Belt and Rope Drives

1. Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power trans-mitted depends upon the following factors :

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and 8. Service conditions.

3. Types of Belt Drives

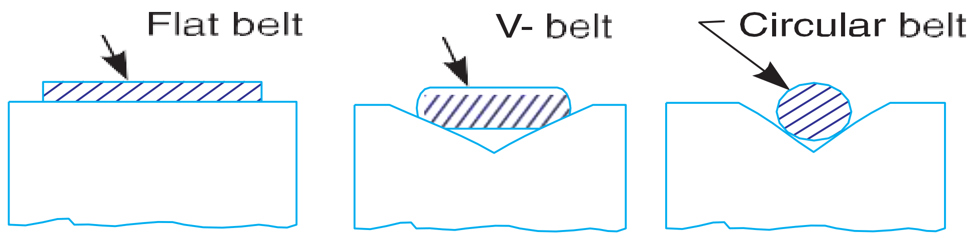
The belt drives are usually classified into the following three groups :

1. ***Light drives***. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.

2. ***Medium drives***. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. ***Heavy drives***. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

4. Types of Belts



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Fig.1 Types of belts.

Though there are many types of belts used these days, yet the following are important from the subject point of view :

1. ***Flat belt.*** The flat belt, as shown in Fig.1 (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.

2. ***V-belt.*** The V-belt, as shown in Fig.1 (*b*), is mostly used in the factories and work- shops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. ***Circular belt or rope***. The circular belt or rope, as shown in Fig.1 (*c*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives:

1. *Open belt drive*. The open belt drive, as shown in Fig. 2, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.* lower side *RQ*) and delivers it to the other side (*i.e.* upper side *L M*). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side***, as shown in Fig. 2

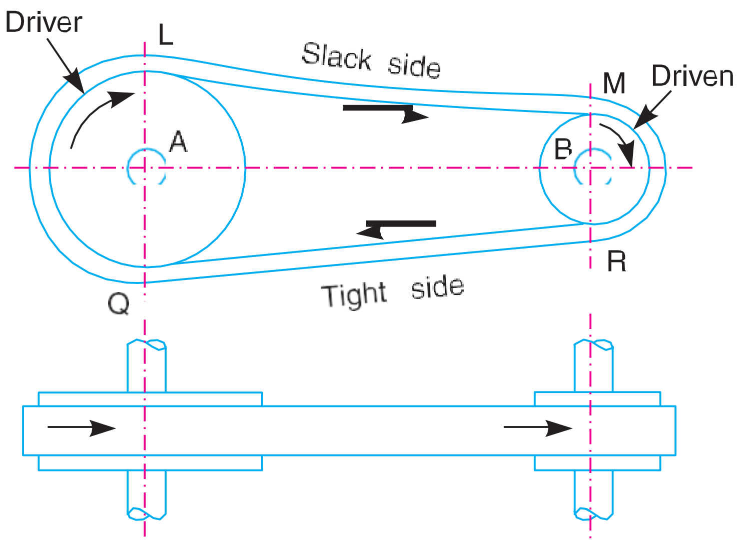


Fig. 2 Open belt drive.

2. ***Crossed or twist belt drive***. The crossed or twist belt drive, as shown in Fig.3, is used with shafts arranged parallel and rotating in the opposite directions.

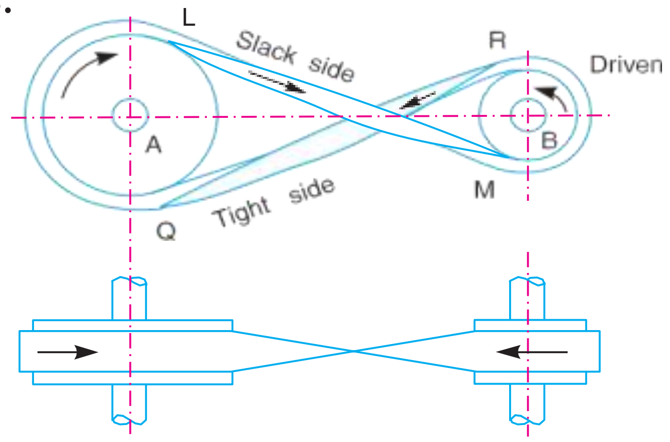
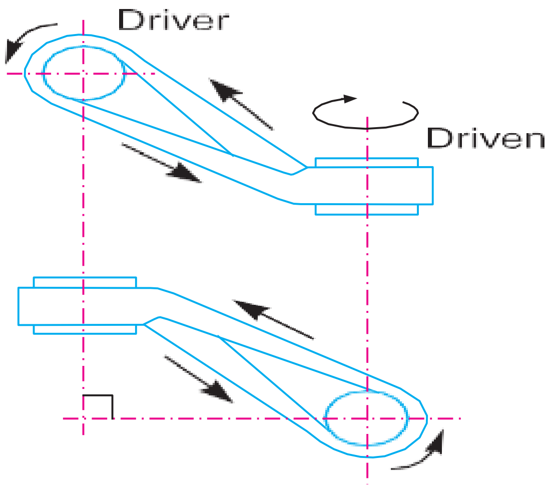


Fig. 3 Crossed or twist belt drive.

