In[1]:= (*Our matrix which we need to decompose*)

```
USecondStage = {{2.8075734707065387`*^-7 + 0.2911826741057892` ⅈ,
   -0.6801709725379373` - 2.9627595301618716`*^-7 ⅈ, 1.0931336462752239`*^-7 +
    0.5794402171339909` ⅈ, 0.3418026664798702` - 1.161407788602322`*^-7 ⅈ},
  {-0.6801743563117079` - 6.146575163143084`*^-7 ⅈ, -6.756311026502853`*^-7 -
    0.29096636701472084` ⅈ, 0.34196390696075846` - 2.409939992161457`*^-7 ⅈ,
   -2.6711700956173756`*^-7 - 0.5794513539198103` ⅈ},
  {-9.920949915016505`*^-8 - 0.5794403152627157` ⅈ,
   -0.341802757902192` + 1.0548185515010019`*^-7 ⅈ, 2.548096760210958`*^-7 +
    0.2911829242607881` ⅈ, -0.6801707395336708` - 2.690876888297288`*^-7 ⅈ},
  {-0.34196827374770844` - 1.606676264063328`*^-7 ⅈ, -1.462238991833953`*^-7 +
    0.5794554254904356` ⅈ, -0.6801633118872569` + 4.097359505815523`*^-7 ⅈ,
   3.7611236884671495`*^-7 - 0.29097669631159595` ⅈ}}
```

Out[1]= $\{\{2.80757\times10^{-7}+0.291183\,i,\ -0.680171-2.96276\times10^{-7}\,i,\ 1.09313\times10^{-7}+0.57944\,i,\ 0.341803-1.16141\times10^{-7}\,i\},\ \{-0.680174-6.14658\times10^{-7}\,i,\ -6.75631\times10^{-7}-0.290966\,i,\ 0.341964-2.40994\times10^{-7}\,i,\ -2.67117\times10^{-7}-0.579451\,i\},\ \{-9.92095\times10^{-8}-0.57944\,i,\ -0.341803+1.05482\times10^{-7}\,i,\ 2.5481\times10^{-7}+0.291183\,i,\ -0.680171-2.69088\times10^{-7}\,i\},\ \{-0.341968-1.60668\times10^{-7}\,i,\ -1.46224\times10^{-7}+0.579455\,i,\ -0.680163+4.09736\times10^{-7}\,i,\ 3.76112\times10^{-7}-0.290977\,i\}\}$

In[2]:= `MatrixForm[Round[USecondStage, 0.00001]]`

Out[2]//MatrixForm=

$$\begin{pmatrix} 0.+0.29118\,i & -0.68017 & 0.+0.57944\,i & 0.3418 \\ -0.68017 & 0.-0.29097\,i & 0.34196 & 0.-0.57945\,i \\ 0.-0.57944\,i & -0.3418 & 0.+0.29118\,i & -0.68017 \\ -0.34197 & 0.+0.57946\,i & -0.68016 & 0.-0.29098\,i \end{pmatrix}$$

In[3]:=
```
(*It is unitary, vectors and eigenvalues are found with high accuracy*)
Transpose[Eigenvectors[USecondStage]].
  {{Eigenvalues[USecondStage][[1]], 0, 0, 0}, {0, Eigenvalues[USecondStage][[2]], 0, 0},
   {0, 0, Eigenvalues[USecondStage][[3]], 0}, {0, 0, 0, Eigenvalues[USecondStage][[4]]}}.
  Conjugate[Eigenvectors[USecondStage]] - USecondStage
ConjugateTranspose[USecondStage].USecondStage
Max[Abs[ConjugateTranspose[USecondStage].USecondStage -
   {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}]]
```

Out[3]= $\{\{3.87268\times10^{-7}+0.0000829953\,i, -0.0000625707+1.73634\times10^{-7}\,i, -1.65842\times10^{-7}-0.0000391922\,i, -0.0000614814+3.94314\times10^{-7}\,i\}, \{1.66794\times10^{-6}+7.77953\times10^{-8}\,i, 5.35763\times10^{-7}-0.0000870917\,i, -0.0000888024-5.20505\times10^{-7}\,i, 1.29418\times10^{-7}+0.0000394315\,i\}, \{1.29332\times10^{-7}+0.0000441734\,i, 0.0000476828+3.91932\times10^{-7}\,i, -5.35663\times10^{-7}+0.0000828494\,i, -0.0000723653+1.4253\times10^{-7}\,i\}, \{0.0000846202-2.6555\times10^{-7}\,i, -1.6574\times10^{-7}-0.0000439341\,i, -8.68539\times10^{-6}+1.08985\times10^{-7}\,i, -3.87368\times10^{-7}-0.0000787531\,i\}\}$

Out[4]= $\{\{1.00012+0.\,i, 2.90852\times10^{-7}-0.000247805\,i, -9.80189\times10^{-7}+3.74323\times10^{-7}\,i, 7.41527\times10^{-7}-0.000012612\,i\}, \{2.90852\times10^{-7}+0.000247805\,i, 0.999892+0.\,i, -1.26447\times10^{-7}-0.0000212256\,i, -7.18752\times10^{-6}+4.82365\times10^{-7}\,i\}, \{-9.80189\times10^{-7}-3.74323\times10^{-7}\,i, -1.26447\times10^{-7}+0.0000212256\,i, 1.0001+0.\,i, -6.08342\times10^{-7}-0.000239882\,i\}, \{7.41527\times10^{-7}+0.000012612\,i, -7.18752\times10^{-6}-4.82365\times10^{-7}\,i, -6.08342\times10^{-7}+0.000239882\,i, 0.999893+0.\,i\}\}$

Out[5]= 0.000247806

In[6]:=
```
(*Do SVD on the UR and UI*)
M = 1/Sqrt[2] {{1, 0, 0, I}, {0, I, 1, 0}, {0, I, -1, 0}, {1, 0, 0, -I}}
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
UP = ConjugateTranspose[M].USecondStage.M
UR = (UP + Conjugate[UP]) / 2
UI = (UP - Conjugate[UP]) / (2 * I)
{a, b, c} = SingularValueDecomposition[UR]
Max[Abs[a.b.ConjugateTranspose[c] - UR]]
{d, e, f} = SingularValueDecomposition[UI]
Max[Abs[d.e.ConjugateTranspose[f] - UI]]
MatrixForm[M]
MatrixForm[M.ConjugateTranspose[M]]
```

Out[6]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}, \left\{0, \frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}, \left\{0, \frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[7]= {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}

Out[8]= $\{\{-0.0000824752+0.00010285\,i,\ -0.579448-0.680167\,i,\ -3.95809\times10^{-6}+7.25117\times10^{-6}\,i,\ -0.29108-0.341886\,i\},\ \{-0.579446+0.680173\,i,\ 0.0000803641+0.000108211\,i,\ -0.291074+0.341884\,i,\ -1.72444\times10^{-6}+5.34654\times10^{-6}\,i\},\ \{-1.89234\times10^{-6}-5.69211\times10^{-6}\,i,\ 0.291075+0.341883\,i,\ -0.0000807849+0.000108346\,i,\ -0.579445-0.680172\,i\},\ \{0.29108-0.341885\,i,\ -3.70256\times10^{-6}-7.95718\times10^{-6}\,i,\ -0.579448+0.680167\,i,\ 0.0000831321+0.000103127\,i\}\}$

Out[9]= $\{\{-0.0000824752+0.\,i,\ -0.579448+0.\,i,\ -3.95809\times10^{-6}+0.\,i,\ -0.29108+0.\,i\},\ \{-0.579446+0.\,i,\ 0.0000803641+0.\,i,\ -0.291074+0.\,i,\ -1.72444\times10^{-6}+0.\,i\},\ \{-1.89234\times10^{-6}+0.\,i,\ 0.291075+0.\,i,\ -0.0000807849+0.\,i,\ -0.579445+0.\,i\},\ \{0.29108+0.\,i,\ -3.70256\times10^{-6}+0.\,i,\ -0.579448+0.\,i,\ 0.0000831321+0.\,i\}\}$

