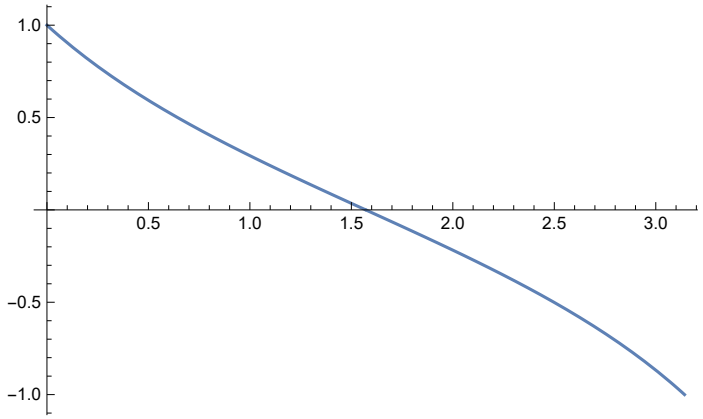
In[1]:=
```
Solve[(1 - x^2) Cos[σ] / (2 * x - 2 Cos[σ] * x^2) == 1, x]
```

Out[1]=
$$\left\{\left\{x \to -\frac{1}{2}\,\text{Sec}[\sigma]\,(-2-2\,\text{Sin}[\sigma])\right\},\ \left\{x \to -\frac{1}{2}\,\text{Sec}[\sigma]\,(-2+2\,\text{Sin}[\sigma])\right\}\right\}$$

In[2]:=
```
Plot[{Sec[σ] (1 - Sin[σ])}, {σ, 0, π}]
```

Out[2]=



In[3]:=
```
ProductHermitian[v1_, v2_, α_, ω_, τ_] :=
 Conjugate[v1].{{Cos[α - ω * τ]^2 + Sin[ω * τ]^2, -2 i Sin[α] Sin[ω * τ]^2},
    {2 i Sin[α] Sin[ω * τ]^2, Cos[α + ω * τ]^2 + Sin[ω * τ]^2}}.Transpose[v2] Sec[α]^2
Evolution = {{Cos[ω * τ - α], -i * Sin[ω * τ]}, {-i * Sin[ω * τ], Cos[α + ω * τ]}} * Sec[α]
EvolutionConjugated =
 Transpose[{{Cos[ω * τ - α], i * Sin[ω * τ]}, {i * Sin[ω * τ], Cos[α + ω * τ]}} * Sec[α]]
v1Ref = {{Cos[1/4 (π - 2 σ)], -i Sin[1/4 (π - 2 σ)]}}
v2Ref = {{Cos[1/4 (π + 2 σ)], -i Sin[1/4 (π + 2 σ)]}}
v1Probe = {{Cos[1/4 (π + 2 δ)], -i Sin[1/4 (π + 2 δ)]}}
```

Out[4]=
$$\{\{\text{Cos}[\alpha-\tau\,\omega]\,\text{Sec}[\alpha],\ -i\,\text{Sec}[\alpha]\,\text{Sin}[\tau\,\omega]\},\ \{-i\,\text{Sec}[\alpha]\,\text{Sin}[\tau\,\omega],\ \text{Cos}[\alpha+\tau\,\omega]\,\text{Sec}[\alpha]\}\}$$

Out[5]=
$$\{\{\text{Cos}[\alpha-\tau\,\omega]\,\text{Sec}[\alpha],\ i\,\text{Sec}[\alpha]\,\text{Sin}[\tau\,\omega]\},\ \{i\,\text{Sec}[\alpha]\,\text{Sin}[\tau\,\omega],\ \text{Cos}[\alpha+\tau\,\omega]\,\text{Sec}[\alpha]\}\}$$

Out[6]=
$$\left\{\left\{\text{Cos}\left[\frac{1}{4}\,(\pi-2\,\sigma)\right],\ -i\,\text{Sin}\left[\frac{1}{4}\,(\pi-2\,\sigma)\right]\right\}\right\}$$

