ASE 389P-7 Problem Set 3

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You need not hand in anything. Instead, be prepared to answer any of these problems—or similar problems—on an upcoming take-home exam. You may discuss your solutions with classmates up until the time that the exam becomes available, but do not swap work (including code).

Readings

All background reading material is found on Canvas. You'll find an intuitive model for ionospheric scintillation in [1]. The receiver-effects scintillation model used in GNSS is presented in [2] and [3]. Some interesting real-world scintillation is examined in [4]. The latest GPS interface specification is given in [5]; you'll use this to implement the broadcast model for ionospheric delay.

Problems

1. In lecture we introduced the concept of code-carrier divergence by considering the effect of the dispersive ionosphere on a simplified signal impinging on a GNSS receiver:

$$r(t) = C(t)\sin(2\pi f_c t)$$

Where $C(t)$ is a spreading waveform with null-to-null bandwidth B_1 of approximately 2 MHz and f_c is a GNSS L-band carrier frequency (somewhere between 1.2 and 1.6 GHz). We subsequently expressed $r(t)$ as a function of the Fourier transform $\tilde{C}(f) = \mathcal{F}[C(t)]$:

$$r(t) = \int_{-\infty}^{\infty} \frac{\tilde{C}(f)}{2j}\left[e^{j2\pi(f+f_c)t} - e^{j2\pi(f-f_c)t}\right] df \tag{1}$$

We see from this equation that $r(t)$ is made up of two monochromatic signals that get weighted by $\tilde{C}(f)/2j$ (a complex number) and integrated over all f. For a particular value of f, the first monochromatic signal looks like

$$\frac{\tilde{C}(f)}{2j}e^{j2\pi(f+f_c)t} \tag{2}$$

If this signal were to pass through the ionosphere, it would experience an excess phase delay given by

$$\Delta\tau_{p,H} = \frac{-K}{(f+f_c)^2}$$

Thus, the received signal corresponding to (2) could be written

$$\frac{\tilde{C}(f)}{2j} e^{j2\pi(f+f_c)(t-\Delta\tau_{p,H})} \tag{3}$$

Following this reasoning, we can rewrite (1) as

$$r(t) = \int_{-\infty}^{\infty} \frac{\tilde{C}(f)}{2j} \left[e^{j2\pi(f+f_c)(t-\Delta\tau_{p,H})} - e^{j2\pi(f-f_c)(t-\Delta\tau_{p,L})} \right] df \tag{4}$$

where $\Delta\tau_{p,H}$ is the ionospheric delay (excess phase delay) for the frequency constituent at the "high" frequency $f + f_c$ and

$$\Delta\tau_{p,L} = \frac{-K}{(f - f_c)^2}$$

is the ionospheric delay for the frequency constituent at the "low" frequency $f - f_c$.

With some work it can be shown that (4) reduces to

$$r(t) = C(t - \Delta\tau_g) \sin\left[2\pi f_c(t - \Delta\tau_p)\right]$$

where $\Delta\tau_g = K/f_c^2$ is the so-called group delay and $\Delta\tau_p = -K/f_c^2$ is the so-called phase delay, with $K = 40.3 \cdot \mathrm{TEC}/c$. Derive this latter expression for $r(t)$ from (4).

Hints: Plug the expressions for $\Delta\tau_{p,L}$ and $\Delta\tau_{p,H}$ into (4). Distribute the $(f + f_c)$ and the $(f - f_c)$. Recognize that the effective range of the integral is small compared to f_c (explain why). This allows you to approximate a term $K/(f_c^2 - f^2)$, which emerges as you simplify, as K/f_c^2 over the range of integration.

2. Show that the group velocity v_g and the phase index of refraction n_p are related by $v_g = cn_p$ for small group and phase velocity departures from the speed of light c.

3. If a carrier wave meets an ionospheric layer with a refractive index of 0, then the group velocity tends to zero and the phase velocity tends to infinity as the wave is reflected off the layer. What electron density n_e would be required for reflection at the GPS L_1 frequency? Research typical electron density values to determine whether there is any chance of such a reflection occurring. What n_e is required for reflection of the Ham Radio band at 50 MHz? What are the chances of reflection at this frequency? For all reflections, assume a normally incident wave (a wave with zenith angle 0). Use the original expression given in lecture for n that involves a square root to calculate n_e, since the approximate expression, based on a first-order Taylor series expansion, will not be valid for such large n_e.