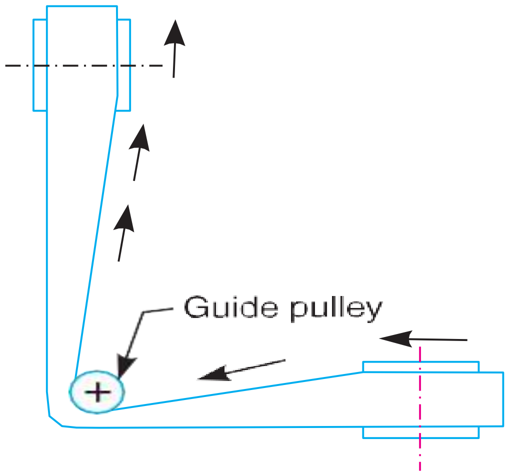
In this case, the driver pulls the belt from one side (*i.e. RQ*) and delivers it to the other side (*i.e. L M*). Thus the tension in the belt *RQ* will be more than that in the belt *L M*. The belt *RQ* (because of more tension) is known as ***tight side,*** whereas the belt *LM* (because of less tension) is known as ***slack side***, as shown in Fig.3

3. *Quarter turn belt drive.* The quarter turn belt drive also known as right angle belt drive, as shown in Fig. 4 (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, where *b* is the width of belt.

In case the pulleys cannot be arranged, as shown in Fig. 4 (*a*), or when the reversible motion is desired, then a ***quarter turn belt drive with guide pulley***, as shown in Fig. 4 (*b*), may be used.



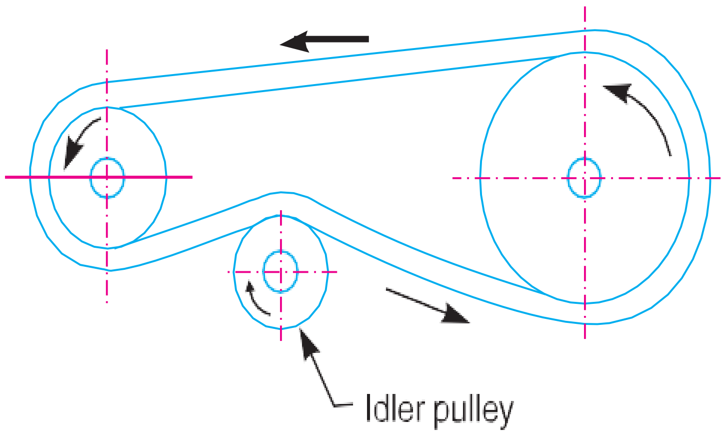
(*a*) Quarter turn belt drive.



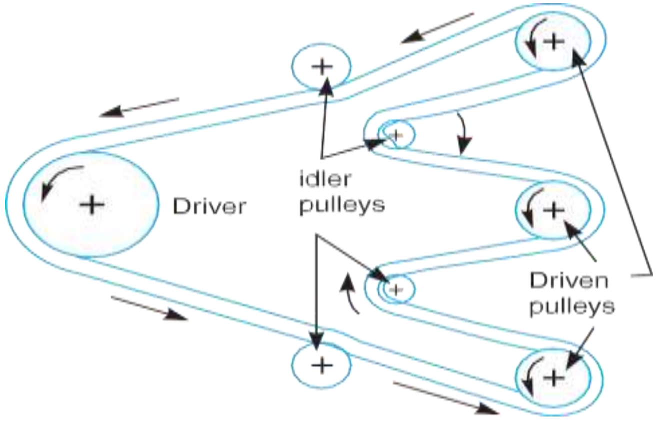
(*b*) Quarter turn belt drive with guide pulley.

Fig. 4

4. *Belt drive with idler pulleys*. A belt drive with an idler pulley, as shown in Fig. 5 (*a*), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.



(*a*) Belt drive with single idler pulley.



(*b*) Belt drive with many idler pulleys.

Fig. 5

When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. 5 (b), may be employed.

5. *Compound belt drive*. A compound belt drive, as shown in Fig. 11.7, is used when power is transmitted from one shaft to another through a number of pulleys.

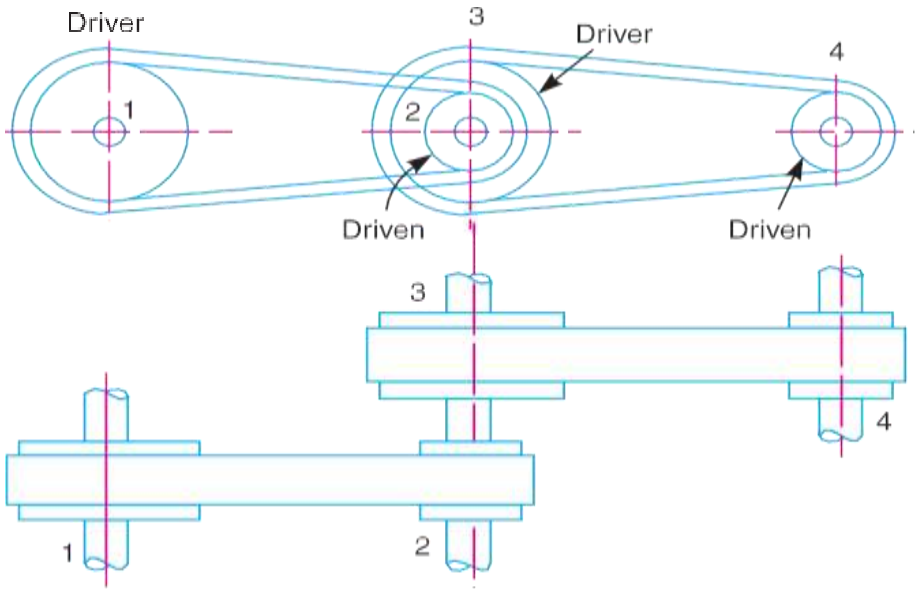


Fig. 6 Compound belt drive.

