Deep learning enhanced quantitative trading strategies

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July 11, 2023

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Outline

An Overview of Research at the Oxford-Man Institute

Classical Time-Series Momentum Strategies

Deep Momentum Networks

Deep Momentum Networks with Changepoint Detection

Momentum Transformer

An Overview of Research at the Oxford-Man Institute

The Oxford-Man Institute of Quantitative Finance

- A unique collaboration between Oxford University and Man Group plc
 - Opened Summer 2007
 - Focus: Machine Learning in quantitative finance & alternative investments
 - · Vision to become world leading research centre
- Man Research Laboratory
 - Embedded commercial research laboratory
 - · Undertakes Man AHL's research
 - Opportunity for collaboration and academic publication







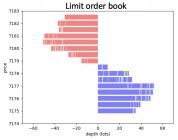






Research Topics: Modelling High-Frequency Data

- Institute intensely engaged in:
 - Modelling of limit order book data
 - Deep learning models for forecasting from HFT observations
 - Key contributions such as <u>DeepLOB</u> have been adopted across industry in market making and execution



Zhao Zhang (5): Stefan Zohren (5): Stephen Roberts (5) All Authors

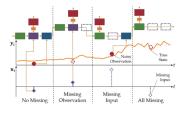
Technology

Man Group-Oxford Quants Say Their Al

Can Predict Stock Moves

Research Topics: Deep Learning for Time Series

- We have done extensive research on deep learning for time-series modelling with applications in finance and beyond
- Members of the group were involved in one of the first transformer based architectures for time-series, the temporal fusion transformer

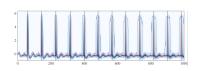


Review articles

Time-series forecasting with deep learning: a survey

Bryan Lim ☑ and Stefan Zohren

Published: 15 February 2021 https://doi.org/10.1098/rsta.2020.0209



Research Topics: Deep Learning for Quant Trading

- A major focus has been on deep learning for quant trading, including work on:
 - Portfolio optimisation
 - Reinforcement learning for trading
 - Autoencoder based risk/fragility measures
 - Enhancing traditional quant trading strategies (e.g. time-series momentum and cross-sectional momentum strategies)

Detecting changes in asset comovement using the

autoencoder reconstruction ratio

ARR aims to anticipate volatility patterns to provide signals for risk management and trading

Deep Learning for Portfolio Optimization

Zihao Zhang, Stefan Zohren and Stephen Roberts
The Journal of Financial Data Science Fall 2020, 2 (4) 8-20: DOI: https://doi.org/10.3905/fds.2020.1.042

financial data science

Building Cross-Sectional Systematic Strategies by Learning to Rank

Daniel Poh, Bryan Lim, Stefan Zohren and Stephen Roberts

The Journal of Financial Data Science Spring 2021, 3 (2) 70-86; DOI: https://doi.org/10.3905/jfds.2021.1.060



Research Topics: Networks and NLP in Finance

- Institute engages in research on:
 - · Financial news networks
 - Sentiment analysis and propagation
 - Virality in Social Media (Reddit, Twitter)
 - Financial word embeddings
- Organised some key workshops in field





Classical Time-Series Momentum Strategies

Momentum Strategies

- ▶ Time-series Momentum (TSMOM) (*Moskowitz et al.* [1]) is derived from the philosophy that strong price trends have a tendency to persist.
- ▶ Often known as 'follow the winner' because it is assumed that winners will continue to be winners in the subsequent period.
- ▶ TSMOM is a univariate approach as opposed to cross-sectional (*Jegadeesh et al.* [2]) momentum strategies, which trade assets against each other and select a portfolio based on relative ranking.
- Strategies involve 1) estimation of a trend, and 2) sizing positions accordingly.
- ▶ Momentum strategies are an important part of alternative investments and are at the heart of commodity trading advisors (CTAs).

