A formalism to specify unambiguous instructions inspired by Mīmāṁsā in computational settings

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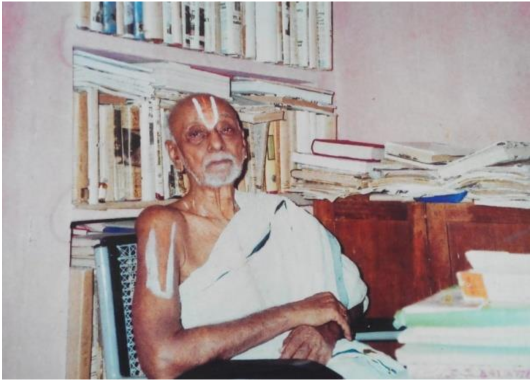
Acknowledgement



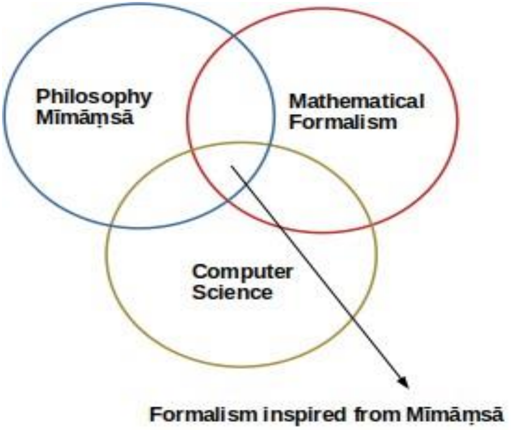
Our teacher - Erudite scholar Vidvān Late Śrī D Gopaladesikan

Mīmāṁsā Sironmani

Outline

Outline

Motivation

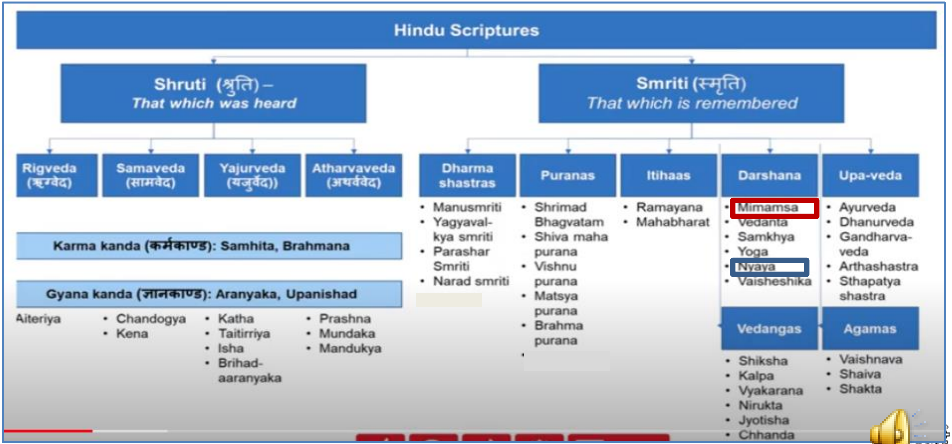


Mīmāṃsā Inspired Representation of Actions (MIRA)

Outline

Overview of Hindu Scriptures

angāni vedās chatvārah, Mīmāṁsā nyāya vistarah |
purānam dharma ṡhāstram cha vidya hyetās chaturdasha | |



Introduction to Mīmāṁsā

- Vedas (*apauruṣeya*) constitute a body of knowledge revealed and passed on through oral traditions

- Mīmāṁsā by Sage Jaimini deals with the interpretation of Vedic injunctions (for the performance of rituals) that signify right action (dharma)
    - Mīmāṁsā - thinking, reflecting back, investigating, interpreting what is said in the Vedas

- Karma Mīmāṁsā / Pūrva Mīmāṁsā - arranged in 12 chapters deals with rituals and actions

The concept of Bhāvanā in Mīmāṁsā

- Bhāvanā - '*Bringing into Being*' - Addresses **motivation** at the psychological level and at the actual activity level

Type of bhāvanā	Objective (What?)	Instrument (How?)	Auxiliaries (Through what means?)
Śābdībhāvanā (Psychological level)	Generating ārthībhāvanā	Knowledge of verbal termination	Arthavāda (Eulogy or Criticism)
Ārthībhāvanā (Actual activity level)	Objective of sacrifice	Actual Performance	Procedure that involves instruction interpretati

