

**3 SEPTEMBER 2024**

# **ASE 367K: FLIGHT DYNAMICS**

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TTH 09:30-11:00  
CMA 2.306

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# Topics for Today

- Topic(s):

- Lift of entire airplane
- Pitch Moment about arbitrary point
- Longitudinal Stability
- Trim

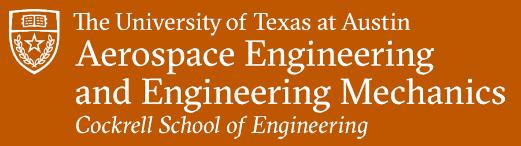
Useful website

airplane characteristic

boeing airport

manual 747-600

Trim = equilibrium



# LIFT OF ENTIRE AIRPLANE

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$$\sum F_x = L_w + D_w \alpha + L_t + D_t \alpha_t - W = 0$$

$$\sum F_x = D_w - L_w \alpha + D_t - L_t \alpha_t + W(\alpha + \gamma) - T = 0$$

$$\sum M = M_p + M_f$$

$$+ M_{\alpha c_w} + (L_w + D_w \alpha)(x_{cg} - x_w) + (L_t + D_t \alpha_t)(x_{cg} - x_t)$$

$$+ M_{\alpha c_f} + (D_w - L_w \alpha)(z_w - z_{cg}) + (D_t - L_t \alpha_t)(z_t - z_{cg})$$

$$D_w = q_s C_{Dw}$$

$$= q_s [C_{Dow} + k C_{lw}]^2$$

$$= q_s [C_{Dow} + k[C'_{low} + C_{\alpha w} \alpha]^2]$$

$$= q_s [C_{Dow} + k[C'_{low}]^2 + 2C'_{low} C_{low} \alpha + C_{\alpha w}^2 \alpha^2]$$

