



UNIVERSIDAD AUTÓNOMA CHAPINGO

POSGRADO EN INGENIERÍA AGRÍCOLA  
Y USO INTEGRAL DEL AGUA

## MODELOS BASADOS EN APRENDIZAJE PROFUNDO

### TAREA 4. RETROPROPAGACIÓN EN EL PERCEPTRÓN MULTICAPA

Presentan:

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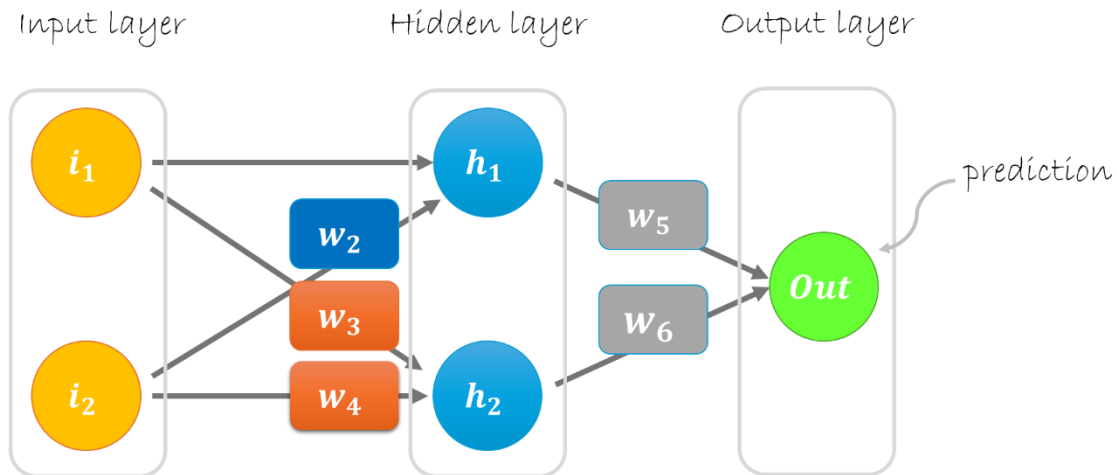
### Datos del problema:

Pesos iniciales:  $w_1 = 0.11$ ,  $w_2 = 0.21$ ,  $w_3 = 0.12$ ,  $w_4 = 0.08$ ,  $w_5 = 0.14$ ,  $w_6 = 0.15$

Entrada:  $[i_1 \ i_2] = [2 \ 3]$

Salida deseada: Output = 1

Learning rate:  $\alpha = 0.05$



### Evaluación 1 entrada - salida

$$\text{Capa oculta} = [i_1 \ i_2] \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = [h_1 \ h_2]; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_1 w_3 + i_2 w_4$$

$$\text{Output} = [h_1 \ h_2] \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa oculta} = [2 \ 3] \begin{bmatrix} 0.11 & 0.12 \\ 0.21 & 0.08 \end{bmatrix} = [0.85 \ 0.48] \implies h_1 = 0.85, h_2 = 0.48$$

$$\text{Output} = [0.85 \ 0.48] \begin{bmatrix} 0.14 \\ 0.15 \end{bmatrix} = [0.191]$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.191 - 1.0)^2 = 0.327$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.191 - 1 = -0.809$$

Fórmula para actualización de pesos en forma matricial:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot \begin{bmatrix} w_5 & w_6 \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

Nuevos pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.15 \end{bmatrix} - (0.05)(-0.809) \begin{bmatrix} 0.85 \\ 0.48 \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.15 \end{bmatrix} - \begin{bmatrix} -0.034 \\ -0.019 \end{bmatrix} = \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.11 & 0.12 \\ 0.21 & 0.08 \end{bmatrix} - \begin{bmatrix} -0.011 & -0.012 \\ -0.017 & -0.018 \end{bmatrix} = \begin{bmatrix} 0.12 & 0.13 \\ 0.23 & 0.10 \end{bmatrix}$$

