In[1]:= (*Our matrix which we need to decompose*)

```
USecondStage = {{-6.206655138647245`*^-8 + 0.20774699595156004` i,
    -0.7547114656008732` + 6.166771923275458`*^-8 i, -4.107575420162629`*^-8 +
     0.5542935245741105` i, 0.28286282477421143` + 4.048690621291911`*^-8 i},
   {-0.7547053706485582` - 1.3855643970486428`*^-8 i, -1.3431780850035077`*^-8 -
     0.2077005694310296` i, 0.28291911871353015` - 8.630859642646052`*^-9 i,
    -8.796505672958518`*^-9 - 0.5542897096356874` i},
   {4.107575420162629`*^-8 - 0.5542935245741105` i,
    -0.28286282477421143` - 4.048690621291911`*^-8 i, -6.206655138647245`*^-8 +
     0.20774699595156004` i, -0.7547114656008732` + 6.166771923275458`*^-8 i},
   {-0.28291911871353015` + 8.630859642646052`*^-9 i, 8.796505672958518`*^-9 +
     0.5542897096356874` i, -0.7547053706485582` - 1.3855643970486428`*^-8 i,
    -1.3431780850035077`*^-8 - 0.2077005694310296` i}}
```

Out[1]= $\{\{-6.20666\times10^{-8}+0.207747\,i,\ -0.754711+6.16677\times10^{-8}\,i,\ -4.10758\times10^{-8}+0.554294\,i,\ 0.282863+4.04869\times10^{-8}\,i\},\ \{-0.754705-1.38556\times10^{-8}\,i,\ -1.34318\times10^{-8}-0.207701\,i,\ 0.282919-8.63086\times10^{-9}\,i,\ -8.79651\times10^{-9}-0.55429\,i\},\ \{4.10758\times10^{-8}-0.554294\,i,\ -0.282863-4.04869\times10^{-8}\,i,\ -6.20666\times10^{-8}+0.207747\,i,\ -0.754711+6.16677\times10^{-8}\,i\},\ \{-0.282919+8.63086\times10^{-9}\,i,\ 8.79651\times10^{-9}+0.55429\,i,\ -0.754705-1.38556\times10^{-8}\,i,\ -1.34318\times10^{-8}-0.207701\,i\}\}$

In[2]:= MatrixForm[Round[USecondStage, 0.00001]]

Out[2]//MatrixForm=

$$\begin{pmatrix} 0.+0.20775\,i & -0.75471 & 0.+0.55429\,i & 0.28286 \\ -0.75471 & 0.-0.2077\,i & 0.28292 & 0.-0.55429\,i \\ 0.-0.55429\,i & -0.28286 & 0.+0.20775\,i & -0.75471 \\ -0.28292 & 0.+0.55429\,i & -0.75471 & 0.-0.2077\,i \end{pmatrix}$$

In[3]:=
```
(*It is unitary, vectors and eigenvalues are found with high accuracy*)
Transpose[Eigenvectors[USecondStage]].
  {{Eigenvalues[USecondStage][[1]], 0, 0, 0}, {0, Eigenvalues[USecondStage][[2]], 0, 0},
   {0, 0, Eigenvalues[USecondStage][[3]], 0}, {0, 0, 0, Eigenvalues[USecondStage][[4]]}}.
  Conjugate[Eigenvectors[USecondStage]] - USecondStage
ConjugateTranspose[USecondStage].USecondStage
Max[Abs[ConjugateTranspose[USecondStage].USecondStage -
   {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}]]
```

Out[3]= $\{\{8.20482\times10^{-9}+0.0000250612\,i,\ -0.0000134767-2.87986\times10^{-8}\,i,\ 2.18888\times10^{-8}-9.39388\times10^{-6}\,i,\ -0.0000159408-2.79203\times10^{-8}\,i\},\ \{-3.18973\times10^{-7}-3.35127\times10^{-9}\,i,\ -8.20482\times10^{-9}-0.0000250612\,i,\ -0.0000208717+3.99676\times10^{-8}\,i,\ -2.18888\times10^{-8}+9.39388\times10^{-6}\,i\},\ \{-2.18888\times10^{-8}+9.39388\times10^{-6}\,i,\ 0.0000159408+2.79203\times10^{-8}\,i,\ 8.20482\times10^{-9}+0.0000250612\,i,\ -0.0000134767-2.87986\times10^{-8}\,i\},\ \{0.0000208717-3.99676\times10^{-8}\,i,\ 2.18888\times10^{-8}-9.39388\times10^{-6}\,i,\ -3.18973\times10^{-7}-3.35127\times10^{-9}\,i,\ -8.20482\times10^{-9}-0.0000250612\,i\}\}$

Out[4]= $\{\{1.00002+0.\,i,\ 8.57866\times10^{-8}-0.0000664133\,i,\ 0.-3.08719\times10^{-8}\,i,\ -5.80014\times10^{-8}-7.6976\times10^{-6}\,i\},\ \{8.57866\times10^{-8}+0.0000664133\,i,\ 0.999977+0.\,i,\ 5.80014\times10^{-8}-7.6976\times10^{-6}\,i,\ 0.-8.47628\times10^{-8}\,i\},\ \{0.+3.08719\times10^{-8}\,i,\ 5.80014\times10^{-8}+7.6976\times10^{-6}\,i,\ 1.00002+0.\,i,\ 8.57866\times10^{-8}-0.0000664133\,i\},\ \{-5.80014\times10^{-8}+7.6976\times10^{-6}\,i,\ 0.+8.47628\times10^{-8}\,i,\ 8.57866\times10^{-8}+0.0000664133\,i,\ 0.999977+0.\,i\}\}$

Out[5]= 0.0000664133

In[6]:=
```
(*Do SVD on the UR and UI*)
M = 1/Sqrt[2] {{1, 0, 0, I}, {0, I, 1, 0}, {0, I, -1, 0}, {1, 0, 0, -I}}
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
UP = ConjugateTranspose[M].USecondStage.M
UR = (UP + Conjugate[UP]) / 2
UI = (UP - Conjugate[UP]) / (2 * I)
{a, b, c} = SingularValueDecomposition[UR]
Max[Abs[a.b.ConjugateTranspose[c] - UR]]
{d, e, f} = SingularValueDecomposition[UI]
Max[Abs[d.e.ConjugateTranspose[f] - UI]]
MatrixForm[M]
MatrixForm[M.ConjugateTranspose[M]]
```

Out[6]= $\{\{\frac{1}{\sqrt{2}},0,0,\frac{i}{\sqrt{2}}\},\{0,\frac{i}{\sqrt{2}},\frac{1}{\sqrt{2}},0\},\{0,\frac{i}{\sqrt{2}},-\frac{1}{\sqrt{2}},0\},\{\frac{1}{\sqrt{2}},0,0,-\frac{i}{\sqrt{2}}\}\}$

Out[7]= {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}

Out[8]=
$$\{\{-0.0000281847+0.0000232378\,\mathrm{i},\,-0.554292-0.754708\,\mathrm{i},\,-3.02254\times10^{-6}-1.86971\times10^{-6}\,\mathrm{i},\,-0.207724-0.282891\,\mathrm{i}\},\\ \{-0.554292+0.754708\,\mathrm{i},\,0.0000281092+0.0000231887\,\mathrm{i},\,-0.207724+0.282891\,\mathrm{i},\,3.07241\times10^{-6}-1.94523\times10^{-6}\,\mathrm{i}\},\,\{3.02254\times10^{-6}+1.86971\times10^{-6}\,\mathrm{i},\,0.207724+0.282891\,\mathrm{i},\\ -0.0000281847+0.0000232378\,\mathrm{i},\,-0.554292-0.754708\,\mathrm{i}\},\,\{0.207724-0.282891\,\mathrm{i},\\ -3.07241\times10^{-6}+1.94523\times10^{-6}\,\mathrm{i},\,-0.554292+0.754708\,\mathrm{i},\,0.0000281092+0.0000231887\,\mathrm{i}\}\}$$

Out[9]=
$$\{\{-0.0000281847+0.\,\mathrm{i},\,-0.554292+0.\,\mathrm{i},\,-3.02254\times10^{-6}+0.\,\mathrm{i},\,-0.207724+0.\,\mathrm{i}\},\\ \{-0.554292+0.\,\mathrm{i},\,0.0000281092+0.\,\mathrm{i},\,-0.207724+0.\,\mathrm{i},\,3.07241\times10^{-6}+0.\,\mathrm{i}\},\\ \{3.02254\times10^{-6}+0.\,\mathrm{i},\,0.207724+0.\,\mathrm{i},\,-0.0000281847+0.\,\mathrm{i},\,-0.554292+0.\,\mathrm{i}\},\\ \{0.207724+0.\,\mathrm{i},\,-3.07241\times10^{-6}+0.\,\mathrm{i},\,-0.554292+0.\,\mathrm{i},\,0.0000281092+0.\,\mathrm{i}\}\}$$

