hypothesis test for a proportion

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Hypothesis testing for a single proportion:

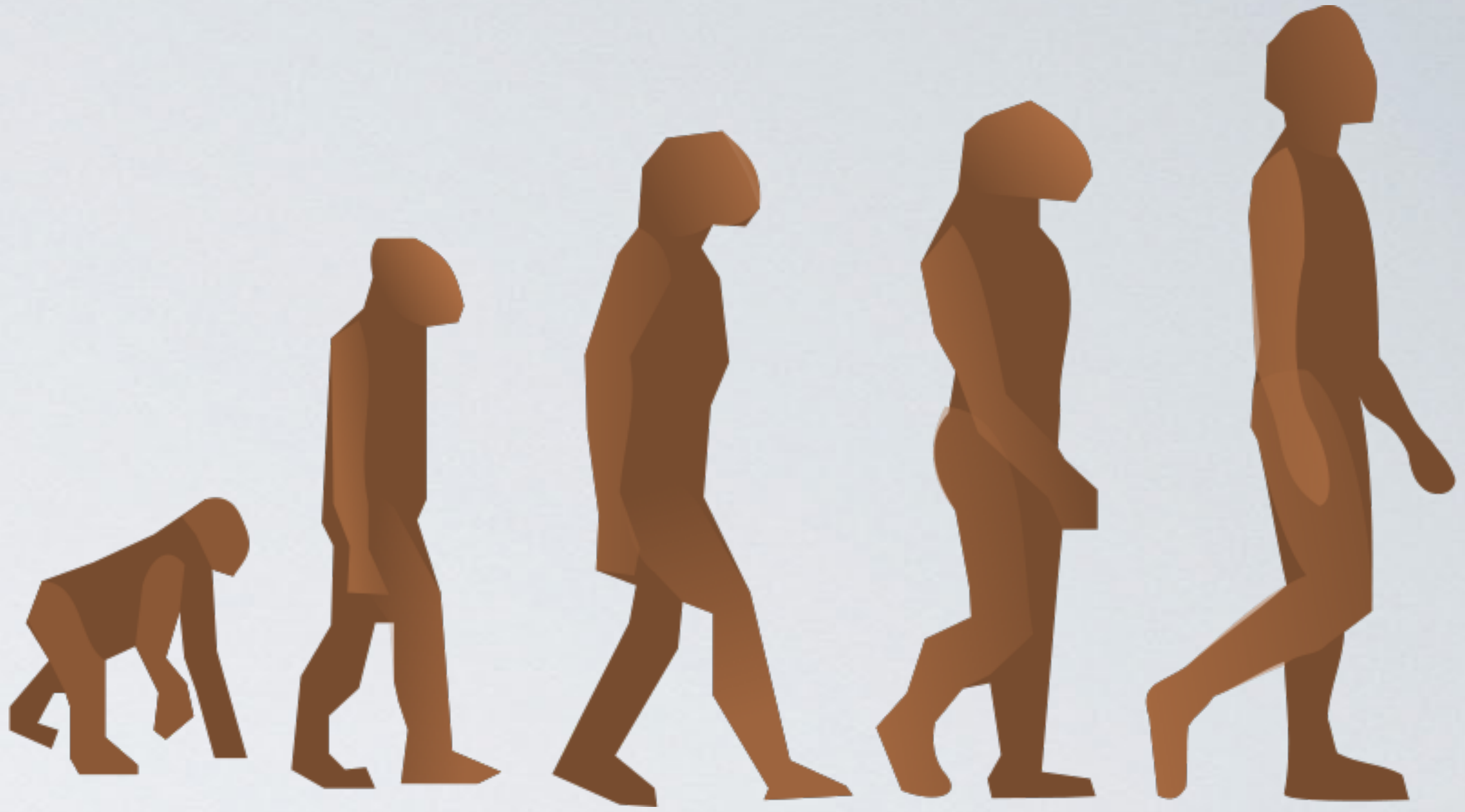
1. Set the hypotheses: $H_0 : p = null\ value$
   $H_A : p < \ or \ > \ or \ \neq \ null\ value$
2. Calculate the point estimate: $\hat{p}$
3. Check conditions:
   1. ***Independence:*** Sampled observations must be independent (random sample/assignment & if sampling without replacement, n < 10% of population)
   2. ***Sample size/skew:*** *np* ≥ *10* and *n(1-p)* ≥ *10*
4. Draw sampling distribution, shade p-value, calculate test statistic $Z = \frac{\hat{p} - p}{SE}, \quad SE = \sqrt{\frac{p(1-p)}{n}}$
5. Make a decision, and interpret it in context of the research question:
   - If p-value < α, reject H_0; the data provide convincing evidence for H_A.
   - If p-value > α, fail to reject H_0 the data *do not* provide convincing evidence for H_A.