## Evaluación 2 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0.12 & 0.13 \\ 0.23 & 0.10 \end{bmatrix} = \begin{bmatrix} 0.92 & 0.56 \end{bmatrix} \implies h_1 = 0.92, h_2 = 0.56$$

$$\text{Output} = \begin{bmatrix} 0.92 & 0.56 \end{bmatrix} \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix} = \begin{bmatrix} 0.26 \end{bmatrix}$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.26 - 1.0)^2 = 0.2738$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.26 - 1 = -0.7400$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.17 \\ 0.17 \end{bmatrix} - (0.05)(-0.7400) \begin{bmatrix} 0.92 \\ 0.56 \end{bmatrix} = \begin{bmatrix} 0.2040 \\ 0.1907 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot \begin{bmatrix} w_5 & w_6 \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.12 & 0.13 \\ 0.23 & 0.10 \end{bmatrix} - (0.05)(-0.7400) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 0.2040 & 0.1907 \end{bmatrix} = \begin{bmatrix} 0.1351 & 0.1441 \\ 0.2526 & 0.1212 \end{bmatrix}$$

### Evaluación 3 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0.1351 & 0.1441 \\ 0.2526 & 0.1212 \end{bmatrix} = \begin{bmatrix} 1.0280 & 0.6518 \end{bmatrix} \Rightarrow h_1 = 1.0280, h_2 = 0.6518$$

$$\text{Output} = \begin{bmatrix} 1.0280 & 0.6518 \end{bmatrix} \begin{bmatrix} 0.2040 \\ 0.1907 \end{bmatrix} = \begin{bmatrix} 0.3340 \end{bmatrix}$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.3340 - 1.0)^2 = 0.2218$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.3340 - 1.0 = -0.6660$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.2040 \\ 0.1907 \end{bmatrix} - (0.05)(-0.6660) \begin{bmatrix} 1.0280 \\ 0.6518 \end{bmatrix} = \begin{bmatrix} 0.2382 \\ 0.2114 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot [w_5 \ w_6] = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.1351 & 0.1441 \\ 0.2526 & 0.1212 \end{bmatrix} - (0.05)(-0.6660) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot [0.2382 \ 0.2114] =$$

$$\begin{bmatrix} 0.1351 & 0.1441 \\ 0.2526 & 0.1212 \end{bmatrix}$$

#### Evaluación 4 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0.1351 & 0.1441 \\ 0.2526 & 0.1212 \end{bmatrix} = \begin{bmatrix} 1.0280 & 0.6518 \end{bmatrix} \implies h_1 = 1.0280, h_2 = 0.6518$$

$$\text{Output} = \begin{bmatrix} 1.0280 & 0.6518 \end{bmatrix} \begin{bmatrix} 0.2382 \\ 0.2114 \end{bmatrix} = \begin{bmatrix} 0.3827 \end{bmatrix}$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.3827 - 1.0)^2 = 0.1905$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.3827 - 1 = -0.6173$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.2382 \\ 0.2114 \end{bmatrix} - (0.05)(-0.6173) \begin{bmatrix} 1.0280 \\ 0.6518 \end{bmatrix} = \begin{bmatrix} 0.2699 \\ 0.2315 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot [w_5 \quad w_6] = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.1351 & 0.1441 \\ 0.2526 & 0.1212 \end{bmatrix} - (0.05)(-0.6173) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot [0.2699 \quad 0.2315] = \begin{bmatrix} 0.1518 & 0.1584 \\ 0.2776 & 0.1426 \end{bmatrix}$$

### Evaluación 5 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0.1518 & 0.1584 \\ 0.2776 & 0.1426 \end{bmatrix} = \begin{bmatrix} 1.1363 & 0.7447 \end{bmatrix} \implies h_1 = 1.1363, h_2 = 0.7447$$

$$\text{Output} = \begin{bmatrix} 1.1363 & 0.7447 \end{bmatrix} \begin{bmatrix} 0.2699 \\ 0.2315 \end{bmatrix} = \begin{bmatrix} 0.4791 \end{bmatrix}$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.4791 - 1.0)^2 = 0.1357$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.4791 - 1 = -0.5209$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.2699 \\ 0.2315 \end{bmatrix} - (0.05)(-0.5209) \begin{bmatrix} 1.1363 \\ 0.7447 \end{bmatrix} = \begin{bmatrix} 0.2995 \\ 0.2509 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot [w_5 \quad w_6] = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.1518 & 0.1584 \\ 0.2776 & 0.1426 \end{bmatrix} - (0.05)(-0.5209) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot [0.2995 \quad 0.2509] =$$