***6. Stepped or cone pulley drive*.** A stepped or cone pulley drive, as shown in Fig. 7, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

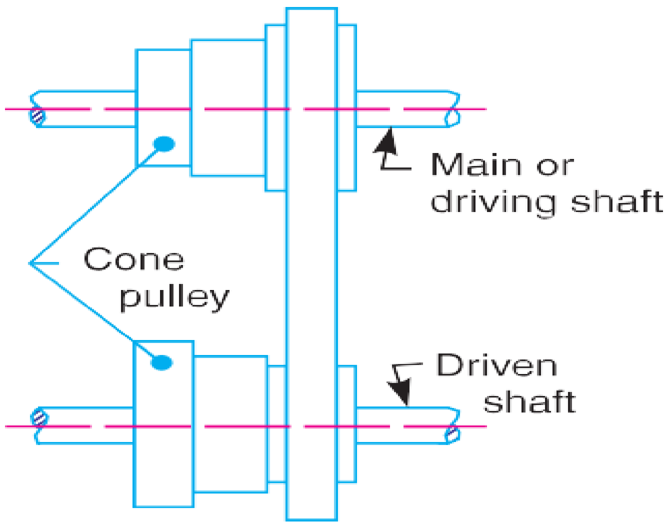


Fig. 7 Stepped or cone pulley drive.

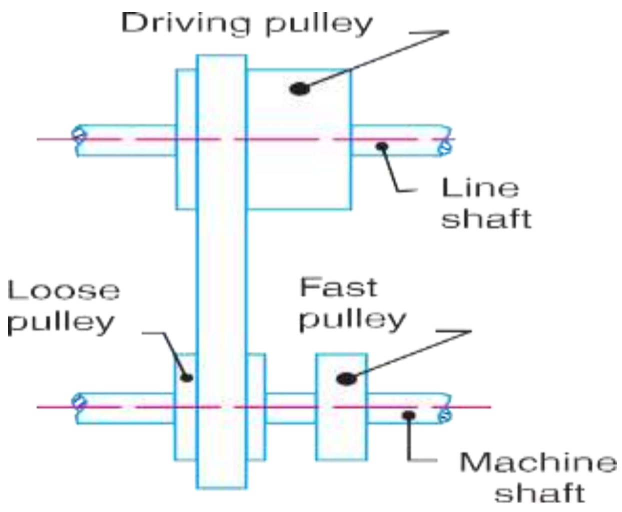


Fig. 8 Fast and loose pulley drive.

7. Fast and loose pulley drive. A fast and loose pulley drive, as shown in Fig. 8, is used when the driven or machine shaft is to be started or stopped when ever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called ***fast pulley*** and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

Velocity Ratio of Belt Drive

It is the **ratio between the velocities of the driver and the follower or driven.** It may be expressed, mathematically, as discussed below :

d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m., and

N_2 = Speed of the follower in r.p.m.

$\therefore$ Length of the belt that passes over the driver, in one minute

$$= \pi\, d_1 . N_1$$

Similarly, length of the belt that passes over the follower, in one minute

$$= \pi\, d_2 . N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\pi\, d_1 . N_1 = \pi\, d_2 . N_2$$

$\therefore$ Velocity ratio, $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

When the thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Note: The velocity ratio of a belt drive may also be obtained as discussed below :

We know that peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi\, d_1 . N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven or follower pulley,

$$v_2 = \frac{\pi\, d_2 . N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi\, d_1 . N_1}{60} = \frac{\pi\, d_2 . N_2}{60} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys as shown in Fig. 11.7. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.

Let d_1 = Diameter of the pulley 1,

N_1 = Speed of the pulley 1 in r.p.m.,

d_2, d_3, d_4, and N_2, N_3, N_4 = Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{...}(i)$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{...}(ii)$$

Multiplying equations (*i*) and (*ii*),

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

or $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \ldots(\because N_2 = N_3, \text{being keyed to the same shaft})$$

A little consideration will show, that if there are six pulleys, then

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

or $$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

Slip of Belt

we have discussed the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called ***slip of the belt*** and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let $s_1\%$ = Slip between the driver and the belt, and

$s_2\%$ = Slip between the belt and the follower.

$\therefore$ Velocity of the belt passing over the driver per second

$$v = \frac{\pi\, d_1 \,.\, N_1}{60} - \frac{\pi\, d_1 \,.\, N_1}{60} \times \frac{s_1}{100} = \frac{\pi\, d_1 \,.\, N_1}{60}\left(1 - \frac{s_1}{100}\right) \quad \text{...}(i)$$

and velocity of the belt passing over the follower per second,

$$\frac{\pi\, d_2 \,.\, N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi\, d_2\, N_2}{60} = \frac{\pi\, d_1\, N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \quad \text{...}\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

... (where $s = s_1 + s_2$, *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Example. *An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when* **1.** *there is no slip, and* **2.** *there is a slip of 2% at each drive.*

Solution. Given : $N_1 = 150$ r.p.m. ; $d_1 = 750$ mm ; $d_2 = 450$ mm ; $d_3 = 900$ mm ; $d_4 = 150$ mm

The arrangement of belt drive is shown in Fig. 11.10.

Let N_4 = Speed of the dynamo shaft .

