Tradition of Mathematical Astronomy in India

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Astronomy in the *Vedas*

- One finds a list of all the 27 stars in the Vedas: अश्विनि भरणि etc.
- There is also mention of Sun, Moon and to a lesser extent planets.
- Similarly, the names of the 12 Months and *ṛtu* s mentioned : मधु माधव (वसन्त) etc.
- Concept of *Adhikamāsa* already there. Mention of 2 more months than the 12. (*samsarpa*, *amhaspati*.)
- There are references to Eclipses too.

Vedanga Jyotisa **1200 BC**

- Calendar: **a short Yuga of 5 years** consists of
  - 1830 days $\Longrightarrow$ A solar year has 366 days (Little more than actual)
  - 62 Lunar months $\Longrightarrow$ Average lunar month has nearly 29.48 days (Little less than actual)
- Length of the day and variation etc. also discussed. Nothing on planetary motion
- Long gap before *Aryabhatīya* in 499 AD. Tradition : 18 *Siddhānta*-s were there before *Aryabhatīya* . 5 of them summarised in *Pañcasiddhāntikā* of Varahamihira (6th Century)

Aryabhatīya of *Aryabhata*

- It is mentioned in the text itself that it was composed 3600 years after the begining of *Kaliyuga*.
- This corresponds to 499 AD.
- Further it is stated that *Aryabhata*was 23 at the time of composition.
- *Aryabhata*has composed one more work – *Aryabhatasiddhanta*.
- *Aryabhatīya* has only 121 stanzas, and has 4 parts –

- ***Gitikapada***
- ***Ganitapada***
- ***Kalakriyapada*** and
- ***Golapada***

Aryabhatīya of *Aryabhata*

- *Gītikāpāda* in 13 stanzas deals with
  - Basic definitions
  - Revolution numbers of planets and Parameters associated with them.
- *Gaṇitapāda* in 33 stanzas deals with mathematical problems such as –
  - Squaring, squareroot, Cubing and cuberoot
  - Areas of plane figures, Volumes, Value of π
  - Methods for finding Sines and constructing sine table
  - Summation of Series, **Kuttaka**

Aryabhatīya of *Aryabhata*

- *Kālakriyāpāda* in 25 stanzas deals with
  - Reckoning of Time
  - Planetary model : Epicycle and Eccentric circle theory
  - Procedure for calculation of planetary positions etc.
- *Golapāda* in 50 stanzas deals with
  - Spherical Astronomy
  - *Bhagola* (Celestial sphere) as seen at different latitudes

- Diurnal problems associated with motion of Sun, Moon and planets on the celestial sphere
- Parallax, Lunar eclipse, Solar eclipse and so on.

Salient features of *Aryabhatīya*

- While discussing revolution numbers of planets in a Yuga of 43,20,000 years, it is mentioned that the number for earth is 1,58,22,37,500.
- Then the sidereal period of rotation of the Earth would be 23 hr, 56 min, 4.1 sec, compared with the modern value 23 hr, 56 min, 4.091 sec.
- Again that Earth rotates through one minute of arc in one *prana* (4 sidereal seconds)
- Among other things we highlight two very important features described in *Aryabhatīya* – **Rotation of the Earth, Value of** Π

Salient features of *Aryabhatīya*

Rotation of the Earth

अनुलोमगतिर्नौस्थः पश्यत्यचलं विलोमगं यद्वत् ।
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ॥

Just as a man in a boat moving forward sees the stationary objects as moving backward, just so are the stationary stars seen by people at Lanka (on the equator), as moving exactly towards the west.

Salient features of *Aryabhatīya*

Value of π

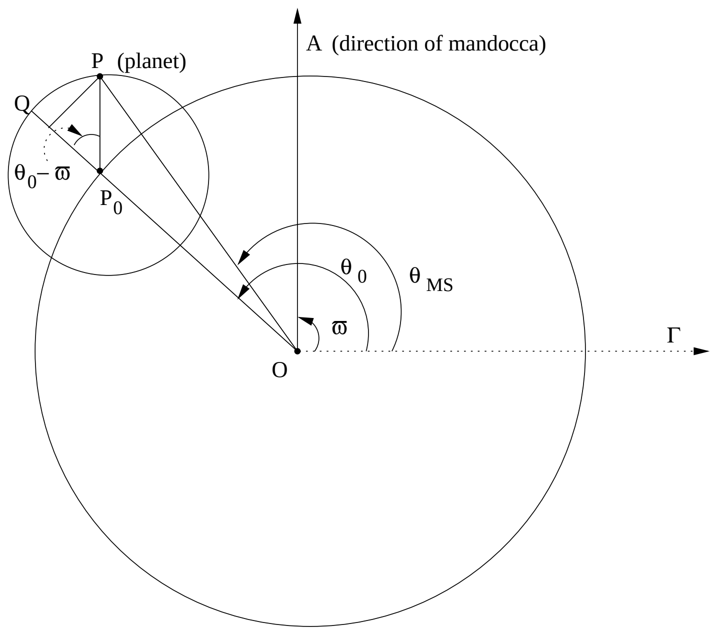
The value of π is described in the following verse –

