The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

17 SEPTEMBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Yaw Stiffness
  - Yaw Control
  - Yaw Stiffness/Control Problem
  - Roll Stiffness

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YAW STIFFNESS

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Yaw Stiffness (1)

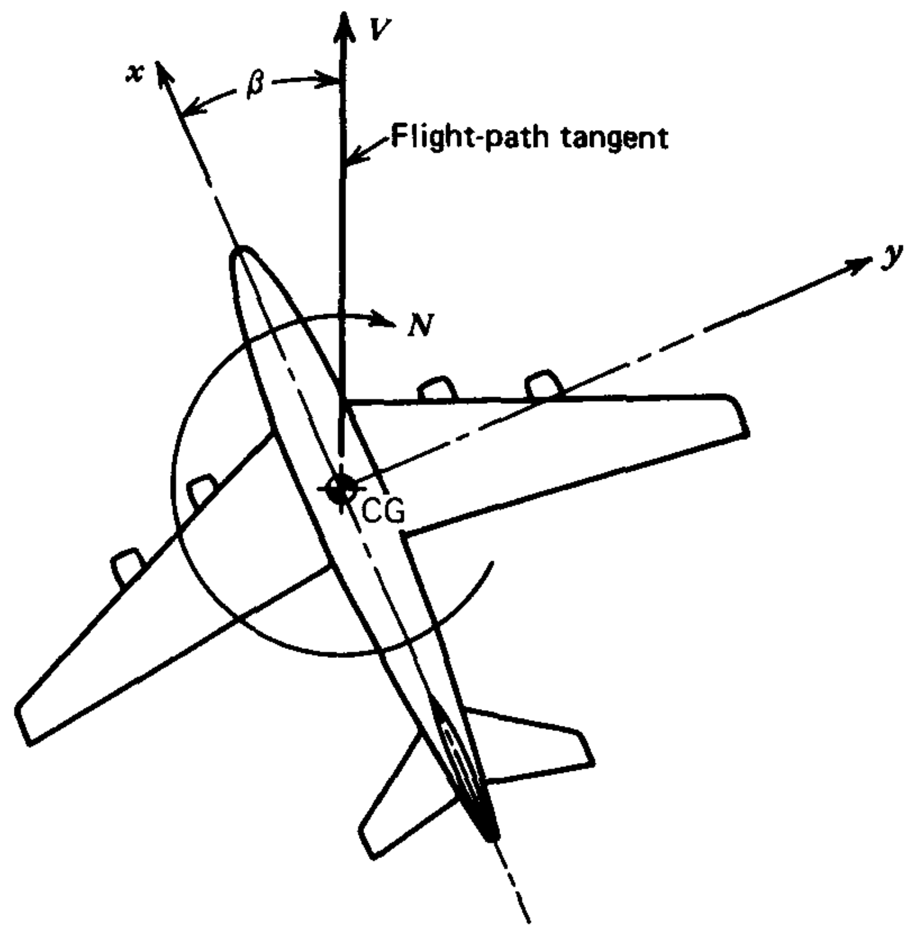
L, M, N
C_l C_m C_n

- The yawing moment N is positive when it induces the nose to turn to the right.
  - The z axis is point down through the bottom of the aircraft so per the “right-hand rule” a turn to the right is positive.
- The nondimensional coefficient of N is...

$$C_n = \frac{N}{\frac{1}{2}\rho V^2 S b}$$

- The derivative of this coefficient with respect to the sideslip angle β is…

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$



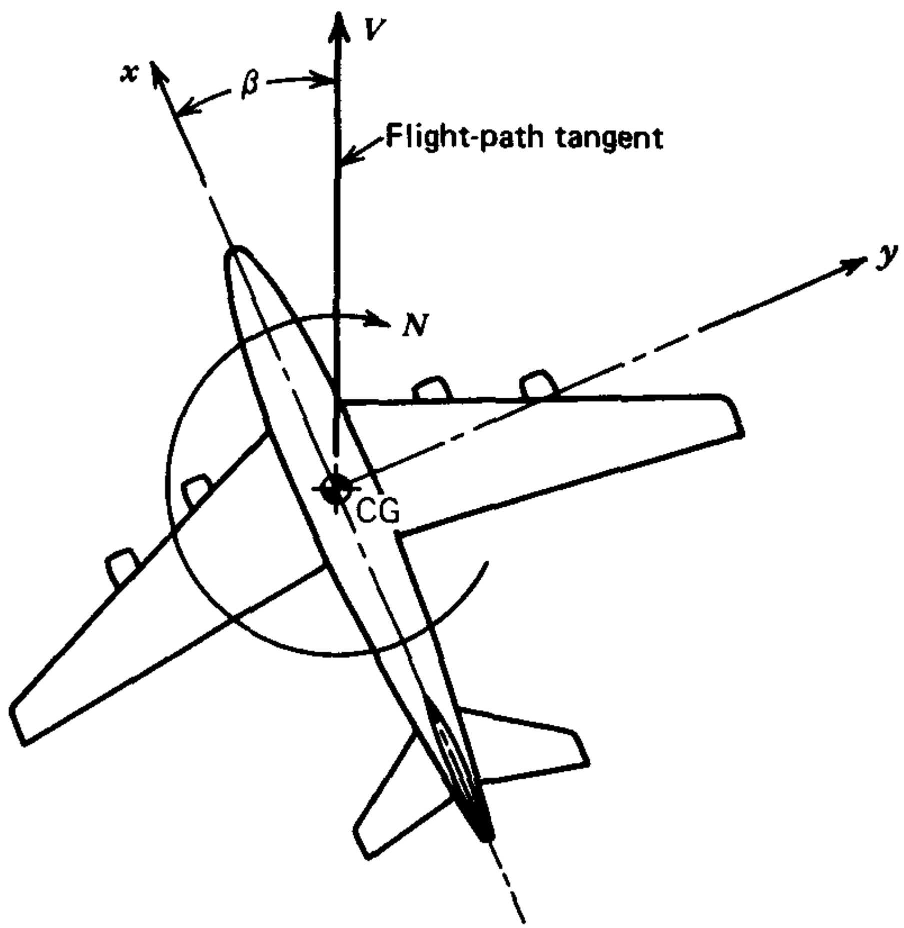
Yaw Stiffness (2)

- We know that the ”weathercocks” are very stable in the presence of winds and we would like airplane fins to be the same.

Yaw Stiffness (3)

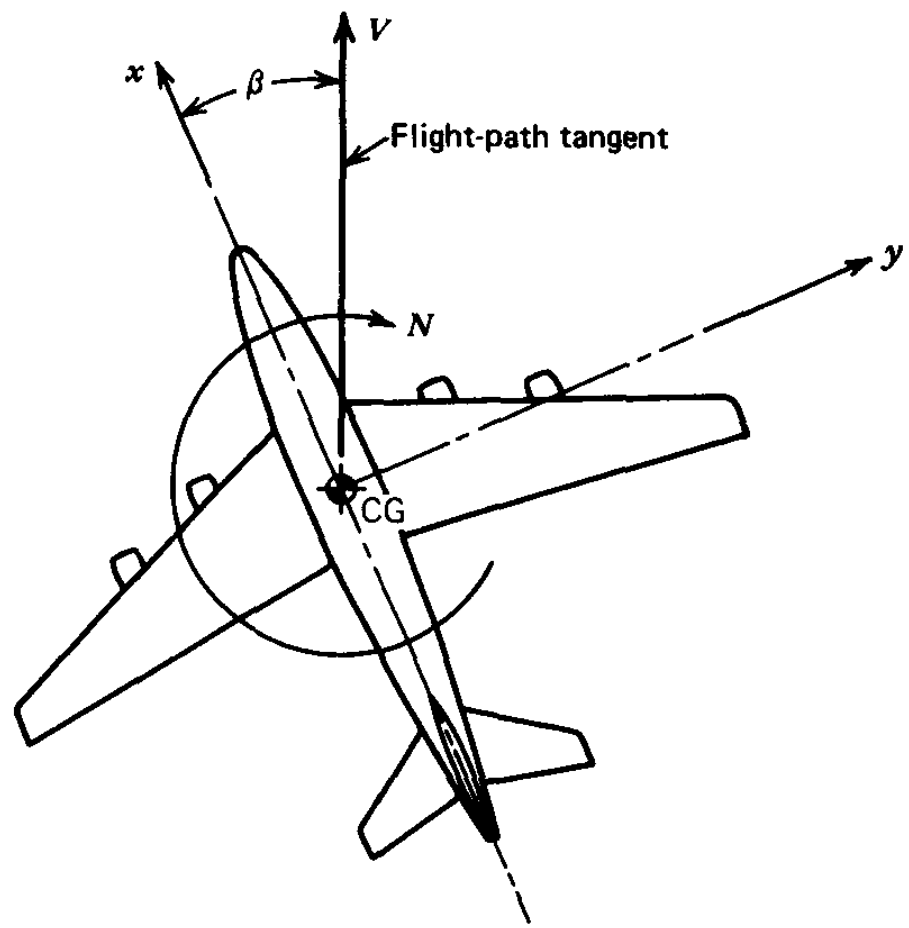
- Application of the static stability principle to rotation about the z axis suggests that a stable airplane should have "weathercock stability."
  - When the airplane is at an angle of sideslip relative to its flight path, the yawing moment produced should be such as to tend to restore it to symmetric flight.
  - Otherwise, the airplane would keep spinning!
- Since β is positive when the sideslip is to the right, and you want the yawing moment to be to the right (i.e., positive) then...

