bayesian inference

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set-up

win: ≥4

6-sided

12-sided

Image sources:

6-sided: http://commons.wikimedia.org/wiki/File:Sixsided_Dice_inJapan.jpg

12-sided: http://commons.wikimedia.org/wiki/File:12-sided_die.jpg

probabilities

What is the probability of rolling ≥4 with a 6-sided die?

What is the probability of rolling ≥4 with a 12-sided die?

probabilities

What is the probability of rolling ≥4 with a 6-sided die?

$S = \{1,2,3,\boxed{4,5,6}\}$

$P(\geq 4) = 3/6 = 1/2 = 0.5$

What is the probability of rolling ≥4 with a 12-sided die?

$S = \{1,2,3,\boxed{4,5,6,7,8,9,10,11,12}\}$

$P(\geq 4) = 9/12 = 3/4 = 0.75$

“good die”

Say you're playing a game where the goal is to roll ≥ 4. If you could get your pick, which die would you prefer to play this game with?

(a) 

6-sided

(b) 

12-sided

“good die”

Say you're playing a game where the goal is to roll ≥ 4. If you could get your pick, which die would you prefer to play this game with?

good die

(a) 

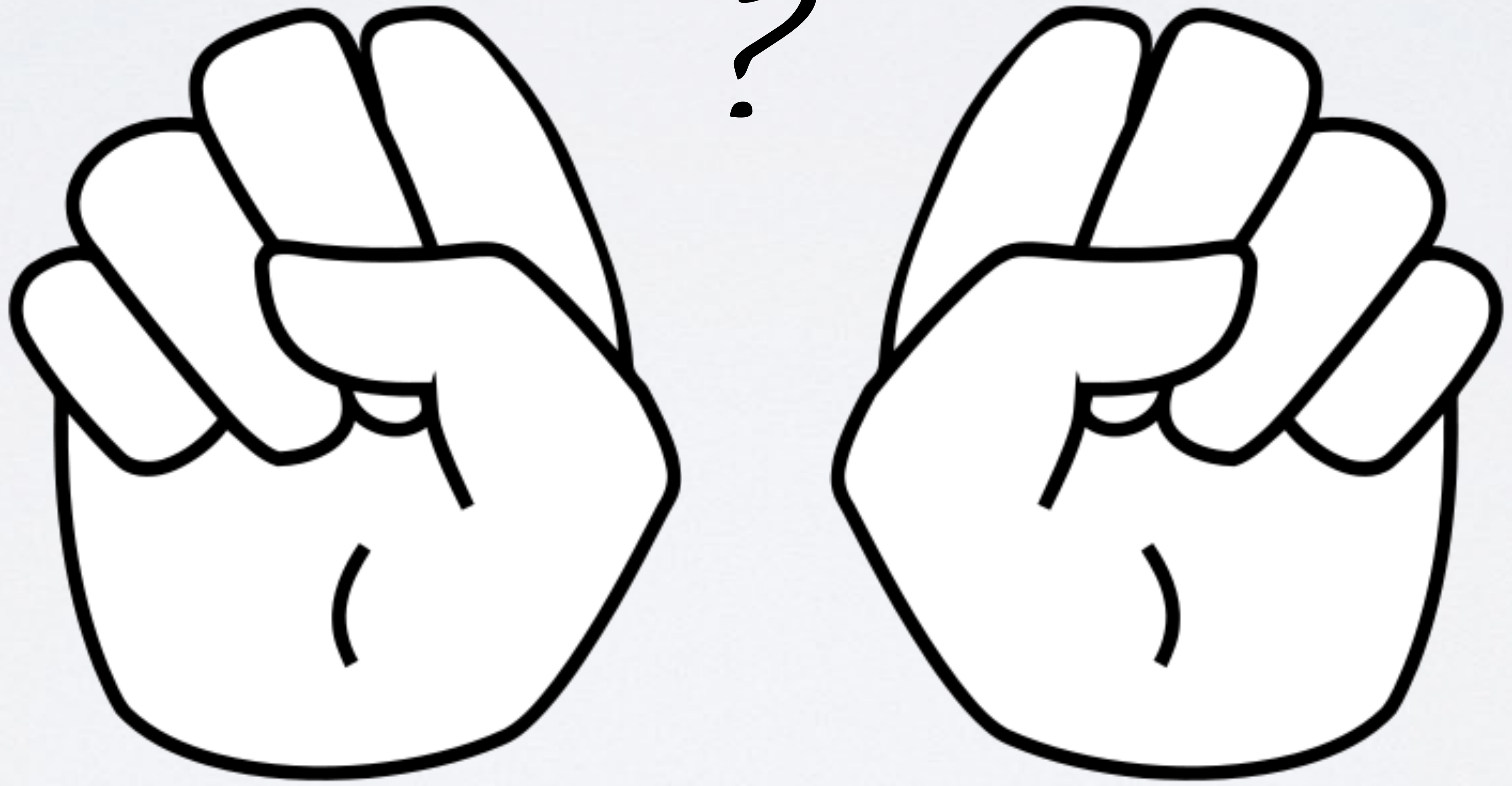
(b) 