$$= q_s [C_{Dow} + k[C'_{low}]^2 + 2C'_{low} C_{\alpha w} \alpha + HUT]$$

$$\approx q_s [C_{Dow} + k[C'_{low}]^2 + 2k C'_{low} C_{\alpha w} \alpha]$$

$$\approx q_s [C'_{Dow} + 2k C'_{low} C_{\alpha w} \alpha]$$

where  $C'_{Dow} = C_{Dow} + k[C'_{low}]^2$

$\times i \in T$

higher order terms

$$\approx q_s C_{Dow} + 2q_s k C'_{low} C_{\alpha w} \alpha$$

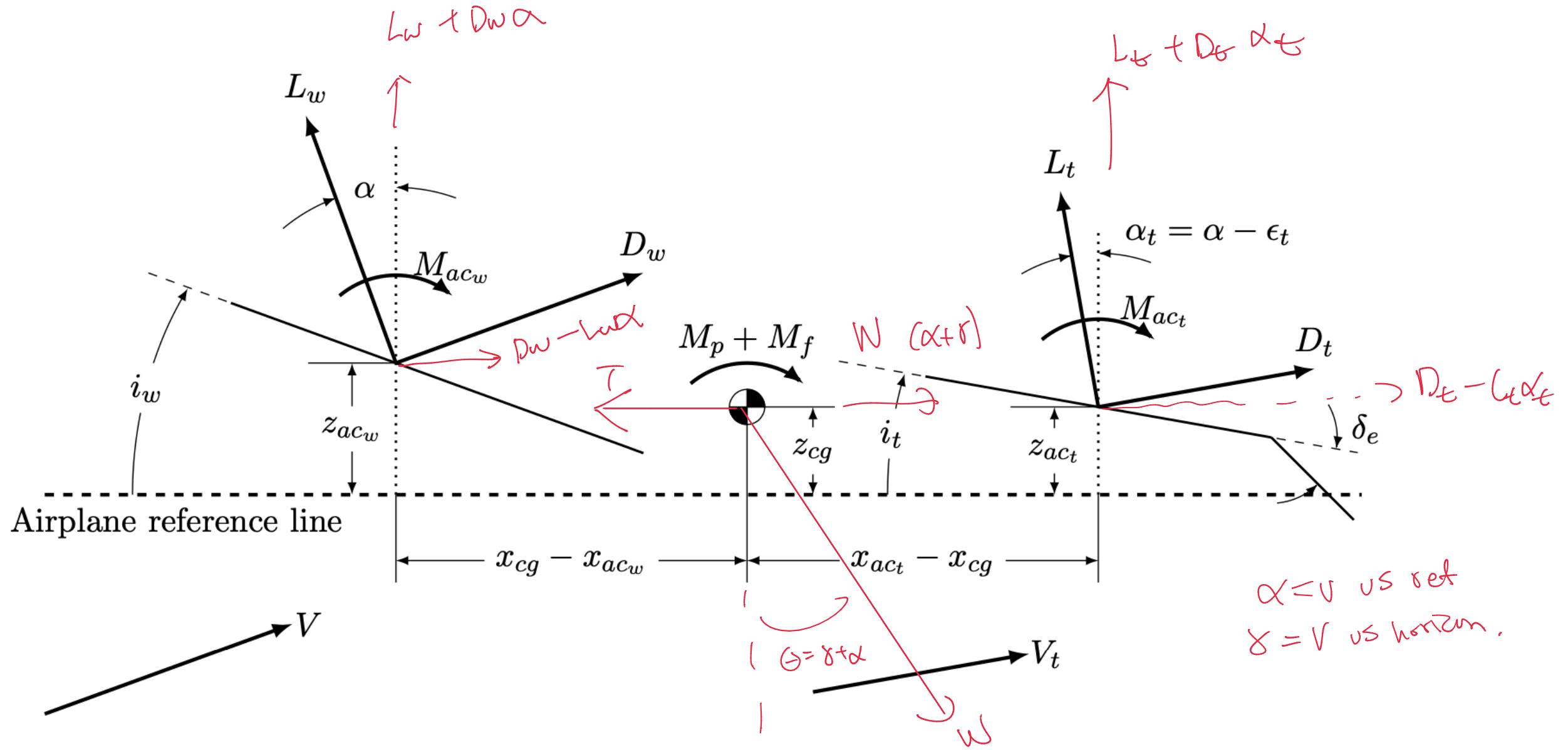


Figure 3.1: Simplified geometry of the wing and tail sections of an aircraft

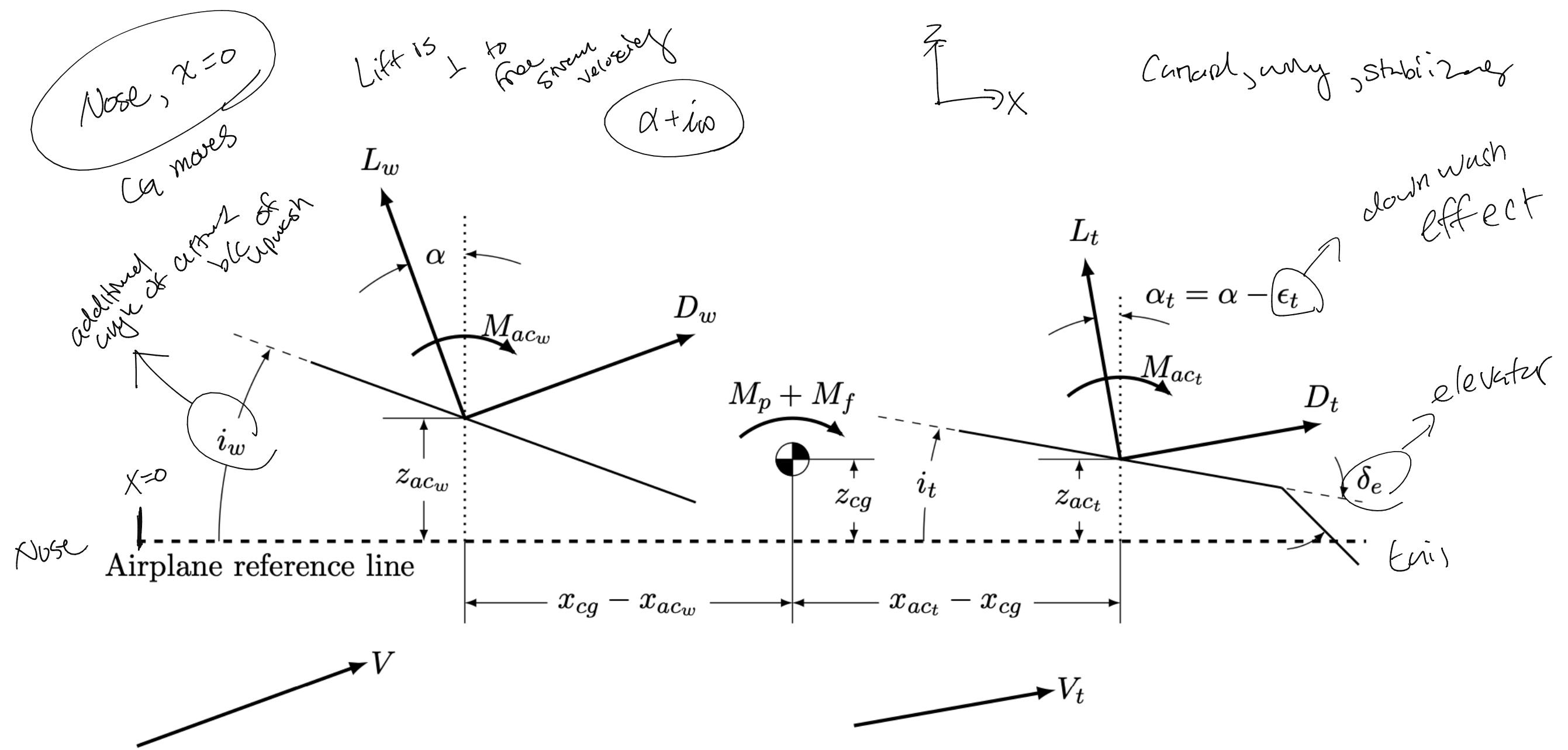


Figure 3.1: Simplified geometry of the wing and tail sections of an aircraft

$$L = L_w + L_t$$

where  $L_w = qSC_{L_w} = qS [C_{L_{0W}} + C_{L\alpha_W}(\alpha + i_w)]$

$$L_t = q_t S_t C_{L_t} = q_t S_t [C_{L_{0t}} + C_{L\alpha_t}(\alpha_t + i_t) + C_{L\delta_e} \delta_e]$$

$$= q_t S_t [C_{L_{0t}} + C_{L\alpha_t}(\alpha - \epsilon_t + i_t) + C_{L\delta_e} \delta_e]$$

... but the downwash angle  $\epsilon_t$  is a function of the angle of attack

$$\epsilon_t = \epsilon_{0t} + \epsilon_{\alpha t} \alpha$$

thus

$$L_t = q_t S_t \left[ C_{L_{0t}} + C_{L_{\alpha t}} [i_t - \epsilon_{0t} + (1 - \epsilon_{\alpha t}) \alpha] + C_{L_{\delta_e}} \delta_e \right]$$

Combining the expressions for  $L_w$  and  $L_t$  we get...

$$L = qS \left[ C_{L0_W} + C_{L\alpha_W}(\alpha + i_w) + \frac{q_t S_t}{qS} \left[ C_{L0_t} + C_{L\alpha_t}[i_t - \epsilon_{0t} + (1 - \epsilon_{\alpha_t})\alpha] + C_{L\delta_e}\delta_e \right] \right]$$

... and an overall lift coefficient

$$C_L = C_{L0_W} + C_{L\alpha_W}(\alpha + i_w) + \frac{q_t S_t}{qS} \left[ C_{L0_t} + C_{L\alpha_t}[i_t - \epsilon_{0t} + (1 - \epsilon_{\alpha_t})\alpha] + C_{L\delta_e}\delta_e \right]$$

