ASE 367K HW 6

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October 23 2024

```
Number 1
Short Period
----------------------------
Damped Frequency: 0.682998 [rad/s]
Natural Frequency: 0.853890 [rad/s]
Damping Ratio: 0.600178
Time to Damp to Half the initial Amplitude: 1.352520 [sec]
Cycle to Damp to Half the initial Amplitude: 0.147022 [cycle]

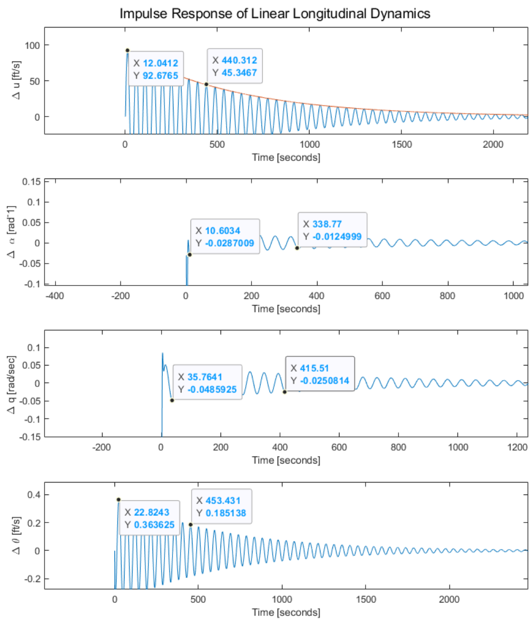
Long Period
----------------------------
Damped Frequency: 0.132028 [rad/s]
Natural Frequency: 0.132039 [rad/s]
Damping Ratio: 0.012647
Time to damp to half the initial Amplitude: 415.083320 [sec]
Cycle to damp to half the initial Amplitude: 8.722113 [cycle]

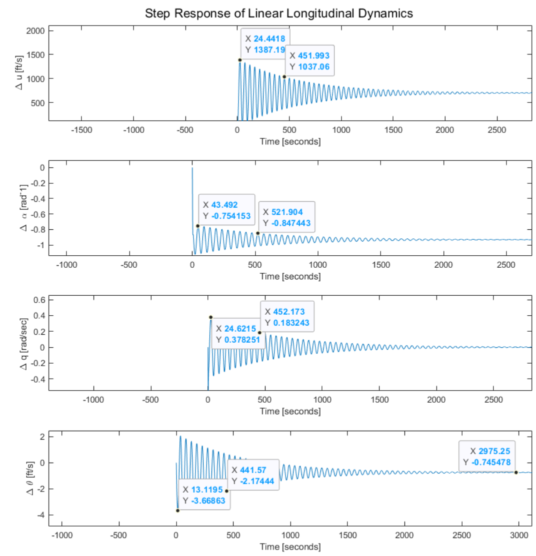
Number 2
----------------------------
Damped Frequency: 0.702656 [rad/s]
Natural Frequency: 0.870800 [rad/s]
Damping Ratio: 0.590676
Time to Damp to Half the initial Amplitude: 1.347591 [sec]
Cycle to Damp to Half the initial Amplitude: 0.150703 [cycle]

Number 3
----------------------------
Damped Frequency: 0.146263 [rad/s]
Natural Frequency: 0.146565 [rad/s]
Damping Ratio: 0.010795
Time to Damp to Half the initial Amplitude: 73.739062 [sec]
Cycle to Damp to Half the initial Amplitude: 1.716537 [cycle]
```