$$C_{n_\beta} > 0$$



Yaw Stiffness (4)

- A positive value of C_{n_β} does not guarantee lateral stability.
- Lateral stability can only be determined by a full dynamic analysis.
  - As was the case in longitudinal static stability, yaw stiffness only indicates the tendency of the airplane to return to its equilibrium if it is held in a deviated state (in this case at a constant sideslip angle) and then released from rest.

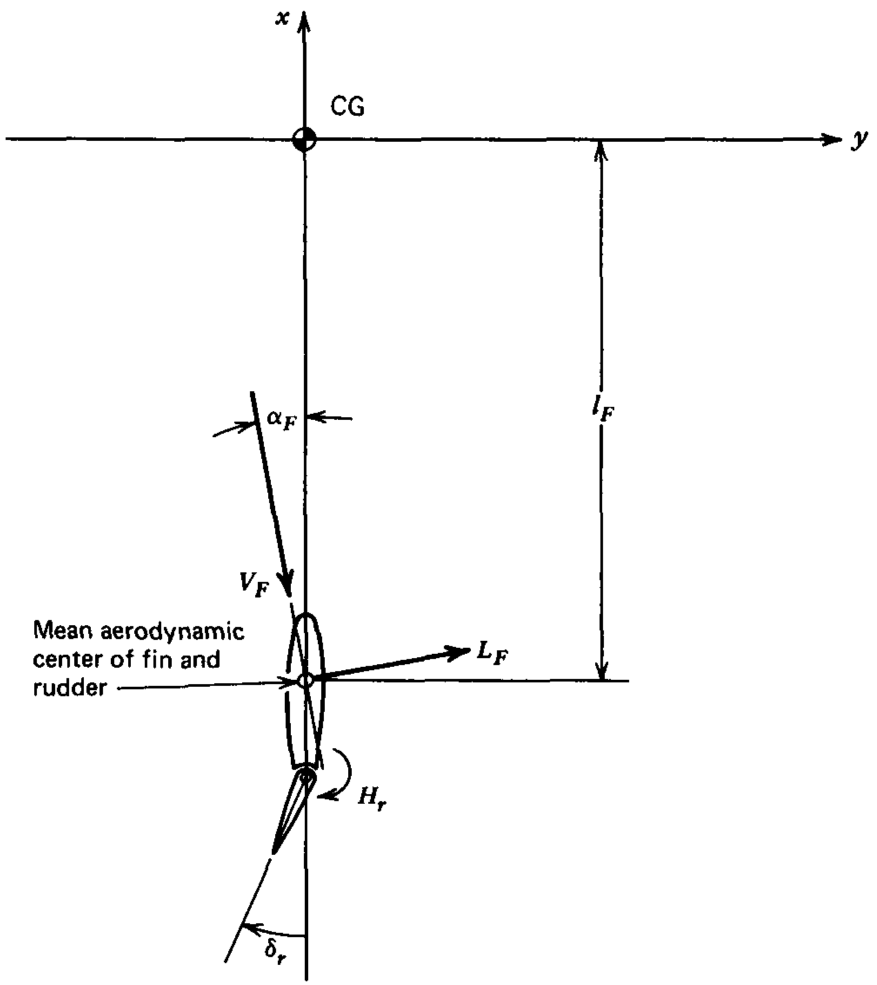


Yaw Stiffness (5)

- The body and the vertical-tail surface (i.e., the fin) provide the principal contributions to C_{n_β}.
  - The contribution of the body is complex and is usually estimated from empirical data, as in Appendix B of Etkin and Reid.
- The angle of attack of the fin is…

$$\alpha_F = -\beta + \sigma$$

where σ is the "sidewash" angle, which is is positive when the flow of the sidewash is in the y direction.



Yaw Stiffness (6)

- The fin is an airfoil with a coefficient of lift…

$$C_{L_F} = a_F(-\beta + \sigma) + a_r\delta_r$$

lift curve slope

where a_F and a_r are the partial derivative of C_{L_F} with respect to α_F and rudder deflection δ_r.

- The resulting lift of the fin (a force to the side) is …

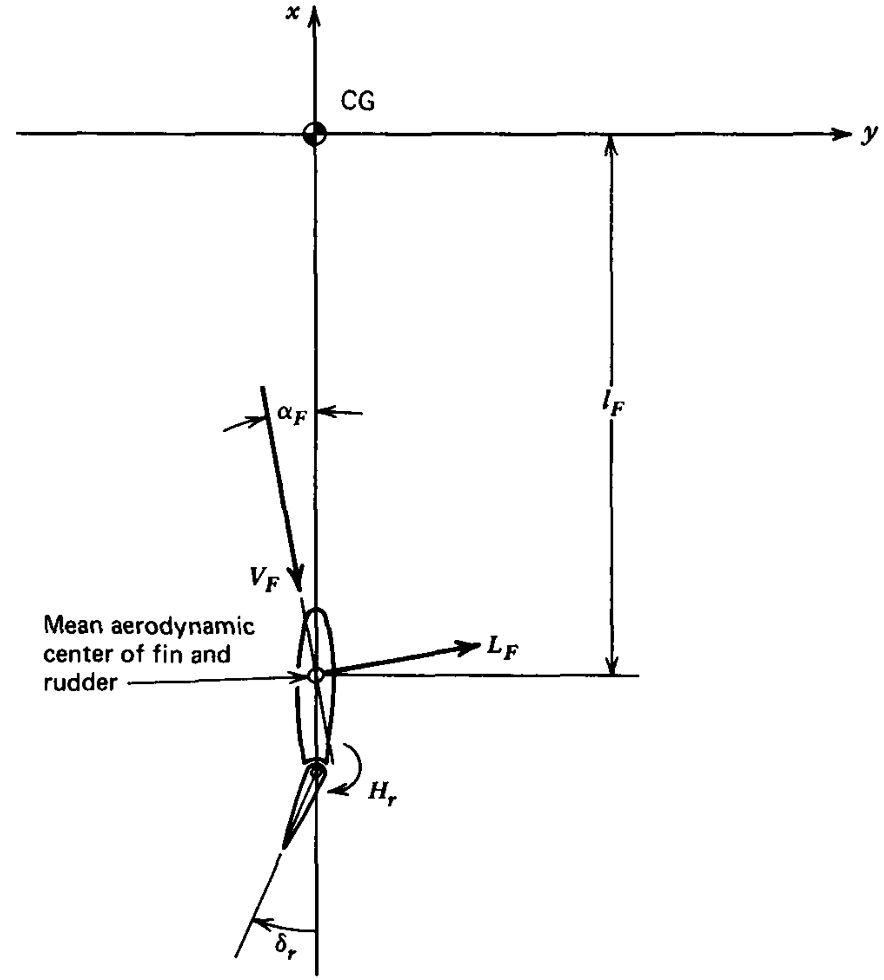
$$L_F = \frac{1}{2}\rho V_F^2 S_F C_{L_F}$$

and the corresponding yawing moment is…

$$N_F = \frac{1}{2}\rho V_F^2 S_F C_{L_F} l_F$$

ignore effect of drag

where l_F is the distance between the center of gravity and the mean aerodynamic center of the fin.



