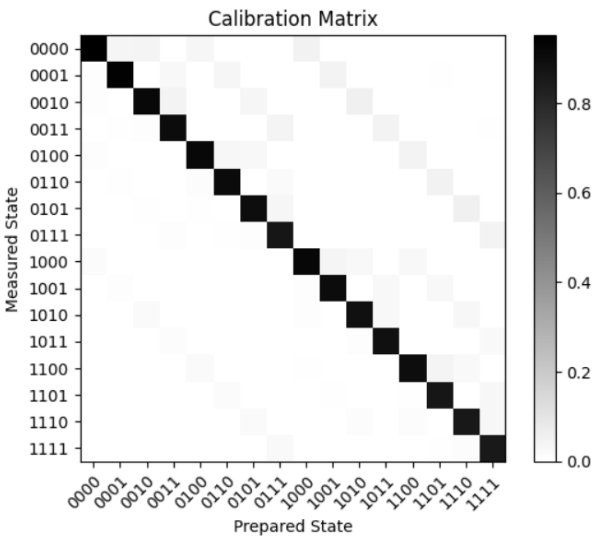

Figure 7: Calibration Matrix on 4 qubits

to obtain the average read-out fidelity, which is a measure that quantifies the probability of obtaining the correct measurement outcome when performing a measurement on a quantum state. In this case, the result was equal to 0.9.
Once I obtained the calibration matrix, I applied this matrix as a filter to my data, obtaining the counts after mitigation for each state. The histogram was computed using 10000 shots.
Summarizing the results:

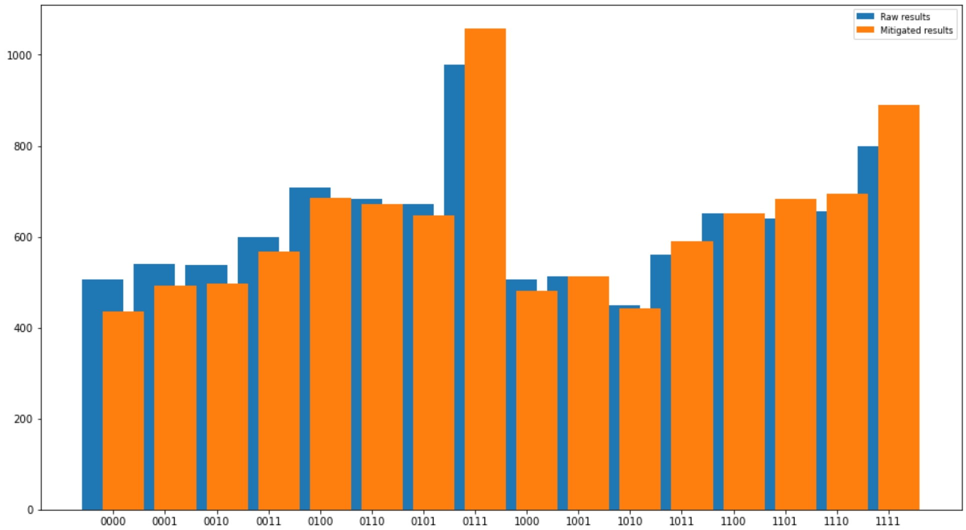

Figure 8: Mitigated and Raw counts

N° of Qubits	State	Acc. w/out Mitigation	Acc. w/ Mitigation	Improvement (%)
3	011	0.59	0.64	8.5
4	0111	0.098	0.11	21

Table 1: Accuracy assessment for the Grover Algorithm

Conclusions

This report applied a measurement error mitigation model to the Grover quantum-search algorithm. The algorithm is implemented for up to 4-qubits with and without measurement error mitigation. As the number of qubits rises from three to four, the circuit complexity increases, leading to more measurement errors when measuring. The accuracy of the Grover's algorithm is significantly improved when it is implemented with a measurement error mitigation technique, compared to Grover's algorithm implementations with measurement errors.

The implementation is quite extensive and nearly reaches the circuit capacity of QISKIT. As the number of circuits rises, both the execution time and gate errors increase. These two variables likely have an impact on the algorithm's accuracy, which explains why patterns are not easily discernible from the data. Since the simulator does not experience these issues and its results are consistent with mathematical theory, it further reinforces the notion that decoherence and gate errors are what make the LIMA results so unpredictable. The comparison between the LIMA simulation results and the execution results indicates that the current hardware is not yet adequate for circuits with the complexity needed for a 4-qubit Grover implementation.