The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

14 NOVEMBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Term Project
  - Launch Vehicle Dynamics
  - Pre-Reading for Next Class

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TERM PROJECT

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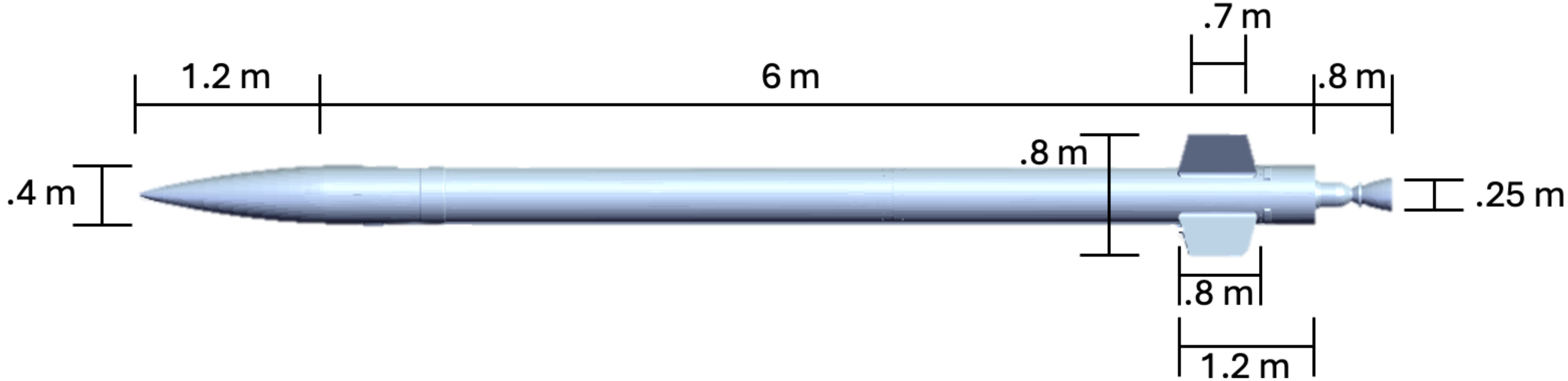
The Rocket

- Static Thrust: 15.5 kN
- Tolerances: +/- 2mm



The Challenge

how likely is that value going to be.

as you burn fuel, mass will change.

- Compute pdf (probability density function) for the location of the CG and CP as a function of remaining rocket mass.
  - Include the effects of uncertainty in dimensions and mass of the rocket on the location of the points.

What does CP and CG mean for our staging.

turbulence

- Develop model for the atmosphere as a function of altitude including turbulence (e.g., Dryden Wind Turbulence Model)
  - Include the effects of turbulence on the winds at various altitudes.
- Develop Monte Carlo simulation model for the rocket trajectory.

Central limit theorem:

If you measure noise so many

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LAUNCH VEHICLE DYNAMICS

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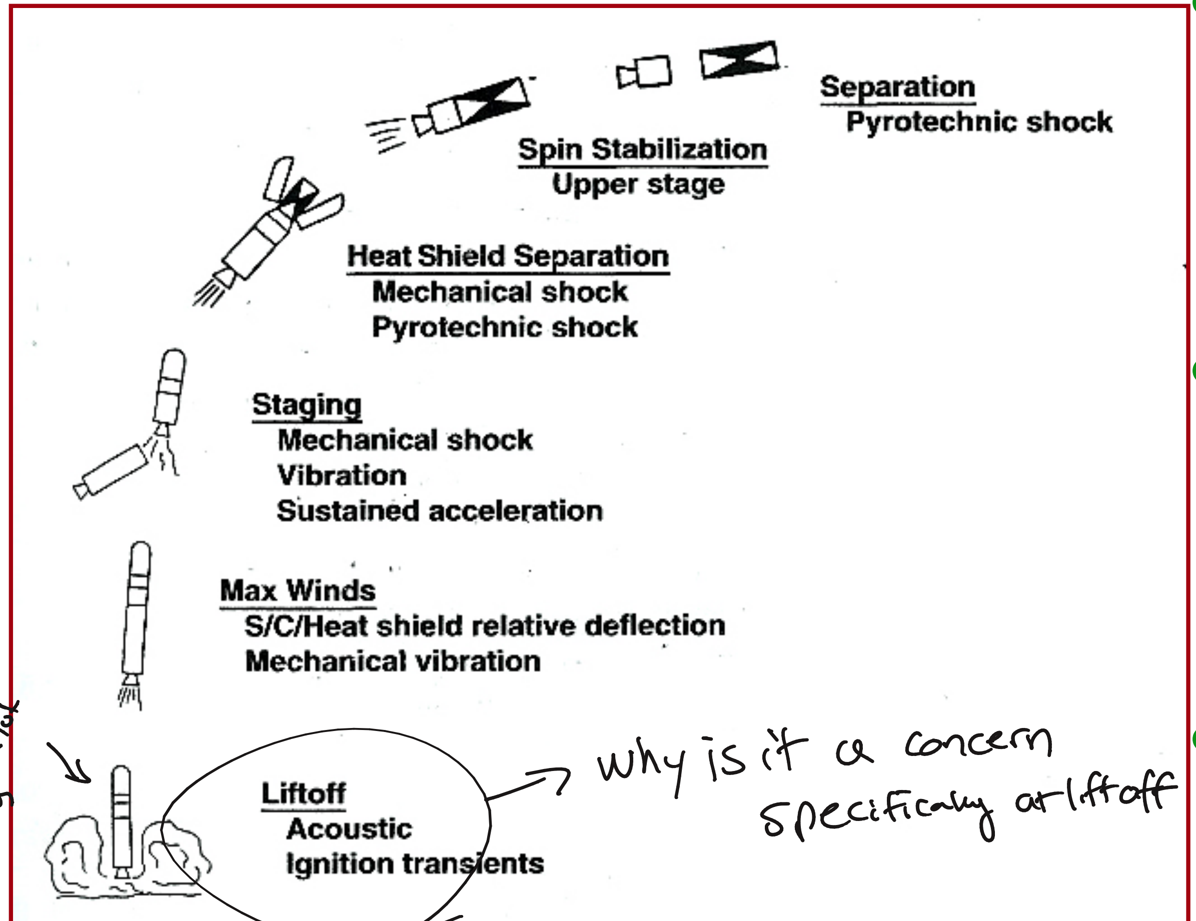
Launch Phases and Loading Issues-1