A few interpretation tenets in Mīmāṁsā

- Types of injunctions
- Classification of injunctions
- Connection between injunctive and non-injunctive statements
- Determination of the sequence
- Rules denoting mandatory and optional performance of actions

Mīmāṁsā - Vidhi, Niṣedha and Arthavāda

- Injunctive statements (Vidhi): These direct a person to perform actions
  - Example: ‘Tell the truth’
- Prohibitory statements (Niṣedha): These prohibit a person from performing an action
  - Example: ‘Do not harm others’
- Eulogy and Criticism (Arthavāda): These type of statements support injunctions
  - Example: 'Vāyu being a God of speed will help you achieve the results faster (Vayūr Vai Kshepishta Devata..’)

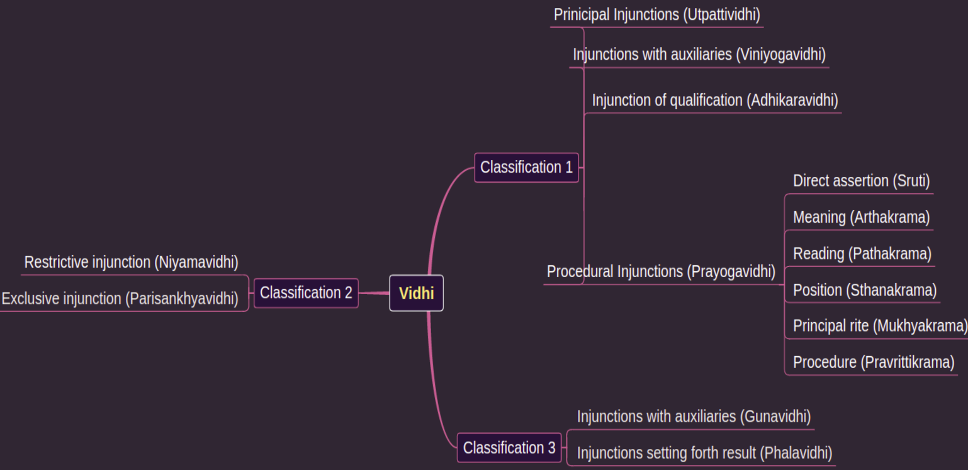
Mīmāṁsā – Arthavāda contd.

- Connection between injunctive and non-injunctive statements (hetu-hetumatbhāva)

- Injunctions with conditions or reason

- Example:
  - "If it is raining, take an umbrella"
    - the two parts have different verbal forms
  - "take an umbrella", is the injunction
  - " If it is raining" expresses ground or reason (hetu) for the injunction

Various Classifications of Vidhi



Restrictive and Exclusive Injunctions

- **Restrictive injunction (Niyamavidhi)** : When two or more methods are possible to reach the goal, making one of these mandatory is a restrictive injunction
- Example: "Pound the corn to remove the husk" - the husk can normally be removed in more than one way.
    - But, this injunction mandates that the husk is to be removed only by the action of pounding
- **Exclusive injunction (Parisaṅkhyāvidhi)**: Exclusion of one from two items that are present simultaneously is an exclusive injunction
- "When two alternatives become simultaneously available, the rule which excludes one of them is the injunction of exclusion"
- Example: "Only five animals with five toes may be eaten", implies that five-toed animals other than the stipulated five must not be eaten

Mandatory and optional rules

- **Obligatory Rule (samuccayah):** If several accessories in the form of instructions are detailed, leading to several subgoals, which thereby lead to a single goal, Mīmāṁsā mandates performing all those
- Example: 'Take pencil to draw and take pen to write to get good grade in examination'

- **Optional Rule (vikalpa):** If several optional instructions are laid out to achieve the same purpose, Mīmāṁsā suggests choosing one among those instructions.
- Example: 'Go by cycle or go by bus to reach the destination'

Basis of the formalism

Type of bhāvanā	Objective (What?)	Instrument (How?)	Auxiliaries (Through What means?)
Ārthībhāvanā (Actual activity level)	Objective of sacrifice	Actual Performance	Procedure that involves instruction interpretation

- **Motivation:** Intention to achieve the goal is identified as the prime requirement of instruction execution
- **Auxiliaries:** Interpretation is carried out according to Mīmāṁsā principles
- **Instruction Execution:** Carried out with three values
  - Satisfaction (S), Violation (V) and no intention to reach the goal (N)

