Reinforcement Learning
in
Finance and Automated Trading

Rahul Savani

Alan Turing Institute / University of Liverpool

ML x Finance @ OxML, 10th July 2023

Reinforcement Learning
for
Trading in Limit Order Book Environments

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Reinforcement Learning for Trading in Limit Order Book Environments

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Parts based on joint work with:

Andrea Coletta, **Martin Herdegen**, **Joe Jerome**, John Fearnley, Andreas Koukorinis, **Leandro Sánchez-Betancourt**, Tom Spooner, Svitlana Vytrenko

Funding acknowledgements: EPSRC, J.P.Morgan AI Research

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Outline

Applications of Reinforcement Learning in Finance

- Portfolio Allocation / Optimization
- Hedging (and Option Pricing)
- Optimal Execution (and Smart Order Routing)
- Market Making

Excellent survey

Recent advances in reinforcement learning in finance
Ben Hambly, Renyuan Xu, Huining Yang
Mathematical Finance (2023) – 67 pages

Our focus – limit order books

- To train good policies with RL **lots of experience is needed** (**sample inefficiency**)
- **Limit order books** (LOBs) are a **widespread market mechanism** relevant for **data-rich high frequency finance**
- **Intro to LOBs**: First, we will look at the limit order books as a market mechanism in detail
- In this context, we will also explain the role and challenge faced by a market maker
- **Ways to model LOBs**: Then, we will then discuss different ways to create **LOB environments** for RL
- **Hands-on RL demo**: Finally, focusing on one of these ways, I will give an RL demo using `mbt_gym` (freely available)

Order driven, aka limit order book markets

- In **order-driven markets** (as opposed to quote-driven markets):

  all buyers and sellers display prices at which they wish to buy or sell a particular security, as well as the amounts of the security desired to be bought or sold
- **Buy** orders are called **bids**; **sell** orders are called **asks** or **offers**
- In the academic literature limit order books markets are sometimes called **continuous double auctions**
- **This market mechanism is extremely widespread**; it is used in, e.g., stock, derivatives, and sports betting markets

Limit and market orders

Two orders types:

market and **limit** orders, both specified by **direction** (buy/sell) & **volume**

Additionally a **limit order** has a **limit price**

Unmatched limit orders sit in the **limit order book**

Bids	Price	Asks
	101.50	5
	101.25	11
	101.00	12
	100.50	
	100.25	22
	100.00	
14	99.75	
30	99.50	
3	99.25	
11	99.00	

Best prices, midprice, and spread

Bids	Price	Asks	
	101.50	5	
	101.25	11	
	101.00	12	
	100.50		
	100.25	22	Best ask
	100.00		
14	99.75		Best bid
30	99.50		
3	99.25		
11	99.00		

The **spread** is the difference between the best ask and best bid:

$$100.25 - 99.75 = 0.5$$

The **midprice** is the average of the best ask and best bid:

$$0.5 \cdot (100.25 + 99.75) = 100$$

Market makers provide liquidity

Bids	Price	Asks	
	101.50	5	
	101.25	11	
	101.00	**12**	MM ask
	100.50		
	100.25	22	Best ask
	100.00		
14	99.75		MM bid
30	99.50		
3	99.25		
11	99.00		

A **market maker provides liquidity** by placing bids and asks

(some or all of the 12 and 14 units of volume in the example)

How do they make money?

	Bids	Price	Asks	
		101.50	5	
		101.25	11	
MM ask		101.00	**12**	
		100.50		
Best ask		100.25	22	
		100.00		
	14	99.75		MM bid
	30	99.50		
	3	99.25		
	11	99.00		

A **market maker** tries to repeatedly **“make the spread”**

How do they make money?

requires others to **"cross the spread"** in **both** directions, i.e.,

someone lifts MM bid; someone **else** lifts MM ask

Market order example

Bids	Price	Asks
	101.50	5
	101.25	11
	101.00	12
	100.50	
	100.25	22
	100.00	
14	99.75	
30	99.50	
3	99.25	
11	99.00	

Market order to buy 40

$$\frac{1}{40}(100.25 \times 22 + 101.00 \times 12 + 101.25 \times 6) = 100.625$$

Spread has widened

Bids	Price	Asks	
	101.50	5	
	101.25	5	
	101.00		last MM sale
	100.50		
	100.25		
	100.00		
14	99.75		old MM bid
30	99.50		
3	99.25		
11	99.00		

What happens next? The spread will narrow typically, but how?