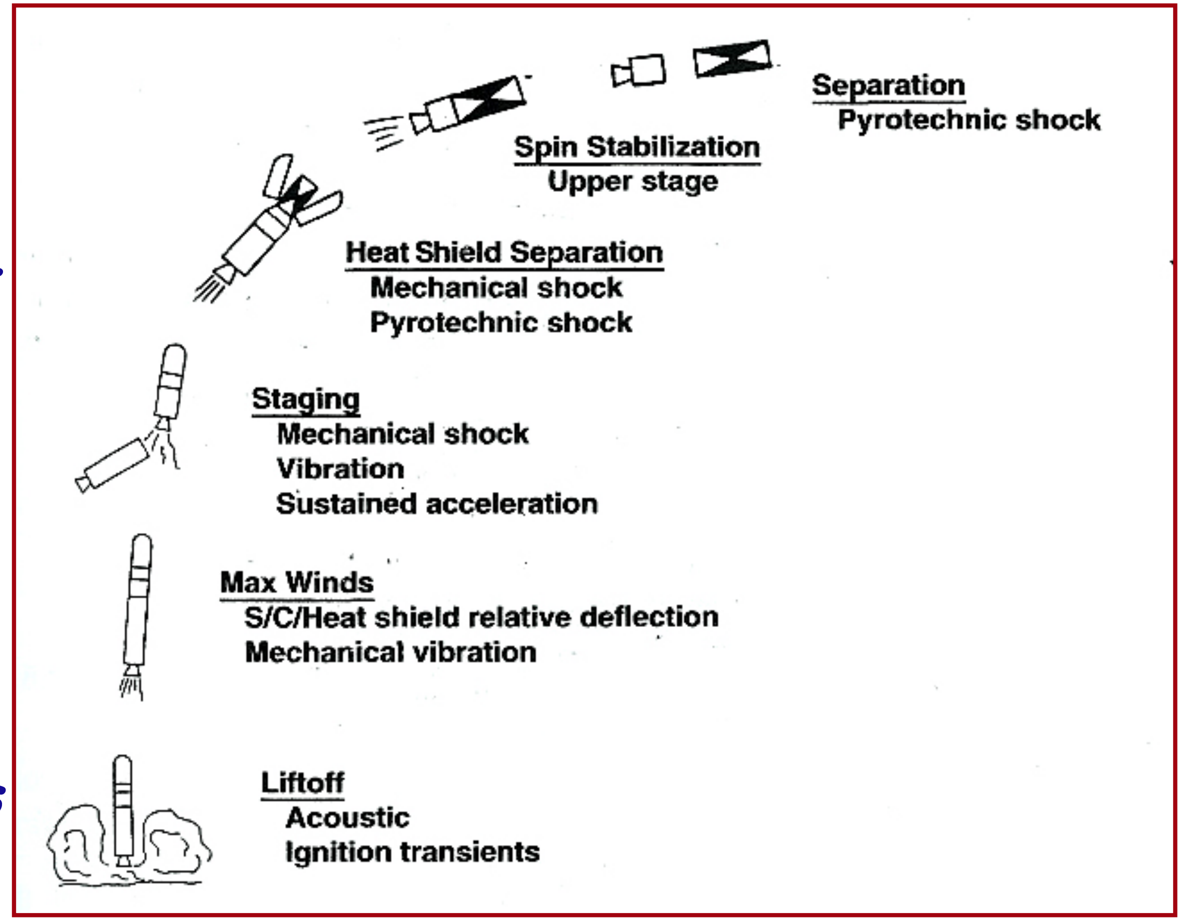
why is it a concern specifically at liftoff

Payload could get a lot of vibrations at this stage

- **Liftoff**
  - Reverberation from the ground
  - Random vibrations
  - Thrust transients
- **Winds and Transonic Aerodynamics**
  - High-altitude jet stream
  - Buffeting
- **Staging**
  - High sustained acceleration
  - Thrust transients

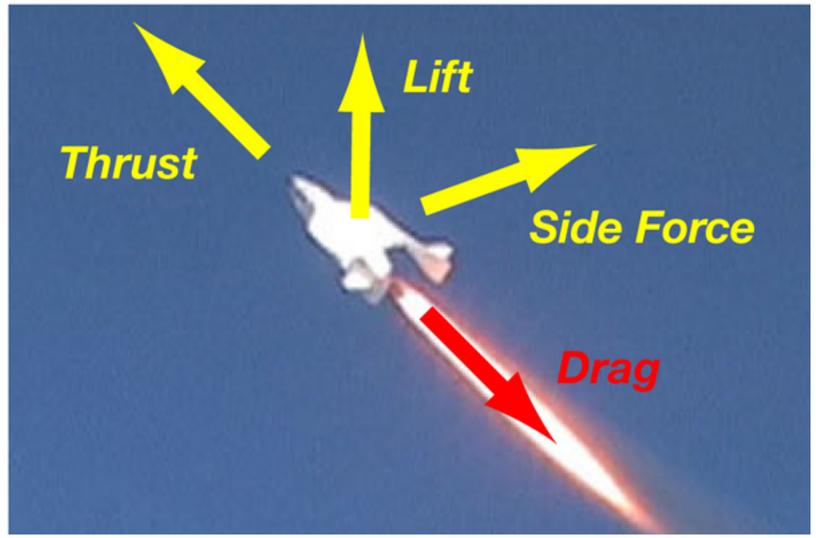
Launch Phases and Loading Issues-2

- Heat shield separation
  - Mechanical and pyrotechnic transients
- Spin stabilization
  - Tangential and centripetal acceleration
  - Steady-state rotation
- Separation
  - Pyrotechnic transients



Aerodynamic Forces

Symmetric



from diameter area of circle

$$\begin{bmatrix} Drag \\ Side\ Force \\ Lift \end{bmatrix} = \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} \frac{1}{2} \rho V^2 S$$

- V = air-relative velocity = velocity w.r.t. air mass
- Drag measured opposite to the air-relative velocity vector
- Lift and side force are perpendicular to the velocity vector

Aerodynamic Force Parameters

$$\rho = \textbf{air density}, \text{ function of height, } h$$

$$= \rho_{sea level} e^{-\beta h}$$

scale height

$$\rho_{sea level} = 1.225\, kg/m^3; \; \beta = 1/9{,}042\, m$$

$$V = \left[v_x^{\,2} + v_y^{\,2} + v_z^{\,2}\right]^{1/2} = \left[\mathbf{v}^T \mathbf{v}\right]^{1/2}, m/s$$