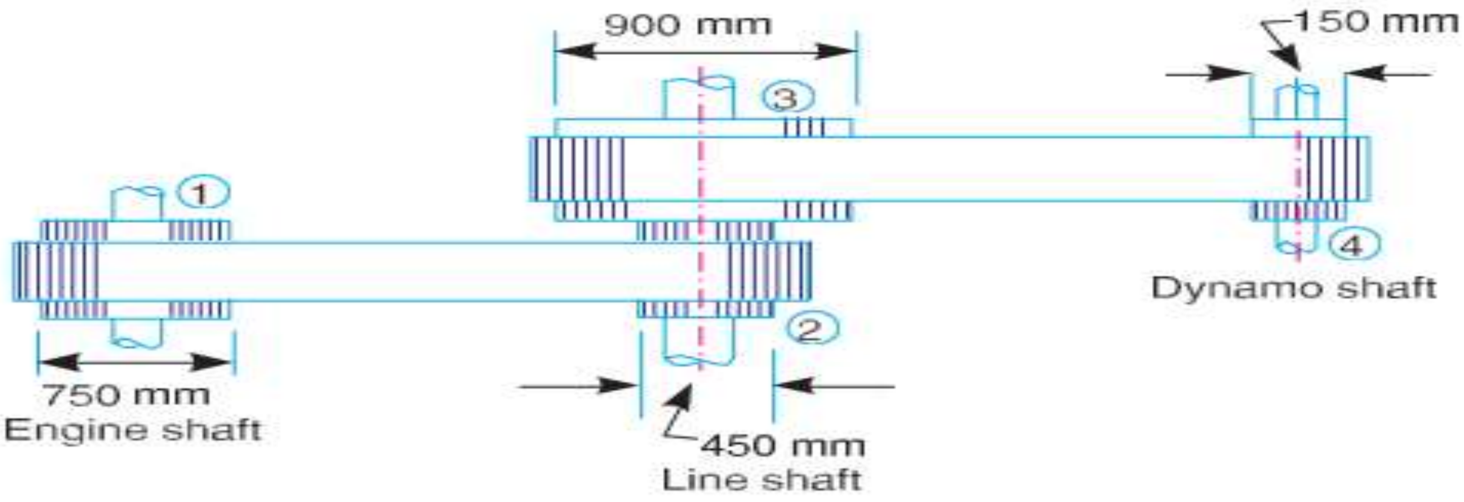


Fig. 11.10

1. *When there is no slip*

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

$$\therefore \quad N_4 = 150 \times 10 = 1500 \text{ r.p.m.}$$ **Ans.**

2. *When there is a slip of 2% at each drive*

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$$\therefore \quad N_4 = 150 \times 9.6 = 1440 \text{ r.p.m.}$$ **Ans.**

Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as ***creep.*** The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering creep, the velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

where σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Example . *The power is transmitted from a pulley 1 m diameter running at 200 r.p.m. to a pulley 2.25 m diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight and slack side of the belt is 1.4 MPa and 0.5 MPa respectively. The Young's modulus for the material of the belt is 100 MPa.*

Solution. Given : $d_1 = 1$ m ; $N_1 = 200$ r.p.m. ; $d_2 = 2.25$ m ; $\sigma_1 = 1.4$ MPa $= 1.4 \times 10^6$ N/m^2; $\sigma_2 = 0.5$ MPa $= 0.5 \times 10^6$ N/m^2 ; $E = 100$ MPa $= 100 \times 10^6$ N/m^2

Let N_2 = Speed of the driven pulley.

Neglecting creep, we know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad N_2 = N_1 \times \frac{d_1}{d_2} = 200 \times \frac{1}{2.25} = 88.9 \text{ r.p.m.}$$

Considering creep, we know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

or

$$N_2 = 200 \times \frac{1}{2.25} \times \frac{100 \times 10^6 + \sqrt{0.5 \times 10^6}}{100 \times 10^6 + \sqrt{1.4 \times 10^6}} = 88.7 \text{ r.p.m.}$$

$\therefore$ Speed lost by driven pulley due to creep

$= 88.9 - 88.7 = 0.2$ r.p.m. **Ans.**

Length of an Open Belt Drive

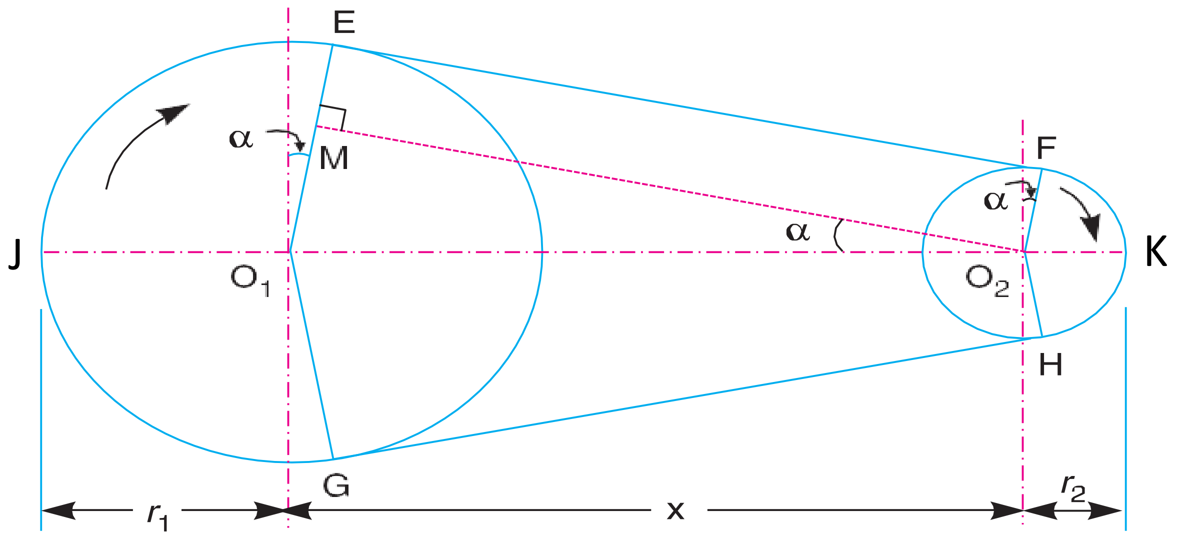


Fig. Length of an open belt drive.

In an open belt drive, both the pulleys rotate in the ***same*** direction as shown in Fig.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. Through O_2, draw $O_2 M$ parallel to FE.

