The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

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ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Longitudinal Static Stability Example Problems
  - Lateral Static Stability Example Problems

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LONGITUDINAL STABILITY EXAMPLES

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Question LONG 1

If the slope of the C_m versus C_L curve is -0.15 and the pitching moment at zero lift is equal to 0.08, determine the trim lift coefficient. If the center of gravity of the airplane is located at $X_{cg}/\bar{c} = 0.3$, determine the stick fixed neutral point.

Question LONG 1 - Solution

$$\frac{dC_m}{dC_L} = -0.15 \qquad C_{m_0} = 0.08 \qquad C_{L_{trim}}\ ?$$

- The pitching moment coefficient can be expressed:

$$C_m = C_{m_0} + \frac{dC_m}{dC_L} C_L$$

At trim $C_{m_{trim}} = 0 = C_{m_0} + \frac{dC_m}{dC_L} C_{L_{trim}}$

$$\Longrightarrow \quad C_{L_{trim}} = -\frac{C_{m_0}}{\frac{dC_m}{dC_L}} = -\frac{0.08}{-0.15} \qquad \boxed{C_{L_{trim}} = 0.53}$$

Question LONG 1 - Solution

- Stick-fixed neutral point ?

$$C_{m_\alpha} = C_{L_\alpha}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right)$$ (as in equation 2.9)

i.e. $$\frac{dC_m}{d\alpha} = \frac{dC_L}{d\alpha}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right)$$

Eq. 2.3 : $$\frac{dC_m}{d\alpha} = \frac{dC_m}{dC_L}\cancel{\frac{dC_L}{d\alpha}} = \cancel{\frac{dC_L}{d\alpha}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right)$$

thus $$\frac{x_{NP}}{\bar{c}} = \frac{x_{cg}}{\bar{c}} - \frac{dC_m}{dC_L} = 0.3 - (-0.15)$$

$$\boxed{\frac{x_{NP}}{\bar{c}} = 0.45}$$

Question LONG 2

The C_m versus, α curve for a large jet transport can be seen in Figure P2.4. Use the figure and the following information to answer questions (a) to (c).

$$C_L = 0.03 + 0.08\alpha \text{ (deg.)}$$

$$-15^\circ \leq \delta_e \leq 20^\circ$$

(a) Estimate the stick fixed neutral point.
(b) Estimate the control power $C_{m_{\delta_e}}$.
(c) Find the forward center of gravity limit. Hint:

$$\frac{dC_{m_{cg}}}{dC_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{NP}}{\bar{c}}$$

Question LONG 2 (cont'd)

