In[1]:=
```
(*Our matrix which we need to decompose*)
USecondStage = {{-5.470718671585391`*^-8 + 0.36039809255631056` ⅈ,
    -0.6210442200926538` + 5.3655281852076075`*^-8 ⅈ, -1.4461179168642063`*^-8 +
     0.5785960063687721` ⅈ, 0.38683850628287325` + 1.4174596511731113`*^-8 ⅈ},
   {-0.6211049467275785` + 1.0861203342954484`*^-10 ⅈ, 1.0726060404447901`*^-10 -
     0.36066959816689237` ⅈ, 0.38660157983019594` + 3.1491373311126614`*^-11 ⅈ,
    2.996696857673897`*^-11 - 0.5785200435920416` ⅈ},
   {-5.182483487492506`*^-9 - 0.5785950959524974` ⅈ,
    -0.3868376052923497` + 5.520881436260963`*^-9 ⅈ, 1.967400970067333`*^-8 +
     0.3603946451957212` ⅈ, -0.6210476317450373` - 2.0920774945970902`*^-8 ⅈ},
   {-0.38660095690105795` + 7.271716690382055`*^-9 ⅈ, 7.3355330863431045`*^-9 +
     0.5785194635910924` ⅈ, -0.6211071014688335` - 2.7556989886604738`*^-8 ⅈ,
    -2.778761162418623`*^-8 - 0.3606675381017591` ⅈ}}
```

Out[1]= $\{\{-5.47072\times10^{-8}+0.360398\,i,\ -0.621044+5.36553\times10^{-8}\,i,\ -1.44612\times10^{-8}+0.578596\,i,\ 0.386839+1.41746\times10^{-8}\,i\},\ \{-0.621105+1.08612\times10^{-10}\,i,\ 1.07261\times10^{-10}-0.36067\,i,\ 0.386602+3.14914\times10^{-11}\,i,\ 2.9967\times10^{-11}-0.57852\,i\},\ \{-5.18248\times10^{-9}-0.578595\,i,\ -0.386838+5.52088\times10^{-9}\,i,\ 1.9674\times10^{-8}+0.360395\,i,\ -0.621048-2.09208\times10^{-8}\,i\},\ \{-0.386601+7.27172\times10^{-9}\,i,\ 7.33553\times10^{-9}+0.578519\,i,\ -0.621107-2.7557\times10^{-8}\,i,\ -2.77876\times10^{-8}-0.360668\,i\}\}$

In[2]:=
```
MatrixForm[Round[USecondStage, 0.0001]]
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 0.+0.3604\,i & -0.621 & 0.+0.5786\,i & 0.3868 \\ -0.6211 & 0.-0.3607\,i & 0.3866 & 0.-0.5785\,i \\ 0.-0.5786\,i & -0.3868 & 0.+0.3604\,i & -0.621 \\ -0.3866 & 0.+0.5785\,i & -0.6211 & 0.-0.3607\,i \end{pmatrix}$$

In[3]:=
```
(*It is unitary, vectors and eigenvalues are found with high accuracy*)
Transpose[Eigenvectors[USecondStage]].
  {{Eigenvalues[USecondStage][[1]], 0, 0, 0}, {0, Eigenvalues[USecondStage][[2]], 0, 0},
   {0, 0, Eigenvalues[USecondStage][[3]], 0}, {0, 0, 0, Eigenvalues[USecondStage][[4]]}}.
  Conjugate[Eigenvectors[USecondStage]] - USecondStage
ConjugateTranspose[USecondStage].USecondStage
Max[Abs[ConjugateTranspose[USecondStage].USecondStage -
   {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}]]
```

Out[3]= $\{\{1.73314\times10^{-8} - 0.000112242\,i, 0.0000937902 - 6.92326\times10^{-9}\,i, 1.00689\times10^{-9} + 0.0000677722\,i, 0.0000851958 + 6.54077\times10^{-9}\,i\}, \{0.0000308663 + 5.02328\times10^{-9}\,i, 1.60615\times10^{-8} + 0.000111703\,i, 0.000124981 - 5.34868\times10^{-9}\,i, -1.04596\times10^{-9} - 0.000067779\,i\}, \{-1.04393\times10^{-9} - 0.0000702397\,i, -0.0000820673 + 2.45771\times10^{-8}\,i, -1.60536\times10^{-8} - 0.000109414\,i, 0.0000977892 - 6.96407\times10^{-9}\,i\}, \{-0.000120695 - 2.57695\times10^{-8}\,i, 1.00459\times10^{-9} + 0.0000702329\,i, 0.0000348807 + 5.03443\times10^{-9}\,i, -1.73393\times10^{-8} + 0.000109953\,i\}\}$

Out[4]= $\{\{0.999891 + 0.\,i, 5.33884\times10^{-8} + 0.000358304\,i, 2.76886\times10^{-6} - 1.81751\times10^{-9}\,i, 7.30748\times10^{-9} + 5.10234\times10^{-6}\,i\}, \{5.33884\times10^{-8} - 0.000358304\,i, 1.00011 + 0.\,i, 1.39365\times10^{-8} + 0.0000100766\,i, 2.16118\times10^{-6} - 4.65857\times10^{-9}\,i\}, \{2.76886\times10^{-6} + 1.81751\times10^{-9}\,i, 1.39365\times10^{-8} - 0.0000100766\,i, 0.999892 + 0.\,i, 1.00404\times10^{-8} + 0.000355432\,i\}, \{7.30748\times10^{-9} - 5.10234\times10^{-6}\,i, 2.16118\times10^{-6} + 4.65857\times10^{-9}\,i, 1.00404\times10^{-8} - 0.000355432\,i, 1.00011 + 0.\,i\}\}$

Out[5]= 0.000358304

In[6]:=
```
(*Do SVD on the UR and UI*)
M = 1/Sqrt[2] {{1, 0, 0, I}, {0, I, 1, 0}, {0, I, -1, 0}, {1, 0, 0, -I}}
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
UP = ConjugateTranspose[M].USecondStage.M
UR = (UP + Conjugate[UP]) / 2
UI = (UP - Conjugate[UP]) / (2 * I)
{a, b, c} = SingularValueDecomposition[UR]
Max[Abs[a.b.ConjugateTranspose[c] - UR]]
{d, e, f} = SingularValueDecomposition[UI]
Max[Abs[d.e.ConjugateTranspose[f] - UI]]
MatrixForm[M]
MatrixForm[M.ConjugateTranspose[M]]
```

Out[6]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}, \left\{0, \frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}, \left\{0, \frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[7]= {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}

Out[8]= {{0.000118733 - 0.000134712 i, -0.578558 - 0.621076 i,
0.0000314516 - 0.0000382308 i, -0.360533 - 0.38672 i}, {-0.578558 + 0.621076 i,
-0.000118003 - 0.000137474 i, -0.360532 + 0.38672 i, -0.0000286601 - 0.0000375157 i},
{-0.0000286549 + 0.0000375367 i, 0.360532 + 0.38672 i, 0.000118023 - 0.000137479 i,
-0.578558 - 0.621076 i}, {0.360533 - 0.38672 i, 0.0000314298 + 0.000038312 i,
-0.578558 + 0.621076 i, -0.000118816 - 0.000134733 i}}

Out[9]= {{0.000118733 + 0. i, -0.578558 + 0. i, 0.0000314516 + 0. i, -0.360533 + 0. i},
{-0.578558 + 0. i, -0.000118003 + 0. i, -0.360532 + 0. i, -0.0000286601 + 0. i},
{-0.0000286549 + 0. i, 0.360532 + 0. i, 0.000118023 + 0. i, -0.578558 + 0. i},
{0.360533 + 0. i, 0.0000314298 + 0. i, -0.578558 + 0. i, -0.000118816 + 0. i}}

Out[10]= {{-0.000134712 + 0. i, -0.621076 + 0. i, -0.0000382308 + 0. i, -0.38672 + 0. i},
{0.621076 + 0. i, -0.000137474 + 0. i, 0.38672 + 0. i, -0.0000375157 + 0. i},
{0.0000375367 + 0. i, 0.38672 + 0. i, -0.000137479 + 0. i, -0.621076 + 0. i},
{-0.38672 + 0. i, 0.000038312 + 0. i, 0.621076 + 0. i, -0.000134733 + 0. i}}

Out[11]= {{{-0.358836 + 0. i, 0.609401 + 0. i, -0.67747 + 0. i, -0.202243 + 0. i},
{0.609269 + 0. i, 0.359007 + 0. i, 0.202455 + 0. i, -0.677434 + 0. i},
{-0.609176 + 0. i, -0.358901 + 0. i, 0.202125 + 0. i, -0.677672 + 0. i},
{-0.359072 + 0. i, 0.609044 + 0. i, 0.677637 + 0. i, 0.202338 + 0. i}},
{{0.681737, 0., 0., 0.}, {0., 0.681734, 0., 0.},
{0., 0., 0.681663, 0.}, {0., 0., 0., 0.68166}},
{{-0.706988 + 0. i, 0.0175391 + 0. i, 0.186444 + 0. i, 0.681982 + 0. i},
{-0.0177537 + 0. i, -0.707009 + 0. i, 0.681899 + 0. i, -0.186644 + 0. i},
{-0.0176025 + 0. i, -0.706762 + 0. i, -0.682216 + 0. i, 0.186437 + 0. i},
{0.706784 + 0. i, -0.0178171 + 0. i, 0.186636 + 0. i, 0.682134 + 0. i}}}

Out[12]= 6.66134×10^{-16}

Out[13]= {{{-0.185237 + 0. i, 0.682511 + 0. i, -0.624778 + 0. i, 0.330935 + 0. i},
{-0.682533 + 0. i, -0.185032 + 0. i, -0.33077 + 0. i, -0.624902 + 0. i},
{0.68232 + 0. i, 0.185339 + 0. i, -0.330877 + 0. i, -0.624987 + 0. i},
{-0.185134 + 0. i, 0.682342 + 0. i, 0.625111 + 0. i, -0.330713 + 0. i}},
{{0.731675, 0., 0., 0.}, {0., 0.731672, 0., 0.},
{0., 0., 0.731595, 0.}, {0., 0., 0., 0.731592}},
{{-0.481443 + 0. i, -0.517827 + 0. i, -0.611136 + 0. i, -0.355781 + 0. i},
{0.517989 + 0. i, -0.481316 + 0. i, 0.355589 + 0. i, -0.611211 + 0. i},
{-0.518015 + 0. i, 0.481335 + 0. i, 0.355928 + 0. i, -0.610977 + 0. i},
{-0.481208 + 0. i, -0.518177 + 0. i, 0.611051 + 0. i, 0.355736 + 0. i}}}

Out[14]= 7.77156×10^{-16}

Out[15]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}$$

Out[16]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[17]:=
```
(*Real and symmetric matrices, with high accuracy*)
MatrixForm[UI.ConjugateTranspose[UR]]
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0.498753 + 0.\,i & 0.000176094 + 0.\,i & -0.000178097 + 0.\,i & -2.12086 \times 10^{-8} + 0.\,i \\ 0.000178968 + 0.\,i & -0.498753 + 0.\,i & -1.3925 \times 10^{-8} + 0.\,i & 0.000178774 + 0.\,i \\ 0.000178769 + 0.\,i & 1.46479 \times 10^{-8} + 0.\,i & 0.498753 + 0.\,i & 0.000179021 + 0.\,i \\ 2.73955 \times 10^{-8} + 0.\,i & -0.000178099 + 0.\,i & 0.000176146 + 0.\,i & -0.498753 + 0.\,i \end{pmatrix}$$