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- $I = (I^v \cup I^n)$, where I^v denotes positive unconditional imperatives (*vidhi*), $\{i_1^+, i_2^+, \cdots, i_n^+\}$, I^n denotes negative unconditional imperatives (*niṣedha*) $\{i_1', i_2', \cdots, i_n'\}$, I includes both I^v and I^n

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- B indicates the binary connectives $\{\wedge, \oplus, \rightarrow_r, \rightarrow_i, \rightarrow_p\}$

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- If $i \in I$, $j \in I$, $p_1 \in P$ and $p_2 \in P$, then $(i \rightarrow_p p_1) \wedge (j \rightarrow_p p_2) \in \mathscr{F}_i$, where $i \neq j$ and $p_1 \neq p_2$.

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- If $\varphi \in \mathscr{F}_i$, and $\psi \in \mathscr{F}_i$, then $\varphi \rightarrow_i \psi \in \mathscr{F}_i$, where $\varphi \neq \psi$.

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- If $\varphi \in \mathscr{F}_i$, and $\psi \in \mathscr{F}_i$, then $\varphi \rightarrow_i \psi \in \mathscr{F}_i$, where $\varphi \neq \psi$.
- If $\tau \in R$ and $\varphi \in \mathscr{F}_i$, then $\tau \rightarrow_r \varphi \in \mathscr{F}_i$.

$$\mathscr{F}_i = i|(i \rightarrow_p p)|(i \rightarrow_p p_1) \wedge (j \rightarrow_p p_2)|(i \rightarrow_p \theta) \oplus (j \rightarrow_p \theta)|(\varphi \rightarrow_i \psi)|(\tau \rightarrow_r \varphi)$$

Deduction rules

Let $i^+, j^+ \in I^+; i', j' \in I^n, i \in I; \tau \in R; \varphi, \psi \in \mathscr{F}_i;$ *and* $\theta \in P$.

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$$\frac{(i^+ \rightarrow_p \theta) \oplus (j^+ \rightarrow_p \theta) \qquad (i^+ \rightarrow_p \theta)}{(j' \rightarrow_p \theta)}\ de\_pv$$

ci - conjunction introduction, *ce* - conjunction elimination, *de_nv* - Niyamavidhi, *de_pv* - *Parisaṅkhyāvidhi*

Deduction rules contd.

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$$\dfrac{\begin{array}{c}[\tau] \\ \vdots \\ \varphi\end{array}}{\tau \rightarrow_r \varphi}$$

Deduction rules contd.

$$\dfrac{\begin{matrix}[\tau] \\ \vdots \\ \varphi\end{matrix}}{\tau \rightarrow_r \varphi} \qquad \dfrac{\begin{matrix}[\varphi] \\ \vdots \\ \psi\end{matrix}}{\varphi \rightarrow_i \psi}$$

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cni - conditional introduction

1. $X = \tau$, $Y = \varphi$ and $w = r$.
2. $X = \varphi$, $Y = \psi$ and $w = i$.
3. $X = \varphi$, $Y = \theta$ and $w = p$.

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cne - conditional elimination

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$$\mathscr{E}(\varphi) \in \{S, V, N\}$$

φ - imperatives

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- Satisfaction (S)
- Violation (V)
- No intention to reach the goal (N)

$$\mathscr{E}(\varphi) \in \{S, V, N\}$$

φ - imperatives

- τ and θ are propositional formulas taking the value of *True* or *False*

$$\mathscr{E}(\tau) \in \{\top, \bot\}$$

$$\mathscr{E}(\theta) \in \{\top, \bot\}$$

τ - reasons, θ - goal

Evaluation of i^+ and i'

Let $i^+ \in I^v$ and $i' \in I^n$.

Evaluation of i^+ and $i^{'}$

Let $i^+ \in I^v$ and $i' \in I^n$.

If $\mathscr{E}(i^+) = S$, $\mathscr{E}(i') = V$; If $\mathscr{E}(i') = V$, $\mathscr{E}(i^+) = S$.

Evaluation of i^{+} and $i^{'}$

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If $\mathscr{E}(i^{+}) = S$, $\mathscr{E}(i') = V$; If $\mathscr{E}(i') = V$, $\mathscr{E}(i^{+}) = S$.