Yaw Stiffness (7)

close to 1.

vertical tail volume ratio

- The coefficient for the yawing moment due to the fin is…

$$C_{n_F} = -C_{L_F}\frac{S_F \overset{l_F}{\cancel{C_{L_F}}}}{Sb}\left(\frac{V_F}{V}\right)^2$$

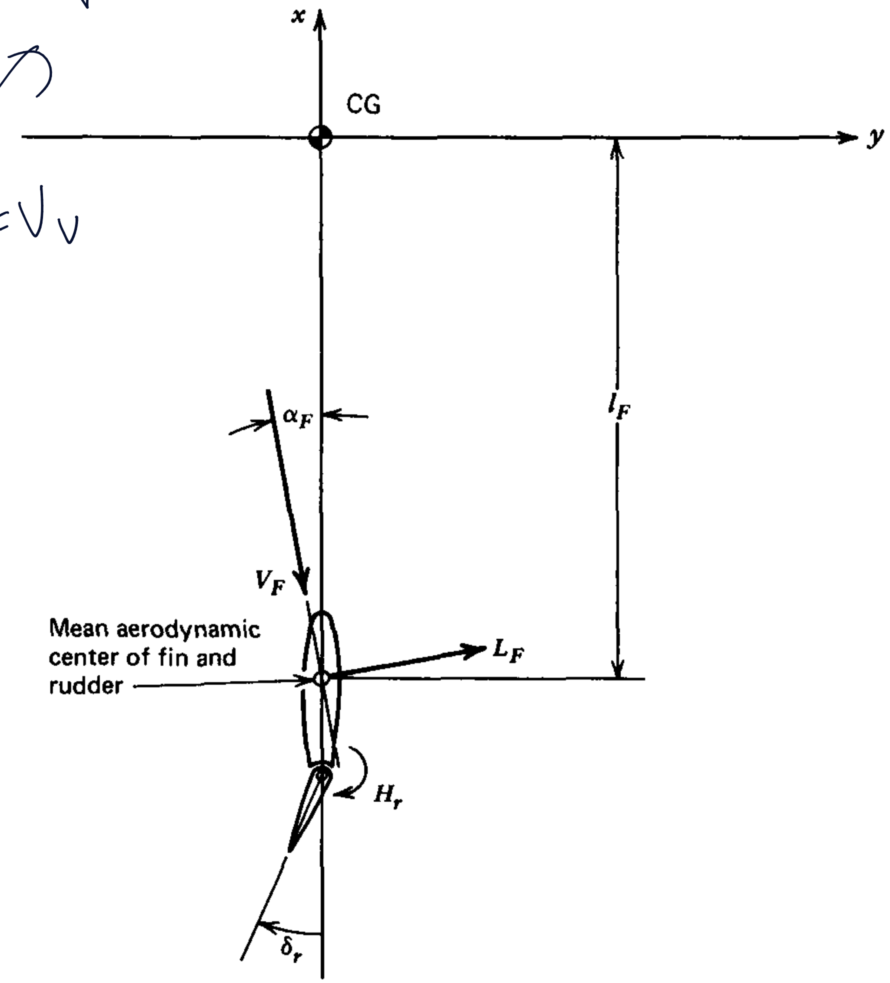
$$\frac{S_F l_F}{s \cdot b} = V_V$$

where $\frac{S_F \overset{l_F}{\cancel{C_{L_F}}}}{Sb}$ is the vertical-tail volume ratio V_V and the equation above can be re-written as…

$$C_{n_F} = -V_V\left(\frac{V_F}{V}\right)^2 C_{L_F}$$

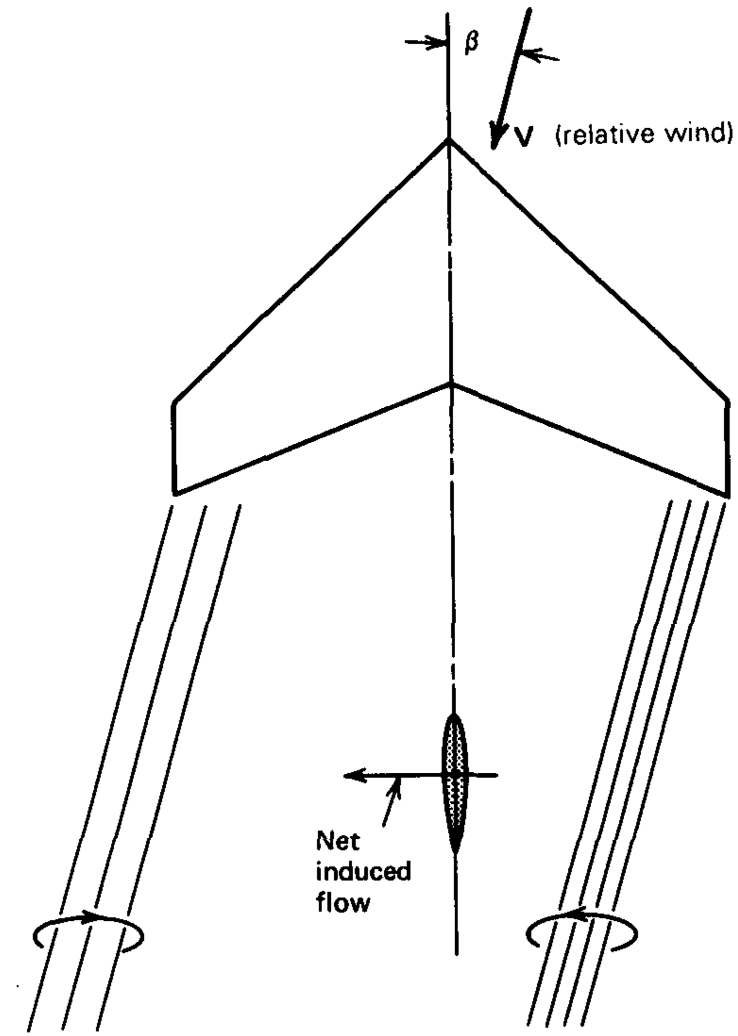
and the contribution of the fin to weathercock stability is…

$$\frac{\partial C_{n_F}}{\partial \beta} = \underset{(+)}{-V_V}\left(\frac{V_F}{\underset{(+)}{V}}\right)^2 \underset{(-)}{\frac{\partial C_{L_F}}{\partial \beta}} = a_F V_V\left(\frac{V_F}{V}\right)^2\left(1-\frac{\partial \sigma}{\partial \beta}\right)$$



Yaw Stiffness (8)

- The velocity ratio $\frac{V_F}{V} = 1$ when the fin is not in a propeller slipstream.
- The sidewash factor $\frac{\partial \sigma}{\partial \beta}$ (a.k.a. sidewash derivative) is difficult to estimate analytically.
  - Fuselage becomes a lifting body when yawed.
  - Vortex wake from wings (esp. low-aspect-ratio swept wings) induces a lateral-flow field at tail.
  - Sidewash from the propeller is associated with the side force which acts upon it when yawed.
- Sidewash typically estimated from wind-tunnel tests.



Yaw Stiffness(9)

- The normal force that acts on the yawed propeller produces a yawing moment…

$$\Delta \frac{\partial C_n}{\partial \beta} = C_{n_F} = -\frac{x_p}{b}\frac{S_p}{S}\frac{\partial C_{n_p}}{\partial \alpha_p}$$

  where x_p is the distance from the CG to the propeller and α_p is the angle of attack at the propeller.