The MM would like the asks to be **replenished**...

Midprice goes up

Bids	Price	Asks	
	101.50	5	
	101.25	5	
4	101.00		last MM sale
10	100.50		
	100.25		
7	100.00		
14	99.75		old MM bid
30	99.50		
3	99.25		
11	99.00		

...but instead **bids may fill the spread**

Toxic order flow and inventory risk

Bids	Price	Asks	
	101.50	5	
	101.25	5	
4	101.00		last MM sale
10	100.50		
	100.25		
7	100.00		
14	99.75		old MM bid
30	99.50		
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11	99.00		

Inventory risk: may amass undesired position in trending markets

Toxic order flow and inventory risk

Bids	Price	Asks	
	101.50	5	
	101.25	5	
4	101.00		last MM sale
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11	99.00		

Toxic order flow: **adverse selection** of
trading against better-informed counterparties

Developing an automated market making strategy

	Price	Asks
	101.50	5
	101.25	5
4	101.00	
10	100.50	
	100.25	
7	100.00	
14	99.75	
30	99.50	
3	99.25	
11	99.00	
Bids		

To effectively develop and test a market making strategy, one would like to be able to simulate **how the market reacts** to the actions of the market maker

Developing an automated market making strategy

Bids	Price	Asks
	101.50	5
	101.25	5
4	101.00	
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Reinforcement Learning from scratch through live trading is generally infeasible; some form of **simulated environment** is used

Developing an automated market making strategy

	Price	Asks
	101.50	5
	101.25	5
4	101.00	
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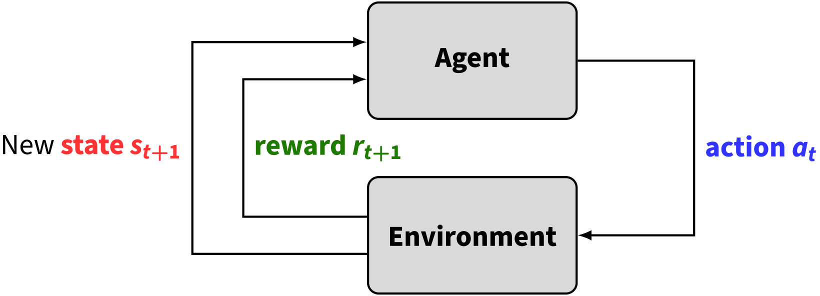
Bids		

Reinforcement Learning is able to effectively **exploit weaknesses** in the **environment** (simulator)

Reinforcement learning environment



How should an **agent** take **actions** in an **environment** so as to maximize some notion of **reward**?

LOB environments: Three main possiblities

1. Backtesting – market replay
2. Conditional generative models
3. Mathematical models
4. (Agent-based models)

Data-replay based LOB simulator (“backtesting”)

- **Replay historic** limit order book **data**
- Simulate a **queue at each level of the order book**
- Allow backtested agent to place orders in the book
- **Process transactions and cancellations** from both fixed data **and** the backtested agent
- Obviously difficulty: **the replayed data does not react to the actions of our backtested agent**

A second issue: cancellations and queue modelling

Market Making via Reinforcement Learning **AAMAS 2018**

Thomas Spooner, John Fearnley, Rahul Savani, Andreas Koukorinis

assumes that cancellations happen uniformly across queue

but, evidence shows they tend to happen near back of queue

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highly realistic simulator using LOBSTER data (but still no reactivity)

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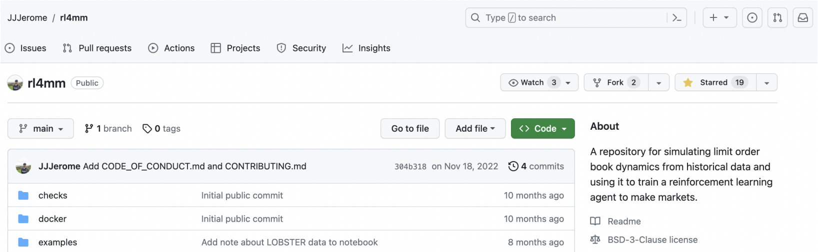
> Ignoring the reactivity issue, many papers that build market making agents do not simulate queues fully realistically

rl4mm repo (requires LOBSTER data)



`https://github.com/JJJerome/rl4mm`

Conditional Generative Model as the simulator

- Simulator based on **conditional generative model** that was trained on real order book data
- **Promises both realism and reactivity**