Classical Strategies

- ▶ Volatility scaling has been proven to play a crucial role in the positive performance of TSMOM strategies (*Kim et al.* [3]).
- Where $X_t^{(i)}$ is position size of the *i*-th asset, *N* the number of assets in our portfolio, $\sigma_t^{(i)}$ the ex-ante volatility estimate and $\sigma_{\rm tgt}$ the target volatilty,

$$R_{t+1}^{\text{TSMOM}} = \frac{1}{N} \sum_{i=1}^{N} R_{t+1}^{(i)}, \quad R_{t+1}^{(i)} = X_t^{(i)} \frac{\sigma_{\text{tgt}}}{\sigma_t^{(i)}} r_{t+1}^{(i)}.$$
 (1)

- ▶ *Moskowitz et al.* [1], selects position as $X_t^{(i)} = \operatorname{sgn}(r_{t-252,t})$, where we are using the volatility scaling framework and $r_{t-252,t}$ is annual return.
- ▶ Moving Average Convergence Divergence (*MACD*) is a volatility normalised trend-following momentum indicator that describes the relationship between two moving averages of a security's price, functioning as a trigger for buy and sell signals (see *Baz et al.* [4]).

Deep Momentum Networks

Deep Momentum Networks

- Previous Deep learning approaches either only learnt the trend or alternatively performed classification, taking a maximum long or short position.
- ▶ In our first work with B. Lim and S. Roberts, we proposed a framework termed as *Deep Momentum Networks* (DMNs) which resulted in significantly better risk-adusted-returns.
- ▶ Rather than estimating the trend and then using a rule based approach to size positions, DMNs learn the trend in a data-driven manner and directly output position sizes.
- A squashing function $tanh(\cdot)$ directly outputs positions $X_t^{(i)} \in (-1, 1)$
- ▶ DMNs also benefit from the volatility scaling framework.
- ▶ Inputs are normalised returns at different timescales and different MACD indicators.



Deep Momentum Networks

- We found that the LSTM, a type of Recurrent Neural Network used for sequence modelling, produced the best results.
- Since we want to maximise risk-adjusted-returns, DMNs use a Sharpe Ratio Loss function

$$\mathcal{L}_{\text{sharpe}}(\boldsymbol{\theta}) = -\frac{\sqrt{252} \, \mathbb{E}_{\Omega} \left[R_t^{(i)} \right]}{\sqrt{\text{Var}_{\Omega} \left[R_t^{(i)} \right]}}.$$
(2)

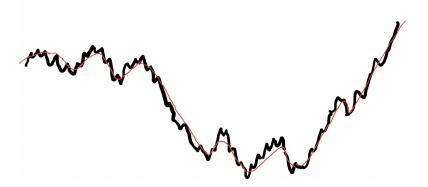
next day returns, with vol. scaling Sharpe loss function tanh tanh dense dense LSTM LSTM encoder encoder inputs, τ steps

Figure: LSTM Deep Momentum Network architecture



Interpreting the results of DMNs

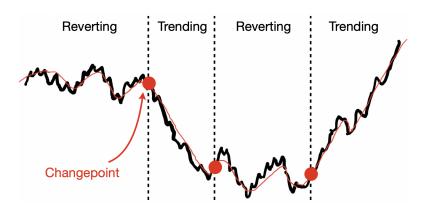
- ► DMN's strong performance attributed to learning simultaneously to exploit slow momentum and fast reversion
- ► Following small reversions in regimes of strong trend can lead to larger transaction costs



Deep Momentum Networks with Changepoint Detection

Momentum Turning Points and Changepoint Detection

- ► Ideally identify regimes trending and reverting market then learn to switch to exploit each state.
- Can be done with changepoint detection as proposed in earlier work with K. Woods and S. Roberts



Motivation from Momentum Turning Points

- ▶ Immediately after momentum turning points, where a trend reverses from an uptrend (downtrend) to a downtrend (uptrend), time-series momentum (TSMOM) strategies are prone to making bad bets.
- We require an approach which is a balancing act between being quick enough to respond to turning points, but not over-reacting to noise.
- ▶ *Garg et al.* [5] proposed an *Intermediate* stategy where a slow momentum signal based on a long lookback window, such as one year, is blended with a fast momentum signal based on a short lookback window, such as one month.

$$X_t = (1 - w)\operatorname{sgn}(r_{t-252,t}) + w\operatorname{sgn}(r_{t-21,t}). \tag{3}$$

► *MACD* can often produce false positives and signal a possible reversal without one actually happening.