- This is known as the propeller fin effect and is negative (i.e., destabilizing) when the propeller is forward of the CG, but is usually positive for pusher propellers.
  - There is a similar effect for jet engines.

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YAW CONTROL

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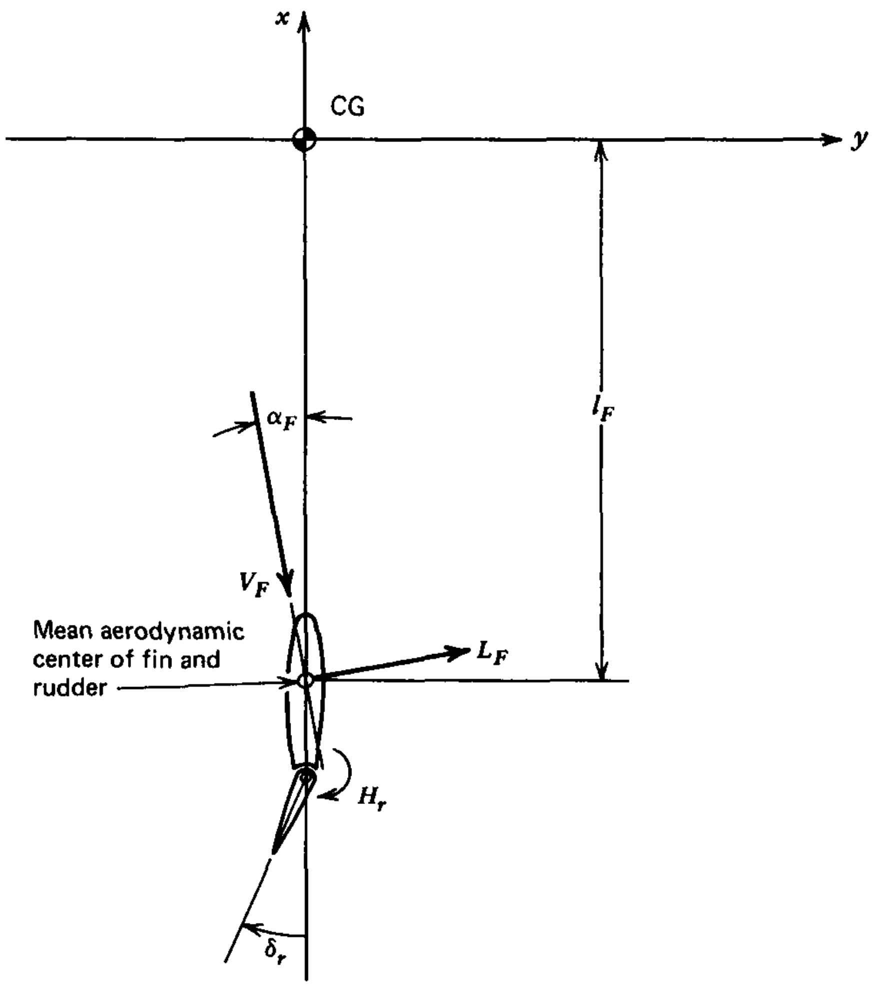
Yaw Control (1)

- The rate of change of yawing moment with respect to rudder deflection is…

$$C_{n_{\delta_r}} = \frac{\partial C_n}{\partial \delta_r}$$
$$= -V_V \left(\frac{V_F}{V}\right)^2 \frac{\partial C_{L_F}}{\partial \delta_r}$$
$$= -a_r V_V \left(\frac{V_F}{V}\right)^2$$

- The rate of change of yawing moment with respect to rudder deflection is also referred to as the "rudder power."



Lecture 12.2:

in trim, $C_n = 0$

Yaw Control (2)

- Sometimes we need to use the rudder to maintain a constant sideslip angle (e.g., during crosswind landing). In this case…

$$C_n = \underbrace{C_{n_\beta}}_{\text{sideslip}}\beta + C_{n_{\delta_r}}\delta_r$$

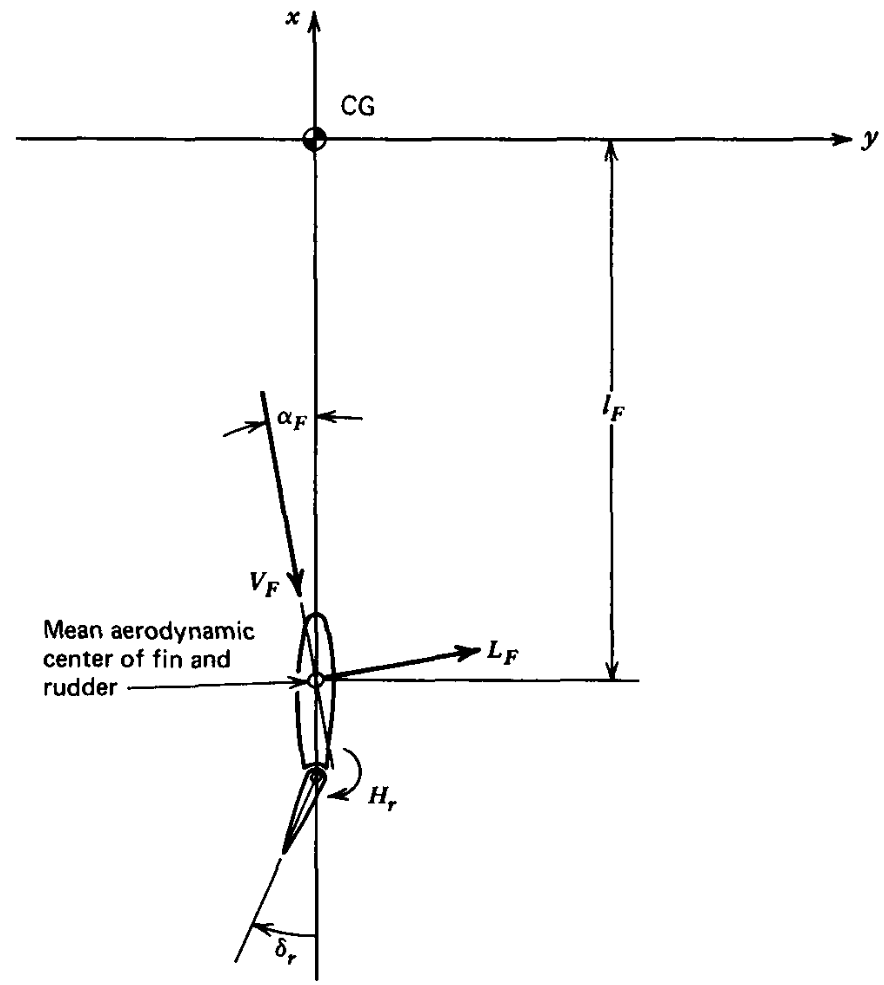
rudder

and the rudder deflection required is $\delta_r = -\dfrac{C_{n_\beta}}{C_{n_{\delta_r}}}\beta$

- Sometimes we need to use the rudder to balance a moment acting on the airplane (e.g., when an engine fails or during a turn). When an engine fails…

$$C_n = N_{engine} + C_{n_{\delta_r}}\delta_r$$

and the rudder deflection required is $\delta_r = -\dfrac{N_{engine}}{C_{n_{\delta_r}}}$



Yaw Control (3)

- Why have they never replaced the 8 engines on a B52 with 4 engines?

$C_{n_{\delta_r}}$ isn’t big enough!

