In[2]:=
```
Adjuster45 = {{-0.837714578104161`, 0.5461083094329986` i},
  {0.5461083094329986` i, -0.837714578104161`}}

AncillaAdjuster = {{1/Sqrt[3], -Sqrt[2/3]}, {Sqrt[2/3], 1/Sqrt[3]}}

(*Our matrix which we need to decompose*)
UFirstStage = {{0.14255151035802727` - 0.2356629432699872` i,
   -0.6505224884333463` - 0.3935044406748646` i, 0.289700420753269` - 0.478918679313866` i,
   -0.1552669366362766` - 0.09392478469664213` i},
  {-0.650481603268594` - 0.3934780983546269` i, 0.25797870818826696` -
    0.42647790172708877` i, -0.15525704666657789` - 0.09391580760922821` i,
   -0.19407070833203416` + 0.32082463867050615` i},
  {-0.28969622305981935` + 0.4789091231465468` i,
   0.1552844882590119` + 0.09393239821770455` i, 0.14254564812916865` -
    0.2356495966707425` i, -0.6505473768940662` - 0.39351513458229703` i},
  {0.15525045430548745` + 0.09391292831002403` i, 0.19407603261457435` -
    0.3208370951868854` i, -0.65047234292375` - 0.39347409356116836` i,
   0.2579713298010056` - 0.4264601673333378` i}}
```

Out[2]=
$$\{\{-0.837715,\ 0.+0.546108\,i\},\ \{0.+0.546108\,i,\ -0.837715\}\}$$

Out[3]=
$$\left\{\left\{\frac{1}{\sqrt{3}},\ -\sqrt{\frac{2}{3}}\right\},\ \left\{\sqrt{\frac{2}{3}},\ \frac{1}{\sqrt{3}}\right\}\right\}$$

Out[4]=
$$\{\{0.142552-0.235663\,i,\ -0.650522-0.393504\,i,\ 0.2897-0.478919\,i,\ -0.155267-0.0939248\,i\},\ \{-0.650482-0.393478\,i,\ 0.257979-0.426478\,i,\ -0.155257-0.0939158\,i,\ -0.194071+0.320825\,i\},\ \{-0.289696+0.478909\,i,\ 0.155284+0.0939324\,i,\ 0.142546-0.23565\,i,\ -0.650547-0.393515\,i\},\ \{0.15525+0.0939129\,i,\ 0.194076-0.320837\,i,\ -0.650472-0.393474\,i,\ 0.257971-0.42646\,i\}\}$$

In[7]:=
```
TeXForm[MatrixForm[Round[UFirstStage, 0.00001]]]
```

Out[7]//TeXForm=
```
\left(
\begin{array}{cccc}
 0.14255\, -0.23566 i & -0.65052-0.3935 i & 0.2897\,
   -0.47892 i & -0.15527-0.09392 i \\
 -0.65048-0.39348 i & 0.25798\, -0.42648 i &
   -0.15526-0.09392 i & -0.19407+0.32082 i \\
 -0.2897+0.47891 i & 0.15528\, +0.09393 i & 0.14255\,
   -0.23565 i & -0.65055-0.39352 i \\
 0.15525\, +0.09391 i & 0.19408\, -0.32084 i &
   -0.65047-0.39347 i & 0.25797\, -0.42646 i \\
\end{array}
\right)
```

In[ ]:=
```
(*It is unitary, vectors and eigenvalues are found with high accuracy*)
```

```
In[•]:= Transpose[Eigenvectors[UFirstStage]].
          {{Eigenvalues[UFirstStage][[1]], 0, 0, 0}, {0, Eigenvalues[UFirstStage][[2]], 0, 0},
           {0, 0, Eigenvalues[UFirstStage][[3]], 0}, {0, 0, 0, Eigenvalues[UFirstStage][[4]]}}.
          Conjugate[Eigenvectors[UFirstStage]] - UFirstStage
        ConjugateTranspose[UFirstStage].UFirstStage
```

```
Out[•]= {{-0.0000132009 + 0.0000145075 i, 0.0000304514 + 0.0000175108 i,
          1.71963 × 10^-6 + 2.00592 × 10^-6 i, 1.20404 × 10^-6 + 6.86856 × 10^-6 i},
         {-0.0000435284 - 0.0000316839 i, 4.09144 × 10^-6 - 0.0000150216 i,
          4.31454 × 10^-7 - 2.0276 × 10^-6 i, 0.0000117956 - 0.0000107024 i},
         {7.48819 × 10^-6 - 3.58096 × 10^-6 i, -0.0000351023 - 0.0000173809 i,
          -3.22422 × 10^-6 + 0.0000135888 i, 0.0000510349 + 0.0000296823 i},
         {2.19214 × 10^-6 + 2.02518 × 10^-6 i, 6.02612 × 10^-6 - 5.11362 × 10^-6 i,
          -0.0000448163 - 0.0000225953 i, 0.0000123337 - 0.0000130747 i}}
```

```
Out[•]= {{1.00001 + 0. i, 1.55312 × 10^-7 - 0.0000890683 i, 0.0000181754 + 2.44787 × 10^-6 i,
          6.18464 × 10^-6 + 0.0000850625 i}, {1.55312 × 10^-7 + 0.0000890683 i, 1. + 0. i,
          -1.75062 × 10^-8 + 0.0000618652 i, -0.0000197188 + 2.76669 × 10^-6 i},
         {0.0000181754 - 2.44787 × 10^-6 i, -1.75062 × 10^-8 - 0.0000618652 i, 1. + 0. i,
          -1.4253 × 10^-6 - 0.0000963966 i}, {6.18464 × 10^-6 - 0.0000850625 i,
          -0.0000197188 - 2.76669 × 10^-6 i, -1.4253 × 10^-6 + 0.0000963966 i, 1.00001 + 0. i}}
```

```
In[•]:= (*Do SVD on the UR and UI*)
```

```
In[•]:= M = 1/Sqrt[2] {{1, 0, 0, I}, {0, I, 1, 0}, {0, I, -1, 0}, {1, 0, 0, -I}}
        Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
        UP = ConjugateTranspose[M].UFirstStage.M
        UR = (UP + Conjugate[UP]) / 2
        UI = (UP - Conjugate[UP]) / (2 * I)
        {a, b, c} = SingularValueDecomposition[UR]
        Max[Abs[a.b.ConjugateTranspose[c] - UR]]
        {d, e, f} = SingularValueDecomposition[UI]
        Max[Abs[d.e.ConjugateTranspose[f] - UI]]
        MatrixForm[M]
```

```
Out[•]= {{1/Sqrt[2], 0, 0, I/Sqrt[2]}, {0, I/Sqrt[2], 1/Sqrt[2], 0}, {0, I/Sqrt[2], -1/Sqrt[2], 0}, {1/Sqrt[2], 0, 0, -I/Sqrt[2]}}
```

```
Out[•]= {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
```

```
Out[•]= {{0.200253 - 0.331067 i, 0.793367 - 0.408609 i,
          -0.0478373 + 0.0790256 i, -0.189317 + 0.0975488 i},
         {0.00637026 + 0.892398 i, 0.200276 - 0.331055 i, -0.00149005 - 0.212987 i,
          -0.0477799 + 0.0790608 i}, {0.0478456 - 0.0790237 i, 0.189338 - 0.0975542 i,
          0.200248 - 0.331072 i, 0.793363 - 0.408631 i}, {0.00147976 + 0.212969 i,
          0.0477871 - 0.079056 i, 0.00638862 + 0.892386 i, 0.20027 - 0.331056 i}}
```

```
Out[•]= {{0.200253 + 0. i, 0.793367 + 0. i, -0.0478373 + 0. i, -0.189317 + 0. i},
         {0.00637026 + 0. i, 0.200276 + 0. i, -0.00149005 + 0. i, -0.0477799 + 0. i},
         {0.0478456 + 0. i, 0.189338 + 0. i, 0.200248 + 0. i, 0.793363 + 0. i},
         {0.00147976 + 0. i, 0.0477871 + 0. i, 0.00638862 + 0. i, 0.20027 + 0. i}}
```

Out[•]= {{−0.331067 + 0. i, −0.408609 + 0. i, 0.0790256 + 0. i, 0.0975488 + 0. i},
{0.892398 + 0. i, −0.331055 + 0. i, −0.212987 + 0. i, 0.0790608 + 0. i},
{−0.0790237 + 0. i, −0.0975542 + 0. i, −0.331072 + 0. i, −0.408631 + 0. i},
{0.212969 + 0. i, −0.079056 + 0. i, 0.892386 + 0. i, −0.331056 + 0. i}}

Out[•]= {{{−0.680962 + 0. i, 0.694188 + 0. i, −0.215297 + 0. i, 0.0896723 + 0. i},
{−0.163336 + 0. i, 0.166479 + 0. i, 0.897707 + 0. i, −0.373803 + 0. i},
{−0.694188 + 0. i, −0.680962 + 0. i, −0.0896714 + 0. i, −0.215296 + 0. i},
{−0.166478 + 0. i, −0.163336 + 0. i, 0.373803 + 0. i, 0.897707 + 0. i}},
{{0.865034, 0., 0., 0.}, {0., 0.865011, 0., 0.},
{0., 0., 0.0428411, 0.}, {0., 0., 0., 0.042818}},
{{−0.197525 + 0. i, 0.123988 + 0. i, −0.960118 + 0. i, 0.154219 + 0. i},
{−0.823502 + 0. i, 0.517161 + 0. i, 0.230255 + 0. i, −0.0370335 + 0. i},
{−0.123989 + 0. i, −0.197525 + 0. i, −0.154219 + 0. i, −0.960118 + 0. i},
{−0.517161 + 0. i, −0.823502 + 0. i, 0.0370344 + 0. i, 0.230256 + 0. i}}}

Out[•]= 4.44089×10^{-16}

Out[•]= {{{0.215109 + 0. i, −0.0901164 + 0. i, 0.0142926 + 0. i, 0.972318 + 0. i},
{−0.896854 + 0. i, 0.375845 + 0. i, 0.003399 + 0. i, 0.233198 + 0. i},
{−0.0901166 + 0. i, −0.215114 + 0. i, −0.972317 + 0. i, 0.0142921 + 0. i},
{0.375845 + 0. i, 0.896853 + 0. i, −0.233202 + 0. i, 0.00340085 + 0. i}},
{{0.999097, 0., 0., 0.}, {0., 0.999074, 0., 0.},
{0., 0., 0.501746, 0.}, {0., 0., 0., 0.501723}},
{{−0.785111 + 0. i, 0.57377 + 0. i, 0.0507684 + 0. i, −0.227622 + 0. i},
{0.188262 + 0. i, −0.137647 + 0. i, 0.211909 + 0. i, −0.949055 + 0. i},
{0.57377 + 0. i, 0.785112 + 0. i, 0.227618 + 0. i, 0.0507712 + 0. i},
{−0.137648 + 0. i, −0.188257 + 0. i, 0.949056 + 0. i, 0.211908 + 0. i}}}