Out[10]=
$$\{\{0.0000232378+0.\,\mathrm{i},\,-0.754708+0.\,\mathrm{i},\,-1.86971\times10^{-6}+0.\,\mathrm{i},\,-0.282891+0.\,\mathrm{i}\},\\ \{0.754708+0.\,\mathrm{i},\,0.0000231887+0.\,\mathrm{i},\,0.282891+0.\,\mathrm{i},\,-1.94523\times10^{-6}+0.\,\mathrm{i}\},\\ \{1.86971\times10^{-6}+0.\,\mathrm{i},\,0.282891+0.\,\mathrm{i},\,0.0000232378+0.\,\mathrm{i},\,-0.754708+0.\,\mathrm{i}\},\\ \{-0.282891+0.\,\mathrm{i},\,1.94523\times10^{-6}+0.\,\mathrm{i},\,0.754708+0.\,\mathrm{i},\,0.0000231887+0.\,\mathrm{i}\}\}$$

Out[11]=
$$\{\{\{0.707952+0.\,\mathrm{i},\,0.+0.\,\mathrm{i},\,0.+0.\,\mathrm{i},\,0.706261+0.\,\mathrm{i}\},\\ \{0.00267438+0.\,\mathrm{i},\,-0.706256+0.\,\mathrm{i},\,-0.707947+0.\,\mathrm{i},\,-0.00268078+0.\,\mathrm{i}\},\\ \{-5.47184\times10^{-13}+0.\,\mathrm{i},\,-0.707952+0.\,\mathrm{i},\,0.706261+0.\,\mathrm{i},\,5.48494\times10^{-13}+0.\,\mathrm{i}\},\\ \{-0.706256+0.\,\mathrm{i},\,-0.00267438+0.\,\mathrm{i},\,-0.00268078+0.\,\mathrm{i},\,0.707947+0.\,\mathrm{i}\}\},\\ \{\{0.591943,\,0.,\,0.,\,0.\},\,\{0.,\,0.591943,\,0.,\,0.\},\\ \{0.,\,0.,\,0.591929,\,0.\},\,\{0.,\,0.,\,0.,\,0.591929\}\},\\ \{\{-0.250376+0.\,\mathrm{i},\,0.660391+0.\,\mathrm{i},\,0.661995+0.\,\mathrm{i},\,0.250914+0.\,\mathrm{i}\},\\ \{-0.662918+0.\,\mathrm{i},\,-0.248467+0.\,\mathrm{i},\,0.247812+0.\,\mathrm{i},\,-0.661357+0.\,\mathrm{i}\},\\ \{0.660391+0.\,\mathrm{i},\,0.250376+0.\,\mathrm{i},\,0.250914+0.\,\mathrm{i},\,-0.661995+0.\,\mathrm{i}\},\\ \{-0.248467+0.\,\mathrm{i},\,0.662918+0.\,\mathrm{i},\,-0.661357+0.\,\mathrm{i},\,-0.247812+0.\,\mathrm{i}\}\}\}$$

Out[12]= 3.06903×10^{-16}

Out[13]=
$$\{\{\{0.+0.\,\mathrm{i},\,-0.707948+0.\,\mathrm{i},\,0.706265+0.\,\mathrm{i},\,0.+0.\,\mathrm{i}\},\\ \{-0.70626+0.\,\mathrm{i},\,-0.00257728+0.\,\mathrm{i},\,-0.00258342+0.\,\mathrm{i},\,0.707943+0.\,\mathrm{i}\},\\ \{-0.707948+0.\,\mathrm{i},\,-1.2496\times10^{-13}+0.\,\mathrm{i},\,-1.25258\times10^{-13}+0.\,\mathrm{i},\,-0.706265+0.\,\mathrm{i}\},\\ \{-0.00257728+0.\,\mathrm{i},\,0.70626+0.\,\mathrm{i},\,0.707943+0.\,\mathrm{i},\,0.00258342+0.\,\mathrm{i}\}\},\\ \{\{0.805995,\,0.,\,0.,\,0.\},\,\{0.,\,0.805995,\,0.,\,0.\},\\ \{0.,\,0.,\,0.805975,\,0.\},\,\{0.,\,0.,\,0.,\,0.805975\}\},\\ \{\{-0.660417+0.\,\mathrm{i},\,-0.250319+0.\,\mathrm{i},\,-0.250881+0.\,\mathrm{i},\,0.662004+0.\,\mathrm{i}\},\\ \{-0.248498+0.\,\mathrm{i},\,0.662902+0.\,\mathrm{i},\,-0.661339+0.\,\mathrm{i},\,-0.247873+0.\,\mathrm{i}\},\\ \{-0.250319+0.\,\mathrm{i},\,0.660417+0.\,\mathrm{i},\,0.662004+0.\,\mathrm{i},\,0.250881+0.\,\mathrm{i}\},\\ \{0.662902+0.\,\mathrm{i},\,0.248498+0.\,\mathrm{i},\,-0.247873+0.\,\mathrm{i},\,0.661339+0.\,\mathrm{i}\}\}\}$$

Out[14]= 5.55112×10^{-16}

Out[15]//MatrixForm=
$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{\mathrm{i}}{\sqrt{2}} \\ 0 & \frac{\mathrm{i}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{\mathrm{i}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{\mathrm{i}}{\sqrt{2}} \end{pmatrix}$$

Out[16]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[17]:=
```
(*Real and symmetric matrices, with high accuracy*)
MatrixForm[UI.ConjugateTranspose[UR]]
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0.477092 + 0.\,i & -0.0000345756 + 0.\,i & 0.0000332358 + 0.\,i & 2.30341 \times 10^{-7} + 0.\,i \\ -0.0000345755 + 0.\,i & -0.477092 + 0.\,i & 2.03004 \times 10^{-7} + 0.\,i & -0.000033178 + 0.\,i \\ -0.0000332358 + 0.\,i & -2.30341 \times 10^{-7} + 0.\,i & 0.477092 + 0.\,i & -0.0000345756 + 0.\,i \\ -2.03004 \times 10^{-7} + 0.\,i & 0.000033178 + 0.\,i & -0.0000345755 + 0.\,i & -0.477092 + 0.\,i \end{pmatrix}$$

In[18]:=
```
Max[Abs[UI.ConjugateTranspose[UR] - Transpose[UI.ConjugateTranspose[UR]]]]
```

Out[18]= 0.0000664716

In[19]:=
```
MatrixForm[UR.ConjugateTranspose[UI]]
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 0.477092 + 0.\,i & -0.0000345755 + 0.\,i & -0.0000332358 + 0.\,i & -2.03004 \times 10^{-7} + 0.\,i \\ -0.0000345756 + 0.\,i & -0.477092 + 0.\,i & -2.30341 \times 10^{-7} + 0.\,i & 0.000033178 + 0.\,i \\ 0.0000332358 + 0.\,i & 2.03004 \times 10^{-7} + 0.\,i & 0.477092 + 0.\,i & -0.0000345755 + 0.\,i \\ 2.30341 \times 10^{-7} + 0.\,i & -0.000033178 + 0.\,i & -0.0000345756 + 0.\,i & -0.477092 + 0.\,i \end{pmatrix}$$

In[20]:=
```
Max[Abs[UR.ConjugateTranspose[UI] - Transpose[UR.ConjugateTranspose[UI]]]]
```

Out[20]= 0.0000664716

In[21]:=
```
A = {{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}
```

Out[21]= {{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}

In[22]:=
```
Solve[c.A == f, {a11, a12, a13, a14, a21, a22, a23, a24, a31, a32, a33, a34, a41, a42, a43, a44}]
```

Out[22]= {{a11 → 0.0000688155 + 0. i, a12 → -0.00238579 + 0. i,
a13 → 0.999997 + 0. i, a14 → -0.0000716202 + 0. i, a21 → 0.00238579 + 0. i,
a22 → 0.0000688155 + 0. i, a23 → 0.0000716202 + 0. i, a24 → 0.999997 + 0. i,
a31 → -0.999997 + 0. i, a32 → -0.0000733207 + 0. i, a33 → 0.0000688114 + 0. i,
a34 → 0.00238579 + 0. i, a41 → 0.0000733207 + 0. i, a42 → -0.999997 + 0. i,
a43 → -0.00238579 + 0. i, a44 → 0.0000688114 + 0. i}}