From the geometry of the figure, we find that $O_2 M$ will be perpendicular to $O_1 E$. Let the angle $MO_2 O_1 =$ α radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \quad \text{...}(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x} \quad \text{...}(ii)$$

$\therefore$

$$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{...}(iii)$$

Similarly

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \quad \text{...}(iv)$$

and

$$EF = MO_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + ...\right] = x - \frac{(r_1 - r_2)^2}{2x} \qquad ...(v)$$

Substituting the values of arc JE from equation (*iii*), arc FK from equation (*iv*) and EF from equation (*v*) in equation (*i*), we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 - r_2}{x}$ from equation (*ii*),

$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$ ...(In terms of pulley radii)

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$ ...(In terms of pulley diameters)

Length of a Cross Belt Drive

In a cross belt drive, both the pulleys rotate in ***opposite*** directions as shown in Fig.

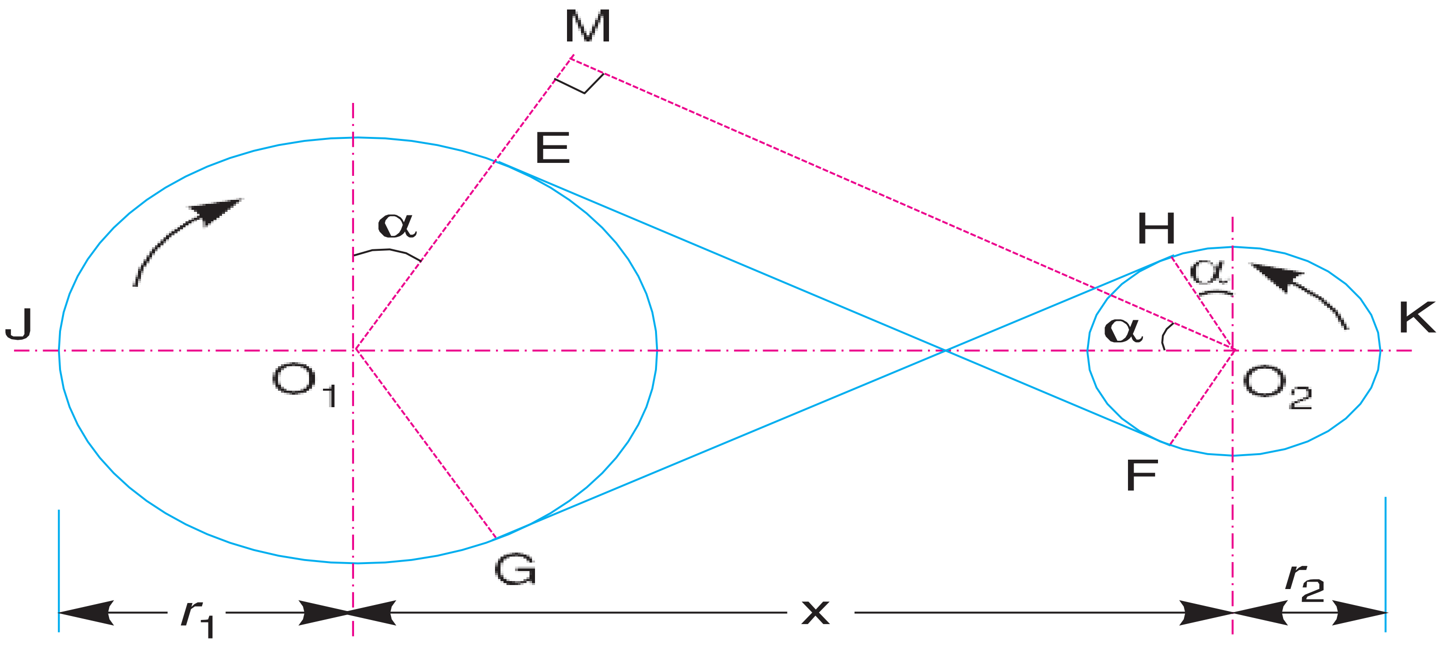


Fig. Length of a cross belt drive.

Let, r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. through O_2, draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E. Let the angle $MO_2 O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \text{...}(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + EM}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha\ (\text{in radians}) = \frac{r_1 + r_2}{x} \qquad \text{...}(ii)$$

$$\therefore \qquad \text{Arc } JE = r_1\left(\frac{\pi}{2}+\alpha\right) \qquad ...(iii)$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2}+\alpha\right) \qquad ...(iv)$$

and
$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1+r_2)^2}$$
$$= x\sqrt{1-\left(\frac{r_1+r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1-\frac{1}{2}\left(\frac{r_1+r_2}{x}\right)^2+...\right] = x - \frac{(r_1+r_2)^2}{2x} \qquad ...(v)$$

Substituting the values of arc JE from equation *(iii)*, arc FK from equation *(iv)* and EF from equation *(v)* in equation *(i)*, we get

$$L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + x - \frac{(r_1+r_2)^2}{2x} + r_2\left(\frac{\pi}{2}+\alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1 . \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2 . \alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 + r_2}{x}$ from equation *(ii)*,

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

Power Transmitted by a Belt

Fig. shows the driving pulley (or driver) A and the driven pulley (or follower) B. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former side (i.e. tight side) will be greater than the latter side (i.e. slack side) as shown in Fig.

Let, T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driver and follower respectively, and

v = Velocity of the belt in m/s.