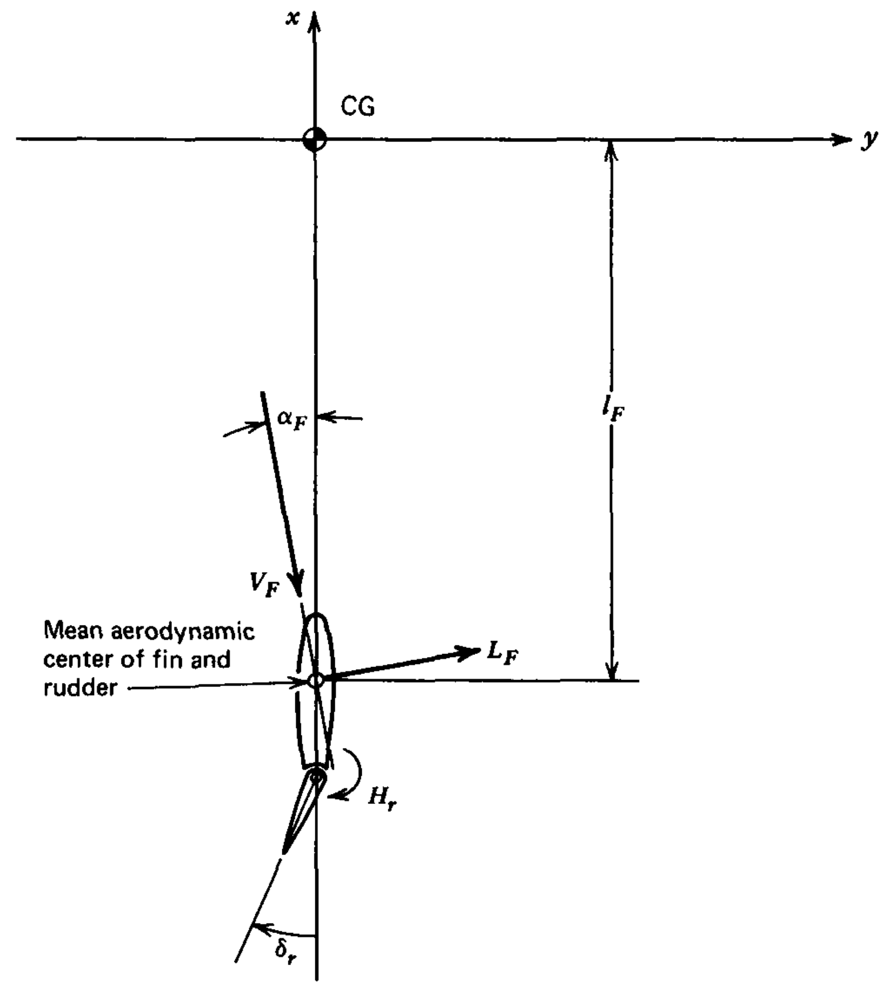
Yaw Control (4)

- The rudder pedal force required to maintain a given rudder deflection is…

$$P = G\,\tfrac{1}{2}\rho V_F^2 S_r c_r (b_1\alpha_F + b_2\delta_r)$$

$$= G\,\tfrac{1}{2}\rho V_F^2 S_r c_r [b_1(-\beta + \sigma) + b_2\delta_r]$$

where G is the rudder system gearing ratio and b_1 and b_2 are constants in the expression for the rudder hinge coefficient $C_{h_r} = b_1\alpha_F + b_2\delta_r$.



Yaw Control (5)

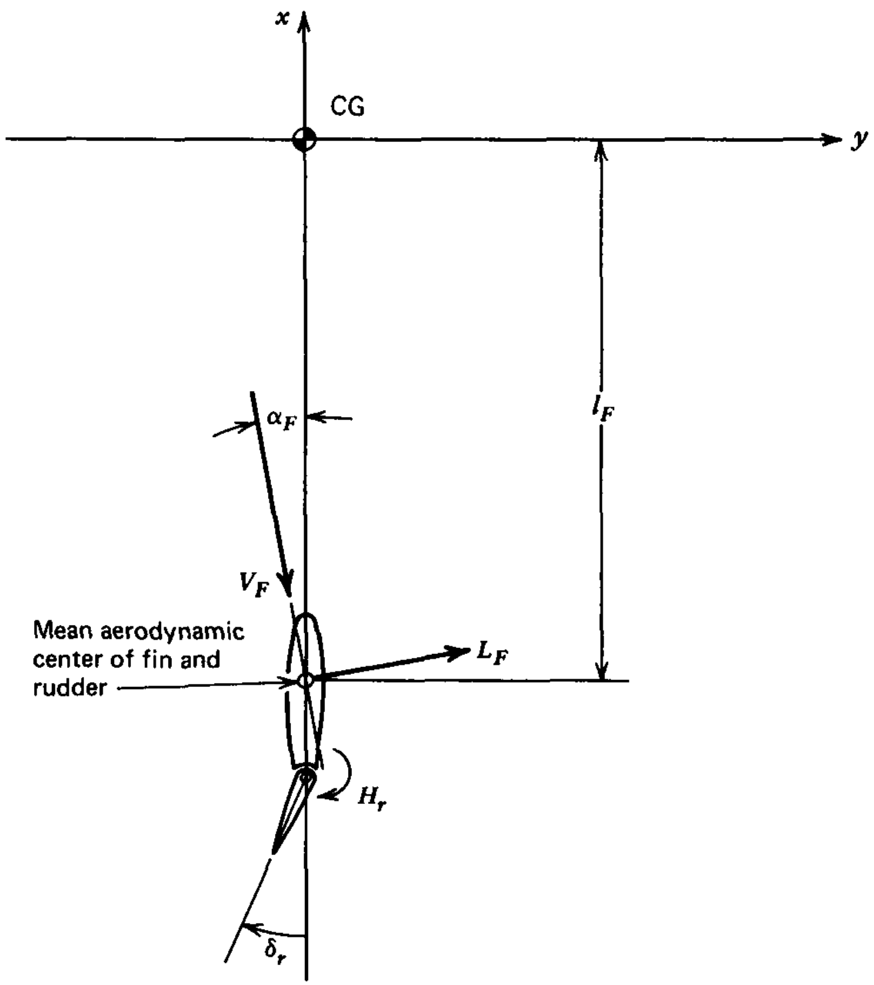
- If no force is applied to the rudder pedals (i.e., you let the rudder go and $\mathrm{P}=0$) the rudder is free to "move with the wind" and settle at the angle…

$$\delta_{r_{free}} = -\frac{b_1}{b_2}\alpha_F$$

- This "free" rudder affects the lift generated but the fin and the corresponding "adjusted" lift is…

$$C_{L_F} = a_F\alpha_F - a_r\frac{b_1}{b_2}\alpha_F$$

$$= a_F\alpha_F\left(1 - \frac{a_r}{a_F}\frac{b_1}{b_2}\right)$$



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YAW STIFFNESS/CONTROL PROBLEM

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Yaw Stiffness/Control Question (1)

For the twin engine airplane shown in Figure P2.16, determine the rudder size to control the airplane if one engine needs to be shut down. Use the flight information shown in the figure and

Wing: $S = 980\ \text{ft}^2$ $b = 93\ \text{ft}$

Vertical tail: $S_v = 330\ \text{ft}^2$ $AR_v = 4.3$ $l_v = 37\ \text{ft}$ $\eta_v = 1.0$

Rudder: $\delta_r = \pm 15°$

Propulsion: $T = 14{,}000$ lb each $y_T = 16\ \text{ft}$

Flight condition: $V = 250\ \text{ft/s}$ $\rho = 0.002378\ \text{slug/ft}^3$

Yaw Stiffness/Control Question (2)

