

29 OCTOBER 2024

ASE 367K: FLIGHT DYNAMICS

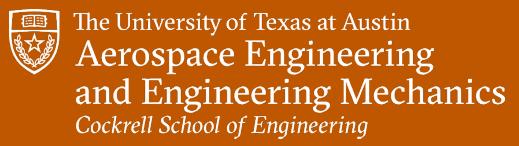
TTH 09:30-11:00
CMA 2.306

JOHN-PAUL CLARKE

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Topics for Today

- Topic(s):
 - High-Lift / High-Drag Devices
 - Takeoff
 - Rotation
 - Landing



HIGH-LIFT / HIGH-DRAG DEVICES

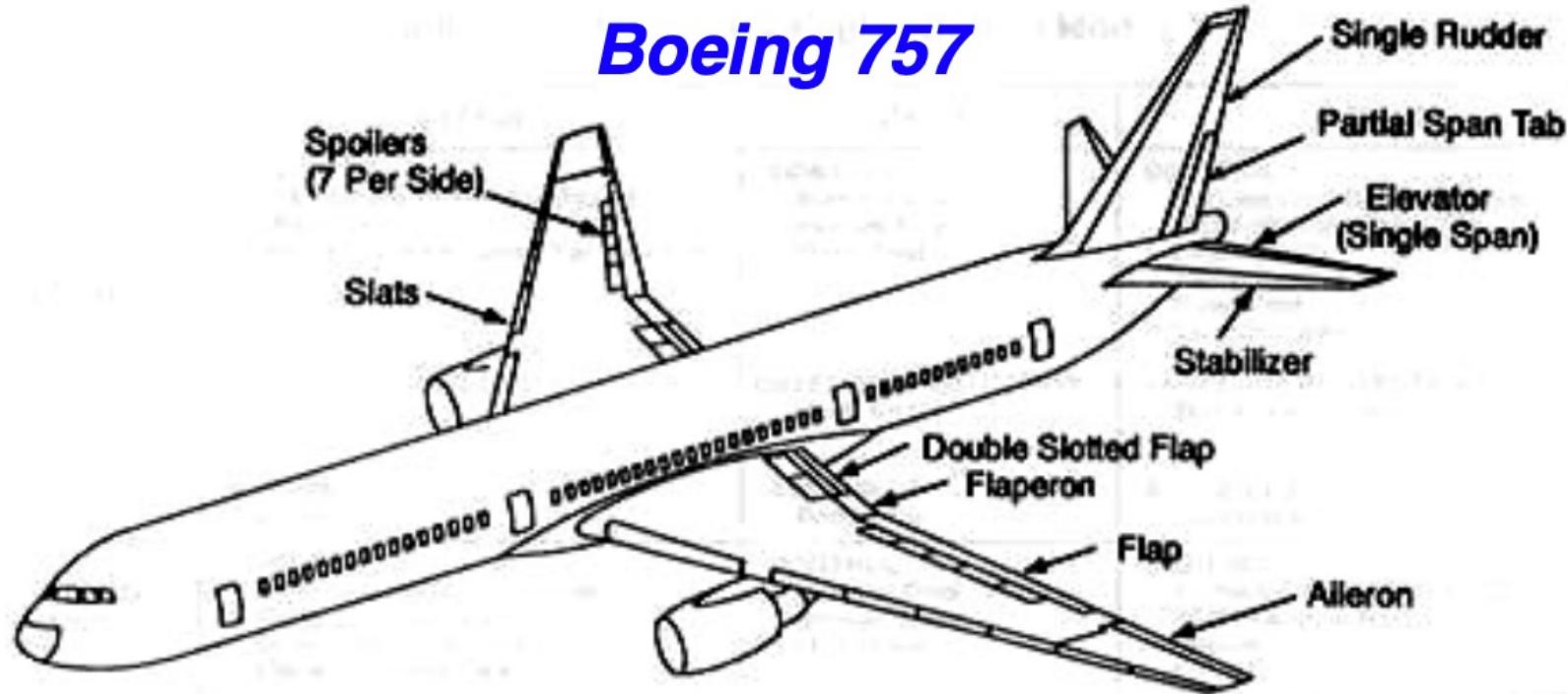
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Design for Control

Boeing 757

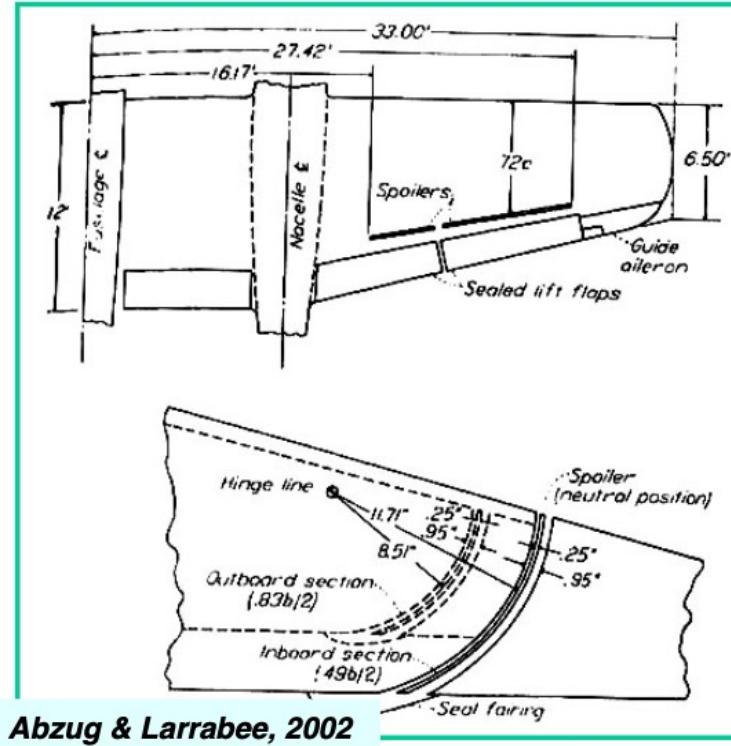


- **Elevator/stabilator: pitch control**
- **Rudder: yaw control**
- **Ailerons: roll control**
- **Trailing-edge flaps: low-angle lift control**
- **Leading-edge flaps/slats: High-angle lift control**
- **Spoilers: Roll, lift, and drag control**
- **Thrust: speed/altitude control**

Spoilers



- **Spoiler reduces lift, increases drag**
 - Speed control
- **Hinged flap has high hinge moment**
- **Differential spoilers**
 - Roll control
 - Avoid twist produced by outboard ailerons on long, slender wings
 - free trailing edge for larger high-lift flaps
- **Plug-slot spoiler on P-61 Black Widow: low control force**