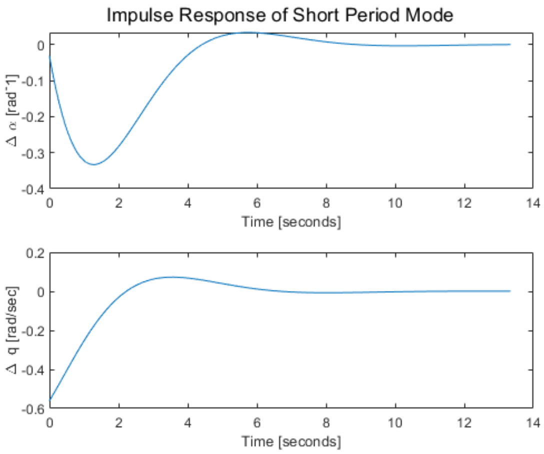
1 Plots

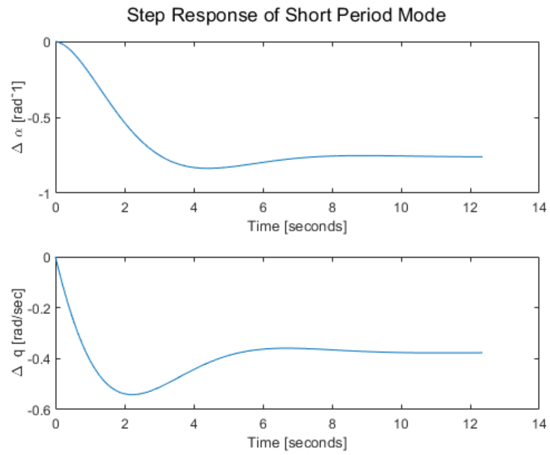
1.1 Problem 1b Longitudinal Dynamics Response



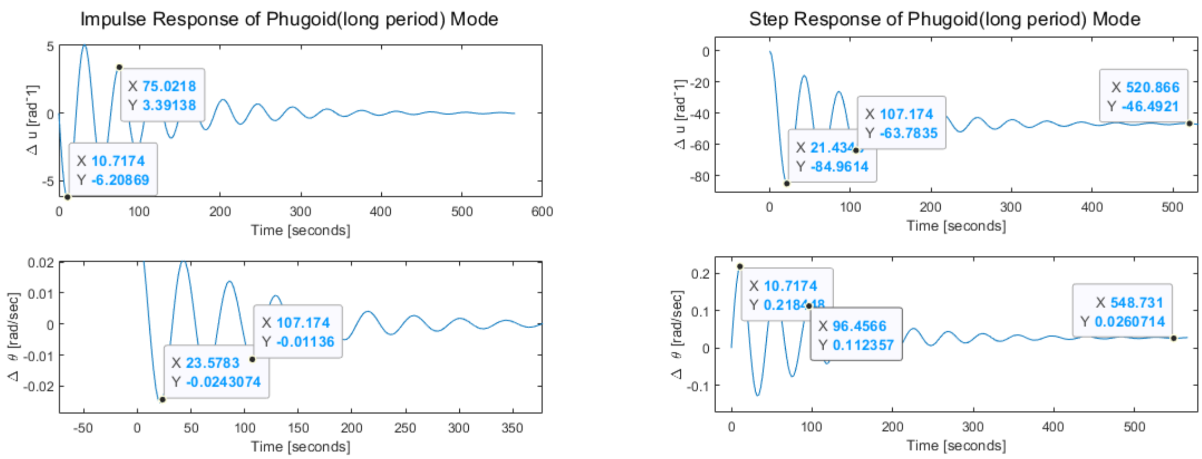


1.2 Problem 2b Short Period Mode





1.3 Problem 3b Phugoid (Long Period) Mode



2 Problem 4

2.1 Explain any differences between the full model and the two approximations in terms of the computed damped frequency, natural frequency, damping ratio, the time to damp to half the initial amplitude, and the number of cycles to damp to half the initial amplitude.

The full model exhibits very similar system properties to short mode approximation. The damped and natural frequencies of full model's short period are only 0.02 rad/s lower and the damping ratio is 0.01 higher than those of short approximation. The time and cycle to damp to half the initial amplitudes are similar too. It is clearly seen from the short approximation response graphs in Section 1.2 that short response damps to a steady state after its first peak within a few seconds, and this response matches with the time to damp to half the initial amplitude calculated as 1.347591 sec.

However, this short period responses are not visible in both step and impulse response for the full model (Section 1.1) because the long period mode dominates the longitudinal dynamics.

The graphs in section Section 1.1 show a prevalent effect of the long period mode. It shows that the time to damp to half the initial amplitude is about 413 seconds and the cycle to damp to half the initial amplitude is about 8 to 9 cycles that are close to the calculated 415.08 seconds and 8.72 cycles, respectively. In the long mode approximation model, although it shows relatively longer time to reach steady state compare to both the short mode response and the short mode approximation, it has much less time and cycle to damp compare to the response from the full model. This long mode approximation has 73.7391 seconds or 1.7156 cycles to damp to half the initial amplitude as shown in Section 1.3.