Out[7]=
$$\left\{\left\{\text{Cos}\left[\frac{1}{4}\,(\pi+2\,\sigma)\right],\ -i\,\text{Sin}\left[\frac{1}{4}\,(\pi+2\,\sigma)\right]\right\}\right\}$$

Out[8]=
$$\left\{\left\{\text{Cos}\left[\frac{1}{4}\,(\pi+2\,\delta)\right],\ -i\,\text{Sin}\left[\frac{1}{4}\,(\pi+2\,\delta)\right]\right\}\right\}$$

In[9]:=
```
τ = π / (2 * ω)
```

Out[9]=
$$\frac{\pi}{2\,\omega}$$

In[10]:=
```
α = ArcSin[(1 - Sin[σ]) / Cos[σ]]
```

Out[10]=
$$\text{ArcSin}\left[\text{Sec}[\sigma]\,(1-\text{Sin}[\sigma])\right]$$

In[11]:=
$$\tau = \frac{\text{ArcSin}\left[\sqrt{\frac{\text{Cos}[\alpha]^2\,\text{Cos}[\sigma]}{2\,\text{Sin}[\alpha]-2\,\text{Cos}[\sigma]\,\text{Sin}[\alpha]^2}}\right]}{\omega}$$

Out[11]=
$$\frac{\text{ArcSin}\left[\sqrt{\frac{\text{Cos}[\sigma]\,\left(1-\text{Sec}[\sigma]^2\,(1-\text{Sin}[\sigma])^2\right)}{2\,\text{Sec}[\sigma]\,(1-\text{Sin}[\sigma])-2\,\text{Sec}[\sigma]\,(1-\text{Sin}[\sigma])^2}}\right]}{\omega}$$

In[12]:=
```
FullSimplify[ArcSin[Sqrt[(Cos[σ] (1 - Sec[σ]^2 (1 - Sin[σ])^2))/(2 Sec[σ] (1 - Sin[σ]) - 2 Sec[σ] (1 - Sin[σ])^2)]]]
```

Out[12]=
$$\frac{\pi}{2}$$

In[13]:=
```
cosFirst = FullSimplify[(Abs[ProductHermitian[v1Ref, v1Probe, α, ω, τ]])^2 /
    (ProductHermitian[v1Ref, v1Ref, α, ω, τ] *
      ProductHermitian[v1Probe, v1Probe, α, ω, τ]), {σ ∈ Reals, δ ∈ Reals}]
```

Out[13]=
$$\left\{\left\{\frac{-1+\text{Cos}[\delta-\sigma]}{-2+2\,\text{Cos}[\delta]\,\text{Cos}[\sigma]}\right\}\right\}$$

In[14]:=
```
cosFirstFunction[δ_, σ_] := (1 - Cos[δ - σ])/(2 - 2 Cos[δ] Cos[σ])
```

In[15]:=
```
EvolutionMT = FullSimplify[{{Cos[α + ω * τ] Sec[α], i Sec[α] Sin[ω * τ]},
    {i Sec[α] Sin[ω * τ], Cos[α - ω * τ] Sec[α]}}, {σ ∈ Reals, δ ∈ Reals, σ > 0}]
```

Out[15]=
$$\left\{\left\{-\text{Cot}[\sigma]\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}},\ \frac{i}{2\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}}}\right\},\left\{\frac{i}{2\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}}},\ \text{Cot}[\sigma]\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}}\right\}\right\}$$

In[16]:=
```
LeftEvolutionMT = FullSimplify[{{Cos[α + ω * τ] Sec[α], -i * Sec[α] Sin[ω * τ]},
    {-i * Sec[α] Sin[ω * τ], Sec[α] Cos[α - ω * τ]}}, {σ ∈ Reals, δ ∈ Reals, σ > 0}]
```

Out[16]=
$$\left\{\left\{-\text{Cot}[\sigma]\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}},\ -\frac{i}{2\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}}}\right\},\left\{-\frac{i}{2\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}}},\ \text{Cot}[\sigma]\sqrt{\frac{1}{2+2\,\text{Csc}[\sigma]}}\right\}\right\}$$

