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Aerospace Engineering
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26 SEPTEMBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Gliding Flight
  - Climbing Flight
  - Flight Envelope
  - Maneuvering

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GLIDING FLIGHT

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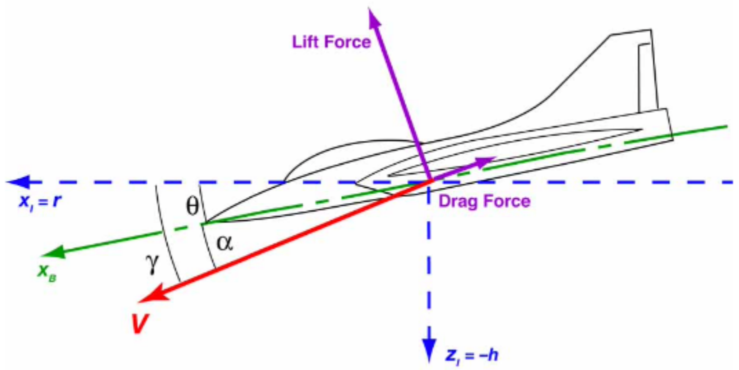
Equilibrium Gliding Flight



$$C_D \frac{1}{2}\rho V^2 S = -W \sin\gamma$$

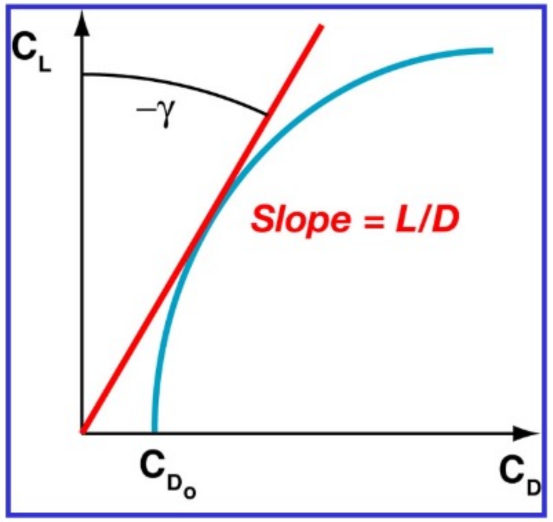
$$C_L \frac{1}{2}\rho V^2 S = W \cos\gamma$$

$$\dot{h} = V \sin\gamma$$

$$\dot{r} = V \cos\gamma$$

Gliding Flight

- Thrust = 0
- Flight path angle < 0 in gliding flight
- Altitude is decreasing
- Airspeed ~ constant
- Air density ~ constant