Out[•]= 4.44089×10^{-16}

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}$$

In[•]:= **(*Real and symmetric matrices, with high accuracy*)**

In[•]:= **MatrixForm[UI.ConjugateTranspose[UR]]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -0.412722 + 0. i & -0.0887222 + 0. i & 0.000010916 + 0. i & 0.0000247711 + 0. i \\ -0.0887218 + 0. i & -0.0640778 + 0. i & 0.0000894365 + 0. i & -0.0000268796 + 0. i \\ -0.0000225069 + 0. i & -0.0000235134 + 0. i & -0.412741 + 0. i & -0.0887303 + 0. i \\ -0.0000874431 + 0. i & 0.0000117579 + 0. i & -0.0887273 + 0. i & -0.064062 + 0. i \end{pmatrix}$$

In[•]:= **Max[Abs[UI.ConjugateTranspose[UR] − Transpose[UI.ConjugateTranspose[UR]]]]**

Out[•]= 0.00011295

In[]:=
```
MatrixForm[UR.ConjugateTranspose[UI]]
```

Out[]//MatrixForm=

$$\begin{pmatrix} -0.412722+0.\,i & -0.0887218+0.\,i & -0.0000225069+0.\,i & -0.0000874431+0.\,i \\ -0.0887222+0.\,i & -0.0640778+0.\,i & -0.0000235134+0.\,i & 0.0000117579+0.\,i \\ 0.000010916+0.\,i & 0.0000894365+0.\,i & -0.412741+0.\,i & -0.0887273+0.\,i \\ 0.0000247711+0.\,i & -0.0000268796+0.\,i & -0.0887303+0.\,i & -0.064062+0.\,i \end{pmatrix}$$

In[]:=
```
Max[Abs[UR.ConjugateTranspose[UI] - Transpose[UR.ConjugateTranspose[UI]]]]
```

Out[]=
```
0.00011295
```

In[]:=
```
A = {{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}
```

Out[]=
```
{{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}
```

In[]:=
```
Solve[f.A == c, {a11, a12, a13, a14, a21, a22, a23, a24, a31, a32, a33, a34, a41, a42, a43, a44}]
```

Out[]=
```
{{a11 → 0.0000897831 + 0. i, a12 → 0.000036652 + 0. i, a13 → 0.703564 + 0. i,
  a14 → -0.710632 + 0. i, a21 → 0.0000323095 + 0. i, a22 → -0.0000946283 + 0. i,
  a23 → -0.710632 + 0. i, a24 → -0.703564 + 0. i, a31 → -0.703572 + 0. i, a32 → -0.710624 + 0. i,
  a33 → 0.0000944007 + 0. i, a34 → -0.000032081 + 0. i, a41 → 0.710624 + 0. i,
  a42 → -0.703572 + 0. i, a43 → 0.0000368826 + 0. i, a44 → 0.00009001 + 0. i}}
```

```
In[•]:= a11 = 0.00008978313338295048` + 0.` ⅈ
        a12 = 0.000036652026357919395` + 0.` ⅈ
        a13 = 0.7035641166283789` + 0.` ⅈ
        a14 = -0.7106317783413194` + 0.` ⅈ
        a21 = 0.00003230949593134145` + 0.` ⅈ
        a22 = -0.00009462830142237808` + 0.` ⅈ
        a23 = -0.7106317777309963` + 0.` ⅈ
        a24 = -0.7035641168226701` + 0.` ⅈ
        a31 = -0.7035723471195443` + 0.` ⅈ
        a32 = -0.7106236292356297` + 0.` ⅈ
        a33 = 0.00009440066086783344` + 0.` ⅈ
        a34 = -0.00003208104252797817` + 0.` ⅈ
        a41 = 0.7106236298236789` + 0.` ⅈ
        a42 = -0.7035723468656908` + 0.` ⅈ
        a43 = 0.00003688261555204474` + 0.` ⅈ
        a44 = 0.00009000997523456999` + 0.` ⅈ
Out[•]= 0.0000897831 + 0. ⅈ

Out[•]= 0.000036652 + 0. ⅈ

Out[•]= 0.703564 + 0. ⅈ

Out[•]= -0.710632 + 0. ⅈ

Out[•]= 0.0000323095 + 0. ⅈ

Out[•]= -0.0000946283 + 0. ⅈ

Out[•]= -0.710632 + 0. ⅈ

Out[•]= -0.703564 + 0. ⅈ

Out[•]= -0.703572 + 0. ⅈ

Out[•]= -0.710624 + 0. ⅈ

Out[•]= 0.0000944007 + 0. ⅈ

Out[•]= -0.000032081 + 0. ⅈ

Out[•]= 0.710624 + 0. ⅈ

Out[•]= -0.703572 + 0. ⅈ

Out[•]= 0.0000368826 + 0. ⅈ

Out[•]= 0.00009001 + 0. ⅈ

In[•]:= Max[Abs[f.A - c]]
        Max[Abs[a.b.ConjugateTranspose[c] - UR]]
Out[•]= 3.33067 × 10^-16

Out[•]= 4.44089 × 10^-16
```

In[•]:=
```
B = {{b11, b12, b13, b14}, {b21, b22, b23, b24}, {b31, b32, b33, b34}, {b41, b42, b43, b44}}
```

Out[•]=
```
{{b11, b12, b13, b14}, {b21, b22, b23, b24}, {b31, b32, b33, b34}, {b41, b42, b43, b44}}
```

In[•]:=
```
Solve[d.B == a, {b11, b12, b13, b14, b21, b22, b23, b24, b31, b32, b33, b34, b41, b42, b43, b44}]
```

Out[•]=
```
{{b11 → -4.09217 × 10^-6 + 0. i, b12 → -4.34988 × 10^-6 + 0. i, b13 → -0.702851 + 0. i,
  b14 → 0.711337 + 0. i, b21 → 1.57155 × 10^-7 + 0. i, b22 → 8.59919 × 10^-6 + 0. i,
  b23 → 0.711337 + 0. i, b24 → 0.702851 + 0. i, b31 → 0.703506 + 0. i, b32 → 0.710689 + 0. i,
  b33 → -8.62209 × 10^-6 + 0. i, b34 → -1.26195 × 10^-7 + 0. i, b41 → -0.710689 + 0. i,
  b42 → 0.703506 + 0. i, b43 → -4.33061 × 10^-6 + 0. i, b44 → -4.0654 × 10^-6 + 0. i}}
```

```
In[•]:= b11 = -4.092172668451582`*^-6 + 0.` ⅈ
        b12 = -4.349882863576696`*^-6 + 0.` ⅈ
        b13 = -0.7028514541922289` + 0.` ⅈ
        b14 = 0.7113366525803387` + 0.` ⅈ
        b21 = 1.571547740263507`*^-7 + 0.` ⅈ
        b22 = 8.599188281626926`*^-6 + 0.` ⅈ
        b23 = 0.7113366525399729` + 0.` ⅈ
        b24 = 0.7028514542058337` + 0.` ⅈ
        b31 = 0.7035063883528934` + 0.` ⅈ
        b32 = 0.7106889343955701` + 0.` ⅈ
        b33 = -8.62209299445916`*^-6 + 0.` ⅈ
        b34 = -1.2619533592834702`*^-7 + 0.` ⅈ
        b41 = -0.7106889344360845` + 0.` ⅈ
        b42 = 0.7035063883397376` + 0.` ⅈ
        b43 = -4.330612018565357`*^-6 + 0.` ⅈ
        b44 = -4.065400530840143`*^-6 + 0.` ⅈ
Out[•]= -4.09217 × 10^-6 + 0. ⅈ

Out[•]= -4.34988 × 10^-6 + 0. ⅈ

Out[•]= -0.702851 + 0. ⅈ

Out[•]= 0.711337 + 0. ⅈ

Out[•]= 1.57155 × 10^-7 + 0. ⅈ

Out[•]= 8.59919 × 10^-6 + 0. ⅈ

Out[•]= 0.711337 + 0. ⅈ

Out[•]= 0.702851 + 0. ⅈ

Out[•]= 0.703506 + 0. ⅈ

Out[•]= 0.710689 + 0. ⅈ

Out[•]= -8.62209 × 10^-6 + 0. ⅈ

Out[•]= -1.26195 × 10^-7 + 0. ⅈ

Out[•]= -0.710689 + 0. ⅈ

Out[•]= 0.703506 + 0. ⅈ

Out[•]= -4.33061 × 10^-6 + 0. ⅈ

Out[•]= -4.0654 × 10^-6 + 0. ⅈ

In[•]:= B.b.ConjugateTranspose[A]

Out[•]= {{-0.0428294 + 0. ⅈ, -0.000031383 + 0. ⅈ, 1.3448 × 10^-6 + 0. ⅈ, 1.76277 × 10^-6 + 0. ⅈ},
         {0.0000544786 + 0. ⅈ, -0.0428296 + 0. ⅈ, -3.47021 × 10^-6 + 0. ⅈ, -1.30405 × 10^-6 + 0. ⅈ},
         {0.0000769141 + 0. ⅈ, -0.0000382446 + 0. ⅈ, -0.865022 + 0. ⅈ, -0.0000687032 + 0. ⅈ},
         {-0.0000328986 + 0. ⅈ, -0.0000771938 + 0. ⅈ, 0.0000918769 + 0. ⅈ, -0.865023 + 0. ⅈ}}
```

In[]:=
```
Inside = B.b.ConjugateTranspose[A] + I * e
```

Out[]=
$$\{\{-0.0428294 + 0.999097\,i,\ -0.000031383 + 0.\,i,\ 1.3448 \times 10^{-6} + 0.\,i,\ 1.76277 \times 10^{-6} + 0.\,i\},$$
$$\{0.0000544786 + 0.\,i,\ -0.0428296 + 0.999074\,i,\ -3.47021 \times 10^{-6} + 0.\,i,\ -1.30405 \times 10^{-6} + 0.\,i\},$$
$$\{0.0000769141 + 0.\,i,\ -0.0000382446 + 0.\,i,\ -0.865022 + 0.501746\,i,\ -0.0000687032 + 0.\,i\},$$
$$\{-0.0000328986 + 0.\,i,\ -0.0000771938 + 0.\,i,\ 0.0000918769 + 0.\,i,\ -0.865023 + 0.501723\,i\}\}$$