This indicates that although the long approximation retains its relationship with the short mode (i.e. long period mode has lower frequency response and lower damping ratio relative to the short mode), the magnitude of response is not accurately represented through long period approximation.

The frequencies and damping ratios of both longe period mode from full model and the long period approximation are similar, there aren't much difference.

Contents


```
clear all;
close all;
clc;

Xu     =-0.0188;
XTu    = 0;
Xa     = 11.5905;
Xde    = 0;
Zu     = -0.1862;
Za     = -149.4408;
Zq     = -6.8045;
Zadot  = -8.4426;
Zde    = -8.7058;
Mu     = 0.0001;
MTu    = 0;
Ma     = -0.5294;
MTa    = 0;
Mq     = -0.4275;
Madot  = -0.0658;
Mde    = -0.5630;
u1     = 279.1094; % ft/s
theta1 = 0;
g = 32.2; % ft/s^2
```

Set Up Matrix A, B C, D

```
R = [Xu+XTu Xa 0 -g*cos(theta1);...
    Zu Za u1+Zq -g*sin(theta1);...
    Mu+MTu Ma+MTa Mq 0;...
    0 0 1 0];

M = [1 0 0 0;
    0 u1-Zadot 0 0;...
    0 -Madot 1 0;...
    0 0 0 1];

F = [Xde; Zde; Mde; 0];

A = inv(M)*R;
B = inv(M)*F;

C = eye(4);
```

```
D = zeros(4,1);
```

eig Vectors and Values

```
[eVec,eVal] = eig(A);

% Make them unitless
eVecUnitless = eVec;
eVecUnitless(1,:) = eVec(1,:)/u1;
eVecUnitless(3,:) = eVec(3,:)/20; % Not sure how we get 20?
% eVecUnitless

% Normalize respect to delta theta

eVec12 = abs(eVecUnitless(:,1)/eVecUnitless(4,1));
eVec34 = abs(eVecUnitless(:,3)/eVecUnitless(4,3));
```

Answer Questions

```
% Short
wd   = imag(eVal(1,1));
wn   = sqrt((real(eVal(1,1)))^2+(wd)^2);
damp = abs(real(eVal(1,1))/wn);
delT = abs(log(2)/(real(eVal(1,1))));
N = abs(log(2)*wd/(2*pi*real(eVal(1,1))));
% charEqShort=conv([1 -eVal(1,1)],[1 -eVal(2,2)]);
% wn   = sqrt(charEqShort(end));
% damp = charEqShort(2)/2/wn;
% wd   = wn*sqrt(1-damp^2);
% beta = atan(sqrt(1-damp^2)/damp);
% tr   = (pi-beta)/wd; % Rise time (0-100%)
% tp   = pi/wd;% Peak Time
% Mp   = exp(-damp*pi/sqrt(1-damp^2)); % Maximum overshoot
% ts   = 4/(damp*wn); % Settling Time (2% criterion)
% delT = log(2)/(damp*wn);
% N = log(2)*wd/(2*pi*damp*wn);


% Long
wdLong   = imag(eVal(3,3));
wnLong   = sqrt((real(eVal(3,3)))^2+(wdLong)^2);
dampLong = abs(real(eVal(3,3))/wnLong);
delTLong = abs(log(2)/(real(eVal(3,3))));
NLong = abs(log(2)*wdLong/(2*pi*real(eVal(3,3))));
```