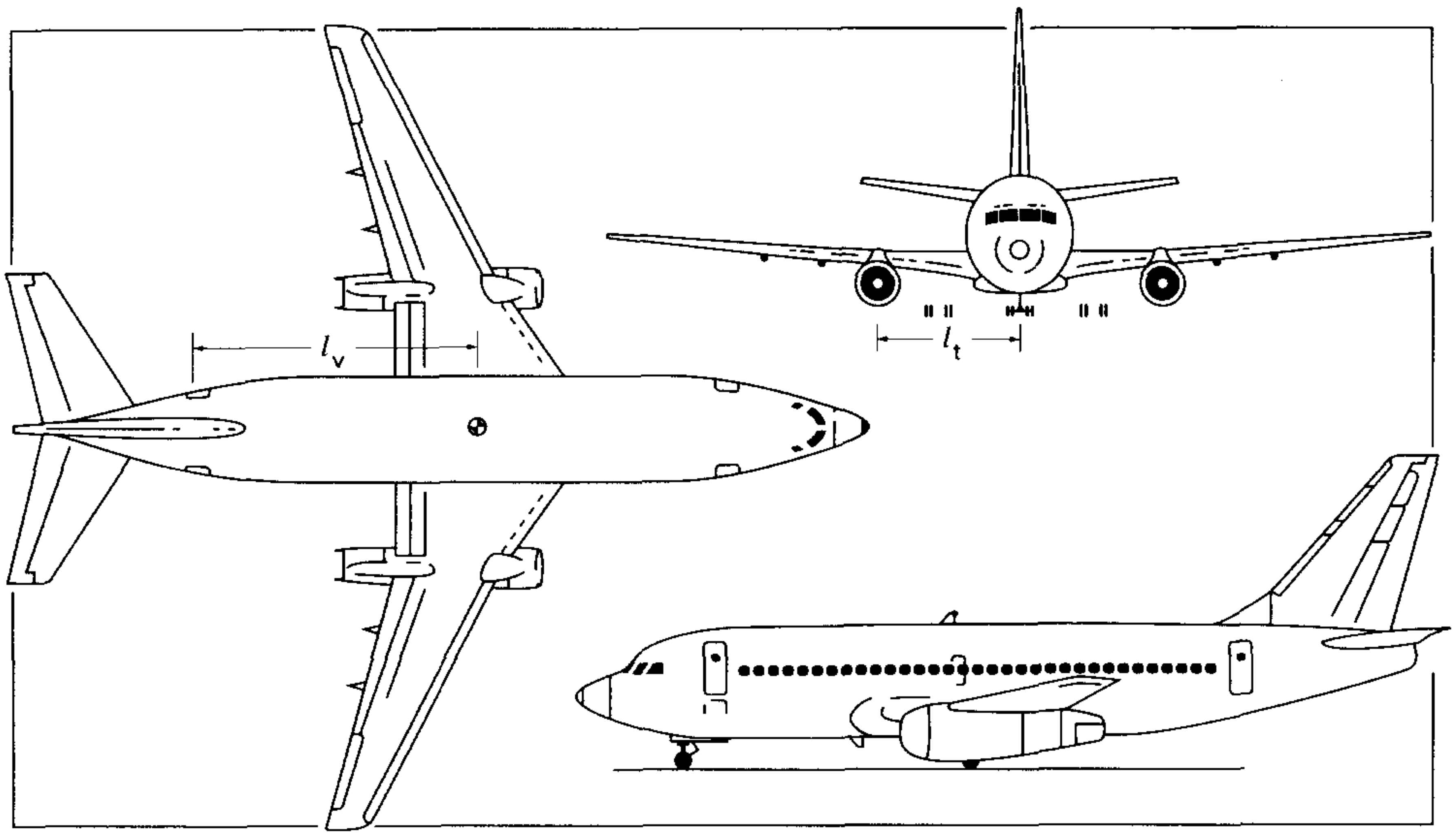


FIGURE P2.16

Yaw Stiffness/Control Question (3)

- Want to control the airplane with one engine shut down ie the total yawing moment should be zero:

$$\underset{\text{(from rudder)}}{N_{aero}} + N_{engine} = 0$$

$$N_{engine} = -Ty_T \Rightarrow N_{aero} = -N_{engine} = Ty_T = 14{,}000 \times 16$$

$$\underline{\underline{N_{aero} = 224{,}000 \text{ lb.ft}}}$$

- By definition $C_n = \dfrac{N_{aero}}{\frac{1}{2}\rho V^2 S b} = \dfrac{224000}{\frac{1}{2}\, 0.002378 \times 250^2 \times 980 \times 93}$

$$\underline{\underline{C_n = 0.0331}}$$

Yaw Stiffness/Control Question (4)

- $C_n = C_{n_{\delta r}} \delta r \Rightarrow C_{n_{\delta r}} = \frac{C_n}{\delta r} = \frac{0.0331}{-15^\circ \times \frac{\pi}{180}} = -0.1264 \ /\text{rad.}$

Moment is related to the flap effectiveness parameter

$$C_{n_{\delta r}} = -\eta_v V_v C_{L_{\alpha_v}} \tau \Rightarrow \tau = -\frac{C_{n_{\delta r}}}{\eta_v V_v C_{L_{\alpha_v}}}$$

* Assume $\eta_v = 1$

* $V_v = \frac{l_v S_v}{S b} = \frac{37 \times 330}{980 \times 93} = 0.134$

* $C_{L_{\alpha_v}} = \frac{C_{l_\alpha}}{1 + C_{l_\alpha}/\pi AR_v}$ assume $C_{l_\alpha} = 0.1/\text{deg} = 5.73/\text{rad.}$

lift curve slope for vertical tail

$AR_v = 4.3$

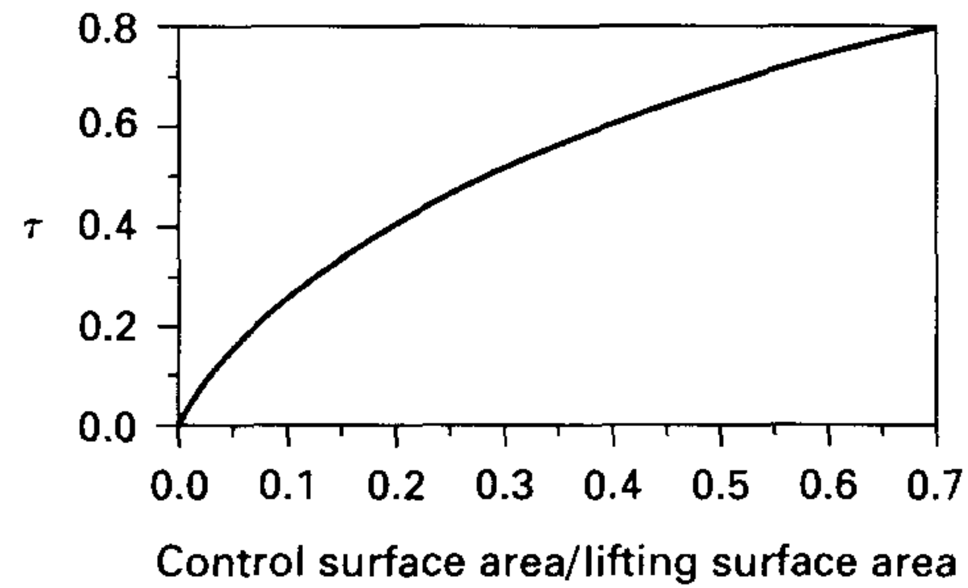
$\Rightarrow \underline{\underline{C_{L_{\alpha_v}} = 4.02 \ /\text{rad}}}$

$$\Rightarrow \tau = -\frac{-0.1264}{0.134 \times 4.02} = \underline{\underline{0.23}}$$

Yaw Stiffness/Control Question (5)

True for canards

Flap effectiveness parameter versus control surface area



$\tau \simeq 0.23 \Rightarrow \frac{S_r}{S_V} \simeq 0.08$

thus $S_r = S_V \times 0.08 = 330 \times 0.08$

$\boxed{S_r = 26.4 \text{ ft}^2}$ small.