In[18]:=
```
Max[Abs[UI.ConjugateTranspose[UR] - Transpose[UI.ConjugateTranspose[UR]]]]
```

Out[18]= 0.000356873

In[19]:=
```
MatrixForm[UR.ConjugateTranspose[UI]]
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 0.498753 + 0.\,i & 0.000178968 + 0.\,i & 0.000178769 + 0.\,i & 2.73955 \times 10^{-8} + 0.\,i \\ 0.000176094 + 0.\,i & -0.498753 + 0.\,i & 1.46479 \times 10^{-8} + 0.\,i & -0.000178099 + 0.\,i \\ -0.000178097 + 0.\,i & -1.3925 \times 10^{-8} + 0.\,i & 0.498753 + 0.\,i & 0.000176146 + 0.\,i \\ -2.12086 \times 10^{-8} + 0.\,i & 0.000178774 + 0.\,i & 0.000179021 + 0.\,i & -0.498753 + 0.\,i \end{pmatrix}$$

In[20]:=
```
Max[Abs[UR.ConjugateTranspose[UI] - Transpose[UR.ConjugateTranspose[UI]]]]
```

Out[20]= 0.000356873

In[21]:=
```
A = {{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}
```

Out[21]= {{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}

In[22]:=
```
Solve[c.A == f, {a11, a12, a13, a14, a21, a22, a23, a24, a31, a32, a33, a34, a41, a42, a43, a44}]
```

Out[22]= $\{\{a11 \to 0.000186193 + 0.\,i,\ a12 \to -0.0000691446 + 0.\,i,\ a13 \to 0.851369 + 0.\,i,\ a14 \to 0.524567 + 0.\,i,\ a21 \to 0.0000197061 + 0.\,i,\ a22 \to 0.000255946 + 0.\,i,\ a23 \to -0.524567 + 0.\,i,\ a24 \to 0.851369 + 0.\,i,\ a31 \to 0.527041 + 0.\,i,\ a32 \to -0.84984 + 0.\,i,\ a33 \to -0.000242226 + 0.\,i,\ a34 \to 0.0000940407 + 0.\,i,\ a41 \to -0.84984 + 0.\,i,\ a42 \to -0.527041 + 0.\,i,\ a43 \to 0.0000241438 + 0.\,i,\ a44 \to 0.000192991 + 0.\,i\}\}$

In[ ]:=

```
In[23]:= a11 = 0.00018619270405096746` + 0.` ⅈ
         a12 = -0.00006914459367213958` + 0.` ⅈ
         a13 = 0.8513689400109632` + 0.` ⅈ
         a14 = 0.5245673346062549` + 0.` ⅈ
         a21 = 0.000019706086442842956` + 0.` ⅈ
         a22 = 0.0002559460369932462` + 0.` ⅈ
         a23 = -0.5245673157262387` + 0.` ⅈ
         a24 = 0.8513689361111698` + 0.` ⅈ
         a31 = 0.527041282981804` + 0.` ⅈ
         a32 = -0.8498396428243356` + 0.` ⅈ
         a33 = -0.0002422260352057725` + 0.` ⅈ
         a34 = 0.0000940406715542057 3` + 0.` ⅈ
         a41 = -0.8498396619226715` + 0.` ⅈ
         a42 = -0.5270412803517932` + 0.` ⅈ
         a43 = 0.000024143843562750908` + 0.` ⅈ
         a44 = 0.00019299061208149975` + 0.` ⅈ