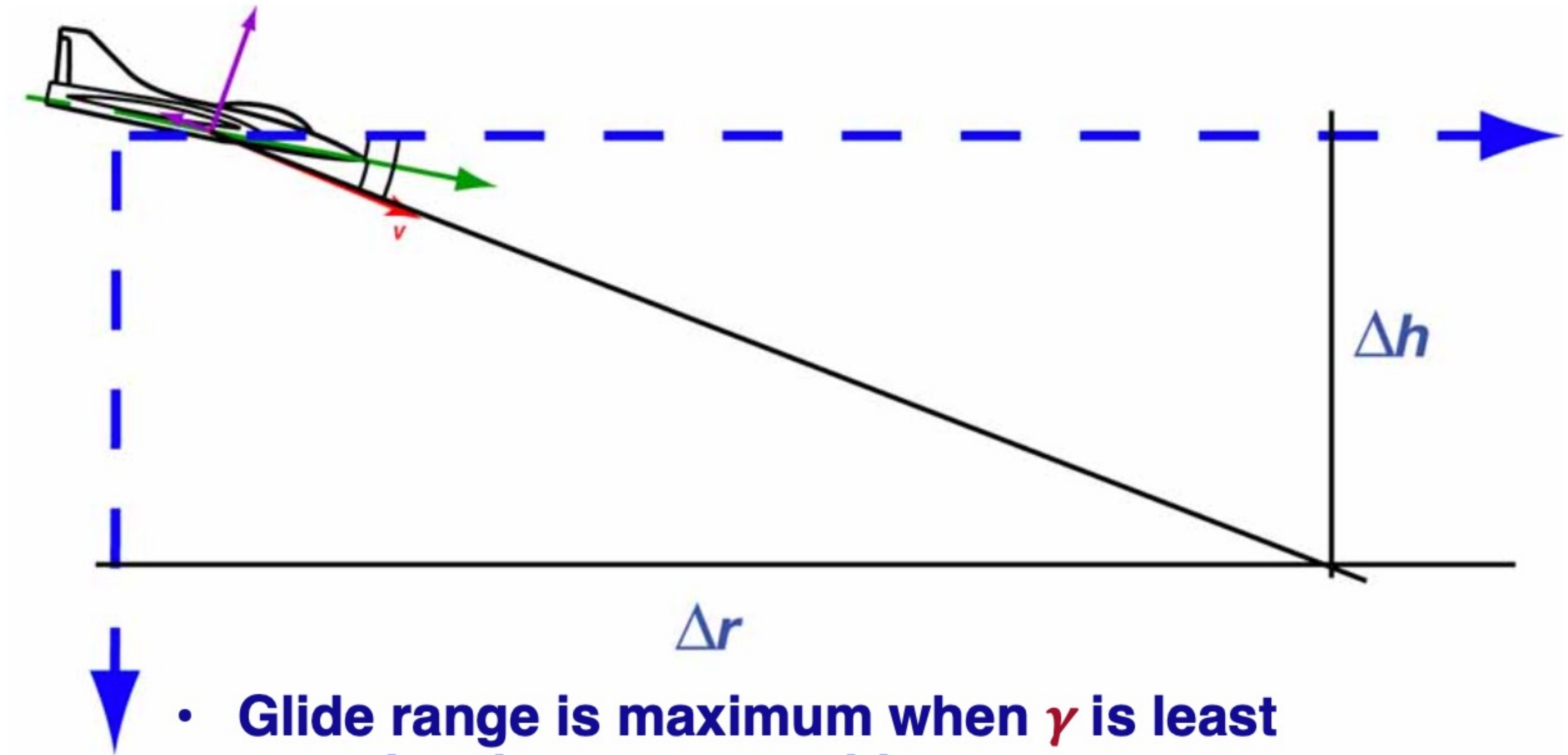
Gliding flight path angle

$$\tan\gamma = -\frac{D}{L} = -\frac{C_D}{C_L} = \frac{\dot{h}}{\dot{r}} = \frac{dh}{dr}; \quad \gamma = -\tan^{-1}\left(\frac{D}{L}\right) = -\cot^{-1}\left(\frac{L}{D}\right)$$

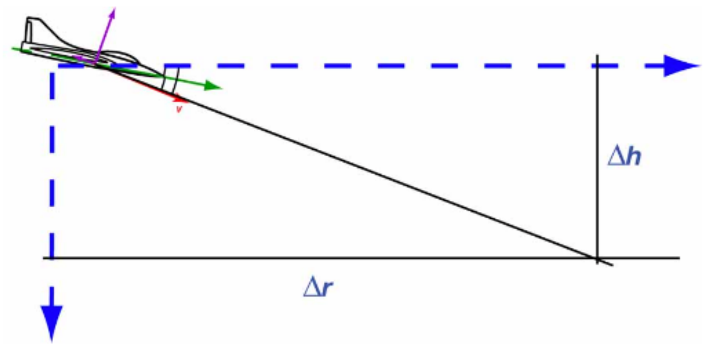
Corresponding airspeed

$$V_{glide} = \sqrt{\frac{2W}{\rho S\sqrt{C_D^2 + C_L^2}}}$$

Maximum Steady Gliding Range



- **Glide range is maximum when γ is least negative, i.e., most positive**
- **This occurs at $(L/D)_{max}$**



Maximum Steady Gliding Range

- Glide range is maximum when γ is least negative, i.e., most positive
- This occurs at $(L/D)_{max}$

$$\boxed{\gamma_{max} = -\tan^{-1}\left(\frac{D}{L}\right)_{min} = -\cot^{-1}\left(\frac{L}{D}\right)_{max}}$$

$$\tan\gamma = \frac{\dot{h}}{\dot{r}} = negative\ constant = \frac{(h - h_o)}{(r - r_o)}$$

$$\Delta r = \frac{\Delta h}{\tan\gamma} = \frac{-\Delta h}{-\tan\gamma} = maximum\ when\ \frac{L}{D} = maximum$$

Sink Rate, m/s

Lift and drag define γ and V in gliding equilibrium

$$D = C_D \frac{1}{2}\rho V^2 S = -W \sin\gamma$$

$$\sin\gamma = -\frac{D}{W}$$

$$L = C_L \frac{1}{2}\rho V^2 S = W \cos\gamma$$

$$V = \sqrt{\frac{2W\cos\gamma}{C_L \rho S}}$$

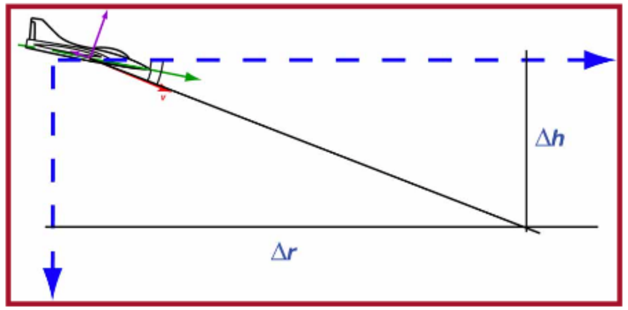
Sink rate = altitude rate, *dh/dt* (negative)

$$\dot{h} = V \sin\gamma$$

$$= -\sqrt{\frac{2W\cos\gamma}{C_L \rho S}}\left(\frac{D}{W}\right) = -\sqrt{\frac{2W\cos\gamma}{C_L \rho S}}\left(\frac{L}{W}\right)\left(\frac{D}{L}\right)$$

$$= -\sqrt{\frac{2W\cos\gamma}{C_L \rho S}}\cos\gamma\left(\frac{1}{L/D}\right)$$

Conditions for Minimum Steady Sink Rate



- **Minimum sink rate provides maximum endurance**
- **Minimize sink rate by setting $\partial(dh/dt)/\partial C_L = 0$ ($\cos\gamma \sim 1$)**

$$\dot{h} = -\sqrt{\frac{2W\cos\gamma}{C_L \rho S}}\cos\gamma\left(\frac{C_D}{C_L}\right)$$

$$= -\sqrt{\frac{2W\cos^3\gamma}{\rho S}}\left(\frac{C_D}{C_L^{3/2}}\right) \approx -\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)}\left(\frac{C_D}{C_L^{3/2}}\right)$$

$$C_{L_{ME}} = \sqrt{\frac{3C_{D_o}}{\varepsilon}} \quad and \quad C_{D_{ME}} = 4C_{D_o}$$