# Total Lift Summary

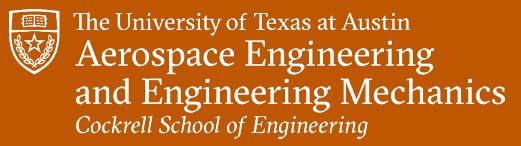
$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{i_t}} i_t + C_{L_{\delta_e}} \delta_e$$

where  $C_{L_0} = C_{L_{0w}} + C_{L_{\alpha w}} i_w + \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (C_{L_{0t}} - C_{L_{\alpha t}} \epsilon_{0t})$

$$C_{L_\alpha} = C_{L_{\alpha w}} + \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} C_{L_{\alpha t}} (1 - \epsilon_{\alpha t})$$

$$C_{L_{i_t}} = \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} C_{L_{\alpha t}}$$

$$C_{L_{\delta_e}} = \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} C_{L_{\delta_e t}}$$



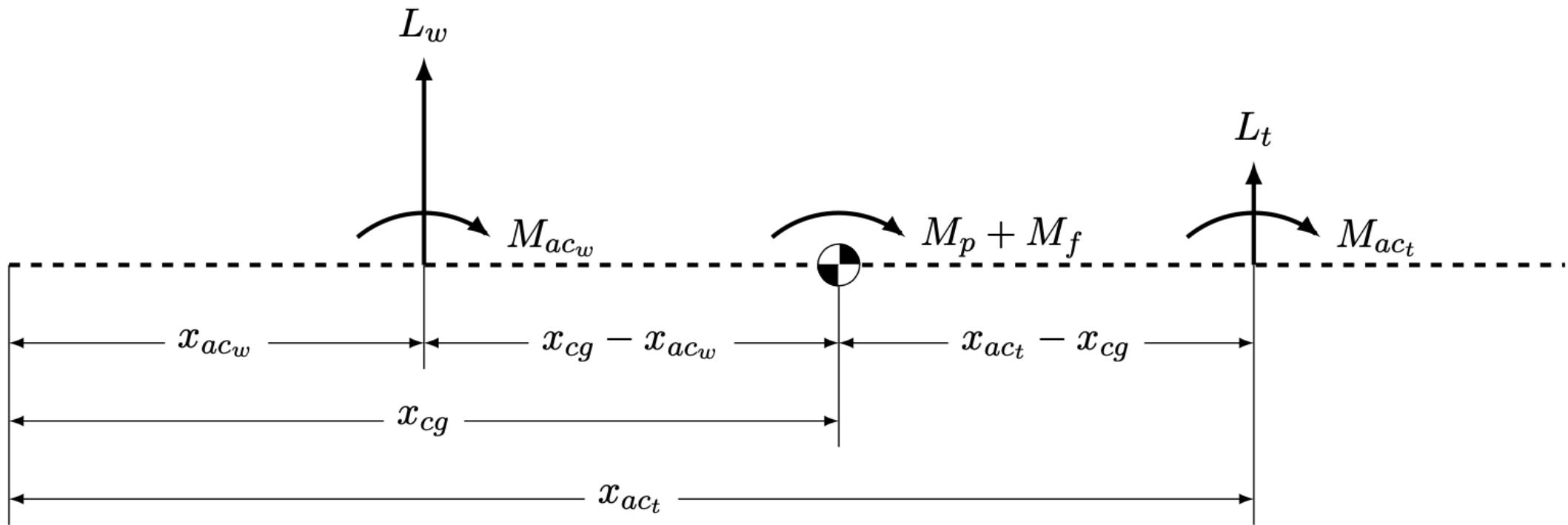
# PITCH MOMENT ABOUT ARBITRARY POINT

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$\Sigma M \text{ Eq} = \text{The } M \text{ is met u}$   
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The moment about an arbitrary reference point along the body  $x$  axis is given by...

$$M_r = M_{ac_w} + L_w(x_r - x_{ac_w}) + M_{act} + L_t(x_r - x_{act}) + M_p + M_f$$

where  $M_{ac_w} = qS\bar{c}C_{M_{ac_w}}$

$$M_{act} = q_t S_t \bar{c}_t C_{M_{act}}$$

$$M_p = qS\bar{c}C_{M_p}$$

mean aero dynamic chord

$$M_f = qS\bar{c}C_{M_f}$$

The corresponding coefficient is given by...

$$\begin{aligned}
 C_{M_r} = & C_{Mac_w} + \left[ C_{L_{0W}} + C_{L_{\alpha_W}} (\alpha + i_w) \right] (\bar{x}_r - \bar{x}_{ac_w}) + \frac{q_t S_t \bar{c}_t}{q S \bar{c}} C_{Mact} \\
 & + \left[ \frac{q_t S_t}{q S} \left[ C_{L_{0t}} + C_{L_{\alpha_t}} [i_t - \epsilon_{0t} + (1 - \epsilon_{\alpha_t})\alpha] + C_{L_{\delta_e}} \delta_e \right] \right] (\bar{x}_r - \bar{x}_{act}) \\
 & + C_{M_{0p}} + C_{M_{\alpha p}} \alpha + C_{M_{0f}} + C_{M_{\alpha f}} \alpha
 \end{aligned}$$

where

$$\bar{x}_r = \frac{x_r}{\bar{c}}$$

$$\bar{x}_{ac_w} = \frac{x_{ac_w}}{\bar{c}}$$

$$\bar{x}_{act} = \frac{x_{act}}{\bar{c}}$$

# Pitch Moment Summary

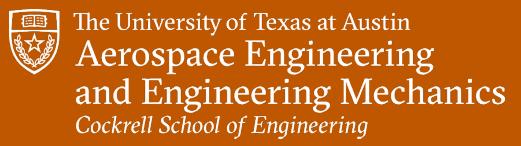
$$C_{M_r} = C_{M_{0r}} + C_{M_{\alpha r}} \alpha + C_{M_{i_{tr}}} i_t + C_{M_{\delta_{er}}} \delta_e$$

$$\begin{aligned} C_{M_{0r}} &= C_{M_{acw}} + \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} \frac{\bar{c}_t}{\bar{c}} C_{M_{act}} + C_{M_{0p}} + C_{M_{0f}} + (\bar{x}_r - \bar{x}_{acw}) (C_{L_{0w}} + C_{L_{\alpha w}} i_w) \\ &\quad + \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_r - \bar{x}_{act}) (C_{L_{0t}} - C_{L_{\alpha t}} \epsilon_{0t}) \end{aligned}$$

$$C_{M_{\alpha r}} = C_{M_{\alpha p}} + C_{M_{\alpha f}} + (\bar{x}_r - \bar{x}_{acw}) C_{L_{\alpha w}} + \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_r - \bar{x}_{act}) C_{L_{\alpha t}} (1 - \epsilon_{\alpha t})$$

$$C_{M_{i_{tr}}} = \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_r - \bar{x}_{act}) C_{L_{\alpha t}}$$

$$C_{M_{\delta_{er}}} = \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_r - \bar{x}_{act}) C_{L_{\delta_{et}}}$$



# LONGITUDINAL STABILITY

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# Aerodynamic Center of Aircraft

- What is so unique about the aerodynamic center?