Out[10]= $\{\{0.00010285+0.\,i,\ -0.680167+0.\,i,\ 7.25117\times10^{-6}+0.\,i,\ -0.341886+0.\,i\},\ \{0.680173+0.\,i,\ 0.000108211+0.\,i,\ 0.341884+0.\,i,\ 5.34654\times10^{-6}+0.\,i\},\ \{-5.69211\times10^{-6}+0.\,i,\ 0.341883+0.\,i,\ 0.000108346+0.\,i,\ -0.680172+0.\,i\},\ \{-0.341885+0.\,i,\ -7.95718\times10^{-6}+0.\,i,\ 0.680167+0.\,i,\ 0.000103127+0.\,i\}\}$

Out[11]= $\{\{\{0.498562+0.\,i,\ -0.502268+0.\,i,\ -0.705322+0.\,i,\ -0.0410291+0.\,i\},\ \{0.503003+0.\,i,\ 0.501056+0.\,i,\ -0.0421484+0.\,i,\ 0.702961+0.\,i\},\ \{0.497074+0.\,i,\ 0.498838+0.\,i,\ 0.0373753+0.\,i,\ -0.709001+0.\,i\},\ \{-0.50134+0.\,i,\ 0.497826+0.\,i,\ -0.706645+0.\,i,\ -0.0384762+0.\,i\}\},\ \{\{0.648488,\ 0.,\ 0.,\ 0.\},\ \{0.,\ 0.648479,\ 0.,\ 0.\},\ \{0.,\ 0.,\ 0.648416,\ 0.\},\ \{0.,\ 0.,\ 0.,\ 0.648408\}\},\ \{\{-0.674546+0.\,i,\ -0.224198+0.\,i,\ -0.279464+0.\,i,\ -0.645463+0.\,i\},\ \{-0.222306+0.\,i,\ 0.672767+0.\,i,\ 0.647078+0.\,i,\ -0.281522+0.\,i\},\ \{0.222127+0.\,i,\ -0.669793+0.\,i,\ 0.650403+0.\,i,\ -0.281091+0.\,i\},\ \{-0.668002+0.\,i,\ -0.220221+0.\,i,\ 0.283135+0.\,i,\ 0.652005+0.\,i\}\}\}$

Out[12]= 5.55112×10^{-16}

Out[13]= $\{\{\{-0.0245783+0.\,i,\ 0.707132+0.\,i,\ -0.509306+0.\,i,\ -0.489864+0.\,i\},\ \{-0.709606+0.\,i,\ -0.0262009+0.\,i,\ 0.486992+0.\,i,\ -0.508538+0.\,i\},\ \{-0.703793+0.\,i,\ -0.021372+0.\,i,\ -0.490008+0.\,i,\ 0.513917+0.\,i\},\ \{0.0230133+0.\,i,\ -0.706272+0.\,i,\ -0.513165+0.\,i,\ -0.487146+0.\,i\}\},\ \{\{0.761305,\ 0.,\ 0.,\ 0.\},\ \{0.,\ 0.761297,\ 0.,\ 0.\},\ \{0.,\ 0.,\ 0.761222,\ 0.\},\ \{0.,\ 0.,\ 0.,\ 0.761214\}\},\ \{\{-0.644316+0.\,i,\ 0.293862+0.\,i,\ 0.665552+0.\,i,\ -0.235675+0.\,i\},\ \{-0.294198+0.\,i,\ -0.641369+0.\,i,\ 0.235076+0.\,i,\ 0.668456+0.\,i\},\ \{-0.298207+0.\,i,\ -0.642769+0.\,i,\ -0.239877+0.\,i,\ -0.663611+0.\,i\},\ \{0.639825+0.\,i,\ -0.298562+0.\,i,\ 0.666513+0.\,i,\ -0.239259+0.\,i\}\}\}$

Out[14]= 7.77156×10^{-16}

Out[15]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}$$

Out[16]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[17]:=
```
(*Real and symmetric matrices, with high accuracy*)
MatrixForm[UI.ConjugateTranspose[UR]]
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0.493637 + 0.\, i & -0.000115778 + 0.\, i & 0.000124454 + 0.\, i & -1.67277 \times 10^{-7} + 0.\, i \\ -0.000121709 + 0.\, i & -0.493637 + 0.\, i & -5.06754 \times 10^{-7} + 0.\, i & -0.000119323 + 0.\, i \\ -0.000118959 + 0.\, i & 4.09437 \times 10^{-7} + 0.\, i & 0.493636 + 0.\, i & -0.000122248 + 0.\, i \\ 9.74163 \times 10^{-8} + 0.\, i & 0.000124947 + 0.\, i & -0.000116373 + 0.\, i & -0.493637 + 0.\, i \end{pmatrix}$$

In[18]:=
```
Max[Abs[UI.ConjugateTranspose[UR] - Transpose[UI.ConjugateTranspose[UR]]]]
```

Out[18]= 0.00024427

In[19]:=
```
MatrixForm[UR.ConjugateTranspose[UI]]
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 0.493637 + 0.\, i & -0.000121709 + 0.\, i & -0.000118959 + 0.\, i & 9.74163 \times 10^{-8} + 0.\, i \\ -0.000115778 + 0.\, i & -0.493637 + 0.\, i & 4.09437 \times 10^{-7} + 0.\, i & 0.000124947 + 0.\, i \\ 0.000124454 + 0.\, i & -5.06754 \times 10^{-7} + 0.\, i & 0.493636 + 0.\, i & -0.000116373 + 0.\, i \\ -1.67277 \times 10^{-7} + 0.\, i & -0.000119323 + 0.\, i & -0.000122248 + 0.\, i & -0.493637 + 0.\, i \end{pmatrix}$$

In[20]:=
```
Max[Abs[UR.ConjugateTranspose[UI] - Transpose[UR.ConjugateTranspose[UI]]]]
```

Out[20]= 0.00024427

In[21]:=
```
A = {{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}
```

Out[21]= {{a11, a12, a13, a14}, {a21, a22, a23, a24}, {a31, a32, a33, a34}, {a41, a42, a43, a44}}

In[22]:=
```
Solve[c.A == f, {a11, a12, a13, a14, a21, a22, a23, a24, a31, a32, a33, a34, a41, a42, a43, a44}]
```

Out[22]= $\{\{a11 \to 0.00637882 + 0.\, i,\ a12 \to 0.00102052 + 0.\, i,\ a13 \to -0.999719 + 0.\, i,\ a14 \to 0.022791 + 0.\, i,$
$a21 \to 0.00536138 + 0.\, i,\ a22 \to -0.00110288 + 0.\, i,\ a23 \to 0.0228242 + 0.\, i,$
$a24 \to 0.999725 + 0.\, i,\ a31 \to -0.0231034 + 0.\, i,\ a32 \to -0.999732 + 0.\, i,$
$a33 \to -0.00118964 + 0.\, i,\ a34 \to -0.000951828 + 0.\, i,\ a41 \to 0.999698 + 0.\, i,$
$a42 \to -0.0231048 + 0.\, i,\ a43 \to 0.00622906 + 0.\, i,\ a44 \to -0.00552894 + 0.\, i\}\}$

In[ ]:=

In[23]:=
```
a11 = 0.006378823687439644` + 0.` ⅈ
a12 = 0.001020516951554195` + 0.` ⅈ
a13 = -0.9997193810193498` + 0.` ⅈ
a14 = 0.022790971190469144` + 0.` ⅈ
a21 = 0.005361376918133426` + 0.` ⅈ
a22 = -0.0011028802838845496` + 0.` ⅈ
a23 = 0.022824171178452732` + 0.` ⅈ
a24 = 0.9997245103040328` + 0.` ⅈ
a31 = -0.02310339406778138` + 0.` ⅈ
a32 = -0.9997319200443886` + 0.` ⅈ
a33 = -0.0011896427748351098` + 0.` ⅈ
a34 = -0.0009518282286368085` + 0.` ⅈ
a41 = 0.9996983542191388` + 0.` ⅈ
a42 = -0.023104766707126863` + 0.` ⅈ
a43 = 0.0062290588674059205` + 0.` ⅈ
a44 = -0.005528937744319864` + 0.` ⅈ
```