Out[23]= 0.000186193 + 0. ⅈ

Out[24]= -0.0000691446 + 0. ⅈ

Out[25]= 0.851369 + 0. ⅈ

Out[26]= 0.524567 + 0. ⅈ

Out[27]= 0.0000197061 + 0. ⅈ

Out[28]= 0.000255946 + 0. ⅈ

Out[29]= -0.524567 + 0. ⅈ

Out[30]= 0.851369 + 0. ⅈ

Out[31]= 0.527041 + 0. ⅈ

Out[32]= -0.84984 + 0. ⅈ

Out[33]= -0.000242226 + 0. ⅈ

Out[34]= 0.0000940407 + 0. ⅈ

Out[35]= -0.84984 + 0. ⅈ

Out[36]= -0.527041 + 0. ⅈ

Out[37]= 0.0000241438 + 0. ⅈ

Out[38]= 0.000192991 + 0. ⅈ

In[39]:= d.e.ConjugateTranspose[A].ConjugateTranspose[c]

Out[39]= {{-0.000134712 + 0. ⅈ, -0.621076 + 0. ⅈ, -0.0000382308 + 0. ⅈ, -0.38672 + 0. ⅈ},
          {0.621076 + 0. ⅈ, -0.000137474 + 0. ⅈ, 0.38672 + 0. ⅈ, -0.0000375157 + 0. ⅈ},
          {0.0000375367 + 0. ⅈ, 0.38672 + 0. ⅈ, -0.000137479 + 0. ⅈ, -0.621076 + 0. ⅈ},
          {-0.38672 + 0. ⅈ, 0.000038312 + 0. ⅈ, 0.621076 + 0. ⅈ, -0.000134733 + 0. ⅈ}}
```

In[40]:= **UI**

Out[40]= {{-0.000134712 + 0. i, -0.621076 + 0. i, -0.0000382308 + 0. i, -0.38672 + 0. i},
{0.621076 + 0. i, -0.000137474 + 0. i, 0.38672 + 0. i, -0.0000375157 + 0. i},
{0.0000375367 + 0. i, 0.38672 + 0. i, -0.000137479 + 0. i, -0.621076 + 0. i},
{-0.38672 + 0. i, 0.000038312 + 0. i, 0.621076 + 0. i, -0.000134733 + 0. i}}

In[41]:= **Max[Abs[d.e.ConjugateTranspose[A].ConjugateTranspose[c] - UI]]**

Out[41]= 6.66134×10^{-16}

In[42]:= **F = {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}**

Out[42]= {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}

In[43]:= **MatrixForm[a]**

Out[43]//MatrixForm=

$$\begin{pmatrix} -0.358836+0.\,i & 0.609401+0.\,i & -0.67747+0.\,i & -0.202243+0.\,i \\ 0.609269+0.\,i & 0.359007+0.\,i & 0.202455+0.\,i & -0.677434+0.\,i \\ -0.609176+0.\,i & -0.358901+0.\,i & 0.202125+0.\,i & -0.677672+0.\,i \\ -0.359072+0.\,i & 0.609044+0.\,i & 0.677637+0.\,i & 0.202338+0.\,i \end{pmatrix}$$

In[44]:= **MatrixForm[F.d.ConjugateTranspose[A]]**

Out[44]//MatrixForm=

$$\begin{pmatrix} -0.358401+0.\,i & 0.609657+0.\,i & -0.67747+0.\,i & -0.202241+0.\,i \\ 0.609525+0.\,i & 0.358572+0.\,i & 0.202454+0.\,i & -0.677435+0.\,i \\ -0.609432+0.\,i & -0.358466+0.\,i & 0.202124+0.\,i & -0.677673+0.\,i \\ -0.358637+0.\,i & 0.6093+0.\,i & 0.677637+0.\,i & 0.202337+0.\,i \end{pmatrix}$$

In[45]:= **Max[Abs[F.d.ConjugateTranspose[A] - a]]**

Out[45]= 0.000434969

In[46]:= **a.Inverse[d.ConjugateTranspose[A]]**

Out[46]= $\{\{1. + 0.\,i,\ 0.000355949 + 0.\,i,\ 0.000357759 + 0.\,i,\ 1.33566 \times 10^{-7} + 0.\,i\},$
$\{0.000355949 + 0.\,i,\ -1. + 0.\,i,\ 1.28088 \times 10^{-7} + 0.\,i,\ -0.000357765 + 0.\,i\},$
$\{-0.000357759 + 0.\,i,\ -1.2664 \times 10^{-7} + 0.\,i,\ 1. + 0.\,i,\ 0.000356055 + 0.\,i\},$
$\{-1.21162 \times 10^{-7} + 0.\,i,\ 0.000357765 + 0.\,i,\ 0.000356055 + 0.\,i,\ -1. + 0.\,i\}\}$

In[47]:= **a.b.ConjugateTranspose[c] + I * d.ConjugateTranspose[A].e.ConjugateTranspose[c] - UP**

Out[47]= $\{\{5.13478 \times 10^{-16} + 0.0000410171\,i,\ 3.33067 \times 10^{-16} - 4.09211 \times 10^{-7}\,i,$
$4.85723 \times 10^{-16} - 0.0000657849\,i,\ -4.44089 \times 10^{-16} - 2.44451 \times 10^{-7}\,i\},$
$\{0. - 3.74619 \times 10^{-7}\,i,\ 2.77556 \times 10^{-17} + 0.0000430601\,i,$
$-1.66533 \times 10^{-16} - 2.99337 \times 10^{-7}\,i,\ 2.77556 \times 10^{-16} - 0.0000692439\,i\},$
$\{1.66533 \times 10^{-16} + 0.0000692685\,i,\ -1.66533 \times 10^{-16} - 2.69833 \times 10^{-7}\,i,$
$-2.77556 \times 10^{-17} + 0.0000430206\,i,\ 6.66134 \times 10^{-16} + 3.94214 \times 10^{-7}\,i\},$
$\{-3.88578 \times 10^{-16} - 2.74679 \times 10^{-7}\,i,\ -4.19803 \times 10^{-16} + 0.0000657604\,i,$
$-6.66134 \times 10^{-16} + 3.90779 \times 10^{-7}\,i,\ 2.63678 \times 10^{-16} + 0.0000410566\,i\}\}$

In[48]:= **Max[**
Abs[a.b.ConjugateTranspose[c] + I * d.ConjugateTranspose[A].e.ConjugateTranspose[c] - UP]]

Out[48]= 0.0000692685

In[49]:=
```
{b, e}
```

Out[49]=
```
{{{0.681737, 0., 0., 0.}, {0., 0.681734, 0., 0.},
  {0., 0., 0.681663, 0.}, {0., 0., 0., 0.68166}}, {{0.731675, 0., 0., 0.},
  {0., 0.731672, 0., 0.}, {0., 0., 0.731595, 0.}, {0., 0., 0., 0.731592}}}
```

In[50]:=
```
Max[Abs[b.a.ConjugateTranspose[c] + I * F.a.e.ConjugateTranspose[c] - UP]]
```

Out[50]= 0.000264846

In[51]:=
```
Max[Abs[(b + I * F.e).a.ConjugateTranspose[c] - UP]]
```

Out[51]= 0.00024472

In[52]:=
```
DiagonalPart = b + I * F.e
```

Out[52]=
```
{{0.681737 + 0.731675 i, 0. + 0. i, 0. + 0. i, 0. + 0. i},
 {0. + 0. i, 0.681734 - 0.731672 i, 0. + 0. i, 0. + 0. i},
 {0. + 0. i, 0. + 0. i, 0.681663 + 0.731595 i, 0. + 0. i},
 {0. + 0. i, 0. + 0. i, 0. + 0. i, 0.68166 - 0.731592 i}}
```

In[53]:=
```
ℭ = a.ConjugateTranspose[c]
```

Out[53]=
```
{{0.000144835 + 0. i, -0.8487 + 0. i, 0.0000903089 + 0. i, -0.528874 + 0. i},
 {-0.8487 + 0. i, -0.000144854 + 0. i, -0.528874 + 0. i, -0.0000902795 + 0. i},
 {-0.0000902891 + 0. i, 0.528874 + 0. i, 0.00014491 + 0. i, -0.8487 + 0. i},
 {0.528874 + 0. i, 0.0000902596 + 0. i, -0.8487 + 0. i, -0.000144929 + 0. i}}
```

In[54]:=
```
Max[Abs[DiagonalPart.ℭ - UP]]
```

Out[54]= 0.00024472

In[55]:=
```
Max[Abs[USecondStage - M.UP.ConjugateTranspose[M]]]
```

Out[55]= 2.48157×10^{-16}

In[56]:=
```
Max[Abs[USecondStage - M.DiagonalPart.ConjugateTranspose[M].M.ℭ.ConjugateTranspose[M]]]
```

Out[56]= 0.000244652

In[57]:=
```
(*Now we decompose this happiness*)
Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]
```

Out[57]=
```
{{UA11 UB11, UA11 UB12, UA12 UB11, UA12 UB12}, {UA11 UB21, UA11 UB22, UA12 UB21, UA12 UB22},
 {UA21 UB11, UA21 UB12, UA22 UB11, UA22 UB12}, {UA21 UB21, UA21 UB22, UA22 UB21, UA22 UB22}}
```

In[ ]:=