There is a point where moment is constant.

$$C_{M\alpha ac} = 0$$

moment gradient of moment coeff. is 0.

Moment is constant at this point

Lift and drag are acting

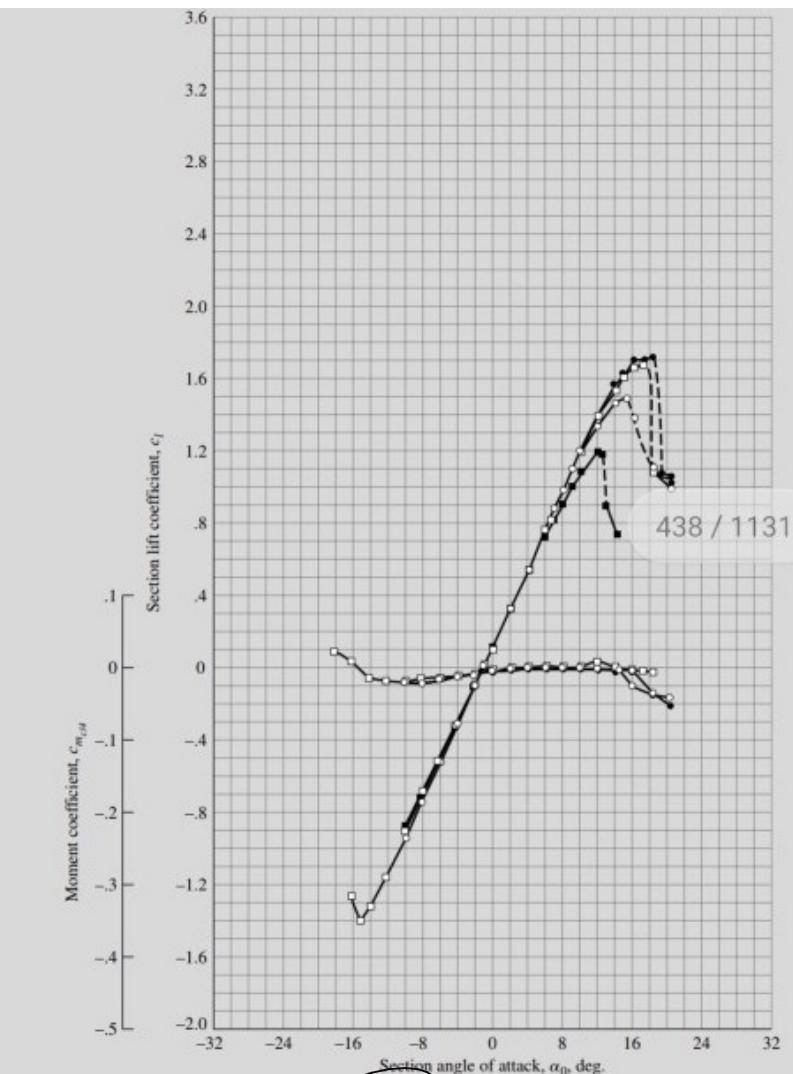
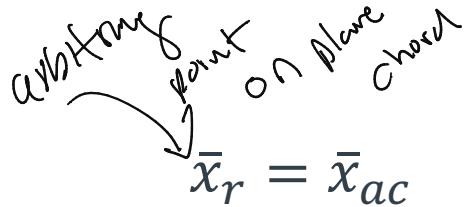


Figure 5.2a Lift coefficient and moment coefficient about the quarter-chord for the NACA 23015 airfoil.

# Aerodynamic Center of Aircraft

- How do we determine the aerodynamic center?



The diagram shows a horizontal line representing a chord of an airfoil. A point is marked on this chord, labeled "arbitrary point on chord". A curved arrow points from this label to the point on the chord.

$$\bar{x}_r = \bar{x}_{ac}$$

$$C_{M_{\alpha_{ac}}} = C_{M_{\alpha_p}} + C_{M_{\alpha_f}} + (\bar{x}_{ac} - \bar{x}_{ac_w}) C_{L_{\alpha_w}} + \frac{q_t S_t}{qS} (\bar{x}_{ac} - \bar{x}_{ac_t}) C_{L_{\alpha_t}} (1 - \epsilon_{\alpha_t}) = 0$$

$$\rightarrow \bar{x}_{ac} \left[ C_{L_{\alpha_w}} + \frac{q_t S_t}{qS} C_{L_{\alpha_t}} (1 - \epsilon_{\alpha_t}) \right] = \bar{x}_{ac_w} C_{L_{\alpha_w}} + \frac{q_t S_t}{qS} \bar{x}_{ac_t} C_{L_{\alpha_t}} (1 - \epsilon_{\alpha_t}) - C_{M_{\alpha_p}} - C_{M_{\alpha_f}}$$

$$\rightarrow \bar{x}_{ac} C_{L_\alpha} = \bar{x}_{ac_w} C_{L_{\alpha_w}} + \frac{q_t S_t}{qS} \bar{x}_{ac_t} C_{L_{\alpha_t}} (1 - \epsilon_{\alpha_t}) - C_{M_{\alpha_p}} - C_{M_{\alpha_f}}$$

$$\rightarrow \bar{x}_{ac} = \frac{1}{C_{L_\alpha}} \left[ \bar{x}_{ac_w} C_{L_{\alpha_w}} + \frac{q_t S_t}{qS} \bar{x}_{ac_t} C_{L_{\alpha_t}} (1 - \epsilon_{\alpha_t}) - C_{M_{\alpha_p}} - C_{M_{\alpha_f}} \right]$$