Out[23]= 0.00637882 + 0. ⅈ

Out[24]= 0.00102052 + 0. ⅈ

Out[25]= -0.999719 + 0. ⅈ

Out[26]= 0.022791 + 0. ⅈ

Out[27]= 0.00536138 + 0. ⅈ

Out[28]= -0.00110288 + 0. ⅈ

Out[29]= 0.0228242 + 0. ⅈ

Out[30]= 0.999725 + 0. ⅈ

Out[31]= -0.0231034 + 0. ⅈ

Out[32]= -0.999732 + 0. ⅈ

Out[33]= -0.00118964 + 0. ⅈ

Out[34]= -0.000951828 + 0. ⅈ

Out[35]= 0.999698 + 0. ⅈ

Out[36]= -0.0231048 + 0. ⅈ

Out[37]= 0.00622906 + 0. ⅈ

Out[38]= -0.00552894 + 0. ⅈ

In[39]:=
```
d.e.ConjugateTranspose[A].ConjugateTranspose[c]
```

Out[39]=
$$\{\{0.00010285 + 0.\,i,\ -0.680167 + 0.\,i,\ 7.25117\times10^{-6} + 0.\,i,\ -0.341886 + 0.\,i\},$$
$$\{0.680173 + 0.\,i,\ 0.000108211 + 0.\,i,\ 0.341884 + 0.\,i,\ 5.34654\times10^{-6} + 0.\,i\},$$
$$\{-5.69211\times10^{-6} + 0.\,i,\ 0.341883 + 0.\,i,\ 0.000108346 + 0.\,i,\ -0.680172 + 0.\,i\},$$
$$\{-0.341885 + 0.\,i,\ -7.95718\times10^{-6} + 0.\,i,\ 0.680167 + 0.\,i,\ 0.000103127 + 0.\,i\}\}$$

In[40]:= **UI**

Out[40]= $\{\{0.00010285 + 0.\,i,\ -0.680167 + 0.\,i,\ 7.25117 \times 10^{-6} + 0.\,i,\ -0.341886 + 0.\,i\},$
$\{0.680173 + 0.\,i,\ 0.000108211 + 0.\,i,\ 0.341884 + 0.\,i,\ 5.34654 \times 10^{-6} + 0.\,i\},$
$\{-5.69211 \times 10^{-6} + 0.\,i,\ 0.341883 + 0.\,i,\ 0.000108346 + 0.\,i,\ -0.680172 + 0.\,i\},$
$\{-0.341885 + 0.\,i,\ -7.95718 \times 10^{-6} + 0.\,i,\ 0.680167 + 0.\,i,\ 0.000103127 + 0.\,i\}\}$

In[41]:= **Max[Abs[d.e.ConjugateTranspose[A].ConjugateTranspose[c] - UI]]**

Out[41]= 7.77156×10^{-16}

In[42]:= **F = {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}**

Out[42]= {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}

In[43]:= **MatrixForm[a]**

Out[43]//MatrixForm=

$$\begin{pmatrix} 0.498562 + 0.\,i & -0.502268 + 0.\,i & -0.705322 + 0.\,i & -0.0410291 + 0.\,i \\ 0.503003 + 0.\,i & 0.501056 + 0.\,i & -0.0421484 + 0.\,i & 0.702961 + 0.\,i \\ 0.497074 + 0.\,i & 0.498838 + 0.\,i & 0.0373753 + 0.\,i & -0.709001 + 0.\,i \\ -0.50134 + 0.\,i & 0.497826 + 0.\,i & -0.706645 + 0.\,i & -0.0384762 + 0.\,i \end{pmatrix}$$

In[44]:= **MatrixForm[F.d.ConjugateTranspose[A]]**

Out[44]//MatrixForm=

$$\begin{pmatrix} 0.498564 + 0.\,i & -0.502266 + 0.\,i & -0.705303 + 0.\,i & -0.041373 + 0.\,i \\ 0.502999 + 0.\,i & 0.501059 + 0.\,i & -0.0424929 + 0.\,i & 0.702942 + 0.\,i \\ 0.497072 + 0.\,i & 0.498841 + 0.\,i & 0.03772 + 0.\,i & -0.708981 + 0.\,i \\ -0.501344 + 0.\,i & 0.497822 + 0.\,i & -0.706626 + 0.\,i & -0.0388215 + 0.\,i \end{pmatrix}$$

In[45]:= **Max[Abs[F.d.ConjugateTranspose[A] - a]]**

Out[45]= 0.000345287

In[46]:= **a.Inverse[d.ConjugateTranspose[A]]**

Out[46]= $\{\{1. + 0.\,i,\ -0.000240549 + 0.\,i,\ -0.000246551 + 0.\,i,\ -1.12221 \times 10^{-8} + 0.\,i\},$
$\{-0.000240549 + 0.\,i,\ -1. + 0.\,i,\ -3.89759 \times 10^{-8} + 0.\,i,\ 0.000247419 + 0.\,i\},$
$\{0.000246551 + 0.\,i,\ -1.58084 \times 10^{-7} + 0.\,i,\ 1. + 0.\,i,\ -0.000241697 + 0.\,i\},$
$\{-1.30329 \times 10^{-7} + 0.\,i,\ -0.000247419 + 0.\,i,\ -0.000241697 + 0.\,i,\ -1. + 0.\,i\}\}$

In[47]:= **a.b.ConjugateTranspose[c] + I * d.ConjugateTranspose[A].e.ConjugateTranspose[c] - UP**

Out[47]= $\{\{0. - 0.0000357015\,i,\ 0. - 3.90177 \times 10^{-6}\,i,\ -3.55618 \times 10^{-17} + 0.0000704905\,i,$
$0. - 1.07444 \times 10^{-6}\,i\},\ \{-5.55112 \times 10^{-16} - 3.51585 \times 10^{-6}\,i,\ 3.05311 \times 10^{-16} - 0.0000389891\,i,$
$4.44089 \times 10^{-16} - 1.83298 \times 10^{-6}\,i,\ -1.11022 \times 10^{-16} + 0.0000781045\,i\},$
$\{1.11022 \times 10^{-16} - 0.000078217\,i,\ -1.11022 \times 10^{-16} - 1.08417 \times 10^{-6}\,i,$
$-3.33067 \times 10^{-16} - 0.0000387655\,i,\ 0. + 3.87378 \times 10^{-6}\,i\},$
$\{3.88578 \times 10^{-16} - 1.80471 \times 10^{-6}\,i,\ -2.5327 \times 10^{-16} - 0.0000703782\,i,$
$-3.33067 \times 10^{-16} + 3.50696 \times 10^{-6}\,i,\ -1.63064 \times 10^{-16} - 0.0000359252\,i\}\}$

In[48]:= **Max[**
Abs[a.b.ConjugateTranspose[c] + I * d.ConjugateTranspose[A].e.ConjugateTranspose[c] - UP]]

Out[48]= 0.000078217

In[49]:=
```
{b, e}
```
Out[49]=
```
{{{0.648488, 0., 0., 0.}, {0., 0.648479, 0., 0.},
  {0., 0., 0.648416, 0.}, {0., 0., 0., 0.648408}}, {{0.761305, 0., 0., 0.},
  {0., 0.761297, 0., 0.}, {0., 0., 0.761222, 0.}, {0., 0., 0., 0.761214}}}
```
In[50]:=
```
Max[Abs[b.a.ConjugateTranspose[c] + I * F.a.e.ConjugateTranspose[c] - UP]]
```
Out[50]=
```
0.000203742
```
In[51]:=
```
Max[Abs[(b + I * F.e).a.ConjugateTranspose[c] - UP]]
```
Out[51]=
```
0.000190791
```
In[52]:=
```
DiagonalPart = b + I * F.e
```
Out[52]=
```
{{0.648488 + 0.761305 i, 0. + 0. i, 0. + 0. i, 0. + 0. i},
 {0. + 0. i, 0.648479 - 0.761297 i, 0. + 0. i, 0. + 0. i},
 {0. + 0. i, 0. + 0. i, 0.648416 + 0.761222 i, 0. + 0. i},
 {0. + 0. i, 0. + 0. i, 0. + 0. i, 0.648408 - 0.761214 i}}
```
In[53]:=
```
ξ = a.ConjugateTranspose[c]
```
Out[53]=
```
{{-0.000100771 + 0. i, -0.89359 + 0. i, -0.0000510722 + 0. i, -0.448883 + 0. i},
 {-0.893591 + 0. i, 0.000100834 + 0. i, -0.448883 + 0. i, 0.0000509453 + 0. i},
 {0.0000507457 + 0. i, 0.448883 + 0. i, -0.000101602 + 0. i, -0.89359 + 0. i},
 {0.448883 + 0. i, -0.0000506187 + 0. i, -0.893591 + 0. i, 0.000101665 + 0. i}}
```
In[54]:=
```
Max[Abs[DiagonalPart.ξ - UP]]
```
Out[54]=
```
0.000190791
```
In[55]:=
```
Max[Abs[USecondStage - M.UP.ConjugateTranspose[M]]]
```
Out[55]= 3.42342×10^{-16}