चतुरधिकं शतमष्टगुणं द्वाशष्टिस्तथा सहस्राणां ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाःअः ॥

The circumference of a circle of diameter 20000 is **nearly** 62832.

The above $\Longrightarrow \pi = \frac{62832}{20000} = 3.1416.$

Use of Sine function in Astronomy



Here P_0 is the "mean planet" and P is the "true planet".

Longitude of mean planet = $\theta_0 = \Gamma\hat{O}P_0$. True planet is on a circle of radius ('epicycle') r with P_0 as centre. Its longitude, $\theta_{MS} = \Gamma\hat{O}P$.

$$\sin(\theta_0 - \theta_{MS}) = \frac{r\sin(\theta_0 - \varpi)}{[(R + r\cos(\theta_0 - \varpi))^2 + r^2\sin^2(\theta_0 - \varpi)]^{\frac{1}{2}}}$$

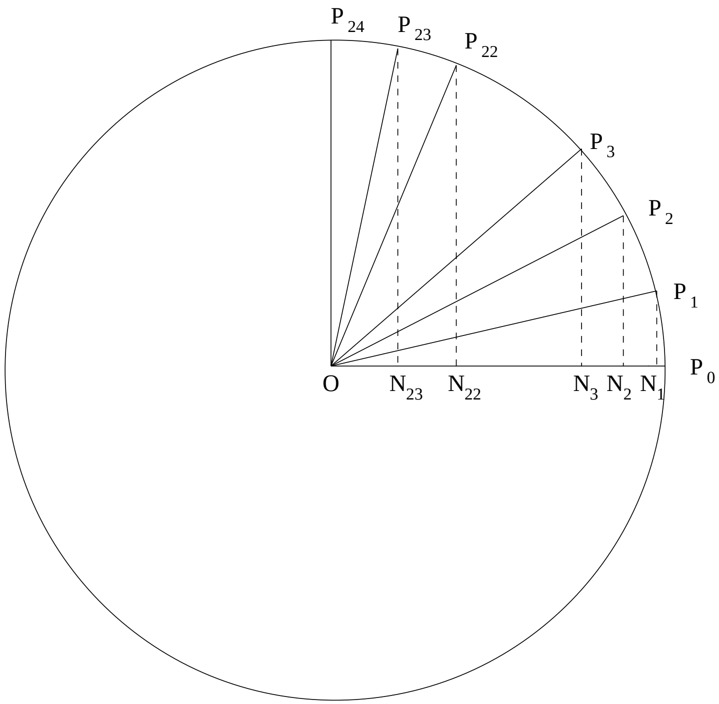
In fact r varies in the Indian models, and it turns out that

$$sin(\theta_{MS} - \theta_0) = -\frac{r_0}{R}\sin(\theta_0 - \varpi).$$

, where r_0 is a constant, or

$$(\theta_{MS} - \theta_0) = -sin^{-1}(\frac{r_0}{R}\sin(\theta_0 - \varpi).)$$

Construction of the Sine-table



Normally a quadrant is divided into 24 equal parts, so that each arc bit $\alpha = \frac{90}{24} = 3^0 45' = 225'$. Then the procedure for finding $R \sin i\alpha, \quad i = 1, 2, \ldots 24$ is explicitly given.R = 3438.The R sines of the intermediate angles are to be determined by interpolation.

Recursive relation for the construction of sine-table

The text *Aryabhatīya* gives the explicit algorithm for constructing the sine-table[a]:

प्रथमाच्चापज्यार्धाद्यैरूनं खण्डितं द्वितीयार्धम् ।
तत्प्रथमज्यार्धांशैस्तैस्तैरूनानि शेषाणि ॥

The first Rsine divided by itself and then diminished by the quotient gives the second Rsine difference. The same first

[a] *Gaṇitapāda)*, verse 12.

Rsine diminished by the quotients obtained by dividing each of the preceding R-sines by the first Rsine gives the remaining Rsine-differences.