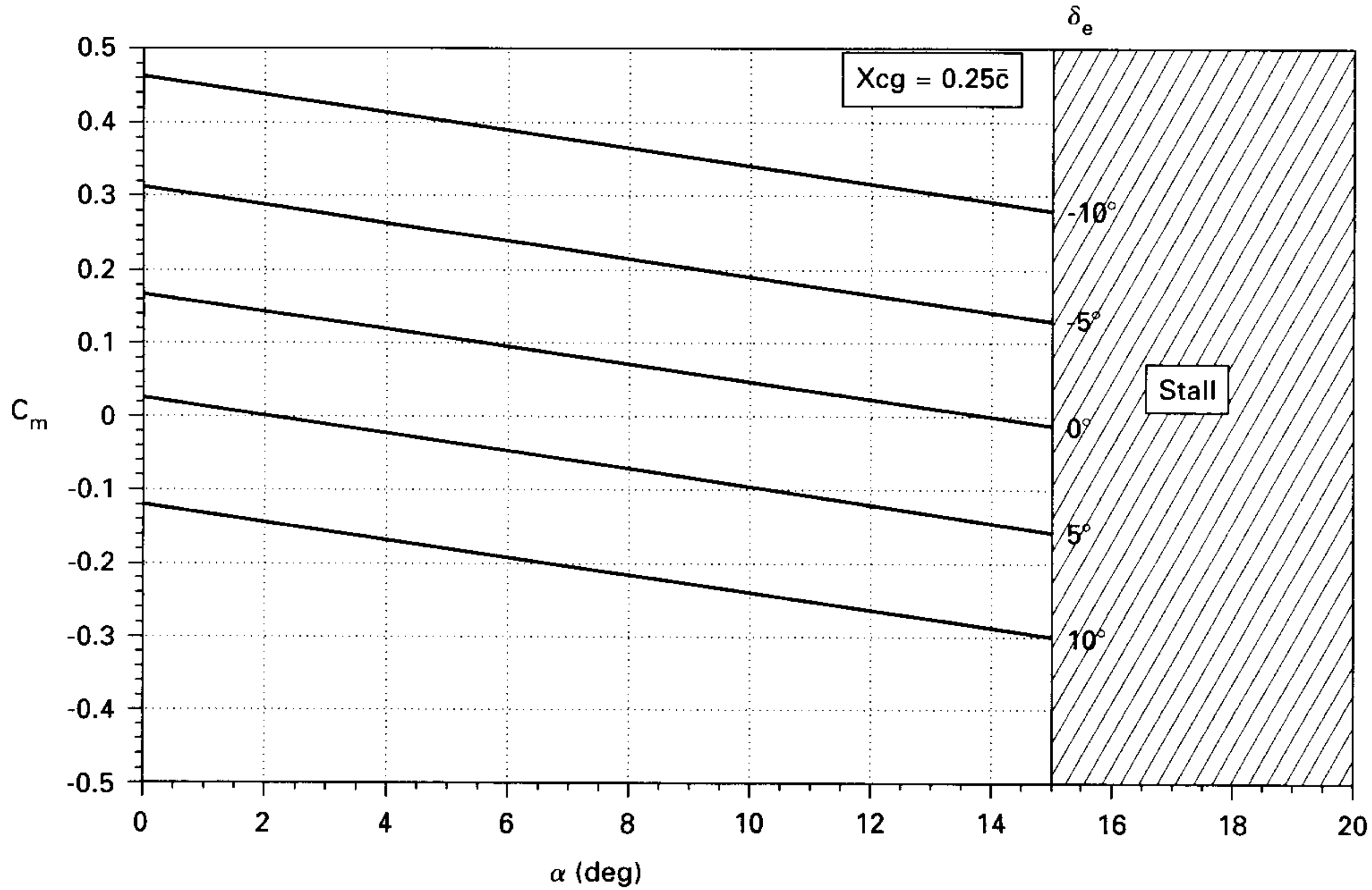

FIGURE P2.4

Question LONG 2 - Solution

(a) Estimate the stick-fixed neutral point. $\frac{x_{NP}}{\bar{c}}$?

Use $\frac{x_{NP}}{\bar{c}} = \frac{x_{cg}}{\bar{c}} - \frac{dC_{m_{cg}}}{dC_L}$

- For large jet transport

$$\frac{x_{cg}}{\bar{c}} = 0.25$$

- From figure P.2.4 $\frac{dC_m}{d\alpha} \approx \frac{\Delta C_m}{\Delta \alpha} = \frac{0.04 - 0.17}{10^\circ - 0^\circ} = -0.013 \, / \deg$

$\frac{dC_{m_{cg}}}{dC_L} = \frac{dC_{m_{cg}}}{d\alpha} \frac{1}{\frac{dC_L}{d\alpha}}$ and since $\frac{dC_L}{d\alpha} = 0.08 \, / \deg$

$$\frac{dC_m}{dC_L} = -\frac{0.013}{0.08} = 0.1625$$

therefore $\frac{x_{NP}}{\bar{c}} = 0.25 + 0.1625 \Rightarrow \boxed{\frac{x_{NP}}{\bar{c}} = 0.4125}$

Question LONG 2 - Solution

(b) $C_{m_{\delta_e}}$? $C_{m_{\delta_e}} = \frac{dC_m}{d\delta_e} \simeq \frac{\Delta C_m}{\Delta \delta_e}$

From figure P2.4 at $\alpha = 0$ $\frac{\Delta C_m}{\Delta \delta_e} = \frac{0.17 - 0.46}{0^\circ - (-10^\circ)}$

$$\boxed{C_{m_{\delta_e}} \simeq -0.029/\text{deg}}$$

(c) Forward c-g limit?

The forward c-g limit is determined by the requirement to trim the airplane at a high C_L (landing).