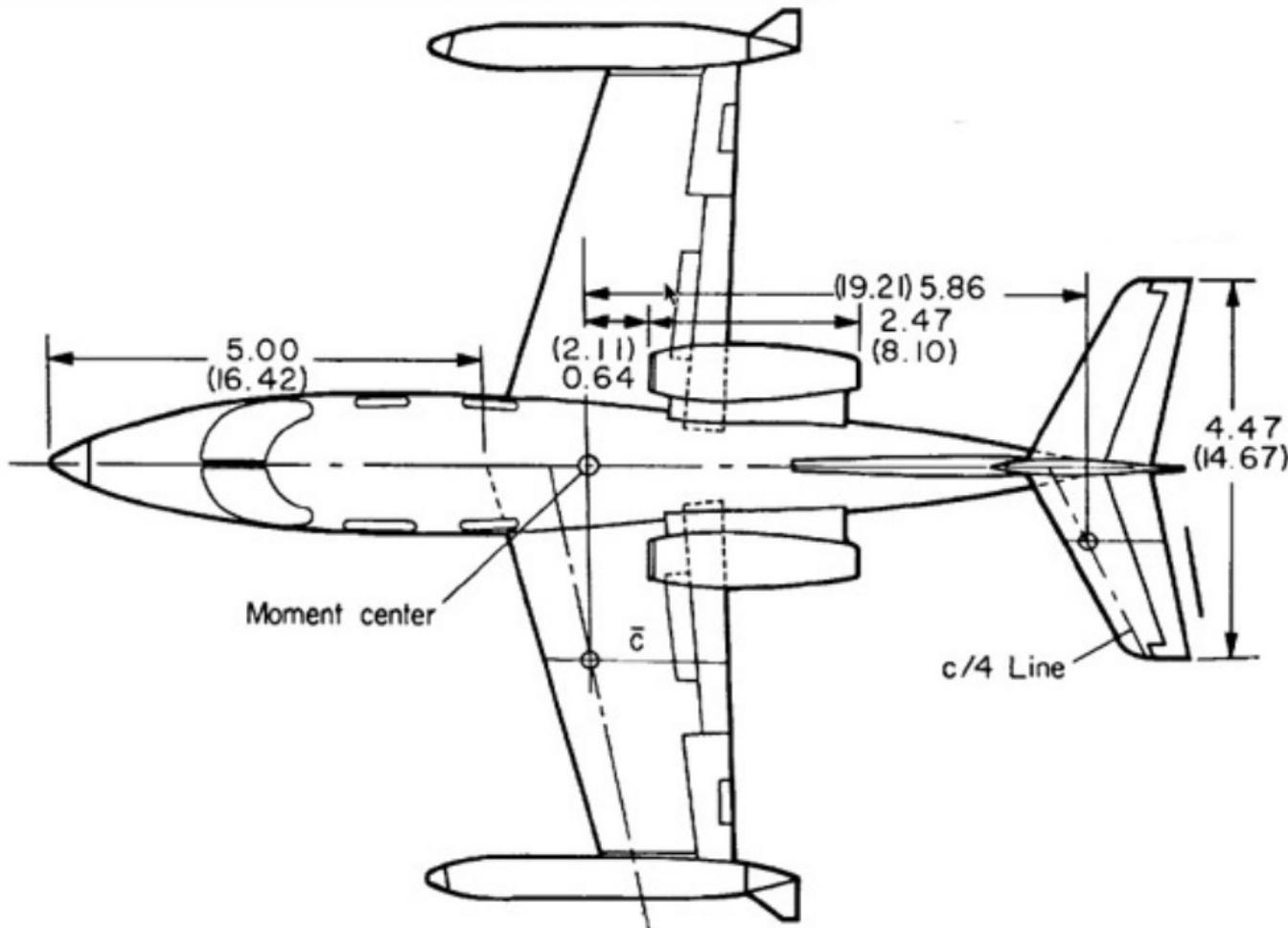
Abzug & Larrabee, 2002



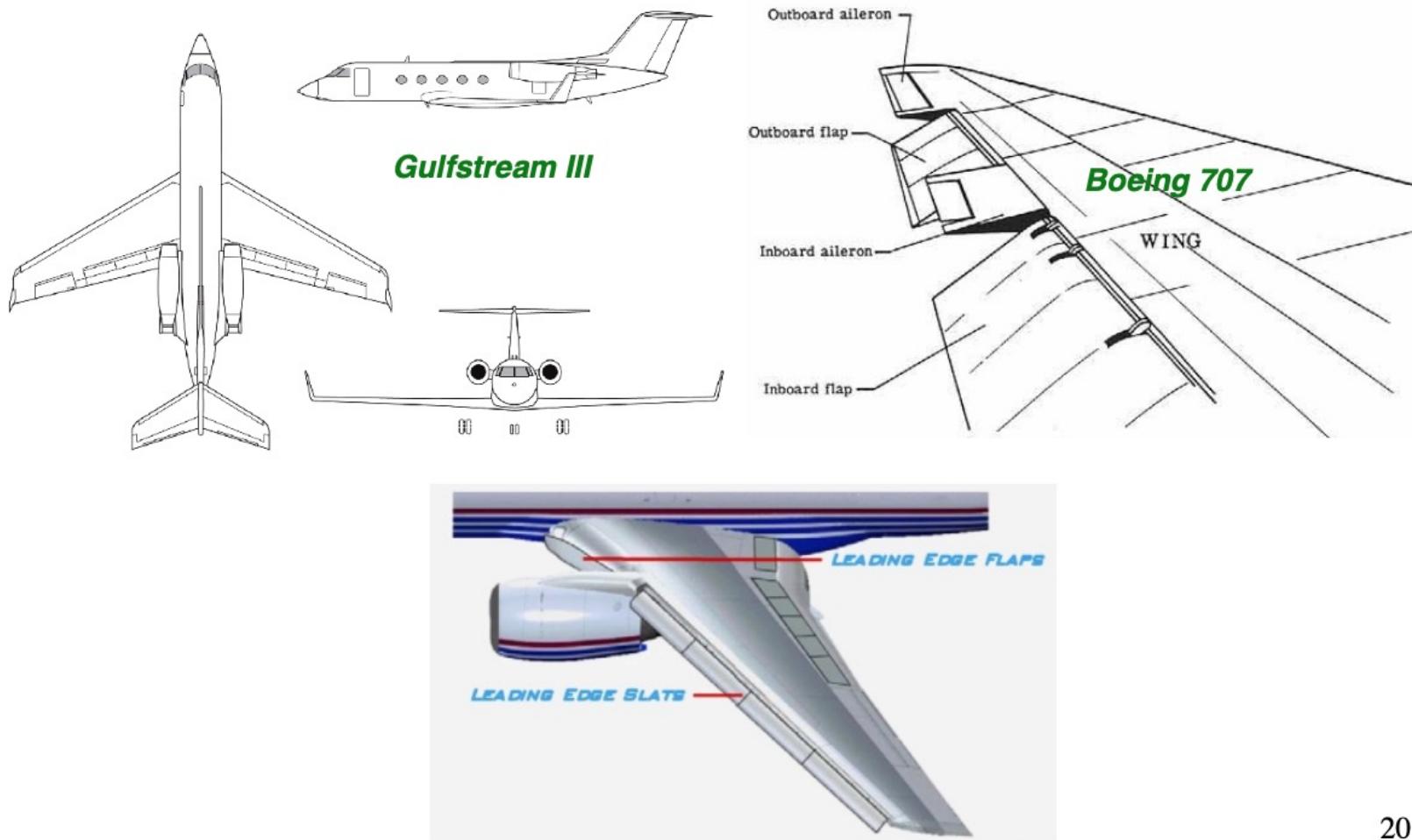
Why?

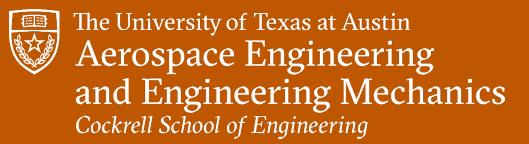
Business Jet Plan View

- Ailerons insensitive at high-speed cruise
- Differential spoilers provide more effective roll control