In[]:=
```
(*Absolute values of eigenvalues is 1, as it should be*)
Abs[Inside[[1]][[1]]]
Abs[Inside[[2]][[2]]]
Abs[Inside[[3]][[3]]]
Abs[Inside[[4]][[4]]]
```

Out[]= 1.00001

Out[]= 0.999992

Out[]= 1.00001

Out[]= 0.999995

In[]:=
```
DiagonalPart = {{Inside[[1]][[1]], 0, 0, 0}, {0, Inside[[2]][[2]], 0, 0},
  {0, 0, Inside[[3]][[3]], 0}, {0, 0, 0, Inside[[4]][[4]]}}
```

Out[]=
$$\{\{-0.0428294 + 0.999097\,i,\ 0,\ 0,\ 0\},\ \{0,\ -0.0428296 + 0.999074\,i,\ 0,\ 0\},$$
$$\{0,\ 0,\ -0.865022 + 0.501746\,i,\ 0\},\ \{0,\ 0,\ 0,\ -0.865023 + 0.501723\,i\}\}$$

In[]:=
```
Max[Abs[Abs[Inside] - {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}]]
```

Out[]= 0.0000918769

In[]:=
```
(*Phases are:*)
Φ0 = Log[Inside[[1]][[1]]] / I
Φ1 = Log[Inside[[2]][[2]]] / I
Φ2 = Log[Inside[[3]][[3]]] / I
Φ3 = Log[Inside[[4]][[4]]] / I
```

Out[]= $1.61364 - 0.0000146166\,i$

Out[]= $1.61364 + 8.11255 \times 10^{-6}\,i$

Out[]= $2.61598 - 6.19075 \times 10^{-6}\,i$

Out[]= $2.616 + 5.08183 \times 10^{-6}\,i$

In[]:=
```
Max[Abs[UP - d.Inside.ConjugateTranspose[f]]]
```

Out[]= 6.75322×10^{-16}

In[]:=
```
Max[Abs[M.UP.ConjugateTranspose[M] - UFirstStage]]
```

Out[]= 2.77556×10^{-16}

In[]:=
```
Max[Abs[M.d.ConjugateTranspose[M].M.DiagonalPart.ConjugateTranspose[M].
    M.ConjugateTranspose[f].ConjugateTranspose[M] - UFirstStage]]
```

Out[]= 0.0000681932

```
In[•]:= (*Now we find UA and UB*)

In[•]:= Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]

Out[•]= {{UA11 UB11, UA11 UB12, UA12 UB11, UA12 UB12}, {UA11 UB21, UA11 UB22, UA12 UB21, UA12 UB22},
  {UA21 UB11, UA21 UB12, UA22 UB11, UA22 UB12}, {UA21 UB21, UA21 UB22, UA22 UB21, UA22 UB22}}

In[•]:= WeHave = M.d.ConjugateTranspose[M]
        Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]
        UAMatr = {{UA11, UA12}, {UA21, UA22}}
        UBMatr = {{UB11, UB12}, {UB21, UB22}}
        Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[2]], Search[[1]][[1]] / Search[[1]][[2]]}]
        Reverse[{WeHave[[1]][[3]] / WeHave[[1]][[4]], Search[[1]][[3]] / Search[[1]][[4]]}]
        Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[3]], Search[[1]][[1]] / Search[[1]][[3]]}]
        Reverse[{WeHave[[1]][[2]] / WeHave[[1]][[4]], Search[[1]][[2]] / Search[[1]][[4]]}]
        Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[2]], Search[[2]][[1]] / Search[[2]][[2]]}]
        Reverse[{WeHave[[2]][[3]] / WeHave[[2]][[4]], Search[[2]][[3]] / Search[[2]][[4]]}]
        Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[3]], Search[[2]][[1]] / Search[[2]][[3]]}]
        Reverse[{WeHave[[2]][[2]] / WeHave[[2]][[4]], Search[[2]][[2]] / Search[[2]][[4]]}]
        Reverse[{WeHave[[3]][[1]] / WeHave[[3]][[2]], Search[[3]][[1]] / Search[[3]][[2]]}]
        Reverse[{WeHave[[3]][[3]] / WeHave[[3]][[4]], Search[[3]][[3]] / Search[[3]][[4]]}]
        Reverse[{WeHave[[4]][[1]] / WeHave[[4]][[2]], Search[[4]][[1]] / Search[[4]][[2]]}]
        Reverse[{WeHave[[4]][[3]] / WeHave[[4]][[4]], Search[[4]][[3]] / Search[[4]][[4]]}]
        Reverse[{WeHave[[3]][[1]] / WeHave[[1]][[4]], Search[[3]][[1]] / Search[[1]][[4]]}]
        Reverse[{WeHave[[4]][[1]] / WeHave[[1]][[4]], Search[[4]][[1]] / Search[[1]][[4]]}]
        Reverse[{WeHave[[4]][[2]] / WeHave[[3]][[3]], Search[[4]][[2]] / Search[[3]][[3]]}]
        Reverse[{WeHave[[1]][[2]] / WeHave[[3]][[1]], Search[[1]][[2]] / Search[[3]][[1]]}]

Out[•]= {{0.109255 - 0.298237 i, 0.455573 - 0.071543 i, 0.44128 + 0.161659 i, 0.105854 + 0.674082 i},
  {0.0715405 - 0.455573 i, -0.298236 + 0.109256 i,
   0.674081 + 0.105857 i, -0.161657 - 0.441281 i}, {0.161657 - 0.441281 i,
   0.674081 - 0.105857 i, -0.298236 - 0.109256 i, -0.0715405 - 0.455573 i},
  {0.105854 - 0.674082 i, -0.44128 + 0.161659 i, -0.455573 - 0.071543 i, 0.109255 + 0.298237 i}}

Out[•]= {{UA11 UB11, UA11 UB12, UA12 UB11, UA12 UB12}, {UA11 UB21, UA11 UB22, UA12 UB21, UA12 UB22},
  {UA21 UB11, UA21 UB12, UA22 UB11, UA22 UB12}, {UA21 UB21, UA21 UB22, UA22 UB21, UA22 UB22}}

Out[•]= {{UA11, UA12}, {UA21, UA22}}

Out[•]= {{UB11, UB12}, {UB21, UB22}}

Out[•]= {UB11/UB12, 0.334377 - 0.602131 i}

Out[•]= {UB11/UB12, 0.334377 - 0.602131 i}

Out[•]= {UA11/UA12, -3.59848 × 10^-6 - 0.675843 i}

Out[•]= {UA11/UA12, -3.59848 × 10^-6 - 0.675843 i}
```

Out[•]= $\left\{\frac{\text{UB21}}{\text{UB22}}, -0.704887 + 1.26933\,i\right\}$

Out[•]= $\left\{\frac{\text{UB21}}{\text{UB22}}, -0.704887 + 1.26933\,i\right\}$

Out[•]= $\left\{\frac{\text{UA11}}{\text{UA12}}, -3.59848 \times 10^{-6} - 0.675843\,i\right\}$

Out[•]= $\left\{\frac{\text{UA11}}{\text{UA12}}, -3.59848 \times 10^{-6} - 0.675843\,i\right\}$

Out[•]= $\left\{\frac{\text{UB11}}{\text{UB12}}, 0.334377 - 0.602131\,i\right\}$

Out[•]= $\left\{\frac{\text{UB11}}{\text{UB12}}, 0.334377 - 0.602131\,i\right\}$

Out[•]= $\left\{\frac{\text{UB21}}{\text{UB22}}, -0.704887 + 1.26933\,i\right\}$

Out[•]= $\left\{\frac{\text{UB21}}{\text{UB22}}, -0.704887 + 1.26933\,i\right\}$

Out[•]= $\left\{\frac{\text{UA21 UB11}}{\text{UA12 UB12}}, -0.602132 - 0.334374\,i\right\}$

Out[•]= $\left\{\frac{\text{UA21 UB21}}{\text{UA12 UB12}}, -0.951867 - 0.30651\,i\right\}$

Out[•]= $\left\{\frac{\text{UA21 UB22}}{\text{UA22 UB11}}, 1.12948 - 0.955826\,i\right\}$

Out[•]= $\left\{\frac{\text{UA11 UB12}}{\text{UA21 UB11}}, 0.476393 + 0.857867\,i\right\}$

In[•]:= **UB11 = (0.3343766999311674` - 0.6021306191753054` i) * UB12**

Out[•]= $(0.334377 - 0.602131\,i)\ \text{UB12}$

In[•]:= **UA11 = UA12 ***
(-3.5984807505717864`\*^-6 - 0.6758427521645463` i) / (1.` + 5.551115123125783`*^-17 i)

Out[•]= $(-3.59848 \times 10^{-6} - 0.675843\,i)\ \text{UA12}$

In[•]:= **UB21 = UB22 * (-0.7048872452335632` + 1.2693294524069183` i) / (1.` + 0.` i)**

Out[•]= $(-0.704887 + 1.26933\,i)\ \text{UB22}$

In[•]:= **UA21 = UA12 * (-0.6021323492125987` - 0.33437358453833077` i) /**
(0.3343766999311674` - 0.6021306191753054` i)

Out[•]= $(-5.17394 \times 10^{-6} - 1.\,i)\ \text{UA12}$

In[•]:= **UB22 = UB12 * (-0.9518674673297047` - 0.3065099095287677` i) /**
(1.269333099435022` + 0.704880677788299` i)