# Pitch Moment about the CG

$$C_M = C_{M_0} + C_{M_\alpha} \alpha + C_{M_{i_t}} i_t + C_{M_{\delta_e}} \delta_e$$

$$C_{M_0} = C_{M_{ac_w}} + \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} \frac{\bar{c}_t}{\bar{c}} C_{M_{act}} + C_{M_{0_p}} + C_{M_{0_f}} + (\bar{x}_{cg} - \bar{x}_{ac_w}) (C_{L_{0_w}} + C_{L_{\alpha_w}} i_w)$$

$$+ \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_{cg} - \bar{x}_{act}) (C_{L_{0_t}} - C_{L_{\alpha_t}} \epsilon_{0_t})$$

$$C_{M_\alpha} = C_{M_{\alpha_p}} + C_{M_{\alpha_f}} + (\bar{x}_{cg} - \bar{x}_{ac_w}) C_{L_{\alpha_w}} + \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_{cg} - \bar{x}_{act}) C_{L_{\alpha_t}} (1 - \epsilon_{\alpha_t})$$

$$C_{M_{i_t}} = \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_{cg} - \bar{x}_{act}) C_{L_{\alpha_t}}$$

$$C_{M_{\delta_e}} = \frac{\bar{q}_t}{\bar{q}} \frac{S_t}{S} (\bar{x}_{cg} - \bar{x}_{act}) C_{L_{\delta_{e_t}}}$$

# Static Margin

- Can we simplify the equation for  $C_{M_\alpha}$  ?

$$C_{M_\alpha} = C_{M_{\alpha p}} + C_{M_{\alpha f}} + (\bar{x}_{cg} - \bar{x}_{ac_w})C_{L_{\alpha w}} + \frac{q_t S_t}{qS} (\bar{x}_{cg} - \bar{x}_{ac_t})C_{L_{\alpha t}}(1 - \epsilon_{\alpha_t})$$

$$= \bar{x}_{cg} \left[ C_{L_{\alpha w}} + \frac{q_t S_t}{qS} C_{L_{\alpha t}}(1 - \epsilon_{\alpha_t}) \right] - \left[ \bar{x}_{ac_w} C_{L_{\alpha w}} + \frac{q_t S_t}{qS} \bar{x}_{ac_t} C_{L_{\alpha t}}(1 - \epsilon_{\alpha_t}) - C_{M_{\alpha p}} - C_{M_{\alpha f}} \right]$$

$$= \bar{x}_{cg} C_{L_\alpha} - \bar{x}_{ac} C_{L_\alpha}$$

$$= -(\bar{x}_{ac} - \bar{x}_{cg}) C_{L_\alpha}$$

$\bar{x}_{ac}$  needs to be behind  $\bar{x}_{cg}$

$$= \textcircled{-SM} \cdot C_{L_\alpha} \quad \text{where SM is the "Static Margin"}$$

Static margin

# Why is the Static Margin important?

- Which of the following aircraft is stable?

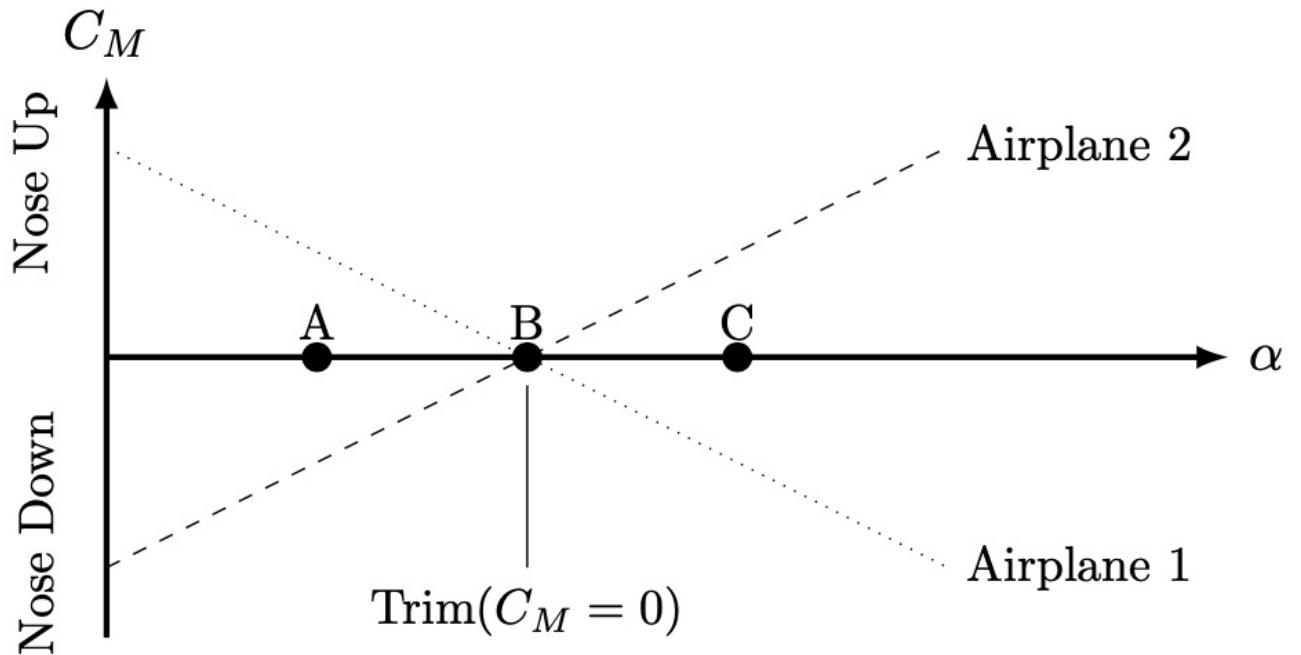


Figure 3.4: Pitch moment coefficient vs. angle of attack

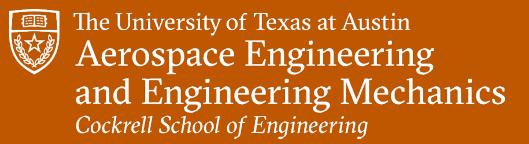
For a trimmed aircraft...