L/D and V_{ME} for Minimum Sink Rate

$$\left(L/D\right)_{ME} = \frac{1}{4}\sqrt{\frac{3}{\varepsilon C_{D_o}}} = \frac{\sqrt{3}}{2}\left(L/D\right)_{\max} \approx 0.86\left(L/D\right)_{\max}$$

$$V_{ME} = \sqrt{\frac{2W}{\rho S\sqrt{C_{D_{ME}}^2 + C_{L_{ME}}^2}}} \approx \sqrt{\frac{2\left(W/S\right)}{\rho}\sqrt{\frac{\varepsilon}{3C_{D_o}}}} \approx 0.76 V_{L/D_{\max}}$$

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CLIMBING FLIGHT

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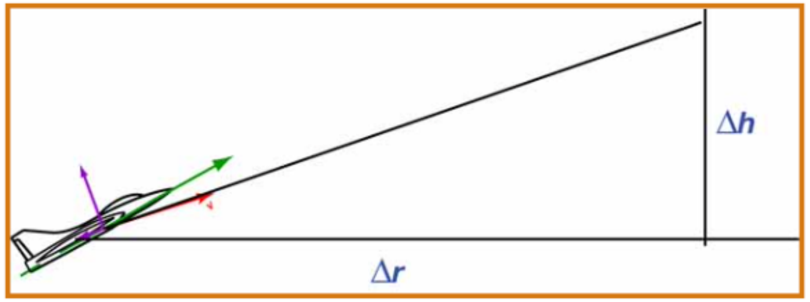


Climbing Flight

- **Flight path angle**

$$\dot{V} = 0 = \frac{(T - D - W\sin\gamma)}{m}$$

$$\sin\gamma = \frac{(T - D)}{W}; \quad \gamma = \sin^{-1}\frac{(T - D)}{W}$$

- **Required lift**

$$\dot{\gamma} = 0 = \frac{(L - W\cos\gamma)}{mV}$$

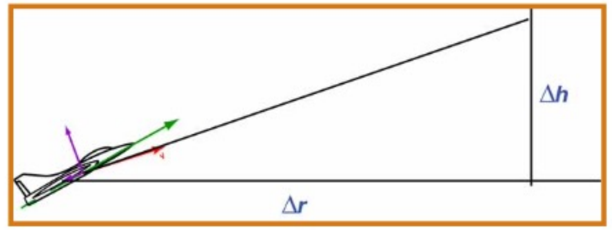
$$L = W\cos\gamma$$

Rate of climb, *dh/dt* = Specific Excess Power

$$\dot{h} = V\sin\gamma = V\frac{(T - D)}{W} = \frac{(P_{thrust} - P_{drag})}{W}$$

$$\textit{Specific Excess Power (SEP)} = \frac{\textit{Excess Power}}{\textit{Unit Weight}} \equiv \frac{(P_{thrust} - P_{drag})}{W}$$

Steady Rate of Climb



Climb rate

$$\dot{h} = V\sin\gamma = V\left[\left(\frac{T}{W}\right) - \frac{\left(C_{D_o} + \varepsilon C_L^2\right)\bar{q}}{(W/S)}\right]$$

$$L = C_L\bar{q}S = W\cos\gamma$$

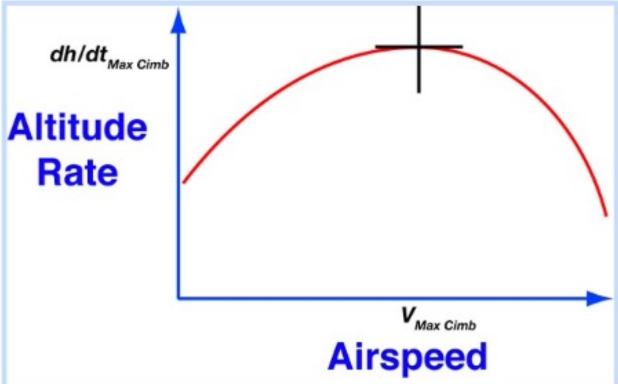
$$C_L = \left(\frac{W}{S}\right)\frac{\cos\gamma}{\bar{q}}$$

$$V = \sqrt{2\left(\frac{W}{S}\right)\frac{\cos\gamma}{C_L\rho}}$$

Note significance of thrust-to-weight ratio and wing loading

$$\dot{h} = V\left[\left(\frac{T}{W}\right) - \frac{C_{D_o}\bar{q}}{(W/S)} - \frac{\varepsilon(W/S)\cos^2\gamma}{\bar{q}}\right]$$

$$= V\left(\frac{T(h)}{W}\right) - \frac{C_{D_o}\rho(h)V^3}{2(W/S)} - \frac{2\varepsilon(W/S)\cos^2\gamma}{\rho(h)V}$$

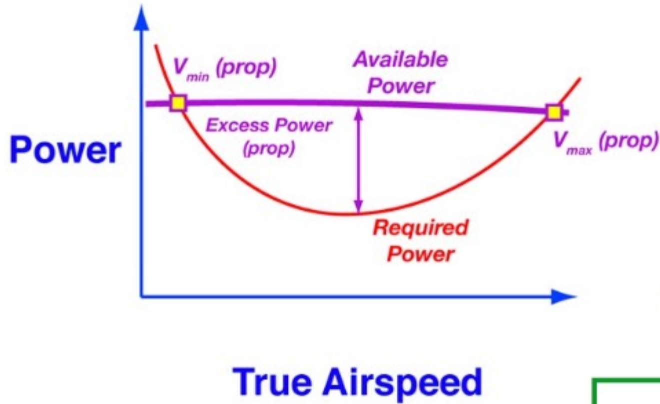


Condition for Maximum Steady Rate of Climb

$$\dot{h} = V\left(\frac{T}{W}\right) - \frac{C_{D_o}\rho V^3}{2(W/S)} - \frac{2\varepsilon (W/S)\cos^2\gamma}{\rho V}$$