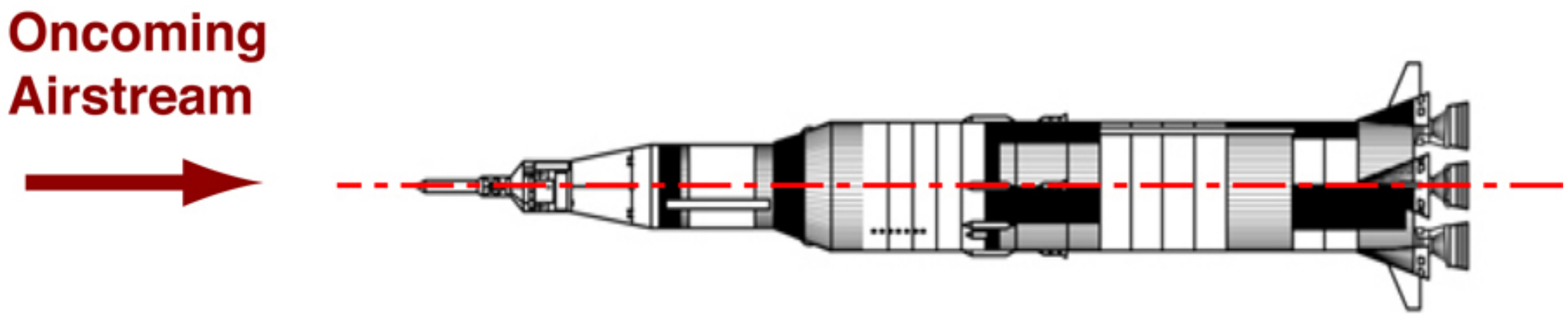
$$\bar{q} = \frac{1}{2}\rho V^2 = \textbf{Dynamic pressure}, N/m^2$$

$$S = \textbf{reference area}, \; m^2$$

$$\begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} = \textbf{non-dimensional aerodynamic coefficients}$$

Aerodynamic Drag

$$\boxed{Drag = C_D \frac{1}{2} \rho V^2 S}$$

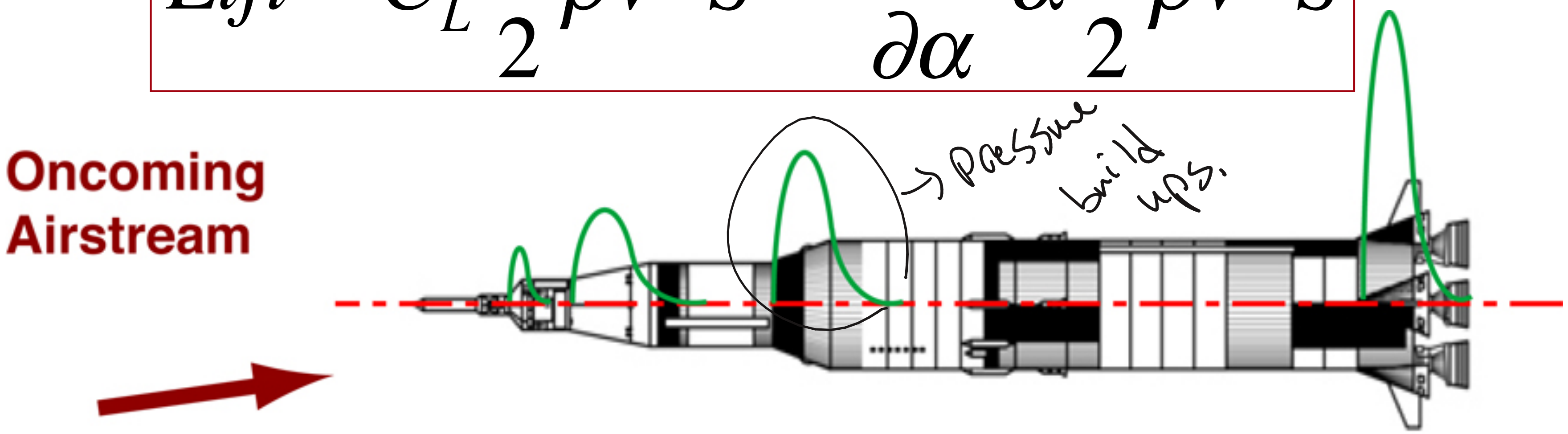


- **Drag components sum to produce total drag**
  - Parallel to airstream
  - Skin friction — No slip condition $V_{stream} = 0$ at the skin.
  - Base pressure differential
  - Forebody pressure differential ($M > 1$)

Aerodynamic Lift Force

$$Lift = C_L \frac{1}{2}\rho V^2 S \approx \frac{\partial C_L}{\partial \alpha}\alpha\frac{1}{2}\rho V^2 S$$

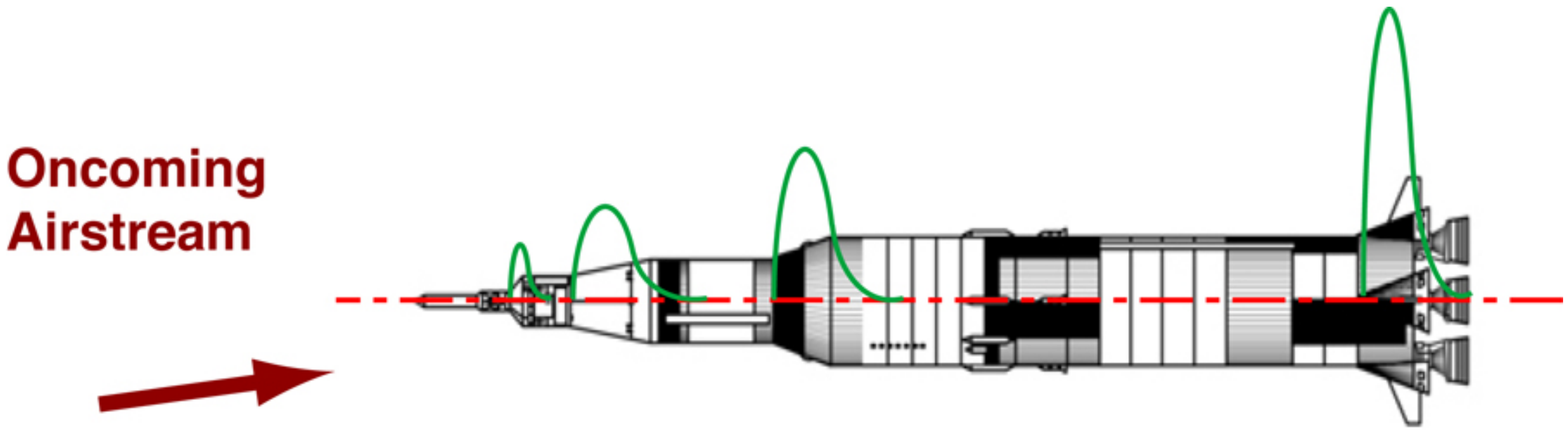
if $\alpha = 0$ Symmetry causes No lift



- *Perpendicular to airstream*
- *Angle between x axis and airstream = angle of attack, α*
- *Lift components integrate over length to produce net lift*
  - *Increase in cross-sectional area*
  - *Tail fins*
- *For symmetric vehicle, lift = 0 if α = 0*

