independence

- independent events
- assessing independence
- multiplication rule for independent events

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independence

two processes are independent if knowing the outcome of one provides no useful information about the outcome of the other.

1st toss 2nd toss

P(H) = 0.5 P(T) = 0.5

outcomes of two tosses of a coin are independent

1st draw 2nd draw

P(A) = 3/51 P(J) = 4/51

outcomes of two draws from a deck of cards (without replacement) are dependent

Image sources:

Coin: http://commons.wikimedia.org/wiki/File:1913_Eliasberg_Liberty_Head_Nickel.png

Card: Open Clip Art Library (http://openclipart.org/cgi-bin/navigate/recreation/games/cards/white)

Checking for independence:

$P(A \mid B) = P(A)$, then A and B are independent.

given

practice

In 2013, SurveyUSA interviewed a random sample of 500 NC residents asking them whether they think widespread gun ownership protects law abiding citizens from crime, or makes society more dangerous.

- 58% of all respondents said it protects citizens.
- 67% of White respondents,
- 28% of Black respondents,
- and 64% of Hispanic respondents shared this view.

Opinion on gun ownership and race ethnicity are most likely _____?

(a) complementary
(b) mutually exclusive
(c) independent
(d) dependent
(e) disjoint

$P(\text{protects citizens}) = 0.58$

$P(\text{protects citizens} \mid \text{White}) = 0.67$

$P(\text{protects citizens} \mid \text{Black}) = 0.28$

$P(\text{protects citizens} \mid \text{Hispanic}) = 0.64$

Link to poll: http://www.surveyusa.com/client/PollReport.aspx?g=a5f460ef-bba9-484b-8579-1101ea26421b

determining dependence based on sample data

observed difference between conditional probabilities → dependence → hypothesis test

if difference is large, there is stronger evidence that the difference is real

if sample size is large, even a small difference can provide strong evidence of a real difference

Product rule for independent events:

If A and B are independent, $P(A \text{ and } B) = P(A) \times P(B)$

You toss a coin twice, what is the probability of getting two tails in a row?

P(two tails in a row) =

= P(T on the 1st toss) × P(T on the 2nd toss)

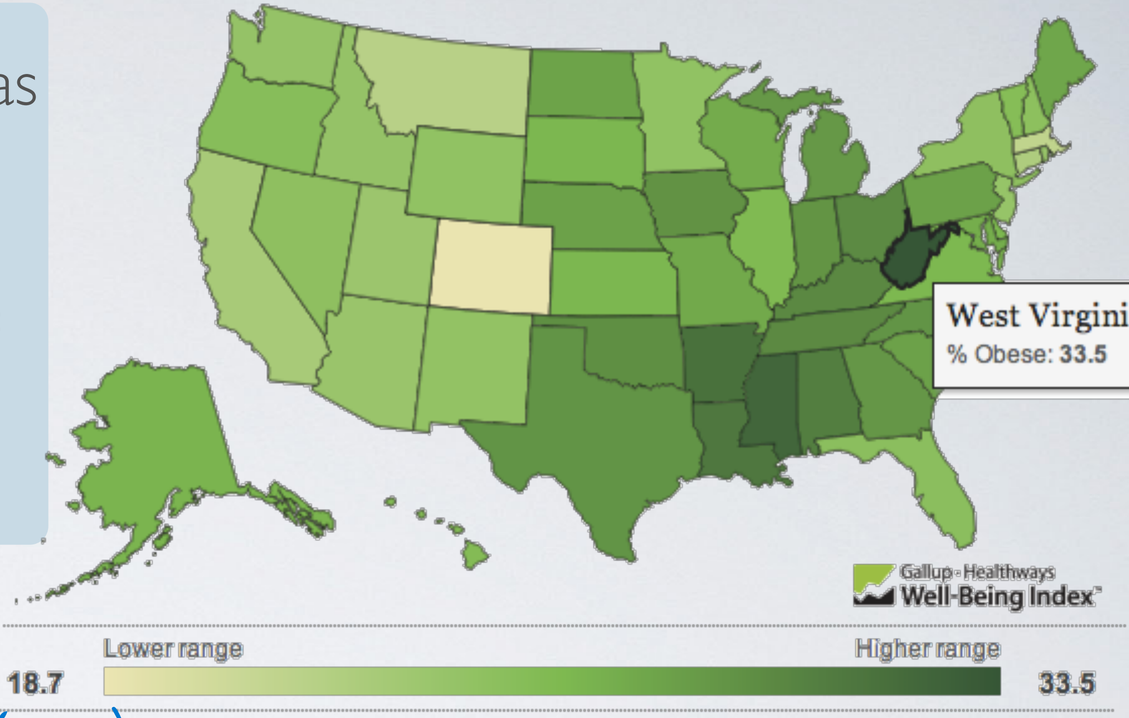
= (1/2) × (1/2)

= 1/4

Note: If $A_1, A_2, \ldots, A_k$ are independent, $P(A_1 \text{ and } A_2 \text{ and } \ldots A_k) = P(A_1) \times P(A_2) \times \ldots \times P(A_k)$

practice

A 2012 Gallup poll suggests that West Virginia has the highest obesity rate among US states, with 33.5% of West Virginians being obese. Assuming that the obesity rate stayed constant, what is the probability that two randomly selected West Virginians are both obese? *independent*



$P(\text{obese}) = 0.335$

$P(\text{both obese}) = P(\text{1st obese}) \times P(\text{2nd obese})$

$= 0.335 \times 0.335$

≈ 0.11

Image source + Link to poll: http://www.surveyusa.com/client/PollReport.aspx?g=a5f460ef-bba9-484b-8579-1101ea26421