$\hat{p}$ vs. p

	confidence interval	hypothesis test
success-failure condition	$n\hat{p} \geq 10$ $n(1-\hat{p}) \geq 10$	$np \geq 10$ $n(1-p) \geq 10$
standard error	$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$SE = \sqrt{\frac{p(1-p)}{n}}$

practice

A 2013 Pew Research poll found that 60% of 1,983 randomly sampled American adults believe in evolution. Does this provide convincing evidence that majority of Americans believe in evolution?



$H_0: p = 0.5$

$H_A: p > 0.5$

$\hat{p} = 0.6$

$n = 1983$

1. independence: 1983 < 10% of Americans & random sample

Whether one American in the sample believes in evolution is independent of another.

2. sample size / skew: $1983 \times 0.5 = 991.5 > 10$

S-F condition met → nearly normal sampling distribution

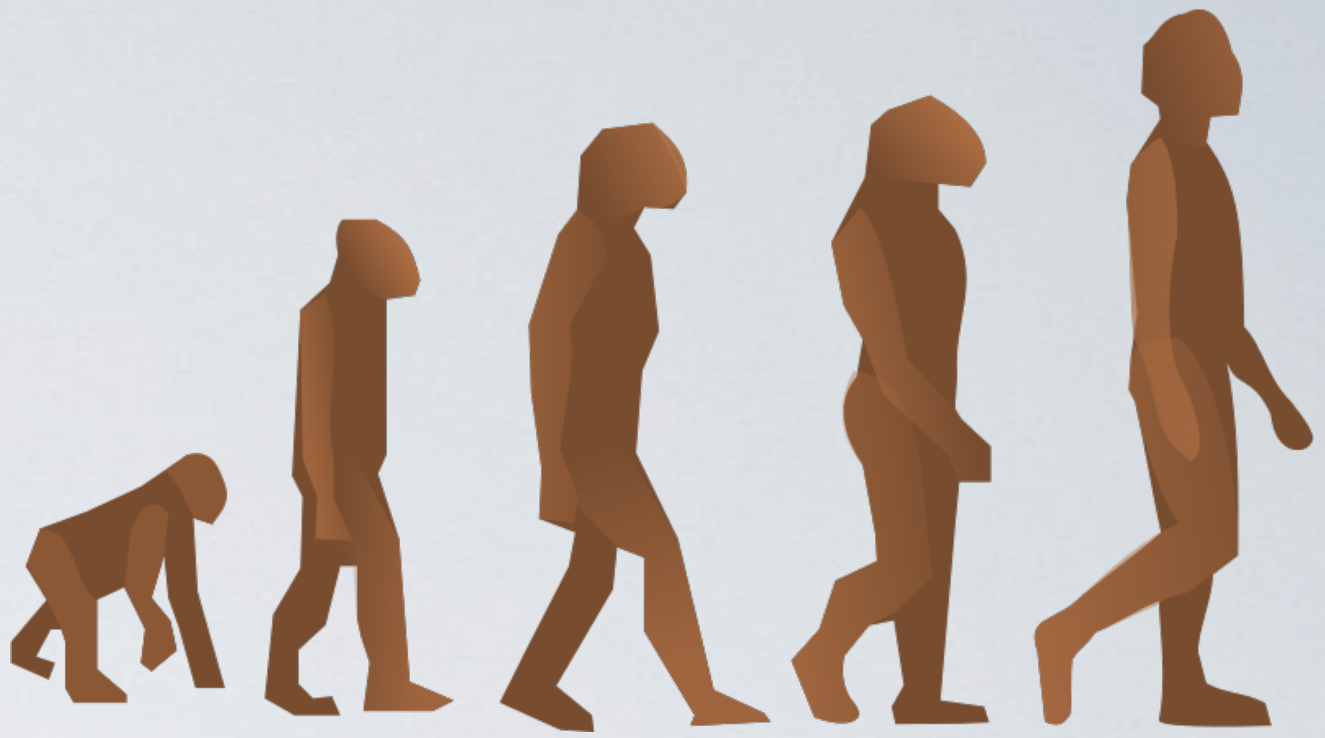
Image source: http://openclipart.org/detail/12755/evolution-steps-by-anonymous-12755