Normal Force about equal to Lift

$$Normal\ Force = C_N \frac{1}{2}\rho V^2 S \approx \frac{\partial C_N}{\partial \alpha}\alpha \frac{1}{2}\rho V^2 S$$



- *Perpendicular to body centerline*
- *For small angle of attack, normal force is approximately the same as lift*

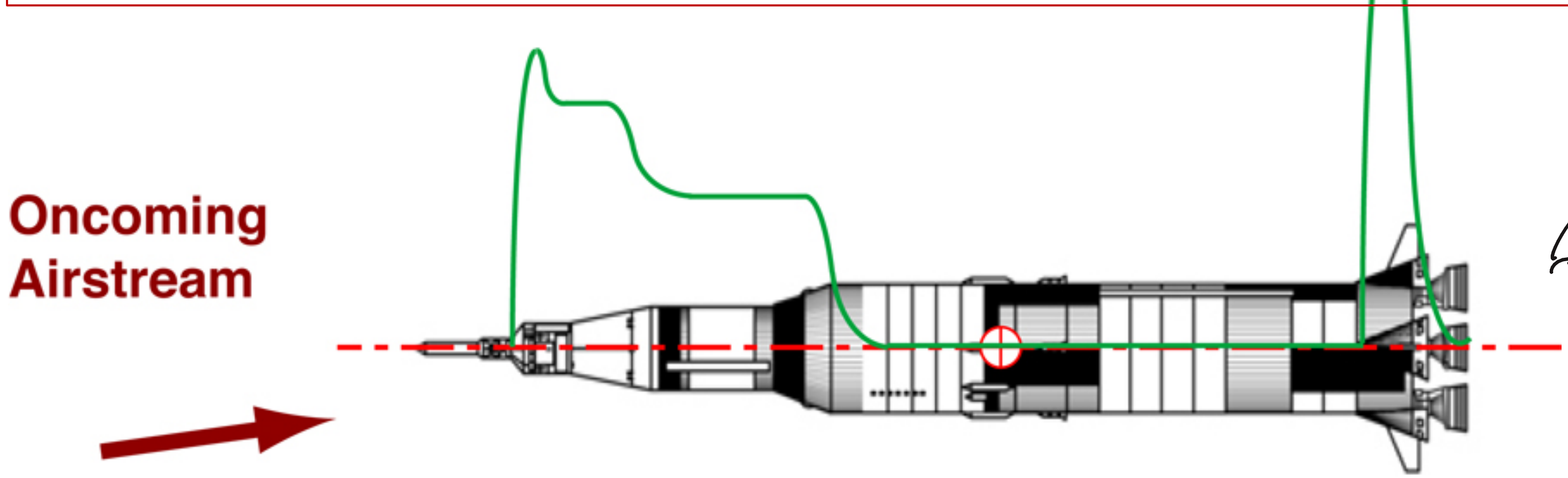
Aerodynamic Pitching Moment

I_B = constant

$$Pitching\ \ Moment = C_m \frac{1}{2}\rho V^2 Sr \approx \frac{\partial C_m}{\partial \alpha}\alpha\frac{1}{2}\rho V^2 Sr$$

$$r = \text{Reference Length}$$

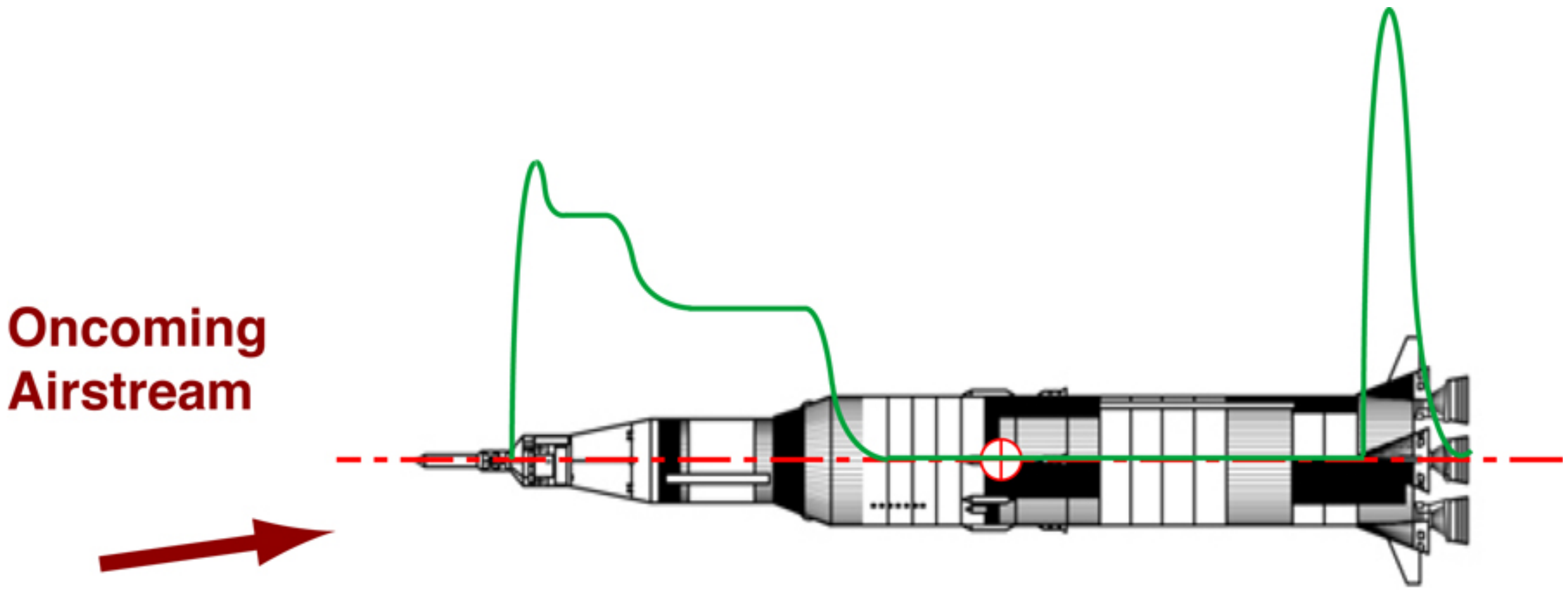
lift × moment arm



- *Pitching moment components integrate over length to produce net pitching moment*
  - *Increase in cross-sectional area*
  - *Tail fins*