Changepoint Detection

- ► Changepoint detection (CPD) is a field which involves the identification of abrupt changes in sequential data.
- ➤ To enable us to respond to CPD in real time, we require an 'online' algorithm, which processes each data point as it becomes available.
- ▶ First introduced by *Adams et al.* [6], Bayesian approaches to online CPD, which naturally accommodate to noisy, uncertain and incomplete time-series data, have proven to be very successful.
- ▶ We focus on approaches using Gaussian Processes (GPs) (Williams et al. [7]) a Bayesian non-parametric model which has a proven track record for time-series forecasting, is principled and is robust to noisy inputs.

Changepoint Detection with Gaussian Processes

For daily return $\hat{r}_t^{(i)}$, normalised over some look-back window (we'll revisit this), we define the GP as a distribution over functions,

$$\hat{r}_t^{(i)} = f(t) + \epsilon_t, f \sim \mathcal{GP}(0, k_{\xi}), \epsilon_t \sim \mathcal{N}(0, \sigma_n^2), \tag{4}$$

where ϵ is an additive noise process and \mathcal{GP} is specified by a covariance function $k_{\xi}(\cdot,\cdot)$, which is in turn parameterised by a set of hyperparameters ξ . Noise variance σ_n , helps to deal with noisy outputs which are uncorrelated.

- ▶ The Matérn 3/2 kernel is a good choice of covariance function for noisy financial data, with kernel hyperparameters $\xi_M = (\lambda, \sigma_h, \sigma_n)$, with λ the input scale and σ_h the output scale.
- ► A changepoint can either be a drastic change in covariance, a sudden change in the input scale, or a sudden change in the output scale

Changepoint Kernel

▶ *Garnett et al.* introduced the **Region-switching kernel**, where it is assumed there is a drastic change, or changepoint, at $c \in \{t-l+1, t-l+2, \ldots, t-1\}$, after which all observations before c are completely uninformative about the observations after this point,

$$k_{\xi_{R}}(x, x') = \begin{cases} k_{\xi_{1}}(x, x') & x, x' < c \\ k_{\xi_{2}}(x, x') & x, x' \ge c \\ 0 & \text{otherwise.} \end{cases}$$
 (5)

- ► The lookback window (LBW) *l* for this approach needs to be prespecified and it is assumed to contain a single changepoint.
- A more flexible approach is the **Changepoint kernel**, where $c \in (t-l,t)$ is the changepoint location, s > 0 is the steepness parameter and $\sigma(x,x') = \sigma(x)\sigma(x')$, $\bar{\sigma}(x,x')(1-\sigma(x))(1-\sigma(x'))$,

$$k_{\xi_C}(x, x') = k_{\xi_1}(x, x')\sigma(x, x') + k_{\xi_2}(x, x')\bar{\sigma}(x, x').$$
 (6)

▶ We use Matérn 3/2 for the left and right kernels.



Changepoint Detection Module Outputs

- We consider the series $\{r_{t'}^{(i)}\}_{t'=t-l}^t$, with lookback horizon l from time t. For every CPD window, where $\mathcal{T} = \{t-l, t-l+1, \ldots, t\}$, we standardise our returns for consistency.
- ▶ For each time step, our changepoint detection module outputs,
 - 1. changepoint detection location $\gamma_t^{(i)} \in (0,1)$, indicating how far in the past the changepoint is, and,
 - 2. changepoint score $\nu_t^{(i)} \in (0,1)$, which measures the level of disequilibrium, measured by the reduction in negative log marginal likelihood achieved via the introduction of the changepoint kernel hyperparameters.

$$\nu_t^{(i)}(l) = 1 - \frac{1}{1 + e^{-(\text{nlmn}_{\xi_C} - \text{nlmn}_{\xi_M})}}, \quad \gamma_t^{(i)}(l) = \frac{c - (t - l)}{l}, \quad (7)$$

▶ Both values are normalised to help improve stability and performance of our LSTM module.