In[ ]:=

```
In[23]:= a11 = 0.0000688154514616224` + 0.` i
         a12 = -0.002385792710397109` + 0.` i
         a13 = 0.9999971490599949` + 0.` i
         a14 = -0.00007162023874931548` + 0.` i
         a21 = 0.002385792722504498` + 0.` i
         a22 = 0.00006881545086480508` + 0.` i
         a23 = 0.00007162023874684468` + 0.` i
         a24 = 0.9999971490599664` + 0.` i
         a31 = -0.9999971489367318` + 0.` i
         a32 = -0.00007332070267749527` + 0.` i
         a33 = 0.00006881139449641641` + 0.` i
         a34 = 0.002385792839519523` + 0.` i
         a41 = 0.00007332070267698777` + 0.` i
         a42 = -0.9999971489367613` + 0.` i
         a43 = -0.00238579282741257` + 0.` i
         a44 = 0.00006881139389743788` + 0.` i
```

Out[23]= 0.0000688155 + 0. i

Out[24]= -0.00238579 + 0. i

Out[25]= 0.999997 + 0. i

Out[26]= -0.0000716202 + 0. i

Out[27]= 0.00238579 + 0. i

Out[28]= 0.0000688155 + 0. i

Out[29]= 0.0000716202 + 0. i

Out[30]= 0.999997 + 0. i

Out[31]= -0.999997 + 0. i

Out[32]= -0.0000733207 + 0. i

Out[33]= 0.0000688114 + 0. i

Out[34]= 0.00238579 + 0. i

Out[35]= 0.0000733207 + 0. i

Out[36]= -0.999997 + 0. i

Out[37]= -0.00238579 + 0. i

Out[38]= 0.0000688114 + 0. i

```
In[39]:= d.e.ConjugateTranspose[A].ConjugateTranspose[c]
```

Out[39]= $\{\{0.0000232378 + 0.\,i,\ -0.754708 + 0.\,i,\ -1.86971\times10^{-6} + 0.\,i,\ -0.282891 + 0.\,i\},$
$\{0.754708 + 0.\,i,\ 0.0000231887 + 0.\,i,\ 0.282891 + 0.\,i,\ -1.94523\times10^{-6} + 0.\,i\},$
$\{1.86971\times10^{-6} + 0.\,i,\ 0.282891 + 0.\,i,\ 0.0000232378 + 0.\,i,\ -0.754708 + 0.\,i\},$
$\{-0.282891 + 0.\,i,\ 1.94523\times10^{-6} + 0.\,i,\ 0.754708 + 0.\,i,\ 0.0000231887 + 0.\,i\}\}$

In[40]:=
```
UI
```

Out[40]= $\{\{0.0000232378 + 0.\,i, -0.754708 + 0.\,i, -1.86971\times10^{-6} + 0.\,i, -0.282891 + 0.\,i\},$
$\{0.754708 + 0.\,i, 0.0000231887 + 0.\,i, 0.282891 + 0.\,i, -1.94523\times10^{-6} + 0.\,i\},$
$\{1.86971\times10^{-6} + 0.\,i, 0.282891 + 0.\,i, 0.0000232378 + 0.\,i, -0.754708 + 0.\,i\},$
$\{-0.282891 + 0.\,i, 1.94523\times10^{-6} + 0.\,i, 0.754708 + 0.\,i, 0.0000231887 + 0.\,i\}\}$

In[41]:=
```
Max[Abs[d.e.ConjugateTranspose[A].ConjugateTranspose[c] - UI]]
```

Out[41]= 6.37945×10^{-16}

In[42]:=
```
F = {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}
```

Out[42]= {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}

In[43]:=
```
MatrixForm[a]
```

Out[43]//MatrixForm=

$$\begin{pmatrix} 0.707952 + 0.\,i & 0. + 0.\,i & 0. + 0.\,i & 0.706261 + 0.\,i \\ 0.00267438 + 0.\,i & -0.706256 + 0.\,i & -0.707947 + 0.\,i & -0.00268078 + 0.\,i \\ -5.47184\times10^{-13} + 0.\,i & -0.707952 + 0.\,i & 0.706261 + 0.\,i & 5.48494\times10^{-13} + 0.\,i \\ -0.706256 + 0.\,i & -0.00267438 + 0.\,i & -0.00268078 + 0.\,i & 0.707947 + 0.\,i \end{pmatrix}$$

In[44]:=
```
MatrixForm[F.d.ConjugateTranspose[A]]
```

Out[44]//MatrixForm=

$$\begin{pmatrix} 0.707952 + 0.\,i & 1.86508\times10^{-6} + 0.\,i & 0.000100506 + 0.\,i & 0.706261 + 0.\,i \\ 0.00267657 + 0.\,i & -0.706256 + 0.\,i & -0.707947 + 0.\,i & -0.00258037 + 0.\,i \\ 1.86508\times10^{-6} + 0.\,i & -0.707952 + 0.\,i & 0.706261 + 0.\,i & -0.000100506 + 0.\,i \\ -0.706256 + 0.\,i & -0.00267657 + 0.\,i & -0.00258037 + 0.\,i & 0.707947 + 0.\,i \end{pmatrix}$$

In[45]:=
```
Max[Abs[F.d.ConjugateTranspose[A] - a]]
```

Out[45]= 0.000100506

In[46]:=
```
a.Inverse[d.ConjugateTranspose[A]]
```

Out[46]= $\{\{1. + 0.\,i, -0.0000724715 + 0.\,i, -0.0000696633 + 0.\,i, 3.36947\times10^{-8} + 0.\,i\},$
$\{-0.0000724715 + 0.\,i, -1. + 0.\,i, -2.36063\times10^{-8} + 0.\,i, 0.0000695421 + 0.\,i\},$
$\{0.0000696633 + 0.\,i, -3.36947\times10^{-8} + 0.\,i, 1. + 0.\,i, -0.0000724715 + 0.\,i\},$
$\{2.36063\times10^{-8} + 0.\,i, -0.0000695421 + 0.\,i, -0.0000724715 + 0.\,i, -1. + 0.\,i\}\}$

In[47]:=
```
a.b.ConjugateTranspose[c] + I * d.ConjugateTranspose[A].e.ConjugateTranspose[c] - UP
```

Out[47]= $\{\{-4.16334\times10^{-17} - 7.04237\times10^{-6}\,i, -2.22045\times10^{-16} - 2.79624\times10^{-10}\,i,$
$-5.55112\times10^{-17} + 0.0000185775\,i, -2.77556\times10^{-17} + 4.49768\times10^{-10}\,i\},$
$\{1.11022\times10^{-16} + 8.4911\times10^{-11}\,i, -1.75207\times10^{-16} - 6.90403\times10^{-6}\,i,$
$-2.77556\times10^{-17} - 5.22753\times10^{-10}\,i, -1.43494\times10^{-16} + 0.0000186293\,i\},$
$\{3.06903\times10^{-16} - 0.0000185775\,i, 1.66533\times10^{-16} - 4.49768\times10^{-10}\,i,$
$-5.99022\times10^{-17} - 7.04237\times10^{-6}\,i, -1.11022\times10^{-16} - 2.79624\times10^{-10}\,i\},$
$\{-1.11022\times10^{-16} + 5.22753\times10^{-10}\,i, 0. - 0.0000186293\,i,$
$0. + 8.49112\times10^{-11}\,i, 5.55112\times10^{-17} - 6.90403\times10^{-6}\,i\}\}$

In[48]:=
```
Max[
 Abs[a.b.ConjugateTranspose[c] + I * d.ConjugateTranspose[A].e.ConjugateTranspose[c] - UP]]
```

Out[48]= 0.0000186293

In[49]:=
```
{b, e}
```

Out[49]=
```
{{{0.591943, 0., 0., 0.}, {0., 0.591943, 0., 0.},
  {0., 0., 0.591929, 0.}, {0., 0., 0., 0.591929}}, {{0.805995, 0., 0., 0.},
  {0., 0.805995, 0., 0.}, {0., 0., 0.805975, 0.}, {0., 0., 0., 0.805975}}}
```

In[50]:=
```
Max[Abs[b.a.ConjugateTranspose[c] + I * F.a.e.ConjugateTranspose[c] - UP]]
```

