THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Aerospace Engineering and Engineering Mechanics

ASE 367K FLIGHT DYNAMICS
Fall 2024

HOMEWORK 8
Due: Sunday 2024-11-10 at 11:59pm via Canvas

Problem 1

Using the data for the 737-800 provided, the U.S. Standard Atmosphere of 1976, and the fact that Austin-Bergstrom International Airport (AUS) is 542 ft MSL…

(a) Develop a numerical integration scheme in MATLAB or Python and use it to compute the acceleration, velocity and position of the aircraft as a function of time (i.e. the trajectory of the aircraft) during its takeoff roll. You may assume that the takeoff roll lasts 24 seconds; the density, pressure and temperature at AUS are standard day values; and that the velocity achieved at the end of the takeoff roll is the rotation velocity V_R (i.e. the velocity at which the pilot will being to increase the pitch of the aircraft – pitch up the nose of the aircraft – so that it can actually lift off the ground).

(b) Develop an analytical expression for the acceleration of the aircraft as a function of velocity, for the case where the weight of aircraft is assumed to be constant, and then integrate this expression (after "separating the variables" of velocity and time) to determine the velocity achieved and the distance traveled during the takeoff roll.

$a(v)$

(c) Develop an analytical expression for the acceleration of the aircraft, for the case where the weight of aircraft and the acceleration of the aircraft are both assumed to be constant, and then integrate this expression to determine the velocity achieved and the distance traveled during the takeoff roll.

(d) Compare and contrast the results of parts (a) through (c), providing explanations and comments where necessary regarding the potential errors that would be introduced via various approximations.

DATA

$C_D(0)$	Drag Coefficient@ $\alpha = 0$	0.025	
$C_L(0)$	Lift Coefficient@ $\alpha = 0$	0.349	
F_N^{STATIC}	Static Takeoff Thrust@S/L	216,000 N	(total for both engines)
$k_0^{h \to F}$	0th Altitude Coefficient	1	
$k_1^{h \to F}$	1st Altitude Coefficient	3.281×10^{-5} m^{-1}	$\leftarrow h$
$k_2^{h \to F}$	2nd Altitude Coefficient	10.764×10^{-9} m^{-2}	h^2
$k_0^{M \to F}$	0th Mach Coefficient	1	
$k_1^{M \to F}$	1st Mach Coefficient	-1.07	
$k_2^{M \to F}$	2nd Mach Coefficient	0.56	
μ	Rolling Friction Coefficient	0.03	
W_0	Initial Takeoff Weight	790,100 N	
$\dot{W}_f$	Fuel Flow Rate	9 N/s	
S	Wing Area	125 m²	

REMEMBER

$$\int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} + C$$

$$\frac{dv}{dt} = av^2 + bv + c \qquad \swarrow v(t)?$$

$$\Rightarrow \quad dv = (av^2 + bv + c)\,dt$$

$$\Rightarrow \quad \int dv = \int (av^2 + bv + c)\,dt$$

$$\uparrow$$
not possible

b)

$W = \text{const}$

$$\frac{W}{g} \dot{V} = \text{const}_F \left[k'_{F_0} + k'_{F_1} V + k'_{F_2} V^2 \right]$$

$$- \text{const}_D \cdot V^2$$

$$- \mu W + \mu \, \text{const}_L V^2$$

$$\frac{W}{g} \dot{V} = V^2 \left(\text{const}_F \, k'_{F_2} - \text{const}_D + \mu \, \text{const}_L \right)$$

$$+ V \left(\text{const}_F \, k'_{F_1} \right)$$

$$+ \text{const}_F \, k'_{F_0} - \mu W$$

$$a = \left(\text{const}_F \, k_{F_2} - \text{const}_D + \mu \, \text{const}_c \right) \frac{g}{W}$$

$$b = \left(\text{const}_F \, k'_{F_1} \right) \frac{g}{W}$$

$$c = \left(\text{const}_F \, k_{F_0}' - \mu W \right) \frac{g}{W}$$

$$\int \frac{1}{a V^2 + b V + c} \, dV = \int dt$$

$$\frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \left(\frac{2aV + b}{4ac - b^2} \right) + C = t$$

$$@ \ t = 0 \qquad V = 0$$

$$C = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \left(\frac{b}{4ac - b^2} \right)$$

C) Weight and acceleration constant

$$a = \frac{g}{W}(T - D - \mu N) = \text{Const.}$$