- The positive and negative imperatives are treated separately since the negation of “Doing A” is not “not doing A”.

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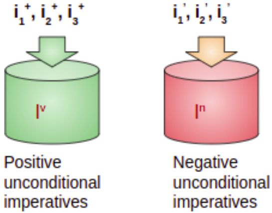
- The positive and negative imperatives are treated separately since the negation of “Doing A” is not “not doing A”.
- Can be imagined as two separate containers for positive and negative imperatives without any connection



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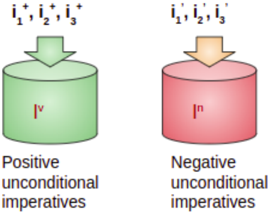
- The positive and negative imperatives are treated separately since the negation of "Doing A" is not "not doing A".
- Can be imagined as two separate containers for positive and negative imperatives without any connection



- Inspired from *vidhi* and *niṣedha*, respectively from Mīmāṁsā.

Imperative enjoining goal

φ	θ	$\varphi \rightarrow_p \theta$
S	$\top$	S
S	$\perp$	N
V	$\top$	V
V	$\perp$	N
N	$\top$	N
N	$\perp$	N

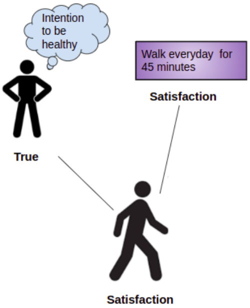
φ - *Walk everyday for 45 minutes*

θ - *To stay healthy*

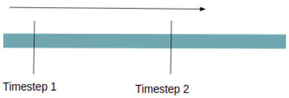
$\top$ - *True*, $\perp$ - *False*

S - Satisfaction, V - Violation

N - Disinterest/ No intention in reaching the goal



Imperative enjoining sequence of actions



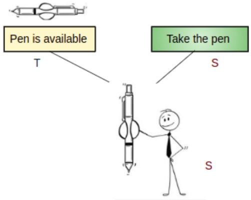
φ^1	ψ^2	$\varphi \rightarrow_i \psi$
S	S	S
S	V	V
V	S	V
V	V	V
N	S/V	N
S/V	N	N

φ - *Take the paper* ψ - *Then write*

Assumption: At timestep1, φ holds one of the values S, V, N

Imperative enjoining reason

τ	φ	$\tau \rightarrow_r \varphi$
$\top$	S	S
$\top$	V	V
$\top$	N	N
$\bot$	S	N
$\bot$	V	N
$\bot$	N	N

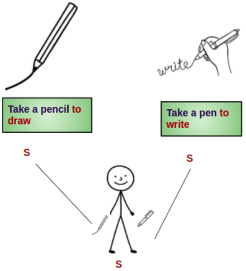


φ- imperative formula, τ - reason

Conjunction of imperatives

$i_1 \to_p p_1$	$i_2 \to_p p_2$	$(i_1 \to_p p_1) \wedge (i_2 \to_p p_2)$
S	S	S
S	V	V
S	N	N
V	S	V
V	V	V
V	N	N
N	S	N
N	V	N
N	N	N

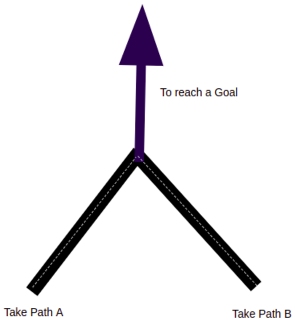
i_1, i_2 - unconditional imperatives, p_1, p_2 - goal



- Take a pencil to draw - $i_1 \to_p p_1$
- Take a pen to write - $i_2 \to_p p_2$

Disjunction of Imperatives

$i \rightarrow_p p$	$j \rightarrow_p p$	$(i \rightarrow_p p) \oplus (j \rightarrow_p p)$
S	S	V
S	V	S
S	N	N
V	S	S
V	V	V
V	N	N
N	S	N
N	V	N
N	N	N



i, j - unconditional imperatives, p - goal

Outline

Soundness - 1

Derivation of conclusion from premises

The process of deriving the conclusion from the given premises is given by:

$$\varphi_1, \varphi_2, \cdots, \varphi_n \vdash \psi$$

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Semantic entailment

If $\varphi_1, \varphi_2, \cdots, \varphi_n$ evaluates to S in the case of imperative formula or $\top$ in the case of propositional formula and ψ evaluates to S, then $\varphi_1, \varphi_2, \cdots, \varphi_n$ semantically entail ψ.