b)

```
x0 = zeros(4,1);
sys = ss(A,B,C,D);
[h, t] = impulse(sys);
figure(1)
sgtitle('Impulse Response of Linear Longitudinal Dynamics')
subplot(4,1,1)
```

```
plot(t,h(:,1))
ylabel('\Delta u [ft/s]');
xlabel('Time [seconds]');
hold on,
[pks, locs] = findpeaks(abs(h(:,1)));
a2     =-log(pks(1)/pks(end)) /(t(1) - t(end));
a1     = pks(end)/(exp(-a2*t(end)));
dampeq =a1*exp(-a2*t);
plot(t,dampeq)

delTGraphu = log(pks(1)/2/a1)/-a2 ;


subplot(4,1,2)
plot(t,h(:,2))
ylabel('\Delta  \alpha [rad^-1]');
xlabel('Time [seconds]');
xlim([0, 200])
subplot(4,1,3)
plot(t,h(:,3))
ylabel('\Delta q [rad/sec] ');
xlabel('Time [seconds]')
subplot(4,1,4)
plot(t,h(:,4))
ylabel('\Delta \theta [ft/s]');
xlabel('Time [seconds]')

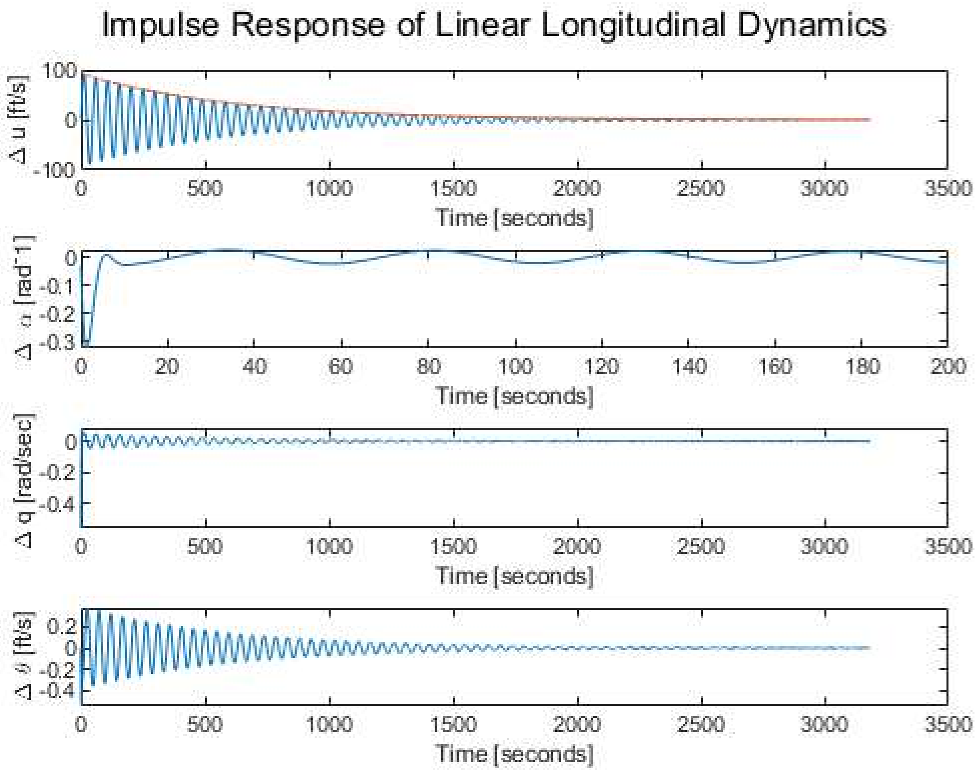


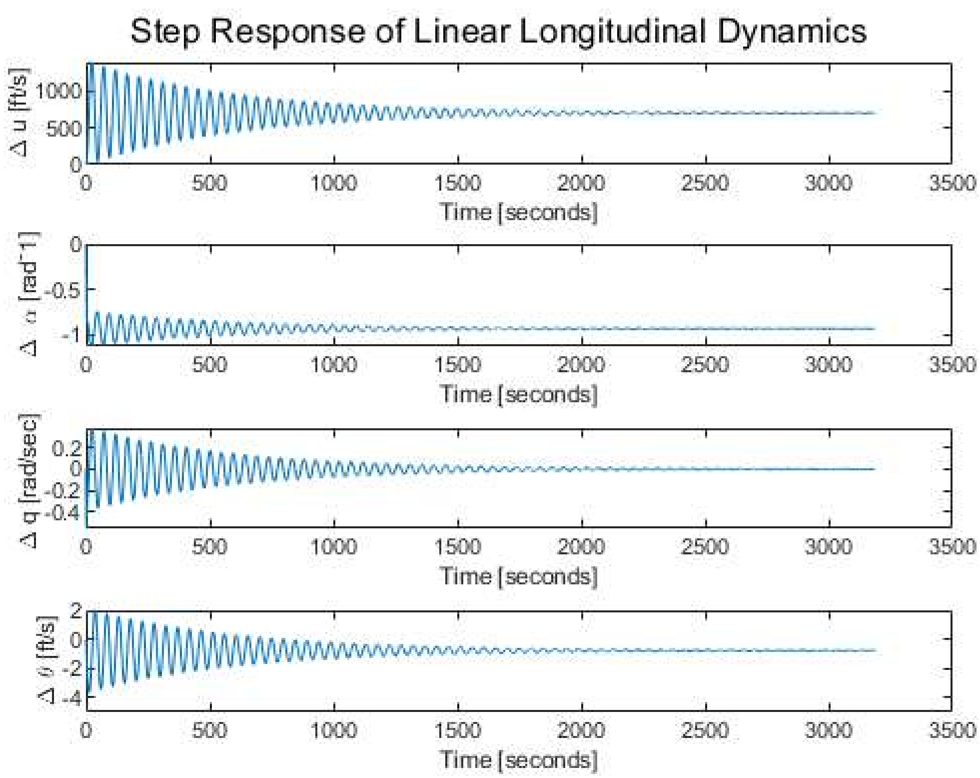






[hs, ts] = step(sys);
figure(2)
sgtitle('Step Response of Linear Longitudinal Dynamics')
subplot(4,1,1)
plot(ts,hs(:,1))
ylabel('\Delta u [ft/s]');
xlabel('Time [seconds]')
subplot(4,1,2)
plot(ts,hs(:,2))
ylabel('\Delta  \alpha [rad^-1]');
xlabel('Time [seconds]')
subplot(4,1,3)
plot(ts,hs(:,3))
ylabel('\Delta q [rad/sec] ');
xlabel('Time [seconds]')
subplot(4,1,4)
plot(ts,hs(:,4))
ylabel('\Delta \theta [ft/s]');
xlabel('Time [seconds]')
```