Out[•]= $(-0.675639 + 0.13372\,i)\ \text{UB12}$

```
In[•]:= UA12 = -UA22 * (1.1294752651880389` - 0.955825701931891` i) /
          (0.7633477689544572` - 0.6459877581133048` i)
```

```
Out[•]= (-1.47963 + 2.22676 × 10^-7 i) UA22
```

```
In[•]:= ConjugateTranspose[{{UA11, UA12}, {UA21, UA22}}].{{UA11, UA12}, {UA21, UA22}}
```

```
Out[•]= {{(3.18932 + 0. i) UA22 Conjugate[UA22],
    (5.7993 × 10^-17 - 2.44249 × 10^-15 i) UA22 Conjugate[UA22]},
   {(5.7993 × 10^-17 + 2.44249 × 10^-15 i) UA22 Conjugate[UA22],
    (3.18932 + 0. i) UA22 Conjugate[UA22]}}
```

```
In[•]:= UA22 = (1 / ((3.1893171471451818` + 0.` i) ^ (1 / 2))) * Exp[I * ψ]
```

```
Out[•]= (0.559952 + 0. i) e^(i ψ)
```

```
In[•]:= ConjugateTranspose[{{UB11, UB12}, {UB21, UB22}}].{{UB11, UB12}, {UB21, UB22}}
```

```
Out[•]= {{(1.47437 + 0. i) UB12 Conjugate[UB12],
    (4.996 × 10^-16 + 3.33067 × 10^-16 i) UB12 Conjugate[UB12]},
   {(4.996 × 10^-16 - 3.33067 × 10^-16 i) UB12 Conjugate[UB12],
    (1.47437 + 0. i) UB12 Conjugate[UB12]}}
```

```
In[•]:= UB12 = (1 / ((1.4743690600052934` + 0.` i) ^ (1 / 2))) * Exp[I * ϕ]
```

```
Out[•]= (0.823563 + 0. i) e^(i ϕ)
```

```
In[•]:= ϕ = 0
```

```
Out[•]= 0
```

```
In[•]:= Search
```

```
Out[•]= {{(0.277677 + 0.154198 i) e^(i ψ), (2.5248 × 10^-6 + 0.461156 i) e^(i ψ),
    (-0.228159 + 0.410859 i) e^(i ψ), (-0.682342 + 1.02688 × 10^-7 i) e^(i ψ)},
   {(0.43896 + 0.141349 i) e^(i ψ), (-0.0616674 - 0.311575 i) e^(i ψ),
    (-0.209148 + 0.649499 i) e^(i ψ), (0.461017 - 0.0912428 i) e^(i ψ)},
   {(0.41086 + 0.228157 i) e^(i ψ), (3.63309 × 10^-6 + 0.682342 i) e^(i ψ), (0.1542 - 0.277676 i) e^(i ψ),
    (0.461156 + 0. i) e^(i ψ)}, {(0.6495 + 0.209145 i) e^(i ψ), (-0.0912452 - 0.461017 i) e^(i ψ),
    (0.141351 - 0.438959 i) e^(i ψ), (-0.311575 + 0.0616657 i) e^(i ψ)}}
```

```
In[•]:= WeHave
```

```
Out[•]= {{0.109255 - 0.298237 i, 0.455573 - 0.071543 i, 0.44128 + 0.161659 i, 0.105854 + 0.674082 i},
  {0.0715405 - 0.455573 i, -0.298236 + 0.109256 i,
   0.674081 + 0.105857 i, -0.161657 - 0.441281 i}, {0.161657 - 0.441281 i,
   0.674081 - 0.105857 i, -0.298236 - 0.109256 i, -0.0715405 - 0.455573 i},
  {0.105854 - 0.674082 i, -0.44128 + 0.161659 i, -0.455573 - 0.071543 i, 0.109255 + 0.298237 i}}
```

```
In[•]:= ψ = Log[WeHave[[1]][[1]] / (0.27767710604452817` + 0.1541983644795603` i)] / I
```

```
Out[•]= -1.72656 + 0. i
```

```
In[•]:= Max[Abs[Search - WeHave]]
```

```
Out[•]= 6.66278 × 10^-16
```

In[•]:=
```
Max[Abs[KroneckerProduct[UAMatr, UBMatr] - WeHave]]
```

Out[•]=
$$6.66278 \times 10^{-16}$$

In[•]:=
```
WeHaveAnother = M.ConjugateTranspose[f].ConjugateTranspose[M]
SearchAnother =
 KroneckerProduct[{{VA11, VA12}, {VA21, VA22}}, {{VB11, VB12}, {VB21, VB22}}]
VAMatr = {{VA11, VA12}, {VA21, VA22}}
VBMatr = {{VB11, VB12}, {VB21, VB22}}
```

Out[•]=
$$\{\{-0.286601 - 0.044987\,i, -0.187643 - 0.0687452\,i, -0.761412 - 0.119516\,i, -0.49851 - 0.182635\,i\}, \{-0.0687441 - 0.187643\,i, 0.0449853 + 0.286602\,i, -0.182632 - 0.49851\,i, 0.119512 + 0.761413\,i\}, \{-0.119512 + 0.761413\,i, -0.182632 + 0.49851\,i, 0.0449853 - 0.286602\,i, 0.0687441 - 0.187643\,i\}, \{-0.49851 + 0.182635\,i, 0.761412 - 0.119516\,i, 0.187643 - 0.0687452\,i, -0.286601 + 0.044987\,i\}\}$$

Out[•]=
$$\{\{(1.42407 - 0.281978\,i)\,VA11\,VB12, VA11\,VB12, (1.42407 - 0.281978\,i)\,VA12\,VB12, VA12\,VB12\}, \{VA11\,VB21, VA11\,VB22, VA12\,VB21, VA12\,VB22\}, \{(1.42407 - 0.281978\,i)\,VA21\,VB12, VA21\,VB12, (1.42407 - 0.281978\,i)\,VA22\,VB12, VA22\,VB12\}, \{VA21\,VB21, VA21\,VB22, VA22\,VB21, VA22\,VB22\}\}$$

Out[•]=
$$\{\{VA11, VA12\}, \{VA21, VA22\}\}$$

Out[•]=
$$\{\{(1.42407 - 0.281978\,i)\,VB12, VB12\}, \{VB21, VB22\}\}$$

In[ ]:=

```
Reverse[{WeHaveAnother[[1]][[1]] / WeHaveAnother[[1]][[2]],
  SearchAnother[[1]][[1]] / SearchAnother[[1]][[2]]}]
Reverse[{WeHaveAnother[[1]][[3]] / WeHaveAnother[[1]][[4]],
  SearchAnother[[1]][[3]] / SearchAnother[[1]][[4]]}]
Reverse[{WeHaveAnother[[1]][[1]] / WeHaveAnother[[1]][[3]],
  SearchAnother[[1]][[1]] / SearchAnother[[1]][[3]]}]
Reverse[{WeHaveAnother[[1]][[2]] / WeHaveAnother[[1]][[4]],
  SearchAnother[[1]][[2]] / SearchAnother[[1]][[4]]}]
Reverse[{WeHaveAnother[[2]][[1]] / WeHaveAnother[[2]][[2]],
  SearchAnother[[2]][[1]] / SearchAnother[[2]][[2]]}]
Reverse[{WeHaveAnother[[2]][[3]] / WeHaveAnother[[2]][[4]],
  SearchAnother[[2]][[3]] / SearchAnother[[2]][[4]]}]
Reverse[{WeHaveAnother[[2]][[1]] / WeHaveAnother[[2]][[3]],
  SearchAnother[[2]][[1]] / SearchAnother[[2]][[3]]}]
Reverse[{WeHaveAnother[[2]][[2]] / WeHaveAnother[[2]][[4]],
  SearchAnother[[2]][[2]] / SearchAnother[[2]][[4]]}]
Reverse[{WeHaveAnother[[3]][[1]] / WeHaveAnother[[3]][[2]],
  SearchAnother[[3]][[1]] / SearchAnother[[3]][[2]]}]
Reverse[{WeHaveAnother[[3]][[3]] / WeHaveAnother[[3]][[4]],
  SearchAnother[[3]][[3]] / SearchAnother[[3]][[4]]}]
Reverse[{WeHaveAnother[[4]][[1]] / WeHaveAnother[[4]][[2]],
  SearchAnother[[4]][[1]] / SearchAnother[[4]][[2]]}]
Reverse[{WeHaveAnother[[4]][[3]] / WeHaveAnother[[4]][[4]],
  SearchAnother[[4]][[3]] / SearchAnother[[4]][[4]]}]
Reverse[{WeHaveAnother[[3]][[1]] / WeHaveAnother[[1]][[4]],
  SearchAnother[[3]][[1]] / SearchAnother[[1]][[4]]}]
Reverse[{WeHaveAnother[[4]][[1]] / WeHaveAnother[[1]][[4]],
  SearchAnother[[4]][[1]] / SearchAnother[[1]][[4]]}]
Reverse[{WeHaveAnother[[4]][[2]] / WeHaveAnother[[3]][[3]],
  SearchAnother[[4]][[2]] / SearchAnother[[3]][[3]]}]
Reverse[{WeHaveAnother[[1]][[2]] / WeHaveAnother[[3]][[1]],
  SearchAnother[[1]][[2]] / SearchAnother[[3]][[1]]}]
Reverse[{WeHaveAnother[[3]][[2]] / WeHaveAnother[[3]][[1]],
  SearchAnother[[3]][[2]] / SearchAnother[[3]][[1]]}]
```

Out[ ]= $\{1.42407 - 0.281978\,i,\ 1.42407 - 0.281978\,i\}$

Out[ ]= $\{1.42407 - 0.281978\,i,\ 1.42407 - 0.281978\,i\}$

Out[ ]= $\left\{\dfrac{\left(1. + 2.77556 \times 10^{-17}\,i\right)\ \text{VA11}}{\text{VA12}},\ 0.376408 + 1.15795 \times 10^{-7}\,i\right\}$

Out[ ]= $\left\{\dfrac{\text{VA11}}{\text{VA12}},\ 0.376408 + 1.15795 \times 10^{-7}\,i\right\}$

Out[ ]= $\left\{\dfrac{\text{VB21}}{\text{VB22}},\ -0.675719 + 0.133798\,i\right\}$

Out[•]= $\left\{\frac{\text{VB21}}{\text{VB22}}, -0.675719 + 0.133798\,i\right\}$

Out[•]= $\left\{\frac{\text{VA11}}{\text{VA12}}, 0.376408 + 1.15795 \times 10^{-7}\,i\right\}$

Out[•]= $\left\{\frac{\text{VA11}}{\text{VA12}}, 0.376408 + 1.15795 \times 10^{-7}\,i\right\}$

Out[•]= $\{1.42407 - 0.281978\,i, 1.42407 - 0.281978\,i\}$

Out[•]= $\{1.42407 - 0.281978\,i, 1.42407 - 0.281978\,i\}$

Out[•]= $\left\{\frac{\text{VB21}}{\text{VB22}}, -0.675719 + 0.133798\,i\right\}$