Trailing-Edge Flaps, Leading-Edge Flaps/Slats

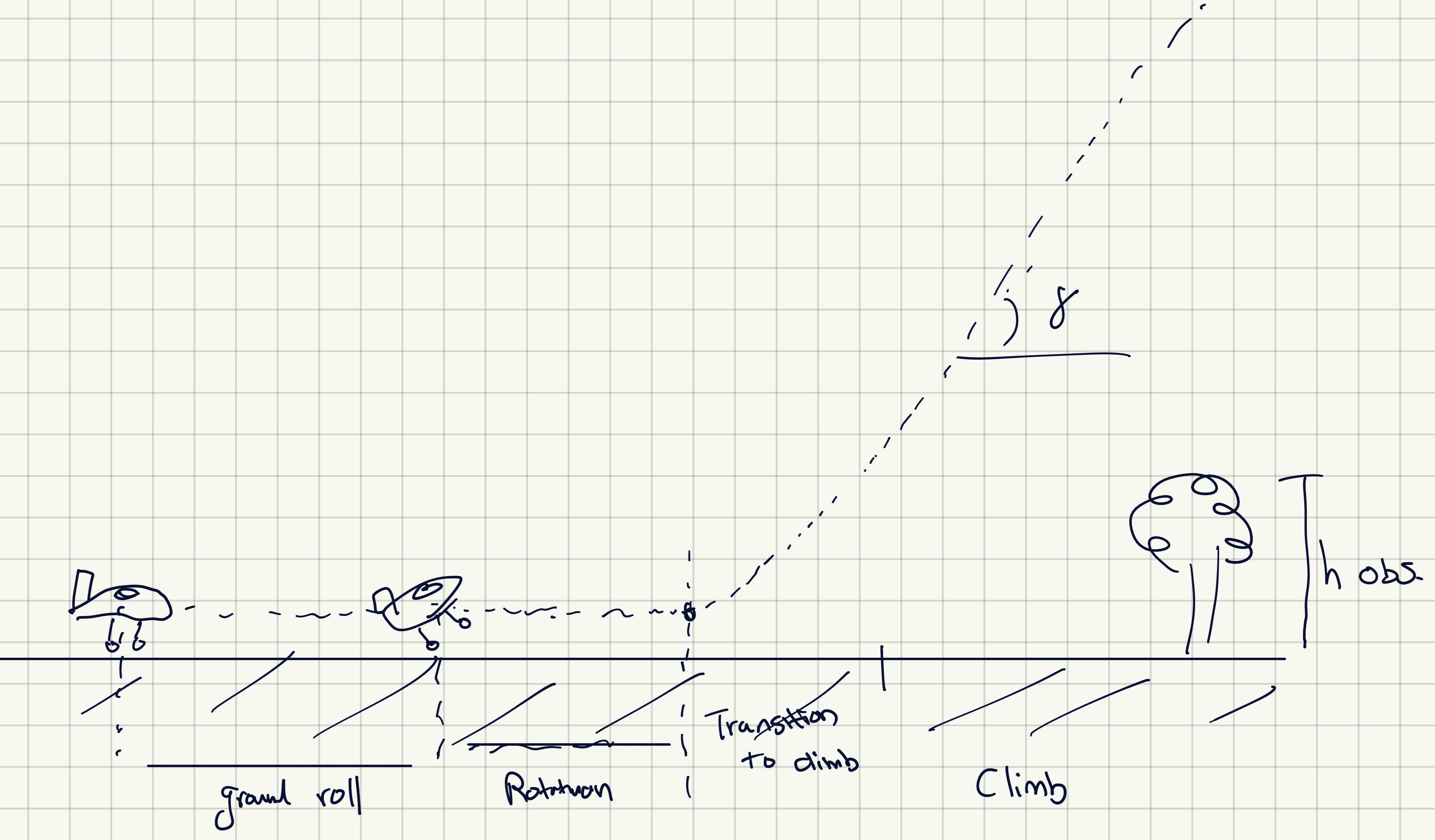




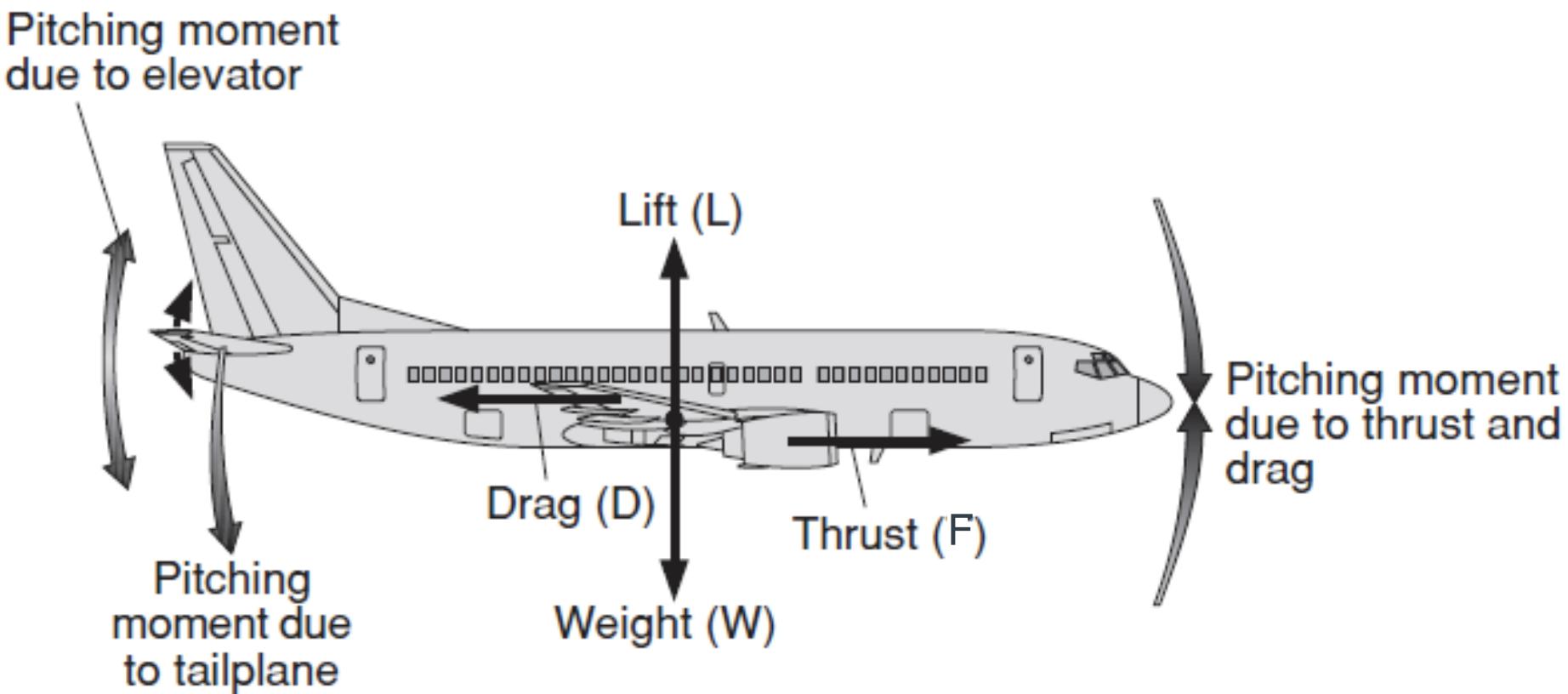
TAKEOFF

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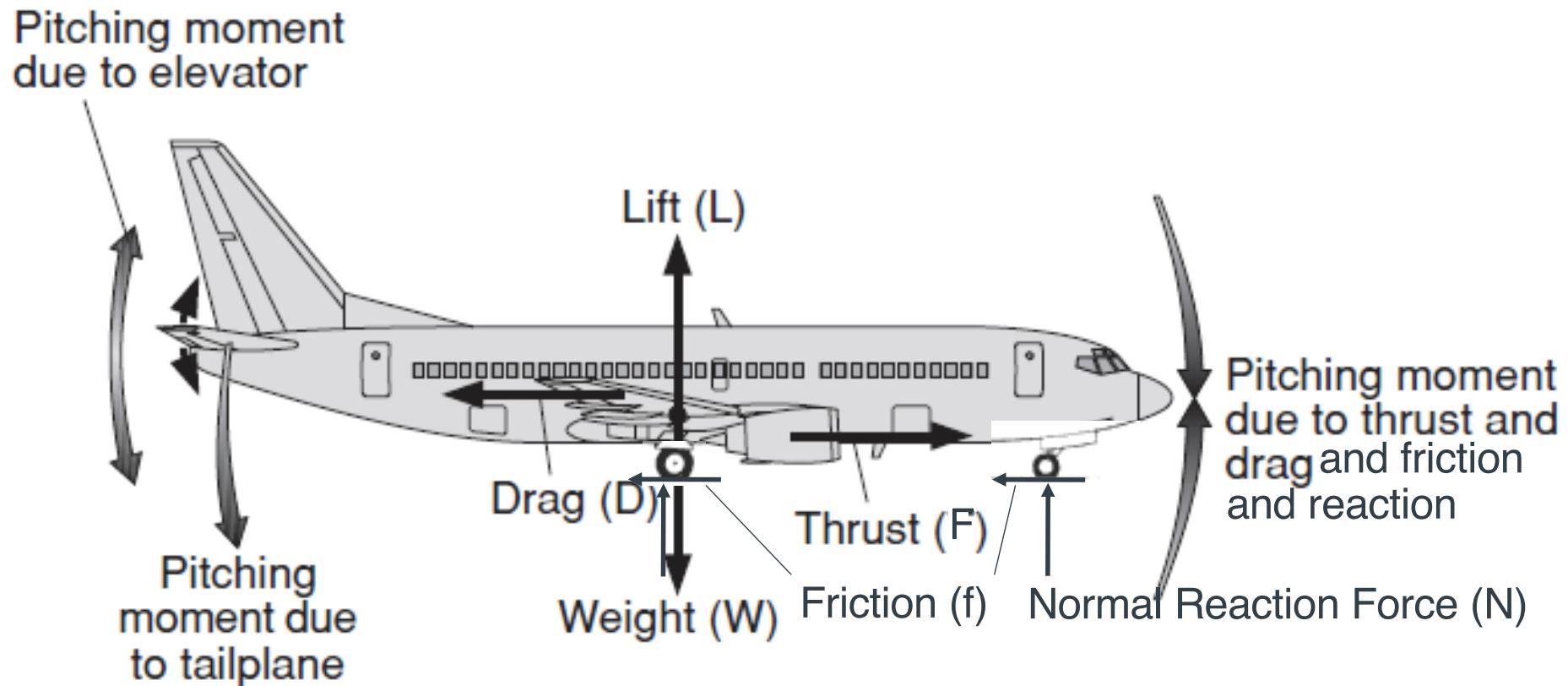
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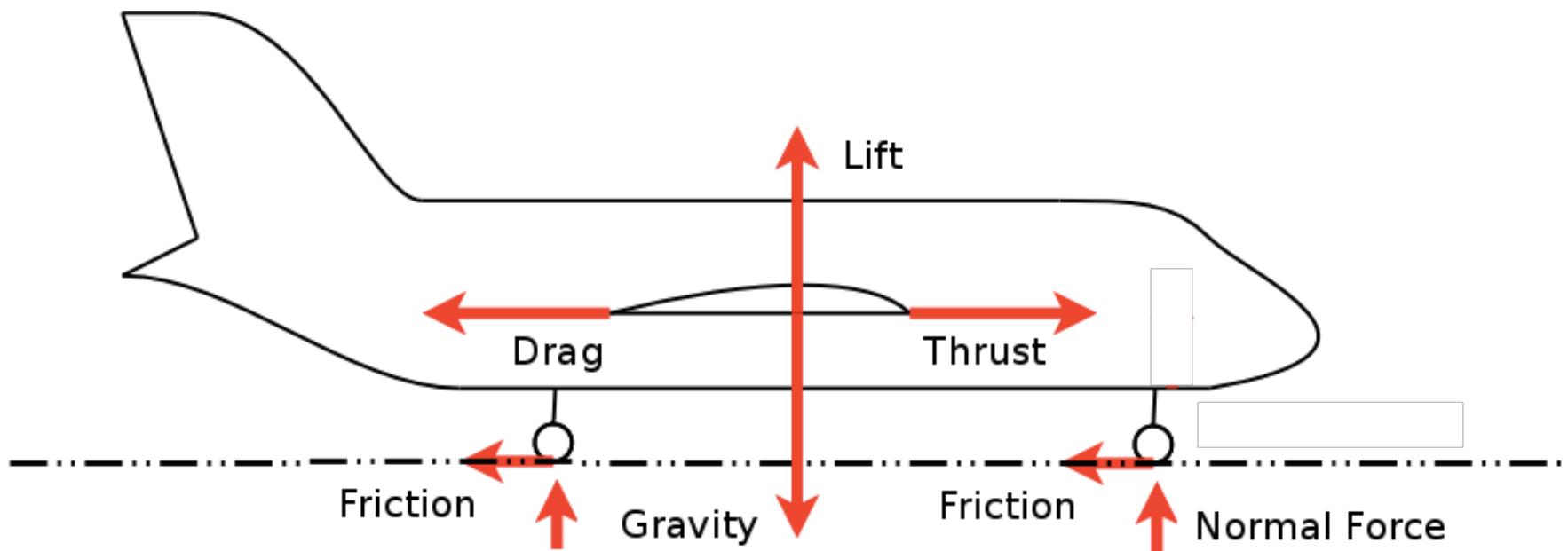
Forces and Moments (in steady flight)