1. If  $C_{M\alpha}$  (the slope of the line) is positive, then an increase in pitch (and thus in the angle of attack) results in a nose up pitching moment.
2. If  $C_{M\alpha}$  is negative, then an increase in pitch (and thus in the angle of attack) results in a nose down pitching moment.
3. Thus, for the aircraft have longitudinal static stability we need  $C_{M\alpha}$  to be negative.
4. Because  $C_{L\alpha}$  is positive...
5. The SM must also be positive, and...
6. The center of gravity must be ahead, i.e., closer to the nose than the aerodynamic center of the aircraft.

**Weight on nose to move the c.g.  
forward for longitudinal stability**



the static margin should be positive to achieve longitudinal static stability;  
this also means that the center of gravity should be ahead of the aerodynamic center to achieve  
longitudinal static stability



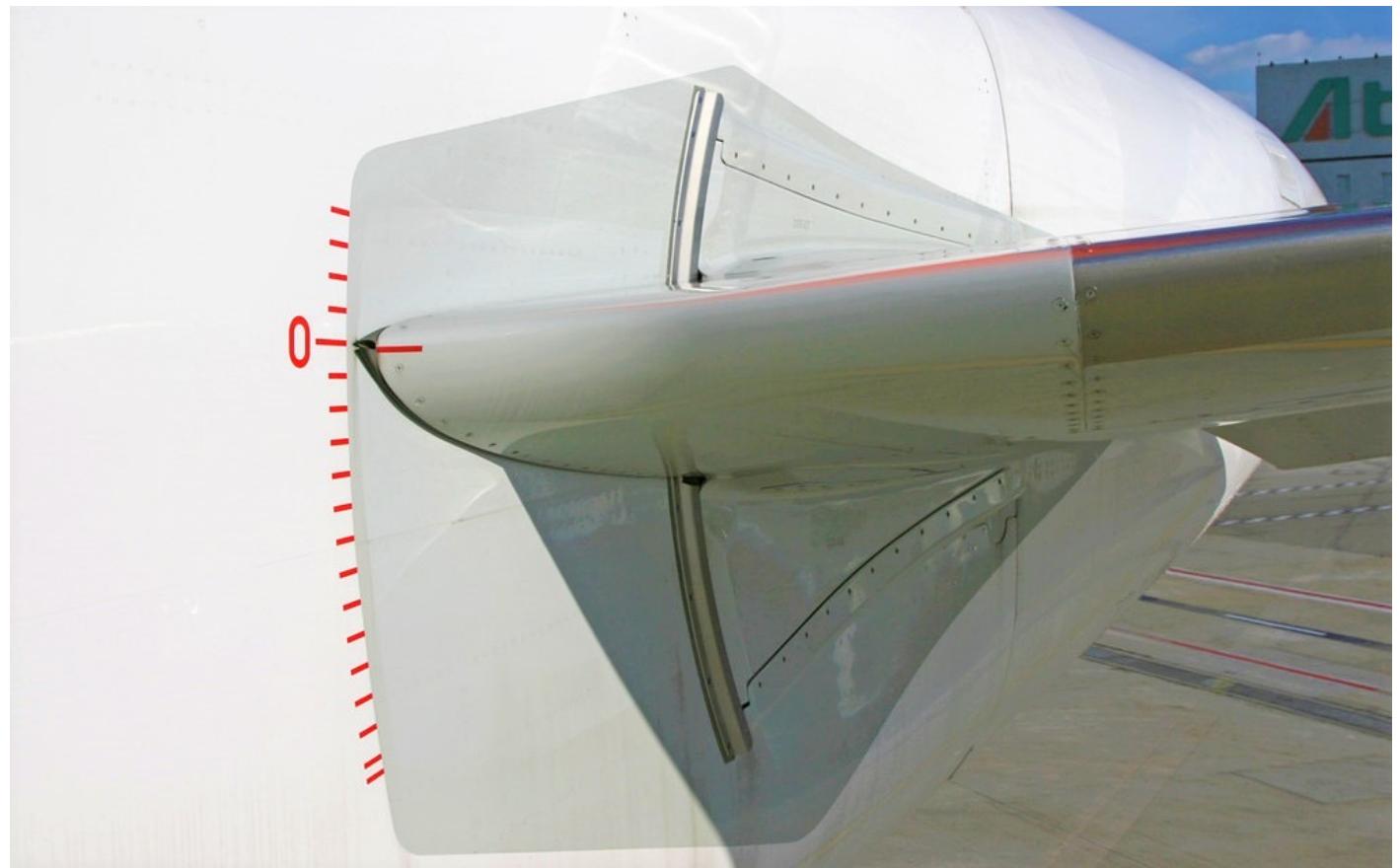
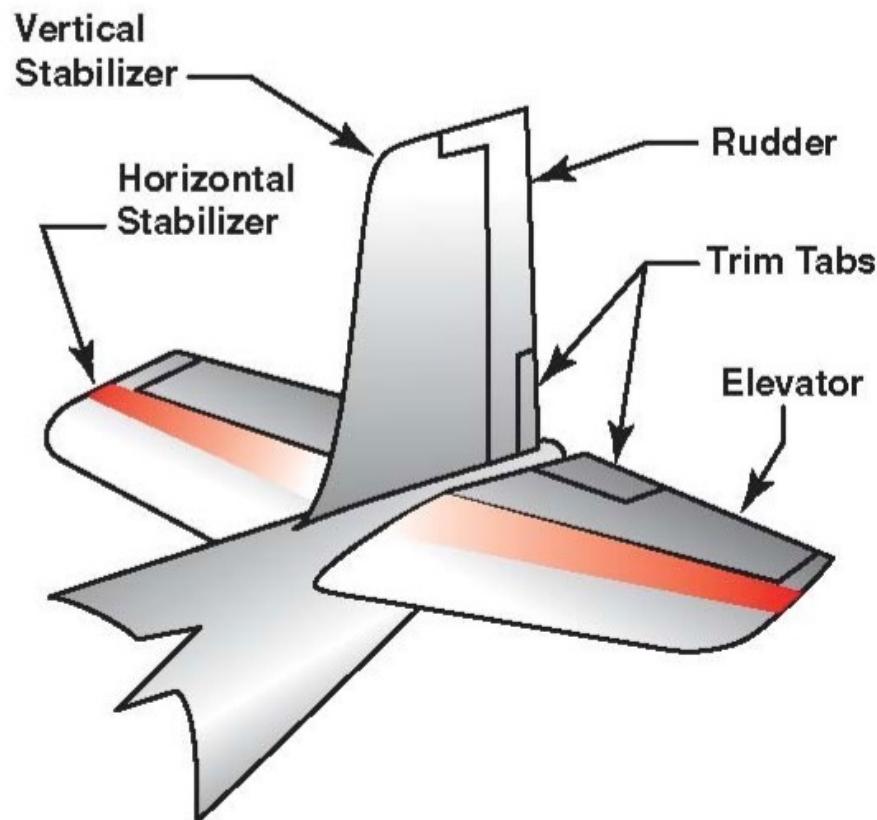
# TRIM