$$\varphi_1, \varphi_2, \cdots, \varphi_n \models \psi$$

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$$\varphi_1, \varphi_2, \cdots, \varphi_n \models \psi$$

Theorem

Let $\varphi_1, \varphi_2, \cdots, \varphi_n$ be imperative or propositional formulas and and ψ be imperative formula. If $\varphi_1, \varphi_2, \cdots, \varphi_n \vdash \psi$, then $\varphi_1, \varphi_2, \cdots, \varphi_n \models \psi$ holds.

Soundness contd.

Theorem

Let $\varphi_1, \varphi_2, \cdots, \varphi_n$ be imperative or propositional formulas and and ψ be imperative formula. If $\varphi_1, \varphi_2, \cdots, \varphi_n \vdash \psi$, then $\varphi_1, \varphi_2, \cdots, \varphi_n \models \psi$ holds.

- The proof is shown as an induction, on the size (number of lines) of the proof.

Soundness contd.

Theorem

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- The proof is shown as an induction, on the size (number of lines) of the proof.
- Let $\varphi = \varphi_1, \varphi_2, \cdots, \varphi_n$. Let k be the number of lines of proof in the expression $\varphi \vdash \psi$, where k is some natural number.

Soundness contd.

Theorem

Let $\varphi_1, \varphi_2, \cdots, \varphi_n$ be imperative or propositional formulas and and ψ be imperative formula. If $\varphi_1, \varphi_2, \cdots, \varphi_n \vdash \psi$, then $\varphi_1, \varphi_2, \cdots, \varphi_n \models \psi$ holds.

- The proof is shown as an induction, on the size (number of lines) of the proof.
- Let $\varphi = \varphi_1, \varphi_2, \cdots, \varphi_n$. Let k be the number of lines of proof in the expression $\varphi \vdash \psi$, where k is some natural number.
- **Base Case** (Axiom): If $k = 1$, then $\varphi \vdash \varphi$. From the action performance table, whenever $\varphi = S$, $\varphi = S$ as well. Therefore, $\varphi \models \varphi$.

Soundness - contd.- Example of a deduction rule *de_pv*

$$\frac{(i^+ \rightarrow_p \theta) \oplus (j^+ \rightarrow_p \theta) \qquad (i^+ \rightarrow_p \theta)}{(j' \rightarrow_p \theta)} \; de\_pv$$

At line k (last line of the proof), if the rule *de_pv* is applied, it results in $(j' \rightarrow p\theta)$, where $\theta = p$.

$i^+ \rightarrow_p p$	$j^+ \rightarrow_p p$	$(i^+ \rightarrow_p p) \oplus (j^+ \rightarrow_p p)$	$j' \rightarrow_p p$
S	S	V	V
S	V	S	S
S	N	N	N
V	S	S	V
V	V	V	S
V	N	N	N
N	S	N	N
N	V	N	N
N	N	N	N

Soundness - contd.- Example of a deduction rule *de_pv*

$$\frac{(i^+ \rightarrow_p \theta) \oplus (j^+ \rightarrow_p \theta) \qquad (i^+ \rightarrow_p \theta)}{(j' \rightarrow_p \theta)} \; de\_pv$$

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S	S	V	V
S	V	S	S
S	N	N	N
V	S	S	V
V	V	V	S
V	N	N	N
N	S	N	N
N	V	N	N
N	N	N	N

Through similar means, other tables can also be verified.

Soundness - contd.- Example of a deduction rule *de_pv*

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$i^+ \rightarrow_p p$	$j^+ \rightarrow_p p$	$(i^+ \rightarrow_p p) \oplus (j^+ \rightarrow_p p)$	$j' \rightarrow_p p$
S	S	V	V
S	V	S	S
S	N	N	N
V	S	S	V
V	V	V	S
V	N	N	N
N	S	N	N
N	V	N	N
N	N	N	N

Through similar means, other tables can also be verified.

Therefore, by induction on the number of lines of proof, when $(\varphi_1, \cdots, \varphi_n) \vdash \psi$, $(\varphi_1, \cdots, \varphi_n) \models \psi$.