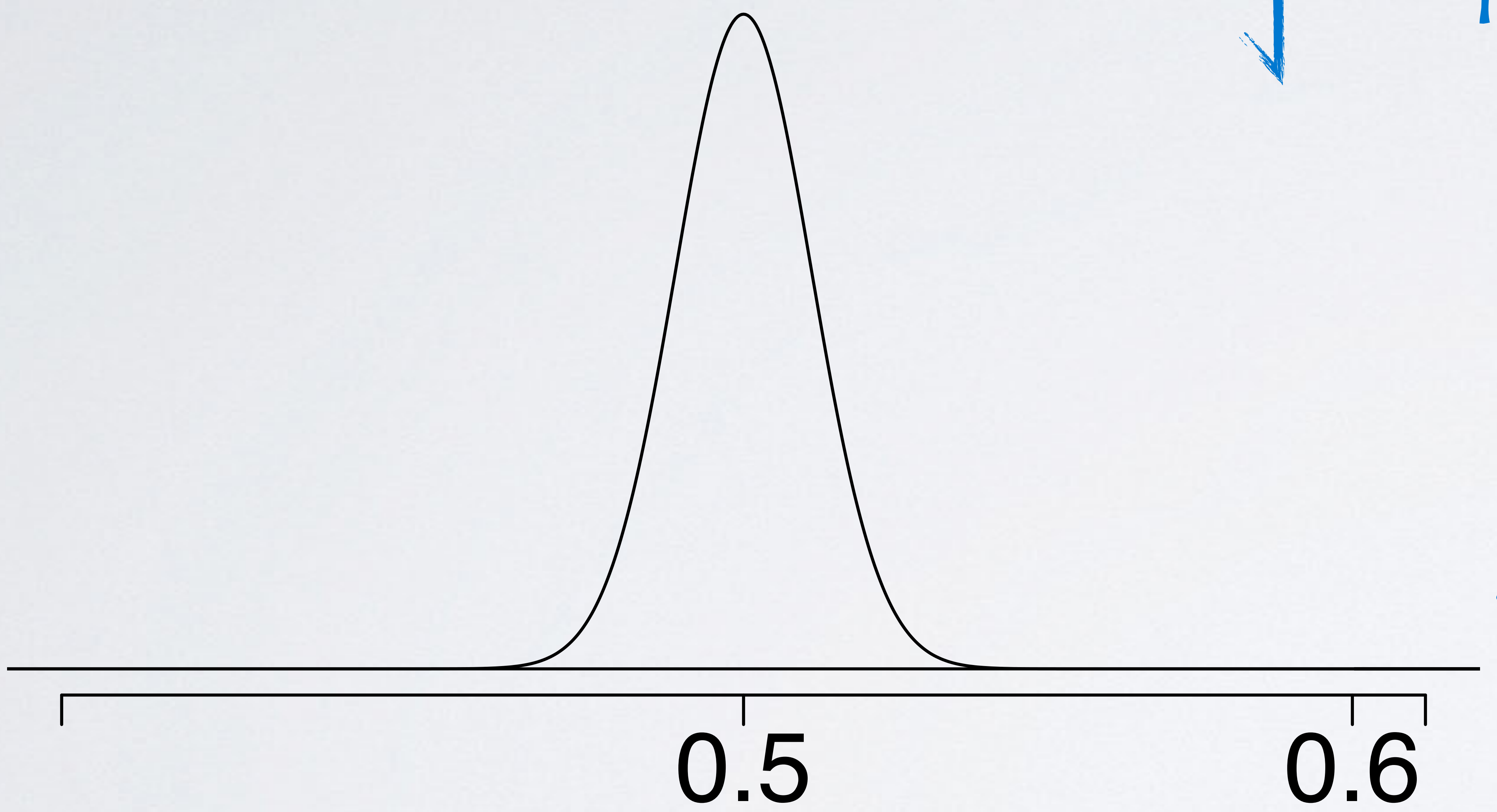
practice

$H_0: p = 0.5$ $\quad \hat{p} = 0.6$

$H_A: p > 0.5$ $\quad n = 1983$

$$\hat{p} \sim N\left(\text{mean} = 0.5,\ SE = \sqrt{\frac{0.5 \times 0.5}{1983}} \approx 0.0112\right)$$





$$Z = \frac{0.6 - 0.5}{0.0112} \approx 8.92$$

$$p\text{-value} = P(Z > 8.92)$$

$$= \text{almost } 0 \rightarrow \text{reject } H_0$$