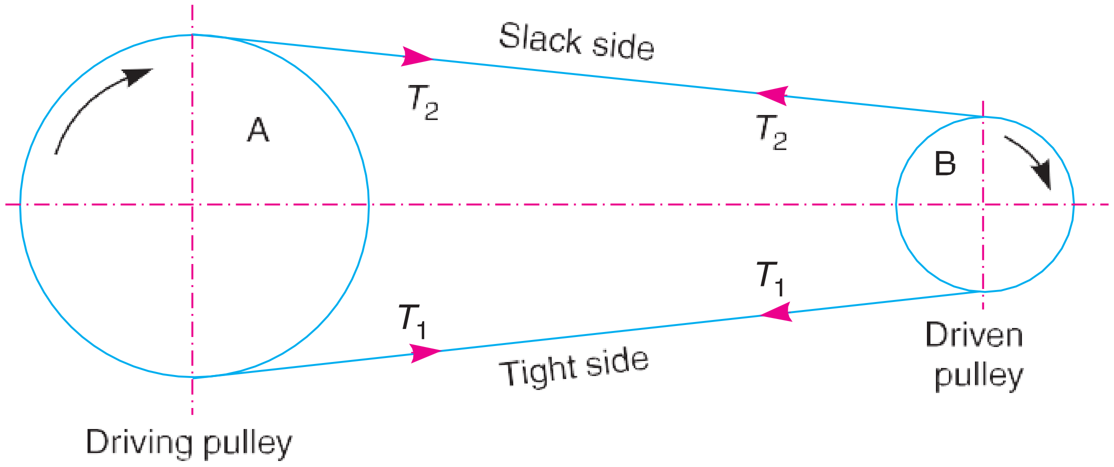


Fig. Power transmitted by a belt.

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (i.e. T1 – T2).

∴ Work done per second = $(T_1 - T_2)\, v$ N-m/s

∴ and power transmitted, $P = (T_1 - T_2)\, v$ W ...(∵ 1 N-m/s = 1 W)

A little consideration will show that the torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley *i.e.* follower is $(T_1 - T_2)\, r_2$.

Ratio of Driving Tensions For Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig.

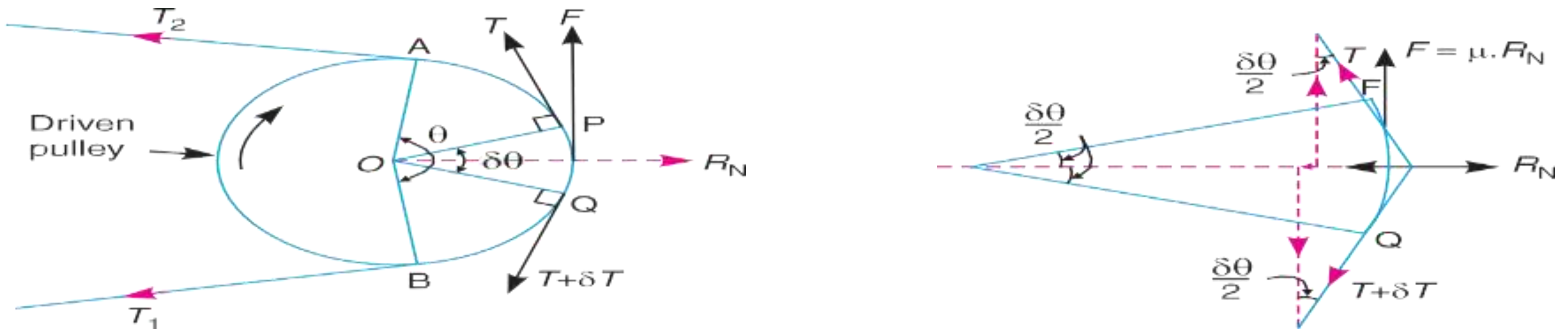


Fig. Ratio of driving tensions for flat belt.

Let, T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *A B*, along which the belt touches the pulley at the centre).

Now consider a small portion of the belt *PQ*, subtending an angle $\delta\theta$ at the centre of the pulley as shown in Fig. The belt *PQ* is in equilibrium under the following forces :

1. Tension *T* in the belt at *P*,
2. Tension $(T + \delta T)$ in the belt at *Q*,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \qquad ...(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta / 2 = \delta\theta / 2$ in equation *(i)*,

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T\times\frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \qquad ...(ii)$$

$$...\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu\times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad ...(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta / 2 = 1$ in equation *(iii)*,

$$\mu\times R_N = T + \delta T - T = \delta T \quad\text{or}\quad R_N = \frac{\delta T}{\mu} \qquad ...(iv)$$

Equating the values of R_N from equations *(ii)* and *(iv)*,

$$T.\delta\theta = \frac{\delta T}{\mu} \quad\text{or}\quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

$$i.e. \quad \int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \quad \text{or} \quad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta} \qquad ...(v)$$

Equation (v) can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

Determination of Angle of Contact

When the two pulleys of different diameters are connected by means of an open belt as shown in Fig., then the angle of contact or lap (θ) at the smaller pulley must be taken into consideration.

Let, r_1 = Radius of larger pulley,

r_2 = Radius of smaller pulley, and

x = Distance between centres of two pulleys (*i.e.* $O_1 O_2$).

From Fig.

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - ME}{O_1 O_2} = \frac{r_1 - r_2}{x} \qquad \ldots(\because ME = O_2 F = r_2)$$