In[17]:=
```
MatrixForm[FullSimplify[LeftEvolutionMT.EvolutionMT]]
```

Out[17]//MatrixForm=
$$\begin{pmatrix} \text{Csc}[\sigma] & -i\,\text{Cot}[\sigma] \\ i\,\text{Cot}[\sigma] & \text{Csc}[\sigma] \end{pmatrix}$$

In[18]:=
```
Unit = {{1, 0}, {0, 1}}
```

Out[18]=
$$\{\{1, 0\}, \{0, 1\}\}$$

In[19]:=
```
Nm = (N0 * FullSimplify[LeftEvolutionMT.EvolutionMT])
ZetaF = MatrixPower[Nm - Unit, 1 / 2]
```

Out[19]=
$$\{\{N0\,\text{Csc}[\sigma],\ -i\,N0\,\text{Cot}[\sigma]\},\ \{i\,N0\,\text{Cot}[\sigma],\ N0\,\text{Csc}[\sigma]\}\}$$

Out[20]=
$$\Bigg\{\Bigg\{\frac{\text{Cot}\left[\frac{\sigma}{2}\right]\sqrt{-1+N0\,\text{Cot}\left[\frac{\sigma}{2}\right]}\ \text{Cot}[\sigma]\left(-\text{Sec}[\sigma]+\text{Cot}\left[\frac{\sigma}{2}\right]\text{Tan}[\sigma]\right)}{-1+\text{Cot}\left[\frac{\sigma}{2}\right]^2}-$$

$$\frac{\text{Cot}\left[\frac{\sigma}{2}\right]\text{Cot}[\sigma]\sqrt{-1+N0\,\text{Tan}\left[\frac{\sigma}{2}\right]}\ \left(-\text{Sec}[\sigma]+\text{Tan}\left[\frac{\sigma}{2}\right]\text{Tan}[\sigma]\right)}{-1+\text{Cot}\left[\frac{\sigma}{2}\right]^2},$$

$$\frac{i\sqrt{-1+N0\,\text{Cot}\left[\frac{\sigma}{2}\right]}\ \left(\text{Csc}[\sigma]-\text{Tan}\left[\frac{\sigma}{2}\right]\right)\text{Tan}\left[\frac{\sigma}{2}\right]\left(-\text{Sec}[\sigma]+\text{Cot}\left[\frac{\sigma}{2}\right]\text{Tan}[\sigma]\right)}{-1+\text{Tan}\left[\frac{\sigma}{2}\right]^2}-$$

$$\frac{i\,\text{Cot}\left[\frac{\sigma}{2}\right]\left(\text{Cot}\left[\frac{\sigma}{2}\right]-\text{Csc}[\sigma]\right)\sqrt{-1+N0\,\text{Tan}\left[\frac{\sigma}{2}\right]}\ \left(-\text{Sec}[\sigma]+\text{Tan}\left[\frac{\sigma}{2}\right]\text{Tan}[\sigma]\right)}{-1+\text{Cot}\left[\frac{\sigma}{2}\right]^2}\Bigg\},$$

$$\Bigg\{\frac{i\,\text{Cot}\left[\frac{\sigma}{2}\right]\sqrt{-1+N0\,\text{Cot}\left[\frac{\sigma}{2}\right]}\ \text{Cot}[\sigma]}{-1+\text{Cot}\left[\frac{\sigma}{2}\right]^2}-\frac{i\,\text{Cot}\left[\frac{\sigma}{2}\right]\text{Cot}[\sigma]\sqrt{-1+N0\,\text{Tan}\left[\frac{\sigma}{2}\right]}}{-1+\text{Cot}\left[\frac{\sigma}{2}\right]^2},$$

$$\frac{\text{Cot}\left[\frac{\sigma}{2}\right]\left(\text{Cot}\left[\frac{\sigma}{2}\right]-\text{Csc}[\sigma]\right)\sqrt{-1+N0\,\text{Tan}\left[\frac{\sigma}{2}\right]}}{-1+\text{Cot}\left[\frac{\sigma}{2}\right]^2}-\frac{\sqrt{-1+N0\,\text{Cot}\left[\frac{\sigma}{2}\right]}\ \left(\text{Csc}[\sigma]-\text{Tan}\left[\frac{\sigma}{2}\right]\right)\text{Tan}\left[\frac{\sigma}{2}\right]}{-1+\text{Tan}\left[\frac{\sigma}{2}\right]^2}\Bigg\}\Bigg\}$$