Changepoint Kernel

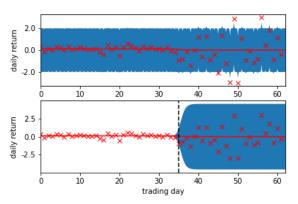


Figure: Plots of daily returns for S&P 500, composite ratio-adjusted continuous futures contract during the first quarter of 2020, where returns have been standardised. The top plot fits a GP, using the Matérn 3/2 kernel and the bottom using the Changepoint kernel specified in (6).

DMNs with Changepoint Detection Model

- In our second paper with K. Wood and S. Roberts, we introduce online CPD based on GPs into DMNs.
- Precisely, the input $u_t^{(i)}$ for each time-step of LSTM sequence consists of past returns, MACD signals, as well as changepoint location and severity scores:
 - 1. $\left\{ r_{t-t',t}^{(i)} / \sigma_t^{(i)} \sqrt{t'} \mid t' \in \{1, 21, 63, 126, 252\} \right\}$
 - 2. $\{MACD(i, t, S, L) | (S.L) \in \{(8, 24), (16, 28), (32, 96)\}\},\$
 - 3. $\nu_t^{(i)}(l)$ and $\gamma_t^{(i)}(l)$ for $l \in \{10, 21, 63, 126, 252\}$
- ▶ The LSTM is not complex enough to handle multiple CPD lookback-windows (LBW) and we optimise *l* as part of the hyperparameter tuning process.
- Later work also demonstrats that multiple LBWs (short and long) work well in conjunction with a variable selection network.



Data and Experimental Setting

- ▶ Portfolio consisting of 50 of the most liquid, ratio-adjusted continuous futures contracts over the period 1990–2020.
- ► Includes daily Commodities, FX, Fixed Income and Equities data, extracted from the Pinnacle Data Corp CLC database.
- ▶ We use an expanding window approach, where we start by using 1990–1995 for training/validation, then test out-of-sample on the period 1995–2000. With each successive iteration, we expand the training/validation window by an additional five years.
- ▶ We use a 90%/10% split for training/validation data, training on the Sharpe loss function via minibatch Stochastic Gradient Descent (SGD), using the validation set to tune the hyper-parameters and for early stopping.
- ► The outer optimisation loop tunes dropout rate, hidden layer size, minibatch size, learning rate, max gradient norm and CPD LBW length, with 50 iterations of random grid search.

Performance Results

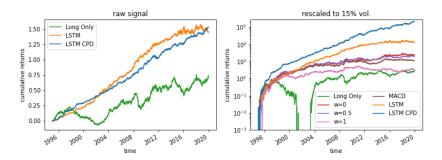


Figure: Benchmarking DMNs against *Intermediate* strategy $w \in \{0, 0.5, 1\}$, *Long Only* and *MACD*.

