CSE-B

DIP

Assignment No. 03 Unit No. 03 Date :

1. Explain about weiner filter in noise removal?

Weiner filter in noise removal:

In this filtering process, it incorporates both the degrade function and statistical characteristics of noise.

* The images and noise in this method are considered as random variables.

* The objectives is to find an estimate $\hat{f}$ of the uncorrupted image f such that the mean square error between them is minimized.

* This error is measured by

$$e^2 = E\{(f-\hat{f})^2\}$$

* where $E\{\}$ is the expected value of the argument.

* It is assumed that noise and image are uncorrelated; one (or) the other has zero mean; the intensity levels in the estimate are a linear function of the levels in the degraded image.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\,S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_n(u,v)}\right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}\right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}\right] G(u,v)$$

* This result is known as "weiner filter".

* The terms inside the bracket is commonly refered as the minimum mean square error filter.

$H(u,v)$ = degraded function

$H^*(u,v)$ = Complex conjugate of $H(u,v)$

$S_n(u,v) = |N(u,v)|^2$ = power spectrum of the noise.

$S_f(u,v) = |F(u,v)|^2$ = power spectrum of the undegraded image

* the Signal to noise ratio in frequency domain.

$$SNR = \frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |F(u,v)|^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |N(u,v)|^2}$$

* The Signal to noise ratio in Spatial domain.

$$SNR = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{n-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{n-1} \left[f(x,y) - \hat{f}(x,y)\right]^2}$$

2) Compare Various mean filters?

Arithmetic mean filter:

* It is the simplest mean filter.
* let S_{xy} represents the set of cordinates in the Subimage of size $m*n$ centered at point (x,y).
* The arithmetic mean filter computes the average value of the corrupted image $g(x,y)$ in the area defined by S_{xy}.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

* this operation can be using a convolution mask in which all coefficients have value $1/mn$.

Geometric Mean filter:

- An image restored using a geometric mean filter is given by the expression.

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t)\right]^{1/mn}$$

* Here, each restored pixel is by the product of the pixel in the Sub-image window, raised to the power $1/mn$.
* A Geometric means filter achieves smoothing Comparable to the arithmetic mean filter, but it tends to lose image details in the process.

Harmonic mean filter:

The harmonic mean filtering operation is given by the expression.

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

* The harmonic mean filter works well for salt noise but fails for pepper noise.

Contra harmonic mean filter:

The Contra harmonic mean filter yields a restored image based on the expression.

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

* where Q is called the order of the filter and this filter is well suited for reducing the effects of salt ~~s~~ and pepper noise.

3) Explain adaptative filters?

Adaptive filters:

Adaptive filter whose behaviour changes based on the characteristics of the image inside the filter region S_{xy}.

Adaptive, local noise reduction filter:

The simplest stastical measures of a random variable are its mean and variance.

* The mean gives a measure of average intensity in the region over which the mean is Computed and the variance gives a measure of contrast in that region.
* let the filter is operate on a local region S_{xy}.
* The response of the filter at any point (x,y) is based on four

quantities: (a) $g(x,y)$, the value of noisy image at (x,y): (b) σ_η^2 the variance of the noise corrupting $f(x,y)$ to form $g(x,y)$: (c) m_L, the local mean of the pixels in S_{xy}: and (d) σ_L^2; the local variance of the pixels in S_{xy}.

* Hence the behaviour of the filter is,
  → If σ_η^2 is zero, the filter should return simply the value of $g(x,y)$.
  → If the local variance is high relative to σ_η^2 that means $(\sigma_L^2 > \sigma_\eta^2)$, the filter should return a value close to $g(x,y)$.
* An adaptive filter for obtaining the restored image is:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

Adaptive median filter:

Adaptive median filters are used to preserve the details while smoothing non impulse.

→ let us consider the following parameters,

Z_{min} = minimum intensity value in S_{xy}

Z_{max} = maximum intensity value in S_{xy}.

Z_{med} = median of intensity values in S_{xy}.

Z_{xy} = intensity value at co-ordinates (x,y)

S_{max} = maximum allowed size of S_{xy}.

4) Discuss about noise models?

Noise models:

* The principle Sources of noise in digital image are due to image acquistion and transmission.
* During image transmission, the images are corrupted due to the interference introduced in the channel used for transmission.