In[•]:=
```
N0 = Cot[σ / 2]
```

Out[•]=
$$\text{Cot}\left[\frac{\sigma}{2}\right]$$

In[•]:=
```
FullSimplify[ZetaF, 0 < σ < π / 2]
```

Out[•]=
$$\left\{\left\{\frac{1}{2}\sqrt{\text{Cos}[\sigma]}\ \text{Csc}\left[\frac{\sigma}{2}\right],\ -\frac{1}{2}i\sqrt{\text{Cos}[\sigma]}\ \text{Csc}\left[\frac{\sigma}{2}\right]\right\},\ \left\{\frac{1}{2}i\sqrt{\text{Cos}[\sigma]}\ \text{Csc}\left[\frac{\sigma}{2}\right],\ \frac{1}{2}\sqrt{\text{Cos}[\sigma]}\ \text{Csc}\left[\frac{\sigma}{2}\right]\right\}\right\}$$

In[•]:=
```
EvolvedFirst = FullSimplify[Evolution.Transpose[v1Probe]]
```

Out[•]=
$$\left\{\left\{\frac{-\text{Sin}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]+\text{Cos}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]\left(\text{Sec}\left[\sigma\right]-\text{Tan}\left[\sigma\right]\right)}{\sqrt{2}\,\sqrt{\frac{1}{1+\text{Csc}\left[\sigma\right]}}}\right\},\right.$$
$$\left.\left\{-\frac{\dot{\imath}\left(\text{Cos}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]+\text{Sin}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]\left(-\text{Sec}\left[\sigma\right]+\text{Tan}\left[\sigma\right]\right)\right)}{\sqrt{2}\,\sqrt{\frac{1}{1+\text{Csc}\left[\sigma\right]}}}\right\}\right\}$$

In[•]:=
```
ZetaEvolvedFirst = FullSimplify[ZetaF.Evolution.Transpose[v1Probe]]
```

Out[•]=
$$\left\{\left\{-\frac{\text{Cos}\left[\frac{\delta}{2}\right]\left(-1+\text{Cot}\left[\frac{\sigma}{2}\right]\right)}{\sqrt{\text{Cos}\left[\sigma\right]\,\text{Csc}\left[\frac{\sigma}{2}\right]^2}\,\sqrt{\frac{1}{1+\text{Csc}\left[\sigma\right]}}}\right\},\ \left\{-\frac{\dot{\imath}\,\text{Cos}\left[\frac{\delta}{2}\right]\left(-1+\text{Cot}\left[\frac{\sigma}{2}\right]\right)}{\sqrt{\text{Cos}\left[\sigma\right]\,\text{Csc}\left[\frac{\sigma}{2}\right]^2}\,\sqrt{\frac{1}{1+\text{Csc}\left[\sigma\right]}}}\right\}\right\}$$

In[•]:=
```
EvolvedFirstNormSq =
 FullSimplify[Abs[EvolvedFirst[[1]][[1]]]^2 + Abs[EvolvedFirst[[2]][[1]]]^2,
  {σ ∈ Reals, δ ∈ Reals, σ > 0}]
```

Out[•]=
$$\frac{1}{2}\,\text{Abs}\left[1+\text{Csc}\left[\sigma\right]\right]\left(\text{Abs}\left[\text{Sin}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]+\text{Cos}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]\left(-\text{Sec}\left[\sigma\right]+\text{Tan}\left[\sigma\right]\right)\right]^2+\right.$$
$$\left.\text{Abs}\left[\text{Cos}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]+\text{Sin}\left[\frac{1}{4}\left(\pi+2\,\delta\right)\right]\left(-\text{Sec}\left[\sigma\right]+\text{Tan}\left[\sigma\right]\right)\right]^2\right)$$