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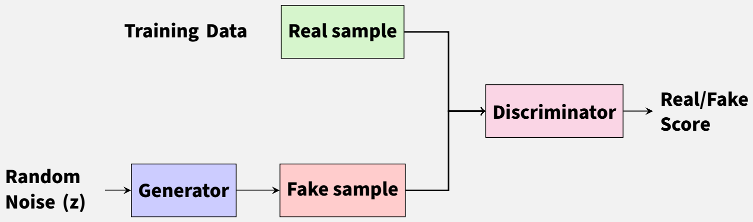
(unconditional) Generative Adversarial Network (GAN)



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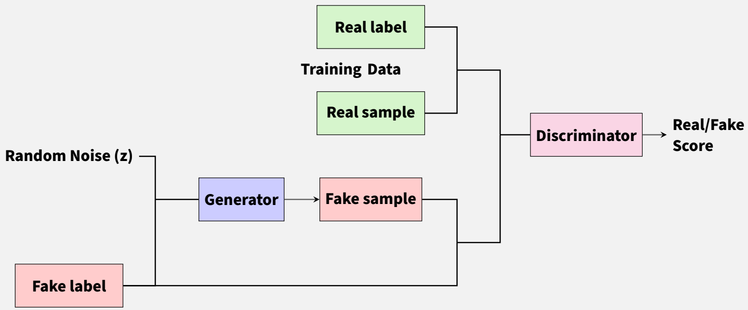
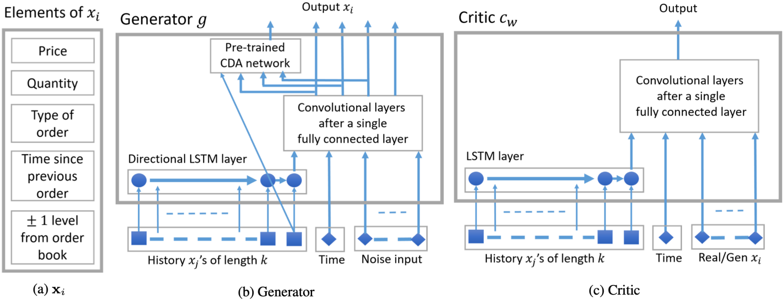
- **Promises both realism and reactivity**

Conditional Generative Adversarial Network (CGAN)



Example setup

Generating Realistic Stock Market Order Streams [Li et al, AAAI'20]



- Primarily tested by realism of outputs **in isolation**
- For building a trading agent using the CGAN we want: **interactive realism**

More work to be done

Conditional Generators for Limit Order Book Environments: Explainability, Challenges, and Robustness

Andrea Coletta, Joseph Jerome, Rahul Savani, Svitlana Vyetrenko

CoRR abs/2306.12806 (2023)

More work to be done

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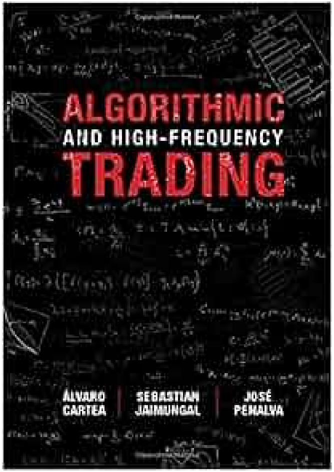
- While the CGAN we analysed had impressive **realism in isolation**
- We we able to construct hand-crafted and RL-based trading agents that **exploited its features and mechanism** to earn unrealistic profits
- That is, more work is needed on **interactive realism**
- This is an **important and exciting direction**

Algorithmic and High-Frequency Trading (2015)
Álvaro Cartea, Sebastian Jaimungal, José Penalva



Financial applications of Reinforcement Learning (RL)

Introduction to Limit Order Books

Types of Limit Order Book RL environment

- Market replay
- Conditional generative models
- Mathematical models

Mathematical models of limit order book trading

Tutorial on mbt-gym (model-based-trading gym)

A family of LOB trading models

The AS model – the first of many related models

High-frequency trading in a limit order book
Marco Avellaneda and Sasha Stoikov
Quantitative Finance 2008

Recurring model structure

- Controls: bid and ask quote **depths** $\delta^{\pm} = (\delta_t^{\pm})_{t \in [0,T]}$

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- Conditional upon arrivals, limit orders are filled according to a **fill probability model**