```
In[58]:= WeHave = M.ξ.ConjugateTranspose[M]
Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]
UAMatr = {{UA11, UA12}, {UA21, UA22}}
UBMatr = {{UB11, UB12}, {UB21, UB22}}
Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[2]], Search[[1]][[1]] / Search[[1]][[2]]}]
Reverse[{WeHave[[1]][[3]] / WeHave[[1]][[4]], Search[[1]][[3]] / Search[[1]][[4]]}]
Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[3]], Search[[1]][[1]] / Search[[1]][[3]]}]
Reverse[{WeHave[[1]][[2]] / WeHave[[1]][[4]], Search[[1]][[2]] / Search[[1]][[4]]}]
Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[2]], Search[[2]][[1]] / Search[[2]][[2]]}]
Reverse[{WeHave[[2]][[3]] / WeHave[[2]][[4]], Search[[2]][[3]] / Search[[2]][[4]]}]
Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[3]], Search[[2]][[1]] / Search[[2]][[3]]}]
Reverse[{WeHave[[2]][[2]] / WeHave[[2]][[4]], Search[[2]][[2]] / Search[[2]][[4]]}]
Reverse[{WeHave[[3]][[1]] / WeHave[[3]][[2]], Search[[3]][[1]] / Search[[3]][[2]]}]
Reverse[{WeHave[[3]][[3]] / WeHave[[3]][[4]], Search[[3]][[3]] / Search[[3]][[4]]}]
Reverse[{WeHave[[4]][[1]] / WeHave[[4]][[2]], Search[[4]][[1]] / Search[[4]][[2]]}]
Reverse[{WeHave[[4]][[3]] / WeHave[[4]][[4]], Search[[4]][[3]] / Search[[4]][[4]]}]
Reverse[{WeHave[[3]][[1]] / WeHave[[1]][[4]], Search[[3]][[1]] / Search[[1]][[4]]}]
Reverse[{WeHave[[4]][[1]] / WeHave[[1]][[4]], Search[[4]][[1]] / Search[[1]][[4]]}]
Reverse[{WeHave[[4]][[2]] / WeHave[[3]][[3]], Search[[4]][[2]] / Search[[3]][[3]]}]
Reverse[{WeHave[[1]][[2]] / WeHave[[3]][[1]], Search[[1]][[2]] / Search[[3]][[1]]}]
```

Out[58]= $\{\{-4.66737\times10^{-8}+0.528874\,i,\ 0.0000902843+7.81271\times10^{-10}\,i,\ -2.46099\times10^{-8}+0.8487\,i,\ 0.000144882+1.24515\times10^{-9}\,i\},\ \{-0.0000902843+7.68464\times10^{-10}\,i,\ 2.83478\times10^{-8}-0.528874\,i,\ -0.000144882+1.24176\times10^{-9}\,i,\ -4.79813\times10^{-9}-0.8487\,i\},\ \{4.79813\times10^{-9}-0.8487\,i,\ -0.000144882-1.24176\times10^{-9}\,i,\ 2.83478\times10^{-8}+0.528874\,i,\ 0.0000902843+7.68464\times10^{-10}\,i\},\ \{0.000144882-1.24515\times10^{-9}\,i,\ 2.46099\times10^{-8}+0.8487\,i,\ -0.0000902843+7.81271\times10^{-10}\,i,\ -4.66737\times10^{-8}-0.528874\,i\}\}$

Out[59]= {{UA11 UB11, UA11 UB12, UA12 UB11, UA12 UB12}, {UA11 UB21, UA11 UB22, UA12 UB21, UA12 UB22}, {UA21 UB11, UA21 UB12, UA22 UB11, UA22 UB12}, {UA21 UB21, UA21 UB22, UA22 UB21, UA22 UB22}}

Out[60]= {{UA11, UA12}, {UA21, UA22}}

Out[61]= {{UB11, UB12}, {UB21, UB22}}

Out[62]= $\left\{\frac{UB11}{UB12},\ 0.0501739+5857.88\,i\right\}$

Out[63]= $\left\{\frac{UB11}{UB12},\ 0.0501739+5857.88\,i\right\}$

Out[64]= $\left\{\frac{UA11}{UA12},\ 0.623157+3.69245\times10^{-8}\,i\right\}$

Out[65]= $\left\{\frac{UA11}{UA12},\ 0.623157+3.69237\times10^{-8}\,i\right\}$

Out[66]= $\left\{\frac{UB21}{UB22},\ -1.46217\times10^{-9}-0.00017071\,i\right\}$

Out[67]= $\left\{\frac{UB21}{UB22},\ -1.46217\times10^{-9}-0.00017071\,i\right\}$

Out[68]= $\left\{\frac{\text{UA11}}{\text{UA12}}, 0.623157 + 3.69256 \times 10^{-8}\, i\right\}$

Out[69]= $\left\{\frac{\text{UA11}}{\text{UA12}}, 0.623157 + 3.69245 \times 10^{-8}\, i\right\}$

Out[70]= $\left\{\frac{\text{UB11}}{\text{UB12}}, 0.0501739 + 5857.88\, i\right\}$

Out[71]= $\left\{\frac{\text{UB11}}{\text{UB12}}, 0.0501739 + 5857.88\, i\right\}$

Out[72]= $\left\{\frac{\text{UB21}}{\text{UB22}}, -1.46217 \times 10^{-9} - 0.00017071\, i\right\}$

Out[73]= $\left\{\frac{\text{UB21}}{\text{UB22}}, -1.46217 \times 10^{-9} - 0.00017071\, i\right\}$

Out[74]= $\left\{\frac{\text{UA21 UB11}}{\text{UA12 UB12}}, -0.0503107 - 5857.88\, i\right\}$

Out[75]= $\left\{\frac{\text{UA21 UB21}}{\text{UA12 UB12}}, 1. - 0.0000171884\, i\right\}$

Out[76]= $\left\{\frac{\text{UA21 UB22}}{\text{UA22 UB11}}, 1.60473 + 3.94816 \times 10^{-8}\, i\right\}$

Out[77]= $\left\{\frac{\text{UA11 UB12}}{\text{UA21 UB11}}, -9.19948 \times 10^{-10} + 0.000106379\, i\right\}$

In[78]:=
```
UB11 = (0.05017391501168136` + 5857.875152268168` i) * UB12
UA11 =
 UA12 * (0.6231573630726742` + 3.692448279261223`*^-8 i) / (1.` - 1.6940658945086007`*^-21 i)
UB21 = UB22 * (-1.462169698759667`*^-9 - 0.0001707103640708271` i) /
   (1.` - 6.617444900424222`*^-24 i)
UA21 = UA12 * (-0.050310666712533654` - 5857.875152264804` i) /
   (0.05017391501168136` + 5857.875152268168` i)
UB22 = UB12 * (0.9999999998522792` - 0.000017188411471334102` i) /
   (1.4661549207101146`*^-9 + 0.00017071036407072907` i)
UA12 = UA22 * (1.6047310988498664` + 3.9481565561747896`*^-8 i) /
   (1.0000000000014846` - 5.799662605071732`*^-8 i)
```

Out[78]= $(0.0501739 + 5857.88\, i)\ \text{UB12}$

Out[79]= $(0.623157 + 3.69245 \times 10^{-8}\, i)\ \text{UA12}$

Out[80]= $(-1.46217 \times 10^{-9} - 0.00017071\, i)\ \text{UB22}$

Out[81]= $(-1. + 2.33449 \times 10^{-8}\, i)\ \text{UA12}$

Out[82]= $(-0.0503769 - 5857.88\, i)\ \text{UB12}$

Out[83]= $(1.60473 + 1.32551 \times 10^{-7}\, i)\ \text{UA22}$

