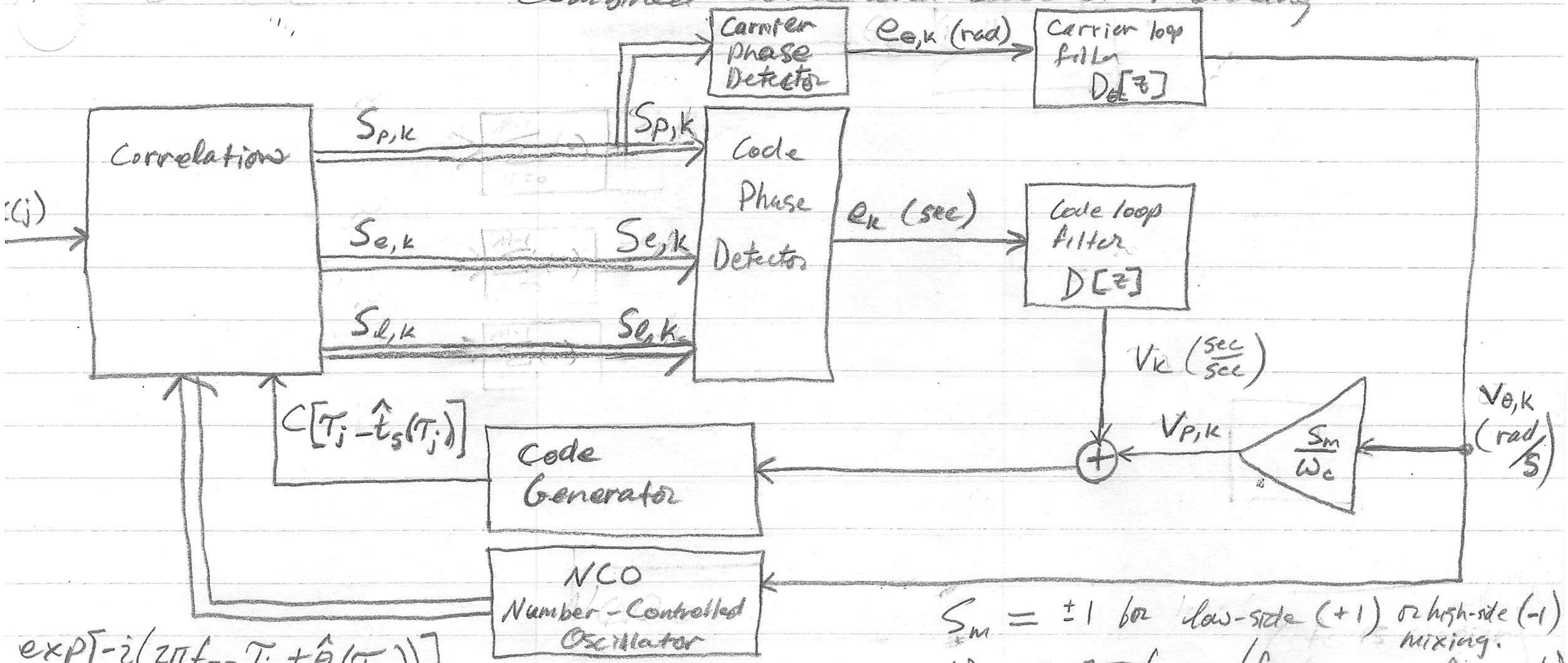
Combined Code and Carrier Tracking

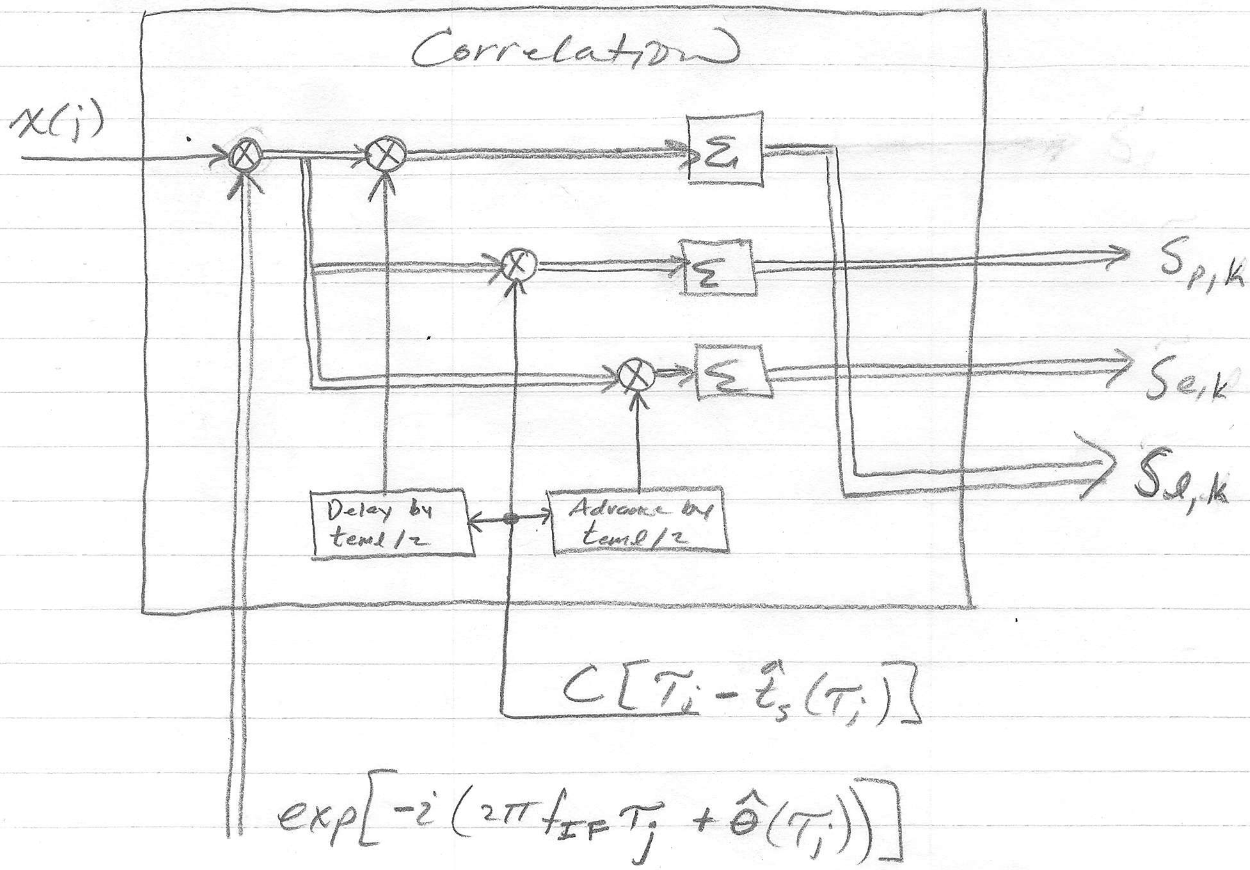


$S_m = \pm 1$ for low-side (+1) or high-side (-1) mixing.

$\omega_c = 2\pi f_c$ (fc is carrier freq. in Hz)