Performance Results

	Returns	Vol.	Sharpe	Downside Deviation	Sortino	MDD	Calmar	% of +ve Returns	Ave. P Ave. L
Reference									
Long Only	2.30%	5.22%	0.44	3.59%	0.64	3.12%	0.79	52.45%	0.975
MACD	2.65%	3.58%	0.77	2.57%	1.09	2.56%	0.95	53.34%	1.002
TSMOM									
$\overline{w=0}$	4.41%	4.80%	0.94	3.44%	1.32	3.22%	1.35	54.28%	0.990
w = 0.5	3.29%	3.78%	0.89	2.80%	1.23	2.70%	1.16	53.88%	0.998
w = 1	2.17%	4.71%	0.48	3.29%	0.68	3.24%	0.67	51.48%	1.026
LSTM	3.53%	2.52%	1.62	1.71%	2.46	1.72%	2.79	55.23%	1.075
LSTM w/ CPD									
10 day LBW	3.04%	1.57%	1.77	1.07%	2.74	1.09%	2.78	55.50%	1.096
21 day LBW	3.68%	1.81%	2.04	1.21%	3.07	1.08%	3.75	56.43%	1.095
63 day LBW	3.51%	1.72%	2.08	1.10%	3.27	1.06%	3.58	55.61%	1.140
126 day LBW	3.37%	2.28%	1.75	1.59%	2.66	1.52%	2.88	54.95%	1.117
252 day LBW	2.81%	2.24%	1.45	1.57%	2.19	1.54%	2.32	54.00%	1.101
LBW Optimised	3.64%	1.73%	2.16	1.17%	3.33	1.14%	3.50	56.22%	1.133

Figure: Strategy performance benchmark for raw signal output.

Slow Momentum with Fast Reversion

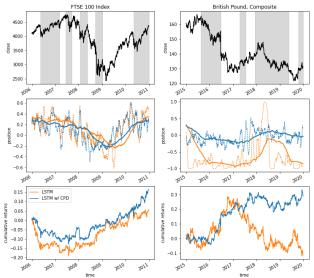


Figure: Slow momentum and fast reversion happening simultaneously.

Momentum Transformer

Transformers

- ▶ Based on the concept 'attention is all you need', doing away with convolutions and recurrent neural networks (RNNs).
- ► The attention-based architecture allows the network to focus on significant time steps in the past and longer-term patterns
- ▶ Have led to state-of-the-art performance in diverse fields, such as of natural language processing, computer vision, and speech processing (see *Lin et al.* [8]).
- ▶ Have recently have been harnessed for time-series modelling (*Li* et al. [9] *Lim* et al. [10], *Zhuo* et al. [11]).
- ► Naturally adapts to new market regimes, such as during the SARS-CoV-2 crisis.

Attention and Self-Attention

- From a time series modelling perspective, attention provides two key benefits.
 - 1. Networks with attention are able to directly attend to any significant events that occur.
 - 2. Attention-based networks can also learn regime-specific temporal dynamics, by using distinct attention weight patterns for each regime (*Lim. et. al.* [10]).

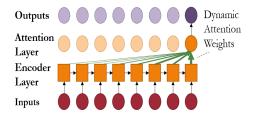


Figure: Attention model.

Base Architectures Tested in the Momentum Transformer

- ▶ **Transformer:** (*Vaswani et al.* [12]) consists of encoder and decoder each consisting of *l* identical layers of a (multi) self-attention mechanism, followed by a position-wise feed-forward network and a residual connection between these two components.
- ▶ **Decoder-Only Transformer:** (*Li et al.* [9]) only the decoder side.
- ➤ **Convolutional Transformer:** *Li et al.* [9] incorporates convolutional and log-sparse self-attention.
- ▶ **Informer Transformer:** *Zhuo et al.* [11] replaces the naive sparsity rule of the Conv. Transformer with a measurement based on the Kullback-Leibler divergence to distinguish essential queries, referred to as *ProbSparse* self-attention.
- ▶ Decoder-Only Temporal Fusion Transformer (TFT): an attention-LSTM hybrid which uses recurrent LSTM layers for local processing and interpretable self-attention layers for long-term dependencies. We consider the Decoder-Only version of the original TFT (*Lim et al.* [10]).