```
In[84]:= ConjugateTranspose[{{UA11, UA12}, {UA21, UA22}}].{{UA11, UA12}, {UA21, UA22}}
Out[84]= {{(3.57516 + 0. i) UA22 Conjugate[UA22],
    (-1.78479 × 10^-12 + 1.69435 × 10^-12 i) UA22 Conjugate[UA22]},
   {(-1.78479 × 10^-12 - 1.69435 × 10^-12 i) UA22 Conjugate[UA22],
    (3.57516 + 0. i) UA22 Conjugate[UA22]}}

In[85]:= UA22 = (1 / ((3.5751618996033474` + 0.` i) ^ (1 / 2))) * Exp[I * ψ]
Out[85]= (0.528874 + 0. i) e^(i ψ)

In[86]:= ConjugateTranspose[{{UB11, UB12}, {UB21, UB22}}].{{UB11, UB12}, {UB21, UB22}}
Out[86]= {{(3.43147 × 10^7 + 0. i) UB12 Conjugate[UB12],
    (-1.45439 × 10^-9 + 1.30885 × 10^-8 i) UB12 Conjugate[UB12]},
   {(-1.45439 × 10^-9 - 1.30885 × 10^-8 i) UB12 Conjugate[UB12],
    (3.43147 × 10^7 + 0. i) UB12 Conjugate[UB12]}}

In[87]:= UB12 = (1 / ((3.431470230207823`*^7 + 0.` i) ^ (1 / 2))) * Exp[I * ϕ]
Out[87]= (0.00017071 + 0. i) e^(i ϕ)

In[88]:= ϕ = 0
Out[88]= 0

In[89]:= Search
Out[89]= {{(4.45489 × 10^-6 + 0.528874 i) e^(i ψ), (0.0000902843 + 1.28072 × 10^-11 i) e^(i ψ),
    (7.19919 × 10^-6 + 0.8487 i) e^(i ψ), (0.000144882 + 1.19672 × 10^-11 i) e^(i ψ)},
   {(-0.0000902843 + 1.53693 × 10^-9 i) e^(i ψ), (-4.47322 × 10^-6 - 0.528874 i) e^(i ψ),
    (-0.000144882 + 2.47494 × 10^-9 i) e^(i ψ), (-7.2286 × 10^-6 - 0.8487 i) e^(i ψ)},
   {(-7.21901 × 10^-6 - 0.8487 i) e^(i ψ), (-0.000144882 - 8.58497 × 10^-12 i) e^(i ψ),
    (4.52991 × 10^-6 + 0.528874 i) e^(i ψ), (0.0000902843 + 0. i) e^(i ψ)},
   {(0.000144882 - 2.47832 × 10^-9 i) e^(i ψ), (7.24841 × 10^-6 + 0.8487 i) e^(i ψ),
    (-0.0000902843 + 1.54974 × 10^-9 i) e^(i ψ), (-4.54824 × 10^-6 - 0.528874 i) e^(i ψ)}}

In[90]:= ψ = Log[WeHave[[1]][[1]] / (4.45489187780763`*^-6 + 0.5288739023065214` i)] / I
Out[90]= 8.5116 × 10^-6 + 2.69118 × 10^-13 i

In[91]:= Max[Abs[Search - WeHave]]
Out[91]= 2.52897 × 10^-12

In[92]:= UAMatr
Out[92]= {{0.528874 + 4.57659 × 10^-6 i, 0.8487 + 7.2939 × 10^-6 i},
   {-0.8487 - 7.27409 × 10^-6 i, 0.528874 + 4.50157 × 10^-6 i}}

In[93]:= UBMatr
Out[93]= {{8.56521 × 10^-6 + 1. i, 0.00017071 + 0. i},
   {-0.00017071 + 2.93025 × 10^-9 i, -8.59986 × 10^-6 - 1. i}}
```

In[94]:=
```
MatrixForm[UAMatr]
```

Out[94]//MatrixForm=

$$\begin{pmatrix} 0.528874 + 4.57659 \times 10^{-6}\, i & 0.8487 + 7.2939 \times 10^{-6}\, i \\ -0.8487 - 7.27409 \times 10^{-6}\, i & 0.528874 + 4.50157 \times 10^{-6}\, i \end{pmatrix}$$

In[95]:=
```
MatrixForm[UBMatr]
```

Out[95]//MatrixForm=

$$\begin{pmatrix} 8.56521 \times 10^{-6} + 1.\, i & 0.00017071 + 0.\, i \\ -0.00017071 + 2.93025 \times 10^{-9}\, i & -8.59986 \times 10^{-6} - 1.\, i \end{pmatrix}$$

In[96]:=
```
Max[Abs[Search - KroneckerProduct[UAMatr, UBMatr]]]
```

Out[96]= 0.

In[97]:=
```
Max[Abs[Search - M.ℰ.ConjugateTranspose[M]]]
```

Out[97]= 2.52897×10^{-12}

In[98]:=
```
Max[Abs[USecondStage - M.DiagonalPart.ConjugateTranspose[M].M.ℰ.ConjugateTranspose[M]]]
```

Out[98]= 0.000244652

In[99]:=
```
Max[Abs[
  USecondStage - M.DiagonalPart.ConjugateTranspose[M].KroneckerProduct[UAMatr, UBMatr]]]
```

Out[99]= 0.000244652

In[100]:=
```
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
ϕ1 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] + KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ2 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] - KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ3 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] - KroneckerProduct[{{0, 1}}, {{1, 0}}])
ϕ4 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] + KroneckerProduct[{{0, 1}}, {{1, 0}}])
DMatrix =
 {{Exp[I * Φ0], 0, 0, 0}, {0, Exp[I * Φ1], 0, 0}, {0, 0, Exp[I * Φ2], 0}, {0, 0, 0, Exp[I * Φ3]}}
FullSimplify[M.DMatrix.ConjugateTranspose[M] -
  Exp[I * θ0] * MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] +
       θ2 * KroneckerProduct[σy, σy] + θ3 * KroneckerProduct[σz, σz])]]
```

Out[100]= {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}

Out[101]= $\frac{\Phi 0}{4}+\frac{\Phi 1}{4}+\frac{\Phi 2}{4}+\frac{\Phi 3}{4}$

Out[102]= $\frac{\Phi 0}{4}+\frac{\Phi 1}{4}-\frac{\Phi 2}{4}-\frac{\Phi 3}{4}$

Out[103]= $-\frac{\Phi 0}{4}+\frac{\Phi 1}{4}-\frac{\Phi 2}{4}+\frac{\Phi 3}{4}$

Out[104]= $\frac{\Phi 0}{4}-\frac{\Phi 1}{4}-\frac{\Phi 2}{4}+\frac{\Phi 3}{4}$

Out[105]= $\{\{0, 1\}, \{1, 0\}\}$

Out[106]= $\{\{0, -i\}, \{i, 0\}\}$

Out[107]= $\{\{1, 0\}, \{0, -1\}\}$

Out[108]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[109]= $\left\{\left\{-\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[110]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[111]= $\left\{\left\{0, -\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right\}\right\}$

Out[112]= $\left\{\left\{e^{i \Phi 0}, 0, 0, 0\right\}, \left\{0, e^{i \Phi 1}, 0, 0\right\}, \left\{0, 0, e^{i \Phi 2}, 0\right\}, \left\{0, 0, 0, e^{i \Phi 3}\right\}\right\}$

Out[113]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