Out[•]= $\left\{\frac{\text{VB21}}{\text{VB22}}, -0.675719 + 0.133798\,i\right\}$

Out[•]= $\left\{\frac{(1.42407 - 0.281978\,i)\ \text{VA21}}{\text{VA12}}, -0.281985 - 1.42407\,i\right\}$

Out[•]= $\left\{\frac{\text{VA21 VB21}}{\text{VA12 VB12}}, 0.763325 - 0.646015\,i\right\}$

Out[•]= $\left\{\frac{(0.675719 + 0.133798\,i)\ \text{VA21 VB22}}{\text{VA22 VB12}}, 0.813957 + 2.52893\,i\right\}$

Out[•]= $\left\{\frac{(0.675719 + 0.133798\,i)\ \text{VA11}}{\text{VA21}}, -0.050364 + 0.254345\,i\right\}$

Out[•]= $\{0.675719 + 0.133798\,i, 0.675719 + 0.133798\,i\}$

In[1]:=
```
VB11 = (1.4240716946570489` - 0.2819779681453262` i) * VB12
```

Out[1]= $(1.42407 - 0.281978\,i)\ \text{VB12}$

In[•]:=
```
VA11 =
 VA12 * (0.3764075799296912` + 1.15795329609647`*^-7 i) / (1.` + 2.7755575615628914`*^-17 i)
```

Out[•]= $(0.376408 + 1.15795 \times 10^{-7}\,i)\ \text{VA12}$

In[•]:=
```
VB21 = VB22 * (-0.6757187466174656` + 0.13379789790343855` i) /
    (0.9999999999999999` - 5.293955920339377`*^-23 i)
```

Out[•]= $(-0.675719 + 0.133798\,i)\ \text{VB22}$

In[•]:=
```
VA21 = VA12 * (-0.28198541482963374` - 1.4240702201317619` i) /
    (1.4240716946570489` - 0.2819779681453262` i)
```

Out[•]= $(-5.22915 \times 10^{-6} - 1.\,i)\ \text{VA12}$

In[•]:=
```
VB22 = VB12 * (0.763324786791514` - 0.6460149145876506` i) /
    (0.13380143133803013` + 0.6757180469586054` i)
```

Out[•]= $(-0.704724 - 1.26919\,i)\ \text{VB12}$

```
In[•]:= VA12 = -VA22 * (0.8139565523223147` + 2.5289328066133243` i) /
          (0.9519077811705007` - 0.306384686541375` i)
```

```
Out[•]= (0.0000147095 - 2.65669 i) VA22
```

```
In[•]:= VAMatr.ConjugateTranspose[VAMatr]
```

```
Out[•]= {{(8.05803 + 0. i) VA22 Conjugate[VA22],
          (-1.38795 × 10^-15 - 3.55271 × 10^-15 i) VA22 Conjugate[VA22]},
         {(-1.38795 × 10^-15 + 3.55271 × 10^-15 i) VA22 Conjugate[VA22],
          (8.05803 + 0. i) VA22 Conjugate[VA22]}}
```

```
In[•]:= VA22 = Exp[I * ϕV] / ((8.058026409433554` + 0.` i) ^ (1 / 2))
```

```
Out[•]= (0.352278 + 0. i) ℯ^(i ϕV)
```

```
In[•]:= VBMatr.ConjugateTranspose[VBMatr]
```

```
Out[•]= {{(3.10749 + 0. i) VB12 Conjugate[VB12],
          (5.55112 × 10^-16 + 2.44249 × 10^-15 i) VB12 Conjugate[VB12]},
         {(5.55112 × 10^-16 - 2.44249 × 10^-15 i) VB12 Conjugate[VB12],
          (3.10749 + 0. i) VB12 Conjugate[VB12]}}
```

```
In[•]:= VB12 = Exp[I * ψV] / ((3.1074917660427652` + 0.` i) ^ (1 / 2))
```

```
Out[•]= (0.567277 + 0. i) ℯ^(i ψV)
```

```
In[•]:= WeHaveAnother
```

```
Out[•]= {{-0.286601 - 0.044987 i, -0.187643 - 0.0687452 i, -0.761412 - 0.119516 i,
          -0.49851 - 0.182635 i}, {-0.0687441 - 0.187643 i, 0.0449853 + 0.286602 i,
          -0.182632 - 0.49851 i, 0.119512 + 0.761413 i}, {-0.119512 + 0.761413 i,
          -0.182632 + 0.49851 i, 0.0449853 - 0.286602 i, 0.0687441 - 0.187643 i},
         {-0.49851 + 0.182635 i, 0.761412 - 0.119516 i, 0.187643 - 0.0687452 i, -0.286601 + 0.044987 i}}
```

```
In[•]:= SearchAnother
```

```
Out[•]= {{(-0.0563486 - 0.284586 i) ℯ^(i ϕV+i ψV), (1.16794 × 10^-6 - 0.199839 i) ℯ^(i ϕV+i ψV),
          (-0.149701 - 0.756057 i) ℯ^(i ϕV+i ψV), (2.93954 × 10^-6 - 0.530912 i) ℯ^(i ϕV+i ψV)},
         {(0.152544 - 0.129097 i) ℯ^(i ϕV+i ψV), (-0.253636 + 0.14083 i) ℯ^(i ϕV+i ψV),
          (0.405262 - 0.342973 i) ℯ^(i ϕV+i ψV), (-0.673832 + 0.374143 i) ℯ^(i ϕV+i ψV)},
         {(-0.756056 + 0.149705 i) ℯ^(i ϕV+i ψV), (-0.530912 - 1.63326 × 10^-7 i) ℯ^(i ϕV+i ψV),
          (0.284585 - 0.0563502 i) ℯ^(i ϕV+i ψV), (0.199839 + 0. i) ℯ^(i ϕV+i ψV)},
         {(-0.342975 - 0.40526 i) ℯ^(i ϕV+i ψV), (0.374146 + 0.673831 i) ℯ^(i ϕV+i ψV),
          (0.129098 + 0.152543 i) ℯ^(i ϕV+i ψV), (-0.140832 - 0.253635 i) ℯ^(i ϕV+i ψV)}}
```

```
In[•]:= ϕV = 0
```

```
Out[•]= 0
```

```
In[•]:= ψV = Log[WeHaveAnother[[1]][[1]] / (-0.05634858587695057` - 0.2845856651180952` i)] / I
```

```
Out[•]= -1.21963 + 0. i
```

```
In[•]:= Max[Abs[SearchAnother - WeHaveAnother]]
Out[•]= 1.40878 × 10^-15

In[•]:=

In[•]:= Max[Abs[UFirstStage - KroneckerProduct[UAMatr, UBMatr].M.
      DiagonalPart.ConjugateTranspose[M].KroneckerProduct[VAMatr, VBMatr]]]
Out[•]= 0.0000681932

In[•]:=

In[•]:= Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
    Θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
    Θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
    Θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
    Θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
    σx = {{0, 1}, {1, 0}}
    σy = {{0, -I}, {I, 0}}
    σz = {{1, 0}, {0, -1}}
    ϕ1 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] + KroneckerProduct[{{0, 1}}, {{0, 1}}])
    ϕ2 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] - KroneckerProduct[{{0, 1}}, {{0, 1}}])
    ϕ3 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] - KroneckerProduct[{{0, 1}}, {{1, 0}}])
    ϕ4 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] + KroneckerProduct[{{0, 1}}, {{1, 0}}])
    DMatrix =
     {{Exp[I * Φ0], 0, 0, 0}, {0, Exp[I * Φ1], 0, 0}, {0, 0, Exp[I * Φ2], 0}, {0, 0, 0, Exp[I * Φ3]}}
    FullSimplify[M.DMatrix.ConjugateTranspose[M] -
      Exp[I * Θ0] * MatrixExp[I * (Θ1 * KroneckerProduct[σx, σx] +
          Θ2 * KroneckerProduct[σy, σy] + Θ3 * KroneckerProduct[σz, σz])]]
Out[•]= {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}

Out[•]= 2.11481 - 1.90325 × 10^-6 i

Out[•]= -0.501176 - 1.34879 × 10^-6 i

Out[•]= 5.2822 × 10^-6 + 8.50044 × 10^-6 i

Out[•]= 4.66778 × 10^-6 - 2.86414 × 10^-6 i

Out[•]= {{0, 1}, {1, 0}}

Out[•]= {{0, -i}, {i, 0}}

Out[•]= {{1, 0}, {0, -1}}

Out[•]= {{1/Sqrt[2], 0, 0, 1/Sqrt[2]}}
```

Out[ ]= $\left\{\left\{-\frac{\text{i}}{\sqrt{2}}, 0, 0, \frac{\text{i}}{\sqrt{2}}\right\}\right\}$

Out[ ]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[ ]= $\left\{\left\{0, -\frac{\text{i}}{\sqrt{2}}, -\frac{\text{i}}{\sqrt{2}}, 0\right\}\right\}$

Out[ ]= $\{\{-0.0428294 + 0.999097\,\text{i}, 0, 0, 0\}, \{0, -0.0428296 + 0.999074\,\text{i}, 0, 0\},$
$\{0, 0, -0.865022 + 0.501746\,\text{i}, 0\}, \{0, 0, 0, -0.865023 + 0.501723\,\text{i}\}\}$

Out[ ]= $\left\{\left\{2.22045 \times 10^{-16} + 1.11022 \times 10^{-16}\,\text{i}, 0. + 0.\,\text{i}, 0. + 0.\,\text{i}, 0. - 2.22045 \times 10^{-16}\,\text{i}\right\},\right.$
$\left\{0. + 0.\,\text{i}, 1.66533 \times 10^{-16} + 2.22045 \times 10^{-16}\,\text{i}, 5.55112 \times 10^{-17} + 0.\,\text{i}, 0. + 0.\,\text{i}\right\},$
$\left\{0. + 0.\,\text{i}, 0. - 2.77556 \times 10^{-17}\,\text{i}, 1.11022 \times 10^{-16} + 3.33067 \times 10^{-16}\,\text{i}, 0. + 0.\,\text{i}\right\},$
$\left.\left\{0. - 2.22045 \times 10^{-16}\,\text{i}, 0. + 0.\,\text{i}, 0. + 0.\,\text{i}, 2.77556 \times 10^{-16} + 0.\,\text{i}\right\}\right\}$