Forces and Moments (on the ground)



Forces and Moments (during takeoff roll)



Relevant Equations



■ Lift

$$L = \frac{1}{2} \rho V^2 S C_L = L(t) = \frac{1}{2} \rho(h) V(t)^2 S C_L$$

■ Drag

$$D = \frac{1}{2} \rho V^2 S C_D = D(t) = \frac{1}{2} \rho(h) V(t)^2 S C_D$$

■ Thrust

$$F = F(t) = k_F \cdot f_1(h) \cdot f_2(T) \cdot f_3(M(V(t), T))$$

■ Weight

$$W = W(t) = W_0 - \int_0^t [TSFC \cdot F(\tau)] d\tau$$

■ Normal Force

$$N = N(t) = W(t) - L(t)$$

■ Friction

$$f = \mu N = f(t) = \mu [W(t) - L(t)]$$

Key Questions

- How do the forces really change as a function of altitude, temperature and Mach number?
atmosphere conditions
- How does weight really change as a function of time? *← fuel*
- What parameters remain constant throughout the takeoff roll?
← C_L, C_D are constant
- Can we reduce the problem to a relatively simple numerical integration?

Lift and Drag During Takeoff Roll

- The following parameters are constant:

- Altitude

$$h = h_{APT} \quad \text{← airport}$$

- Angle of attack

$$\alpha = 0$$

- Thus the following are constant...

- Density

$$\rho(h_{APT})$$

- Coefficient of Lift

$$C_L(0)$$

- Coefficient of Drag

$$C_D(0)$$

- And lift and drag are given by...

$$L = \left[\frac{1}{2} \rho(h_{APT}) \cdot S \cdot C_L(0) \right] \cdot V^2 = const_L \cdot V^2$$

$$D = \left[\frac{1}{2} \rho(h_{APT}) \cdot S \cdot C_D(0) \right] \cdot V^2 = const_D \cdot V^2$$

Thrust During Takeoff Roll

- Thrust is typically corrected for installation effects and normalized by the pressure ratio...

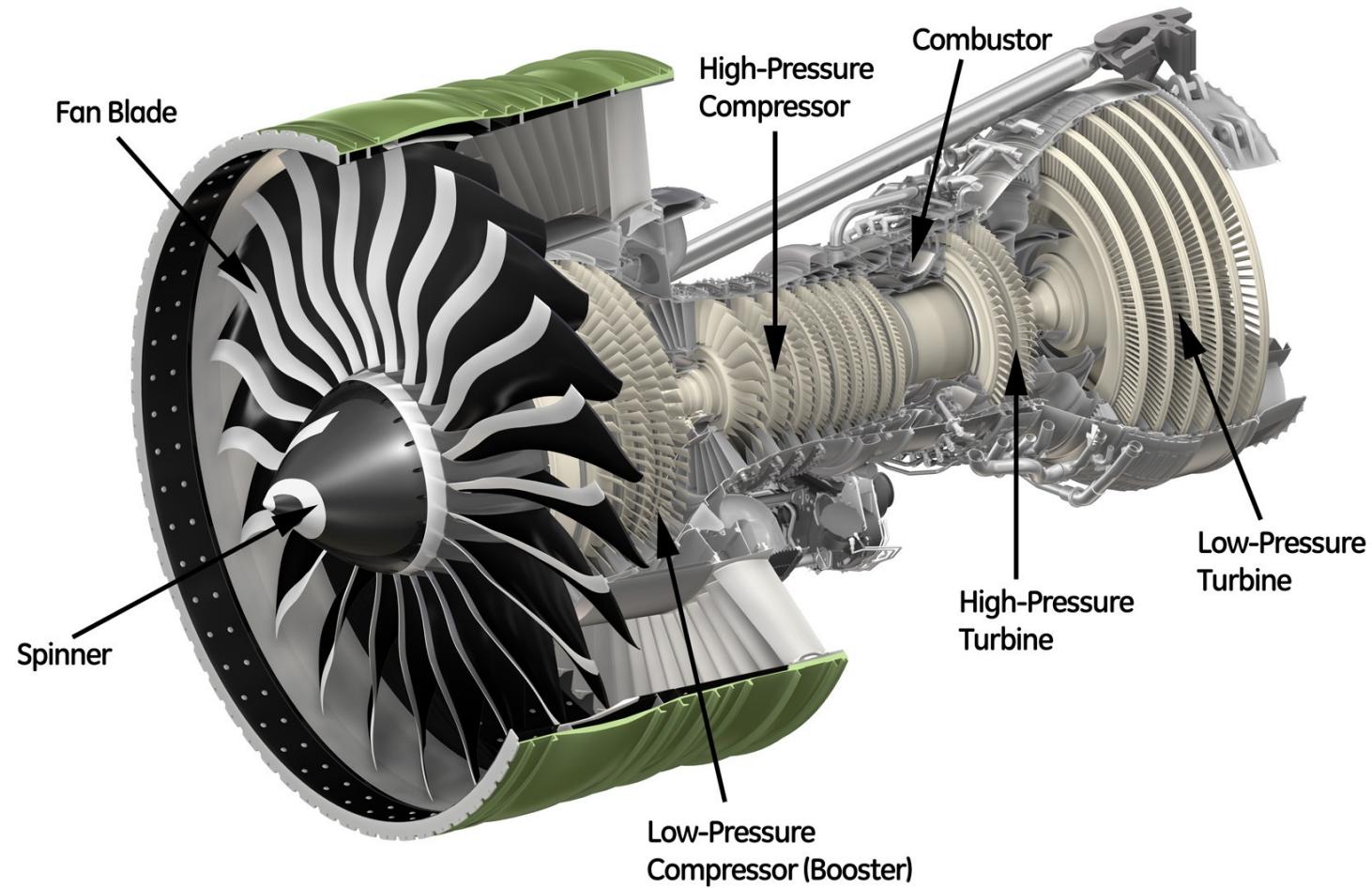
$$\delta = P / P_0$$

- The resulting value is the net correct thrust...

$$F_N / \delta$$

- Net corrected thrust is typically...
 - Quadratic function of altitude;
 - Quadratic function of Mach number;
 - Constant with Temperature until the “break temperature” after which it decreases

Thrust During Takeoff Roll



Thrust During Takeoff Roll

- The following parameters are constant:

- Altitude $h = h_{APT}$

- Temperature $T = T_{APT}$

- Thus the following are constant...

- Pressure Ratio $\delta = P_{APT} / P_0$

- Speed of Sound $a = \sqrt{\gamma R T_{APT}}$

- And thrust is given by...

$$F_N = [\delta(P_{APT}) \cdot k_F \cdot f_1(h_{APT}) \cdot f_2(T_{APT})] \cdot \left[k_{F0} + k_{F1} \frac{V}{a} + k_{F2} \frac{V^2}{a^2} \right]$$
$$= cont_F \cdot \left[k'_{F0} + k'_{F1} V + k'_{F2} V^2 \right]$$

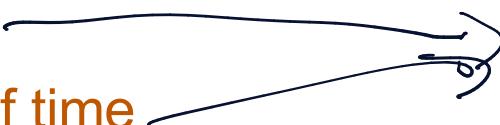
c) Velocity ~ 55 °

$a^2 [\delta(P_{APT}) \cdot k_F f_1 f_2]$

h/s

Weight During Takeoff Roll

- Fuel flow is related to throttle setting
 - Fuel flow typically proportional to throttle lever angle
 - Fuel flow only proportional to thrust (i.e., TSFC is constant) when throttle lever angle is constant and both Mach number and atmospheric conditions are constant
- During the takeoff roll...
 - Speed (and thus Mach number) changes significantly
 - Throttle lever angle constant
- Thus...
 - Fuel flow rate is constant
 - Weight is purely a function of time



$$W = W_0 - \dot{W}_f \cdot t$$

Equations of Motion for Takeoff Roll

- Mass times the acceleration is equal to the sum of the forces along runway ...

$$\frac{(W_0 - \dot{W}_f \cdot t)}{g} \cdot \dot{V} = \underbrace{\left(k_o^m + k_{F1}^m V + k_{F2}^m V^2 \right) - const_D \cdot V^2}_{- \mu \cdot [(W_0 - \dot{W}_f \cdot t) - const_L \cdot V^2]} - D \xrightarrow{\text{friction}} N$$

- Simple numerical integration?