```
In[114]:= σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
MatrixForm[CNOT1]
MatrixForm[CNOT2]
Ry[θ_] := {{Cos[θ/2], Sin[θ/2]}, {-Sin[θ/2], Cos[θ/2]}}
Rz[α_] := {{ⅇ^(ⅈα/2), 0}, {0, ⅇ^(-ⅈα/2)}}
Unit2 = {{1, 0}, {0, 1}}
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
ϕ1 = 1/√2 * (KroneckerProduct[{{1, 0}}, {{1, 0}}] + KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ2 = -I/√2 * (KroneckerProduct[{{1, 0}}, {{1, 0}}] - KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ3 = 1/√2 * (KroneckerProduct[{{1, 0}}, {{0, 1}}] - KroneckerProduct[{{0, 1}}, {{1, 0}}])
ϕ4 = -I/√2 * (KroneckerProduct[{{1, 0}}, {{0, 1}}] + KroneckerProduct[{{0, 1}}, {{1, 0}}])
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
H = 1/√2 * {{1, 1}, {1, -1}}
CNOT2Trial = KroneckerProduct[H, H].CNOT1.KroneckerProduct[H, H]
CNOT2Trial - CNOT2
U3[θ_, ϕ_, λ_] :=
  {{Cos[θ/2], -Exp[I * λ] * Sin[θ/2]}, {Exp[I * ϕ] * Sin[θ/2], Exp[I * (ϕ + λ)] * Cos[θ/2]}}
FullSimplify[Exp[I * π / 4] * KroneckerProduct[Rz[-π/2], Unit2].CNOT2.
    KroneckerProduct[Unit2, Ry[2 * θ2 - π/2]].CNOT1.KroneckerProduct[
      Rz[2 * θ3 - π/2], Ry[π/2 - 2 * θ1]].CNOT2.KroneckerProduct[Unit2, Rz[π/2]] -
  MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] + θ2 * KroneckerProduct[σy, σy] +
        θ3 * KroneckerProduct[σz, σz])]]