In[56]:=
```
Max[Abs[USecondStage - M.DiagonalPart.ConjugateTranspose[M].M.ξ.ConjugateTranspose[M]]]
```
Out[56]=
```
0.000188548
```
In[57]:=
```
(*Now we decompose this happiness*)
Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]
```
Out[57]=
```
{{UA11 UB11, UA11 UB12, UA12 UB11, UA12 UB12}, {UA11 UB21, UA11 UB22, UA12 UB21, UA12 UB22},
 {UA21 UB11, UA21 UB12, UA22 UB11, UA22 UB12}, {UA21 UB21, UA21 UB22, UA22 UB21, UA22 UB22}}
```
In[ ]:=

In[58]:=
```
WeHave = M.ξ.ConjugateTranspose[M]
Search = KroneckerProduct[{{UA11, UA12}, {UA21, UA22}}, {{UB11, UB12}, {UB21, UB22}}]
UAMatr = {{UA11, UA12}, {UA21, UA22}}
UBMatr = {{UB11, UB12}, {UB21, UB22}}
Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[2]], Search[[1]][[1]] / Search[[1]][[2]]}]
Reverse[{WeHave[[1]][[3]] / WeHave[[1]][[4]], Search[[1]][[3]] / Search[[1]][[4]]}]
Reverse[{WeHave[[1]][[1]] / WeHave[[1]][[3]], Search[[1]][[1]] / Search[[1]][[3]]}]
Reverse[{WeHave[[1]][[2]] / WeHave[[1]][[4]], Search[[1]][[2]] / Search[[1]][[4]]}]
Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[2]], Search[[2]][[1]] / Search[[2]][[2]]}]
Reverse[{WeHave[[2]][[3]] / WeHave[[2]][[4]], Search[[2]][[3]] / Search[[2]][[4]]}]
Reverse[{WeHave[[2]][[1]] / WeHave[[2]][[3]], Search[[2]][[1]] / Search[[2]][[3]]}]
Reverse[{WeHave[[2]][[2]] / WeHave[[2]][[4]], Search[[2]][[2]] / Search[[2]][[4]]}]
Reverse[{WeHave[[3]][[1]] / WeHave[[3]][[2]], Search[[3]][[1]] / Search[[3]][[2]]}]
Reverse[{WeHave[[3]][[3]] / WeHave[[3]][[4]], Search[[3]][[3]] / Search[[3]][[4]]}]
Reverse[{WeHave[[4]][[1]] / WeHave[[4]][[2]], Search[[4]][[1]] / Search[[4]][[2]]}]
Reverse[{WeHave[[4]][[3]] / WeHave[[4]][[4]], Search[[4]][[3]] / Search[[4]][[4]]}]
Reverse[{WeHave[[3]][[1]] / WeHave[[1]][[4]], Search[[3]][[1]] / Search[[1]][[4]]}]
Reverse[{WeHave[[4]][[1]] / WeHave[[1]][[4]], Search[[4]][[1]] / Search[[1]][[4]]}]
Reverse[{WeHave[[4]][[2]] / WeHave[[3]][[3]], Search[[4]][[2]] / Search[[3]][[3]]}]
Reverse[{WeHave[[1]][[2]] / WeHave[[3]][[1]], Search[[1]][[2]] / Search[[3]][[1]]}]
```

Out[58]= $\{\{4.47421\times10^{-7}+0.448883\,i,\ -0.0000508455-1.06488\times10^{-7}\,i,\ 2.2675\times10^{-7}+0.893591\,i,\ -0.000101218-2.1206\times10^{-7}\,i\},\ \{0.0000508455-1.06582\times10^{-7}\,i,\ -3.83645\times10^{-7}-0.448883\,i,\ 0.000101218-2.12097\times10^{-7}\,i,\ -9.97915\times10^{-8}-0.893591\,i\},\ \{9.97915\times10^{-8}-0.893591\,i,\ 0.000101218+2.12097\times10^{-7}\,i,\ -3.83645\times10^{-7}+0.448883\,i,\ -0.0000508455-1.06582\times10^{-7}\,i\},\ \{-0.000101218+2.1206\times10^{-7}\,i,\ -2.2675\times10^{-7}+0.893591\,i,\ 0.0000508455-1.06488\times10^{-7}\,i,\ 4.47421\times10^{-7}-0.448883\,i\}\}$

Out[59]= {{UA11 UB11, UA11 UB12, UA12 UB11, UA12 UB12}, {UA11 UB21, UA11 UB22, UA12 UB21, UA12 UB22}, {UA21 UB11, UA21 UB12, UA22 UB11, UA22 UB12}, {UA21 UB21, UA21 UB22, UA22 UB21, UA22 UB22}}

Out[60]= {{UA11, UA12}, {UA21, UA22}}

Out[61]= {{UB11, UB12}, {UB21, UB22}}

Out[62]= $\left\{\frac{UB11}{UB12},\ -18.4983-8828.34\,i\right\}$

Out[63]= $\left\{\frac{UB11}{UB12},\ -18.4983-8828.34\,i\right\}$

Out[64]= $\left\{\frac{UA11}{UA12},\ 0.502336-3.73232\times10^{-7}\,i\right\}$

Out[65]= $\left\{\frac{UA11}{UA12},\ 0.502336-3.7323\times10^{-7}\,i\right\}$

Out[66]= $\left\{\frac{UB21}{UB22},\ 2.37341\times10^{-7}+0.000113271\,i\right\}$

Out[67]= $\left\{\frac{UB21}{UB22},\ 2.37341\times10^{-7}+0.000113271\,i\right\}$

Out[68]= $\left\{\frac{\text{UA11}}{\text{UA12}}, 0.502336 - 3.73233\times 10^{-7}\, i\right\}$

Out[69]= $\left\{\frac{\text{UA11}}{\text{UA12}}, 0.502336 - 3.73232\times 10^{-7}\, i\right\}$

Out[70]= $\left\{\frac{\text{UB11}}{\text{UB12}}, -18.4983 - 8828.34\, i\right\}$

Out[71]= $\left\{\frac{\text{UB11}}{\text{UB12}}, -18.4983 - 8828.34\, i\right\}$

Out[72]= $\left\{\frac{\text{UB21}}{\text{UB22}}, 2.37341\times 10^{-7} + 0.000113271\, i\right\}$

Out[73]= $\left\{\frac{\text{UB21}}{\text{UB22}}, 2.37341\times 10^{-7} + 0.000113271\, i\right\}$

Out[74]= $\left\{\frac{\text{UA21 UB11}}{\text{UA12 UB12}}, 18.4951 + 8828.34\, i\right\}$

Out[75]= $\left\{\frac{\text{UA21 UB21}}{\text{UA12 UB12}}, 0.999991 - 0.00419015\, i\right\}$

Out[76]= $\left\{\frac{\text{UA21 UB22}}{\text{UA22 UB11}}, 1.9907 - 1.19624\times 10^{-6}\, i\right\}$

Out[77]= $\left\{\frac{\text{UA11 UB12}}{\text{UA21 UB11}}, 1.19162\times 10^{-7} - 0.0000569002\, i\right\}$