In[ ]:=
```
Max[Abs[M.DMatrix.ConjugateTranspose[M] -
    Exp[I * θ0] * MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] +
        θ2 * KroneckerProduct[σy, σy] + θ3 * KroneckerProduct[σz, σz])]]]
```

Out[ ]= 3.51083×10^{-16}

In[ ]:=
```
Max[Abs[M.DiagonalPart.ConjugateTranspose[M] -
    Exp[I * θ0] * MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] +
        θ2 * KroneckerProduct[σy, σy] + θ3 * KroneckerProduct[σz, σz])]]]
```

Out[ ]= 3.51083×10^{-16}

In[•]:=
```
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
MatrixForm[CNOT1]
MatrixForm[CNOT2]
Ry[θ_] := {{Cos[θ/2], Sin[θ/2]}, {-Sin[θ/2], Cos[θ/2]}}
Rz[α_] := {{e^(i α/2), 0}, {0, e^(-i α/2)}}
Unit2 = {{1, 0}, {0, 1}}
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
ϕ1 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] + KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ2 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] - KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ3 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] - KroneckerProduct[{{0, 1}}, {{1, 0}}])
ϕ4 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] + KroneckerProduct[{{0, 1}}, {{1, 0}}])
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
H = 1/Sqrt[2] * {{1, 1}, {1, -1}}
CNOT2Trial = KroneckerProduct[H, H].CNOT1.KroneckerProduct[H, H]
CNOT2Trial - CNOT2
U3[θ_, ϕ_, λ_] :=
 {{Cos[θ/2], -Exp[I * λ] * Sin[θ/2]}, {Exp[I * ϕ] * Sin[θ/2], Exp[I * (ϕ + λ)] * Cos[θ/2]}}
FullSimplify[Exp[I * π / 4] * KroneckerProduct[Rz[-π/2], Unit2].CNOT2.
     KroneckerProduct[Unit2, Ry[2 * θ2 - π/2]].CNOT1.KroneckerProduct[
      Rz[2 * θ3 - π/2], Ry[π/2 - 2 * θ1]].CNOT2.KroneckerProduct[Unit2, Rz[π/2]] -
  MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] + θ2 * KroneckerProduct[σy, σy] +
       θ3 * KroneckerProduct[σz, σz])]]
Max[Abs[Exp[I * π / 4] * KroneckerProduct[Rz[-π/2], Unit2].CNOT2.
      KroneckerProduct[Unit2, Ry[2 * θ2 - π/2]].CNOT1.KroneckerProduct[
       Rz[2 * θ3 - π/2], Ry[π/2 - 2 * θ1]].CNOT2.KroneckerProduct[Unit2, Rz[π/2]] -
   MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] + θ2 * KroneckerProduct[σy, σy] +
        θ3 * KroneckerProduct[σz, σz])]]]
```

Out[•]=
```
{{0, 1}, {1, 0}}
```

Out[•]= $\{\{0, -i\}, \{i, 0\}\}$

Out[•]= $\{\{1, 0\}, \{0, -1\}\}$

Out[•]= $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}$

Out[•]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Out[•]= $\{\{1, 0\}, \{0, 1\}\}$

Out[•]= $\{\{0, 1\}, \{1, 0\}\}$

Out[•]= $\{\{0, -i\}, \{i, 0\}\}$

Out[•]= $\{\{1, 0\}, \{0, -1\}\}$

Out[•]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[•]= $\left\{\left\{-\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[•]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[•]= $\left\{\left\{0, -\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right\}\right\}$

Out[•]= $\{\{1, 1, -1, 1\}, \{1, 1, 1, -1\}, \{1, -1, -1, -1\}, \{1, -1, 1, 1\}\}$

Out[•]= $2.11481 - 1.90325 \times 10^{-6}\, i$

Out[•]= $-0.501176 - 1.34879 \times 10^{-6}\, i$

Out[•]= $5.2822 \times 10^{-6} + 8.50044 \times 10^{-6}\, i$

Out[•]= $4.66778 \times 10^{-6} - 2.86414 \times 10^{-6}\, i$

Out[•]= $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}$

Out[•]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[•]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[•]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[•]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[•]= $\{\{-1.11022\times10^{-16}+2.25128\times10^{-16}\,i,\ 0.+0.\,i,\ 0.+0.\,i,\ 7.7717\times10^{-17}+1.66533\times10^{-16}\,i\},$
$\{0.+0.\,i,\ 1.11022\times10^{-16}-3.88126\times10^{-17}\,i,\ 3.57482\times10^{-17}-5.55112\times10^{-17}\,i,\ 0.+0.\,i\},$
$\{0.+0.\,i,\ 3.57482\times10^{-17}+0.\,i,\ 2.22045\times10^{-16}-3.88122\times10^{-17}\,i,\ 0.+0.\,i\},$
$\{7.77187\times10^{-17}+1.66533\times10^{-16}\,i,\ 0.+0.\,i,\ 0.+0.\,i,\ -2.22045\times10^{-16}+2.25128\times10^{-16}\,i\}\}$

Out[•]= 3.16206×10^{-16}

In[•]:=
```
Max[Abs[UFirstStage - KroneckerProduct[UAMatr, UBMatr].M.
    DiagonalPart.ConjugateTranspose[M].KroneckerProduct[VAMatr, VBMatr]]]
```

Out[•]= 0.0000681932

In[•]:=
```
Max[Abs[M.DiagonalPart.ConjugateTranspose[M] -
   Exp[I * θ0] * MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] +
        θ2 * KroneckerProduct[σy, σy] + θ3 * KroneckerProduct[σz, σz])]]]
```

Out[•]= 3.51083×10^{-16}

In[•]:=
```
Max[Abs[UFirstStage - Exp[I * θ0 + I * π / 4] * KroneckerProduct[UAMatr, UBMatr].
     KroneckerProduct[Rz[-π / 2], Unit2].CNOT2.KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
     CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].CNOT2.
     KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[VAMatr, VBMatr]]]
```

Out[•]= 0.0000681932

In[•]:=

In[•]:=
```
Max[Abs[UFirstStage - Exp[I * θ0 + I * π / 4] * KroneckerProduct[UAMatr, UBMatr].
     KroneckerProduct[Rz[-π / 2], Unit2].CNOT2.KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
     CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].CNOT2.
     KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[VAMatr, VBMatr]]]
```

Out[•]= 0.0000681932

In[•]:=
```
Max[Abs[UFirstStage - Exp[I * θ0 + I * π / 4] * KroneckerProduct[UAMatr, UBMatr].
     KroneckerProduct[Rz[-π / 2].H, H].CNOT1.KroneckerProduct[H, H.Ry[2 * θ2 - π / 2]].
     CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2].H, Ry[π / 2 - 2 * θ1].H].CNOT1.
     KroneckerProduct[H, H.Rz[π / 2]].KroneckerProduct[VAMatr, VBMatr]]]
```

Out[•]= 0.0000681932

In[•]:=
```
Max[Abs[
  UFirstStage - Exp[I * θ0 + I * π / 4] * KroneckerProduct[UAMatr.Rz[-π / 2].H, UBMatr.H].CNOT1.
     KroneckerProduct[H, H.Ry[2 * θ2 - π / 2]].CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2].H,
      Ry[π / 2 - 2 * θ1].H].CNOT1.KroneckerProduct[H.VAMatr, H.Rz[π / 2].VBMatr]]]
```

Out[•]= 0.0000681932

```
In[•]:= U11 = UAMatr.Rz[-π / 2].H
        U12 = UBMatr.H
        U21 = H
        U22 = H.Ry[2 * θ2 - π / 2]
        U31 = Rz[2 * θ3 - π / 2].H
        U32 = Ry[π / 2 - 2 * θ1].H
        U41 = H.VAMatr
        U42 = H.Rz[π / 2].VBMatr
```

```
Out[•]= {{-0.11183 + 0.153491 i, 0.578133 - 0.793534 i}, {0.578132 - 0.793535 i, 0.111826 - 0.153494 i}}
```

```
Out[•]= {{0.77707 - 0.350649 i, -0.387624 - 0.350649 i},
         {-0.214958 - 0.476445 i, 0.571954 - 0.632188 i}}
```

$$\text{Out[•]= } \left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$$

$$\text{Out[•]= } \left\{\left\{1. - 4.4901 \times 10^{-11}\, i,\ 5.2822 \times 10^{-6} + 8.50044 \times 10^{-6}\, i\right\},\ \left\{5.2822 \times 10^{-6} + 8.50044 \times 10^{-6}\, i,\ -1. + 4.4901 \times 10^{-11}\, i\right\}\right\}$$

```
Out[•]= {{0.500004 - 0.499999 i, 0.500004 - 0.499999 i},
         {0.500001 + 0.499996 i, -0.500001 - 0.499996 i}}
```

$$\text{Out[•]= } \left\{\left\{0.877018 - 6.48035 \times 10^{-7}\, i,\ -0.480457 - 1.18291 \times 10^{-6}\, i\right\},\ \left\{-0.480457 - 1.18291 \times 10^{-6}\, i,\ -0.877018 + 6.48035 \times 10^{-7}\, i\right\}\right\}$$

```
Out[•]= {{-0.661777 - 0.249098 i, 0.249102 - 0.661778 i},
         {0.661779 - 0.249098 i, -0.249095 - 0.661778 i}}
```

```
Out[•]= {{0.639388 - 0.706834 i, 0.0209676 + 0.301877 i},
         {0.301877 + 0.0209659 i, 0.70683 - 0.639392 i}}
```

```
In[•]:= Max[Abs[UFirstStage -
            Exp[I * (θ0 + π / 4)] * KroneckerProduct[U11, U12].CNOT1.KroneckerProduct[U21, U22].
              CNOT1.KroneckerProduct[U31, U32].CNOT1.KroneckerProduct[U41, U42]]]
```

```
Out[•]= 0.0000681932
```

```
In[•]:= U12P = Adjuster45.U12
```

```
Out[•]= {{-0.390773 + 0.176353 i, 0.669961 + 0.606093 i},
         {0.371566 + 0.823489 i, -0.287642 + 0.317908 i}}
```

```
In[•]:= AncillaAdjuster
```

$$\text{Out[•]= } \left\{\left\{\frac{1}{\sqrt{3}}, -\sqrt{\frac{2}{3}}\right\}, \left\{\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right\}\right\}$$