Here the $C_{L_{max}}$ occurs when α is max i.e. $\alpha = 15^\circ$

Question LONG 2 - Solution

For $\alpha = 15^{\circ}$ $\quad C_{L_{max}} = 0.03 + 0.08 \times 15^{\circ} = \underline{\underline{1.23}}$

Recall $\dfrac{dC_{m_{cg}}}{dC_L} = \dfrac{x_{cg}}{\bar{c}} - \dfrac{x_{NP}}{\bar{c}}$

becomes more negative $\Leftarrow$ $\downarrow$ as cg moves forward

Ie C_m vs C_L or C_m vs α becomes steeper.

To trim the airplane at $C_{L_{max}}$ a <u>positive</u> moment ΔC_m from the control needs to be applied.

Question LONG 2 - Solution

$$\Delta C_{m_{control}} = C_{m_{\delta e}} \, \delta_{e_{max}} = -0.029 \,/\mathrm{deg} \times (-15^{\circ})$$

$$\Delta C_m = 0.435$$

At trim

$$\left.\frac{dC_{m_{cg}}}{dC_L}\right|_{trim} = \frac{\overset{=0}{\cancel{C_{m_{trim}}}} - (C_{m_0} + \Delta C_{m_{control}})}{C_{L_{max}} - C_{L_0}}$$

$$= \frac{-(0.17 + 0.435)}{1.23 - 0.03} = -0.504.$$

at the limit

$$\frac{x_{cg}}{\bar{c}} = \left.\frac{dC_{m_{cg}}}{dC_L}\right|_{trim} - \frac{x_{NP}}{\bar{c}} = -0.504 + 0.4125$$

$$\boxed{\left.\frac{x_{cg}}{\bar{c}}\right._{\substack{max\\forward}} = -0.0915}$$

Question LONG 3

An airplane has the following pitching moment characteristics at the center of gravity position:

$$x_{cg}/\bar{c} = 0.3.$$

$$C_{m_{cg}} = C_{m_0} + \frac{dC_{m_{cg}}}{dC_L} C_L + C_{m_{\delta_e}} \delta_e$$

where $\quad C_{m_0} = 0.05 \quad \frac{dC_{m_{cg}}}{dC_L} = -0.1 \quad C_{m_{\delta_e}} = -0.01/\text{deg}$

$$\frac{dC_{m_{cg}}}{dC_L} = \left[\frac{X_{cg}}{\bar{c}}\right] - \left[\frac{X_{NP}}{\bar{c}}\right]$$

If the airplane is loaded so that the center of gravity position moves to $x_{cg}/\bar{c} = 0.10$, can the airplane be trimmed during landing, $C_L = 1.0$? Assume that C_{m_0} and $C_{m_{\delta_e}}$ are unaffected by the center of gravity travel and that $\delta_{e_{max}} = \pm 20°$.

Question LONG 3 - Solution

$$C_{m_{cg}} = C_{m_0} + \frac{dC_{m_{cg}}}{dC_L} C_L + C_{m_{\delta e}} \delta_e$$

- Find the location of the neutral point:

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{cg}}{\bar{c}} - \frac{dC_{m_{cg}}}{dC_L}$$; from the initial setting we have:

$$\frac{x_{NP}}{\bar{c}} = 0.3 - (-0.1) = 0.4$$ ← this does not change when we move the c.g

- For the new c.g position, what is the new value of $\frac{dC_{m_{cg}}}{dC_L}$?

$$\frac{dC_{m_{cg}}^{*}}{dC_L} = \frac{x_{cg}^{*}}{\bar{c}} - \frac{x_{NP}}{\bar{c}} = 0.1 - 0.4 = -0.3$$

Question LONG 3 - Solution

- Can the airplane be trimmed? For $C_L = 1.0$

  → if the airplane was trimmed, would the δ_e required be within the limits?