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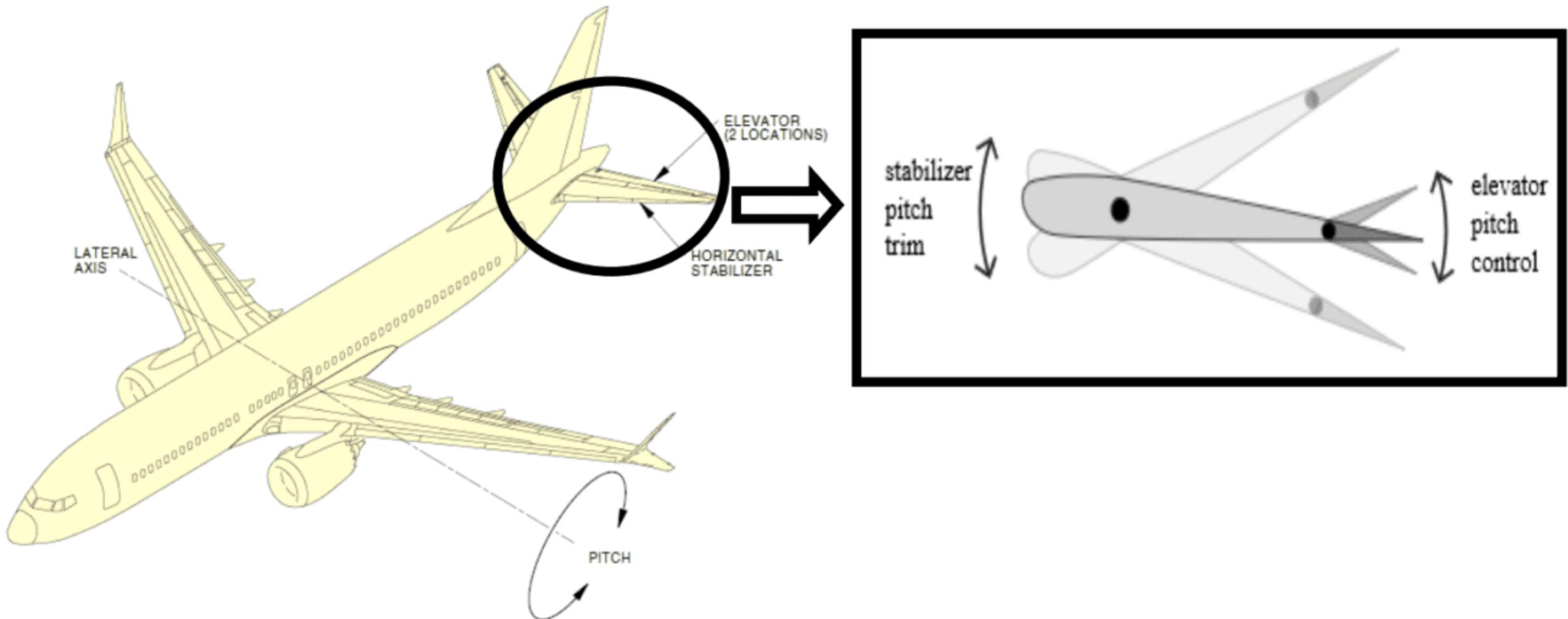
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# Fixed v. Variable-Position Horizontal Stabilizer



# Variable-Position Horizontal Stabilizer (1)



# Variable-Position Horizontal Stabilizer (2)

1. Set desired speed,  $V$ , and fix  $\bar{q} = \rho V^2/2$ . Assume that  $\delta_e = 0$  at this speed.
2. Determine  $\alpha_{\text{trim}}$  and  $i_{t_{\text{trim}}}$  from

$$\frac{W \cos \gamma}{\bar{q} S} = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{i_t}} i_t$$
$$0 = C_{M_0} + C_{M_\alpha} \alpha + C_{M_{i_t}} i_t$$

which is equivalent to solving the linear system of equations

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{i_t}} \\ C_{M_\alpha} & C_{M_{i_t}} \end{bmatrix} \begin{bmatrix} \alpha_{\text{trim}} \\ i_{t_{\text{trim}}} \end{bmatrix} = \begin{bmatrix} \frac{W \cos \gamma}{\bar{q} S} - C_{L_0} \\ -C_{M_0} \end{bmatrix}$$

# Variable-Position Horizontal Stabilizer (3)

and has the solution

$$\alpha_{\text{trim}_{\delta_e=0}} = - \frac{C_{M_{i_t}} \left( \frac{W \cos \gamma}{\bar{q} S} - C_{L_0} \right) + C_{L_{i_t}} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

$$i_{t_{\text{trim}}} = \frac{C_{M_\alpha} \left( \frac{W \cos \gamma}{\bar{q} S} - C_{L_0} \right) + C_{L_\alpha} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