Necessary condition for a maximum with respect to airspeed

$$\frac{\partial \dot{h}}{\partial V} = 0 = \left[\left(\frac{T}{W}\right) + V\left(\frac{\partial T / \partial V}{W}\right)\right] - \frac{3C_{D_o}\rho V^2}{2(W/S)} + \frac{2\varepsilon (W/S)\cos^2\gamma}{\rho V^2}$$



Maximum Steady Rate of Climb:

Propeller-Driven Aircraft

- **At constant power**

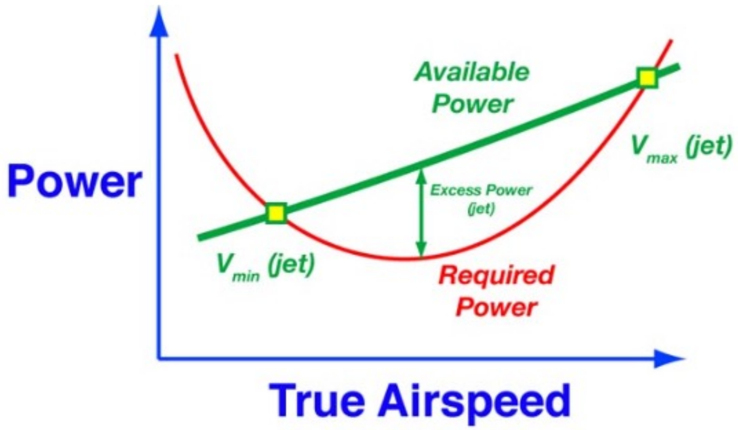
$$\boxed{\frac{\partial P_{thrust}}{\partial V} = 0 = \left[\left(\frac{T}{W}\right) + V\left(\frac{\partial T / \partial V}{W}\right)\right]}$$

- **With $\cos^2\gamma \sim 1$, optimality condition reduces to**

$$\boxed{\frac{\partial \dot{h}}{\partial V} = 0 = -\frac{3C_{D_o}\rho V^2}{2(W/S)} + \frac{2\varepsilon(W/S)}{\rho V^2}}$$

- **Airspeed for maximum rate of climb at maximum power, P_{max}**

$$\boxed{V^4 = \left(\frac{4}{3}\right)\frac{\varepsilon(W/S)^2}{C_{D_o}\rho^2}; \quad V = \sqrt{2\frac{(W/S)}{\rho}\sqrt{\frac{\varepsilon}{3C_{D_o}}}} = V_{ME}}$$



Maximum Steady Rate of Climb: Jet-Driven Aircraft

Condition for a maximum at constant thrust and $\cos^2\gamma \sim 1$

$$\boxed{\frac{\partial \dot{h}}{\partial V} = 0}$$

$$\boxed{-\frac{3C_{D_o}\rho}{2(W/S)}V^4 + \left(\frac{T}{W}\right)V^2 + \frac{2\varepsilon(W/S)}{\rho} = 0}$$

$$\boxed{-\frac{3C_{D_o}\rho}{2(W/S)}\left(V^2\right)^2 + \left(\frac{T}{W}\right)\left(V^2\right) + \frac{2\varepsilon(W/S)}{\rho} = 0}$$

