ASE 389P-7 Problem Set 1

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You need not hand in anything. Instead, be prepared to answer any of these problems—or similar problems—on an upcoming take-home exam. You may discuss your solutions with classmates up until the time that the exam becomes available, but do not swap work.

Readings

All background reading material is found on Canvas. You might enjoy reading about the origins of GNSS in [1]. Polaris submarines were the U.S.'s second strike capability. Second strike as a stabilizing concept in confrontations is discussed in [2].

Problems

1. Write a function in Matlab that simulates the train-horn-Doppler scenario discussed in lecture. Assume that the train tracks are rectilinear. Your function should adhere to the following interface:

```
function [fDVec,tVec] = ...
      simulateTrainDoppler(fc, vTrain, t0, x0, xObs, dObs, delt, N, vs)
% simulateTrainDoppler : Simulate the train horn Doppler shift scenario.
%
% INPUTS
%
% fc ------ train horn frequency, in Hz
%
% vTrain -- constant along-track train speed, in m/s
%
% t0 ------ time at which train passed the along-track coordinate x0, in
%           seconds
%
% x0 ------ scalar along-track coordinate of train at time t0, in meters
%
% xObs ---- scalar along-track coordinate of observer, in meters
%
% dObs ---- scalar cross-track coordinate of observer, in meters (i.e.,
%           shortest distance of observer from tracks)
%
% delt ---- measurement interval, in seconds
%
% N ------- number of measurements
%
% vs ------ speed of sound, in m/s
%
%
% OUTPUTS
```

```
%
% fDVec --- N-by-1 vector of apparent Doppler frequency shift measurements as
%           sensed by observer at the time points in tVec
%
% tVec ---- N-by-1 vector of time points starting at t0 and spaced by delt
%           corresponding to the measurements in fDVec
%
%+------------------------------------------------------------------------------+
% References:
%
%
% Author:
%+==============================================================================+
```

Be sure to account for the nonzero time of flight δt_{TOF} as discussed in lecture. The effect of $\delta t_{\text{TOF}} > 0$ is that the stationary observer will discern an f_D at time t_k that relates to the train's line-of-sight velocity at time $t_k - \delta t_{\text{TOF}}$. More precisely, the apparent frequency of the train horn at the location of the observer at time t_k is given by

$$f_r(t_k) = \frac{f_c}{1 + v_{los}(t_k)/v_s}$$

where f_c is the nominal horn frequency, $v_{los}(t_k)$ is the line-of-sight velocity at t_k, and v_s is the speed of the signal in the medium. Note that the line-of-sight geometry used to calculate $v_{los}(t_k)$ is between the observer at time t_k and the horn at time $t_k - \delta t_{\text{TOF}}$.

Plot the output of your function for various input values. For convenience, you may wish to use the script `topSimulateTrainDoppler_temp.m` on Canvas to set up the problem and call your function. This script also lets you listen to and record the audio time history.

2. Download the audio file `trainout.wav` from Canvas. This file was created with the following input argument values:

```
fh = 440;
vTrain = 20;
t0 = 0;
x0 = 0;
delt = 0.01;
N = 1000;
vs = 343;
```

Set up your simulator with these same values. Estimate the values of `xObs` and `dObs` by adjusting them in your simulation until you get an apparent received frequency profile that matches the one you hear in the audio file.

Extra Credit: Refine your "by ear" estimates of `xObs` and `dObs` by figuring out a way to extract a frequency time history from the `trainout.wav` audio file and matching your function output to this time history.