$$\boxed{\Sigma} = \boxed{\sum_{j=i_k}^{j_k+N_k-1} (\cdot)}$$

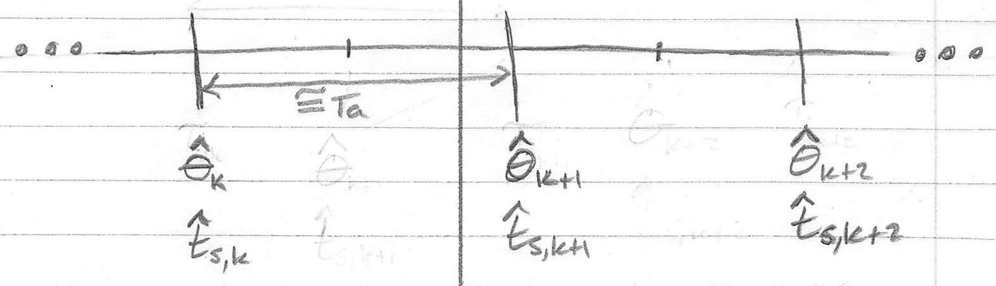
$$x(j) = A(\tau_j)D[\tau_j - t_d(\tau_j)]C[\tau_j - t_s(\tau_j)]\cos[2\pi f_{IF}\tau_j + \theta(\tau_j)] + n(j)$$