* During image acquistion, the performance of image Sensors gets affected by a variety of factors such as environmental conditions and the quality of Sensing elements.
* The most common PDFs found in digital image processing applications are given below:

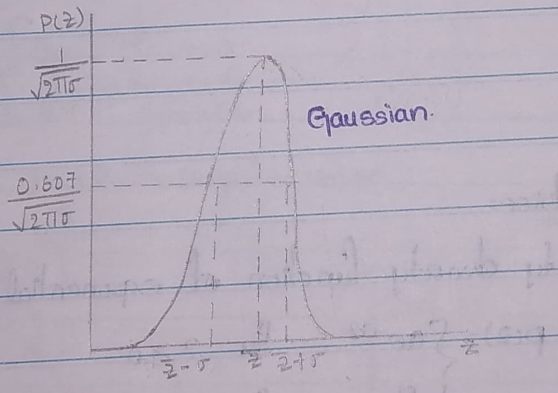
~~Guassian~~

Gaussian Noise:

Gaussian Noise is also known as 'normal' noise. the probability density function of a Gaussian random variable z is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

The values of Gaussian Noise is approximately 70% will be in the range $[(\bar{z}-\sigma), (\bar{z}+\sigma)]$ and 95% will be in the range $[(\bar{z}-2\sigma), (\bar{z}+2\sigma)]$.

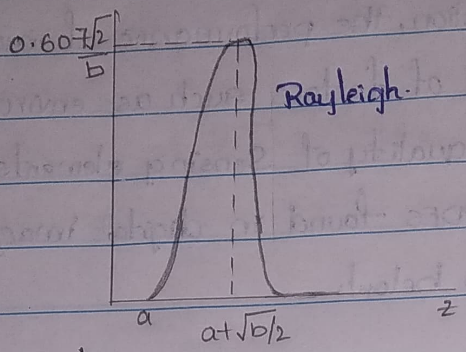


Rayleigh Noise:

The PDF of Rayleigh Noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Mean: $\bar{z} = a + \sqrt{\pi b/4}$ Variance: $\sigma^2 = \frac{b(4-\pi)}{4}$

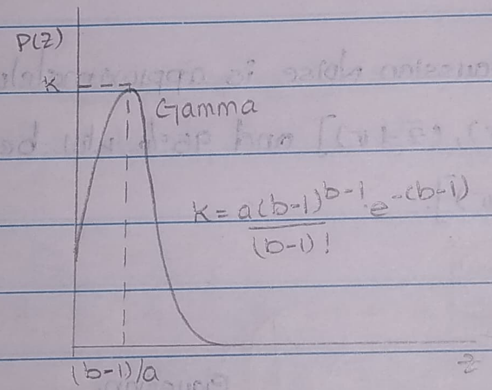


Erlang (gamma) Noise:

The probability density function of Erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad a > 0, \; b \text{ positive Integer}$$

Mean: $\bar{z} = \frac{b}{a}$ Variance: $\sigma^2 = \frac{b}{a^2}$



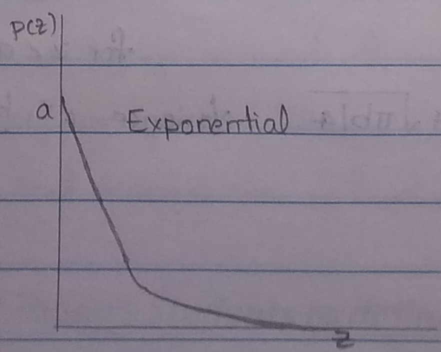
Exponential Noise:

The probability density function of exponential Noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$p(z)$ is maximum at $z=0$

Mean: $\bar{z} = 1/a$ Variance: $\sigma^2 = 1/a^2$

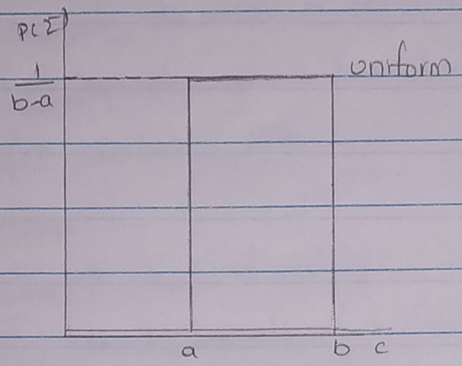


Uniform Noise:

The probability density function of uniform noise is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\bar{z} = \frac{a+b}{2}$ Variance: $\sigma^2 = \frac{(b-a)^2}{12}$

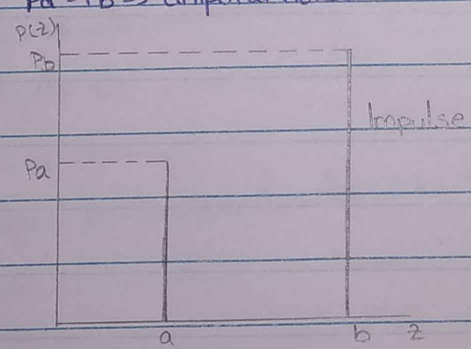


Salt and pepper Noise (Impulse Noise):

The probability density function of salt and impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

$P_a = P_b \Rightarrow$ unipolar noise



periodic Noise:

periodic Noise in an image occured from electrical (or) electromechanical interference during image acquistion.

* The mean and variance are defined as

$$\bar{z} = \sum_{i=0}^{L-1} z_i P_S(z_i)$$

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 P_S(z_i)$$