Problem 2

```
M2 = [u1 0;
    -Madot 1];
R2 = [Za u1;
```

```
    Ma Mq];
A2 = inv(M2)*R2;

[eVec2,eVal2] = eig(A2);

wd2   = imag(eVal2(1,1));
wn2   = sqrt((real(eVal2(1,1)))^2+(wd2)^2);
damp2 = abs(real(eVal2(1,1))/wn2);
delT2 = abs(log(2)/(real(eVal2(1,1))));
N2    = abs(log(2)*wd2/(2*pi*real(eVal2(1,1))));



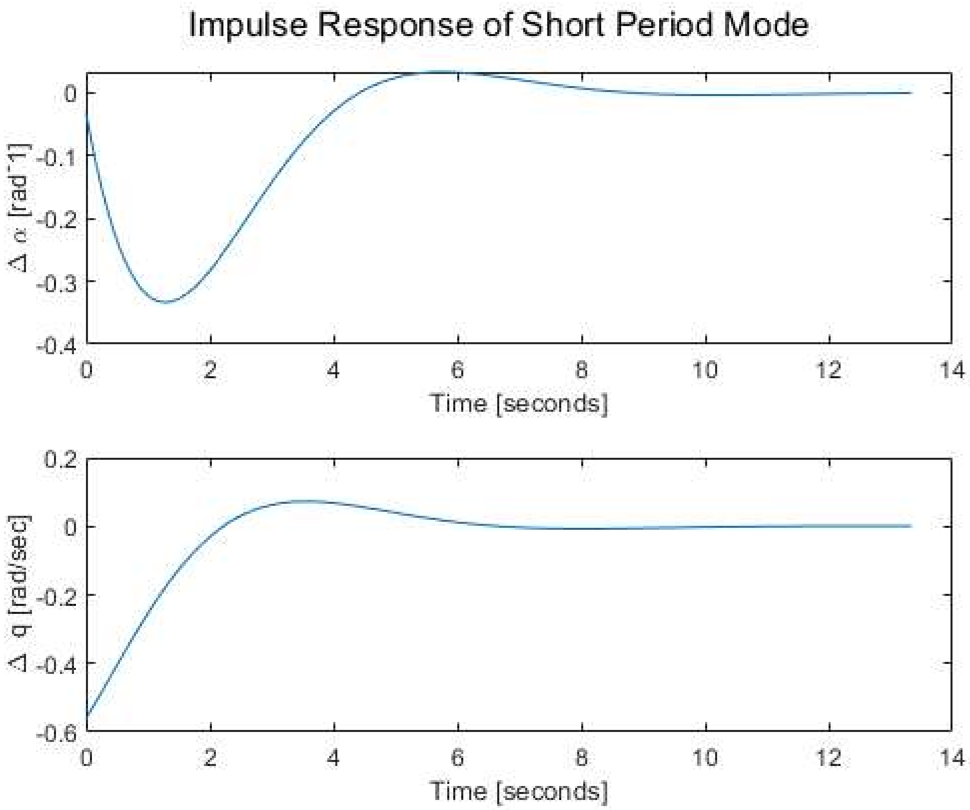
% b)

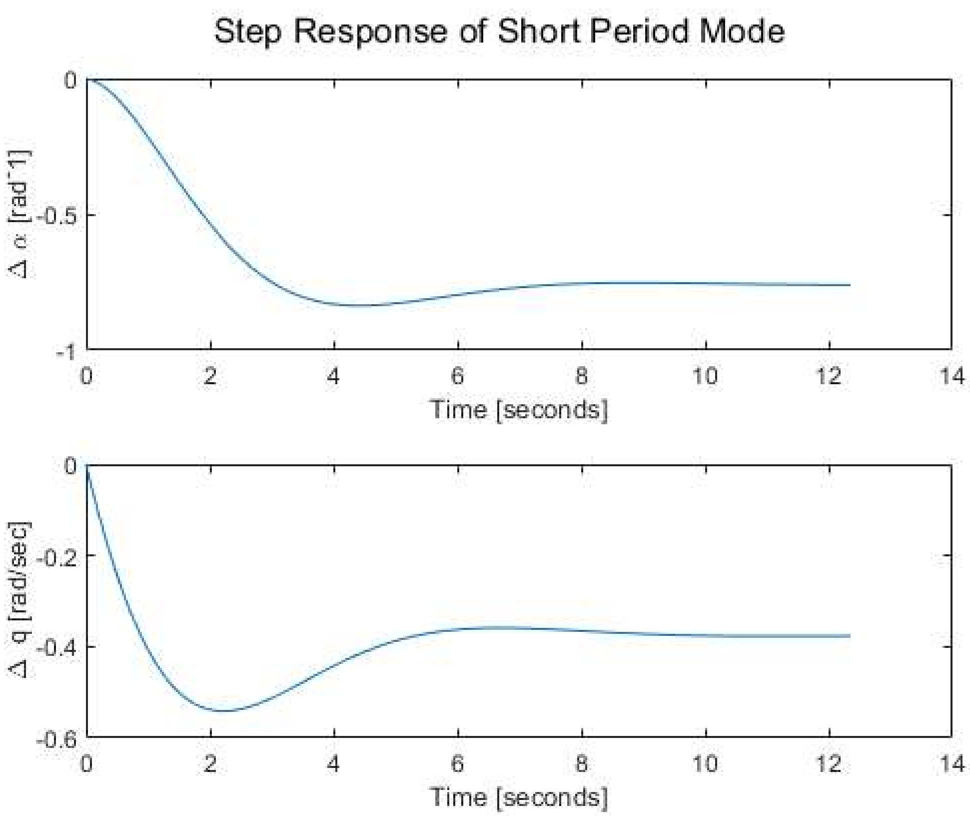
F2 = [Zde;
    Mde];
B2 = inv(M2)*F2;
C2 =eye(2);
D2 = zeros(2,1);

sys2 = ss(A2,B2,C2,D2);

[h2, t2] = impulse(sys2);
figure(3)
sgtitle('Impulse Response of Short Period Mode')
subplot(2,1,1)
plot(t2,h2(:,1))
ylabel('\Delta \alpha [rad^-1]');
xlabel('Time [seconds]')
subplot(2,1,2)
plot(t2,h2(:,2))
ylabel('\Delta  q [rad/sec]');
xlabel('Time [seconds]')

[h2s, t2s] = step(sys2);
figure(4)
sgtitle('Step Response of Short Period Mode')
subplot(2,1,1)
plot(t2s,h2s(:,1))
ylabel('\Delta \alpha [rad^-1]');
xlabel('Time [seconds]')
subplot(2,1,2)
plot(t2s,h2s(:,2))
ylabel('\Delta  q [rad/sec]');
xlabel('Time [seconds]')
```