In[•]:=
```
EvolvedFirstNormSq =
 FullSimplify[1/2 * (1 + Csc[σ]) * ((Sin[1/4 (π + 2 δ)] + Cos[1/4 (π + 2 δ)] (-Sec[σ] + Tan[σ]))^2 +
   (Cos[1/4 (π + 2 δ)] + Sin[1/4 (π + 2 δ)] (-Sec[σ] + Tan[σ]))^2)]
```

Out[•]=
$$-\text{Cos}\left[\delta\right]\,\text{Cot}\left[\sigma\right]+\text{Csc}\left[\sigma\right]$$

In[•]:=
```
ZetaEvolvedFirstNormSq =
 FullSimplify[Abs[ZetaEvolvedFirst[[1]][[1]]]^2 + Abs[ZetaEvolvedFirst[[2]][[1]]]^2,
  {σ ∈ Reals, δ ∈ Reals, σ > 0}]
```

Out[•]=
$$\frac{2\,\text{Abs}\left[\text{Cos}\left[\frac{\delta}{2}\right]^2\left(-1+\text{Cot}\left[\frac{\sigma}{2}\right]\right)^2\left(1+\text{Csc}\left[\sigma\right]\right)\right]}{\text{Abs}\left[\text{Cos}\left[\sigma\right]\,\text{Csc}\left[\frac{\sigma}{2}\right]^2\right]}$$

In[•]:=
```
ZetaEvolvedFirstNormSq = 2 * Cos[δ/2]^2 (-1 + Cot[σ/2])^2 (1 + Csc[σ]) / (Cos[σ] Csc[σ/2]^2)
```

Out[•]=
$$2\,\text{Cos}\left[\frac{\delta}{2}\right]^2\left(-1+\text{Cot}\left[\frac{\sigma}{2}\right]\right)^2\left(1+\text{Csc}\left[\sigma\right]\right)\,\text{Sec}\left[\sigma\right]\,\text{Sin}\left[\frac{\sigma}{2}\right]^2$$

In[•]:=
```
ZetaEvolvedFirstNormSq = FullSimplify[ 2 * Cos[δ/2]^2 (-1 + Cot[σ/2])^2 (1 + Csc[σ]) / (Cos[σ] Csc[σ/2]^2) ]
```

Out[•]=
$$2 \cos\left[\frac{\delta}{2}\right]^2 \left(-1 + \csc[\sigma]\right) \left(1 + \csc[\sigma]\right) \tan[\sigma]$$

In[•]:=
```
DecisivenessFirst =
 FullSimplify[EvolvedFirstNormSq / (EvolvedFirstNormSq + ZetaEvolvedFirstNormSq),
  {0 < σ < π/2, 0 < δ < π/2}]
```

Out[•]=
$$2 \csc[\sigma] \left(-\cos[\delta] \cot[\sigma] + \csc[\sigma]\right) \sin\left[\frac{\sigma}{2}\right]^2$$

In[•]:=
```
FullSimplify[DecisivenessFirst - (1/2) * (1 - Cos[δ] * Cos[σ]) Sec[σ/2]^2]
```

Out[•]=
$$0$$

In[•]:=
```
DecisivenessFunction[δ_, σ_] := 2 Csc[σ] (-Cos[δ] Cot[σ] + Csc[σ]) Sin[σ/2]^2
```

In[•]:=
```
FullSimplify[DecisivenessFunction[σ, σ]]
```

Out[•]=
$$1 - \cos[\sigma]$$

In[•]:=
```
FullSimplify[DecisivenessFunction[-σ, σ]]
```

Out[•]=
$$1 - \cos[\sigma]$$

In[23]:=
```
N0 = Tan[σ/2]
```

Out[23]=
$$\tan\left[\frac{\sigma}{2}\right]$$

In[24]:=
```
FullSimplify[ZetaF, π/2 < σ < π]
```

Out[24]=
$$\left\{\left\{\frac{1}{\sqrt{2}\ \sqrt{-1 - \sec[\sigma]}}, \frac{i}{\sqrt{2}\ \sqrt{-1 - \sec[\sigma]}}\right\}, \left\{-\frac{i}{\sqrt{2}\ \sqrt{-1 - \sec[\sigma]}}, \frac{1}{\sqrt{2}\ \sqrt{-1 - \sec[\sigma]}}\right\}\right\}$$

```
Eigenvalues[
 {{1/(Sqrt[2] Sqrt[-1 - Sec[σ]]), i/(Sqrt[2] Sqrt[-1 - Sec[σ]])}, {-i/(Sqrt[2] Sqrt[-1 - Sec[σ]]), 1/(Sqrt[2] Sqrt[-1 - Sec[σ]])}}]
```