Outline

Completeness - 1

Theorem

Let $\varphi_1, \varphi_2, \cdots, \varphi_n$ and ψ be imperative formulas. If $\varphi_1, \varphi_2, \cdots, \varphi_n \models \psi$, then $\varphi_1, \varphi_2, \cdots, \varphi_n \vdash \psi$.

This theorem is proved on the basis of induction with the help of action performance tables.

- Let Π be the imperative formula with an associated action performance table with formulas $\varphi_1, \varphi_2, ..., \varphi_n$, leading to ψ
- The proof is with six cases based on action performance tables and deduction rules

Completeness - 2 contd.

1. **Case 1: Π in the form i**
   If $\mathscr{E}(i) = S$, then $i \vdash i$, where $i \in I$. Thus $i \vdash \Pi$

Completeness - 2 contd.

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   If $\mathscr{E}(i) = S$, then $i \vdash i$, where $i \in I$. Thus $i \vdash \Pi$
2. **Case 2: Π in the form $(i \rightarrow_p p)$**
   The two formulas in this case are i and p, where $i \in \mathscr{F}_i$ is an imperative and p, a proposition formula with proposition atoms $p_1, p_2, \ldots, p_r$. We proceed in two steps through induction hypothesis and deduction rule.
   - By induction hypothesis and by Case 1, $i \vdash i$. By induction hypothesis and by proposition logic, $p_1, p_2, \ldots, p_r \vdash p$.
   - By deduction rule *cni* where $X = i$, $Y = p$ and $w = p$,

$$i, p \vdash (i \rightarrow_p p)$$

Completeness - 3 contd.

3. **Case 3: Π is of the form** $(i_1 \rightarrow_p p_1) \wedge (i_2 \rightarrow_p p_2)$
   Here, four formulas are i_1, i_2, p_1 and p_2. $i_1, i_2 \in \mathscr{F}_i$ are imperative atoms and p_1, p_2 are goals, represented as proposition formulas with $p_1 \neq p_2$.
   - By induction hypothesis and by Case 2,

     $$i_1, p_1 \vdash (i_1 \rightarrow_p p_1)$$

     $$i_2, p_2 \vdash (i_2 \rightarrow_p p_2)$$

     .
   - By deduction rule *ci*,

     $$i_1, i_2, p_1, p_2 \vdash (i_1 \rightarrow_p p_1) \wedge (i_2 \rightarrow_p p_2)$$

     This rule is applicable when the goals are different, i.e. $p_1 \neq p_2$.

Completeness - 4 contd.

4. **Case 4: Π is of the form** $(i_1 \rightarrow_p p) \oplus (i_2 \rightarrow_p p)$ In this case, there are three formulas i_1, i_2 and p, where $i_1, i_2 \in I$ are imperative atoms and p is goal represented as a proposition formula. We proceed with the induction hypothesis and a Lemma.
   - By induction hypothesis and Case 2,

$$i_1, p \vdash (i_1 \rightarrow_p p)$$

$$i_2, p \vdash (i_2 \rightarrow_p p)$$

   .
   - Let $i_1 = i_1^+$ and $i_2 = i_2^+$
     Lemma:
     - If $(i_1^+ \rightarrow_p p)$ occurs with i_2', then $(i_1^+ \rightarrow_p p) \oplus (i_2^+ \rightarrow_p p)$ can be deduced.
     - Similarly, if $(i_2^+ \rightarrow_p p)$ occurs with i_1', then $(i_1^+ \rightarrow_p p) \oplus (i_2^+ \rightarrow_p p)$ can be deduced.

Completeness - 5 contd.

5. **Case 5: Π is of the form $(\tau \rightarrow_r \varphi)$**
   Here, τ is a proposition formula and φ, an imperative formula. Let τ have proposition atoms $q_1, q_2, \ldots, q_r$ and φ have imperative formulas $\varphi_1, \varphi_2, \ldots, \varphi_s$. In this case, the imperative formula can also be of the type $(\tau \rightarrow_r \varphi)$. To address this aspect Case 5a and Case 5b are considered.

Completeness - 5 contd.

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- **Case 5a: Imperative formulas other than type $(\tau \rightarrow_r \varphi)$**
  - By induction hypothesis and proposition logic, $q_1, q_2, \ldots, q_r \vdash \tau$. By induction hypothesis and through any of the Cases (Case 1,2,3,4,6), $\varphi_1, \varphi_2, \ldots, \varphi_s \vdash \varphi$.
  - By deduction rule *cni*, where $X = \tau$ and $Y = \varphi$ with $w = r$,

$$\tau, \varphi \vdash (\tau \rightarrow_r \varphi)$$

Completeness - 5 contd.