$P(\geq 4) = 0.5$

$P(\geq 4) = 0.75$

rules

LEFT ? RIGHT



Hands: http://commons.wikimedia.org/wiki/File:Hand_0_Alternativ.svg

hypotheses and decisions

		Truth	
		Right good, Left bad	Right bad, Left good
Decision	pick Right	**You win the game!**	**You lose :(**
	pick Left	**You lose :(**	**You win the game!**

before you collect data

Before we collect any data, you have no idea if I am holding the good die (12-sided) on the right hand or the left hand. Then, what are the probabilities associated with the following hypotheses?

H_1: good die on the Right (bad die on the Left)

H_2: good die on the Left (bad die on the Right)

	P(H_1: good die on the Right)	P(H_2: good die on the Left)
(a)	0.33	0.67
(b)	0.5	0.5
(c)	0	1
(d)	0.25	0.75

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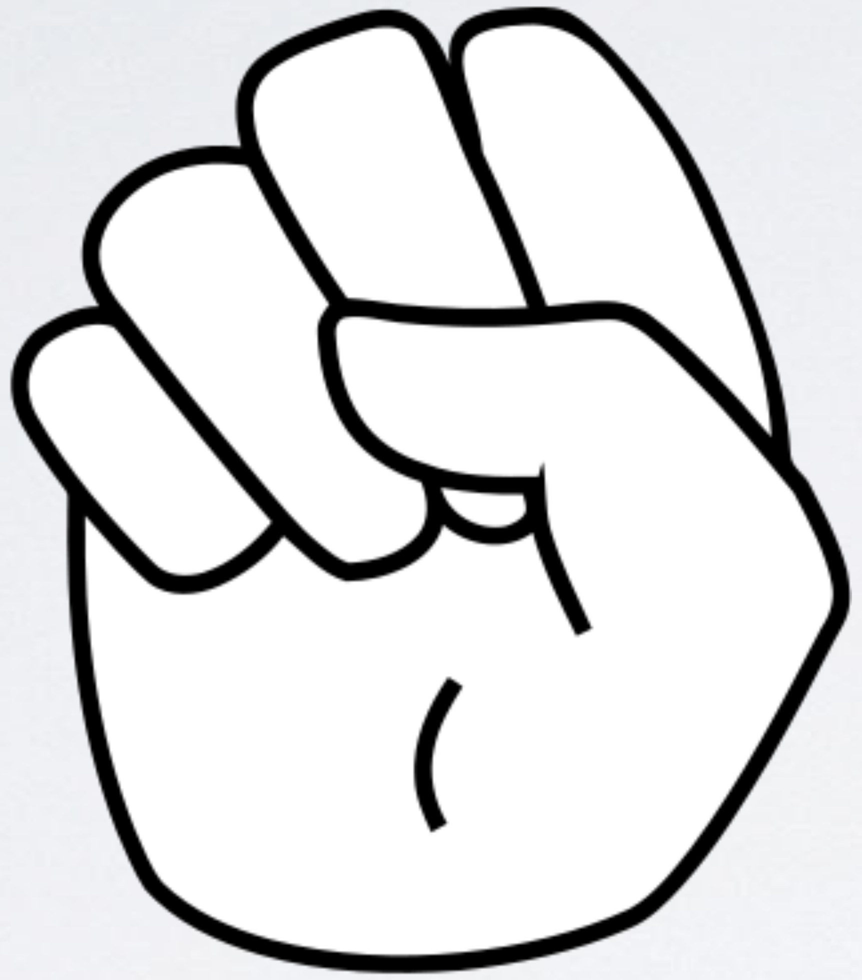
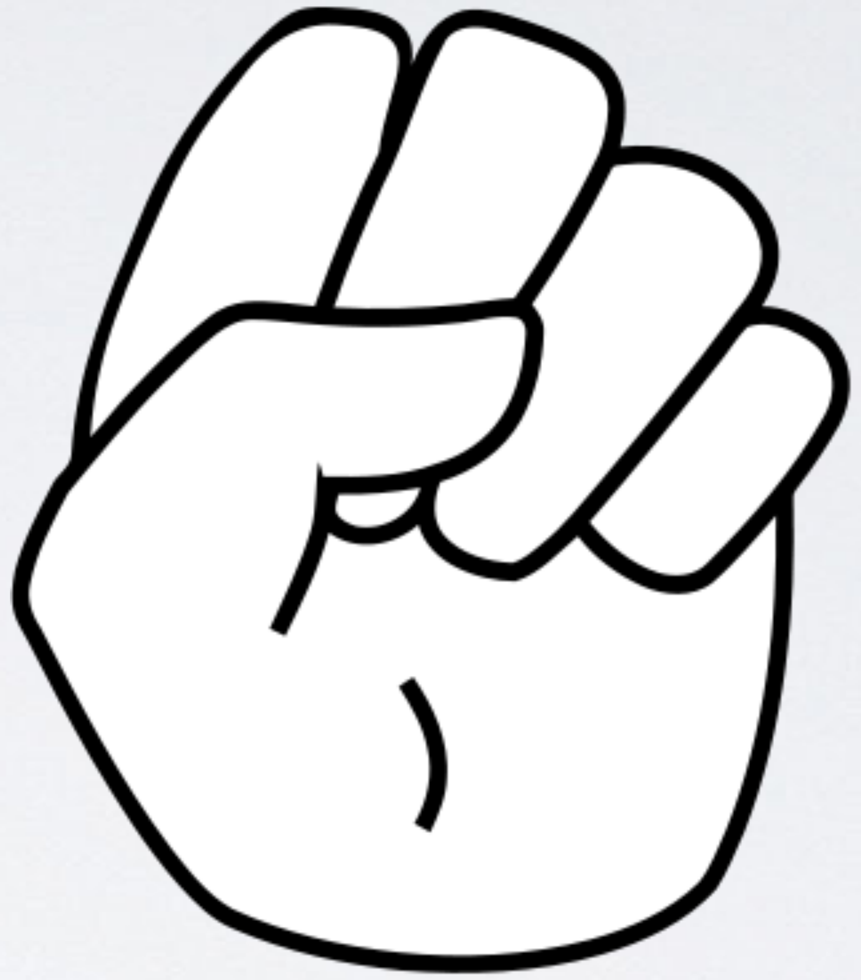
→ prior

data collection: round #1

LEFT



RIGHT



≥4

after you see the data

You chose the right hand, and you won (rolled a number ≥4). Having observed this data point how, if at all, do the probabilities you assign to the same set of hypotheses change?