4. Let I_ρ be the ionospheric group delay, in meters, as measured using pseudorange observables. Let I_ϕ be the ionospheric phase delay, in meters, as measured using carrier phase observables. In the absence of measurement errors, noise, and biases, these are related to the quantities $\Delta\tau_g$ and $\Delta\tau_p$ introduced in lecture by

$$I_\rho = c \cdot \Delta\tau_g, \quad I_\phi = c \cdot \Delta\tau_p + b_\phi$$

where b_ϕ is a (potentially large) bias due to the integer-cycle phase ambiguity of the carrier measurement (the receiver does not know which cycle of the carrier it is tracking).

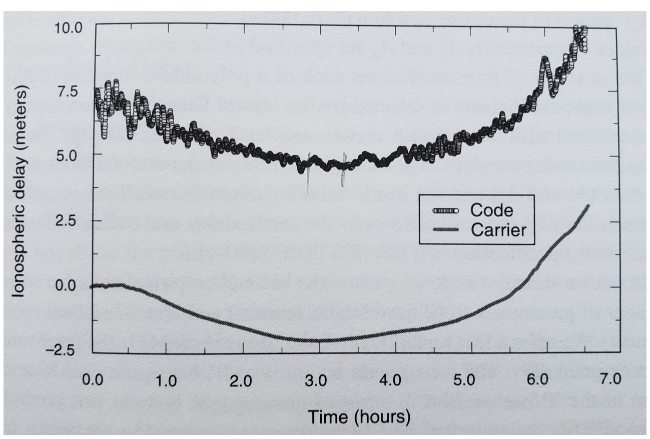
Recall that total electron content (TEC) is related to $\Delta\tau_g$ and $\Delta\tau_p$ by

$$\Delta\tau_g = -\Delta\tau_p = \frac{40.3 \cdot \text{TEC}}{cf^2}$$

where f is the frequency at which $\Delta\tau_g$ and $\Delta\tau_p$ are measured

Download the gzipped archive `gridDataUt6.tar.gz` from `http://radionavlab.ae.utexas.edu/datastore/gnssSigProcCourse`. Un-zip and un-tar this file to access the underlying files. The `*.log` files are columnar-format log files produced by the GRID receiver, a powerful software-defined radionavigation receiver under development in the UT Radionavigation Laboratory. The `*def.txt` files describe the data in the corresponding `*.log` files. For now, you'll only need to pay attention to `channel.log`, whose format is described in `channeldef.txt`.

You'll find that in this data set there were four L2C-capable GPS satellites visible over the entire 27-minute data capture interval: those with TXIDs (PRN identifiers) 7, 8, 19, and 28. For this data set, GRID tracked the signal type `GPS_L1_CA` on L1 and `GPS_L2_CL` on L2. Draw the file `channel.log` (or, equivalently, the file `channel.mat` but with the included matrix transposed) into Matlab and operate on the data to generate, for TXID 7, a plot similar to the one shown below.



From Misra and Enge, 2011.

The plot should include both code-measured ionospheric group delay and carrier-measured ionospheric phase delay time histories at the L1 frequency. The ionospheric phase delay should be inverted so that its shape matches that of the ionospheric group delay, and should be shifted so that its time history begins at zero as shown in the figure above.

The code-measured ionospheric group delay at L1 in meters is given by

$$I_{\rho,\text{L1}} = \frac{f_{\text{L2}}^2}{f_{\text{L1}}^2 - f_{\text{L2}}^2}(\rho_{\text{L2}} - \rho_{\text{L1}})$$

where ρ_{L1} and ρ_{L2} are the pseudorange measurements at L1 and L2, in meters.

The carrier-measured ionospheric phase delay at L1 in meters is given by

$$I_{\phi,\mathrm{L1}} = \frac{f_{\mathrm{L2}}^2}{f_{\mathrm{L2}}^2 - f_{\mathrm{L1}}^2}(\lambda_{\mathrm{L1}}\phi_{\mathrm{L1}} - \lambda_{\mathrm{L2}}\phi_{\mathrm{L2}})$$

where ϕ_{L1} and ϕ_{L2} are the beat carrier phase measurements at L1 and L2, in cycles, and λ_{L1} and λ_{L2} are the respective wavelengths, in meters.

Generate a second plot showing both the code- and carrier-measured TEC seen by the signals from the satellite with TXID 7. Express the value of TEC in TECU. Add a constant offset to the carrier-measured TEC so that it best matches the code-measured TEC in a least-squares sense.

Answer the following questions:

(a) What could explain the negative values in the final TEC estimates? Is this physically possible?

(b) The ionospheric delay for TXID 7 changes significantly over the 27-minute data capture interval. How is this possible if the GPS satellites move on slow 12-sidereal-hour orbits? (You may wish to inspect the file `navsol.log` for additional clues.)

Hints:

(a) Make sure you match pseudorange and carrier phase measurements from the L1 C/A signals with their L2 CL counterparts taken simultaneously.

(b) You'll want to write a Matlab script to automate the process of drawing in the data and generating the plots for this problem because you'll need to repeat the procedure in a later problem.