3. In this problem, we'll examine the effects of a sampling clock frequency bias on Doppler estimation. Assume that some oscillator with a perfect clock produces a pure sinusoid of the form $x(t) = \cos(2\pi f_c t)$. The period of this signal is $T = 1/f_c$ seconds. Suppose that you receive this signal and make a measurement f_m of the signal's frequency as follows. You sample the signal with a local clock having a true sampling

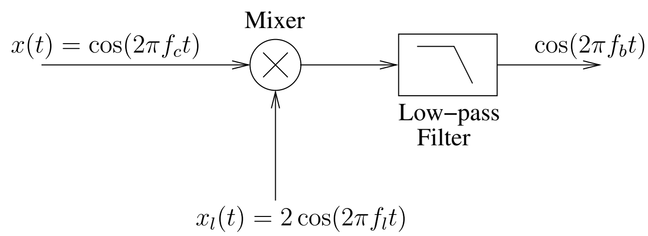


Figure 1: A single-stage frequency conversion (mixing) operation.

interval $\Delta t \ll T$ and you observe the number of sample intervals N (including fractional intervals) that fit within one period T. Thus, $N = T/\Delta t$. However, you don't know the true sampling interval Δt of your local clock; you only know its *nominal* value, Δt_{nom}, which is the sampling interval that would obtain if your clock were oscillating exactly at its *advertised* frequency (i.e., the one printed on the clock's data sheet). Thus, you take as your measurement of the period of the incoming signal the value $T_m = N \cdot \Delta t_{\text{nom}}$, from which you can obtain the measured frequency as $f_m = 1/T_m$. The apparent Doppler is obtained by subtracting the known frequency from your measured frequency:

$$f_D = f_m - f_c$$

Assume there is no motion between your receiver and the signal transmitter; thus, if your receiver's sampling clock were perfect, you would measure $f_D = 0$. But as with all physical clocks, your local clock has a frequency offset (bias). We characterize this offset with a ratio $\beta = \Delta f/f$ called the *fractional frequency error*. For example, suppose your clock is advertised to oscillate at 1000 Hz, but it actually oscillates at 1001 Hz. Then its fractional frequency error is $\beta = \Delta f/f = 0.001$.

Derive an expression for the apparent Doppler f_D in terms of f_c and β.

4. In this problem, we'll examine the effects of clock frequency bias on Doppler derived from frequency downconversion and sampling.

   Usually, we don't measure signal properties directly at the signal's incoming frequency. Instead, we convert incoming signals to a lower center frequency by a process called frequency conversion. To understand frequency conversion, recall that

$$\cos(x)\cos(y) = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

   Hence, by multiplying an incoming signal $x(t) = \cos(2\pi f_c t)$ by a local signal $x_l(t) = 2\cos(2\pi f_l t)$ and then low-pass filtering the result to eliminate the high frequency $f_c + f_l$ component, we convert the high frequency signal down to a lower frequency $f_b = f_c - f_l$. Processes such as amplification, transmission, filtering, delaying, recording, and sampling are easier to do at lower frequencies. Figure 1 shows a diagram of the frequency conversion process.

   Assume that some oscillator with a perfect clock produces a pure sinusoid of the form $x(t) = \cos(2\pi f_c t)$. Suppose that you receive this signal and make a measurement f_m of the signal's frequency by (1) performing frequency conversion to translate the signal to a nominal center frequency $f_{b,\text{nom}} = 0$ Hz and then (2) sampling the signal with nominal sampling interval Δt and observing the number of samples per period. Assume there is no motion between your receiver and the signal transmitter. Suppose that the local clock you're using to perform frequency conversion and sampling has a fractional frequency error of $\beta < 0$.

   Derive an expression for the apparent Doppler f_D resulting from frequency conversion and sampling with a clock that has this fractional frequency error. Express your result in terms of f_c and β. How does this expression differ from the one derived in problem 3?