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- **Reward functions**

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- **Reward functions** (AS: $R_T = \exp\left(-\gamma \mathrm{PnL}_T\right)$)

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Some models covered by `mbt_gym`

Examples of models from the literature:

- Avellaneda and Stoikov [AS08]
- Market-making with limit orders [CJP15, Section 10.2]
- Market-making at the touch [CJP15, Section 10.2.2]
- Cartea, Jaimungal, and Ricci (CJR) [CJR14]
- Guéant, Lehalle, and Fernandez-Tapia [GLFT13] – section 5.2

Examples of extensions:

- Optimal execution with limit and market orders [CJP15, Section 8.4]
- Avellaneda and Stoikov with Hawkes' order flows
- [CJR14] with pure jump price process
- [CJR14] with exponential utility

... and many more!

Solving these models

- For many of these models **optimal solutions** are known
- Found typically via the **Hamilton-Jacobi-Bellman equations** (whose solution is the value function of the control problem)
- RL can go wrong in many ways
- Having these optimal solutions to hand allows us to

  compare the performance of RL in terms of both the reward (value) and the policy itself

Learning to make a market using PPO

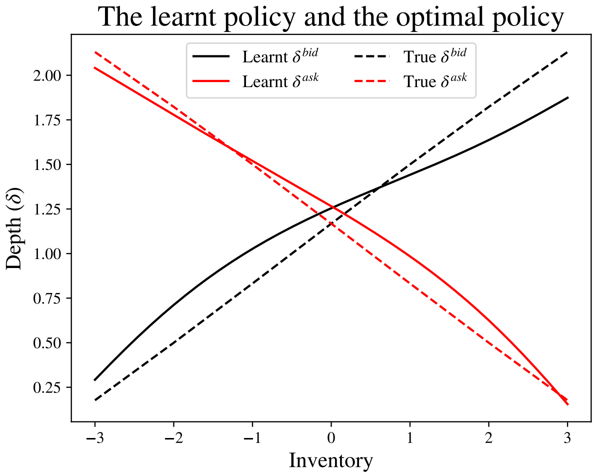


Figure: Learnt PPO agent versus `CarteaJaimungalAgent`.

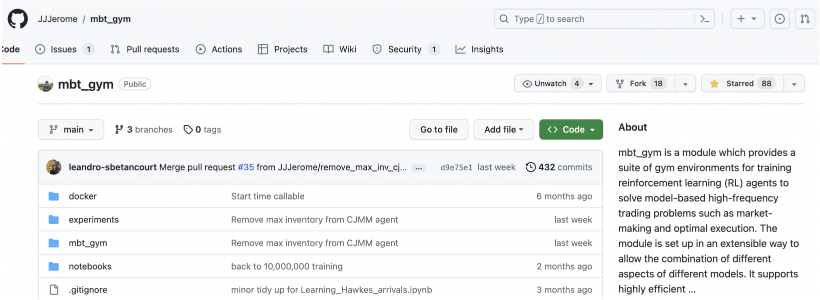
Ultimate vision

- Often model assumptions are driven by the **mathematical tractability** of finding optimal policies
- RL can be explored in contexts where optimal solutions are known (to make sure learning works well)
- Then **apply it settings where mathematical derivation is intractable**
- However, there are serious challenges on how one might use RL to effectively find (approximately) optimal solutions as a function of model parameters...

References

- Marco Avellaneda and Sasha Stoikov, *High-frequency trading in a limit order book*, Quantitative Finance **8** (2008), no. 3, 217–224.
- Álvaro Cartea, Sebastian Jaimungal, and José Penalva, *Algorithmic and High-Frequency Trading*, Cambridge University Press, 2015.
- Álvaro Cartea, Sebastian Jaimungal, and Jason Ricci, *Buy low, sell high: A high frequency trading perspective*, SIAM Journal on Financial Mathematics **5** (2014), no. 1, 415–444.
- Olivier Guéant, Charles-Albert Lehalle, and Joaquin Fernandez-Tapia, *Dealing with the Inventory Risk: A solution to the market making problem*, Mathematics and Financial Economics **7** (2013), no. 4, 477–507.

mbt_gym repo



https://github.com/JJerome/mbt_gym

Before we explore `mbt_gym`, any questions?

mbt_gym repo

https://github.com/JJJerome/mbt_gym

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For the **two notebooks** we will now go through go to branch: **oxml**