Quadratic in V^2

Airspeed for maximum rate of climb at maximum thrust, T_{max}

$$\boxed{0 = ax^2 + bx + c \ and \quad V = +\sqrt{x}}$$

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FLIGHT ENVELOPE

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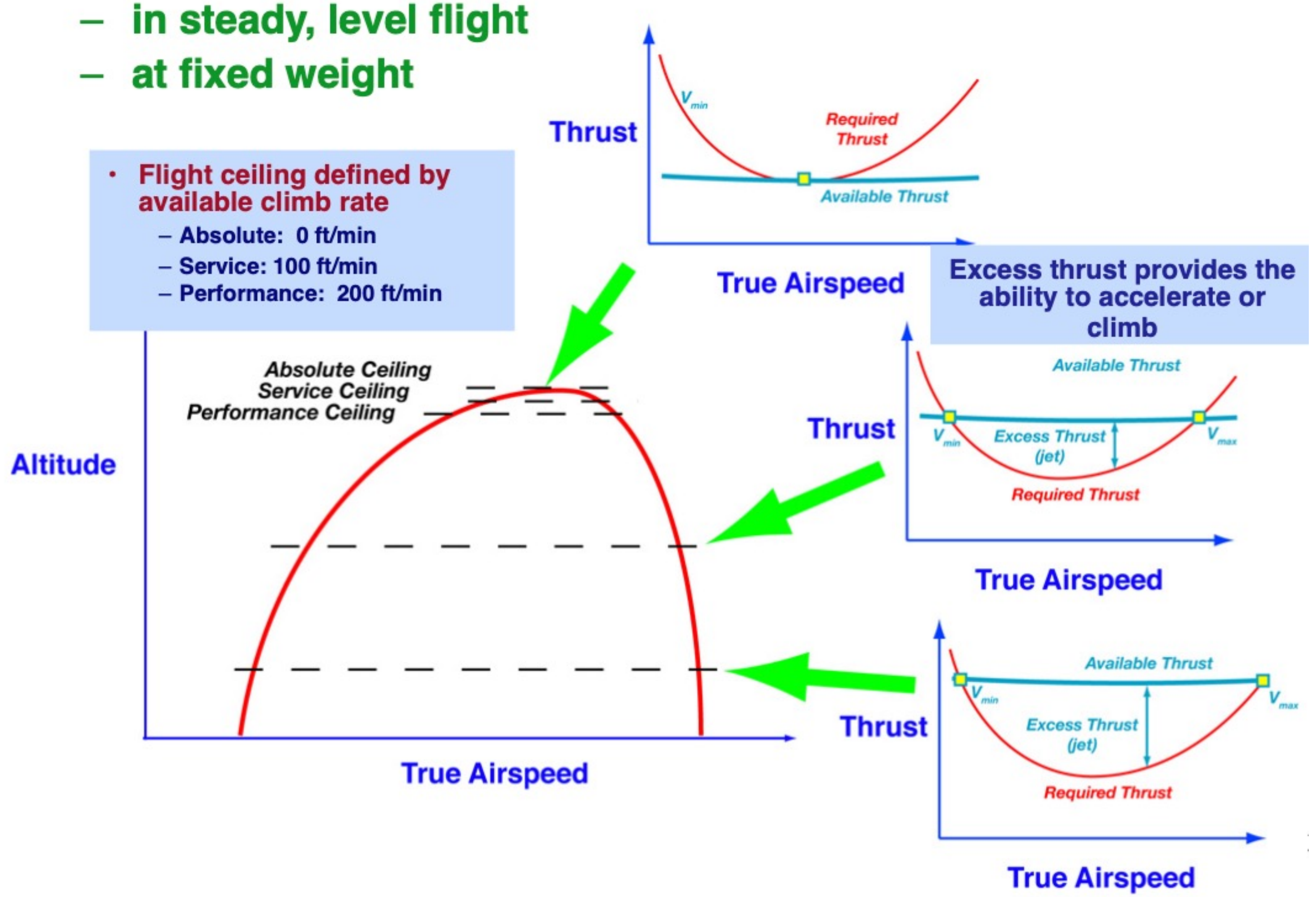
Flight Envelope Determined by Available Thrust

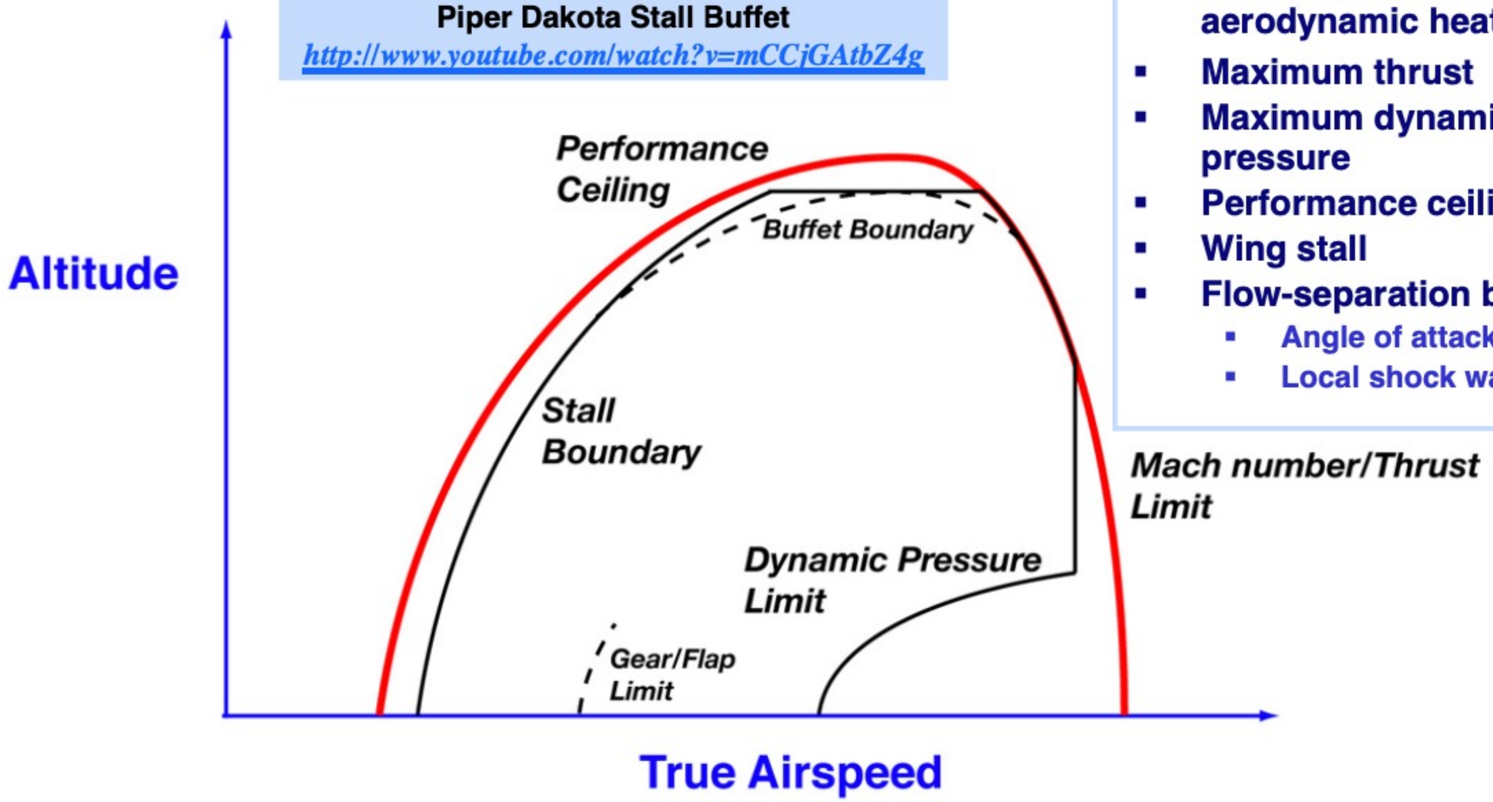
- **All altitudes and airspeeds at which an aircraft can fly**
  - in steady, level flight
  - at fixed weight