Approximation in the Recursive relation

- The content of the verse is equivalent to the relation:

$$R\sin(i+1)\alpha - R\sin i\alpha = R\sin i\alpha - R\sin(i-1)\alpha - \frac{R\sin i\alpha}{R\sin\alpha}.$$

- In fact, the values of the 24 $Rsines$ themselves are explicitly noted in another verse.

- The exact recursion relation for the Rsine differences is:

$$R\sin(i+1)\alpha - R\sin i\alpha = R\sin i\alpha - R\sin(i-1)\alpha - R\sin i\alpha \;\; 2(1-\cos\alpha).$$

- While, $2(1-\cos\alpha) = 0.0042822$

$$\frac{1}{R\sin\alpha} = \frac{1}{225} = 0.00444444$$

Infinite series for $sin\theta$

Infinite series for $sin\theta$, $cos\theta$, and $tan^{-1}\theta$ in Kerala mathematics from Madhava (14th century) onwards. In particular,

$$sin\theta = \theta - \theta^3/3! + \theta^5/5! - \theta^7/7! + -...$$

Upto and including θ^{11} term used by Sankara varier in his *laghuvivṛtti* (around 1530) to construct sine table. Attributes the results to Madhava.

Sine Table according to Indian texts

θ in min.	Aryabhatiya	$R \sin\theta$ according to Govindaswami	$R \sin\theta$ according to Madhava (also Modern)
225	225	224 50 23	224 50 22
450	449	448 42 53	448 42 58
675	671	670 40 11	670 40 16
900	890	889 45 08	889 45 15
1125	1105	1105 01 30	1105 01 39
....	....	....	....
4950	3409	3408 19 42	3408 20 11
5175	3431	3430 22 42	3430 23 11
5400	3438	3437 44 19	3437 44 48

Derivative of $Sin^{-1}\theta$

According to an "Epicycle" model, the true longitude of a planet is

$$\theta = \theta_0 - sin^{-1}(r/RsinM)$$

,

where θ_0 is the mean longitude, θ is the true longitude, R is the radius of the deferent circle , r is the radius of the epicycle, M is the 'anomaly'. Then

$$\frac{d\theta}{dt} = \frac{d\theta_0}{dt} - \frac{rcosM dM/dt}{\sqrt{R^2 - r^2 sin^2 M}}$$

.

This is expressed in Nilakantha's *Tantrasaṅgraha* (1500 AD) thus:

चन्द्रबाहुफलवर्गशोधितत्रिज्याकृतिपदेन संहरेत्।
तत्र कोतिफललिप्तिकाहतां केन्द्रभुक्तिरिह यच्च ल भ्यते॥
तद्विशोध्य . गतेः ..

Let the product of the *koṭiphala* (rcosM) and the daily motion of the *mandakendra* ($\frac{dM}{dt}$) be divided by the square root of the square of the *bāhuphala* of the Moon subtracted from the square of *trijyā* ($\sqrt{R^2 - r^2 sin^2 M}$). The result thus obtained has to be subtracted from the daily motion ..

Evolution after *Aryabhatīya*

- *Aryabhatīya* is extremely cryptic
- Considering the fact that *Aryabhatīya* is very brief, there was need to explain its contents in commentaries.
- Bhaskara I was one of the first to write a commentary on *Aryabhatīya* called *Āryabhaṭīyabhāṣya* He also wrote independent works: *Mahābhāskārīya* (around 640 AD).
- Continuous evolution in mathematical astronomy with innovations, improvements in procedures, parameters, new concepts.

- Brahmagupta (7th century), Manjula (10th century), Bhaskara II (12th century) are some of the important names.
- Kerala emerged as an important centre for mathematics and astronomy during 1350- 1600 CE with important innovations in the planetary model initiated by Parameswara and formulated clearly by Nilakantha Somayaji in his works.

Yuktibhāṣā

Yuktibhāṣā composed by Jyestadeva around 1530 CE contains proofs and demonstrations on mathematics and astronomy. It is well known that this work contains the proofs for infinites series fro π and sine, cosine and inverse tan functions. The astronomy part of this work is not so well known. All the procedures of Indian astronomy involving planetary computations,spherical astronomy, eclipses etc. including the innovations of the Kerala school are explained in great detail here.