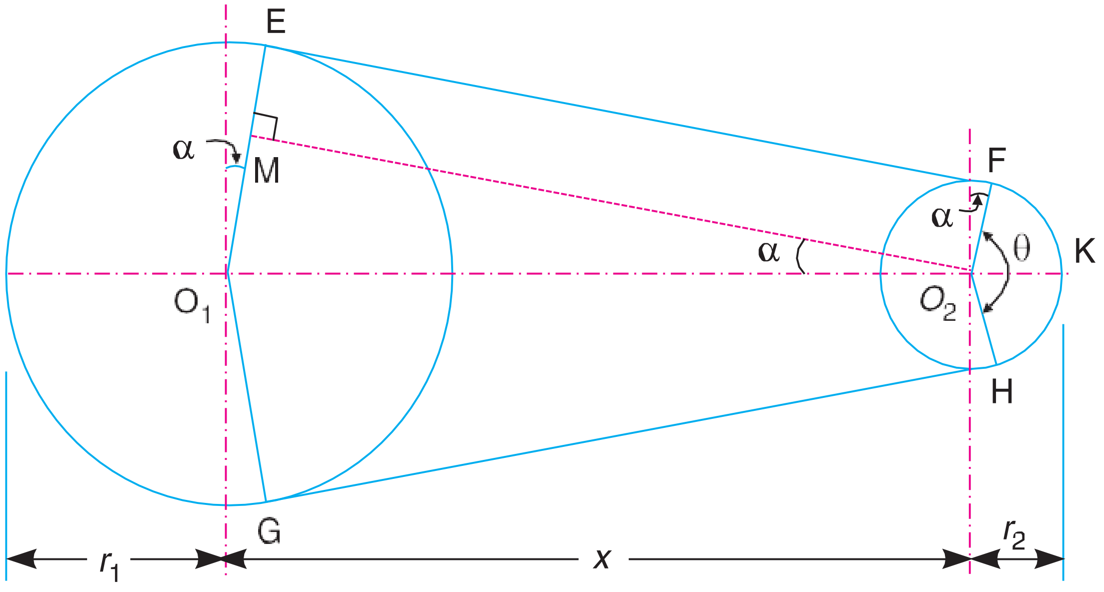
$\therefore$ Angle of contact or lap,

$$\theta = (180° - 2\alpha)\frac{\pi}{180}\text{ rad}$$

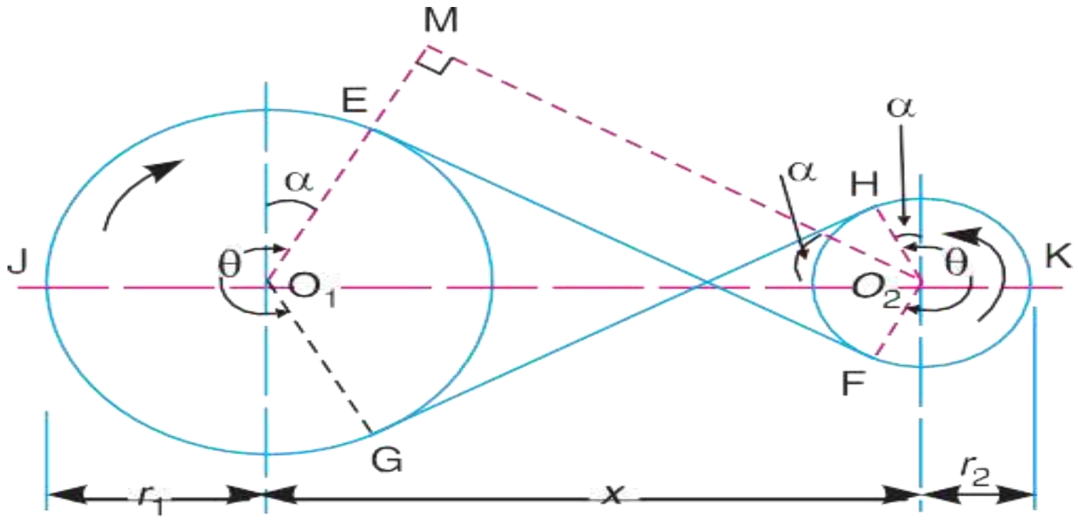
when the two pulleys are connected by means of a crossed belt as shown in Fig., then the angle of contact or lap (θ) on both the pulleys is same. From Fig.

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + ME}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

$\therefore$ Angle of contact or lap, $\theta = (180° + 2\alpha)\dfrac{\pi}{180}$ rad



(*a*) Open belt drive.



(*b*) Crossed belt drive.

Fig.

Initial Tension in the Belt

When a belt is wound round the two pulleys (i.e. driver and follower), its two ends are joined together ; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called initial tension.

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) and delivers it to the other side (decreasing the tension in the belt on that side). The increased tension in one side of the belt is called tension in tight side and the decreased tension in the other side of the belt is called tension in the slack side.

Let,

T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

The increase of tension in the tight side = $T_1 - T_0$

and increase in the length of the belt on the tight side = $\alpha (T_1 - T_0)$ ...*(i)*

Similarly, decrease in tension in the slack side = $T_0 - T_2$

and decrease in the length of the belt on the slack side = $\alpha (T_0 - T_2)$ ...*(ii)*

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations ***(i)*** and ***(ii)*,**

$$\alpha (T_1 - T_0) = \alpha (T_0 - T_2) \text{ or } T_1 - T_0 = T_0 - T_2$$

$$\alpha (T_1 - T_0) = \alpha (T_0 - T_2) \quad \text{or} \quad T_1 - T_0 = T_0 - T_2$$

$$\therefore \quad T_0 = \frac{T_1 + T_2}{2} \quad \text{...(Neglecting centrifugal tension)}$$

$$= \frac{T_1 + T_2 + 2T_C}{2} \quad \text{...(Considering centrifugal tension)}$$

Rope Drive

The rope drives are widely used where a large amount of power is to be transmitted, from one pulley to another, over a considerable distance. It may be noted that the use of flat belts is limited for the transmission of moderate power from one pulley to another when the two pulleys are not more than 8 metres apart. If large amounts of power are to be transmitted by the flat belt, then it would result in excessive belt cross-section. It may be noted that frictional grip in case of rope drives is more than that in V-drive. One of the main advantage of rope drives is that a number of separate drives may be taken from the one driving pulley. For example, in many spinning mills, the line shaft on each floor is driven by ropes passing directly from the main engine pulley on the ground floor.

The rope drives use the following two types of ropes :

1. Fibre ropes, and 2. Wire ropes.

The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are up to 150 metres apart.