In[26]:=
```
EvolvedFirst = FullSimplify[Evolution.Transpose[v1Probe], π/2 < σ < π]
```

Out[26]=
$$\left\{\left\{-\frac{\sqrt{1 + \csc[\sigma]}\ \left(\sin\left[\frac{1}{4}\left(\pi + 2\delta\right)\right] + \cos\left[\frac{1}{4}\left(\pi + 2\delta\right)\right] \left(-\sec[\sigma] + \tan[\sigma]\right)\right)}{\sqrt{2}}\right\},\right.$$
$$\left.\left\{-\frac{i\sqrt{1 + \csc[\sigma]}\ \left(\cos\left[\frac{1}{4}\left(\pi + 2\delta\right)\right] + \sin\left[\frac{1}{4}\left(\pi + 2\delta\right)\right] \left(-\sec[\sigma] + \tan[\sigma]\right)\right)}{\sqrt{2}}\right\}\right\}$$

In[30]:=
```
ZetaEvolvedFirst = FullSimplify[ZetaF.Evolution.Transpose[v1Probe], π/2 < σ < π]
```

Out[30]=
$$\left\{\left\{-\frac{\sqrt{2}\,\text{Cos}\left[\frac{\sigma}{2}\right]\sqrt{-\frac{\text{Cos}[\sigma]+\text{Cot}[\sigma]}{1+\text{Cos}[\sigma]}}\,\text{Sin}\left[\frac{\delta}{2}\right]}{\text{Cos}\left[\frac{\sigma}{2}\right]+\text{Sin}\left[\frac{\sigma}{2}\right]}\right\},\right.$$
$$\left.\left\{i\sqrt{2}\,\text{Cos}\left[\frac{\sigma}{2}\right]\left(-1+\text{Cot}\left[\frac{\sigma}{2}\right]\right)\sqrt{-\frac{\text{Cos}[\sigma]+\text{Cot}[\sigma]}{1+\text{Cos}[\sigma]}}\,\text{Sec}[\sigma]\,\text{Sin}\left[\frac{\delta}{2}\right]\text{Sin}\left[\frac{\sigma}{2}\right]\right\}\right\}$$

In[28]:=
```
EvolvedFirstNormSq =
 FullSimplify[Abs[EvolvedFirst[[1]][[1]]]^2 + Abs[EvolvedFirst[[2]][[1]]]^2,
  {π/2 < σ < π, δ ∈ Reals}]
```

Out[28]=
$$-\text{Cos}[\delta]\,\text{Cot}[\sigma]+\text{Csc}[\sigma]$$

In[31]:=
```
ZetaEvolvedFirstNormSq =
 FullSimplify[Abs[ZetaEvolvedFirst[[1]][[1]]]^2 + Abs[ZetaEvolvedFirst[[2]][[1]]]^2,
  {π/2 < σ < π, δ ∈ Reals}]
```

Out[31]=
$$\left(-1+\text{Cos}[\delta]\right)\text{Cot}[\sigma]$$

In[32]:=
```
DecisivenessFirst = FullSimplify[
  EvolvedFirstNormSq / (EvolvedFirstNormSq + ZetaEvolvedFirstNormSq), {π/2 < σ < π}]
```

Out[32]=
$$\frac{\text{Cos}[\delta]\,\text{Cot}[\sigma]-\text{Csc}[\sigma]}{\text{Cot}[\sigma]-\text{Csc}[\sigma]}$$

In[33]:=
```
FullSimplify[DecisivenessFirst - (1/2) * (1 - Cos[δ] * Cos[σ]) Csc[σ/2]^2]
```

Out[33]=
$$0$$