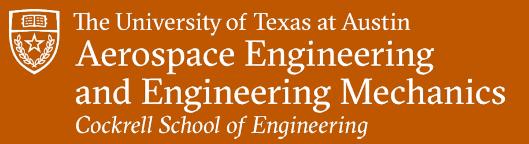
Ground Roll

→ b/c we are not accelerating in vertical acceleration.

Vertical force balance: $L - W + N = 0 \Rightarrow N = W - L$

Horizontal forces : $T - D - \mu N = m a_x = \frac{\omega}{g} \frac{dv}{dt}$

$$T - D - \mu (W - L) = \frac{\omega}{g} \frac{dv}{dt}$$



ROTATION

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Basic Assumptions

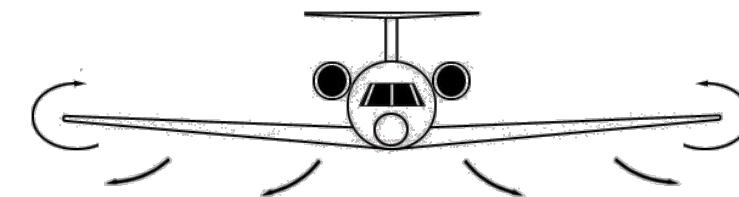
- Pitch rate $\dot{\theta} = q = \text{constant}$
- Maximum allowed pitch is 15°
- Fuel flow rate (and thus derivative of weight with respect to time) is constant and the same as during the takeoff roll
- Ground effect must be considered

Ground Effect

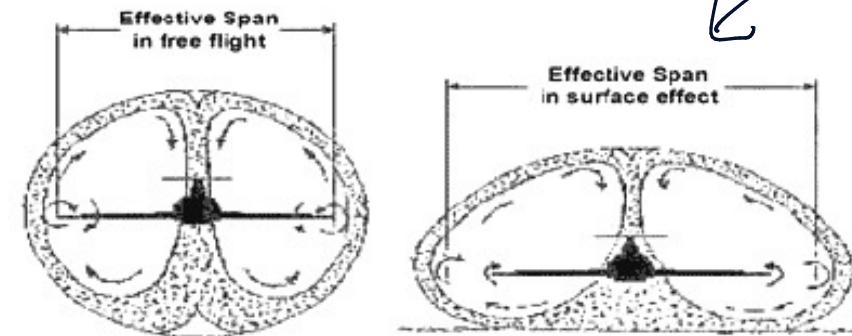
- Change in aerodynamic performance due to proximity of aircraft to the ground.
- Modeled solely a function of height above the ground and angle of attack.
- Increases the slope of the lift versus angle of attack curve.



Vortices fully formed at altitude

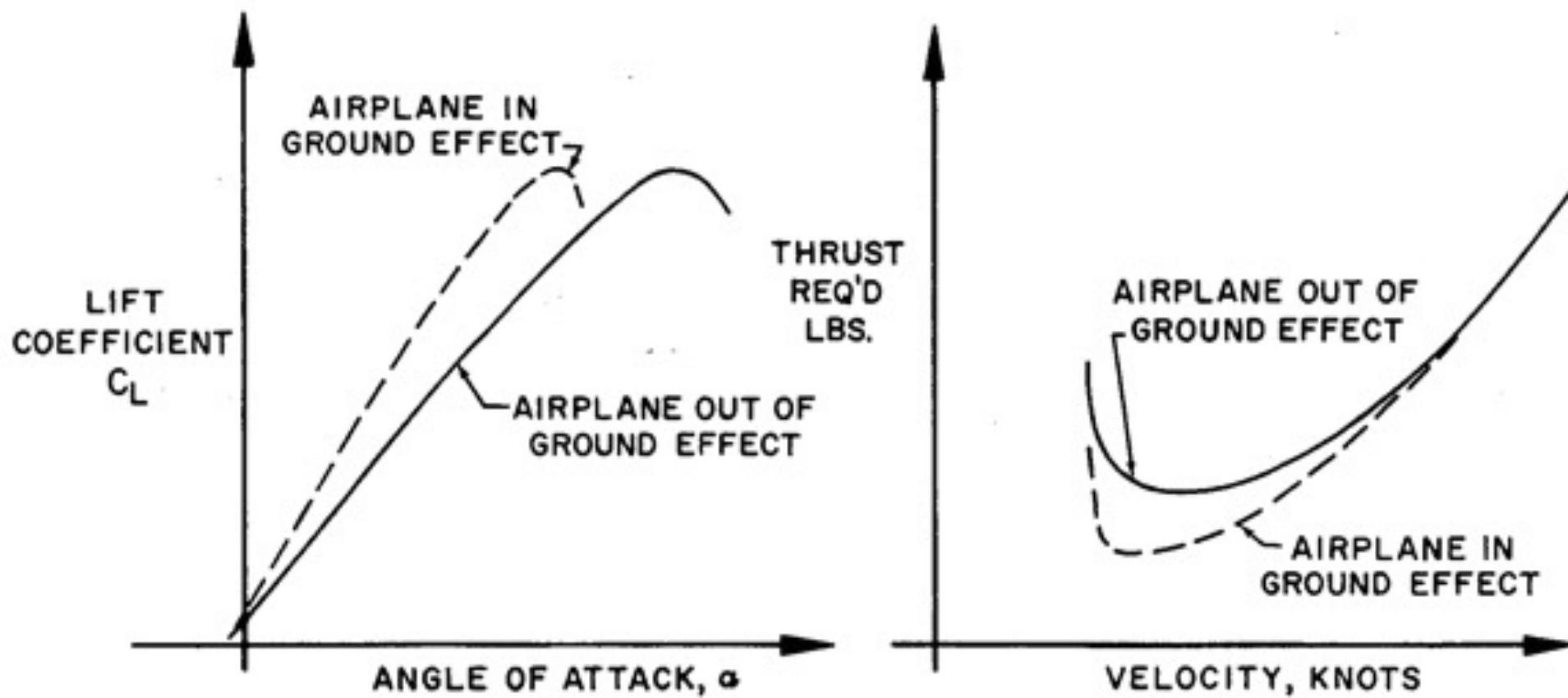


Vortices "compressed" near the ground



Ground effect
reduces
induced
drag.