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(a)	0.5	0.5
(b)	more than 0.5	less than 0.5
(c)	less than 0.5	more than 0.5

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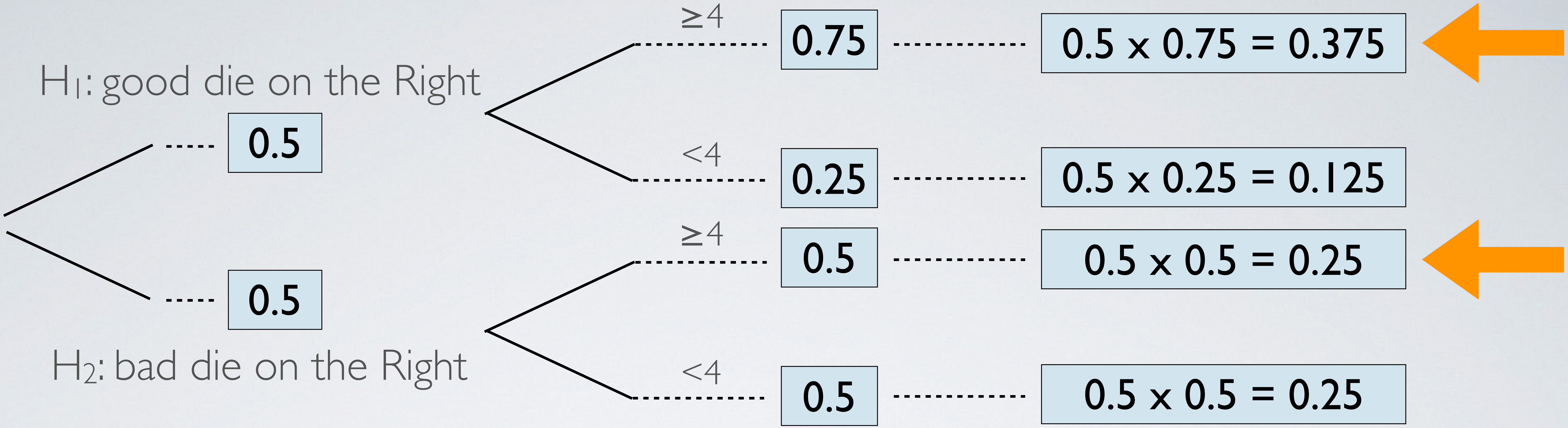

$P(H_1\text{: good die on the Right} \mid \text{you rolled} \geq 4 \text{ with the die on the Right}) =$

$$= \frac{P(\text{good Right} \,\&\, \geq 4 \text{ Right})}{P(\geq 4 \text{ Right})} = \frac{0.375}{0.375 + 0.25} = 0.6$$

posterior

- The probability we just calculated is also called the posterior probability.

  P(H_1: good die on the Right | you rolled $\geq$4 with the die on the Right)
- Posterior probability is generally defined as P(hypothesis | data).
- It tells us the probability of a hypothesis we set forth, given the data we just observed.
- It depends on both the prior probability we set and the observed data.
- This is different than what we calculated at the end of the randomization test on gender discrimination – the probability of observed or more extreme data given the null hypothesis being true, i.e. P(data | hypothesis), also called a p-value.

updating the prior

- In the Bayesian approach, we evaluate claims iteratively as we collect more data.
- In the next iteration (roll) we get to take advantage of what we learned from the data.
- In other words, we update our prior with our posterior probability from the previous iteration.

updated:

$P(H_1$: good die on the Right)	$P(H_2$: good die on the Left)
0.6	0.4

recap

- Take advantage of prior information, like a previously published study or a physical model.
- Naturally integrate data as you collect it, and update your priors.
- Avoid the counter-intuitive definition of a p-value:

  P(observed or more extreme outcome | H0 is true)
- Instead base decisions on the posterior probability:

  P(hypothesis is true | observed data)
- A good prior helps, a bad prior hurts, but the prior matters less the more data you have.
- More advanced Bayesian techniques offer flexibility not present in Frequentist models.