Impulse Response of Short Period Mode
Δ α [rad^-1]
Time [seconds]
Δ q [rad/sec]
Time [seconds]


Step Response of Short Period Mode
Δ α [rad^-1]
Time [seconds]
Δ q [rad/sec]
Time [seconds]

Problem 3

a)

```
A3 = [Xu+XTu -g;
    -Zu/u1 0];
F3 = [Xde;
    -Zde/u1];
B3 = F3;
C3 = eye(2);
D3 = 0;

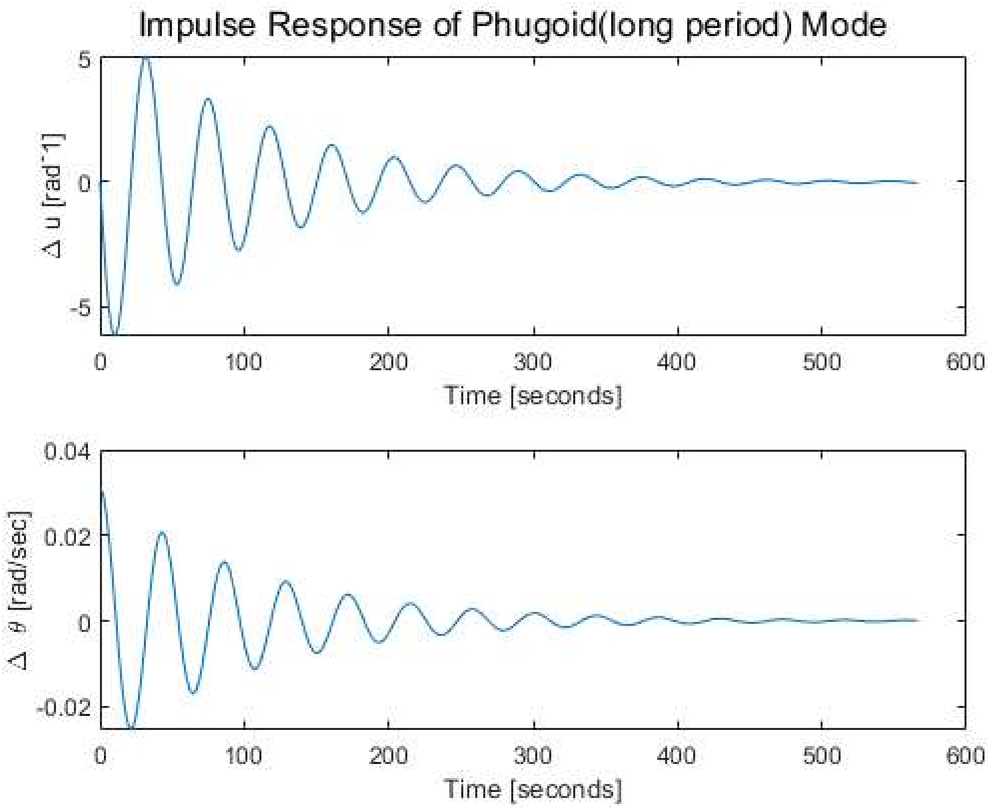
[eVec3,eVal3] = eig(A3);

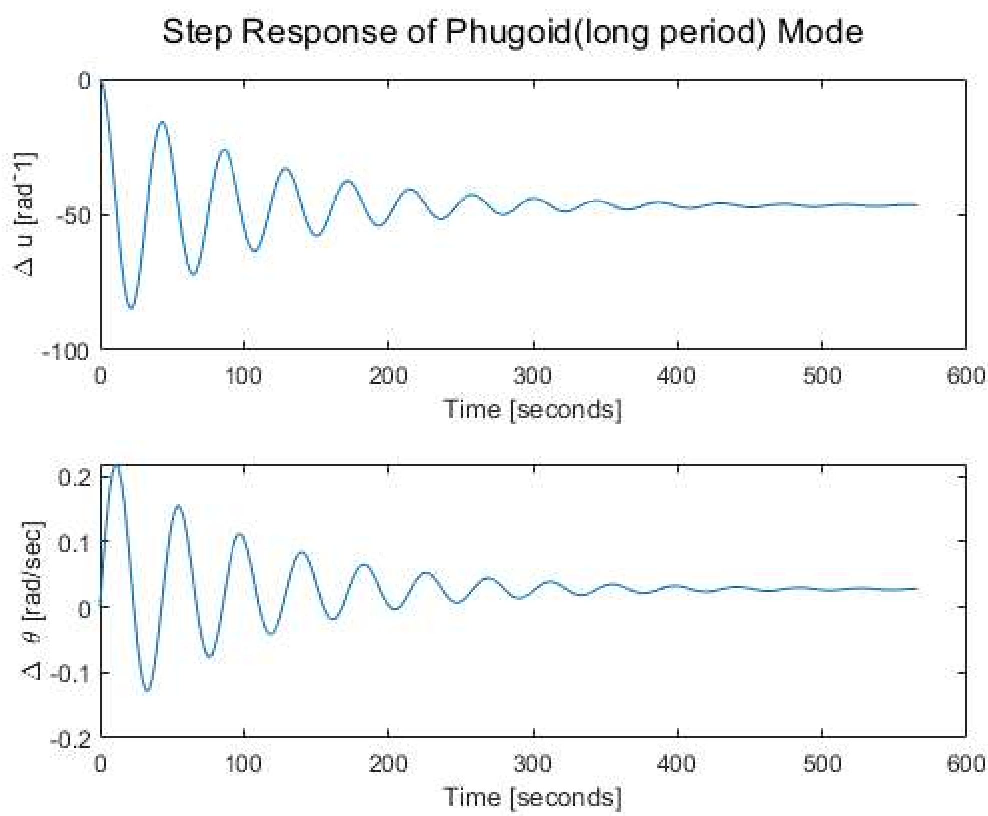
wd3   = imag(eVal3(1,1));
wn3   = sqrt((real(eVal3(1,1)))^2+(wd3)^2);
damp3 = abs(real(eVal3(1,1))/wn2);
delT3 = abs(log(2)/(real(eVal3(1,1))));
N3    = abs(log(2)*wd3/(2*pi*real(eVal3(1,1))));


sys3 = ss(A3,B3,C3,D3);

[h3, t3] = impulse(sys3);
figure(5)
sgtitle('Impulse Response of Phugoid(long period) Mode')
subplot(2,1,1)
plot(t3,h3(:,1))
ylabel('\Delta u [rad^-1]');
xlabel('Time [seconds]')
subplot(2,1,2)
plot(t3,h3(:,2))
ylabel('\Delta  \theta [rad/sec]');
xlabel('Time [seconds]')

[h3s, t3s] = step(sys3);
figure(6)
sgtitle('Step Response of Phugoid(long period) Mode')
subplot(2,1,1)
plot(t3s,h3s(:,1))
ylabel('\Delta u [rad^-1]');
xlabel('Time [seconds]')
subplot(2,1,2)
plot(t3s,h3s(:,2))
ylabel('\Delta  \theta [rad/sec]');
xlabel('Time [seconds]')
```