$$\begin{bmatrix} 0.1674 & 0.1715 \\ 0.3010 & 0.1622 \end{bmatrix}$$

### Evaluación 6 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = [2 \quad 3] \begin{bmatrix} 0.1674 & 0.1715 \\ 0.3010 & 0.1622 \end{bmatrix} = [1.2378 \quad 0.8296] \implies h_1 = 1.2378, h_2 = 0.8296$$

$$\text{Output} = [1.2378 \quad 0.8296] \begin{bmatrix} 0.2995 \\ 0.2509 \end{bmatrix} = [0.5789]$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.5789 - 1.0)^2 = 0.0887$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.5789 - 1 = -0.4211$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.2995 \\ 0.2509 \end{bmatrix} - (0.05)(-0.4211) \begin{bmatrix} 1.2378 \\ 0.8296 \end{bmatrix} = \begin{bmatrix} 0.3256 \\ 0.2684 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot [w_5 \quad w_6] = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.1674 & 0.1715 \\ 0.3010 & 0.1622 \end{bmatrix} - (0.05)(-0.4211) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot [0.3256 \quad 0.2684] =$$

$$\begin{bmatrix} 0.1811 & 0.1828 \\ 0.3216 & 0.1792 \end{bmatrix}$$

### Evaluación 7 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = [2 \quad 3] \begin{bmatrix} 0.1811 & 0.1828 \\ 0.3216 & 0.1792 \end{bmatrix} = [1.3270 \quad 0.9032] \implies h_1 = 1.3270, h_2 = 0.9032$$

$$\text{Output} = [1.3270 \quad 0.9032] \begin{bmatrix} 0.3256 \\ 0.2684 \end{bmatrix} = [0.6745]$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.6745 - 1.0)^2 = 0.0530$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.6745 - 1 = -0.3255$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.3256 \\ 0.2684 \end{bmatrix} - (0.05)(-0.3255) \begin{bmatrix} 1.3270 \\ 0.9032 \end{bmatrix} = \begin{bmatrix} 0.3472 \\ 0.2831 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot [w_5 \quad w_6] = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$



$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.1811 & 0.1828 \\ 0.3216 & 0.1792 \end{bmatrix} - (0.05)(-0.3255) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 0.3472 & 0.2831 \end{bmatrix} = \begin{bmatrix} 0.1924 & 0.1920 \\ 0.3386 & 0.1930 \end{bmatrix}$$

### Evaluación 8 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0.1924 & 0.1920 \\ 0.3386 & 0.1930 \end{bmatrix} = \begin{bmatrix} 1.4006 & 0.9630 \end{bmatrix} \Rightarrow h_1 = 1.4006, h_2 = 0.9630$$

$$\text{Output} = \begin{bmatrix} 1.4006 & 0.9630 \end{bmatrix} \begin{bmatrix} 0.3472 \\ 0.2831 \end{bmatrix} = \begin{bmatrix} 0.7589 \end{bmatrix}.$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.7589 - 1.0)^2 = 0.0291$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.7589 - 1 = -0.2411$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.3472 \\ 0.2831 \end{bmatrix} - (0.05)(-0.2411) \begin{bmatrix} 1.4006 \\ 0.9630 \end{bmatrix} = \begin{bmatrix} 0.3641 \\ 0.2947 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot \begin{bmatrix} w_5 & w_6 \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.1924 & 0.1920 \\ 0.3386 & 0.1930 \end{bmatrix} - (0.05)(-0.2411) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot [0.3641 \quad 0.2947] =$$

$$\begin{bmatrix} 0.2012 & 0.1991 \\ 0.3518 & 0.2037 \end{bmatrix}$$

### Evaluación 9 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = [2 \quad 3] \begin{bmatrix} 0.2012 & 0.1991 \\ 0.3518 & 0.2037 \end{bmatrix} = [1.4578 \quad 1.0093] \implies h_1 = 1.4578, h_2 = 1.0093$$