Ground Effect



Ground Effect

wingspan
and
from ground,
you can ignore
ground effect

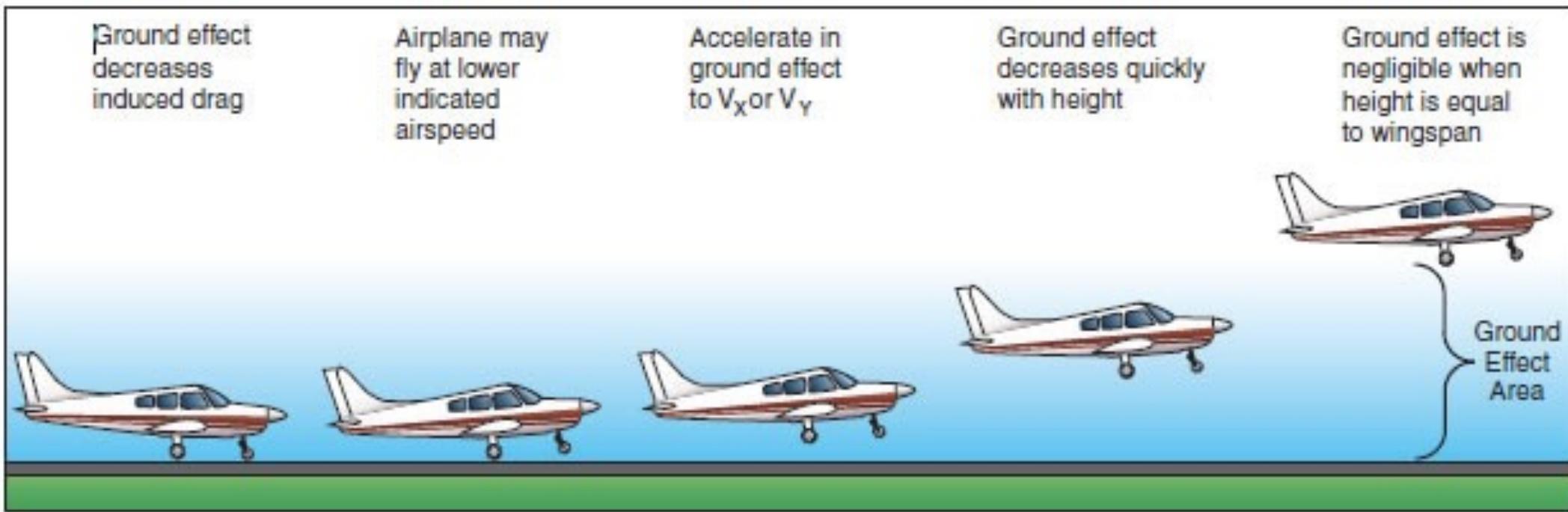
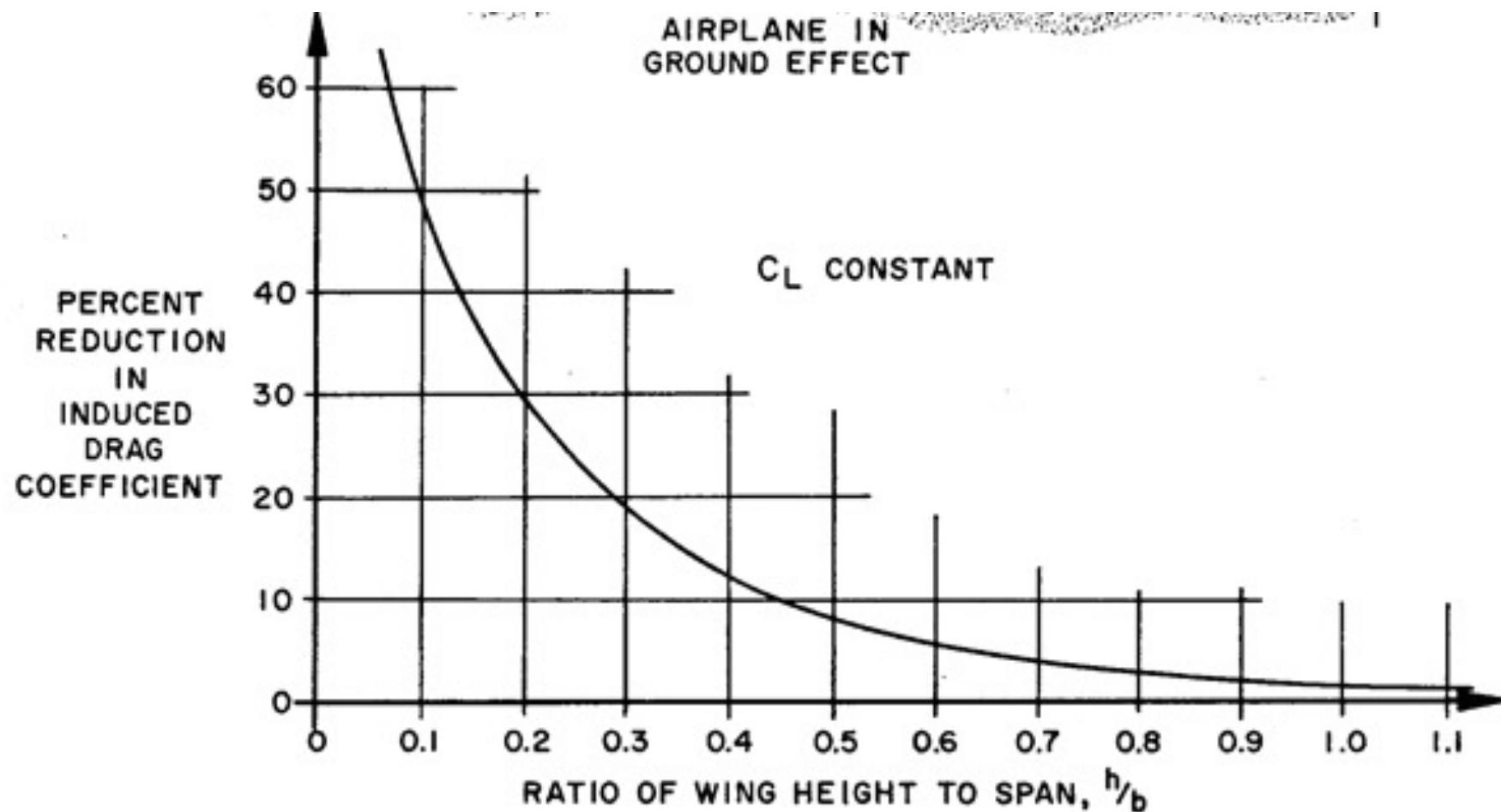


Figure 5-6. Takeoff in ground effect area.

Ground Effect

- Function of height above ground



Forces During Rotation

■ Lift

$$L = \frac{1}{2} \rho_{APT} \cdot V^2 \cdot S \cdot \left[C_L(0) + \frac{dC_L}{d\alpha} \cdot \dot{\alpha} \cdot (t - t_R) \right]$$

C_L
 α

■ Drag

$$D = \frac{1}{2} \rho_{APT} \cdot V^2 \cdot S \cdot \left[C_D(0) + \frac{dC_D}{d\alpha} \cdot \dot{\alpha} \cdot (t - t_R) \right]$$

Ground
friction
on
inert

■ Thrust

$$F_N = F_N^{STATIC} \cdot \left[a_{APT}^2 \cdot k_0^{M \rightarrow F} + a_{APT} \cdot k_1^{M \rightarrow F} \cdot V + k_2^{M \rightarrow F} \cdot V^2 \right]$$

■ Weight

$$W = W_0 - \dot{W} \cdot t$$

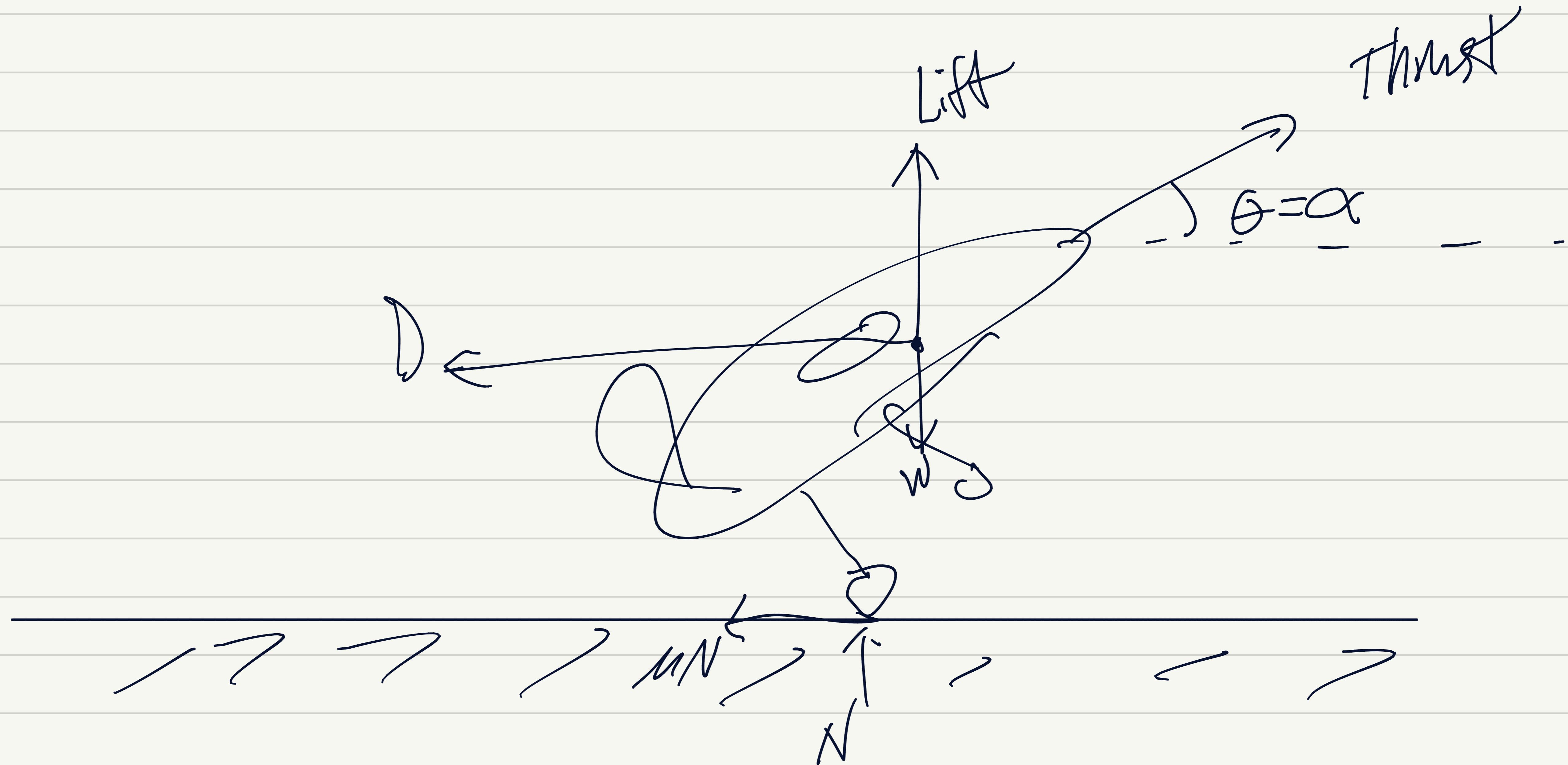
■ Normal Force

$$N = W - L - F_N \sin \theta$$

■ Friction

$$f = \mu N$$

Roktion



$$L - W + N + T \sin \theta$$

$$T \cos \theta - D - \mu N = ma$$
$$= \frac{w}{g} \frac{dv}{dt}$$

$$C_L = C_{L0} + C_{L\alpha} \alpha$$

$$\theta = \dot{\theta}(t - t_R) = \alpha$$

Equations of Motion

- In the vertical...