In[•]:=
```
(*U11*)
U11OverallPhase = Arg[U11[[1]][[1]]]
U11Dephased = U11 * Exp[-I * U11OverallPhase]
θU11 = 2 * ArcTan[Abs[U11Dephased[[1]][[2]]] / Abs[U11Dephased[[1]][[1]]]]
ϕU11 = Arg[U11Dephased[[2]][[1]]]
λU11 = Arg[U11Dephased[[2]][[2]]] - ϕU11
Max[N[Abs[Exp[I * U11OverallPhase] * U3[θU11, ϕU11, λU11] - U11]]]
(*U12P*)
U12POverallPhase = Arg[U12P[[1]][[1]]]
U12PDephased = U12P * Exp[-I * U12POverallPhase]
θU12P = 2 * ArcTan[Abs[U12PDephased[[1]][[2]]] / Abs[U12PDephased[[1]][[1]]]]
ϕU12P = Arg[U12PDephased[[2]][[1]]]
λU12P = Arg[U12PDephased[[2]][[2]]] - ϕU12P
Max[N[Abs[Exp[I * U12POverallPhase] * U3[θU12P, ϕU12P, λU12P] - U12P]]]
(*U21*)
U21OverallPhase = Arg[U21[[1]][[1]]]
U21Dephased = U21 * Exp[-I * U21OverallPhase]
θU21 = 2 * ArcTan[Abs[U21Dephased[[1]][[2]]] / Abs[U21Dephased[[1]][[1]]]]
ϕU21 = Arg[U21Dephased[[2]][[1]]]
λU21 = Arg[U21Dephased[[2]][[2]]] - ϕU21
Max[N[Abs[Exp[I * U21OverallPhase] * U3[θU21, ϕU21, λU21] - U21]]]
(*U22*)
U22OverallPhase = Arg[U22[[1]][[1]]]
U22Dephased = U22 * Exp[-I * U22OverallPhase]
θU22 = 2 * ArcTan[Abs[U22Dephased[[1]][[2]]] / Abs[U22Dephased[[1]][[1]]]]
ϕU22 = Arg[U22Dephased[[2]][[1]]]
λU22 = Arg[U22Dephased[[2]][[2]]] - ϕU22
Max[N[Abs[Exp[I * U22OverallPhase] * U3[θU22, ϕU22, λU22] - U22]]]
(*U31*)
U31OverallPhase = Arg[U31[[1]][[1]]]
U31Dephased = U31 * Exp[-I * U31OverallPhase]
θU31 = 2 * ArcTan[Abs[U31Dephased[[1]][[2]]] / Abs[U31Dephased[[1]][[1]]]]
ϕU31 = Arg[U31Dephased[[2]][[1]]]
λU31 = Arg[U31Dephased[[2]][[2]]] - ϕU31
Max[N[Abs[Exp[I * U31OverallPhase] * U3[θU31, ϕU31, λU31] - U31]]]
(*U32*)
U32OverallPhase = Arg[U32[[1]][[1]]]
U32Dephased = U32 * Exp[-I * U32OverallPhase]
θU32 = 2 * ArcTan[Abs[U32Dephased[[1]][[2]]] / Abs[U32Dephased[[1]][[1]]]]
ϕU32 = Arg[U32Dephased[[2]][[1]]]
λU32 = Arg[U32Dephased[[2]][[2]]] - ϕU32
Max[N[Abs[Exp[I * U32OverallPhase] * U3[θU32, ϕU32, λU32] - U32]]]
(*U41R*)
U41OverallPhaseR = Arg[(U41.Rrot)[[1]][[1]]]
U41DephasedR = (U41.Rrot) * Exp[-I * U41OverallPhaseR]
θU41R = 2 * ArcTan[Abs[U41DephasedR[[1]][[2]]] / Abs[U41DephasedR[[1]][[1]]]]
ϕU41R = Arg[U41DephasedR[[2]][[1]]]
λU41R = Arg[U41DephasedR[[2]][[2]]] - ϕU41R
Max[N[Abs[Exp[I * U41OverallPhaseR] * U3[θU41R, ϕU41R, λU41R] - U41.Rrot]]]
```

Out[•]= 2.20044

Out[•]= $\{\{0.189909 + 1.38778 \times 10^{-17}\, i,\ -0.981802 + 0.0000130072\, i\},$
$\{-0.981802 + 0.0000138706\, i,\ -0.189909 + 5.19896 \times 10^{-6}\, i\}\}$

Out[•]= 2.75945

Out[•]= 3.14158

Out[•]= −0.0000132483

Out[•]= 1.00535×10^{-15}

Out[•]= 2.71766

Out[•]= $\{\{0.428723 - 2.77556 \times 10^{-17}\, i,\ -0.361343 - 0.828026\, i\},$
$\{0.0000633625 - 0.903436\, i,\ 0.39295 - 0.171447\, i\}\}$

Out[•]= 2.25543

Out[•]= −1.57073

Out[•]= 1.15932

Out[•]= 8.67112×10^{-16}

Out[•]= 0

Out[•]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[•]= $\frac{\pi}{2}$

Out[•]= 0

Out[•]= π

Out[•]= 0.

Out[•]= -4.4901×10^{-11}

Out[•]= $\{\{1. + 0.\, i,\ 5.2822 \times 10^{-6} + 8.50044 \times 10^{-6}\, i\},\ \{5.2822 \times 10^{-6} + 8.50044 \times 10^{-6}\, i,\ -1. + 0.\, i\}\}$

Out[•]= 0.0000200159

Out[•]= 1.01479

Out[•]= 2.12681

Out[•]= 0.0000170009

Out[•]= −0.785393

Out[•]= $\{\{0.707109 + 5.55112 \times 10^{-17}\, i,\ 0.707109 + 5.55112 \times 10^{-17}\, i\},$
$\{6.60122 \times 10^{-6} + 0.707105\, i,\ -6.60122 \times 10^{-6} - 0.707105\, i\}\}$

Out[•]= 1.5708

Out[•]= 1.57079

Out[•]= -3.14159

Out[•]= 2.02526×10^{-6}

Out[•]= -7.38907×10^{-7}

Out[•]= $\left\{\left\{0.877018 + 0.\,\mathrm{i},\ -0.480457 - 1.53792 \times 10^{-6}\,\mathrm{i}\right\},\ \left\{-0.480457 - 1.53792 \times 10^{-6}\,\mathrm{i},\ -0.877018 + 0.\,\mathrm{i}\right\}\right\}$

Out[•]= 1.00235

Out[•]= -3.14159

Out[•]= 6.28318

Out[•]= 3.07585×10^{-6}

Out[•]= -1.82627

Out[•]= $\left\{\left\{0.707106 - 2.77556 \times 10^{-17}\,\mathrm{i},\ 3.34256 \times 10^{-6} + 0.707108\,\mathrm{i}\right\},\right.$
$\left.\left\{0.616796 + 0.345781\,\mathrm{i},\ 0.345777 - 0.616796\,\mathrm{i}\right\}\right\}$

Out[•]= 1.5708

Out[•]= 0.510951

Out[•]= -1.5708

Out[•]= 3.7238×10^{-16}

In[•]:=
```
(*Constant input*)
{θU11, ϕU11, λU11}
```

Out[•]= $\{2.75945, 3.14158, -0.0000132483\}$

In[•]:=
```
{θU12P, ϕU12P, λU12P}
```

Out[•]= $\{2.25543, -1.57073, 1.15932\}$

In[•]:=
```
{θU21, ϕU21, λU21}
```

Out[•]= $\left\{\frac{\pi}{2}, 0, \pi\right\}$

In[•]:=
```
{θU22, ϕU22, λU22}
```

Out[•]= $\{0.0000200159, 1.01479, 2.12681\}$

In[•]:=
```
{θU31, ϕU31, λU31}
```

Out[•]= $\{1.5708, 1.57079, -3.14159\}$

In[•]:=
```
{θU32, ϕU32, λU32}
```

Out[•]= $\{1.00235, -3.14159, 6.28318\}$

In[•]:=
```
{θU41R, ϕU41R, λU41R}
```

Out[•]= $\{1.5708, 0.510951, -1.5708\}$

```
In[•]:= σ = 4 / 5
```

$$\text{Out[•]=}\ \frac{4}{5}$$

```
In[•]:=
```

```
In[•]:= δ = σ * 1.0
        Initializer =
          {{Cos[1/4 (π + 2 δ)], -i Sin[1/4 (π + 2 δ)]}, {-i Sin[1/4 (π + 2 δ)], Cos[1/4 (π + 2 δ)]}}
        U42POverallPhase = Arg[(U42.Initializer)[[1]][[1]]]
        U42PDephased = (U42.Initializer) * Exp[-I * U42POverallPhase]
        θU42P = 2 * ArcTan[Abs[U42PDephased[[1]][[2]]] / Abs[U42PDephased[[1]][[1]]]]
        ϕU42P = Arg[U42PDephased[[2]][[1]]]
        λU42P = Arg[U42PDephased[[2]][[2]]] - ϕU42P
        Max[N[Abs[Exp[I * U42POverallPhase] * U3[θU42P, ϕU42P, λU42P] - (U42.Initializer)]]]
        {θU42P, ϕU42P, λU42P}
Out[•]= 0.8

Out[•]= {{0.375928, 0. - 0.926649 i}, {0. - 0.926649 i, 0.375928}}

Out[•]= -0.501506

Out[•]= {{0.593137 + 0. i, -0.337141 - 0.731112 i}, {-0.10893 - 0.797699 i, 0.500068 - 0.318972 i}}

Out[•]= 1.87169

Out[•]= -1.70651

Out[•]= 1.13872

Out[•]= 1.33689 × 10^-15

Out[•]= {1.87169, -1.70651, 1.13872}

In[•]:= (* δ = -σ*1.0 *)

In[•]:= {0.5041543015764468`, -0.6583695122207293`, 0.595407730354344`}
Out[•]= {0.504154, -0.65837, 0.595408}

In[•]:= (* δ = -σ*0.8 *)

In[•]:= {0.6072220975861362`, -0.8916492436351763`, 0.7940996749954748`}
Out[•]= {0.607222, -0.891649, 0.7941}

In[•]:= (* δ = -σ*0.6 *)

In[•]:= {0.7288958160644461`, -1.0601065391321145`, 0.9267075946590133`}
Out[•]= {0.728896, -1.06011, 0.926708}

In[•]:= (* δ = -σ*0.4 *)
```