3. With  $i_{t_{\text{trim}}}$  determined for the specified  $\bar{q}$ , define  $C'_{L_0}$  and  $C'_{M_0}$  as

$$C'_{L_0} = C_{L_0} + C_{L_{i_t}} i_{t_{\text{trim}}}$$

$$C'_{M_0} = C_{M_0} + C_{M_{i_t}} i_{t_{\text{trim}}}$$

# Variable-Position Horizontal Stabilizer (4)

4. Determine  $\delta_e$  to trim at other  $\bar{q}$ 's. That is, solve

$$\begin{bmatrix} C_{L\alpha} & C_{L\delta_e} \\ C_{M\alpha} & C_{M\delta_e} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} \frac{W \cos \gamma}{\bar{q}S} - C'_{L_0} \\ -C'_{M_0} \end{bmatrix}$$

to find  $\alpha_{\text{trim}}$  and  $\delta_{e_{\text{trim}}}$ , which gives

$$\alpha_{\text{trim}} = -\frac{C_{M\delta_e} \left( \frac{W \cos \gamma}{\bar{q}S} - C'_{L_0} \right) + C_{L\delta_e} C'_{M_0}}{C_{M\alpha} C_{L\delta_e} - C_{M\delta_e} C_{L\alpha}}$$

$$\delta_{e_{\text{trim}}} = \frac{C_{M\alpha} \left( \frac{W \cos \gamma}{\bar{q}S} - C'_{L_0} \right) + C_{L\alpha} C'_{M_0}}{C_{M\alpha} C_{L\delta_e} - C_{M\delta_e} C_{L\alpha}}$$

# Let's do some number crunching...

- The aircraft is in level flight.
- The gravitational acceleration is  $32.174 \text{ [ft/s}^2\text{]}$ .
- The wing and horizontal stabilizer are trapezoidal surfaces.
- The cg of the aircraft is at  $x_{cg} = 10.56 \text{ [ft]}$ .
- The weight of the aircraft is  $W = 9,500 \text{ [lb]}$ .
- The propulsive moment coefficients are  $C_{M_{0p}} = 0.0$  and  $C_{M_{\alpha p}} = 0.0$ .
- The fuselage moment coefficients are  $C_{M_{0f}} = 0.0$  and  $C_{M_{\alpha f}} = 0.0$ .
- The atmospheric density is  $0.002378 \text{ [slugs/ft}^3\text{]}$ .
- The horizontal tail incidence can only be set between  $-0.5$  and  $-7 \text{ [deg]}$ .

Wing	Horizontal Stabilizer
$S = 232.00 \text{ [ft}^2\text{]}$	$S = 54.00 \text{ [ft}^2\text{]}$
$\bar{c} = 7.04 \text{ [ft]}$	$\bar{c} = 3.83 \text{ [ft]}$
$x_{ac/le} = 4.07 \text{ [ft]}$	$x_{ac/le} = 2.79 \text{ [ft]}$
$x_{le} = 16.40 \text{ [ft]}$	$x_{le} = 36.90 \text{ [ft]}$
$i = 1.00 \text{ [deg]}$	
$C_{L_0} = -0.0443$	$C_{L_0} = 0.0000$
$C_{L_\alpha} = 5.0800$	$C_{L_\alpha} = 4.2600$
$C_{M_{ac}} = -0.0175$	$C_{M_{ac}} = 0.0000$
	$C_{L_{\delta_{et}}} = 1.8000$
	$\epsilon_0 = 0.642 \text{ [deg]}$
	$\epsilon_\alpha = 0.426$
	$\eta = 0.9$

$$\eta = \frac{q_t}{q}$$

# Number Crunching (1)

1. Set desired speed,  $V$ , and fix  $\bar{q} = \rho V^2 / 2$ . Assume that  $\delta_e = 0$  at this speed.

Let's assume aircraft is flying at ... 500 *knots* = 843.9 *ft/s*

given  $\rho = 0.002378 \text{ slugs}/\text{ft}^3$       then  $q = 846.8 \text{ lb}/\text{ft}^2$

# Number Crunching (2)

2. Determine  $\alpha_{\text{trim}}$  and  $i_{t_{\text{trim}}}$  from

$$\alpha_{\text{trim}_{\delta_e=0}} = -\frac{C_{M_{i_t}} \left( \frac{W \cos \gamma}{\bar{q}S} - C_{L_0} \right) + C_{L_{i_t}} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$
$$i_{t_{\text{trim}}} = \frac{C_{M_\alpha} \left( \frac{W \cos \gamma}{\bar{q}S} - C_{L_0} \right) + C_{L_\alpha} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

First we compute the things we don't have ...  $C_{L_{i_t}}$ ,  $C_{M_{i_t}}$ ,  $C_{M_0}$

... then we “*plug and chug*”

## Number Crunching (3)

- With  $i_{t_{\text{trim}}}$  determined for the specified  $\bar{q}$ , define  $C'_{L_0}$  and  $C'_{M_0}$  as

$$C'_{L_0} = C_{L_0} + C_{L_{i_t}} i_{t_{\text{trim}}}$$

$$C'_{M_0} = C_{M_0} + C_{M_{i_t}} i_{t_{\text{trim}}}$$

# Number Crunching (4)

4. Determine  $\delta_e$  to trim at other  $\bar{q}$ 's. That is, solve

$$\alpha_{\text{trim}} = - \frac{C_{M_{\delta_e}} \left( \frac{W \cos \gamma}{\bar{q}S} - C'_{L_0} \right) + C_{L_{\delta_e}} C'_{M_0}}{C_{M_\alpha} C_{L_{\delta_e}} - C_{M_{\delta_e}} C_{L_\alpha}}$$

$$\delta_{e_{\text{trim}}} = \frac{C_{M_\alpha} \left( \frac{W \cos \gamma}{\bar{q}S} - C'_{L_0} \right) + C_{L_\alpha} C'_{M_0}}{C_{M_\alpha} C_{L_{\delta_e}} - C_{M_{\delta_e}} C_{L_\alpha}}$$

# **Why is this all important?**

- Air Midwest Flight 5481

- <https://www.youtube.com/watch?v=MMsbpLjfWlo>
  - <https://www.youtube.com/watch?v=CHj9Lmjo2Ng>

- Crash at Bagram Air Base

- <https://www.youtube.com/watch?v=5fpxm0D46iQ>
  - [https://www.youtube.com/watch?v=wXJ\\_MfAnjgQ](https://www.youtube.com/watch?v=wXJ_MfAnjgQ)



The University of Texas at Austin  
**Aerospace Engineering**  
**and Engineering Mechanics**  
*Cockrell School of Engineering*