Out[114]= {{0, 1}, {1, 0}}

Out[115]= {{0, -ⅈ}, {ⅈ, 0}}

Out[116]= {{1, 0}, {0, -1}}

Out[117]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}

Out[118]= {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
```

Out[119]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Out[120]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Out[123]= $\{\{1, 0\}, \{0, 1\}\}$

Out[124]= $\{\{0, 1\}, \{1, 0\}\}$

Out[125]= $\{\{0, -i\}, \{i, 0\}\}$

Out[126]= $\{\{1, 0\}, \{0, -1\}\}$

Out[127]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[128]= $\left\{\left\{-\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[129]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[130]= $\left\{\left\{0, -\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right\}\right\}$

Out[131]= $\{\{1, 1, -1, 1\}, \{1, 1, 1, -1\}, \{1, -1, -1, -1\}, \{1, -1, 1, 1\}\}$

Out[132]= $\frac{\Phi0}{4} + \frac{\Phi1}{4} + \frac{\Phi2}{4} + \frac{\Phi3}{4}$

Out[133]= $\frac{\Phi0}{4} + \frac{\Phi1}{4} - \frac{\Phi2}{4} - \frac{\Phi3}{4}$

Out[134]= $-\frac{\Phi0}{4} + \frac{\Phi1}{4} - \frac{\Phi2}{4} + \frac{\Phi3}{4}$

Out[135]= $\frac{\Phi0}{4} - \frac{\Phi1}{4} - \frac{\Phi2}{4} + \frac{\Phi3}{4}$

Out[136]= $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}$

Out[137]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[138]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[139]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[140]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[142]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

In[143]:=
```
Φ0 = Log[DiagonalPart[[1]][[1]]] / I
Φ1 = Log[DiagonalPart[[2]][[2]]] / I
Φ2 = Log[DiagonalPart[[3]][[3]]] / I
Φ3 = Log[DiagonalPart[[4]][[4]]] / I
```

Out[143]= 0.820715 - 0.0000565507 i

Out[144]= -0.820715 - 0.000052348 i

Out[145]= 0.820715 + 0.0000522743 i

Out[146]= -0.820715 + 0.0000564031 i

In[147]:=
```
Max[Abs[USecondStage - Exp[I * θ0] * Exp[I * π / 4] *
    KroneckerProduct[Rz[-π / 2], Unit2].CNOT2.KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
     CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].CNOT2.
     KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[UAMatr, UBMatr]]]
```

Out[147]= 0.000244652

In[148]:=
```
Max[Abs[USecondStage -
   Exp[I * θ0] * Exp[I * π / 4] * KroneckerProduct[Rz[-π / 2], Unit2].KroneckerProduct[H, H].
     CNOT1.KroneckerProduct[H, H].KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
     CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].
     KroneckerProduct[H, H].CNOT1.KroneckerProduct[H, H].
     KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[UAMatr, UBMatr]]]
```

Out[148]= 0.000244652

In[149]:=
```
Max[Abs[USecondStage - Exp[I * (θ0 + π / 4)] * KroneckerProduct[Rz[-π / 2].H, Unit2.H].
     CNOT1.KroneckerProduct[H.Unit2, H.Ry[2 * θ2 - π / 2]].CNOT1.
     KroneckerProduct[Rz[2 * θ3 - π / 2].H, Ry[π / 2 - 2 * θ1].H].CNOT1.
     KroneckerProduct[H.Unit2.UAMatr, H.Rz[π / 2].UBMatr]]]
```

Out[149]= 0.000244652

In[150]:=
```
U11 = Rz[-π/2].H
U12 = Unit2.H
U21 = H.Unit2
U22 = H.Ry[2 * θ2 - π/2]
U31 = Rz[2 * θ3 - π/2].H
U32 = Ry[π/2 - 2 * θ1].H
U41 = H.Unit2.UAMatr
U42 = H.Rz[π/2].UBMatr
```

Out[150]= $\left\{\left\{\frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}, \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}\right\}, \left\{\frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}, -\frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}\right\}\right\}$

Out[151]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[152]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[153]= $\{\{0.681698 + 1.5239 \times 10^{-6}\, i, -0.731633 + 1.41989 \times 10^{-6}\, i\}, \{-0.731633 + 1.41989 \times 10^{-6}\, i, -0.681698 - 1.5239 \times 10^{-6}\, i\}\}$

Out[154]= $\{\{0.5 - 0.5\, i, 0.5 - 0.5\, i\}, \{0.5 + 0.5\, i, -0.5 - 0.5\, i\}\}$

Out[155]= $\{\{1. + 8.2124 \times 10^{-14}\, i, 1.5098 \times 10^{-9} - 0.000054394\, i\}, \{1.5098 \times 10^{-9} - 0.000054394\, i, -1. - 8.2124 \times 10^{-14}\, i\}\}$

Out[156]= $\{\{-0.226151 - 1.90742 \times 10^{-6}\, i, 0.974092 + 8.34066 \times 10^{-6}\, i\}, \{0.974092 + 8.3797 \times 10^{-6}\, i, 0.226151 + 1.97448 \times 10^{-6}\, i\}\}$

Out[157]= $\{\{-0.500081 + 0.50009\, i, -0.499919 - 0.49991\, i\}, \{-0.49991 + 0.499919\, i, 0.50009 + 0.500081\, i\}\}$

In[158]:=
```
Max[Abs[USecondStage -
   Exp[I * (θ0 + π / 4)] * KroneckerProduct[U11, U12].CNOT1.KroneckerProduct[U21, U22].
     CNOT1.KroneckerProduct[U31, U32].CNOT1.KroneckerProduct[U41, U42]]]
```

Out[158]= 0.000244652

In[159]:=
```
N0 = 15.338
r = 1
s = 1
θ = π/2 - 0.5
```
$\omega = \sqrt{s^2 - r^2\,\mathrm{Sin}[\theta]^2}$

α = ArcSin[(r / s) * Sin[θ]]

$\tau\mathrm{New} = \dfrac{(\pi/2)}{\omega}$

Out[159]= 15.338

Out[160]= 1

Out[161]= 1

Out[162]= 1.0708

Out[163]= 0.479426

Out[164]= 1.0708

Out[165]= 3.27641

In[166]:=

$\mathrm{vFirst} = \left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}\right\}$

$\mathrm{vSecond} = \left\{\left\{\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}\right\}\right\}$

$\mathrm{vThird} = \left\{\left\{\mathrm{Cos}\left[\frac{\rho}{2}\right], i * \mathrm{Sin}\left[\frac{\rho}{2}\right]\right\}\right\}$

Out[166]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[167]= $\left\{\left\{\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[168]= $\left\{\left\{\mathrm{Cos}\left[\frac{\rho}{2}\right], i\,\mathrm{Sin}\left[\frac{\rho}{2}\right]\right\}\right\}$

In[169]:=
```
z1 = 2 * ArcTan[(N0 - 1) ^ (1 / 2)]
Rrot = MatrixExp[-I * z1 * σy / 2]
Ancilla = N[Rrot.Transpose[{{1, 0}}]]
```

Out[169]= 2.6252

Out[170]= {{0.255338 + 0. i, -0.966852 + 0. i}, {0.966852 + 0. i, 0.255338 + 0. i}}

Out[171]= {{0.255338 + 0. i}, {0.966852 + 0. i}}

In[172]:=
```
Evolution = {{Cos[τ - α], -i * Sin[τ]}, {-i * Sin[τ], Cos[α + τ]}} * Sec[α]
```

Out[172]= {{2.08583 Cos[1.0708 - τ], (0. - 2.08583 i) Sin[τ]},
{(0. - 2.08583 i) Sin[τ], 2.08583 Cos[1.0708 + τ]}}

In[173]:= $\tau = \pi / 2$

Out[173]= $\frac{\pi}{2}$

In[174]:= **Evolution.Transpose[vFirst]**

Out[174]= {{2.76925 + 0. i}, {0. - 2.76925 i}}

In[175]:= **(Evolution.Transpose[vFirst]) [[1]] [[1]]**

Out[175]= 2.76925 + 0. i

In[176]:= **(Evolution.Transpose[vFirst]) [[2]] [[1]]**

Out[176]= 0. - 2.76925 i

In[177]:= **NormalizationOutput = (FullSimplify[Abs[(Evolution.Transpose[vFirst]) [[1]] [[1]]] ^2 + Abs[(Evolution.Transpose[vFirst]) [[2]] [[1]]] ^2]) ^ (1 / 2)**

Out[177]= 3.91632

In[178]:= **FullSimplify[Evolution.Transpose[vFirst] / NormalizationOutput]**

Out[178]= {{0.707107 + 0. i}, {0. - 0.707107 i}}

In[179]:= **FinalRotation** = $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}, \left\{\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[179]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}, \left\{\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[180]:= **FullSimplify[FinalRotation.Evolution.Transpose[vFirst] / NormalizationOutput]**

Out[180]= $\{\{1. + 0.\, i\}, \{0. + 4.2523 \times 10^{-17}\, i\}\}$

In[181]:= **FullSimplify[FinalRotation.Evolution.Transpose[vSecond]]**

Out[181]= {{0. + 0. i}, {0. - 0.255342 i}}

In[182]:= **U12P = FinalRotation.U12**

Out[182]= $\left\{\left\{\frac{1}{2} + \frac{i}{2}, \frac{1}{2} - \frac{i}{2}\right\}, \left\{\frac{1}{2} + \frac{i}{2}, -\frac{1}{2} + \frac{i}{2}\right\}\right\}$

In[183]:=
```
(*U11*)
U11OverallPhase = Arg[U11[[1]][[1]]]
U11Dephased = U11 * Exp[-I * U11OverallPhase]
θU11 = 2 * ArcTan[Abs[U11Dephased[[1]][[2]]] / Abs[U11Dephased[[1]][[1]]]]
ϕU11 = Arg[U11Dephased[[2]][[1]]]
λU11 = Arg[U11Dephased[[2]][[2]]] - ϕU11
Max[N[Abs[Exp[I * U11OverallPhase] * U3[θU11, ϕU11, λU11] - U11]]]
(*U12P*)
U12POverallPhase = Arg[U12P[[1]][[1]]]
U12PDephased = U12P * Exp[-I * U12POverallPhase]
θU12P = 2 * ArcTan[Abs[U12PDephased[[1]][[2]]] / Abs[U12PDephased[[1]][[1]]]]
ϕU12P = Arg[U12PDephased[[2]][[1]]]
λU12P = Arg[U12PDephased[[2]][[2]]] - ϕU12P
Max[N[Abs[Exp[I * U12POverallPhase] * U3[θU12P, ϕU12P, λU12P] - U12P]]]
(*U21*)
U21OverallPhase = Arg[U21[[1]][[1]]]
U21Dephased = U21 * Exp[-I * U21OverallPhase]
θU21 = 2 * ArcTan[Abs[U21Dephased[[1]][[2]]] / Abs[U21Dephased[[1]][[1]]]]
ϕU21 = Arg[U21Dephased[[2]][[1]]]
λU21 = Arg[U21Dephased[[2]][[2]]] - ϕU21
Max[N[Abs[Exp[I * U21OverallPhase] * U3[θU21, ϕU21, λU21] - U21]]]
(*U22*)
U22OverallPhase = Arg[U22[[1]][[1]]]
U22Dephased = U22 * Exp[-I * U22OverallPhase]
θU22 = 2 * ArcTan[Abs[U22Dephased[[1]][[2]]] / Abs[U22Dephased[[1]][[1]]]]
ϕU22 = Arg[U22Dephased[[2]][[1]]]
λU22 = Arg[U22Dephased[[2]][[2]]] - ϕU22
Max[N[Abs[Exp[I * U22OverallPhase] * U3[θU22, ϕU22, λU22] - U22]]]
(*U31*)
U31OverallPhase = Arg[U31[[1]][[1]]]
U31Dephased = U31 * Exp[-I * U31OverallPhase]
θU31 = 2 * ArcTan[Abs[U31Dephased[[1]][[2]]] / Abs[U31Dephased[[1]][[1]]]]
ϕU31 = Arg[U31Dephased[[2]][[1]]]
λU31 = Arg[U31Dephased[[2]][[2]]] - ϕU31
Max[N[Abs[Exp[I * U31OverallPhase] * U3[θU31, ϕU31, λU31] - U31]]]
(*U32*)
U32OverallPhase = Arg[U32[[1]][[1]]]
U32Dephased = U32 * Exp[-I * U32OverallPhase]
θU32 = 2 * ArcTan[Abs[U32Dephased[[1]][[2]]] / Abs[U32Dephased[[1]][[1]]]]
ϕU32 = Arg[U32Dephased[[2]][[1]]]
λU32 = Arg[U32Dephased[[2]][[2]]] - ϕU32
Max[N[Abs[Exp[I * U32OverallPhase] * U3[θU32, ϕU32, λU32] - U32]]]
(*U41R*)
U41OverallPhaseR = Arg[(U41.Rrot)[[1]][[1]]]
U41DephasedR = (U41.Rrot) * Exp[-I * U41OverallPhaseR]
θU41R = 2 * ArcTan[Abs[U41DephasedR[[1]][[2]]] / Abs[U41DephasedR[[1]][[1]]]]
ϕU41R = Arg[U41DephasedR[[2]][[1]]]
λU41R = Arg[U41DephasedR[[2]][[2]]] - ϕU41R
Max[N[Abs[Exp[I * U41OverallPhaseR] * U3[θU41R, ϕU41R, λU41R] - U41.Rrot]]]
```

Out[183]= $-\frac{\pi}{4}$

Out[184]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[185]= $\frac{\pi}{2}$

Out[186]= $\frac{\pi}{2}$

Out[187]= $-\pi$

Out[188]= $0.$

Out[189]= $\frac{\pi}{4}$

Out[190]= $\left\{\left\{\left(\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}, \left(\frac{1}{2}-\frac{i}{2}\right) e^{-\frac{i\pi}{4}}\right\}, \left\{\left(\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}, \left(-\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}\right\}\right\}$

Out[191]= $\frac{\pi}{2}$

Out[192]= 0

Out[193]= $\frac{\pi}{2}$

Out[194]= $0.$

Out[195]= 0

Out[196]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[197]= $\frac{\pi}{2}$

Out[198]= 0

Out[199]= π

Out[200]= $0.$

Out[201]= 2.23545×10^{-6}

Out[202]= $\{\{0.681698 + 0.\, i, -0.731633 + 3.05542 \times 10^{-6}\, i\}, \{-0.731633 + 3.05542 \times 10^{-6}\, i, -0.681698 + 0.\, i\}\}$

Out[203]= 1.64143

Out[204]= 3.14159

Out[205]= 4.17617×10^{-6}

Out[206]= 6.11085×10^{-6}

Out[207]= -0.785398

Out[208]= $\{\{0.707107 + 0.\, i, 0.707107 + 0.\, i\}, \{-6.25483 \times 10^{-8} + 0.707107\, i, 6.25483 \times 10^{-8} - 0.707107\, i\}\}$

Out[209]= 1.5708

Out[210]= 1.5708

Out[211]= -3.14159

Out[212]= 1.30538×10^{-8}

Out[213]= 8.2124×10^{-14}

Out[214]= $\{\{1. + 0.\,i,\ 1.5098 \times 10^{-9} - 0.000054394\,i\},\ \{1.5098 \times 10^{-9} - 0.000054394\,i,\ -1. + 0.\,i\}\}$

Out[215]= 0.000108788

Out[216]= -1.57077

Out[217]= 4.71236

Out[218]= 0.000108788

Out[219]= 8.57087×10^{-6}

Out[220]= $\{\{0.884058 + 8.47033 \times 10^{-22}\,i,\ 0.467378 - 3.19506 \times 10^{-8}\,i\},$
$\{0.467378 + 4.28537 \times 10^{-8}\,i,\ -0.884058 - 2.06225 \times 10^{-8}\,i\}\}$

Out[221]= 0.972645

Out[222]= 9.16896×10^{-8}

Out[223]= -3.14159

Out[224]= 5.83122×10^{-13}

In[225]:=
```
(*Constant input*)
{θU11, ϕU11, λU11}
```

Out[225]= $\{\frac{\pi}{2}, \frac{\pi}{2}, -\pi\}$

In[226]:= **{θU12P, ϕU12P, λU12P}**

Out[226]= $\{\frac{\pi}{2}, 0, \frac{\pi}{2}\}$

In[227]:= **{θU21, ϕU21, λU21}**

Out[227]= $\{\frac{\pi}{2}, 0, \pi\}$

In[228]:= **{θU22, ϕU22, λU22}**

Out[228]= $\{1.64143, 3.14159, 4.17617 \times 10^{-6}\}$

In[229]:= **{θU31, ϕU31, λU31}**

Out[229]= {1.5708, 1.5708, -3.14159}

In[230]:= **{θU32, ϕU32, λU32}**

Out[230]= {0.000108788, -1.57077, 4.71236}

In[231]:=
```
{θU41R, ϕU41R, λU41R}
```

Out[231]= $\{0.972645, 9.16896\times10^{-8}, -3.14159\}$

In[232]:=
```
(*Varying input*)
(*U42*)
ρ = -(π/2) * 1
Initializer = {{Cos[ρ/2], i Sin[ρ/2]}, {i Sin[ρ/2], Cos[ρ/2]}}
U42POverallPhase = Arg[(U42.Initializer)[[1]][[1]]]
U42PDephased = (U42.Initializer) * Exp[-I * U42POverallPhase]
θU42P = 2 * ArcTan[Abs[U42PDephased[[1]][[2]]] / Abs[U42PDephased[[1]][[1]]]]
ϕU42P = Arg[U42PDephased[[2]][[1]]]
λU42P = Arg[U42PDephased[[2]][[2]]] - ϕU42P
Max[N[Abs[Exp[I * U42POverallPhase] * U3[θU42P, ϕU42P, λU42P] - (U42.Initializer)]]]
{θU42P, ϕU42P, λU42P}
```

Out[232]= $-\frac{\pi}{2}$

Out[233]= $\left\{\left\{\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right\}, \left\{-\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[234]= 2.35619

Out[235]= $\{\{1. + 5.55112\times10^{-17}\, i, 1.73261\times10^{-8} - 0.00017071\, i\},$
$\{-0.00017071 + 1.73256\times10^{-8}\, i, 2.93025\times10^{-9} - 1.\, i\}\}$

Out[236]= 0.000341421

Out[237]= 3.14149

Out[238]= −4.71229

Out[239]= 1.86062×10^{-12}

Out[240]= {0.000341421, 3.14149, −4.71229}

In[ ]:=