In[•]:= **{0.8611971304035912`, -1.1865477058876266`, 1.015502320365517`}**

Out[•]= {0.861197, -1.18655, 1.0155}

In[•]:= **(* δ = -σ*0.2 *)**

In[•]:= {0.9997583587197392`, -1.2865448162648347`, 1.075421373170859`}

Out[•]= {0.999758, -1.28654, 1.07542}

In[•]:= **(* δ = σ*0.0 *)**

In[•]:= **{1.14211255688777`, -1.3699097484066178`, 1.115497751449562`}**

Out[•]= {1.14211, -1.36991, 1.1155}

In[•]:= **(* δ = σ*0.2 *)**

In[•]:= **{1.286769012055299`, -1.4429557742284207`, 1.1410574417025805`}**

Out[•]= {1.28677, -1.44296, 1.14106}

In[•]:= **(* δ = σ*0.4 *)**

In[•]:= **{1.4327429584828353`, -1.5100209253395165`, 1.155148617958269`}**

Out[•]= {1.43274, -1.51002, 1.15515}

In[•]:= **(* δ = σ*0.6 *)**

In[•]:= **{1.5793052094763378`, -1.5743976273392355`, 1.1593281772128206`}**

Out[•]= {1.57931, -1.5744, 1.15933}

In[•]:= **(* δ = σ*0.8 *)**

In[•]:= **{1.725832121907423`, -1.6389361301500462`, 1.1540405898492394`}**

Out[•]= {1.72583, -1.63894, 1.15404}

In[•]:= **(* δ = σ*1.0 *)**

In[•]:= **{1.87169386696376`, -1.7065127757118237`, 1.1387222164849355`}**

Out[•]= {1.87169, -1.70651, 1.13872}

In[•]:=

(*Another initializer*)

In[•]:= **σ = 4 / 5**

Out[•]= $\frac{4}{5}$

In[•]:= **{{Cos[1/4 (π + σ)], -i * Sin[1/4 (π + σ)] * Exp[I * ϕ]}}**

Out[•]= $\left\{\left\{\text{Cos}\left[\frac{1}{4}\left(\frac{4}{5}+\pi\right)\right], -i\,\text{Sin}\left[\frac{1}{4}\left(\frac{4}{5}+\pi\right)\right]\right\}\right\}$

In[•]:=
```
Φ = π * 0.8
Initializer = {{Cos[1/4 (π + σ)], -i * Sin[1/4 (π + σ)] * Exp[-I * Φ]},
  {-i * Sin[1/4 (π + σ)] * Exp[I * Φ], Cos[1/4 (π + σ)]}}
N[Initializer.ConjugateTranspose[Initializer]]
U42POverallPhase = Arg[(U42.Initializer)[[1]][[1]]]
U42PDephased = (U42.Initializer) * Exp[-I * U42POverallPhase]
ϴU42P = 2 * ArcTan[Abs[U42PDephased[[1]][[2]]] / Abs[U42PDephased[[1]][[1]]]]
ϕU42P = Arg[U42PDephased[[2]][[1]]]
λU42P = Arg[U42PDephased[[2]][[2]]] - ϕU42P
Max[N[Abs[Exp[I * U42POverallPhase] * U3[ϴU42P, ϕU42P, λU42P] - (U42.Initializer)]]]
Initializer.ConjugateTranspose[Initializer]
{ϴU42P, ϕU42P, λU42P}
```

Out[•]= 2.51327

Out[•]= $\left\{\left\{\text{Cos}\left[\frac{1}{4}\left(\frac{4}{5}+\pi\right)\right], -0.489914+0.674309\,i\right\}, \left\{0.489914+0.674309\,i, \text{Cos}\left[\frac{1}{4}\left(\frac{4}{5}+\pi\right)\right]\right\}\right\}$

Out[•]= {{1. + 0. i, 0. + 0. i}, {0. + 0. i, 1. + 0. i}}

Out[•]= −0.959968

Out[•]= $\left\{\left\{0.278958-2.77556\times10^{-17}\,i, -0.673137+0.684886\,i\right\},\right.$
$\left.\left\{0.398238+0.873836\,i, 0.262129+0.0954273\,i\right\}\right\}$

Out[•]= 2.57617

Out[•]= 1.14318

Out[•]= −0.794049

Out[•]= 3.17738×10^{-15}

Out[•]= {{1. + 0. i, 0. + 0. i}, {0. + 0. i, 1. + 0. i}}

Out[•]= {2.57617, 1.14318, −0.794049}

In[•]:=
```
(* Φ = -π*1.0 *)
{2.347541659876903`, 2.014151926483645`, -0.9754800738409748`}
```

Out[•]= {2.34754, 2.01415, −0.97548}

```
(* Φ = -π*0.8 *)
{1.9946975606470387`, 2.5773847843572724`, -1.5265048735171`}

(* Φ = -π*0.6 *)
{1.6569736803046362`, 3.064282009723725`, -2.2795859679240684`}

(* Φ = -π*0.4 *)
{1.4196712611485087`, -2.6911755489400577`, 3.1058919257992734`}

(* Φ = -π*0.2 *)
{1.3611000693373656`, -2.1070848644646274`, 2.1180195788433895`}

(* Φ = -π*0.0 *)
{1.5059906091990785`, -1.5423626024592276`, 1.158425877812545`}

(* Φ = π*0.2 *)
{1.7976182918404742`, -1.04072017208684`, 0.32029014709129855`}

(* Φ = π*0.4 *)
```

Out[ ]= {1.9947, 2.57738, −1.5265}

Out[ ]= {1.65697, 3.06428, −2.27959}

Out[ ]= {1.41967, −2.69118, 3.10589}

Out[ ]= {1.3611, −2.10708, 2.11802}

Out[ ]= {1.50599, −1.54236, 1.15843}

Out[ ]= {1.79762, −1.04072, 0.32029}

```
{2.1533172120263906`, -0.5411953842293125`, -0.35636943673119204`}
```

Out[ ]= {2.15332, −0.541195, −0.356369}

```
(* Φ = π*0.6 *)
{2.4763256761141634`, 0.12527524353801067`, -0.7700150666664707`}
```

Out[ ]= {2.47633, 0.125275, −0.770015}

```
(* Φ = π*0.8 *)
{2.576173989062413`, 1.143183266478647`, -0.7940490282393208`}
```

Out[ ]= {2.57617, 1.14318, −0.794049}

In[•]:=
```
Φ = π * 0.8
Initializer = {{Cos[1/4 (π + σ/2)], -i * Sin[1/4 (π + σ/2)] * Exp[-I * Φ]},
  {-i * Sin[1/4 (π + σ/2)] * Exp[I * Φ], Cos[1/4 (π + σ/2)]}}
N[Initializer.ConjugateTranspose[Initializer]]
U42POverallPhase = Arg[(U42.Initializer)[[1]][[1]]]
U42PDephased = (U42.Initializer) * Exp[-I * U42POverallPhase]
θU42P = 2 * ArcTan[Abs[U42PDephased[[1]][[2]]] / Abs[U42PDephased[[1]][[1]]]]
ϕU42P = Arg[U42PDephased[[2]][[1]]]
λU42P = Arg[U42PDephased[[2]][[2]]] - ϕU42P
Max[N[Abs[Exp[I * U42POverallPhase] * U3[θU42P, ϕU42P, λU42P] - (U42.Initializer)]]]
Initializer.ConjugateTranspose[Initializer]
{θU42P, ϕU42P, λU42P}
```

Out[•]= 2.51327

Out[•]= $\left\{\left\{\text{Cos}\left[\frac{1}{4}\left(\frac{2}{5}+\pi\right)\right], -0.455044+0.626314\,i\right\}, \left\{0.455044+0.626314\,i, \text{Cos}\left[\frac{1}{4}\left(\frac{2}{5}+\pi\right)\right]\right\}\right\}$

Out[•]= {{1. + 0. i, 0. + 0. i}, {0. + 0. i, 1. + 0. i}}

Out[•]= −0.921913

Out[•]= {{0.372651 + 0. i, −0.627862 + 0.683316 i}, {0.420353 + 0.827306 i, 0.358848 + 0.100484 i}}

Out[•]= 2.37786

Out[•]= 1.10069

Out[•]= −0.827666

Out[•]= 2.31888×10^{-15}

Out[•]= {{1. + 0. i, 0. + 0. i}, {0. + 0. i, 1. + 0. i}}

Out[•]= {2.37786, 1.10069, −0.827666}

In[•]:=
```
(* Φ = -π*1.0 *)
```

In[•]:=
```
{2.176904881652221`, 1.8781793090109806`, -1.0625769894185124`}
```

Out[•]= {2.1769, 1.87818, −1.06258}

In[•]:=
```
(* Φ = -π*0.8 *)
```

In[•]:=
```
{1.8353472760930267`, 2.448792388304744`, -1.5700920205750886`}
```

Out[•]= {1.83535, 2.44879, −1.57009}

In[•]:=
```
(* Φ = -π*0.6 *)
```

In[•]:=
```
{1.4860858389057967`, 2.960244952961612`, -2.2796626206448822`}
```

Out[•]= {1.48609, 2.96024, −2.27966}

```
In[•]:= (* Φ = -π*0.4 *)

In[•]:= {1.227716546051603`, -2.749196491292067`, 3.1201529583917536`}
Out[•]= {1.22772, -2.7492, 3.12015}

In[•]:= (* Φ = -π*0.2 *)

In[•]:= {1.1617961910199859`, -2.089543736392359`, 2.1126526454891104`}
Out[•]= {1.1618, -2.08954, 2.11265}

In[•]:= (* Φ = -π*0.0 *)

In[•]:= {1.32317045701139`, -1.4601417099928056`, 1.1455731572827124`}
Out[•]= {1.32317, -1.46014, 1.14557}

In[•]:= (* Φ = π*0.2 *)

In[•]:= {1.6339329737879875`, -0.924439830571605`, 0.3371644326917308`}
Out[•]= {1.63393, -0.92444, 0.337164}

In[•]:= (* Φ = π*0.4 *)

In[•]:= {1.9923512993405348`, -0.405481149276967`, -0.29077267908756566`}
Out[•]= {1.99235, -0.405481, -0.290773}

In[•]:= (* Φ = π*0.6 *)

In[•]:= {2.2928930621278125`, 0.24208905581408824`, -0.6846981193361248`}
Out[•]= {2.29289, 0.242089, -0.684698}

In[•]:= (* Φ = π*0.8 *)

In[•]:= {2.377864693059344`, 1.1006903067020746`, -0.8276656379683764`}
Out[•]= {2.37786, 1.10069, -0.827666}
```