At trim $C_{m_{cg}} = 0 = 0.05 + (-0.3)\,1.0 + (-0.01)\,\delta_e$

$$\delta_e = \frac{-0.05 + 0.3}{-0.01} = -25^\circ \Rightarrow \delta_e \notin [-\delta_{e\,max}, \delta_{e\,max}] = [-20^\circ, 20^\circ]$$

therefore the airplane cannot be trimmed at this c.g and lift coefficient

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LATERAL STABILITY EXAMPLES

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Question LAT 1

For the twin engine airplane shown in Figure P2.16, determine the rudder size to control the airplane if one engine needs to be shut down. Use the flight information shown in the figure and

Wing: $S = 980\ \text{ft}^2$ $b = 93\ \text{ft}$

Vertical tail: $S_v = 330\ \text{ft}^2$ $AR_v = 4.3$ $l_v = 37\ \text{ft}$ $\eta_v = 1.0$

Rudder: $\delta_r = \pm 15°$

Propulsion: $T = 14{,}000$ lb each $y_T = 16\ \text{ft}$

Flight condition: $V = 250\ \text{ft/s}$ $\rho = 0.002378\ \text{slug/ft}^3$

Question LAT 1 (cont'd)

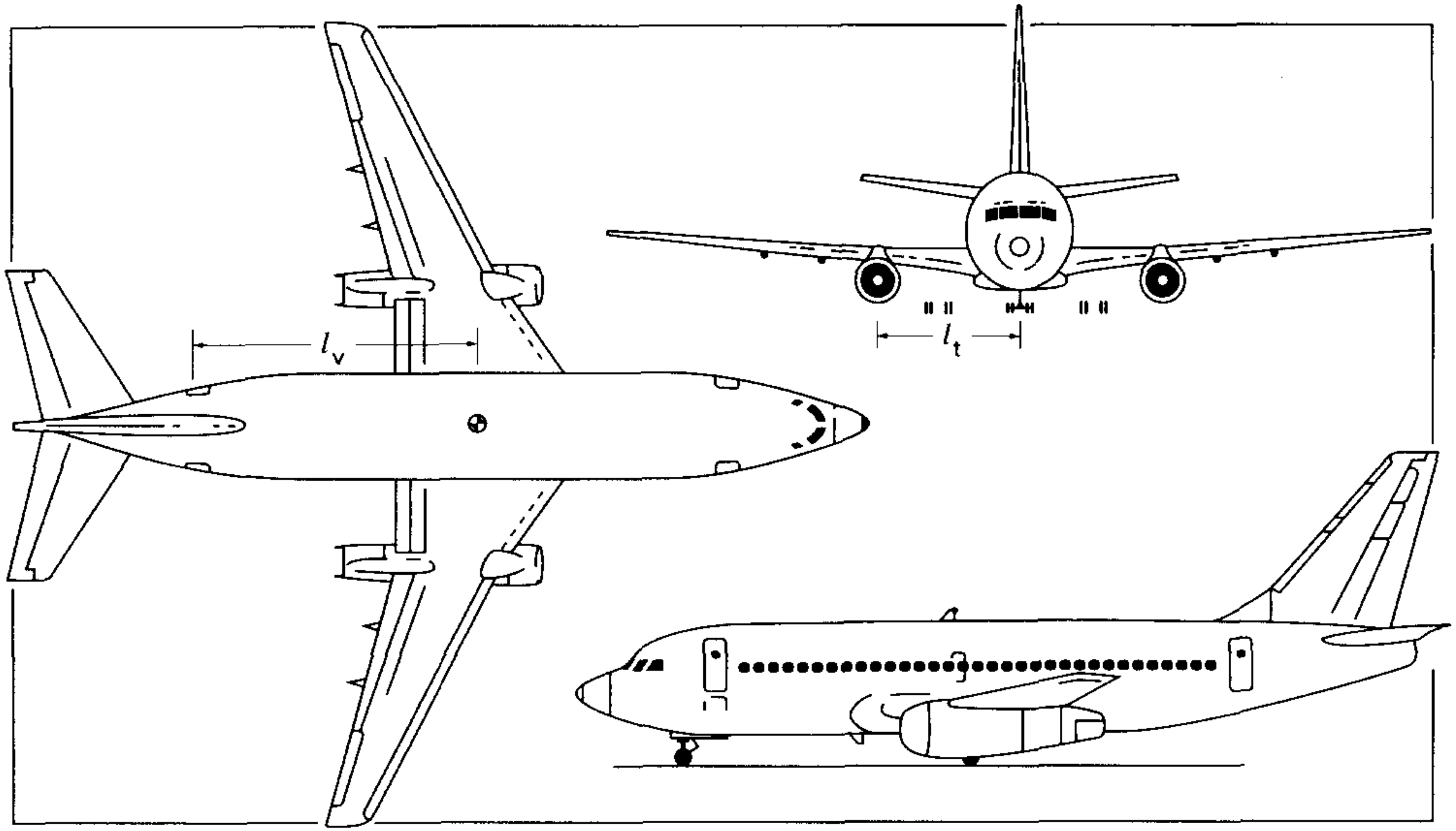


FIGURE P2.16

Question LAT 1 - Solution

- Want to control the airplane with one engine shut down ie the total yawing moment should be zero:

$$\underset{\text{(from rudder)}}{N_{aero}} + N_{engine} = 0$$

$$N_{engine} = -T y_T \Rightarrow N_{aero} = -N_{engine} = T y_T = 14{,}000 \times 16$$

$$\underline{\underline{N_{aero} = 224{,}000 \text{ lb.ft}}}$$

- By definition $C_n = \dfrac{N_{aero}}{\frac{1}{2}\rho V^2 S b} = \dfrac{224\,000}{\frac{1}{2}\, 0.002378 \times 250^2 \times 980 \times 93}$

$$\underline{\underline{C_n = 0.0331}}$$

Question LAT 1 - Solution

$$C_n = C_{n_{\delta r}} \delta r \Longrightarrow C_{n_{\delta r}} = \frac{C_n}{\delta r} = \frac{0.0331}{-15^\circ \times \frac{\pi}{180}} = -0.1264 \text{ /rad.}$$

Moment is related to the flap effectiveness parameter

$$C_{n_{\delta r}} = -\eta_v V_v C_{L_{\alpha_v}} \tau \Longrightarrow \tau = -\frac{C_{n_{\delta r}}}{\eta_v V_v C_{L_{\alpha v}}}$$

* Assume $\eta_v = 1$

* $V_v = \frac{l_v S_v}{S b} = \frac{37 \times 330}{980 \times 93} = 0.134$

* $C_{L_{\alpha v}} = \frac{C_{l\alpha}}{1 + C_{l\alpha}/\pi AR_v}$ assume $C_{l_\alpha} = 0.1/\text{deg} = 5.73/\text{rad.}$

lift curve slope for vertical tail

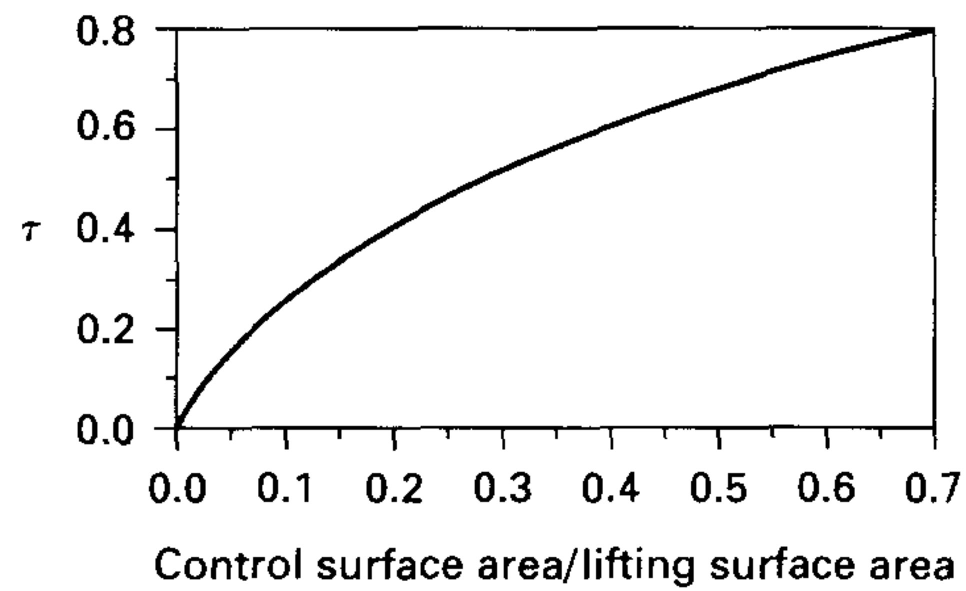
$AR_v = 4.3$

$\Rightarrow \underline{\underline{C_{L_{\alpha_v}} = 4.02 \text{ /rad}}}$

$$\Rightarrow \tau = -\frac{-0.1264}{0.134 \times 4.02} = \underline{\underline{0.23}}$$

Question LAT 1 - Solution

Flap effectiveness parameter versus control surface area



$\tau = 0.23 \Rightarrow \frac{S_r}{S_v} \simeq 0.08$

thus $S_r = S_v \times 0.08 = 330 \times 0.08$

$\boxed{S_r = 26.4 \ ft^2}$ small.