[Show full closed-loop tracking system as roadmap]

Filter-Based Est. of t_s, θ

The ML estimators for t_s, θ discussed earlier perform batch estimation — they produce a single estimate from a given interval of data. To perform ongoing estimation of t_s, θ, we could do batch estimation on consecutive segments of data:

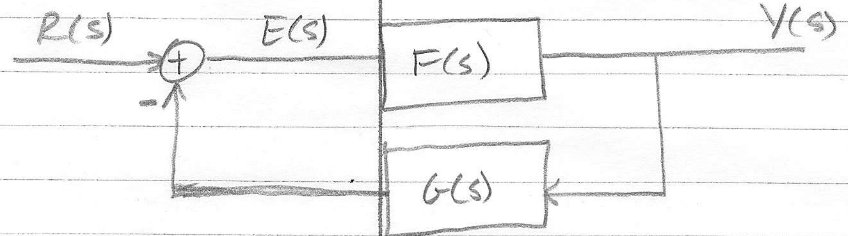


But this approach throws away information from previous segments. Better would be a recursive estimator such as a Kalman Filter.

A simple approximation to the KF: feedback control loop. We'll design separate, but coupled, loops for carrier and code.

Basic Feedback Control Loop theory

Let $R(s)$ (Laplace tfm.) be a reference signal we wish to track. Std. feedback loop:



$F(s)$ ≝ loop filter
$G(s)$ = feedback gain (=1 for unity gain)
$E(s)$ = error signal
$Y(s)$ = output signal
$F(s)\,G(s)$ = open loop transfer function

$$E(s) = R(s) - F(s)\,G(s)\,E(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + F(s)\,G(s)}$$

$$Y(s) = F(s)\,E(s)$$

$$\Rightarrow H(s) \equiv \frac{Y(s)}{R(s)} = \frac{F(s)}{1 + F(s)\,G(s)}$$ closed-loop transfer function

A study of $H(s)$ reveals the performance and stability of the feedback system.

$$R(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

← Insert output desc.

5.3 PERFORMANCE OF A SECOND-ORDER SYSTEM

(Modern Control Sys. Dorf / Bishop)

Let us consider a single-loop second-order system and determine its response to a unit step input. A closed-loop feedback control system is shown in Fig. 5.4. The closed-loop output is

$F(s) = \frac{K}{s(s+p)}$, $G(s) = 1$

$$Y(s) = \frac{G(s)}{1 + G(s)} R(s) = \frac{K}{s^2 + ps + K} R(s). \tag{5.6}$$

Utilizing the generalized notation of Section 2.4, we may rewrite Eq. (5.6) as

$$\boxed{Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s).} \tag{5.7}$$

With a unit step input, we obtain

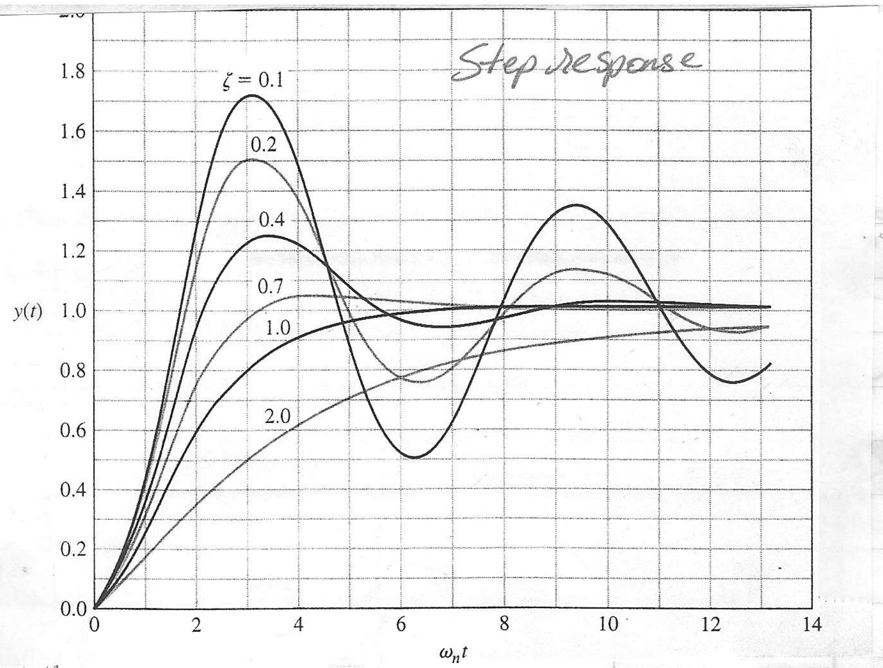
$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}, \tag{5.8}$$