Out[50]=
```
0.0000617957
```

In[51]:=
```
Max[Abs[(b + I * F.e).a.ConjugateTranspose[c] - UP]]
```

Out[51]=
```
0.000058278
```

In[52]:=
```
DiagonalPart = b + I * F.e
```

Out[52]=
```
{{0.591943 + 0.805995 i, 0. + 0. i, 0. + 0. i, 0. + 0. i},
 {0. + 0. i, 0.591943 - 0.805995 i, 0. + 0. i, 0. + 0. i},
 {0. + 0. i, 0. + 0. i, 0.591929 + 0.805975 i, 0. + 0. i},
 {0. + 0. i, 0. + 0. i, 0. + 0. i, 0.591929 - 0.805975 i}}
```

In[53]:=
```
ξ = a.ConjugateTranspose[c]
```

Out[53]=
```
{{-0.0000434084 + 0. i, -0.936404 + 0. i, -0.0000162015 + 0. i, -0.350923 + 0. i},
 {-0.936404 + 0. i, 0.000043365 + 0. i, -0.350923 + 0. i, 0.0000163173 + 0. i},
 {0.0000162015 + 0. i, 0.350923 + 0. i, -0.0000434084 + 0. i, -0.936404 + 0. i},
 {0.350923 + 0. i, -0.0000163173 + 0. i, -0.936404 + 0. i, 0.000043365 + 0. i}}
```

In[54]:=
```
Max[Abs[DiagonalPart.ξ - UP]]
```

Out[54]=
```
0.000058278
```

In[55]:=
```
Max[Abs[USecondStage - M.UP.ConjugateTranspose[M]]]
```

Out[55]= 3.3392×10^{-16}

In[56]:=
```
Max[Abs[USecondStage - M.DiagonalPart.ConjugateTranspose[M].M.ξ.ConjugateTranspose[M]]]
```

Out[56]=
```
0.0000582918
```

In[57]:=
```
(*Now we decompose this happiness*)
Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]
```

Out[57]=
```
{{UA11 UB11, UA11 UB12, UA12 UB11, UA12 UB12}, {UA11 UB21, UA11 UB22, UA12 UB21, UA12 UB22},
 {UA21 UB11, UA21 UB12, UA22 UB11, UA22 UB12}, {UA21 UB21, UA21 UB22, UA22 UB21, UA22 UB22}}
```

In[ ]:=

In[160]:=
```
WeHave = M.ℰ.ConjugateTranspose[M]
Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]
UAMatr = {{UA11, UA12}, {UA21, UA22}}
UBMatr = {{UB11, UB12}, {UB21, UB22}}
Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[2]], Search[[1]][[1]] / Search[[1]][[2]]}]
Reverse[{WeHave[[1]][[3]] / WeHave[[1]][[4]], Search[[1]][[3]] / Search[[1]][[4]]}]
Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[3]], Search[[1]][[1]] / Search[[1]][[3]]}]
Reverse[{WeHave[[1]][[2]] / WeHave[[1]][[4]], Search[[1]][[2]] / Search[[1]][[4]]}]
Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[2]], Search[[2]][[1]] / Search[[2]][[2]]}]
Reverse[{WeHave[[2]][[3]] / WeHave[[2]][[4]], Search[[2]][[3]] / Search[[2]][[4]]}]
Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[3]], Search[[2]][[1]] / Search[[2]][[3]]}]
Reverse[{WeHave[[2]][[2]] / WeHave[[2]][[4]], Search[[2]][[2]] / Search[[2]][[4]]}]
Reverse[{WeHave[[3]][[1]] / WeHave[[3]][[2]], Search[[3]][[1]] / Search[[3]][[2]]}]
Reverse[{WeHave[[3]][[3]] / WeHave[[3]][[4]], Search[[3]][[3]] / Search[[3]][[4]]}]
Reverse[{WeHave[[4]][[1]] / WeHave[[4]][[2]], Search[[4]][[1]] / Search[[4]][[2]]}]
Reverse[{WeHave[[4]][[3]] / WeHave[[4]][[4]], Search[[4]][[3]] / Search[[4]][[4]]}]
Reverse[{WeHave[[3]][[1]] / WeHave[[1]][[4]], Search[[3]][[1]] / Search[[1]][[4]]}]
Reverse[{WeHave[[4]][[1]] / WeHave[[1]][[4]], Search[[4]][[1]] / Search[[1]][[4]]}]
Reverse[{WeHave[[4]][[2]] / WeHave[[3]][[3]], Search[[4]][[2]] / Search[[3]][[3]]}]
Reverse[{WeHave[[1]][[2]] / WeHave[[3]][[1]], Search[[1]][[2]] / Search[[3]][[1]]}]
```

Out[160]= $\{\{-2.16963\times10^{-8}+0.350923\,i,\ -0.0000162594+1.38157\times10^{-8}\,i,\ -5.78946\times10^{-8}+0.936404\,i,\ -0.0000433867+3.68658\times10^{-8}\,i\},\ \{0.0000162594+1.38157\times10^{-8}\,i,\ -2.16963\times10^{-8}-0.350923\,i,\ 0.0000433867+3.68658\times10^{-8}\,i,\ -5.78946\times10^{-8}-0.936404\,i\},\ \{5.78946\times10^{-8}-0.936404\,i,\ 0.0000433867-3.68658\times10^{-8}\,i,\ -2.16963\times10^{-8}+0.350923\,i,\ -0.0000162594+1.38157\times10^{-8}\,i\},\ \{-0.0000433867-3.68658\times10^{-8}\,i,\ 5.78946\times10^{-8}+0.936404\,i,\ 0.0000162594+1.38157\times10^{-8}\,i,\ -2.16963\times10^{-8}-0.350923\,i\}\}$

Out[161]= $\{\{(18.3402-21582.7\,i)\ \mathrm{UA22\,UB12},\ (1.+1.93081\times10^{-11}\,i)\ \mathrm{UA22\,UB12},\ (48.9391-57591.5\,i)\ \mathrm{UA22\,UB12},\ (2.66841+5.15221\times10^{-11}\,i)\ \mathrm{UA22\,UB12}\},\ \{(-0.999999-0.0016994\,i)\ \mathrm{UA22\,UB12},\ (-18.3375+21582.7\,i)\ \mathrm{UA22\,UB12},\ (-2.6684-0.0045347\,i)\ \mathrm{UA22\,UB12},\ (-48.932+57591.5\,i)\ \mathrm{UA22\,UB12}\},\ \{(-48.9391+57591.5\,i)\ \mathrm{UA22\,UB12},\ (-2.66841-4.46869\times10^{-11}\,i)\ \mathrm{UA22\,UB12},\ (18.3402-21582.7\,i)\ \mathrm{UA22\,UB12},\ \mathrm{UA22\,UB12}\},\ \{(2.6684+0.0045347\,i)\ \mathrm{UA22\,UB12},\ (48.932-57591.5\,i)\ \mathrm{UA22\,UB12},\ (-0.999999-0.0016994\,i)\ \mathrm{UA22\,UB12},\ (-18.3375+21582.7\,i)\ \mathrm{UA22\,UB12}\}\}$

Out[162]= $\{\{(1.+1.93081\times10^{-11}\,i)\ \mathrm{UA22},\ (2.66841+5.15221\times10^{-11}\,i)\ \mathrm{UA22}\},\ \{(-2.66841-4.46869\times10^{-11}\,i)\ \mathrm{UA22},\ \mathrm{UA22}\}\}$

Out[163]= $\{\{(18.3402-21582.7\,i)\ \mathrm{UB12},\ \mathrm{UB12}\},\ \{(-0.999999-0.0016994\,i)\ \mathrm{UB12},\ (-18.3375+21582.7\,i)\ \mathrm{UB12}\}\}$

Out[164]= $\{18.3402-21582.7\,i,\ 18.3402-21582.7\,i\}$

Out[165]= $\{18.3402-21582.7\,i,\ 18.3402-21582.7\,i\}$

Out[166]= $\{0.374755-5.41017\times10^{-17}\,i,\ 0.374755-5.40679\times10^{-17}\,i\}$

Out[167]= $\{0.374755 - 5.40679\times10^{-17}\,i,\ 0.374755 - 6.27621\times10^{-12}\,i\}$

Out[168]= $\{-3.93724\times10^{-8} + 0.0000463333\,i,\ -3.93724\times10^{-8} + 0.0000463333\,i\}$

Out[169]= $\{-3.93724\times10^{-8} + 0.0000463333\,i,\ -3.93724\times10^{-8} + 0.0000463333\,i\}$

Out[170]= $\{0.374755 - 5.41017\times10^{-17}\,i,\ 0.374755 + 2.75687\times10^{-12}\,i\}$