Fibre Ropes

The ropes for transmitting power are usually made from fibrous materials such as hemp, manila and cotton. Since the hemp and manila fibres are rough, therefore the ropes made from these fibres are not very flexible and possesses poor mechanical properties. The hemp ropes have less strength as compared to manila ropes. When the hemp and manila ropes are bent over the sheave (or pulley), there is some sliding of fibres, causing the rope to wear and chafe internally. In order to minimize this defect, the rope fibres are lubricated with a tar, tallow or graphite. The lubrication also makes the rope moisture proof. The hemp ropes are suitable only for hand operated hoisting machinery and as tie ropes for lifting tackle, hooks etc.

The cotton ropes are very soft and smooth. The lubrication of cotton ropes is not necessary. But if it is done, it reduces the external wear between the rope and the grooves of its sheaves. It may be noted that manila ropes are more durable and stronger than cotton ropes. The cotton ropes are costlier than manila ropes.

Note : The diameter of manila and cotton ropes usually ranges from 38 mm to 50 mm. The size of the rope is usually designated by its circumference or 'girth'.

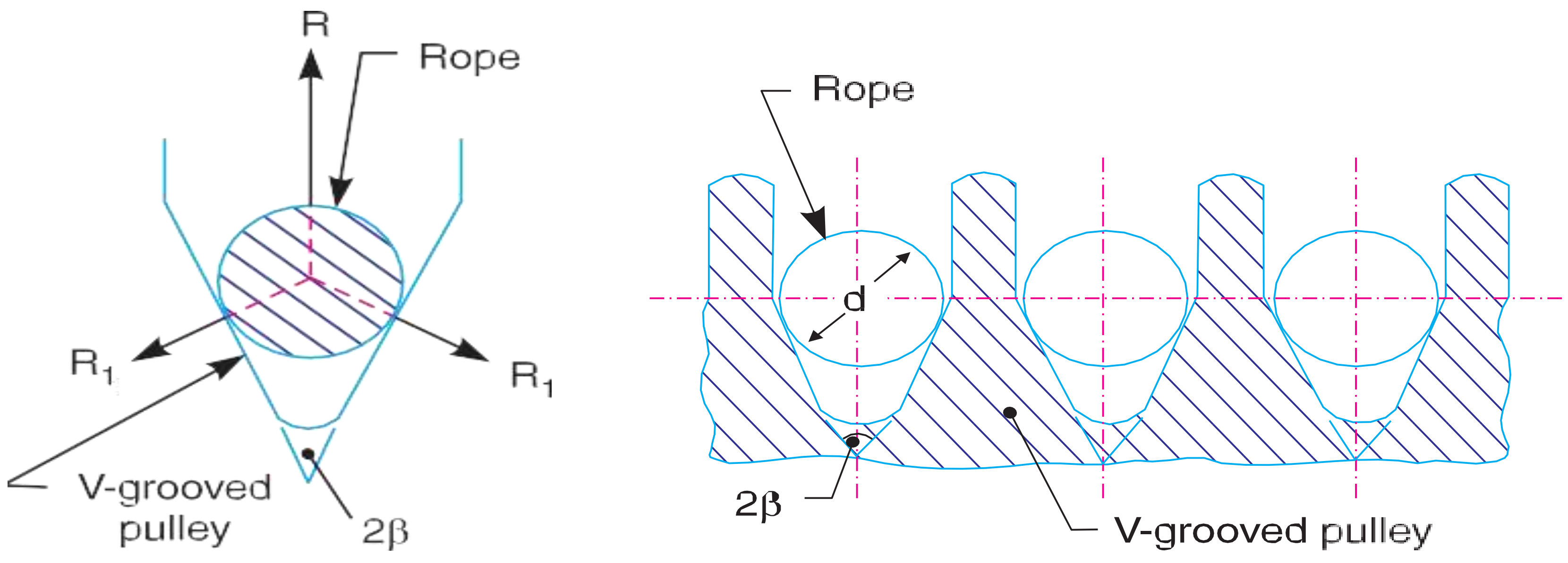
Advantages of Fibre Rope Drives

The fibre rope drives have the following advantages :

1. They give smooth, steady and quiet service.
2. They are little affected by out door conditions.
3. The shafts may be out of strict alignment.
4. The power may be taken off in any direction and in fractional parts of the whole amount.
5. They give high mechanical efficiency.

Sheave for Fibre Ropes

The fibre ropes are usually circular in cross-section as shown in Fig. The sheave for the fibre ropes is shown in Fig. The groove angle of the pulley for rope drives is usually 45°. The grooves in the pulleys are made narrow at the bottom and the rope is pinched between the edges of the V-groove to increase the holding power of the rope on the pulley.



(a) Cross-section of a rope. (*b*) Sheave (Grooved pulley) for ropes.

Fig. Rope and sheave.

Wire Ropes

When a large amount of power is to be transmitted over long distances from one pulley to another (i.e. when the pulleys are up to 150 metres apart), then wire ropes are used. The wire ropes are widely used in elevators, mine hoists, cranes, conveyors, hauling devices and suspension bridges. The wire ropes run on grooved pulleys but they rest on the bottom of the *grooves and are not wedged between the sides of the grooves. The wire ropes have the following advantage over cotton ropes.

1. These are lighter in weight, **2.** These offer silent operation, **3.** These can withstand shock loads, **4.** These are more reliable, **5.** They do not fail suddenly, **6.** These are more durable, **7.** The efficiency is high, and **8.** The cost is low.

Ratio of Driving Tensions for Rope Drive

The ratio of driving tensions for the rope drive may be obtained in the similar way as V-belts. We have discussed, that the ratio of driving tensions is

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta \operatorname{cosec} \beta$$

where, μ, θ and β have usual meanings.