Additional Factors Define the Flight Envelope

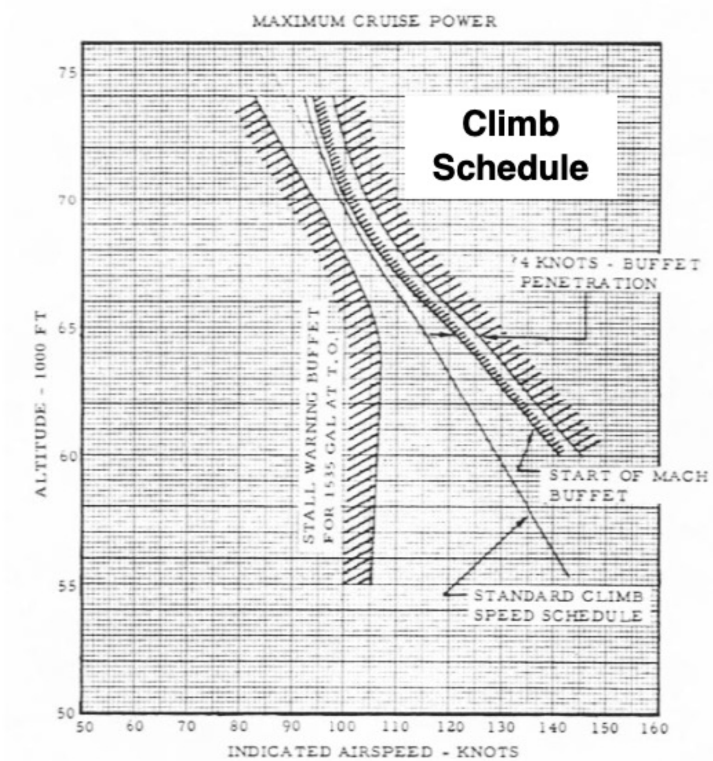
Piper Dakota Stall Buffet
http://www.youtube.com/watch?v=mCCjGAtbZ4g

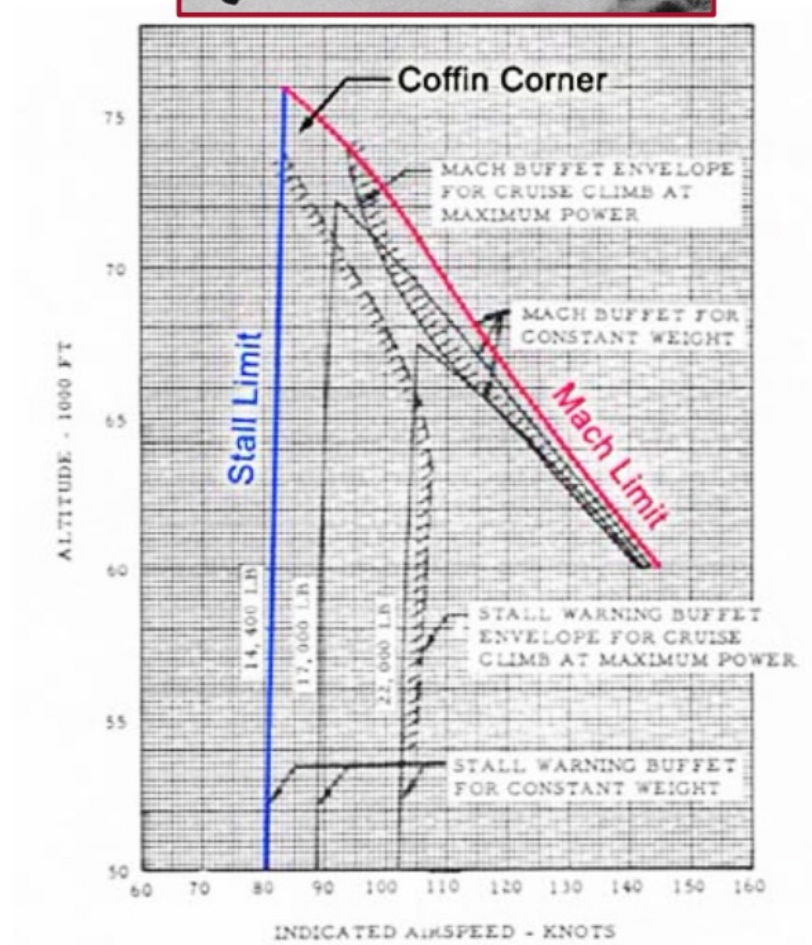
- Maximum Mach number
- Maximum allowable aerodynamic heating
- Maximum thrust
- Maximum dynamic pressure
- Performance ceiling
- Wing stall
- Flow-separation buffet
  - Angle of attack
  - Local shock waves