(c) The Matlab files `*.mat` contain the same data as the corresponding `*.log` files but in a convenient Matlab format. For example, the following Matlab command sequence (1) draws `channel.mat` directly into Matlab, (2) stores the contents of the file in the matrix `M`, (3) finds all the row indices corresponding to the L1 C/A signal of the satellite with TXID 5, (4) loads all the C/N_0 measurements from these rows into `C_N0Vec_txid05`:

```
>> load channel.mat
>> M = channel';     // Transpose to get in columnar format
>> iidum = find(M(:,13) == 0 & M(:,14) == 5);
>> C_N0Vec_txid05 = M(iidum,9);
```

5. Write a function in Matlab for computing the ionospheric delay from a model of the ionosphere. Your function should adhere to the interface described on the next page (which you can copy and paste as comments to your function). Only develop calculations for the broadcast (Klobuchar) model. The function can later be augmented to accommodate other model types.

   You can learn about the broadcast model from Figure 20-4 of the GPS interface specification (IS) `IS-GPS-200L.pdf` posted on Canvas. You will also need to write your own function for computing satellite elevation and azimuth angles. Assume the WGS84 model for the shape of the Earth.

   Note that the GPS IS uses *semicircles* as its angular measure, which is a strange convention employed back in the 1970s to reduce memory and computation requirement. The sin and cos functions in the IS (e.g., in Figure 20-4) are meant to operate on semicircles, unless otherwise indicated. Thus, when the IS writes, e.g., $\cos(\lambda_i - 1.1617)$, where λ_i is given in semicircles, you can implement this in Matlab as $\cos((\lambda_i - 1.1617)\pi)$.

```
function [delTauG] = getIonoDelay(ionodata,fc,rRx,rSv,tGPS,model)
% getIonoDelay : Return a model-based estimate of the ionospheric delay
%                experienced by a transionospheric GNSS signal as it
%                propagates from a GNSS SV to the antenna of a terrestrial
%                GNSS receiver.
%
% INPUTS
%
% ionodata ------- Structure containing a parameterization of the
%                  ionosphere that is valid at time tGPS.  The structure is
%                  defined differently depending on what ionospheric model
%                  is selected:
%
%                  broadcast --- For the broadcast (Klobuchar) model, ionodata
%                  is a structure containing the following fields:
%
%                     alpha0 ... alpha3 -- power series expansion coefficients
%                     for amplitude of ionospheric delay
%
%                     beta0 ... beta3 -- power series expansion coefficients
%                     for period of ionospheric plasma density cycle
%
%
%                  Other models TBD ...
%
% fc ------------- Carrier frequency of the GNSS signal, in Hz.
%
% rRx ------------ A 3-by-1 vector representing the receiver antenna position
%                  at the time of receipt of the signal, expressed in meters
%                  in the ECEF reference frame.
%
% rSv ------------ A 3-by-1 vector representing the space vehicle antenna
%                  position at the time of transmission of the signal,
%                  expressed in meters in the ECEF reference frame.
%
% tGPS ----------- A structure containing the true GPS time of receipt of
%                  the signal.  The structure has the following fields:
%
%                  week -- unambiguous GPS week number
%
%                  seconds -- seconds (including fractional seconds) of the
%                  GPS week
%
% model ---------- A string identifying the model to be used in the
%                  computation of the ionospheric delay:
%
%                  broadcast --- The broadcast (Klobuchar) model.
%
%                  Other models TBD ...
%
% OUTPUTS
%
```

```
% delTauG -------- Modeled scalar excess group ionospheric delay experienced
%                  by the transionospheric GNSS signal, in seconds.
%
%+------------------------------------------------------------------------------+
% References: For the broadcast (Klobuchar) model, see IS-GPS-200x
% Figure 20-4.
%
%+==============================================================================+
```

6. Download the gzipped archive `gridDataUt1.tar.gz` from `http://radionavlab.ae.utexas.edu/datastore/gnssSigProcCourse/`. Repeat the steps in problem 4 for this data set. Do this for TXID 29 and for TXID 31. Note that the data capture interval is much shorter, and the ionospheric delay changes much less significantly, than in the original data set for problem 4.