Momentum Transformer (Decoder-Only TFT)

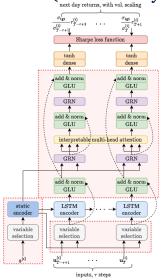


Figure: Decoder-Only TFT

Results

	Returns	Vol.	Sharpe	Downside Deviation	Sortino	MDD	Calmar	% of +ve Returns	Ave. I
Average 2015-2020									
Long-Only	1.73%	5.00%	0.37	3.59%	0.51	11.41%	0.15	51.97%	0.982
TSMOM	0.97%	4.38%	0.24	3.19%	0.33	8.25%	0.12	52.82%	0.93
LSTM	1.23%	1.85%	0.82	1.32%	1.19	3.55%	0.66	53.38%	1.00
Transformer	1.98%	1.29%	1.53	0.85%	2.32	1.07%	1.86	54.76%	1.07
Decoder-Only Trans.	1.37%	1.97%	0.72	1.37%	1.03	2.63%	0.60	52.76%	1.013
Conv. Transformer	1.85%	1.92%	0.98	1.30%	1.47	3.14%	0.77	52.93%	1.05
Informer	1.67%	1.09%	1.51	0.72%	2.30	1.17%	1.44	54.39%	1.089
Decoder-Only TFT	1.99%	1.23%	1.71	0.82%	2.61	1.17%	2.06	55.72%	1.07
Decoder-Only TFT CPD	2.06%	1.02%	2.00	0.66%	3.10	0.82%	2.53	55.74%	1.12
SARS-CoV-2									
Long-Only	-1.46%	6.73%	-0.19	5.64%	-0.22	12.32%	-0.12	57.28%	0.72
TSMOM	0.90%	4.73%	0.21	3.14%	0.32	4.17%	0.22	50.00%	1.04
LSTM	-4.15%	2.82%	-1.50	2.52%	-1.67	5.35%	-0.78	52.29%	0.64
Transformer	4.42%	1.28%	3.38	0.83%	5.55	0.84%	7.31	64.85%	1.06
Decoder-Only Trans.	8.02%	2.58%	3.01	1.42%	5.55	1.05%	8.56	58.83%	1.24
Conv. Transformer	3.13%	1.99%	1.81	1.40%	2.74	1.61%	3.17	57.48%	1.05
Informer	4.30%	1.60%	2.71	1.00%	4.45	1.07%	4.28	59.61%	1.13
Decoder-Only TFT	1.81%	1.75%	1.22	1.37%	1.74	2.14%	1.57	60.39%	0.83
Decoder-Only TFT CPD	3.39%	1.51%	2.47	1.03%	4.08	1.15%	5.92	59.90%	1.06

Figure: Strategy Performance Benchmark – Raw Signal Output

Results

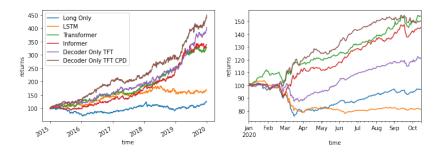


Figure: These plots benchmark our strategy performance for the 2015–2020 scenario (left) and the SARS-CoV-2 scenario (right). For each plot we start with \$100 and we re-scale returns to 15% volatility. Since we ran each experiment five times, we plot the repeat which resulted in the median Sharpe ratio, across the entire experiment.

Attention Patterns

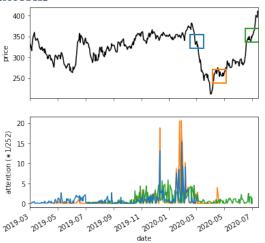


Figure: Lumber future price during SARS-CoV-2 crisis and the associated attention pattern when making a prediction at 1 March 2020 (blue), 21 April 2020 (orange), and 2 July 2020 (green).

Attention Patterns

- We observe significant structure in attention patterns.
- The attention on momentum turning points is pronounced, segmenting the time series into regimes.
- Our model focuses on previous time-steps which are in a similar regime.

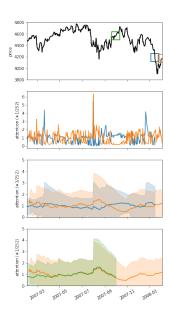


Figure: FTSE 100 future prior to 2008.



Variable Importance

- Our model intelligently blends different classical strategies at different points in time.
- ▶ We observe that the strategy changes with the addition of CPD, placing left emphasis on returns at timescales in between daily (shortest) and annual (longest).