Lockheed U-2 "Coffin Corner"

Stall buffeting and Mach buffeting are limiting factors
Narrow corridor for safe flight





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MANEUVERING

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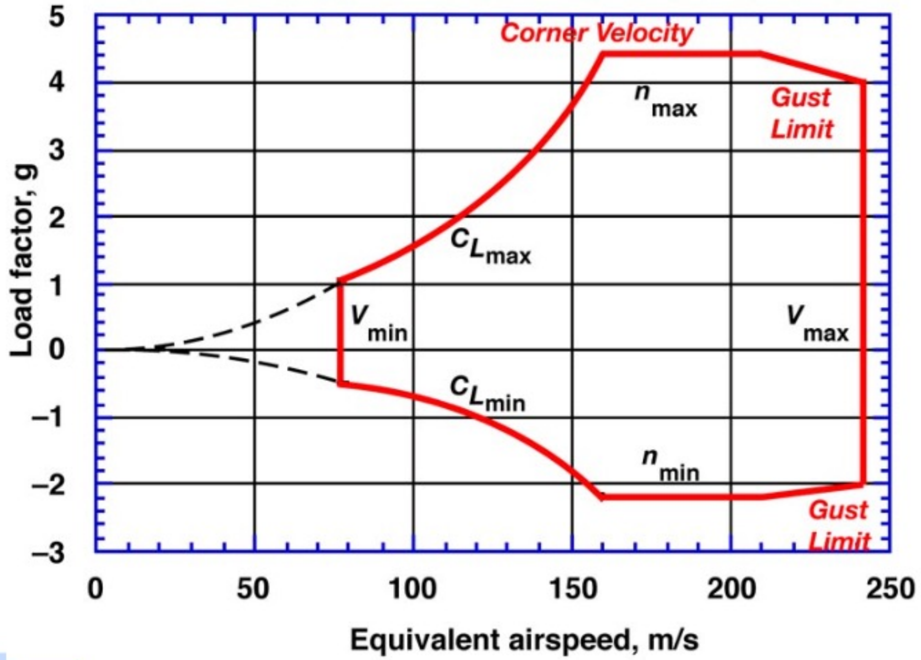
Typical Maneuvering Envelope: *V-n* Diagram

- **Maneuvering envelope**: limits on normal load factor and allowable equivalent airspeed
  - Structural factors
  - Maximum and minimum achievable lift coefficients
  - Maximum and minimum airspeeds
  - Protection against overstressing due to gusts
  - Corner Velocity: Intersection of maximum lift coefficient and maximum load factor



- Typical positive load factor limits
  - Transport: > 2.5
  - Utility: > 4.4
  - Aerobatic: > 6.3
  - Fighter: > 9

- Typical negative load factor limits
  - Transport: < –1
  - Others: < –1 to –3

Level Turning Flight

- Level flight = constant altitude
- Sideslip angle = 0
- Vertical force equilibrium

$$\boxed{L\cos\mu = W}$$

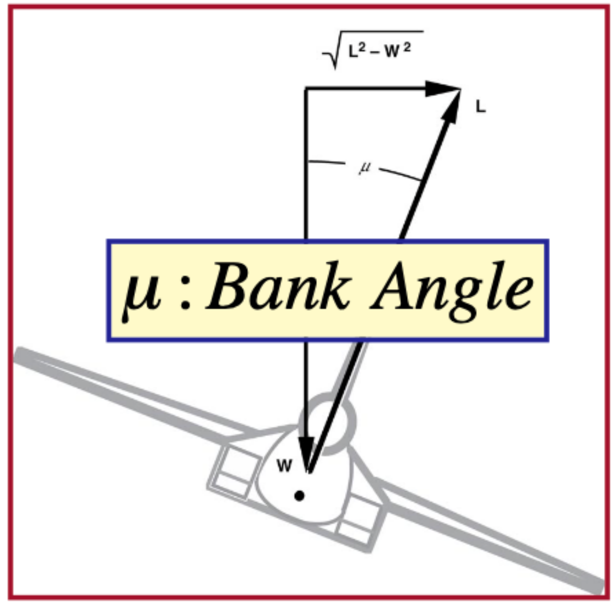
- Load factor

$$\boxed{n = L/W = L/mg = \sec\mu, \text{"}g\text{"}s}$$

- Thrust required to maintain level flight

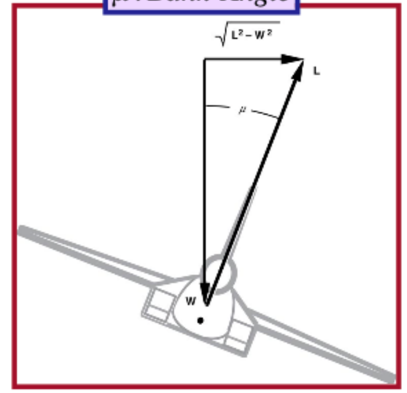
$$T_{req} = \left(C_{D_o} + \varepsilon C_L{}^2\right)\frac{1}{2}\rho V^2 S = D_o + \frac{2\varepsilon}{\rho V^2 S}\left(\frac{W}{\cos\mu}\right)^2$$

$$= D_o + \frac{2\varepsilon}{\rho V^2 S}(nW)^2$$



Maximum Bank Angle in Steady Level Flight

$\mu : Bank\ Angle$



Bank angle

$$\cos\mu = \frac{W}{C_L \bar{q} S} = \frac{1}{n} = W\sqrt{\frac{2\varepsilon}{\left(T_{req} - D_o\right)\rho V^2 S}}$$

$$\mu = \cos^{-1}\left(\frac{W}{C_L \bar{q} S}\right) = \cos^{-1}\left(\frac{1}{n}\right) = \cos^{-1}\left[W\sqrt{\frac{2\varepsilon}{\left(T_{req} - D_o\right)\rho V^2 S}}\right]$$