   The following broadcast ionospheric parameters are valid for the data in `gridDataUt1.tar.gz`:

   ```
   alpha0: 4.6566e-009
   alpha1: 1.4901e-008
   alpha2: -5.9605e-008
   alpha3: -5.9605e-008
    beta0: 79872
    beta1: 65536
    beta2: -65536
    beta3: -393220
   ```

   Assume a true GPS time of receipt given by

   ```
   week = 1490
   seconds = 146238.774036515
   ```

   and a static ECEF receiver antenna location given in meters by

   ```
   X =  1101972.5309609
   Y = -4583489.78279095
   Z =  4282244.3010423
   ```

   Use the function you wrote for problem 5 to determine the ionospheric delay as calculated by the broadcast model for TXIDs 29 and 31, with the ECEF SV position of TXID 29 at time of transmission given in meters by

   ```
   X = 24597807.6872883
   Y = -3065999.1384585
   Z =  9611346.77939927
   ```

   and the ECEF SV position of TXID 31 at time of transmission given in meters by

```
X =   2339172.27088689
Y = -16191391.3551878
Z =  21104185.0481546
```

Compare the ionospheric delay as calculated by the broadcast model for TXIDs 29 and 31 with your corresponding plots for these two. How much difference is there between the model-calculated and the empirical ionospheric delays? To what do you attribute this discrepancy?

7. In lecture, we noted that the phase shift across an ionospheric blob (irregularity) of size a (in meters) in excess of the phase shift across a layer of average background density of the same size is

$$\Delta\phi_0 = a r_e \lambda \Delta n_e$$

where r_e is the classical electron radius in meters, λ is the wavelength of the carrier in meters, and Δn_e is the excess electron density within the blob as compared to the average background density, in electrons per cubic meter. Derive this expression from expressions for the refractive index n and the excess phase delay $\Delta\tau_p$ given in lecture.

8. What is the scale size of the ionospheric irregularities that cause the strongest scintillation effects at the GPS L_1 frequency? Note all the assumptions behind your calculation. Repeat your calculation for the GPS L_2 frequency.

9. Download the Cornell Scintillation Simulation Toolkit from `http://radionavlab.ae.utexas.edu/datastore/websiteFiles/scintSim.zip`.
Read the instructions in `UserGuide.pdf` for generating synthetic scintillation. Experiment with the graphical user interface by typing `guiscint` at the Matlab prompt. You'll be able to generate a time history of the complex channel response function $z(t)$ by setting the parameters S_4 and τ_0 and pressing 'Simulate'. The quantity T_e above the bar chart is the mean time between differentially-detected bit errors, which serves as a proxy for the mean time between cycle slips. Notice how T_e changes as you experiment with different values of S_4, τ_0, and C/N_0.

   Write your own Matlab function to calculate the S_4 index and the decorrelation time τ_0 corresponding to a given scintillation time history produced by the scintillation simulator and stored in `scintDat.mat`. Test your function to see how well your calculated S_4 and τ_0 values match the values that were used as parameters in generating the data. Your function should adhere to the following interface:

```
function [S4,tau0] = computeS4AndTau0(zkhist,tkhist)
% computeS4AndTau0 : Compute the scintillation index S4 and the decorrelation
%                    time tau0 corresponding to the input complex channel
%                    response function time history zkhist.
%
%
% INPUTS
%
% zkhist ----- Nt-by-1 vector containing the normalized complex scintillation
%              time history in the form of averages over Ts with sampling
%              interval Ts.  zkhist(kp1) is the average over tk to tkp1.
%
% tkhist ----- Nt-by-1 vector of time points corresponding to zkhist.
%
%
% OUTPUTS
%
% S4 --------- Intensity scintillation index of the scintillation time history
%              in zkhist, equal to the mean-normalized standard deviation of
%              the intensity abs(zkhist).^2.
%
% tau0 ------- The decorrelation time of the scintillation time history in
%              zkhist, in seconds.
%
%
%+------------------------------------------------------------------------------+
% References:
%
%
%+==============================================================================+
```

References

[1] B. H. Briggs, "Ionospheric irregularities and radio scintillations," *Contemporary Physics*, vol. 16, no. 5, pp. 469–488, 1975.

[2] T. E. Humphreys, M. L. Psiaki, and P. M. Kintner, "Modeling the effects of ionospheric scintillation on GPS carrier phase tracking," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 4, pp. 1624–1637, Oct. 2010.

[3] T. E. Humphreys, M. L. Psiaki, J. C. Hinks, B. O'Hanlon, and P. M. Kintner, Jr., "Simulating ionosphere-induced scintillation for testing GPS receiver phase tracking loops," *IEEE Journal of Selected Topics in Signal Processing*, vol. 3, no. 4, pp. 707–715, Aug. 2009.

[4] T. E. Humphreys, M. L. Psiaki, B. M. Ledvina, A. P. Cerruti, and P. M. Kintner, Jr., "A data-driven testbed for evaluating GPS carrier tracking loops in ionospheric scintillation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 4, pp. 1609–1623, Oct. 2010.

[5] GPS Directorate, "Systems engineering and integration Interface Specification IS-GPS-200L," 2020, http://www.gps.gov/technical/icwg/.