In[78]:=
```
UB11 = (-18.498331697023218` - 8828.338006857459` i) * UB12
UA11 =
 UA12 * (0.5023362746131667` - 3.7323171592453836`*^-7 i) / (0.9999999999999999` + 0.` i)
UB21 = UB22 * (2.373409908124606`*^-7 + 0.00011327110593311901` i) /
   (1.` - 2.117582368135751`*^-22 i)
UA21 = -UA12 * (18.49510556103598` + 8828.338013640034` i) /
   (18.498331697023218` + 8828.338006857459` i)
UB22 = -UB12 * (0.9999912213019237` - 0.004190145473233464` i) /
   (2.372995981980318`*^-7 + 0.00011327110602014214` i)
UA12 = UA22 * (1.9906983638989886` - 1.1962411133088268`*^-6 i) /
   (0.9999999999993658` + 5.07510234580566`*^-7 i)
```

Out[78]= $(-18.4983 - 8828.34\, i)\ \text{UB12}$

Out[79]= $(0.502336 - 3.73232\times 10^{-7}\, i)\ \text{UA12}$

Out[80]= $(2.37341\times 10^{-7} + 0.000113271\, i)\ \text{UB22}$

Out[81]= $(-1. - 3.6543\times 10^{-7}\, i)\ \text{UA12}$

Out[82]= $(18.4971 + 8828.34\, i)\ \text{UB12}$

Out[83]= $(1.9907 - 2.20654\times 10^{-6}\, i)\ \text{UA22}$

In[84]:= `ConjugateTranspose[{{UA11, UA12}, {UA21, UA22}}].{{UA11, UA12}, {UA21, UA22}}`

Out[84]= $\{\{(4.96288 + 0.\,i)\ \text{UA22 Conjugate[UA22]},$
$(-4.25193\times10^{-12} - 8.30477\times10^{-12}\,i)\ \text{UA22 Conjugate[UA22]}\},$
$\{(-4.25193\times10^{-12} + 8.30477\times10^{-12}\,i)\ \text{UA22 Conjugate[UA22]},$
$(4.96288 + 0.\,i)\ \text{UA22 Conjugate[UA22]}\}\}$

In[85]:= `UA22 = (1/((4.962879976057498` + 0.` i)^(1/2))) * Exp[I * ψ]`

Out[85]= $(0.448883 + 0.\,i)\ e^{i\psi}$

In[86]:= `ConjugateTranspose[{{UB11, UB12}, {UB21, UB22}}].{{UB11, UB12}, {UB21, UB22}}`

Out[86]= $\{\{(7.79399\times10^{7} + 0.\,i)\ \text{UB12 Conjugate[UB12]},$
$(4.61927\times10^{-9} + 5.11209\times10^{-8}\,i)\ \text{UB12 Conjugate[UB12]}\},$
$\{(4.61927\times10^{-9} - 5.11209\times10^{-8}\,i)\ \text{UB12 Conjugate[UB12]},$
$(7.79399\times10^{7} + 0.\,i)\ \text{UB12 Conjugate[UB12]}\}\}$

In[87]:= `UB12 = (1/((7.793989515159951`*^7 + 0.` i)^(1/2))) * Exp[I * ϕ]`

Out[87]= $(0.000113271 + 0.\,i)\ e^{i\phi}$

In[88]:= `ϕ = 0`

Out[88]= 0

In[89]:= `Search`

Out[89]= $\{\{(-0.000941389 - 0.448882\,i)\ e^{i\psi},\ (0.0000508456 - 9.41364\times10^{-11}\,i)\ e^{i\psi},$
$(-0.00187336 - 0.893589\,i)\ e^{i\psi},\ (0.000101218 - 1.12193\times10^{-10}\,i)\ e^{i\psi}\},$
$\{(-0.0000508451 + 2.13163\times10^{-7}\,i)\ e^{i\psi},\ (0.000941326 + 0.448882\,i)\ e^{i\psi},$
$(-0.000101217 + 4.24268\times10^{-7}\,i)\ e^{i\psi},\ (0.00187323 + 0.893589\,i)\ e^{i\psi}\},$
$\{(0.00187303 + 0.893589\,i)\ e^{i\psi},\ (-0.000101218 + 7.52047\times10^{-11}\,i)\ e^{i\psi},$
$(-0.000940558 - 0.448882\,i)\ e^{i\psi},\ (0.0000508456 + 0.\,i)\ e^{i\psi}\},$
$\{(0.000101217 - 4.24231\times10^{-7}\,i)\ e^{i\psi},\ (-0.00187291 - 0.893589\,i)\ e^{i\psi},$
$(-0.0000508451 + 2.13069\times10^{-7}\,i)\ e^{i\psi},\ (0.000940495 + 0.448882\,i)\ e^{i\psi}\}\}$

In[90]:= `ψ = Log[WeHave[[1]][[1]] / (-0.0009413894710387319` - 0.448881964191114` i)] / I`

Out[90]= $-3.1395 - 2.11031\times10^{-12}\,i$

In[91]:= `Max[Abs[Search - WeHave]]`

Out[91]= 6.99146×10^{-12}

In[92]:= `UAMatr`

Out[92]= $\{\{-0.448882 - 0.000940111\,i,\ -0.893589 - 0.00187214\,i\},$
$\{0.893589 + 0.00187247\,i,\ -0.448882 - 0.000940942\,i\}\}$

In[93]:= `UBMatr`

Out[93]= $\{\{-0.00209533 - 0.999998\,i,\ 0.000113271 + 0.\,i\},$
$\{-0.00011327 + 4.74665\times10^{-7}\,i,\ 0.00209519 + 0.999998\,i\}\}$

In[94]:=
```
Max[Abs[Search - KroneckerProduct[UAMatr, UBMatr]]]
```
Out[94]= 0.

In[95]:=
```
Max[Abs[Search - M.ξ.ConjugateTranspose[M]]]
```
Out[95]= 6.99146×10^{-12}

In[96]:=
```
Max[Abs[USecondStage - M.DiagonalPart.ConjugateTranspose[M].M.ξ.ConjugateTranspose[M]]]
```
Out[96]= 0.000188548

In[97]:=
```
Max[Abs[
  USecondStage - M.DiagonalPart.ConjugateTranspose[M].KroneckerProduct[UAMatr, UBMatr]]]
```
Out[97]= 0.000188548

In[98]:=
```
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
ϕ1 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] + KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ2 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] - KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ3 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] - KroneckerProduct[{{0, 1}}, {{1, 0}}])
ϕ4 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] + KroneckerProduct[{{0, 1}}, {{1, 0}}])
DMatrix =
 {{Exp[I * Φ0], 0, 0, 0}, {0, Exp[I * Φ1], 0, 0}, {0, 0, Exp[I * Φ2], 0}, {0, 0, 0, Exp[I * Φ3]}}
FullSimplify[M.DMatrix.ConjugateTranspose[M] -
  Exp[I * θ0] * MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] +
       θ2 * KroneckerProduct[σy, σy] + θ3 * KroneckerProduct[σz, σz])]]
```
Out[98]= {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}

Out[99]= $\frac{\Phi0}{4} + \frac{\Phi1}{4} + \frac{\Phi2}{4} + \frac{\Phi3}{4}$

Out[100]= $\frac{\Phi0}{4} + \frac{\Phi1}{4} - \frac{\Phi2}{4} - \frac{\Phi3}{4}$

Out[101]= $-\frac{\Phi0}{4} + \frac{\Phi1}{4} - \frac{\Phi2}{4} + \frac{\Phi3}{4}$

Out[102]= $\frac{\Phi0}{4} - \frac{\Phi1}{4} - \frac{\Phi2}{4} + \frac{\Phi3}{4}$

Out[103]= $\{\{0, 1\}, \{1, 0\}\}$

Out[104]= $\{\{0, -i\}, \{i, 0\}\}$

Out[105]= $\{\{1, 0\}, \{0, -1\}\}$

Out[106]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[107]= $\left\{\left\{-\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[108]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[109]= $\left\{\left\{0, -\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right\}\right\}$