- *... plus pitching moment due to thrust vectoring for control*

Pitching Moment Distribution Causes Large Bending Effects

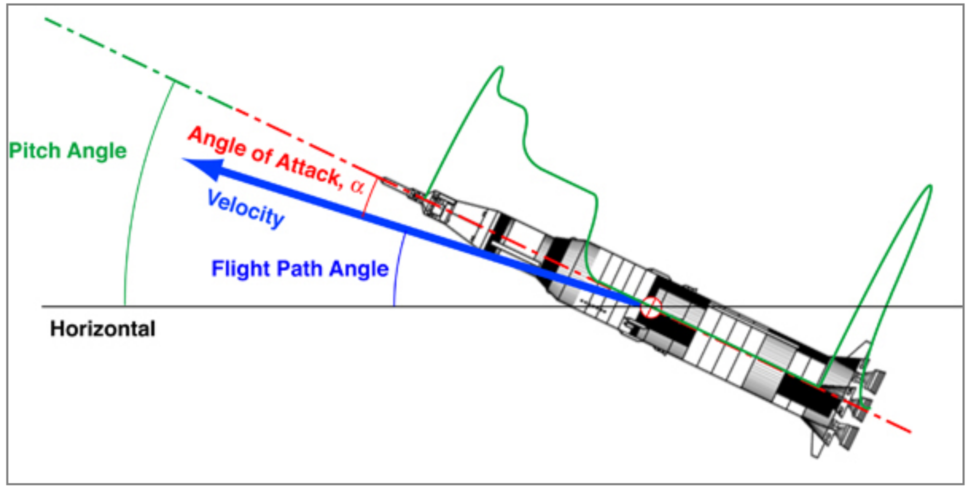


ignore it

Aerodynamic and thrust-vectoring effects
<u>bend the vehicle</u>

Trajectory shaped to reduce structural loads

Angular Attitude Perturbations



- ***Pitch-angle perturbation, $\Delta\theta$, is about the same as angle-of-attack perturbation, $\Delta\alpha$***

Compute lift distributions w.r.t. α
moment

$$\boxed{\Delta\ddot{\theta} \approx \Delta\ddot{\alpha} = \frac{Net\ Pitching\ Moment}{Pitching\ Moment\ of\ Inertia}}$$

$$\boxed{\Delta\ddot{\alpha} = \frac{M_{y_{aero}} + M_{y_{thrust}}}{I_{yy}} \approx \frac{1}{I_{yy}}\left[\frac{\partial M_{y_{net}}}{\partial\dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M_{y_{net}}}{\partial\alpha}\Delta\alpha\right]}$$

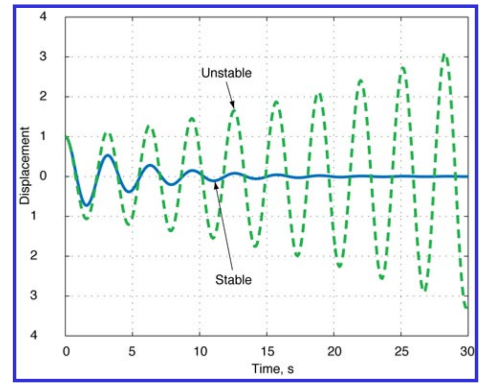
damping

Attitude Stability

$$\Delta\ddot{\alpha} = \frac{1}{I_{yy}}\left[\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M_{y_{net}}}{\partial \alpha}\Delta\alpha\right]$$

- **Attitude perturbations are stable if**

$$\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}} < 0, \quad \frac{\partial M_{y_{net}}}{\partial \alpha} < 0$$



- **Oscillatory divergence if**

$$\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}} > 0 \quad \textit{Dynamic Instability}$$

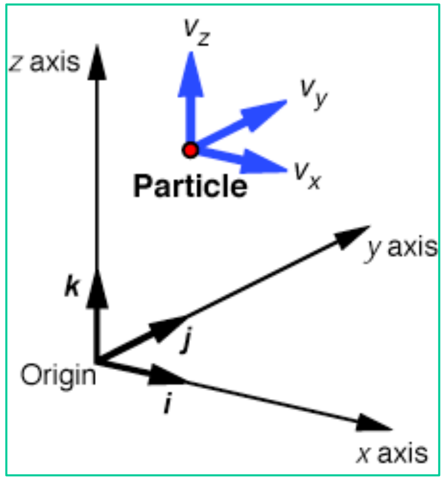
- **Non-oscillatory divergence if**

$$\frac{\partial M_{y_{net}}}{\partial \alpha} > 0 \quad \textit{Static Instability}$$

Thrust-vector feedback control normally required to provide static and dynamic stability

Equations of Motion for a Point Mass

$$\frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \mathbf{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$



$$\frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \frac{1}{m}\mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

Equations of Motion for a Point Mass

Velocity and position dynamics expressed in a single equation

$$\boxed{\dot{\mathbf{x}}(t) = \frac{d\mathbf{x}(t)}{dt} = \mathbf{f}[\mathbf{x}(t), \mathbf{F}]}$$

function of position

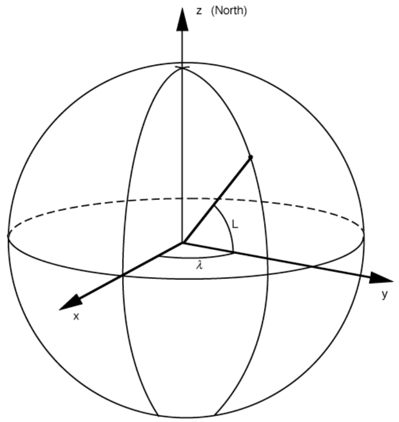
$$\mathbf{x} \equiv \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix}$$

Combined Equations of Motion for a Point Mass

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{v}_x\\ \dot{v}_y\\ \dot{v}_z\end{bmatrix}_I = \begin{bmatrix}v_x\\ v_y\\ v_z\\ f_x/m\\ f_y/m\\ f_z/m\end{bmatrix}_I = \begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ v_x\\ v_y\\ v_z\end{bmatrix}_I + \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\1/m&0&0\\0&1/m&0\\0&0&1/m\end{bmatrix}\begin{bmatrix}f_x\\ f_y\\ f_z\end{bmatrix}_I$$

With

$$\mathbf{F}_I = \begin{bmatrix}f_x\\ f_y\\ f_z\end{bmatrix}_I = \left[\mathbf{F}_{gravity} + \mathbf{F}_{aerodynamics} + \mathbf{F}_{thrust}\right]_I$$



Math Models of Gravity

$\Omega^2 z - \Omega^2 \cdot r$

gravity decreases as inverse-square law

- **Flat-earth approximation**
  - g is gravitational acceleration
  - mg is gravitational force
  - Independent of position
- **Round, rotating earth**
  - Inverse-square gravitation
  - "Centripetal acceleration"
  - Non-linear function of position
  - $\mu = 3.986 \times 10^5 \ km^3/s^2$
  - $\Omega = 7.29 \times 10^{-5}$ rad/s