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ROLL STIFFNESS

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Roll Stiffness (1)

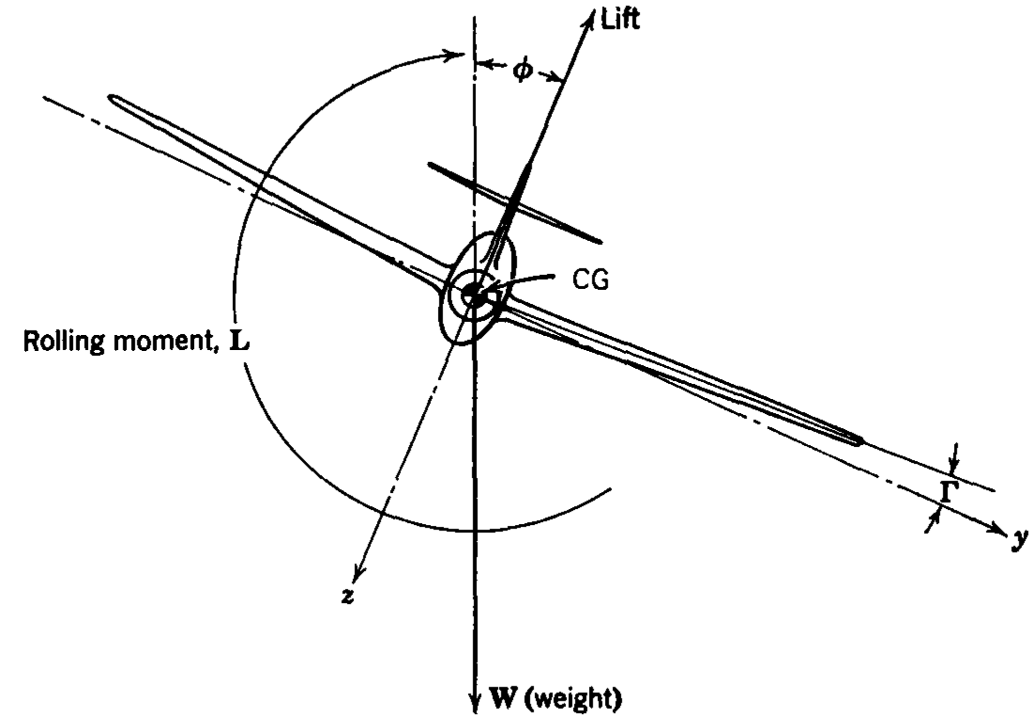
- An airplane in steady flight with wings level will have a velocity...

$$\boldsymbol{V}_1 = \begin{bmatrix} V\cos\alpha_x \\ 0 \\ V\sin\alpha_x \end{bmatrix}$$

where α_x is the angle of attack of the x axis.

- After the aircraft rotates about its x axis, the resulting velocity in the new reference frame after the rotation is…

$$\boldsymbol{V}_2 = \boldsymbol{L}_1(\phi)\boldsymbol{V}_1 = \begin{bmatrix} V\cos\alpha_x \\ V\sin\alpha_x\sin\phi \\ V\sin\alpha_x\cos\phi \end{bmatrix}$$



Roll Stiffness (2)

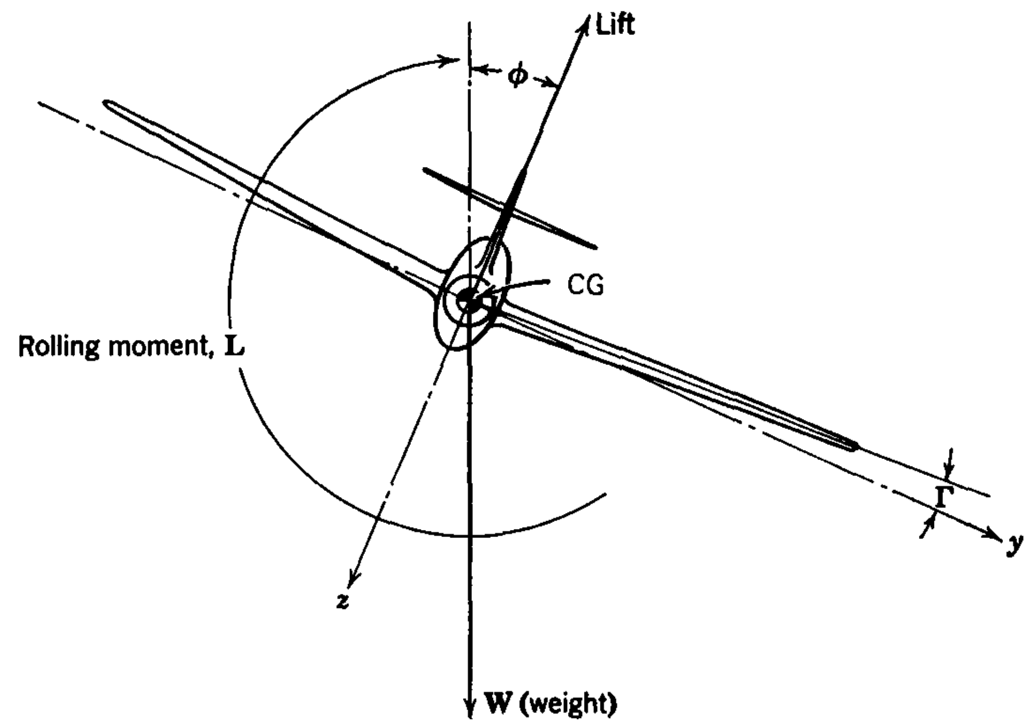
- The resulting sideslip angle is...

$$\beta = \sin^{-1}\left(\frac{v}{V}\right) = \sin^{-1}(\sin\alpha_x \sin\phi)$$

- If α_x and ϕ (i.e., the airplane is pitched up and rolls to the right) and the derivative $C_{l_\beta} = \frac{\partial C_l}{\partial \beta} < 0$ then the resulting moment about the x axis…

$$\Delta C_l = C_{l_\beta} \sin^{-1}(\sin\alpha_x \sin\phi) < 0$$

- and the airplane will experience a “restoring” rolling moment to the left (i.e., in the opposite direction to the direction in which it rolled).



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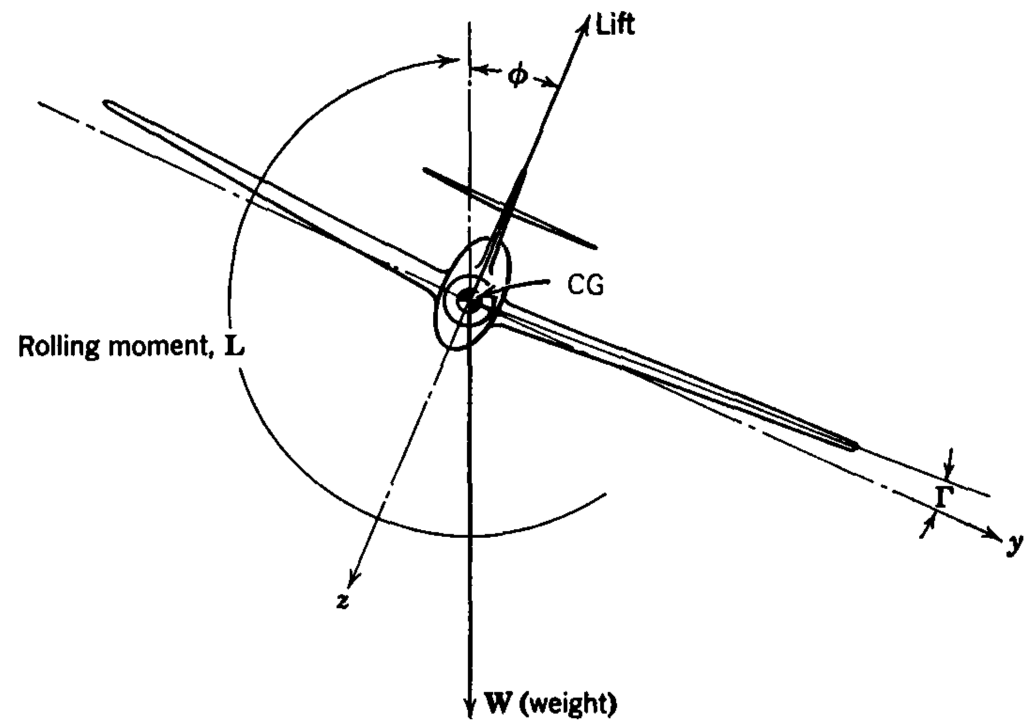
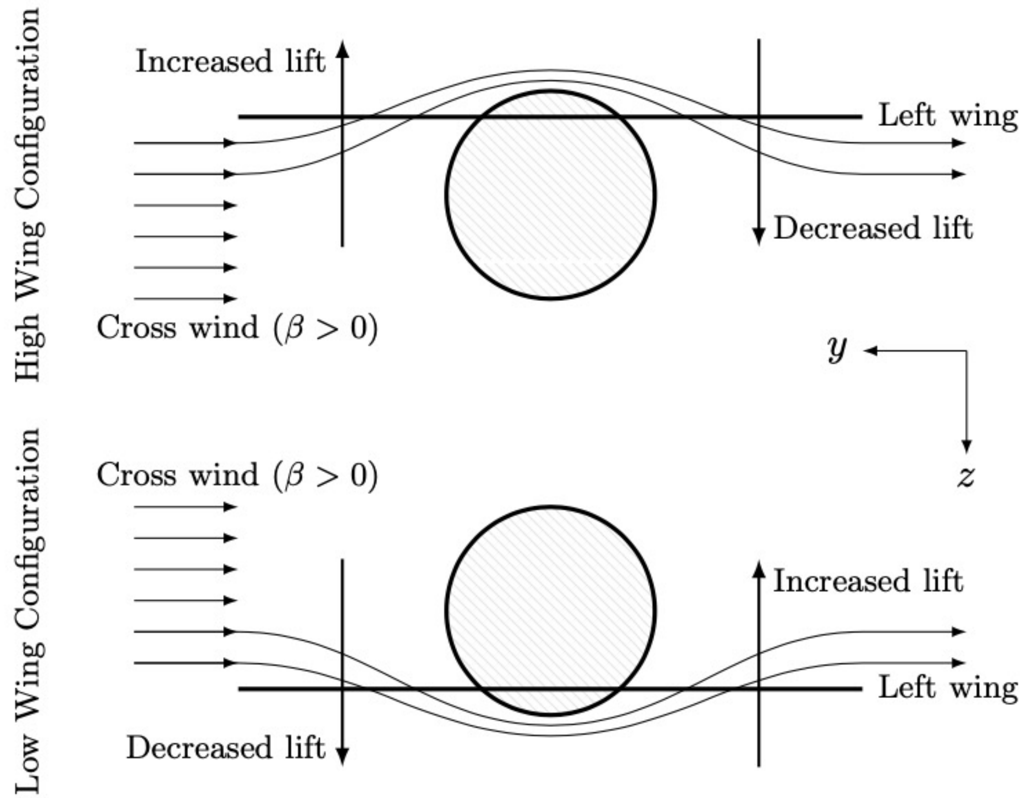