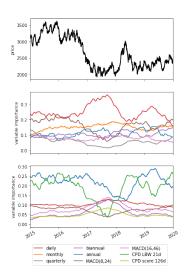


Figure: Variable importance for Cocoa future for Decoder-Only TFT (middle) and with CPD (bottom).



Transaction Cost Impact

C	0bps	0.5bps	1bps	1.5bps	2bps	2.5bps	3bps				
LSTM											
Indv. CM	0.12	0.09	0.05	0.01	-0.02	-0.06	-0.10				
Indv. EQ	0.37	0.32	0.27	0.22	0.16	0.11	0.06				
Indv. FI	0.09	-0.11	-0.32	-0.53	-0.74	-0.94	-1.15				
Indv. FX	0.11	0.01	-0.08	-0.18	-0.27	-0.37	-0.46				
Portfolio	0.82	0.51	0.20	-0.12	-0.43	-0.74	-1.05				
Transformer											
Indv. CM	0.27	0.23	0.19	0.15	0.11	0.08	0.04				
Indv. EQ	0.37	0.33	0.28	0.23	0.19	0.14	0.10				
Indv. FI	0.23	0.03	-0.16	-0.35	-0.55	-0.74	-0.93				
Indv. FX	-0.17	-0.24	-0.32	-0.39	-0.47	-0.55	-0.62				
Portfolio	1.53	1.26	0.99	0.72	0.45	0.18	-0.09				
Informer											
Indv. CM	0.28	0.24	0.19	0.15	0.10	0.06	0.01				
Indv. EQ	0.34	0.28	0.22	0.16	0.10	0.04	-0.02				
Indv. FI	0.08	-0.13	-0.35	-0.56	-0.78	-0.99	-1.20				
Indv. FX	-0.14	-0.24	-0.33	-0.43	-0.53	-0.62	-0.72				
Portfolio	1.51	1.17	0.83	0.49	0.15	-0.19	-0.53				
Decoder-O											
Indv. CM	0.44	0.40	0.35	0.31	0.26	0.22	0.17				
Indv. EQ	0.25	0.19	0.13	0.07	0.02	-0.04	-0.10				
Indv. FI	0.30	0.05	-0.20	-0.45	-0.69	-0.94	-1.18				
Indv. FX	0.28	0.18	0.08	-0.02	-0.12	-0.22	-0.32				
Portfolio	1.71	1.36	1.01	0.67	0.32	-0.03	-0.37				
Decoder-Only TFT CPD											
Indv. CM	0.55	0.50	0.45	0.40	0.35	0.30	0.25				
Indv. EQ	0.18	0.12	0.05	-0.01	-0.07	-0.14	-0.20				
Indv. FI	0.23	-0.03	-0.29	-0.55	-0.81	-1.07	-1.33				
Indv. FX	0.24	0.13	0.02	-0.09	-0.20	-0.30	-0.41				
Portfolio	2.00	1.61	1.22	0.83	0.44	0.04	-0.35				

Figure: Transaction cost impact on Sharpe over 2015–2020 for individual assets, averaged by asset class, and for diversified portfolio.

Conclusions

- Deep Momentum Networks are novel models which directly output trading signals which are optimised for Sharpe ratio
- ► The original deep Momentum Networks based on LSTMs perform well by exploiting a blend of momentum and mean reversion
- We introduce Changepoint detection to this model to more intelligently adapt to changes from trending to more reverting regimes
- We further improve the model by considering transformer based architectures
- ► The attention-based architectures, which we tested, are robust to significant events, such as during the SARS-CoV-2 market crash and tend to focus less on mean-reversion and more on longer term trends.

Thanks!

Papers:

Time Series Forecasting With Deep Learning: A Survey [2004.13408]

Deep Momentum Networks [1904.04912]

DMNs with Changepoints [2105.13727]

Momentum Transform [2112.08534]

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