Out[171]= $\{0.374755 - 5.41017\times10^{-17}\,i,\ 0.374755 - 2.58866\times10^{-16}\,i\}$

Out[172]= $\{18.3402 - 21582.7\,i,\ 18.3402 - 21582.7\,i\}$

Out[173]= $\{18.3402 - 21582.7\,i,\ 18.3402 - 21582.7\,i\}$

Out[174]= $\{-3.93724\times10^{-8} + 0.0000463333\,i,\ -3.93724\times10^{-8} + 0.0000463333\,i\}$

Out[175]= $\{-3.93724\times10^{-8} + 0.0000463333\,i,\ -3.93724\times10^{-8} + 0.0000463333\,i\}$

Out[176]= $\{-18.3402 + 21582.7\,i,\ -18.3402 + 21582.7\,i\}$

Out[177]= $\{0.999999 + 0.0016994\,i,\ 0.999999 + 0.0016994\,i\}$

Out[178]= $\{2.66841 - 3.29957\times10^{-7}\,i,\ 2.66841 - 3.29957\times10^{-7}\,i\}$

Out[179]= $\{-1.4755\times10^{-8} - 0.0000173637\,i,\ -1.4755\times10^{-8} - 0.0000173637\,i\}$

In[153]:=

```
UB11 = (18.340207766383053` - 21582.71608019263` i) * UB12
UA11 = (0.37475541020937153` - 5.4067879881030404`*^-17 i) * UA12
UB11 = (18.340207460191692` - 21582.716080172042` i) * UB12
UB21 = (-3.937238674008911`*^-8 + 0.000046333337944429925` i) * UB22
UA21 = ((-18.340207404848844` + 21582.716080104503` i) /
     ((18.340207460191692` - 21582.716080172042` i))) * UA12
UB22 = UB12 * (0.9999985560146947` + 0.0016994024025000693` i) /
    (3.93723866212802`*^-8 - 0.000046333337944284934` i)
UA12 = UA22 * (2.668407107028297` - 3.299565601603283`*^-7 i) /
    (1.0000000000009581` - 1.2367231425687748`*^-7 i)
```

Out[153]= $(18.3402 - 21582.7\,i)\ \mathrm{UB12}$

Out[154]= $(0.374755 - 5.40679\times10^{-17}\,i)\ \mathrm{UA12}$

Out[155]= $(18.3402 - 21582.7\,i)\ \mathrm{UB12}$

Out[156]= $(-0.999999 - 0.0016994\,i)\ \mathrm{UB12}$

Out[157]= $(-1. + 2.56156\times10^{-12}\,i)\ \mathrm{UA12}$

Out[158]= $(-18.3375 + 21582.7\,i)\ \mathrm{UB12}$

Out[159]= $(2.66841 + 5.15221\times10^{-11}\,i)\ \mathrm{UA22}$

In[180]:=

```
ConjugateTranspose[{{UA11, UA12}, {UA21, UA22}}].{{UA11, UA12}, {UA21, UA22}}
```

Out[180]=

```
{{(8.1204 + 0. i) UA22 Conjugate[UA22],
   (5.7776 × 10^-12 + 4.46872 × 10^-11 i) UA22 Conjugate[UA22]},
 {(5.7776 × 10^-12 - 4.46872 × 10^-11 i) UA22 Conjugate[UA22],
   (8.1204 + 0. i) UA22 Conjugate[UA22]}}
```

In[181]:= UA22 = (1 / ((8.120396488778955` + 0.` i) ^ (1 / 2))) * Exp[I * ψ]

Out[181]= $(0.350923 + 0.\ i)\ e^{i\psi}$

In[182]:= ConjugateTranspose[{{UB11, UB12}, {UB21, UB22}}].{{UB11, UB12}, {UB21, UB22}}

Out[182]= $\{\{(4.65814\times10^{8}+0.\ i)\ \text{UB12 Conjugate[UB12]},$
$(-3.06306\times10^{-7}-1.5576\times10^{-7}\ i)\ \text{UB12 Conjugate[UB12]}\},$
$\{(-3.06306\times10^{-7}+1.5576\times10^{-7}\ i)\ \text{UB12 Conjugate[UB12]},$
$(4.65814\times10^{8}+0.\ i)\ \text{UB12 Conjugate[UB12]}\}\}$

In[183]:= UB12 = (1 / ((4.658139707605265`*^8 + 0.` i) ^ (1 / 2))) * Exp[I * ϕ]

Out[183]= $(0.0000463334 + 0.\ i)\ e^{i\phi}$

In[184]:= ϕ = 0

Out[184]= 0

In[185]:= Search

Out[185]= $\{\{(0.000298201-0.350923\ i)\ e^{i\psi},\ (0.0000162594+3.13938\times10^{-16}\ i)\ e^{i\psi},$
$(0.000795722-0.936404\ i)\ e^{i\psi},\ (0.0000433868+8.3772\times10^{-16}\ i)\ e^{i\psi}\},$
$\{(-0.0000162594-2.76313\times10^{-8}\ i)\ e^{i\psi},\ (-0.000298158+0.350923\ i)\ e^{i\psi},$
$(-0.0000433867-7.37316\times10^{-8}\ i)\ e^{i\psi},\ (-0.000795606+0.936404\ i)\ e^{i\psi}\},$
$\{(-0.000795722+0.936404\ i)\ e^{i\psi},\ (-0.0000433868-7.26583\times10^{-16}\ i)\ e^{i\psi},$
$(0.000298201-0.350923\ i)\ e^{i\psi},\ (0.0000162594+0.\ i)\ e^{i\psi}\},$
$\{(0.0000433867+7.37316\times10^{-8}\ i)\ e^{i\psi},\ (0.000795606-0.936404\ i)\ e^{i\psi},$
$(-0.0000162594-2.76313\times10^{-8}\ i)\ e^{i\psi},\ (-0.000298158+0.350923\ i)\ e^{i\psi}\}\}$

In[186]:= ψ = Log[WeHave[[1]][[1]] / (-0.0002972345461975721` - 0.3466644810136629` i)] / I

Out[186]= $-3.14074 - 0.012208\ i$

In[187]:= Max[Abs[Search - WeHave]]

Out[187]= 0.0116136

In[188]:= UAMatr

Out[188]= $\{\{-0.355233-0.000304603\ i,\ -0.947906-0.000812805\ i\},$
$\{0.947906+0.000812805\ i,\ -0.355233-0.000304603\ i\}\}$

In[189]:= UBMatr

Out[189]= $\{\{0.000849763-1.\ i,\ 0.0000463334+0.\ i\},$
$\{-0.0000463333-7.8739\times10^{-8}\ i,\ -0.00084964+1.\ i\}\}$

In[190]:= Max[Abs[Search - KroneckerProduct[UAMatr, UBMatr]]]

Out[190]= 1.11023×10^{-16}

In[193]:=
```
TeXForm[MatrixForm[Round[UAMatr, 0.00001]]]
```

Out[193]//TeXForm=
```
\left(
\begin{array}{cc}
 -0.35523-0.0003 i & -0.94791-0.00081 i \\
 0.94791\, +0.00081 i & -0.35523-0.0003 i \\
\end{array}
\right)
```

In[194]:=
```
TeXForm[MatrixForm[Round[UBMatr, 0.00001]]]
```

Out[194]//TeXForm=
```
\left(
\begin{array}{cc}
 0.00085\, -1. i & 0.00005 \\
 -0.00005 & -0.00085+1. i \\
\end{array}
\right)
```

In[195]:=
```
Max[Abs[Search - M.ξ.ConjugateTranspose[M]]]
```

Out[195]= 0.0116136

In[196]:=
```
Max[Abs[USecondStage - M.DiagonalPart.ConjugateTranspose[M].M.ξ.ConjugateTranspose[M]]]
```

Out[196]= 0.0000582918

In[197]:=
```
Max[Abs[
  USecondStage - M.DiagonalPart.ConjugateTranspose[M].KroneckerProduct[UAMatr, UBMatr]]]
```

