MMÜ 466 - COMPUTATIONAL MULTI-BODY DYNAMICS - HOMEWORK 3

<u>**Due:**</u> **24 April 2023, Monday**

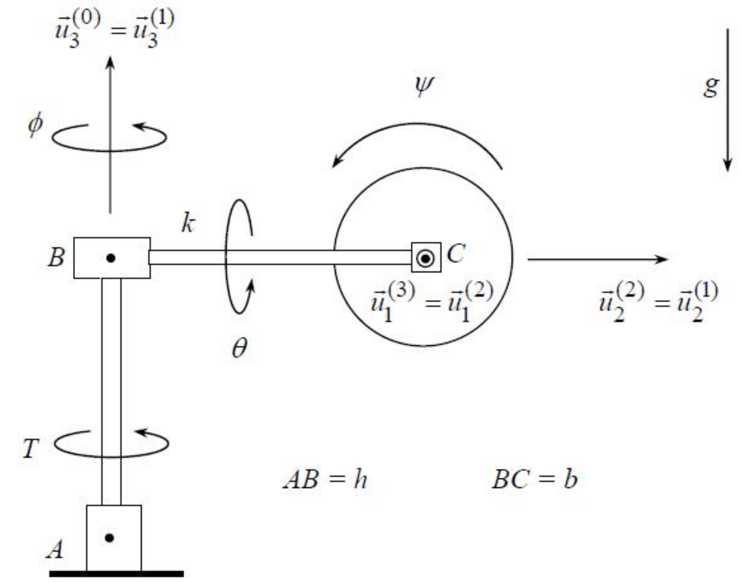
The multi-body system shown below is composed of a vertical shaft (body 1), a horizontal bar (body 2) and a wheel (body 3). A torque T is applied to rotate the vertical shaft by an angle ϕ. The horizontal bar rotates by an angle θ in the bearing B at the upper end of the vertical shaft. A torsional spring with coefficient k resists the rotation θ of the horizontal bar and it exerts no torque when $\theta = 0$. The wheel is spinning freely at a rate $\dot{\psi}$ around an axis which coincides with point C.

The masses of the vertical shaft and the horizontal bar are negligible. The mass of the wheel is m_w, its centre of mass is at point C and and its inertia dyadic about point C is given by $\check{J}_C = J_1\vec{u}_1\vec{u}_1 + J_2(\vec{u}_2\vec{u}_2 + \vec{u}_3\vec{u}_3)$ where $\vec{u}_k = \vec{u}_k^{(2)}$ is the k^{th} unit basis vector of frame $\mathcal{F}_2$ fixed to the horizontal bar. The radius of the wheel is r and the thickness of the wheel is t_w. It can be shown that the differential equations of motion for this system are:

$$\dot{\psi} - \dot{\phi}Sin\theta = constant$$

$$J_2\ddot{\theta} + J_1\dot{\phi}\dot{\psi}Cos\theta - (J_1 - J_2)\dot{\phi}^2 Sin\theta Cos\theta + k\theta = 0$$

$$(m_w b^2 + J_2 Cos^2\theta)\ddot{\phi} - (2J_2 - J_1)\dot{\phi}\dot{\theta}Sin\theta Cos\theta - J_1\dot{\psi}\dot{\theta}Cos\theta = T$$



The numerical values are given below:

$|AB| = h = 0.25\ m,\ |BC| = b = 0.25\ m,\ m_w = 0.1\ kg,\ r = 0.05\ m,\ t_w = 2\ mm = 0.002\ m$

$J_1 = \frac{1}{2}m_w r^2,\quad J_2 = \frac{1}{4}m_w r^2 + \frac{1}{12}m_w(t_w)^2,\quad k = \frac{0.2}{\pi}\ Nm/rad,\quad T = 0.1Sin\left(\frac{\pi}{2}t\right)\ Nm,\quad t\text{: } time$

Using the initial conditions given below, solve the given system of differential equations for $t \in [0,\ 5]$ where t $(time)$ is in seconds. As a result, plot $\phi(t)$, $\theta(t)$ and $\psi(t)$ versus t in three separate figures.

$\phi(0) = 0\ rad,\ \dot{\phi}(0) = 0\ rad/s,\ \theta(0) = \frac{\pi}{2}\ rad,\ \dot{\theta}(0) = 0\ rad/s,\ \psi(0) = \frac{\pi}{4}\ rad,\ \dot{\psi}(0) = 2\pi\ rad/s$

Use the Matlab function `ode23s` to solve this system of differential equations. (Hint: Take the derivative of the first equation).