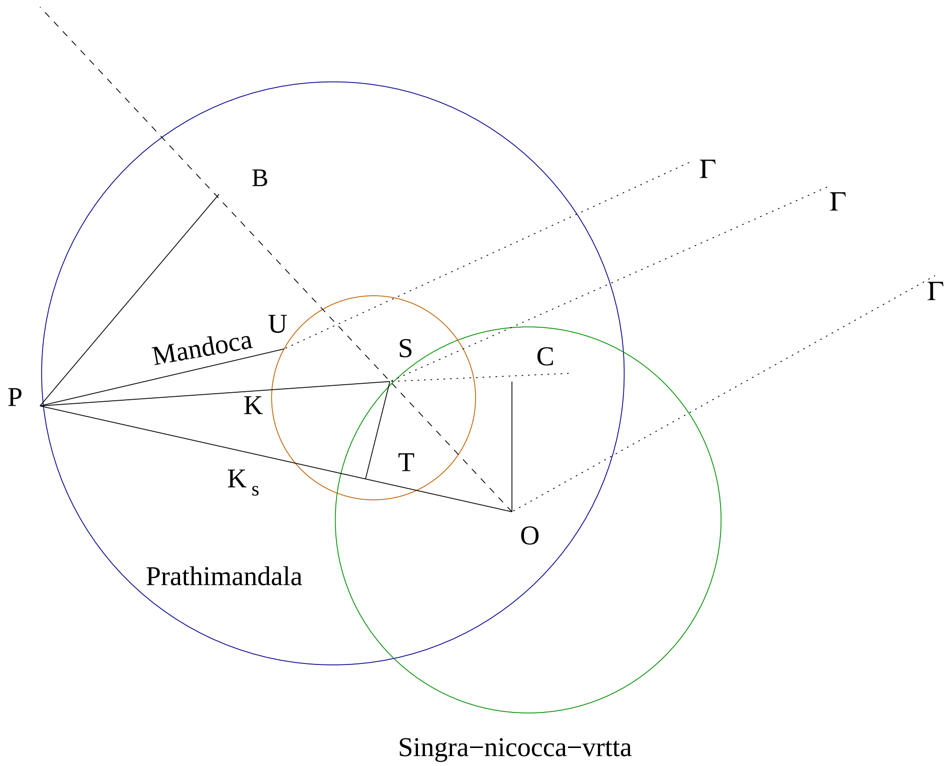
Planetary model

- The model described by $Aryabhata$ "roughly" amounts to – planets orbiting around the Sun in eccentric orbits, with the Sun itself orbiting around the earth. HE DOES NOT STATE IT.
- Significantly, the picture of Latitudes is broadly correct.

Major Innovation by Nilakantha 1500 AD

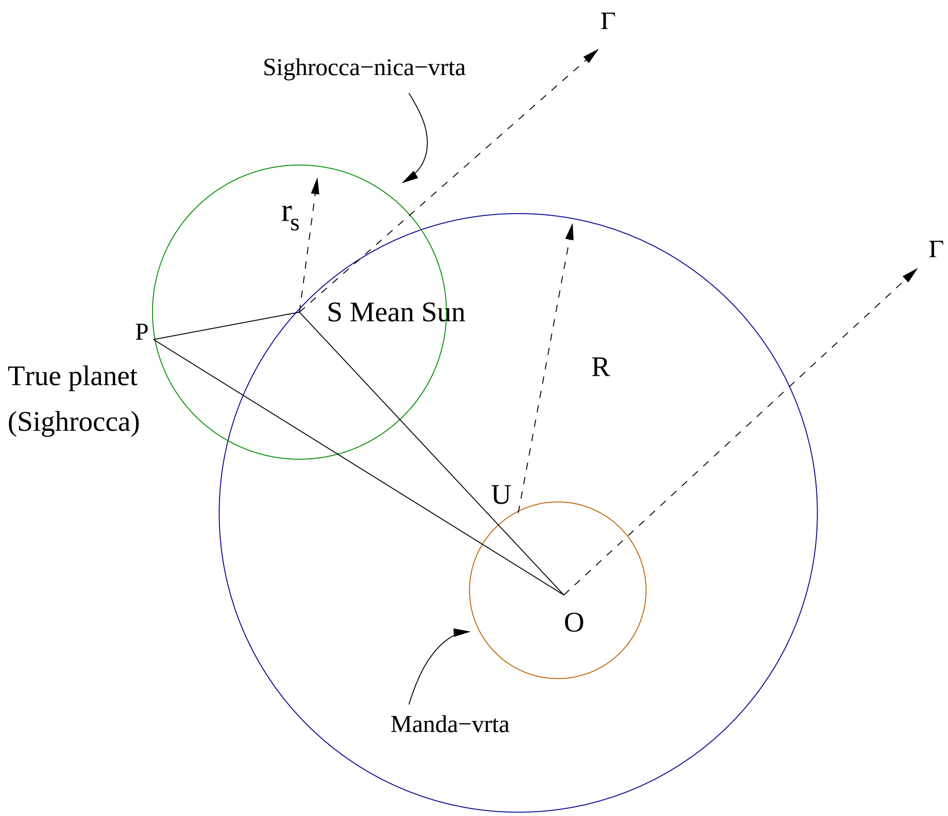
- Nilakantha Somayaji solves the problem of interior planets.Latitudes correct. States the planetary model explicitly:
- All the planets orbit around the Sun in eccentric orbits, which itself orbits around the Earth.
- Model essentially correct from Geocentric frame of refrence.