Out[197]= 0.00938745

In[198]:=
```
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
```

$$\phi 1 = \frac{1}{\sqrt{2}} * (\text{KroneckerProduct}[\{\{1, 0\}\}, \{\{1, 0\}\}] + \text{KroneckerProduct}[\{\{0, 1\}\}, \{\{0, 1\}\}])$$

$$\phi 2 = \frac{-I}{\sqrt{2}} * (\text{KroneckerProduct}[\{\{1, 0\}\}, \{\{1, 0\}\}] - \text{KroneckerProduct}[\{\{0, 1\}\}, \{\{0, 1\}\}])$$

$$\phi 3 = \frac{1}{\sqrt{2}} * (\text{KroneckerProduct}[\{\{1, 0\}\}, \{\{0, 1\}\}] - \text{KroneckerProduct}[\{\{0, 1\}\}, \{\{1, 0\}\}])$$

$$\phi 4 = \frac{-I}{\sqrt{2}} * (\text{KroneckerProduct}[\{\{1, 0\}\}, \{\{0, 1\}\}] + \text{KroneckerProduct}[\{\{0, 1\}\}, \{\{1, 0\}\}])$$

```
DMatrix =
 {{Exp[I * Φ0], 0, 0, 0}, {0, Exp[I * Φ1], 0, 0}, {0, 0, Exp[I * Φ2], 0}, {0, 0, 0, Exp[I * Φ3]}}
FullSimplify[M.DMatrix.ConjugateTranspose[M] -
  Exp[I * θ0] * MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] +
       θ2 * KroneckerProduct[σy, σy] + θ3 * KroneckerProduct[σz, σz])]]
```

Out[198]= $\{\{1, 1, -1, 1\}, \{1, 1, 1, -1\}, \{1, -1, -1, -1\}, \{1, -1, 1, 1\}\}$

Out[199]= $\frac{\Phi 0}{4} + \frac{\Phi 1}{4} + \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[200]= $\frac{\Phi 0}{4} + \frac{\Phi 1}{4} - \frac{\Phi 2}{4} - \frac{\Phi 3}{4}$

Out[201]= $-\frac{\Phi 0}{4} + \frac{\Phi 1}{4} - \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[202]= $\frac{\Phi 0}{4} - \frac{\Phi 1}{4} - \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[203]= $\{\{0, 1\}, \{1, 0\}\}$

Out[204]= $\{\{0, -i\}, \{i, 0\}\}$

Out[205]= $\{\{1, 0\}, \{0, -1\}\}$

Out[206]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[207]= $\left\{\left\{-\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[208]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[209]= $\left\{\left\{0, -\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right\}\right\}$

Out[210]= $\left\{\left\{e^{i \Phi 0}, 0, 0, 0\right\}, \left\{0, e^{i \Phi 1}, 0, 0\right\}, \left\{0, 0, e^{i \Phi 2}, 0\right\}, \left\{0, 0, 0, e^{i \Phi 3}\right\}\right\}$

Out[211]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

```
In[212]:= σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
MatrixForm[CNOT1]
MatrixForm[CNOT2]
Ry[θ_] := {{Cos[θ/2], Sin[θ/2]}, {-Sin[θ/2], Cos[θ/2]}}
Rz[α_] := {{e^(ⅈα/2), 0}, {0, e^(-ⅈα/2)}}
Unit2 = {{1, 0}, {0, 1}}
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
ϕ1 = 1/√2 * (KroneckerProduct[{{1, 0}}, {{1, 0}}] + KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ2 = -I/√2 * (KroneckerProduct[{{1, 0}}, {{1, 0}}] - KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ3 = 1/√2 * (KroneckerProduct[{{1, 0}}, {{0, 1}}] - KroneckerProduct[{{0, 1}}, {{1, 0}}])
ϕ4 = -I/√2 * (KroneckerProduct[{{1, 0}}, {{0, 1}}] + KroneckerProduct[{{0, 1}}, {{1, 0}}])
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
H = 1/√2 * {{1, 1}, {1, -1}}
CNOT2Trial = KroneckerProduct[H, H].CNOT1.KroneckerProduct[H, H]
CNOT2Trial - CNOT2
U3[θ_, ϕ_, λ_] :=
  {{Cos[θ/2], -Exp[I * λ] * Sin[θ/2]}, {Exp[I * ϕ] * Sin[θ/2], Exp[I * (ϕ + λ)] * Cos[θ/2]}}
FullSimplify[Exp[I * π / 4] * KroneckerProduct[Rz[-π/2], Unit2].CNOT2.
    KroneckerProduct[Unit2, Ry[2 * θ2 - π/2]].CNOT1.KroneckerProduct[
      Rz[2 * θ3 - π/2], Ry[π/2 - 2 * θ1]].CNOT2.KroneckerProduct[Unit2, Rz[π/2]] -
   MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] + θ2 * KroneckerProduct[σy, σy] +
       θ3 * KroneckerProduct[σz, σz])]]

Out[212]= {{0, 1}, {1, 0}}

Out[213]= {{0, -ⅈ}, {ⅈ, 0}}

Out[214]= {{1, 0}, {0, -1}}

Out[215]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}

Out[216]= {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
```

Out[217]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Out[218]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Out[221]= $\{\{1, 0\}, \{0, 1\}\}$

Out[222]= $\{\{0, 1\}, \{1, 0\}\}$

Out[223]= $\{\{0, -i\}, \{i, 0\}\}$

Out[224]= $\{\{1, 0\}, \{0, -1\}\}$

Out[225]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[226]= $\left\{\left\{-\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[227]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[228]= $\left\{\left\{0, -\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right\}\right\}$

Out[229]= $\{\{1, 1, -1, 1\}, \{1, 1, 1, -1\}, \{1, -1, -1, -1\}, \{1, -1, 1, 1\}\}$

Out[230]= $\frac{\Phi 0}{4} + \frac{\Phi 1}{4} + \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[231]= $\frac{\Phi 0}{4} + \frac{\Phi 1}{4} - \frac{\Phi 2}{4} - \frac{\Phi 3}{4}$

Out[232]= $-\frac{\Phi 0}{4} + \frac{\Phi 1}{4} - \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[233]= $\frac{\Phi 0}{4} - \frac{\Phi 1}{4} - \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[234]= $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}$

Out[235]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[236]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[237]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[238]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[240]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

In[241]:=
```
Φ0 = Log[DiagonalPart[[1]][[1]]] / I
Φ1 = Log[DiagonalPart[[2]][[2]]] / I
Φ2 = Log[DiagonalPart[[3]][[3]]] / I
Φ3 = Log[DiagonalPart[[4]][[4]]] / I
```

Out[241]= 0.937338 - 0.0000123971 i

Out[242]= -0.937338 - 0.0000123971 i

Out[243]= 0.937337 + 0.0000119311 i

Out[244]= -0.937337 + 0.0000119311 i

In[245]:=
```
Max[Abs[USecondStage - Exp[I * θ0] * Exp[I * π / 4] *
    KroneckerProduct[Rz[-π / 2], Unit2].CNOT2.KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
     CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].CNOT2.
     KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[UAMatr, UBMatr]]]
```

Out[245]= 0.00938745

In[246]:=
```
Max[Abs[USecondStage -
   Exp[I * θ0] * Exp[I * π / 4] * KroneckerProduct[Rz[-π / 2], Unit2].KroneckerProduct[H, H].
     CNOT1.KroneckerProduct[H, H].KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
     CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].
     KroneckerProduct[H, H].CNOT1.KroneckerProduct[H, H].
     KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[UAMatr, UBMatr]]]
```

Out[246]= 0.00938745

In[247]:=
```
Max[Abs[USecondStage - Exp[I * (θ0 + π / 4)] * KroneckerProduct[Rz[-π / 2].H, Unit2.H].
     CNOT1.KroneckerProduct[H.Unit2, H.Ry[2 * θ2 - π / 2]].CNOT1.
     KroneckerProduct[Rz[2 * θ3 - π / 2].H, Ry[π / 2 - 2 * θ1].H].CNOT1.
     KroneckerProduct[H.Unit2.UAMatr, H.Rz[π / 2].UBMatr]]]
```

Out[247]= 0.00938745

In[248]:=
```
U11 = Rz[-π/2].H
U12 = Unit2.H
U21 = H.Unit2
U22 = H.Ry[2*θ2 - π/2]
U31 = Rz[2*θ3 - π/2].H
U32 = Ry[π/2 - 2*θ1].H
U41 = H.Unit2.UAMatr
U42 = H.Rz[π/2].UBMatr
```

Out[248]= $\left\{\left\{\frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}, \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}\right\}, \left\{\frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}, -\frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}\right\}\right\}$

Out[249]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[250]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[251]= $\{\{0.591936 + 4.47412\times10^{-17}\,i, -0.805985 + 3.2859\times10^{-17}\,i\},$
$\{-0.805985 + 3.2859\times10^{-17}\,i, -0.591936 - 4.47412\times10^{-17}\,i\}\}$