Out[110]= $\left\{\left\{e^{i\,\Phi 0}, 0, 0, 0\right\}, \left\{0, e^{i\,\Phi 1}, 0, 0\right\}, \left\{0, 0, e^{i\,\Phi 2}, 0\right\}, \left\{0, 0, 0, e^{i\,\Phi 3}\right\}\right\}$

Out[111]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

In[112]:=
```
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
MatrixForm[CNOT1]
MatrixForm[CNOT2]
Ry[θ_] := {{Cos[θ/2], Sin[θ/2]}, {-Sin[θ/2], Cos[θ/2]}}
Rz[α_] := {{e^(ⅈ α/2), 0}, {0, e^(-ⅈ α/2)}}
Unit2 = {{1, 0}, {0, 1}}
σx = {{0, 1}, {1, 0}}
σy = {{0, -I}, {I, 0}}
σz = {{1, 0}, {0, -1}}
ϕ1 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] + KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ2 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{1, 0}}] - KroneckerProduct[{{0, 1}}, {{0, 1}}])
ϕ3 = 1/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] - KroneckerProduct[{{0, 1}}, {{1, 0}}])
ϕ4 = -I/Sqrt[2] * (KroneckerProduct[{{1, 0}}, {{0, 1}}] + KroneckerProduct[{{0, 1}}, {{1, 0}}])
Λ = {{1, 1, -1, 1}, {1, 1, 1, -1}, {1, -1, -1, -1}, {1, -1, 1, 1}}
θ0 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[1]]
θ1 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[2]]
θ2 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[3]]
θ3 = (Inverse[Λ].{Φ0, Φ1, Φ2, Φ3})[[4]]
CNOT1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
CNOT2 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}
H = 1/Sqrt[2] * {{1, 1}, {1, -1}}
CNOT2Trial = KroneckerProduct[H, H].CNOT1.KroneckerProduct[H, H]
CNOT2Trial - CNOT2
U3[θ_, ϕ_, λ_] :=
  {{Cos[θ/2], -Exp[I * λ] * Sin[θ/2]}, {Exp[I * ϕ] * Sin[θ/2], Exp[I * (ϕ + λ)] * Cos[θ/2]}}
FullSimplify[Exp[I * π / 4] * KroneckerProduct[Rz[-π/2], Unit2].CNOT2.
    KroneckerProduct[Unit2, Ry[2 * θ2 - π/2]].CNOT1.KroneckerProduct[
     Rz[2 * θ3 - π/2], Ry[π/2 - 2 * θ1]].CNOT2.KroneckerProduct[Unit2, Rz[π/2]] -
   MatrixExp[I * (θ1 * KroneckerProduct[σx, σx] + θ2 * KroneckerProduct[σy, σy] +
       θ3 * KroneckerProduct[σz, σz])]]
```

Out[112]= {{0, 1}, {1, 0}}

Out[113]= {{0, -ⅈ}, {ⅈ, 0}}

Out[114]= {{1, 0}, {0, -1}}

Out[115]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}

Out[116]= {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}}

Out[117]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Out[118]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Out[121]= $\{\{1, 0\}, \{0, 1\}\}$

Out[122]= $\{\{0, 1\}, \{1, 0\}\}$

Out[123]= $\{\{0, -i\}, \{i, 0\}\}$

Out[124]= $\{\{1, 0\}, \{0, -1\}\}$

Out[125]= $\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[126]= $\left\{\left\{-\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[127]= $\left\{\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$

Out[128]= $\left\{\left\{0, -\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right\}\right\}$

Out[129]= $\{\{1, 1, -1, 1\}, \{1, 1, 1, -1\}, \{1, -1, -1, -1\}, \{1, -1, 1, 1\}\}$

Out[130]= $\frac{\Phi 0}{4} + \frac{\Phi 1}{4} + \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[131]= $\frac{\Phi 0}{4} + \frac{\Phi 1}{4} - \frac{\Phi 2}{4} - \frac{\Phi 3}{4}$

Out[132]= $-\frac{\Phi 0}{4} + \frac{\Phi 1}{4} - \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[133]= $\frac{\Phi 0}{4} - \frac{\Phi 1}{4} - \frac{\Phi 2}{4} + \frac{\Phi 3}{4}$

Out[134]= $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}$

Out[135]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[136]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[137]= $\{\{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}\}$

Out[138]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[140]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

```
In[141]:= Φ0 = Log[DiagonalPart[[1]][[1]]] / I
          Φ1 = Log[DiagonalPart[[2]][[2]]] / I
          Φ2 = Log[DiagonalPart[[3]][[3]]] / I
          Φ3 = Log[DiagonalPart[[4]][[4]]] / I

Out[141]= 0.865252 - 0.0000608343 i

Out[142]= -0.865253 - 0.0000493725 i

Out[143]= 0.865253 + 0.0000488544 i

Out[144]= -0.865254 + 0.0000603196 i

In[145]:= Max[Abs[USecondStage - Exp[I * θ0] * Exp[I * π / 4] *
            KroneckerProduct[Rz[-π / 2], Unit2].CNOT2.KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
             CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].CNOT2.
             KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[UAMatr, UBMatr]]]

Out[145]= 0.000188548

In[146]:= Max[Abs[USecondStage -
           Exp[I * θ0] * Exp[I * π / 4] * KroneckerProduct[Rz[-π / 2], Unit2].KroneckerProduct[H, H].
             CNOT1.KroneckerProduct[H, H].KroneckerProduct[Unit2, Ry[2 * θ2 - π / 2]].
             CNOT1.KroneckerProduct[Rz[2 * θ3 - π / 2], Ry[π / 2 - 2 * θ1]].
             KroneckerProduct[H, H].CNOT1.KroneckerProduct[H, H].
             KroneckerProduct[Unit2, Rz[π / 2]].KroneckerProduct[UAMatr, UBMatr]]]

Out[146]= 0.000188548

In[147]:= Max[Abs[USecondStage - Exp[I * (θ0 + π / 4)] * KroneckerProduct[Rz[-π / 2].H, Unit2.H].
             CNOT1.KroneckerProduct[H.Unit2, H.Ry[2 * θ2 - π / 2]].CNOT1.
             KroneckerProduct[Rz[2 * θ3 - π / 2].H, Ry[π / 2 - 2 * θ1].H].CNOT1.
             KroneckerProduct[H.Unit2.UAMatr, H.Rz[π / 2].UBMatr]]]

Out[147]= 0.000188548
```

In[148]:=
```
U11 = Rz[-π/2].H
U12 = Unit2.H
U21 = H.Unit2
U22 = H.Ry[2*θ2 - π/2]
U31 = Rz[2*θ3 - π/2].H
U32 = Ry[π/2 - 2*θ1].H
U41 = H.Unit2.UAMatr
U42 = H.Rz[π/2].UBMatr
```

Out[148]= $\left\{\left\{\frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}, \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}\right\}, \left\{\frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}, -\frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}\right\}\right\}$

Out[149]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[150]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[151]= $\{\{0.648448 + 4.36335\times10^{-6}\, i, -0.761259 + 3.71674\times10^{-6}\, i\},$
$\{-0.761259 + 3.71674\times10^{-6}\, i, -0.648448 - 4.36335\times10^{-6}\, i\}\}$

Out[152]= $\{\{0.5 - 0.5\, i, 0.5 - 0.5\, i\}, \{0.5 + 0.5\, i, -0.5 - 0.5\, i\}\}$

Out[153]= $\{\{1. - 1.60914\times10^{-13}\, i, -2.93397\times10^{-9} - 0.0000548452\, i\},$
$\{-2.93397\times10^{-9} - 0.0000548452\, i, -1. + 1.60914\times10^{-13}\, i\}\}$

Out[154]= $\{\{0.314455 + 0.000659276\, i, -0.94927 - 0.00198915\, i\},$
$\{-0.94927 - 0.00198879\, i, -0.314455 - 0.000658457\, i\}\}$

Out[155]= $\{\{0.498895 - 0.50099\, i, 0.501103 + 0.499008\, i\}, \{0.499008 - 0.501103\, i, -0.50099 - 0.498895\, i\}\}$