Question LAT 2

Develop an expression for the wing dihedral effect C_{l_β} for a wing planform that uses dihedral only for the outboard portion of the wing (see Figure P2.18). Clearly state all of your assumptions.

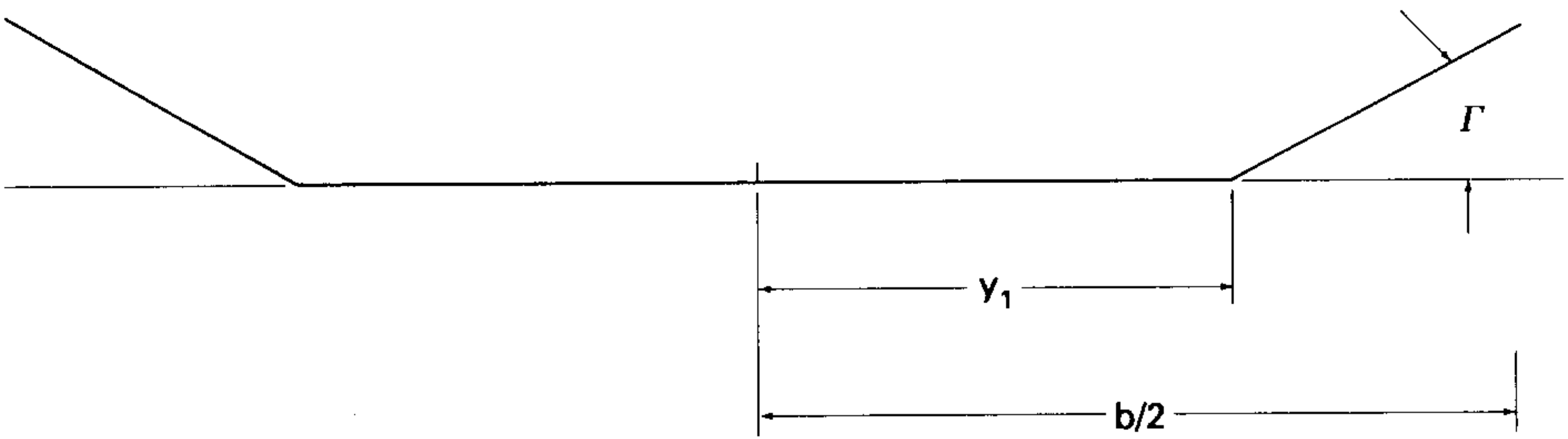


FIGURE P2.18

Question LAT 2 - Solution

The wing dihedral creates a restoring moment if the wing is disturbed from a wings level attitude.

the local change in wing angle of attack was:

$\Delta \alpha = \beta \Gamma$ for the downward wing

$\Delta \alpha = -\beta \Gamma$ —— upward ——

the incremental roll moment can be expressed:

$$dL = -y \, dLift = -y \, C_L Q \, c\, dy = -y \, C_{L_\alpha} \Delta\alpha \, Q \, c \, dy$$

Question LAT 2 - Solution

the non-dimensional coefficient is:

$$dC_\ell = \frac{dL}{QSb} = \frac{-C_{L\alpha}\,\Delta\alpha\,\cancel{Q}\,c\,y\,dy}{\cancel{Q}\,S\,b}$$

$$C_\ell = -2\frac{C_{L\alpha}}{Sb}\beta\Gamma\int_{y_1}^{b/2} c\,y\,dy$$

for both sides of the wing

$$\boxed{C_{\ell_\beta} = -\frac{2C_{L\alpha}}{Sb}\Gamma\int_{y_1}^{b/2} c\,y\,dy}$$

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