Vector triple product prop.

$$m\mathbf{g}_f = m\begin{bmatrix} 0 \\ 0 \\ g_o \end{bmatrix} \quad ; \quad g_o = 9.807\ m/s^2$$

$$\mathbf{g}_r = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \mathbf{g}_{gravity} \quad \text{[non-rotating frame]}$$

Earth rotation rate

$$\mathbf{g}_r = \mathbf{g}_{gravity} + \mathbf{g}_{rotation} \quad \text{[rotating frame]}$$

$\rightarrow \Omega \times (\Omega \times r)$

$$= \frac{-\mu}{r^3}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \Omega^2 \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \quad ; \quad r = \left[x^2 + y^2 + z^2\right]^{1/2}$$

this extra r is from normalizing gravity vector to point to center of earth

$\hat{r} = \frac{\mathbf{r}}{r} = \frac{[x, y, z]^T}{r}$

$\Downarrow$

$\frac{\mu}{r^2}\hat{r}$ unit vector

$\frac{\mu}{r^2}$ = magnitude of gravity

Equations of Motion with Round-Earth Gravity Model
(Inertial, Non-Rotating Frame)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix}_E = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\mu/r^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu/r^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu/r^3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix}_I + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \left[\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}_{aero} + \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}_{thrust} \right]_I$$

Position of the vehicle (in spherical coordinates)

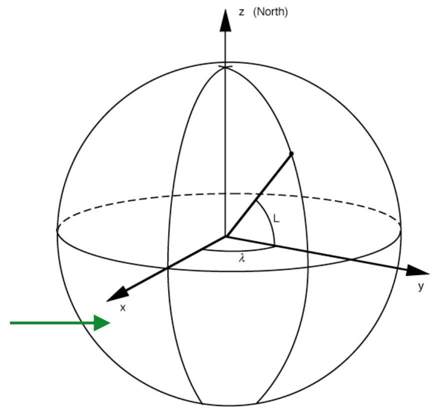
$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos L \cos \lambda \\ \cos L \sin \lambda \\ \sin L \end{bmatrix} (R + h)$$

R : *Earth's radius*

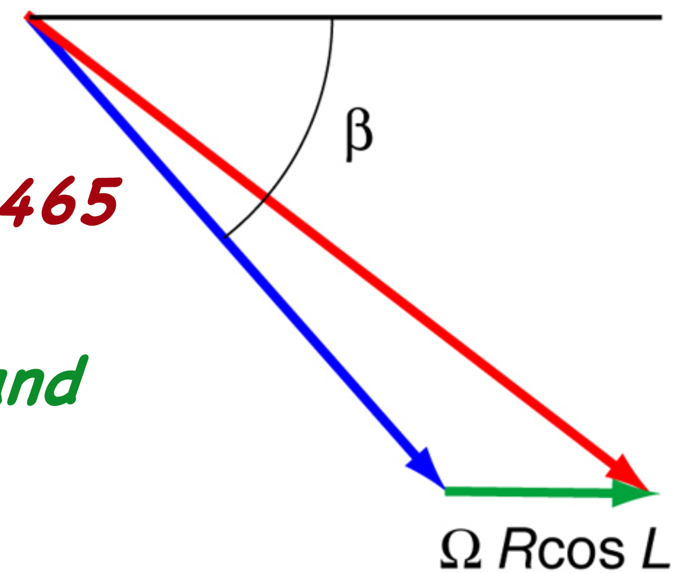
h : *Altitude (height)*

L : *Latitude*

λ : *Longitude*



Effect of Launch Site on Launch Velocity



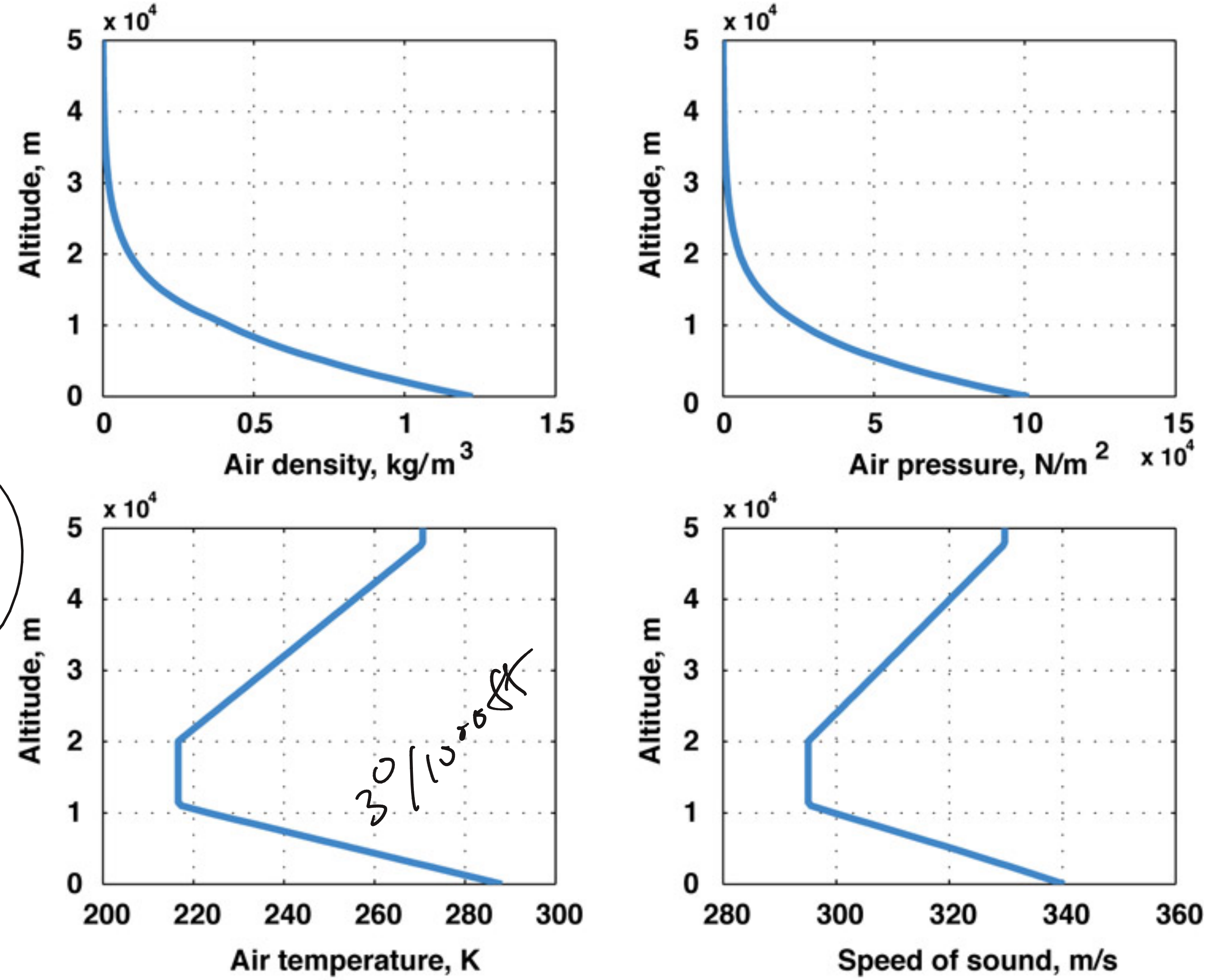
- Launch site and azimuth
  - Earth's rotation adds up to 465 m/s to final inertial velocity
  - Function of launch latitude and azimuth angles