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Here, τ is a proposition formula and φ, an imperative formula. Let τ have proposition atoms $q_1, q_2, \ldots, q_r$ and φ have imperative formulas $\varphi_1, \varphi_2, \ldots, \varphi_s$. In this case, the imperative formula can also be of the type $(\tau \rightarrow_r \varphi)$. To address this aspect Case 5a and Case 5b are considered.
   - **Case 5a: Imperative formulas other than type $(\tau \rightarrow_r \varphi)$**
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     - By deduction rule *cni*, where $X = \tau$ and $Y = \varphi$ with $w = r$,

$$\tau, \varphi \vdash (\tau \rightarrow_r \varphi)$$

   - **Case 5b: Imperative formulas of the type $(\tau \rightarrow \varphi)$**
     - By induction hypothesis and proposition logic, $q_1, q_2, \ldots, q_r \vdash \tau$. By induction hypothesis and through Case 5a, $\varphi_1, \varphi_2, \ldots, \varphi_s \vdash \varphi$.
     - By deduction rule *cni*, where $X = \tau$ and $Y = \varphi$ with $w = r$,

$$\tau, \varphi \vdash (\tau \rightarrow_r \varphi)$$

Completeness - 6 contd.

6. **Case 6: Π is of the form $(\varphi \rightarrow_i \psi)$**
   In this case, φ and ψ are imperatives with formulas $\varphi_1, \varphi_2, \ldots, \varphi_s$ and $\psi_1, \psi_2, \ldots, \psi_n$, denoting actions to be performed at time-steps 1 and 2, respectively. Here, imperatives can also be of the form $(\varphi \rightarrow_i \psi)$. To include this aspect, Case 6a and Case 6b are considered.

Completeness - 6 contd.

6. **Case 6: Π is of the form $(\varphi \rightarrow_i \psi)$**
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   - **Case 6a: Imperative formulas other than type $(\varphi \rightarrow_i \psi)$**
     - By induction hypothesis and through any other cases (Cases 1,2,3,4,5), $\varphi_1, \varphi_2, \ldots, \varphi_s \vdash \varphi$ and $\psi_1, \psi_2, \ldots, \psi_n \vdash \psi$.
     - By deduction rule *cni*, where $X = \varphi$ and $Y = \psi$ with $w = i$,

$$\varphi, \psi \vdash (\varphi \rightarrow_i \psi)$$

Completeness - 6 contd.

6. **Case 6: Π is of the form $(\varphi \rightarrow_i \psi)$**
   In this case, φ and ψ are imperatives with formulas $\varphi_1, \varphi_2, ..., \varphi_s$ and $\psi_1, \psi_2, ..., \psi_n$, denoting actions to be performed at time-steps 1 and 2, respectively. Here, imperatives can also be of the form $(\varphi \rightarrow_i \psi)$. To include this aspect, Case 6a and Case 6b are considered.
   - **Case 6a: Imperative formulas other than type $(\varphi \rightarrow_i \psi)$**
     - By induction hypothesis and through any other cases (Cases 1,2,3,4,5), $\varphi_1, \varphi_2, ..., \varphi_s \vdash \varphi$ and $\psi_1, \psi_2, ..., \psi_n \vdash \psi$.
     - By deduction rule *cni*, where $X = \varphi$ and $Y = \psi$ with $w = i$,

$$\varphi, \psi \vdash (\varphi \rightarrow_i \psi)$$

   - **Case 6b: Imperative formula of type $(\varphi \rightarrow_i \psi)$**
     - By induction hypothesis and through Case 6a, $\varphi_1, \varphi_2, ..., \varphi_s \vdash \varphi$ and $\psi_1, \psi_2, ..., \psi_n \vdash \psi$.
     - By deduction rule *cni*, where $X = \varphi$ and $Y = \psi$ with $w = i$,

$$\varphi, \psi \vdash (\varphi \rightarrow_i \psi)$$

Therefore, when $\varphi_1, \varphi_2, \cdots, \varphi_n \models \psi$, then $\varphi_1, \varphi_2, \cdots, \varphi_n \vdash \psi$