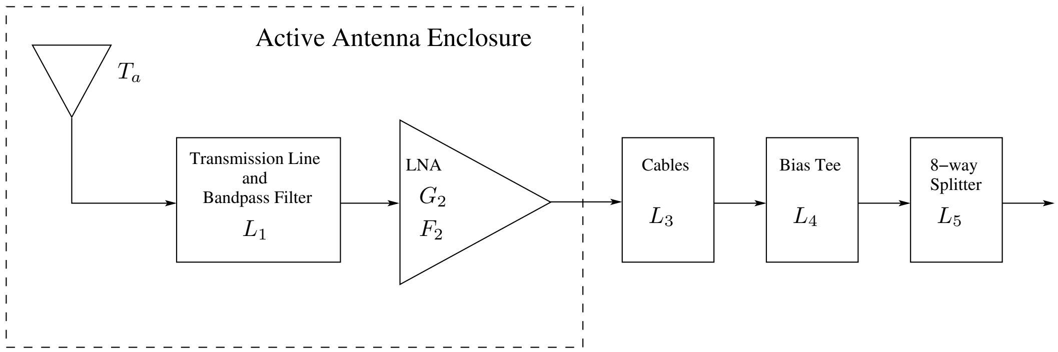


Figure 2: Radionavigation Laboratory GNSS receiver setup.

Note that if $f_l < f_c$, as will be the case for our problem because $\beta < 0$, then $f_b > 0$. This is an example of *low-side mixing*. But if $f_l > f_c$, then $f_b < 0$; this is *high-side mixing*. What would be the meaning of a signal for which $f_b < 0$? Upon sampling a signal with $f_b < 0$ to determine the period, would we measure a *negative* frequency?

5. This problem will walk you through a noise analysis for the UT Radionavigation Laboratory (RNL) GNSS receiver setup. A block diagram of the setup is shown in Fig. 2. A rooftop Trimble Geodetic Zephyr II antenna is followed by cables, a bias tee, and a splitter, with parameters as follows:

   - $T_A = 100K$: The temperature of the passive antenna element within the Trimble antenna.
   - $L_1 = 1$ dB: Before being amplified, signals from the passive antenna element pass through a short transmission line and a low-insertion-loss bandpass filter. L_1 is the combined loss of the line and filter. Because L_1 enters before any gain is applied to the signals, it is very important to make L_1 as low as possible.
   - $G_2 = 50$ dB; $F_2 = 1.5$ dB: The Trimble antenna's built-in low-noise amplifier (LNA) has extraordinary gain and a low noise figure.
   - $L_3 = 6.48$ dB: Signals are routed from the active Trimble antenna to the RNL via 127 feet of C400 cable. The cable is fairly low loss: at 1.5 GHz (near the GPS L1 frequency), the cable loss is 5.1 dB per 100 feet.
   - $L_4 = 0.6$ dB: A bias tee feeds the active antenna with a 5 V supply by biasing the center line of the coaxial cable coming from the antenna to 5 V DC. The DC supply voltage does not affect the high-frequency signals arriving from the antenna. From the perspective of the high-frequency signals, the bias tee looks like a passive component with a 0.6 dB insertion loss.
   - $L_5 = 9.8$ dB: Signals exiting the bias tee are split by a passive 8-way splitter to provide GNSS signals for the many receivers in the RNL. For an ideal splitter, the signal power exiting any one of the 8 splitter outputs is a factor of 8 (9 dB) lower than the input power. In the RNL splitter, the input signal sees an additional 0.8 dB of insertion loss.

   (a) Calculate the noise figure F_1 and effective input temperature T_1 corresponding to L_1. Assume an input temperature $T_{in} = T_o = 290$K.

   (b) Calculate the effective input temperature T_2 corresponding to F_2. Assume F_2 is taken from a device data sheet that assumes $T_{in} = T_o = 290$K.

   (c) Lump losses L_3, L_4, and L_5 into a single loss L_{345}. Calculate an effective input temperature T_{345} corresponding to L_{345}. Assume an input temperature $T_{in} = T_o = 290$K.

   (d) Use the Friis formula to calculate the system temperature $T_S = T_A + T_R$, expressed in degrees K. How many degrees K do the combined losses L_{345} contribute to the system temperature?