$$\Delta V_{launch} \approx \Omega R \cos L \cos \beta$$

β : Launch azimuth angle (rotating frame, from East)

Properties of the Lower Atmosphere

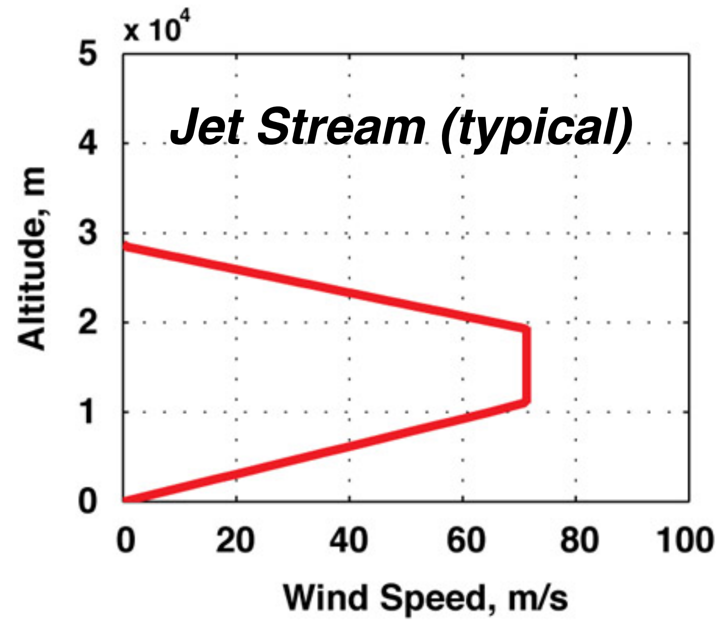
- **Air density** and **pressure** decay exponentially with altitude
- **Air temperature** and **speed of sound** are linear functions of altitude



US Standard Atmosphere, 1976

Lower Atmosphere Rotates With The Earth

- Zero wind at Earth's surface = Inertially rotating air mass
- Wind measured with respect to Earth's rotating surface
- Jet stream magnitude typically peaks at 10-15-km altitude



Flat-Earth 2-D Equations of Motion for a Point Mass

Restrict motions to a vertical plane (i.e., motions in y direction = 0)

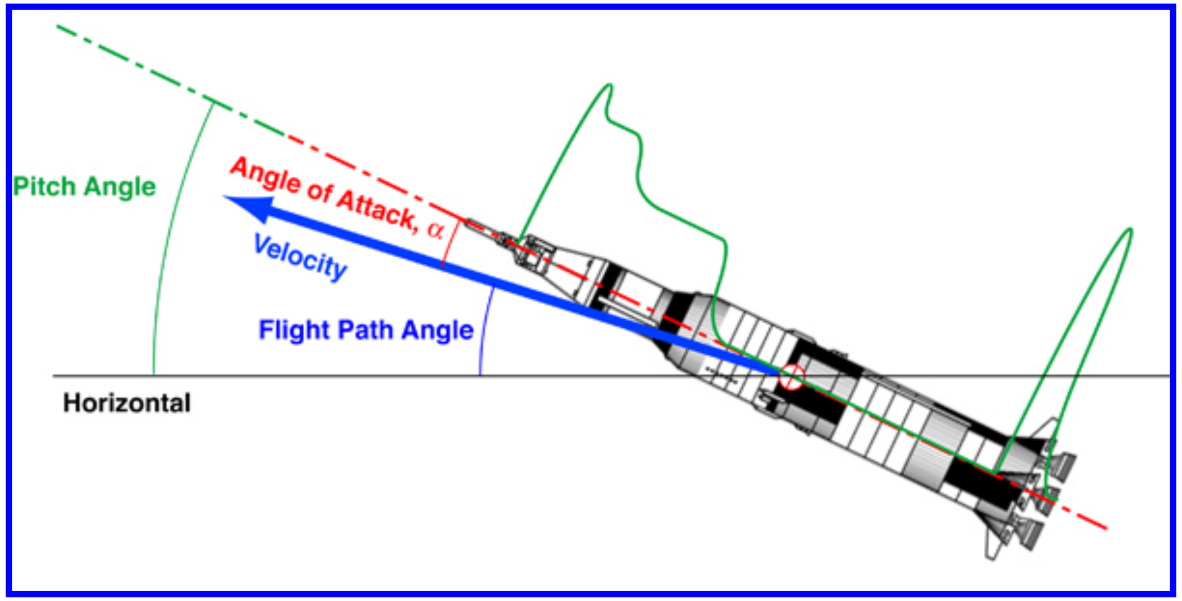
$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_z \\ f_x / m \\ f_z / m \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ v_x \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_z \end{bmatrix}$$

Flat-Earth 2-D Equations of Motion for a Point Mass

Transform velocity from Cartesian to polar coordinates

$$\begin{bmatrix} \dot{x} \\ -\dot{z} \end{bmatrix} \triangleq \begin{bmatrix} \dot{r} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} V\cos\gamma \\ V\sin\gamma \end{bmatrix}$$

$$\begin{bmatrix} V \\ \gamma \end{bmatrix} = \begin{bmatrix} \sqrt{\dot{x}^2 + \dot{z}^2} \\ \sin^{-1}\left(\frac{\dot{h}}{V}\right) \end{bmatrix} = \begin{bmatrix} Velocity \\ Flight\ path\ angle \end{bmatrix}$$



Flat-Earth Model

- Ignore round, rotating Earth effects (!)
- i.e., assume that flat-Earth-relative frame is inertial