Model for Exterior planets.



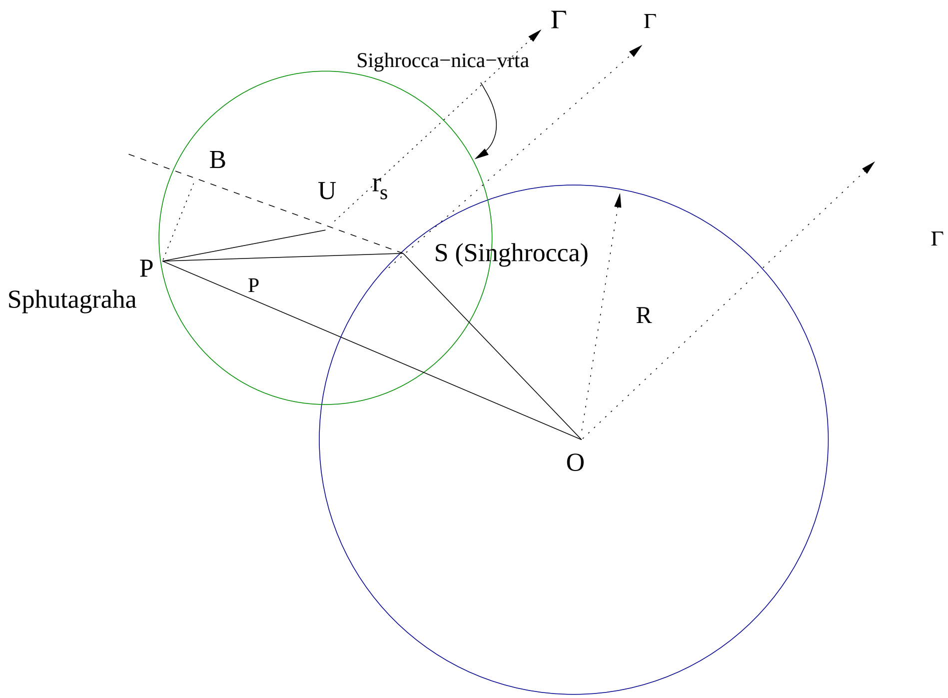
The exterior planet(P) moves in an eccentric orbit around the Sun (S) which itself circles around the Earth(O).

Interior planets before Nilakantha



Conceptually wrong. Equation of centre for interior planets wrongly applied to mean Sun , instead of actual planet.Obtained longitude , not too bad.

Interior planets : Nilakantha's modification



Equation of centre correctly applied to mean heliocentric planet to obtain true heliocentric planet. Then converted to geocentric long.

Copernicus

In the Greko-European tradition this was not achieved even by Copernicus.

In the words of Sverdlow and Neugebauer (historians of science):

"what finally is most notable about Copernicus's treatment of interior planets is that he adapts his models and parameters to Ptolemy's even if it might appear peculiar to do so....."

Or, in the words of Kepler:

"Copernicus, ignorant of his own riches, took it upon himself for the most part to represent Ptolemy, not nature, to which he had nevertheless come the closest of all."

In this tradition, correct picture for the first time by Kepler around 1620 AD.

Concluding Remarks

- Indian astronomy algorthmic in nature.
- Aim is to redduce everything to a few algebraic -trigonometric formulae/tables.
- Different categories of astronomical works :

  Siddhanta s - Theory, *Tantra* s - Less theory,

  Karana s - Even less theory; more tables and short-cut procedures),

  Vākya s - Only tables in verses; this can be used even by people not knowing anything in astronomy and mathematics to calculate positions of planets, eclipses , etc.

- It is because of *Karana*, *Vākya* traditions that *Pañcanga* (Almanac) making was so prevalent in India.
- Evolving tradition, Constant updating of parameters
- Flexibility of approach, so new ideas can be accomodated.
- Steady improvement in mathematics
- Commentaries INDESPENSIBLE for full understanding.