Roll Stiffness (4)



High (straight) wing configuration is statically roll stabilizing …

$C_{\ell_\beta} < 0$ roll stabiliz

moment around x

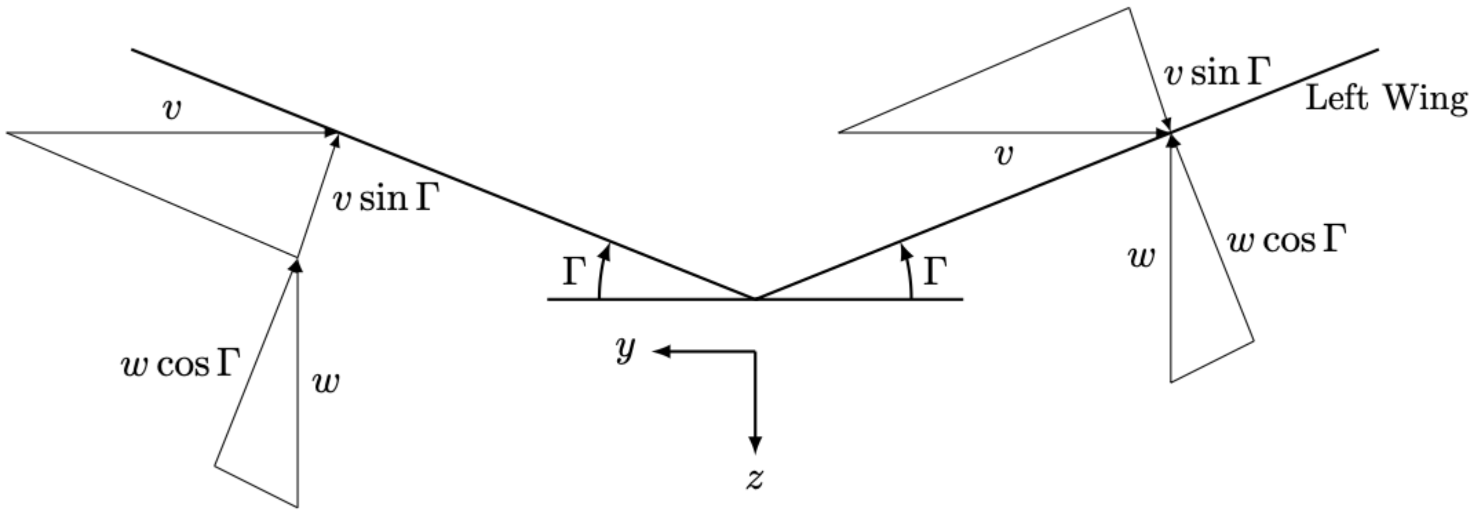
Does the fuselage help or hurt? It depends!

Roll Stiffness (5)

High wing, slight dihedral

Low wing, large dihedral

Roll Stiffness (6)



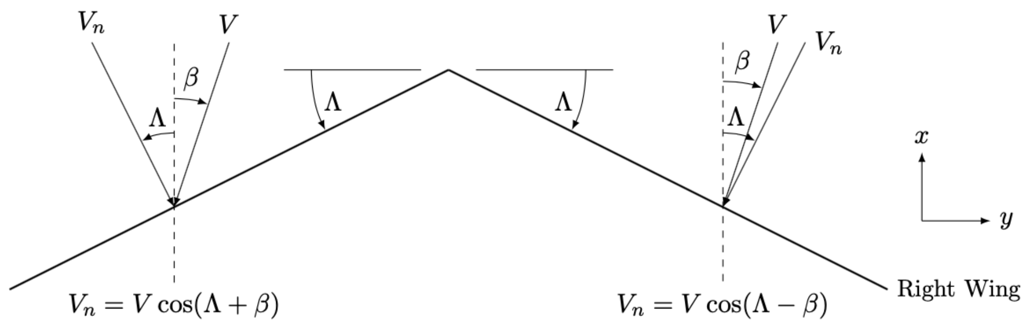
For $C_{\ell_\beta} < 0$

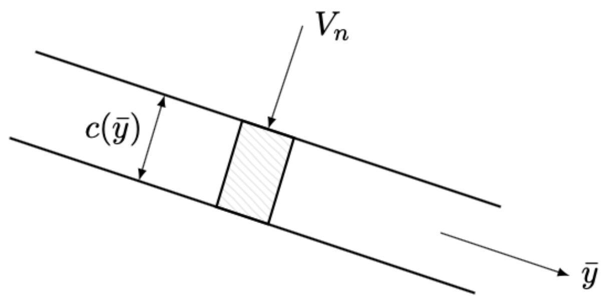
➡ $\Gamma > 0$

$$L = -\frac{1}{2}\bar{q}SC_{L_{\alpha_w}}(\alpha + \Gamma\beta)y_{ac_w} + \frac{1}{2}\bar{q}SC_{L_{\alpha_w}}(\alpha - \Gamma\beta)y_{ac_w}$$
$$= -\bar{q}SC_{L_{\alpha_w}}\Gamma y_{ac_w}\beta$$

➡ $C_\ell = -\frac{y_{ac_w}}{b}C_{L_{\alpha_w}}\Gamma\beta$ and ... $C_{\ell_\beta} = \frac{\partial C_\ell}{\partial \beta} = -\frac{y_{ac_w}}{b}C_{L_{\alpha_w}}\Gamma$

Roll Stiffness (7)





$$V_{n_L} = V\cos(\Lambda + \beta)$$

$$V_{n_R} = V\cos(\Lambda - \beta)$$

Thus …

$$Lift_L = q\frac{S}{2}C_L\cos^2(\Lambda + \beta)$$

$$Lift_R = q\frac{S}{2}C_L\cos^2(\Lambda - \beta)$$

… the net moment $L = -y_{ac_w}q\frac{S}{2}C_L\left[\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right]$

… and $C_{\ell_\beta} = -\frac{y_{ac_w}}{b}C_L\sin(2\Lambda)\beta < 0$ when $\Lambda > 0$

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