$$\text{Output} = [1.4578 \quad 1.0093] \begin{bmatrix} 0.3641 \\ 0.2947 \end{bmatrix} = [0.8282].$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.8282 - 1.0)^2 = 0.0148$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.8282 - 1 = -0.1718$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.3641 \\ 0.2947 \end{bmatrix} - (0.05)(-0.1718) \begin{bmatrix} 1.4578 \\ 1.0093 \end{bmatrix} = \begin{bmatrix} 0.3766 \\ 0.3034 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot [w_5 \quad w_6] = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.2012 & 0.1991 \\ 0.3518 & 0.2037 \end{bmatrix} - (0.05)(-0.1718) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 0.3766 & 0.3034 \end{bmatrix} = \begin{bmatrix} 0.2077 & 0.2043 \\ 0.3615 & 0.2115 \end{bmatrix}$$

### Evaluación 10 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0.2077 & 0.2043 \\ 0.3615 & 0.2115 \end{bmatrix} = \begin{bmatrix} 1.4999 & 1.0431 \end{bmatrix} \Rightarrow h_1 = 1.4999, h_2 = 1.0431$$

$$\text{Output} = \begin{bmatrix} 1.4999 & 1.0431 \end{bmatrix} \begin{bmatrix} 0.3766 \\ 0.3034 \end{bmatrix} = \begin{bmatrix} 0.8813 \end{bmatrix}.$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.8813 - 1.0)^2 = 0.0070$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.8282 - 1 = -0.1187$$

Actualización de pesos:

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - a\Delta \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} - \begin{bmatrix} a\Delta h_1 \\ a\Delta h_2 \end{bmatrix}$$

$$\begin{bmatrix} w_5^* \\ w_6^* \end{bmatrix} = \begin{bmatrix} 0.3766 \\ 0.3034 \end{bmatrix} - (0.05)(-0.1187) \begin{bmatrix} 1.4999 \\ 1.0431 \end{bmatrix} = \begin{bmatrix} 0.3855 \\ 0.3096 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - a\Delta \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \cdot \begin{bmatrix} w_5 & w_6 \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} - \begin{bmatrix} a\Delta i_1 w_5 & a\Delta i_2 w_6 \\ a\Delta i_1 w_5 & a\Delta i_2 w_6 \end{bmatrix}$$

$$\begin{bmatrix} w_1^* & w_3^* \\ w_2^* & w_4^* \end{bmatrix} = \begin{bmatrix} 0.2077 & 0.2043 \\ 0.3615 & 0.2115 \end{bmatrix} - (0.05)(-0.1187) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot [0.3855 \quad 0.3096] =$$

$$\begin{bmatrix} 0.2123 & 0.2080 \\ 0.3684 & 0.2170 \end{bmatrix}$$

### Evaluación 11 entrada - salida

$$\text{Capa oculta} = \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix}; \quad h_1 = i_1 w_1 + i_2 w_2 \text{ y } h_2 = i_3 w_3 + i_2 w_4$$

$$\text{Output} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} w_5 \\ w_6 \end{bmatrix}$$

$$\text{Capa Oculta} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0.2123 & 0.2080 \\ 0.3684 & 0.2170 \end{bmatrix} = \begin{bmatrix} 1.5296 & 1.0670 \end{bmatrix} \implies h_1 = 1.5296, h_2 = 1.0670$$

$$\text{Output} = \begin{bmatrix} 1.5296 & 1.0670 \end{bmatrix} \begin{bmatrix} 0.3855 \\ 0.3096 \end{bmatrix} = \begin{bmatrix} 0.9200 \end{bmatrix}.$$

$$\text{Cálculo del error: Error} = \frac{1}{2}(\text{prediction} - \text{actual})^2$$

$$\text{Error} = \frac{1}{2}(0.9200 - 1.0)^2 = 0.0070$$

$$\Delta = \text{prediction} - \text{actual}$$

$$\Delta = 0.8282 - 1 = -0.0800$$

**Se observa que el modelo mejora su capacidad de predicción conforme se incrementa el número de iteraciones realizadas. A medida que se entrena de forma continua, el error disminuye, lo que indica que el modelo puede converger hacia la solución.**