for which the transient output, as obtained from the Laplace transform table in Appendix A, is

$$y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta), \tag{5.9}$$

where $\beta = \sqrt{1 - \zeta^2}$, $\theta = \cos^{-1}\zeta$, and $0 < \zeta < 1$. The transient response of this second-order system for various values of the damping ratio ζ is shown in Fig. 5.5. As ζ decreases, the closed-loop roots approach the imaginary axis, and the response becomes increasingly oscillatory. The response as a function of ζ and time is also shown in Fig. 5.5(b) for a step input.

Actual oscillations occur at $\omega = \omega_n\sqrt{1-\zeta^2}$
decays as $e^{-\zeta\omega_n t}$



Steady State Tracking Error and Loop Type

The final value theorem states that for some signal $y(t)$,

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,Y(s)$$

We can apply this to the error signal $e(t)$. Let e_{ss} be the steady-state error, then

$$e_{ss} = \lim_{s \to 0} \frac{s\,R(s)}{1 + F(s)G(s)}$$

We commonly test e_{ss} with the following inputs:

Step: $R(s) = \frac{1}{s}$

Ramp: $R(s) = \frac{1}{s^2}$

Quadratic: $R(s) = \frac{1}{s^3}$

Ex: Let $F(s) = \frac{K(s+a)}{s^2}$, $G(s) = 1$

Step: $e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s}}{1 + \frac{K(s+a)}{s^2}} = 0$

Ramp: $e_{ss} = \lim_{s \to 0} \frac{s \cdot 1/s^2}{1 + K(s+a)/s^2} = \lim_{s \to 0} \frac{s}{s^2 + Ks + Ka} = 0$

Quadratic: $e_{ss} = \lim_{s \to 0} \frac{1}{s^2 + Ks + Ka} = \frac{1}{Ka}$

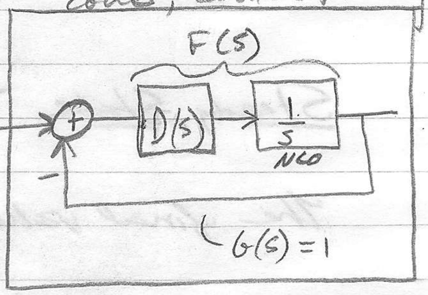
The Type of a system is the number of perfect integrators in $F(s)G(s)$. For example, $F(s)G(s) = \frac{K(s+a)}{s^2}$ is a

Type 2 system.

e_{ss}

Type \ Input	step	ramp	quadratic
0	C_0	∞	∞
1	0	C_1	∞
2	0	0	C_2
3	0	0	0

Linear model for code, carrier tracking



where the C_i are constants.

The orbital dynamics of spacecraft give rise to carrier phase time histories that are closely approximated by quadratic fcns. Thus, we often use Type 3 carrier tracking loops so that $e_{ss} \to 0$.

In code and carrier tracking, $G(s) = 1$ (unity feedback) and there is another block that represents a number controlled oscillator (NCO) which we can model approx. as a pure integrator: $NCO = \frac{1}{s}$.

