Repetitive tutorial

All functions have to be written in Python, and must be iterative

1 Iterations

Exercise 1.1 (Zorglub)

What does the following function compute when called with a strictly positive integer n?

```
function zorglub(integer n) : integer
    variables
        integer    i, j, k
begin
    j ← 1
    k ← 0
    i ← 1
    while i <= n do
        j ← i * j
        k ← j + k
        i ← i + 1
    end while
    return k
end
```

Translate this function in Python.

Exercise 1.2 (Multiplication)

1. Write a function that computes $x \times y$, with $(x, y) \in \mathbb{N}^2$, using only the + and - operators.
2. Write a function that computes $x \times y$, this time with $(x, y) \in \mathbb{Z}^2$.

Exercise 1.3 (Exponentiation)

Write a function that computes x^n, with $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

Exercise 1.4 (Fibonacci)

Write a function that computes the n^{th} term of the Fibonacci sequence.

$$fibo(0) = fibo(1) = 1$$
$$fibo(n) = fibo(n-1) + fibo(n-2)$$

Exercise 1.5 (Sequence)

Let u be a sequence and the function `u(n)` that computes its nth term.

1. Write a function that computes S_n: the sum of the n first terms of u (u_1 to u_n).
2. Without using the previous function, write a function that computes

$$\sum_{i=1}^{n} S_i$$

   where S_i is the sum of the i first terms of u.
3. If the previous function uses two loops, rewrite it with only one.

2 Repetitions

Exercise 2.1 (Divisibility by 11)

This particular test of divisibility by 11 was given in 1897 by Charles L. Dodgson (Lewis Carroll).

Given a number to test, form a new number by:

- removing the unit digit,
- subtracting this digit from the shortened number.

This new number is divisible by 11 if and only if the original number is.

Example :
$24684 \rightarrow 2464 \rightarrow 242 \rightarrow 22$. The number 24684 divisible by 11.

Use the above principle to write a function that checks if a nonzero natural is divisible by 11. The only integer divisions allowed are those by 10 (no division by 11!).

Exercise 2.2 (Euclid)

Write a function that computes the greatest common divisor (gcd) of two nonzero integers a and b using the Euclidean algorithm, the principle of which is reminded below.

Euclidean algorithm :

> If a and b are two nonzero naturals with $a \geq b$, if r is the nonzero remainder of the division of a by b: $a = bq + r$ with $0 < r < b$, then the gcd of a and b is equal to the gcd of b and r.
> If a is divisible by b then the gcd of a and b is equal to b.

Exercise 2.3 (Mirror)

Write a function that returns the "mirror" of a given integer if the latter is positive.

Example: $1278 \rightarrow 8721$.

Remark: if the given integer is 1250, the result will be 521.

Exercise 2.4 (Quotient)

Let a et b be two nonzero naturals. Write a function that computes the quotient a div b (an integer) as well as the remainder. The function must use only additive operators (the allowed operators are `+` and `-`).

Exercise 2.5 (Calculable factorial?)

Write a function that takes an integer *limit* as parameter and computes the greatest even number n such that $n! < limit$. It returns 0 in case the value can not be computed ($limit \leq 0$).

Example: if $limit = 150$, $5! < 150 < 6!$, therefore, the greatest even number whose factorial does not exceed 150 is 4.

Exercise 2.6 (Prime number)

Write a function that checks if an integer greater than 1 is a prime number.

Exercise 2.7 (Bonus: Egyptian multiplication)

Write a function that computes $x \times y$ only using additions, multiplications by 2 and divisions by 2.

Clues :

- $10 \times 13 = 2 \times (5 \times 13)$ because 10 is even.
- $11 \times 13 = 2 \times (5 \times 13) + 13$ because 11 is odd.