Outline

AI Planning

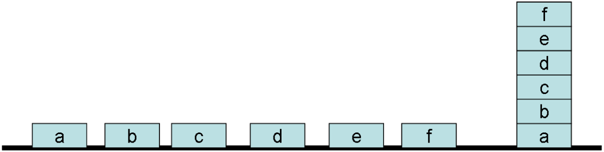


Table: Result of block world

Initial State: Blocks a, b, c	
d, e, f on table	
Final state: b *on* a, c *on* b	
d *on* c, e *on* d, f *on* e	
After step 1	
Sequence: *move b a*, *move c b*	
After step 2	
Sequence: *move d c*, *move e d*	
After step 3	
Sequence: *move f e*	

Table: Comparison

Number of blocks	**Length of optimal plan**	**Length of Plan using MIRA**
20	19	10
25	24	13
30	29	15
35	34	18
40	39	20

Task Analysis for Special Education

Generates a sequence of tasks from the jumbled sets of task

Robotics - 1

Running example: A robot moves from the location *hall* to *porch*.
If the robot senses a *person*, it has to switch on the *camera*.
The locations *hall* and *porch* are adjacent to each other and the robot starts in *hall* with *camera* in *off* position.

Table: **LTLMoP specification with equivalent LTL formulas**

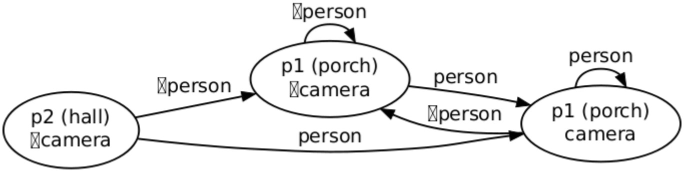
Env starts with false	$\pi_{\neg person}$
Robot with false	$\pi_{\neg camera}$
Robot in hall	φ_{hall}
Do camera if and only if you are sensing the person	$\Box(\bigcirc\pi_{camera} \Leftrightarrow \bigcirc\pi_{person})$
go to porch infinitely often	$\Box\Diamond(\varphi_{porch})$

MIRA specification

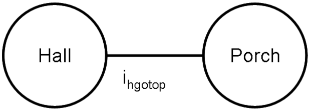
$$
\begin{aligned}
r_{hall} &\rightarrow_r i_{hgotop} \rightarrow_p r_{porch} \\
x_{person} &\rightarrow_r i_{on} \\
x_{\neg person} &\rightarrow_r i_{off}
\end{aligned}
$$

Robotics - 2

For the same example, mentioned above, automaton generated from LTLMoP:

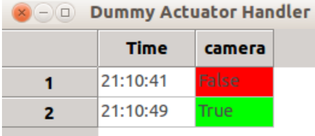


In MIRA based approach, no state space is involved. But the workspace is described through graph.



Robotics - 3

For the example mentioned above, the action of *switching on the camera, on seeing the person* is shown as the transition from *false* value to a *true* value.



Dummy Actuator Handler

	Time	camera
1	21:10:41	False
2	21:10:49	True

Difficulty with true or false values. If the robot is broken and manually placed in porch, the value is *true*.

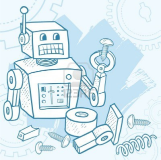


Figure: Courtesy: http://www.123rf.com/ (Royalty free photos)

MIRA based approach: If robot could not move, the value is V.

Instruction classification

Sl. No	Type	Inspired from Mīmāṁsā	Zhang et. al (2012)
1	Negative Instruction	*Niṣedha*	-
2	Mandatory Instruction	*saṃuccayaḥ*	-
3	Optional Instruction	*Vikalpa*	-
4	Goal	*phalavidhi*	Post-condition
5	Reason	*Hetu-hetumatbhāva*	Precondition
6	Instruction with auxiliaries	*Viniyogavidhi*	Instrument
7	Object	derived from *vidhi*	actee
8	Action modifier	-	derived from adjectives

Outline

Conclusion

- A logical formalism is constructed based on Mīmāṁsā principles

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- Provides an unambiguous representation of instructions
- Formalism is unique in different aspects in specifying the intention of goal, composite expression and direct action evaluation from instructions.
- Application in computational areas - AI planning, Robotics, Task analysis for special education and classification of natural language instructions

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Thank You

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