Print Answers

```
fprintf('\nNumber 1 \n');
fprintf('Short Period\n');
fprintf('----------------------------\n');
```

```
fprintf('Damped Frequency: %f [rad/s]\n', wd);
fprintf('Natural Frequency: %f [rad/s]\n', wn);
fprintf('Damping Ratio: %f \n', damp);
fprintf('Time to Damp to Half the initial Amplitude: %f [sec]\n', delT);
fprintf('Cycle to Damp to Half the initial Amplitude: %f [cycle]\n', N);
fprintf('\nLong Period\n');
fprintf('---------------------------\n');
fprintf('Damped Frequency: %f [rad/s]\n', wdLong);
fprintf('Natural Frequency: %f [rad/s]\n', wnLong);
fprintf('Damping Ratio: %f \n', dampLong);
fprintf('Time to damp to half the initial Amplitude: %f [sec]\n', delTLong);
fprintf('Cycle to damp to half the initial Amplitude: %f [cycle]\n', NLong);

fprintf('\nNumber 2 \n');
fprintf('---------------------------\n');
fprintf('Damped Frequency: %f [rad/s]\n', wd2);
fprintf('Natural Frequency: %f [rad/s]\n', wn2);
fprintf('Damping Ratio: %f \n', damp2);
fprintf('Time to Damp to Half the initial Amplitude: %f [sec]\n', delT2);
fprintf('Cycle to Damp to Half the initial Amplitude: %f [cycle]\n', N2);



fprintf('\nNumber 3 \n');
fprintf('---------------------------\n');
fprintf('Damped Frequency: %f [rad/s]\n', wd3);
fprintf('Natural Frequency: %f [rad/s]\n', wn3);
fprintf('Damping Ratio: %f \n', damp3);
fprintf('Time to Damp to Half the initial Amplitude: %f [sec]\n', delT3);
fprintf('Cycle to Damp to Half the initial Amplitude: %f [cycle]\n', N3);
```

```
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