Out[252]= $\{\{0.5 - 0.5\,i, 0.5 - 0.5\,i\}, \{0.5 + 0.5\,i, -0.5 - 0.5\,i\}\}$

Out[253]= $\{\{1. + 0.\,i, 0. - 0.0000121641\,i\}, \{0. - 0.0000121641\,i, -1. + 0.\,i\}\}$

Out[254]= $\{\{0.419083 + 0.000359353\,i, -0.921458 - 0.000790127\,i\},$
$\{-0.921458 - 0.000790127\,i, -0.419083 - 0.000359353\,i\}\}$

Out[255]= $\{\{0.500401 - 0.499552\,i, 0.499598 + 0.500448\,i\},$
$\{0.500448 - 0.499598\,i, -0.499552 - 0.500401\,i\}\}$

In[256]:=
```
Max[Abs[USecondStage -
    Exp[I * (θ0 + π / 4)] * KroneckerProduct[U11, U12].CNOT1.KroneckerProduct[U21, U22].
      CNOT1.KroneckerProduct[U31, U32].CNOT1.KroneckerProduct[U41, U42]]]
```

Out[256]= 0.00938745

```
(*OK, decomposition of USecondStage finished*)
```

In[264]:=
```
N0 = 3.3506871002735
r = 1
s = 1
θ = π / 2 - 1
```
$\omega = \sqrt{s^2 - r^2 \, \mathrm{Sin}[\theta]^2}$

```
α = ArcSin[(r / s) * Sin[θ]]
```
$\tau\mathrm{New} = \frac{(\pi/2)}{\omega}$

Out[264]= 3.35069

Out[265]= 1

Out[266]= 1

Out[267]= $-1 + \frac{\pi}{2}$

Out[268]= $\sqrt{1 - \mathrm{Cos}[1]^2}$

Out[269]= $-1 + \frac{\pi}{2}$

Out[270]= $\frac{\pi}{2\sqrt{1 - \mathrm{Cos}[1]^2}}$

In[271]:= $\mathrm{vFirst} = \left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}\right\}$

$\mathrm{vSecond} = \left\{\left\{\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}\right\}\right\}$

$\mathrm{vThird} = \left\{\left\{\mathrm{Cos}\left[\frac{\rho}{2}\right], i * \mathrm{Sin}\left[\frac{\rho}{2}\right]\right\}\right\}$

Out[271]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[272]= $\left\{\left\{\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[273]= $\left\{\left\{\mathrm{Cos}\left[\frac{\rho}{2}\right], i\,\mathrm{Sin}\left[\frac{\rho}{2}\right]\right\}\right\}$

In[274]:=
```
z1 = 2 * ArcTan[(N0 - 1) ^ (1 / 2)]
Rrot = MatrixExp[-I * z1 * σy / 2]
Ancilla = N[Rrot.Transpose[{{1, 0}}]]
```

Out[274]= 1.98571

Out[275]= {{0.546302 + 0. i, -0.837588 + 0. i}, {0.837588 + 0. i, 0.546302 + 0. i}}

Out[276]= {{0.546302 + 0. i}, {0.837588 + 0. i}}

In[ ]:= **Evolution = {{Cos[τ - α], -i * Sin[τ]}, {-i * Sin[τ], Cos[α + τ]}} * Sec[α]**

Out[ ]= {{Csc[1] Sin[1 + τ], -i Csc[1] Sin[τ]}, {-i Csc[1] Sin[τ], Csc[1] Sin[1 - τ]}}

In[ ]:= **τ = π / 2**

Out[ ]= $\frac{\pi}{2}$

In[ ]:= **Evolution.Transpose[vFirst]**

Out[ ]= $\left\{\left\{\frac{\text{Cot}[1]}{\sqrt{2}}+\frac{\text{Csc}[1]}{\sqrt{2}}\right\},\left\{-\frac{i\,\text{Cot}[1]}{\sqrt{2}}-\frac{i\,\text{Csc}[1]}{\sqrt{2}}\right\}\right\}$

In[ ]:= **(Evolution.Transpose[vFirst])[[1]][[1]]**

Out[ ]= $\frac{\text{Cot}[1]}{\sqrt{2}}+\frac{\text{Csc}[1]}{\sqrt{2}}$

In[ ]:= **(Evolution.Transpose[vFirst])[[2]][[1]]**

Out[ ]= $-\frac{i\,\text{Cot}[1]}{\sqrt{2}}-\frac{i\,\text{Csc}[1]}{\sqrt{2}}$

In[ ]:= **NormalizationOutput = (FullSimplify[Abs[(Evolution.Transpose[vFirst])[[1]][[1]]]^2 + Abs[(Evolution.Transpose[vFirst])[[2]][[1]]]^2])^(1/2)**

Out[ ]= $\text{Cot}\left[\frac{1}{2}\right]$

In[ ]:= **FullSimplify[Evolution.Transpose[vFirst] / NormalizationOutput]**

Out[ ]= $\left\{\left\{\frac{1}{\sqrt{2}}\right\},\left\{-\frac{i}{\sqrt{2}}\right\}\right\}$

In[ ]:= **FinalRotation =** $\left\{\left\{\frac{1}{\sqrt{2}},\frac{i}{\sqrt{2}}\right\},\left\{\frac{i}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\}\right\}$

Out[ ]= $\left\{\left\{\frac{1}{\sqrt{2}},\frac{i}{\sqrt{2}}\right\},\left\{\frac{i}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\}\right\}$

In[ ]:= **FullSimplify[FinalRotation.Evolution.Transpose[vFirst] / NormalizationOutput]**

Out[ ]= {{1}, {0}}

In[ ]:= **FullSimplify[FinalRotation.Evolution.Transpose[vSecond]]**

Out[ ]= $\left\{\{0\},\left\{-i\,\text{Tan}\left[\frac{1}{2}\right]\right\}\right\}$

In[ ]:= **U12P = FinalRotation.U12**

Out[ ]= $\left\{\left\{\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{i}{2}\right\},\left\{\frac{1}{2}+\frac{i}{2},-\frac{1}{2}+\frac{i}{2}\right\}\right\}$

In[ ]:=

In[ ]:=

```
In[•]:= (*U11*)
U11OverallPhase = Arg[U11[[1]][[1]]]
U11Dephased = U11 * Exp[-I * U11OverallPhase]
θU11 = 2 * ArcTan[Abs[U11Dephased[[1]][[2]]] / Abs[U11Dephased[[1]][[1]]]]
ϕU11 = Arg[U11Dephased[[2]][[1]]]
λU11 = Arg[U11Dephased[[2]][[2]]] - ϕU11
Max[N[Abs[Exp[I * U11OverallPhase] * U3[θU11, ϕU11, λU11] - U11]]]
(*U12P*)
U12POverallPhase = Arg[U12P[[1]][[1]]]
U12PDephased = U12P * Exp[-I * U12POverallPhase]
θU12P = 2 * ArcTan[Abs[U12PDephased[[1]][[2]]] / Abs[U12PDephased[[1]][[1]]]]
ϕU12P = Arg[U12PDephased[[2]][[1]]]
λU12P = Arg[U12PDephased[[2]][[2]]] - ϕU12P
Max[N[Abs[Exp[I * U12POverallPhase] * U3[θU12P, ϕU12P, λU12P] - U12P]]]
(*U21*)
U21OverallPhase = Arg[U21[[1]][[1]]]
U21Dephased = U21 * Exp[-I * U21OverallPhase]
θU21 = 2 * ArcTan[Abs[U21Dephased[[1]][[2]]] / Abs[U21Dephased[[1]][[1]]]]
ϕU21 = Arg[U21Dephased[[2]][[1]]]
λU21 = Arg[U21Dephased[[2]][[2]]] - ϕU21
Max[N[Abs[Exp[I * U21OverallPhase] * U3[θU21, ϕU21, λU21] - U21]]]
(*U22*)
U22OverallPhase = Arg[U22[[1]][[1]]]
U22Dephased = U22 * Exp[-I * U22OverallPhase]
θU22 = 2 * ArcTan[Abs[U22Dephased[[1]][[2]]] / Abs[U22Dephased[[1]][[1]]]]
ϕU22 = Arg[U22Dephased[[2]][[1]]]
λU22 = Arg[U22Dephased[[2]][[2]]] - ϕU22
Max[N[Abs[Exp[I * U22OverallPhase] * U3[θU22, ϕU22, λU22] - U22]]]
(*U31*)
U31OverallPhase = Arg[U31[[1]][[1]]]
U31Dephased = U31 * Exp[-I * U31OverallPhase]
θU31 = 2 * ArcTan[Abs[U31Dephased[[1]][[2]]] / Abs[U31Dephased[[1]][[1]]]]
ϕU31 = Arg[U31Dephased[[2]][[1]]]
λU31 = Arg[U31Dephased[[2]][[2]]] - ϕU31
Max[N[Abs[Exp[I * U31OverallPhase] * U3[θU31, ϕU31, λU31] - U31]]]
(*U32*)
U32OverallPhase = Arg[U32[[1]][[1]]]
U32Dephased = U32 * Exp[-I * U32OverallPhase]
θU32 = 2 * ArcTan[Abs[U32Dephased[[1]][[2]]] / Abs[U32Dephased[[1]][[1]]]]
ϕU32 = Arg[U32Dephased[[2]][[1]]]
λU32 = Arg[U32Dephased[[2]][[2]]] - ϕU32
Max[N[Abs[Exp[I * U32OverallPhase] * U3[θU32, ϕU32, λU32] - U32]]]
(*U41R*)
U41OverallPhaseR = Arg[(U41.Rrot)[[1]][[1]]]
U41DephasedR = (U41.Rrot) * Exp[-I * U41OverallPhaseR]
θU41R = 2 * ArcTan[Abs[U41DephasedR[[1]][[2]]] / Abs[U41DephasedR[[1]][[1]]]]
ϕU41R = Arg[U41DephasedR[[2]][[1]]]
λU41R = Arg[U41DephasedR[[2]][[2]]] - ϕU41R
Max[N[Abs[Exp[I * U41OverallPhaseR] * U3[θU41R, ϕU41R, λU41R] - U41.Rrot]]]
```