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_z \\ f_x / m \\ f_z / m \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ v_x \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_z \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} V \cos\gamma \\ -V \sin\gamma \end{bmatrix}$$

$$\begin{bmatrix} V \\ \gamma \end{bmatrix} = \begin{bmatrix} \sqrt{\dot{x}^2 + \dot{z}^2} \\ -\sin^{-1}\left(\dfrac{\dot{z}}{V}\right) \end{bmatrix} = \begin{bmatrix} \mathit{Velocity} \\ \mathit{Flight\ path\ angle} \end{bmatrix}$$

- Adequate model for investigating early phase of launch

Simplified Launch Trajectory Equations of Motion

- **Gravity-turn, flat earth, vertical plane**
  - Thrust aligned with velocity vector ($\alpha = 0$)
  - Lift = 0
  - Round, rotating earth effects neglected

$$\dot{V}(t) = \frac{Thrust - \left[Drag + m(t)g\sin\gamma(t)\right]}{m(t)}$$

$$= \left[\left(Thrust - C_D \frac{1}{2}\rho(h)V^2(t)\right)\Big/ m(t) - g\sin\gamma(t)\right]$$

$$\dot{\gamma}(t) = -g\cos\gamma(t)/V(t)$$

$$\dot{h}(t) = -\dot{z}(t) = V(t)\sin\gamma(t)$$

$$\dot{r}(t) = \dot{x}(t) = V(t)\cos\gamma(t)$$

$V = velocity$

$\gamma = flight\ path\ angle$

$h = height\ (altitude)$

$r = range$

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PRE-READING FOR NEXT CLASS

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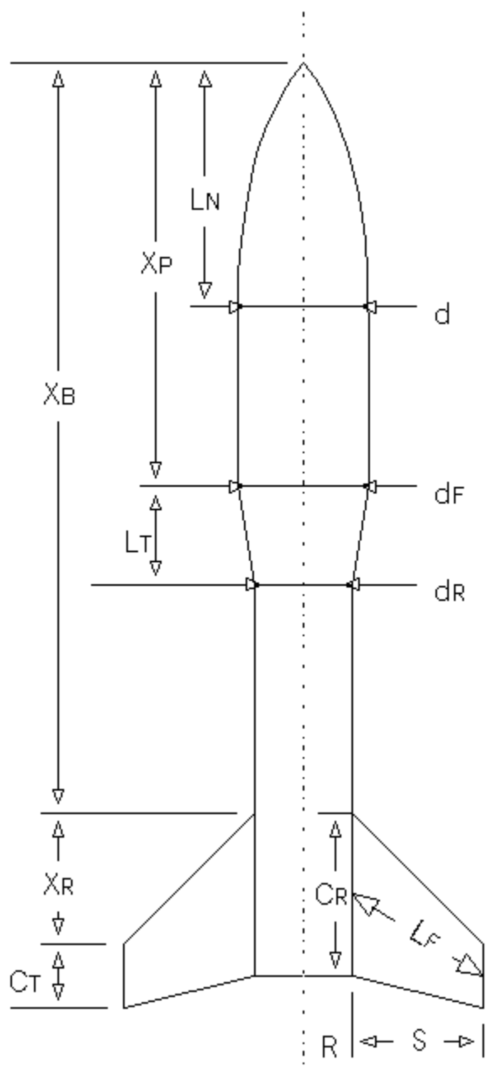
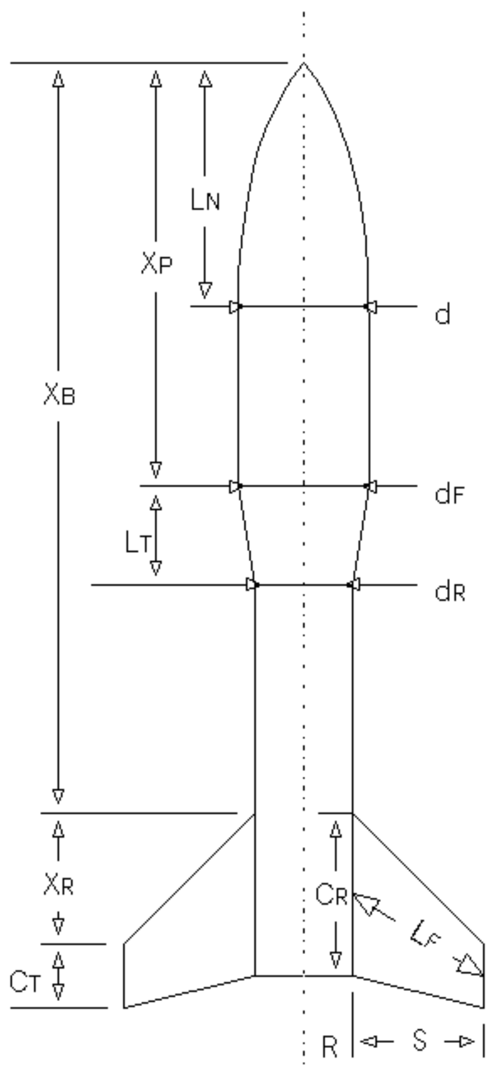
Barrowman Equations



L_N = length of nose
d = diameter at base of nose
d_F = diameter at front of transition
d_R = diameter at rear of transition
L_T = length of transition
X_P = distance from tip of nose to front of transition
C_R = fin root chord
C_T = fin tip chord
S = fin semispan
L_F = length of fin mid-chord line
R = radius of body at aft end
X_R = distance between fin root leading edge and fin tip leading edge parallel to body
X_B = distance from nose tip to fin root chord leading edge
N = number of fins

Barrowman Equations



$$\bar{X} = \frac{(C_N)_N X_N + (C_N)_T X_T + (C_N)_F X_F}{(C_N)_R}$$

NOSE

$(C_N)_N = 2$
For Cone: $X_N = 0.666L_N$
For Ogive: $X_N = 0.466L_N$

TRANSITION

$$(C_N)_T = 2\left[\left(\frac{d_R}{d}\right)^2 - \left(\frac{d_F}{d}\right)^2\right]$$

$$X_T = X_P + \frac{L_T}{3}\left[1 + \frac{1 - \frac{d_F}{d_R}}{1 - \left(\frac{d_F}{d_R}\right)^2}\right]$$

FIN

$$(C_N)_F = \left[1 + \frac{R}{S+R}\right]\left[\frac{4N\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2L_F}{C_R + C_T}\right)^2}}\right]$$

$$X_F = X_B + \frac{X_R}{3}\frac{(C_R + 2C_T)}{(C_R + C_T)} + \frac{1}{6}\left[(C_R + C_T) - \frac{(C_R C_T)}{(C_R + C_T)}\right]$$

Using the Barrowman Equation

- Barrowman Method of Calculating Normal Force
  - https://www.nakka-rocketry.net/RD_Appendix_B.html

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