Thus, $F(s) = \frac{1}{s}D(s)$ and $F(s)G(s) = F(s)$

The loop filter $D(s)$ typically has one of the following forms:

1st-order loop: $D(s) = K$, $H(s) = \frac{K}{s+K}$ (Type I)

$F(s)G(s) = K/s$

2nd-order loop: $D(s) = \frac{K(s+a)}{s}$, $H(s) = \frac{K(s+a)}{s^2 + Ks + Ka}$ (Type II)

3rd-order loop: $D(s) = \frac{K(s^2+as+b)}{s^2}$, $H(s) = \frac{K(s^2+as+b)}{s^3+Ks^2+Kas+Kb}$ (Type III)

HW [Compute e_{ss} for each of these and several different inputs

An important quantity in the study of tracking loops is the loop noise bandwidth, defined in Hz as

$$B_n = \frac{1}{4\pi}\int_{-\infty}^{\infty} |H(j\omega)|^2 \, d\omega$$

(single-sided)

where we're evaluating $H(s=j\omega)$, which applies in steady-state, and we assume $H(0)=1$. B_n is single-sided vers. of B_{neq}, introduced earlier.

order	1st	2nd	3rd
B_n (Hz)	$K/4$	$\frac{K+a}{4}$	$\frac{K(aK+a^2-b)}{4(aK-b)}$
Type	I	II	III

(with $D(s)$ for each order as before)

It's easy to study the passage of noise through a linear feedback loop. Suppose the input $r(t)$ is a Gaussian white noise process with (two-sided) noise density $N_0/2$. Then from lin. sys. thy. the power spectrum of the output $y(t)$ is:

$$R(j\omega) \longrightarrow \boxed{H(j\omega)} \xrightarrow{Y(j\omega)}$$

$$\underbrace{S_y(f)}_{\text{power spectrum of } y(t)} = \frac{N_0}{2}|\tilde{H}(f)|^2 .$$ [For convenience, let $\tilde{H}(f) = H(j2\pi f)$]

then the power in the output $y(t)$ is:

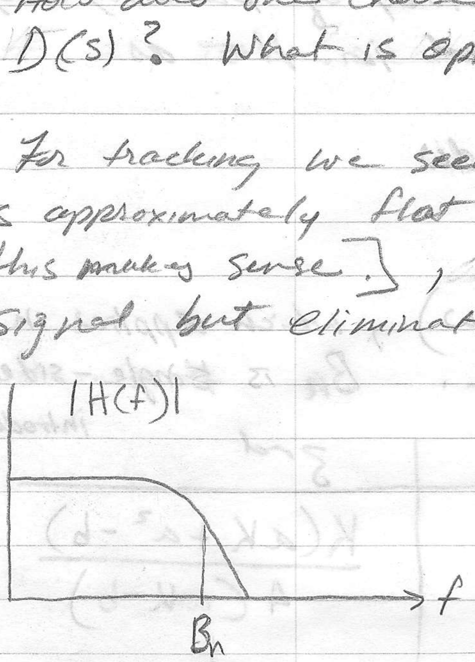
$$P_y = \int_{-\infty}^{\infty} S_y(f)\,df = \frac{N_0}{2}\overbrace{\int_{-\infty}^{\infty} |\tilde{H}(f)|^2\,df}^{2B_n}$$

$$= \frac{N_0}{2}\cdot 2B_n = N_0 B_n$$

3

Q: How does one choose the form and parameters of $D(s)$? What is optimal?

A: For tracking we seek a frequency response that is approximately flat over the passband. [Explain why this makes sense.], capturing the dynamics of the signal but eliminating the high-frequency noise.



One can derive an optimal loop filter $D(s)$ via Wiener Filtering thy. (1949) We minimize the integral square phase error under a bandwidth constraint. The filters $D(s)$ previously introduced are optimal for step, ramp, and parabolic input, respectively.

How to select parameters K, a, b?

Order	
1	$K = 4B_n$
2	$H(s) = \dfrac{K(s+a)}{s^2 + Ks + Ka}$ compare with standard form: $\dfrac{C(s+\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ Set $\zeta = \frac{1}{\sqrt{2}}$ (damping ratio) and recall that $B_n = \frac{K+a}{4}$ Then $K = \frac{8}{3} B_n$ $a = K/2$
3	$a = 1.2\, B_n$ $b = a^2/2$ $K = 2a$

HW — [derive these for 2nd-order sys.]