In[156]:=
```
Max[Abs[USecondStage -
    Exp[I * (θ0 + π / 4)] * KroneckerProduct[U11, U12].CNOT1.KroneckerProduct[U21, U22].
      CNOT1.KroneckerProduct[U31, U32].CNOT1.KroneckerProduct[U41, U42]]]
```

Out[156]= 0.000188548

In[157]:=
```
N0 = 7.51
r = 1
s = 1
θ = π/2 - 0.7
ω = Sqrt[s^2 - r^2 Sin[θ]^2]
α = ArcSin[(r / s) * Sin[θ]]
τNew = (π/2)/ω
```

$$N0 = 7.51$$
$$r = 1$$
$$s = 1$$
$$\theta = \pi/2 - 0.7$$
$$\omega = \sqrt{s^2 - r^2 \,\mathrm{Sin}[\theta]^2}$$
$$\alpha = \mathrm{ArcSin}[(r/s) * \mathrm{Sin}[\theta]]$$
$$\tau New = \frac{(\pi/2)}{\omega}$$

Out[157]= 7.51

Out[158]= 1

Out[159]= 1

Out[160]= 0.870796

Out[161]= 0.644218

Out[162]= 0.870796

Out[163]= 2.4383

In[164]:=

$$vFirst = \left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}\right\}$$
$$vSecond = \left\{\left\{\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}\right\}\right\}$$
$$vThird = \left\{\left\{\mathrm{Cos}\left[\frac{\rho}{2}\right], i * \mathrm{Sin}\left[\frac{\rho}{2}\right]\right\}\right\}$$

Out[164]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}\right\}$

Out[165]= $\left\{\left\{\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[166]= $\left\{\left\{\mathrm{Cos}\left[\frac{\rho}{2}\right], i\,\mathrm{Sin}\left[\frac{\rho}{2}\right]\right\}\right\}$

In[167]:=
```
z1 = 2 * ArcTan[(N0 - 1) ^ (1/2)]
Rrot = MatrixExp[-I * z1 * σy/2]
Ancilla = N[Rrot.Transpose[{{1, 0}}]]
```

Out[167]= 2.39453

Out[168]= {{0.364905 + 0. i, -0.931045 + 0. i}, {0.931045 + 0. i, 0.364905 + 0. i}}

Out[169]= {{0.364905 + 0. i}, {0.931045 + 0. i}}

In[170]:=
```
Evolution = {{Cos[τ - α], -i * Sin[τ]}, {-i * Sin[τ], Cos[α + τ]}} * Sec[α]
```

Out[170]= {{1.55227 Cos[0.870796 - τ], (0. - 1.55227 i) Sin[τ]},
{(0. - 1.55227 i) Sin[τ], 1.55227 Cos[0.870796 + τ]}}

In[172]:= $\tau = \pi/2$

Out[172]= $\frac{\pi}{2}$

In[173]:= `Evolution.Transpose[vFirst]`

Out[173]= `{{1.93713 + 0. i}, {0. - 1.93713 i}}`

In[175]:= `(Evolution.Transpose[vFirst])[[1]][[1]]`

Out[175]= `1.93713 + 0. i`

In[176]:= `(Evolution.Transpose[vFirst])[[2]][[1]]`

Out[176]= `0. - 1.93713 i`

In[174]:=
```
NormalizationOutput = (FullSimplify[Abs[(Evolution.Transpose[vFirst])[[1]][[1]]]^2 +
      Abs[(Evolution.Transpose[vFirst])[[2]][[1]]]^2])^(1/2)
```

Out[174]= `2.73951`

In[177]:= `FullSimplify[Evolution.Transpose[vFirst] / NormalizationOutput]`

Out[177]= `{{0.707107 + 0. i}, {0. - 0.707107 i}}`

In[178]:= FinalRotation = $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}, \left\{\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

Out[178]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right\}, \left\{\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[179]:= `FullSimplify[FinalRotation.Evolution.Transpose[vFirst] / NormalizationOutput]`

Out[179]= $\{\{1. + 0.\, i\}, \{0. + 3.03947 \times 10^{-17}\, i\}\}$

In[180]:= `FullSimplify[FinalRotation.Evolution.Transpose[vSecond]]`

Out[180]= `{{0. + 0. i}, {0. - 0.365028 i}}`

In[181]:= `U12P = FinalRotation.U12`

Out[181]= $\left\{\left\{\frac{1}{2} + \frac{i}{2}, \frac{1}{2} - \frac{i}{2}\right\}, \left\{\frac{1}{2} + \frac{i}{2}, -\frac{1}{2} + \frac{i}{2}\right\}\right\}$

In[182]:=
```
(*U11*)
U11OverallPhase = Arg[U11[[1]][[1]]]
U11Dephased = U11 * Exp[-I * U11OverallPhase]
θU11 = 2 * ArcTan[Abs[U11Dephased[[1]][[2]]] / Abs[U11Dephased[[1]][[1]]]]
ϕU11 = Arg[U11Dephased[[2]][[1]]]
λU11 = Arg[U11Dephased[[2]][[2]]] - ϕU11
Max[N[Abs[Exp[I * U11OverallPhase] * U3[θU11, ϕU11, λU11] - U11]]]
(*U12P*)
U12POverallPhase = Arg[U12P[[1]][[1]]]
U12PDephased = U12P * Exp[-I * U12POverallPhase]
θU12P = 2 * ArcTan[Abs[U12PDephased[[1]][[2]]] / Abs[U12PDephased[[1]][[1]]]]
ϕU12P = Arg[U12PDephased[[2]][[1]]]
λU12P = Arg[U12PDephased[[2]][[2]]] - ϕU12P
Max[N[Abs[Exp[I * U12POverallPhase] * U3[θU12P, ϕU12P, λU12P] - U12P]]]
(*U21*)
U21OverallPhase = Arg[U21[[1]][[1]]]
U21Dephased = U21 * Exp[-I * U21OverallPhase]
θU21 = 2 * ArcTan[Abs[U21Dephased[[1]][[2]]] / Abs[U21Dephased[[1]][[1]]]]
ϕU21 = Arg[U21Dephased[[2]][[1]]]
λU21 = Arg[U21Dephased[[2]][[2]]] - ϕU21
Max[N[Abs[Exp[I * U21OverallPhase] * U3[θU21, ϕU21, λU21] - U21]]]
(*U22*)
U22OverallPhase = Arg[U22[[1]][[1]]]
U22Dephased = U22 * Exp[-I * U22OverallPhase]
θU22 = 2 * ArcTan[Abs[U22Dephased[[1]][[2]]] / Abs[U22Dephased[[1]][[1]]]]
ϕU22 = Arg[U22Dephased[[2]][[1]]]
λU22 = Arg[U22Dephased[[2]][[2]]] - ϕU22
Max[N[Abs[Exp[I * U22OverallPhase] * U3[θU22, ϕU22, λU22] - U22]]]
(*U31*)
U31OverallPhase = Arg[U31[[1]][[1]]]
U31Dephased = U31 * Exp[-I * U31OverallPhase]
θU31 = 2 * ArcTan[Abs[U31Dephased[[1]][[2]]] / Abs[U31Dephased[[1]][[1]]]]
ϕU31 = Arg[U31Dephased[[2]][[1]]]
λU31 = Arg[U31Dephased[[2]][[2]]] - ϕU31
Max[N[Abs[Exp[I * U31OverallPhase] * U3[θU31, ϕU31, λU31] - U31]]]
(*U32*)
U32OverallPhase = Arg[U32[[1]][[1]]]
U32Dephased = U32 * Exp[-I * U32OverallPhase]
θU32 = 2 * ArcTan[Abs[U32Dephased[[1]][[2]]] / Abs[U32Dephased[[1]][[1]]]]
ϕU32 = Arg[U32Dephased[[2]][[1]]]
λU32 = Arg[U32Dephased[[2]][[2]]] - ϕU32
Max[N[Abs[Exp[I * U32OverallPhase] * U3[θU32, ϕU32, λU32] - U32]]]
(*U41R*)
U41OverallPhaseR = Arg[(U41.Rrot)[[1]][[1]]]
U41DephasedR = (U41.Rrot) * Exp[-I * U41OverallPhaseR]
θU41R = 2 * ArcTan[Abs[U41DephasedR[[1]][[2]]] / Abs[U41DephasedR[[1]][[1]]]]
ϕU41R = Arg[U41DephasedR[[2]][[1]]]
λU41R = Arg[U41DephasedR[[2]][[2]]] - ϕU41R
Max[N[Abs[Exp[I * U41OverallPhaseR] * U3[θU41R, ϕU41R, λU41R] - U41.Rrot]]]
```

Out[182]= $-\frac{\pi}{4}$

Out[183]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right\}\right\}$

Out[184]= $\frac{\pi}{2}$

Out[185]= $\frac{\pi}{2}$

Out[186]= $-\pi$

Out[187]= 0.