Out[ ]= $-\frac{\pi}{4}$

Out[ ]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[ ]= $\frac{\pi}{2}$

Out[ ]= $\frac{\pi}{2}$

Out[ ]= $-\pi$

Out[ ]= 0.

Out[ ]= $\frac{\pi}{4}$

Out[ ]= $\left\{\left\{\left(\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}, \left(\frac{1}{2}-\frac{i}{2}\right) e^{-\frac{i\pi}{4}}\right\}, \left\{\left(\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}, \left(-\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}\right\}\right\}$

Out[ ]= $\frac{\pi}{2}$

Out[ ]= 0

Out[ ]= $\frac{\pi}{2}$

Out[ ]= 0.

Out[ ]= 0

Out[ ]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[ ]= $\frac{\pi}{2}$

Out[ ]= 0

Out[ ]= π

Out[ ]= 0.

Out[ ]= 0.0000125403

Out[ ]= $\{\{0.588289 - 8.47033 \times 10^{-22}\, i, -0.808651 + 0.0000155077\, i\},$
$\{-0.808651 + 0.0000155077\, i, -0.588289 + 8.47033 \times 10^{-22}\, i\}\}$

Out[ ]= 1.88371

Out[ ]= 3.14157

Out[ ]= 0.0000191773

Out[ ]= 0.0000310154

Out[ ]= -0.785398

Out[•]= $\{\{0.707107 - 5.55112\times10^{-17}\, i,\ 0.707107 - 5.55112\times10^{-17}\, i\},$
$\{2.37672\times10^{-7} + 0.707107\, i,\ -2.37672\times10^{-7} - 0.707107\, i\}\}$

Out[•]= 1.5708

Out[•]= 1.5708

Out[•]= -3.14159

Out[•]= 3.30202×10^{-9}

Out[•]= -1.13351×10^{-12}

Out[•]= $\{\{1. + 0.\, i,\ -2.71345\times10^{-8} - 0.0000417736\, i\},\ \{-2.71345\times10^{-8} - 0.0000417736\, i,\ -1. + 0.\, i\}\}$

Out[•]= 0.0000835473

Out[•]= -1.57145

Out[•]= 4.71304

Out[•]= 0.0000835473

Out[•]= -3.14074

Out[•]= $\{\{0.532429 - 1.07607\times10^{-16}\, i,\ 0.846475 + 6.49496\times10^{-8}\, i\},$
$\{0.846475 - 1.55472\times10^{-8}\, i,\ -0.532429 - 3.10758\times10^{-8}\, i\}\}$

Out[•]= 2.01866

Out[•]= -1.8367×10^{-8}

Out[•]= -3.14159

Out[•]= 6.97775×10^{-12}

(*Constant input*)

In[•]:= **{θU11, ϕU11, λU11}**

Out[•]= $\left\{\frac{\pi}{2}, \frac{\pi}{2}, -\pi\right\}$

In[•]:= **{θU12P, ϕU12P, λU12P}**

Out[•]= $\left\{\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

In[•]:= **{θU21, ϕU21, λU21}**

Out[•]= $\left\{\frac{\pi}{2}, 0, \pi\right\}$

In[•]:= **{θU22, ϕU22, λU22}**

Out[•]= {1.88371, 3.14157, 0.0000191773}

In[•]:= **{θU31, ϕU31, λU31}**

Out[•]= {1.5708, 1.5708, −3.14159}

In[ ]:= {θU32, ϕU32, λU32}

Out[ ]= {0.0000835473, -1.57145, 4.71304}

In[ ]:= {θU41R, ϕU41R, λU41R}

Out[ ]= $\{2.01866, -1.8367\times10^{-8}, -3.14159\}$

```
(*Varying input*)
```

In[ ]:=
```
(*U42*)
ρ = -(π/2) * 1.0
Initializer = {{Cos[ρ/2], ⅈ Sin[ρ/2]}, {ⅈ Sin[ρ/2], Cos[ρ/2]}}
U42POverallPhase = Arg[(U42.Initializer)[[1]][[1]]]
U42PDephased = (U42.Initializer) * Exp[-I * U42POverallPhase]
θU42P = 2 * ArcTan[Abs[U42PDephased[[1]][[2]]] / Abs[U42PDephased[[1]][[1]]]]
ϕU42P = Arg[U42PDephased[[2]][[1]]]
λU42P = Arg[U42PDephased[[2]][[2]]] - ϕU42P
Max[N[Abs[Exp[I * U42POverallPhase] * U3[θU42P, ϕU42P, λU42P] - (U42.Initializer)]]]
{θU42P, ϕU42P, λU42P}
```

Out[ ]= -1.5708

Out[ ]= {{0.707107, 0. - 0.707107 ⅈ}, {0. - 0.707107 ⅈ, 0.707107}}

Out[ ]= -0.786255

Out[ ]= $\{\{1. + 0.\, i, -1.33441\times10^{-7} + 0.00011429\, i\}, \{0.00011429 - 1.33463\times10^{-7}\, i, -1.95909\times10^{-7} - 1.\, i\}\}$

Out[ ]= 0.00022858

Out[ ]= -0.00116776

Out[ ]= -1.56963

Out[ ]= 1.00367×10^{-11}

Out[ ]= {0.00022858, -0.00116776, -1.56963}

In[ ]:=

```
(* ρ = (π/2)*1.0 *)

{3.1413640738101765`, 3.140424893333628`, -4.713556544857268`}

(* ρ = (π/2) * 0.8 *)

{2.82766196785452`, 6.260768471081834`*^-7, -1.570795462540058`}

(* ρ = (π/2) * 0.6 *)

{2.5135027024956016`, 1.715681921047185`*^-7, -1.5707958726059896`}

(* ρ = (π/2) * 0.4 *)
```

```
{2.1993434371366454`, -1.9453505548842627`*^-9, -1.5707959968563647`}
(* ρ = (π/2) * 0.2 *)
{1.8851841717776803`, -1.0915684202790156`*^-7, -1.5707960461581851`}
(* ρ = (π/2) * 0.0 *)
{1.5710249064187127`, -1.95878356460808`*^-7, -1.5707960599133464`}
(* ρ = - (π/2) * 0.2 *)
{1.2568656410597447`, -2.825869904095626`*^-7, -1.5707960461998678`}
(* ρ = - (π/2) * 0.4 *)
{0.9427063757007798`, -3.8974696018974416`*^-7, -1.570795996965934`}
(* ρ = - (π/2) * 0.6 *)
{0.6285471103418238`, -5.630937743885336`*^-7, -1.570795872891687`}
(* ρ = - (π/2) * 0.8 *)
{0.3143878449829049`, -1.0166779008257722`*^-6, -1.5707954637552022`}
(* ρ = - (π/2) * 1.0 *)
{0.0002285797796166426`, -0.0011677603814344868`, -1.569628762322318`}
```

In[ ]:=

In[ ]:=

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