```
(* ρ = (π/2)*1.0 *)
{3.14125123286507`, -0.00010149118161285882`, -1.570897820906415`}

(* ρ = (π/2) * 0.8 *)
{2.8270919675078474`, -1.0360037804062609`*^-7, -1.5707964388140467`}

(* ρ = (π/2) * 0.6 *)
{2.512932702148869`, -4.4736084381891876`*^-8, -1.5707963857210407`}

(* ρ = (π/2) * 0.4 *)
{2.1987734367898897`, -2.2233880561206133`*^-8, -1.5707963696167426`}

(* ρ = (π/2) * 0.2 *)
{1.884614171430911`, -8.3217619303319`*^-9, -1.5707963632263289`}
```

```
(* ρ = (π/2) * 0.0 *)
{1.5704549060719317`, 2.936170268924922`*^-9, -1.5707963614470914`}
(* ρ = - (π/2) * 0.2 *)
{1.2562956407129526`, 1.4196600767601402`*^-8, -1.5707963632344129`}
(* ρ = - (π/2) * 0.4 *)
{0.9421363753539739`, 2.8118711980511965`*^-8, -1.5707963696379925`}
(* ρ = - (π/2) * 0.6 *)
{0.6279771099949947`, 5.0653250938742075`*^-8, -1.5707963857764484`}
(* ρ = - (π/2) * 0.8 *)
{0.3138178446360164`, 1.0969684877805797`*^-7, -1.5707964390497102`}
(* ρ = - (π/2) * 1.0 *)
{0.00034142072472327444`, 3.141491162408529`, -4.712287486273171`}
```

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