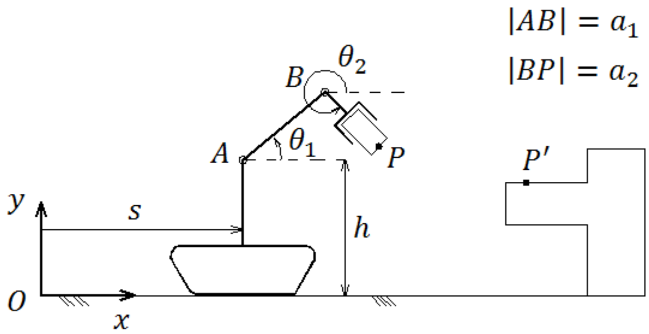
MMÜ 466 - COMPUTATIONAL MULTI-BODY DYNAMICS - HOMEWORK 2

Due: 9 April 2023, Sunday

1) Consider a mobile robot arm used for two-dimensional applications as shown in the below figure. The robot is holding a block at the tip of its arm. The aim of the robot is to place this block on the table such that point P becomes coincident with point P'. The robot is driven by three motors which provide motion for the joint variables s, θ_1 and θ_2. The constant parameters in the robot arm system are,

$$h = 185 \quad , \quad |AB| = a_1 = 150 \quad , \quad |BP| = a_2 = 105$$



The position of the block with respect to the coordinate system xy with origin O is given by $x_p(t)$ and $y_p(t)$ where t is the time. The orientation (i.e. angle) of the block with respect to the coordinate system xy is given by $\theta_p(t)$. The three equations given below describe the kinematics of the system.

$$x_p(t) = s + a_1 Cos(\theta_1) + a_2 Cos(\theta_2)$$

$$y_p(t) = h + a_1 Sin(\theta_1) + a_2 Sin(\theta_2)$$

$$\theta_p(t) = \theta_2$$

Notice that it is straightforward (i.e. very easy) to find x_p, y_p and θ_p when the joint variables s, θ_1 and θ_2 are specified. However, when the motion of the block (x_p, y_p, θ_p) is specified, we need to solve the given system of nonlinear equations to find the required joint motions s, θ_1 and θ_2.

In this problem, the motion of the block is specified as follows (θ_p is in **degrees**).

$$x_p(t) = x_p = 100 Cos\left(\frac{\pi t}{10} + \pi\right) + 580 \quad , \quad y_p(t) = y_p = 22 Cos\left(\frac{\pi t}{10}\right) + 178$$

$$\theta_p(t) = \theta_p = 22 Cos\left(\frac{\pi t}{10}\right) - 68$$

In three separate figures, plot x_p, y_p and θ_p versus time t for $t = 0$ to 10 using a step size of 0.01 .

Then, we would like to calculate the joint variables s, θ_1 and θ_2 to produce the motion of the block as specified above. Apply Matlab's `fsolve` function to calculate the values of $\bar{x} = [s \quad \theta_1 \quad \theta_2\,]^T$ for $t = 0$ to 10 with an increment of 1 (i.e. $t = 0, 1, 2, \cdots\cdots 10$). At $t = 10$, the block is placed on the table. To start the numerical solution at $t = 0$, use the initial guess $\bar{x}_0 = [280 \quad 40° \quad -45°\,]^T$. Note that you must work in **radians** for the angles while using the trigonometric functions in Matlab and while solving the system of equations. In your numerical solution, in order to find the solution vector at a specific t value, choose the initial guess as the solution vector at the previous t value.

In three separate figures, plot the numerical solutions for s, θ_1 and θ_2 together with their exact (i.e. analytical) solutions. In all of your plots, the x-axis must be the time t. The numerical solutions must be plotted as **data-points** (using markers in Matlab) and the exact solution must be plotted using solid lines with an increment of 0.01 for time. Plot θ_1 and θ_2 in degrees.

The exact (i.e. analytical) solution for this problem is:

$$\theta_2 = \theta_p \quad , \quad \theta_1 = atan_2\left(\beta, \sigma\sqrt{1-\beta^2}\right) \quad , \quad \sigma = \mp 1 \quad , \quad \beta = \frac{y_p - h - a_2 Sin(\theta_2)}{a_1}$$

$$s = x_p - a_1 Cos(\theta_1) - a_2 Cos(\theta_2)$$

Choose σ as $+1$ and please read the explanation of the Matlab function `atan2`.