Bank angle is limited by

$$\boxed{C_{L_{\max}} \ or\ T_{\max} \ or\ n_{\max}}$$

Turning Rate and Radius in Level Flight

Turning rate

$$
\begin{aligned}
\dot{\xi} &= \frac{C_L \bar{q} S \sin\mu}{mV} \\
&= \frac{W \tan\mu}{mV} \\
&= \frac{g \tan\mu}{V} \\
&= \frac{\sqrt{L^2 - W^2}}{mV} \\
&= \frac{W\sqrt{n^2 - 1}}{mV} \\
&= \frac{\sqrt{\left(T_{req} - D_o\right)\rho V^2 S / 2\varepsilon - W^2}}{mV}
\end{aligned}
$$

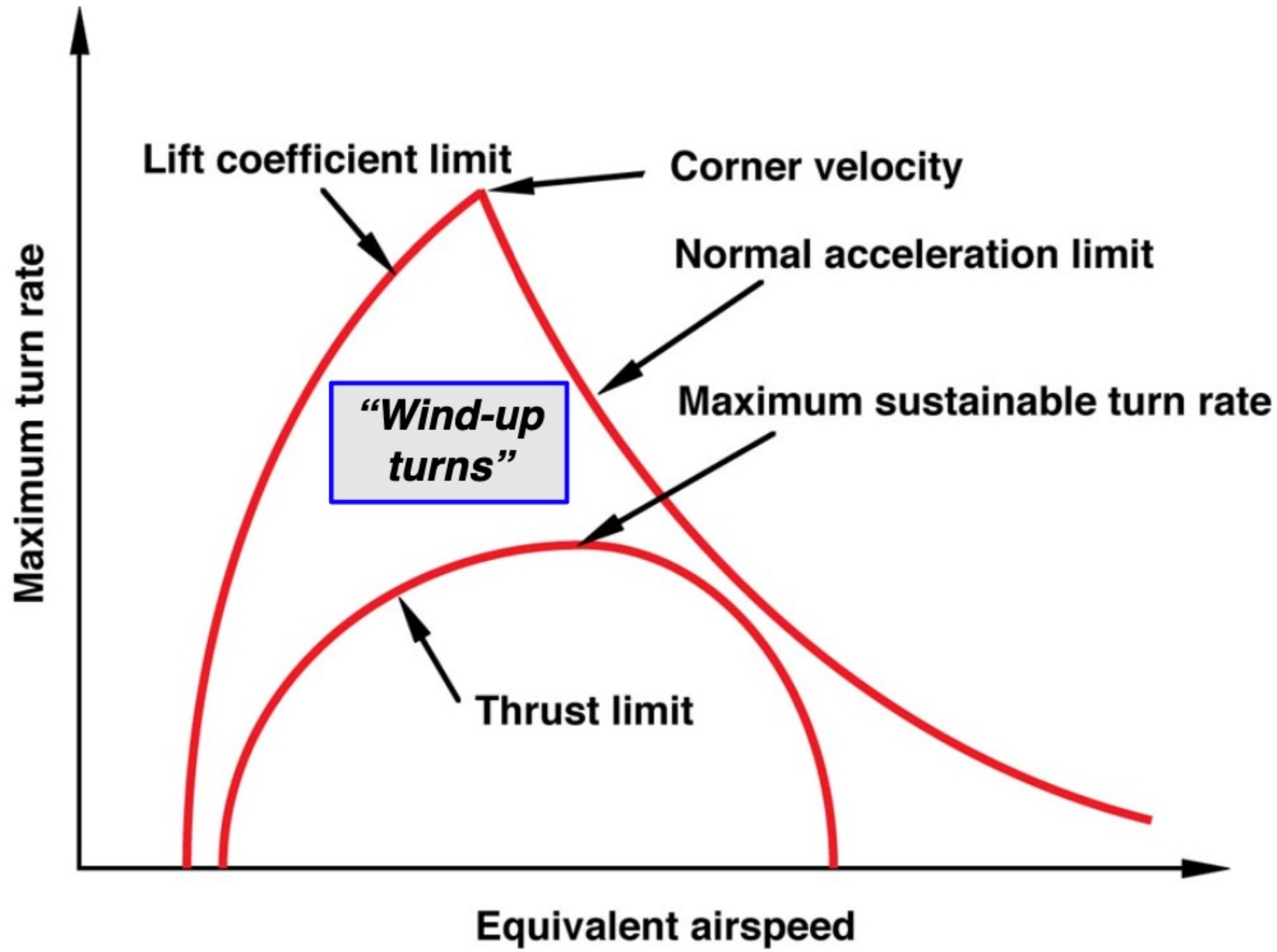
Turning rate is limited by

$$C_{L_{\max}} \; or \; T_{\max} \; or \; n_{\max}$$

Turning radius

$$R_{turn} = \frac{V}{\dot{\xi}} = \frac{V^2}{g\sqrt{n^2 - 1}}$$

Maximum Turn Rates



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