(e) The effective noise floor of the whole cascade, N_0, is related to the system temperature T_S by $N_0 = kT_S$, where k is Boltzmann's constant, equal to -228.6 dBW/K-Hz. The quantity N_0 is the one used in calculating C/N_0, that all-important parameter in GNSS receiver design. For purposes of calculating C/N_0, you can think of the receiver cascade as a chain of ideal gain blocks (no internal noise) with an effective noise density N_0 at the beginning of the cascade (i.e., at the output of the passive antenna element just before the L_1 block). This is precisely the point where the signal power C is defined, so it makes sense to calculate the ratio C/N_0 here. Calculate the value of N_0 in dBW/Hz. For an expected GPS L1 C/A received signal power C ranging from -162.5 to -154.5 dBW, calculate the expected range of carrier-to-noise ratio C/N_0 values. Express your result in dB-Hz.

(f) What is the noise floor $N_{0,sp}$ at the output of the 8-way splitter? Express your answer in dBW/Hz.

(g) **Extra Credit:** Devise an experiment for measuring $N_{0,sp}$ near the GPS L1 frequency.

6. Compare the system temperature of the following two systems: System A is a cable followed by an amplifier followed by a receiver. System B uses the same elements, but in the following order: amplifier, cable, then receiver. The cable has 5 dB of loss; the amplifier has a gain of 20 dB and a noise figure of 2 dB; the receiver has a gain of 50 dB and a noise figure of 7 dB. Assume an antenna temperature $T_A = 100$K. Also compare the cascaded noise figure

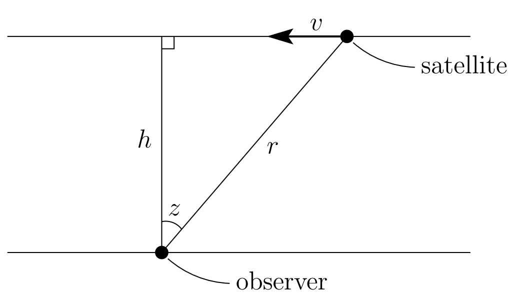
$$F(T_A) \triangleq \frac{\left(\frac{C}{N_0}\right)_{in}}{\left(\frac{C}{N_0}\right)_{out}}$$

for each of the two systems. (Note: This is a variation on problem 10-3 from the Misra and Enge text.)

7. Suppose you wanted to receive and track GPS signals on the surface of the Moon. Assume a total transmitted power $P_t = 30$ W is delivered to a GPS satellite's antenna, and that the satellite's antenna gain in the direction of a receiver on the Moon in $G_t = 10$ dBi. Calculate the signal power received through a $G_r = 6$ dBi antenna from a signal transmitted by a GPS satellite on the opposite side of the Earth (assume a transparent Earth). How many dB weaker is this signal compared to one received on the surface of the earth through a 3 dBi antenna from a GPS satellite directly overhead?

8. Suppose one wishes to amplify a signal after it passes through a long cable whose loss is 1 dB per 100 meters. The amplifier's noise figure is 3 dB and the antenna's noise temperature is 290 K. What is the maximum cable length such that the amplifier's output C/N_0 is at least 5% of the C/N_0 at the input to the cable?

9. Consider a simplified model of low-Earth-orbit (LEO) satellite motion in which a satellite moves with constant velocity v along a straight line at a constant altitude h above the plane of an observer. From the observer's position, the satellite zenith angle is $z(t)$.



Find expressions for the line-of-sight velocity $v_{\text{los}} \triangleq \dot{r}$ and the line of sight acceleration $\dot{v}_{\text{los}} \triangleq \ddot{r}$ in terms of v, h, and z. Draw plots of these as a function of $z \in [0, \pi/2]$. Based on your expressions, calculate the maximum Doppler frequency (in kHz) the observer would measure with a perfect clock when tracking a signal with center frequency $f_0 = 10$ GHz transmitted by a satellite moving at velocity v consistent with a circular Earth orbit with altitude $h = 400$ km.

References

[1] W. H. Guier and G. C. Weiffenbach, "Genesis of satellite navigation," *Johns Hopkins APL technical digest*, vol. 19, no. 1, p. 15, 1998.

[2] T. C. Schelling, "Surprise attack and disarmament," *Bulletin of the Atomic Scientists*, vol. 15, no. 10, pp. 413–418, 1959.