Out[188]= $\frac{\pi}{4}$

Out[189]= $\left\{\left\{\left(\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}, \left(\frac{1}{2}-\frac{i}{2}\right) e^{-\frac{i\pi}{4}}\right\}, \left\{\left(\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}, \left(-\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{i\pi}{4}}\right\}\right\}$

Out[190]= $\frac{\pi}{2}$

Out[191]= 0

Out[192]= $\frac{\pi}{2}$

Out[193]= 0.

Out[194]= 0

Out[195]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$

Out[196]= $\frac{\pi}{2}$

Out[197]= 0

Out[198]= π

Out[199]= 0.

Out[200]= 6.72892×10^{-6}

Out[201]= $\{\{0.648448 + 0.\, i, -0.761259 + 8.83919 \times 10^{-6}\, i\}, \{-0.761259 + 8.83919 \times 10^{-6}\, i, -0.648448 + 0.\, i\}\}$

Out[202]= 1.73051

Out[203]= 3.14158

Out[204]= 0.0000116113

Out[205]= 0.0000176784

Out[206]= −0.785398

Out[207]= $\{\{0.707107 + 5.55112 \times 10^{-17}\,i, 0.707107 + 5.55112 \times 10^{-17}\,i\}, \{-3.76075 \times 10^{-7} + 0.707107\,i, 3.76075 \times 10^{-7} - 0.707107\,i\}\}$

Out[208]= 1.5708

Out[209]= 1.5708

Out[210]= -3.14159

Out[211]= 5.91503×10^{-10}

Out[212]= -1.60914×10^{-13}

Out[213]= $\{\{1. + 0.\,i, -2.93397 \times 10^{-9} - 0.0000548452\,i\}, \{-2.93397 \times 10^{-9} - 0.0000548452\,i, -1. + 0.\,i\}\}$

Out[214]= 0.00010969

Out[215]= -1.57085

Out[216]= 4.71244

Out[217]= 0.00010969

Out[218]= -3.1395

Out[219]= $\{\{0.769068 - 4.14165 \times 10^{-17}\,i, 0.639167 + 4.32243 \times 10^{-7}\,i\}, \{0.639167 - 4.60051 \times 10^{-7}\,i, -0.769068 + 3.34576 \times 10^{-8}\,i\}\}$

Out[220]= 1.38683

Out[221]= -7.19767×10^{-7}

Out[222]= 3.14159

Out[223]= 2.54153×10^{-12}

In[ ]:=
```
(*Constant input*)
{θU11, ϕU11, λU11}
```

Out[ ]= $\{\frac{\pi}{2}, \frac{\pi}{2}, -\pi\}$

In[ ]:=
```
{θU12P, ϕU12P, λU12P}
```

Out[ ]= $\{\frac{\pi}{2}, 0, \frac{\pi}{2}\}$

In[ ]:=
```
{θU21, ϕU21, λU21}
```

Out[ ]= $\{\frac{\pi}{2}, 0, \pi\}$

In[ ]:=
```
{θU22, ϕU22, λU22}
```

Out[ ]= {1.73051, 3.14158, 0.0000116113}

In[ ]:=
```
{θU31, ϕU31, λU31}
```

Out[ ]= {1.5708, 1.5708, -3.14159}

In[]:= {θU32, ϕU32, λU32}

Out[]= {0.00010969, -1.57085, 4.71244}

In[]:= {θU41R, ϕU41R, λU41R}

Out[]= {1.38683, -7.19767×10^{-7}, 3.14159}

In[]:=
```
(*Varying input*)
(*U42*)
ρ = -(π/2) * 1.0
Initializer = {{Cos[ρ/2], ⅈ Sin[ρ/2]}, {ⅈ Sin[ρ/2], Cos[ρ/2]}}
U42POverallPhase = Arg[(U42.Initializer)[[1]][[1]]]
U42PDephased = (U42.Initializer) * Exp[-I * U42POverallPhase]
θU42P = 2 * ArcTan[Abs[U42PDephased[[1]][[2]]] / Abs[U42PDephased[[1]][[1]]]]
ϕU42P = Arg[U42PDephased[[2]][[1]]]
λU42P = Arg[U42PDephased[[2]][[2]]] - ϕU42P
Max[N[Abs[Exp[I * U42POverallPhase] * U3[θU42P, ϕU42P, λU42P] - (U42.Initializer)]]]
{θU42P, ϕU42P, λU42P}
```

Out[]= -1.5708

Out[]= {{0.707107, 0. - 0.707107 ⅈ}, {0. - 0.707107 ⅈ, 0.707107}}

Out[]= -0.787493

Out[]= {{1. + 5.55112×10^{-17} ⅈ, -7.10134×10^{-8} + 0.000113271 ⅈ},
{0.000113271 - 7.10672×10^{-8} ⅈ, -4.74666×10^{-7} - 1. ⅈ}}

Out[]= 0.000226542

Out[]= -0.000627408

Out[]= -1.57017

Out[]= 3.1471×10^{-12}

Out[]= {0.000226542, -0.000627408, -1.57017}

In[]:=
```
(* ρ = (π/2)*1.0 *)
{3.141366111337455`, 3.1409652456479584`, -4.713015913644101`}

(* ρ = (π/2) * 0.8 *)
{2.8276599304386`, -3.7231051927538735`*^-8, -1.5707958668656175`}

(* ρ = (π/2) * 0.6 *)
{2.513500665079638`, -2.7910560774262403`*^-7, -1.5707960850889877`}

(* ρ = (π/2) * 0.4 *)
{2.1993413997206654`, -3.7144425008725`*^-7, -1.5707961512111313`}

(* ρ = (π/2) * 0.2 *)
{1.8851821343616901`, -4.2849907489393627`*^-7, -1.5707961774480153`}
```

```
(* ρ = (π/2) * 0.0 *)
{1.5710228690027142`, -4.746498004648732`*^-7, -1.5707961847680267`}
(* ρ = - (π/2) * 0.2 *)
{1.2568636036437382`, -5.207937323972115`*^-7, -1.5707961774699997`}
(* ρ = - (π/2) * 0.4 *)
{0.942704338284763`, -5.778213828322615`*^-7, -1.5707961512689215`}
(* ρ = - (π/2) * 0.6 *)
{0.62854507292579`, -6.700720866873013`*^-7, -1.5707960852396725`}
(* ρ = - (π/2) * 0.8 *)
{0.3143858075668278`, -9.114590160013453`*^-7, -1.5707958675065188`}
(* ρ = - (π/2) * 1.0 *)
{0.00022654225233811174`, -0.0006274079252995832`, -1.570169393535485`}
```

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In[226]:=

```
MatrixForm[Round[{{UA11, UA12}, {UA21, UA22}}, 0.00001]]
```

Out[226]//MatrixForm=

$$\begin{pmatrix} -0.44888 - 0.00094\,i & -0.89359 - 0.00187\,i \\ 0.89359 + 0.00187\,i & -0.44888 - 0.00094\,i \end{pmatrix}$$

In[ ]:=

In[ ]:=

In[227]:=

```
MatrixForm[Round[{{UB11, UB12}, {UB21, UB22}}, 0.00001]]
```

Out[227]//MatrixForm=

$$\begin{pmatrix} -0.0021 - 1.\,i & 0.00011 \\ -0.00011 & 0.0021 + 1.\,i \end{pmatrix}$$