$$\begin{aligned} \left[\frac{W_0 - \dot{W} \cdot t}{g} \right] \cdot \ddot{h} = 0 &= W_0 - \dot{W} \cdot t \\ &\quad - \frac{1}{2} \rho_{APT} \cdot V^2 \cdot S \cdot \left[C_L(0) + \frac{dC_L}{d\alpha} \cdot \dot{\alpha} \cdot (t - t_R) \right] \\ &\quad - F_N^{STATIC} \cdot [a_{APT}^2 \cdot k_0^{M \rightarrow F} + a_{APT} \cdot k_1^{M \rightarrow F} \cdot V + k_2^{M \rightarrow F} \cdot V^2] \cdot \sin \theta \\ &\quad - N \\ \Rightarrow N &= W_0 - \dot{W} \cdot t \\ &\quad - \frac{1}{2} \rho_{APT} \cdot V^2 \cdot S \cdot \left[C_L(0) + \frac{dC_L}{d\alpha} \cdot \dot{\alpha} \cdot (t - t_R) \right] \\ &\quad - F_N^{STATIC} \cdot [a_{APT}^2 \cdot k_0^{M \rightarrow F} + a_{APT} \cdot k_1^{M \rightarrow F} \cdot V + k_2^{M \rightarrow F} \cdot V^2] \cdot \sin \theta \end{aligned}$$

Equations of Motion

- In the horizontal...

$$\left[\frac{W_0 - \dot{W} \cdot t}{g} \right] \cdot \dot{V} = F_N^{\text{STATIC}} \cdot a_{\text{APT}}^z \cdot k_0^{M \rightarrow F} + a_{\text{APT}} \cdot k_1^{M \rightarrow F} \cdot V + k_2^{M \rightarrow F} \cdot V^2 \cdot \cos \theta$$
$$- \frac{1}{2} \rho_{\text{APT}} \cdot V^2 \cdot S \cdot \left[C_D(0) + \frac{dC_D}{d\alpha} \cdot \dot{\alpha} \cdot (t - t_R) \right]$$
$$- \mu \cdot N$$

unless

Integration Scheme

1. Set $t = t_R$ and $V = V_R$
 2. Compute acceleration
 3. Compute position and velocity after Δt
 4. Update weight, mass, pitch
 5. Compute N
 6. If $N \leq 0$ stop else go to step 2
- for rotation*

Ground Roll Numerical Integration

$$t_0 = 0, \quad x_0 = 0, \quad v_0 = 0$$

Suppose you know t_k, x_k, v_k, a_k, w_k

$$t_{k+1} = t_k + \Delta t$$

$$x_{k+1} = x_k + v_k \Delta t$$

$$v_{k+1} = v_k + a_k \Delta t$$

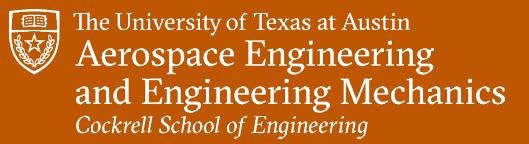
$$w_{k+1} = w_k - \dot{w}_f \Delta t \quad 1 + \frac{m}{m} b + \frac{1}{m^2} \frac{b^2}{g^2}$$

$$\begin{aligned} T_{k+1} &= k_0 + k_1 v_{k+1} + k_2 v_{k+1}^2 \\ L_{k+1} &= \frac{1}{2} \rho v_{k+1}^2 S_C \end{aligned}$$

$$D_{k+1} = \frac{1}{2} \rho v_{k+1}^2 S_C D$$

$$N_{k+1} = w_{k+1} - L_{k+1}$$

$$a_{k+1} = \frac{1}{\rho w_{k+1}} (T_{k+1} - D_{k+1} - \mu N_{k+1})$$



LANDING

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Facts of Life

and
any

There is considerable scatter in landing distances due to
use of spoiler, brakes, reverse thrust, human factors
ground conditions : wet runway , dry runway

FAR-25 Regulations Landing Performance

$V_{\text{approach}} = V_A = 1.3 V_{\text{stall}}$ for civilian aircraft

$V_{\text{approach}} = V_A = 1.2 V_{\text{stall}}$ for military aircraft

$V_{\text{approach}} = V_A = 1.1 V_{\text{stall}}$ for carrier-based aircraft

depends
on
its purpose



$$\text{Total Landing Distance, in feet} = 0.3 (V_{\text{approach}} \text{ in knots})^2$$

These results are empirical, because of variations in pilot technique.

Lift Coefficients for your Design

- For fighter design, use the following $C_{l\max}$
 - With flaps up, 1.2 - 1.8
 - With flaps down, during take-off: 1.4 - 2.0
 - With flaps down, during landing: 1.6 to 2.6
- For transport design, use the following $C_{l\max}$
 - With flaps up, 1.2 - 1.8
 - With flaps down, during take-off: 1.6 - 2.2
 - With flaps down, during landing: 1.8 to 2.8

Increase
wingspan
More space
needed
more drag
for increase
air density
→ heatwing
approach
slower

Landing Distances

Partial flaps if we do full
control surfaces aren't as effective

- Two Varieties
 - Ground Roll
 - Distance over a 50 foot obstacle
 - Also known as Takeoff Distance
 - Distance for aircraft to clear a 50-foot obstacle after a standing start at maximum takeoff power
- Factors
 - ↑ Weight = ↑ Distance (and speed)
 - ↑ Air Density = ↓ Distance
 - ↑ Headwind = ↓ Distance
 - ↑ Slope = ↓ Ground Roll
 - ↑ Flaps = ↓ Distance
 - ↑ Friction = ↓ Ground Roll
- Maximum performance landings are always accomplished with full flaps
 - Stiff crosswinds sometimes dictate the use of partial flaps

dry
reduces drag
more thrust
reduces friction

Cessna 172R

Landing Distance

CESSNA
MODEL 172R

SECTION 5
PERFORMANCE

SHORT FIELD LANDING DISTANCE AT 2450 POUNDS

CONDITIONS:

Flaps 30°
Power Off
Maximum Braking
Paved, level, dry runway
Zero Wind
Speed at 50 Ft: 62 KIAS

$$PV = nRT$$

$$\rho = \frac{P}{RT}$$



Press Alt In Feet	0°C		10°C		20°C		30°C		40°C	
	Grnd Roll Ft	Total Ft To Clear 50 Ft Obst								
S. L.	525	1250	540	1280	560	1310	580	1340	600	1370
1000	545	1280	560	1310	580	1345	600	1375	620	1405
2000	565	1310	585	1345	605	1375	625	1410	645	1440
3000	585	1345	605	1380	625	1415	650	1445	670	1480
4000	605	1380	630	1415	650	1450	670	1485	695	1520
5000	630	1415	650	1455	675	1490	700	1525	720	1560
6000	655	1455	675	1490	700	1530	725	1565	750	1605
7000	680	1495	705	1535	730	1570	755	1610	775	1650
8000	705	1535	730	1575	755	1615	780	1655	810	1695

NOTES:

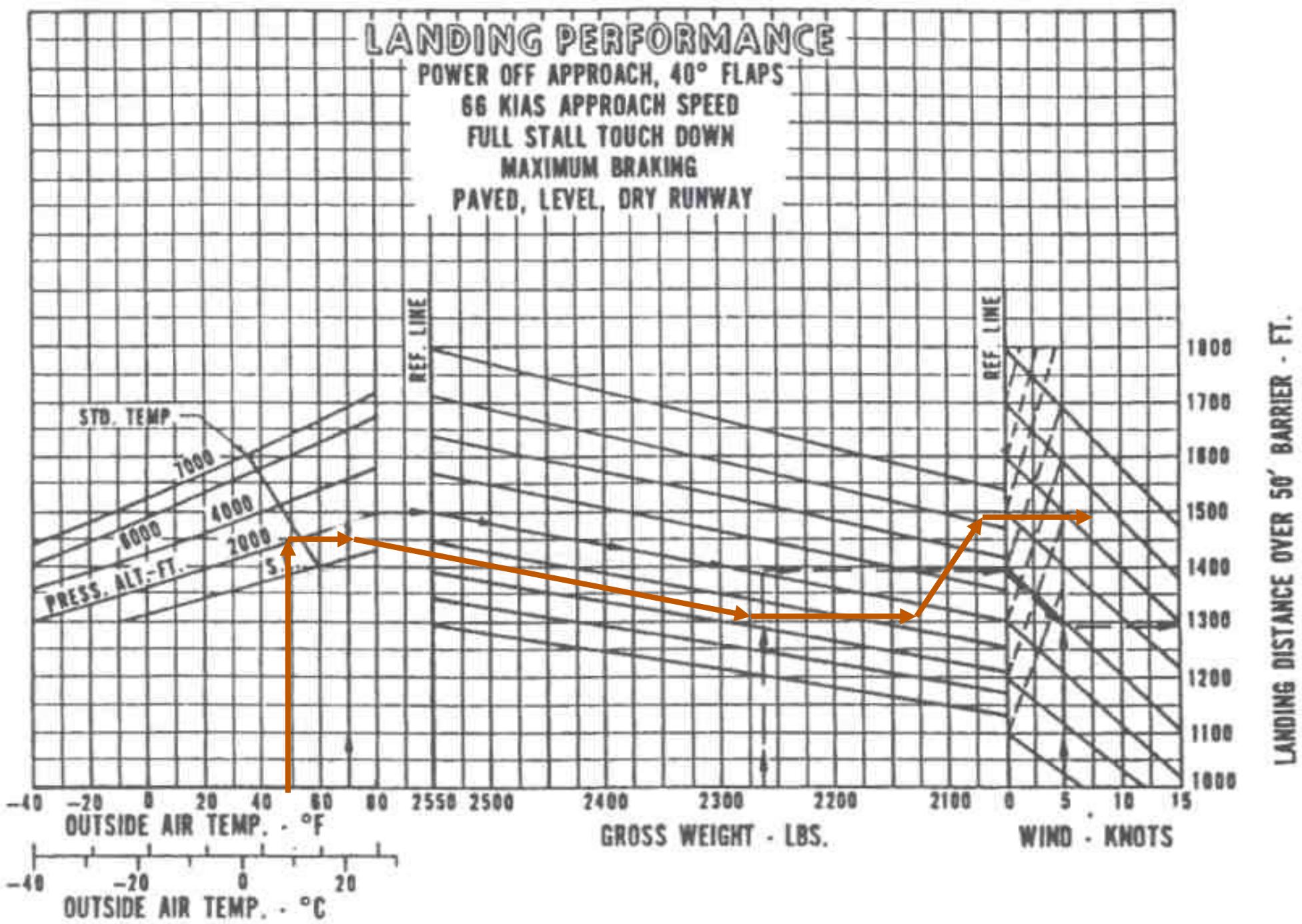
1. Short field technique as specified in Section 4.
2. Decrease distances 10% for each 9 knots headwind. For operation with tail winds up to 10 knots, increase distances by 10% for each 2 knots.
3. For operation on dry, grass runway, increase distances by 45% of the "ground roll" figure.
4. If landing with flaps up, increase the approach speed by 7 KIAS and allow for 35% longer distances.

Figure 5-11. Short Field Landing Distance

PA-28-181

LANDING PERFORMANCE

POWER OFF APPROACH, 40° FLAPS
66 KIAS APPROACH SPEED
FULL STALL TOUCH DOWN
MAXIMUM BRAKING
PAVED, LEVEL, DRY RUNWAY



Boeing 727

Landing Distance

DRY AND ICY RUNWAY LANDING DISTANCE

NOTE

These tables are for guidance only. The dry landing distances are based on crossing threshold, on speed, at 50 feet. Upon landing, use spoilers and maximum braking but no reverse thrust. The "icy" distances assume use of spoilers, maximum braking and reverse thrust.

FLAPS

30

Landing Weight	Landing Distance (Feet)							
	Dry Runway				Icy Runway (Nil Braking)			
	SL	2000	4000	6000	SL	2000	4000	6000
110.0	2310	2400	2500	2600	4600	4850	5130	5400
120.0	2450	2550	2640	2770	4990	5260	5510	5870
130.0	2600	2700	2800	2920	5400	5680	5950	6280
140.0	2740	2850	2970	3120	5790	6115	6420	6830
150.0	2910	3040	3180	3300	6250	6610	7000	7330
160.0	3120	3270	3410	3570	6830	7240	7630	8070

FLAPS

40

Landing Restrictions

- Flaps 40 is not authorized for landing on Stage 3 aircraft

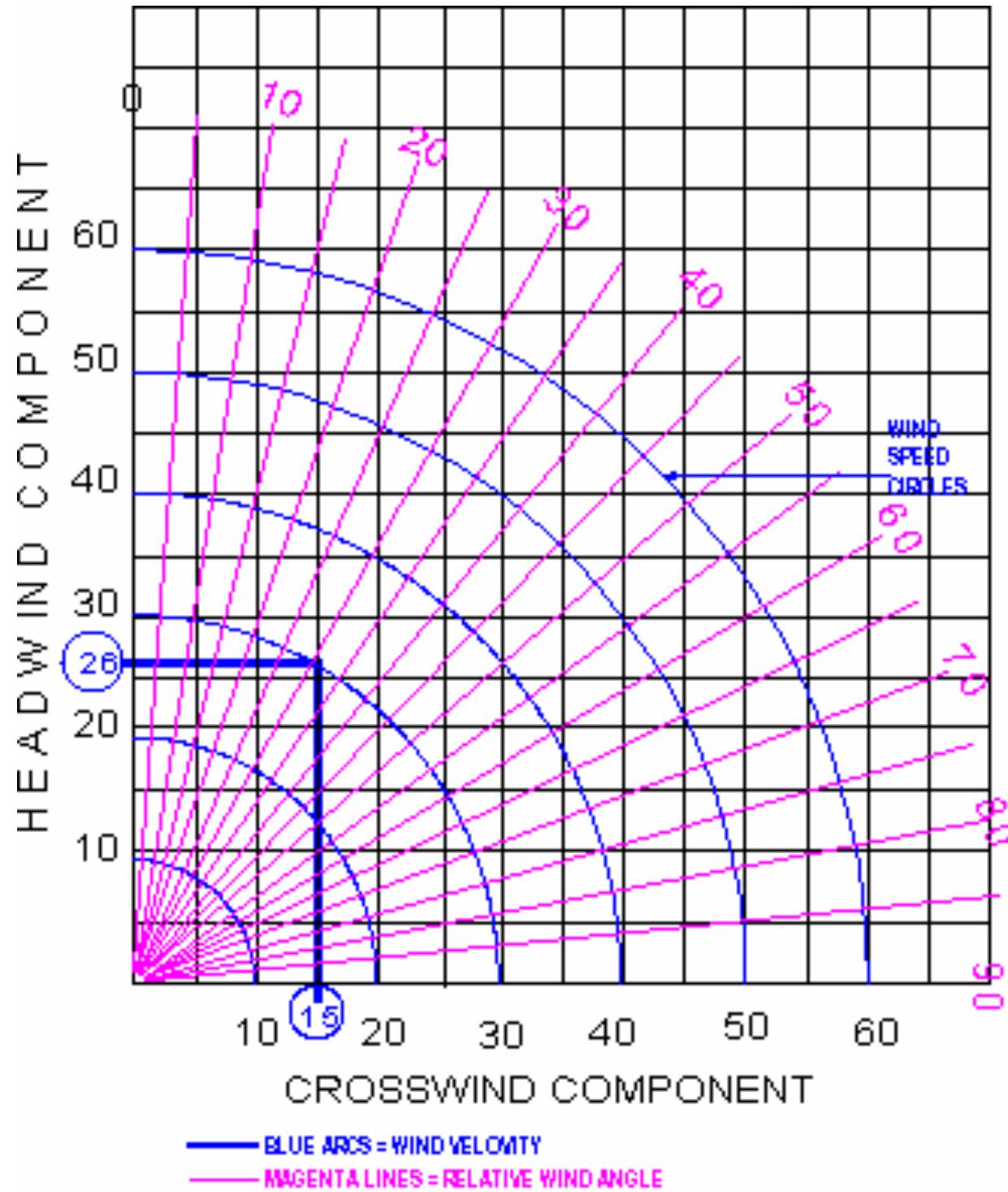
Landing Weight	Landing Distance (Feet)							
	Dry Runway				Icy Runway (Nil Braking)			
	SL	2000	4000	6000	SL	2000	4000	6000
110.0	2130	2220	2300	2400	4110	4360	4580	4850
120.0	2260	2370	2470	2560	4470	4770	5040	5290
130.0	2430	2530	2620	2750	4930	5210	5460	5810
140.0	2580	2700	2790	2920	5350	5680	5920	6280
150.0	2730	2870	2990	3110	5760	6140	6470	6800
160.0	2970	3110	3260	3400	6420	6800	7220	7600

Landing Distances

- Maximum Demonstrated Crosswind Component is the maximum crosswind that existed during FAA certification
 - Often is not close to real maximum capability of aircraft
 - Weak link is generally lack of rudder authority (not enough deflection)
- Often V_X -Wind = 0.3 VS0

Crosswind Chart

30 knot wind,
30 deg off of the nose
= 26 knot headwind
= 15 knot crosswind





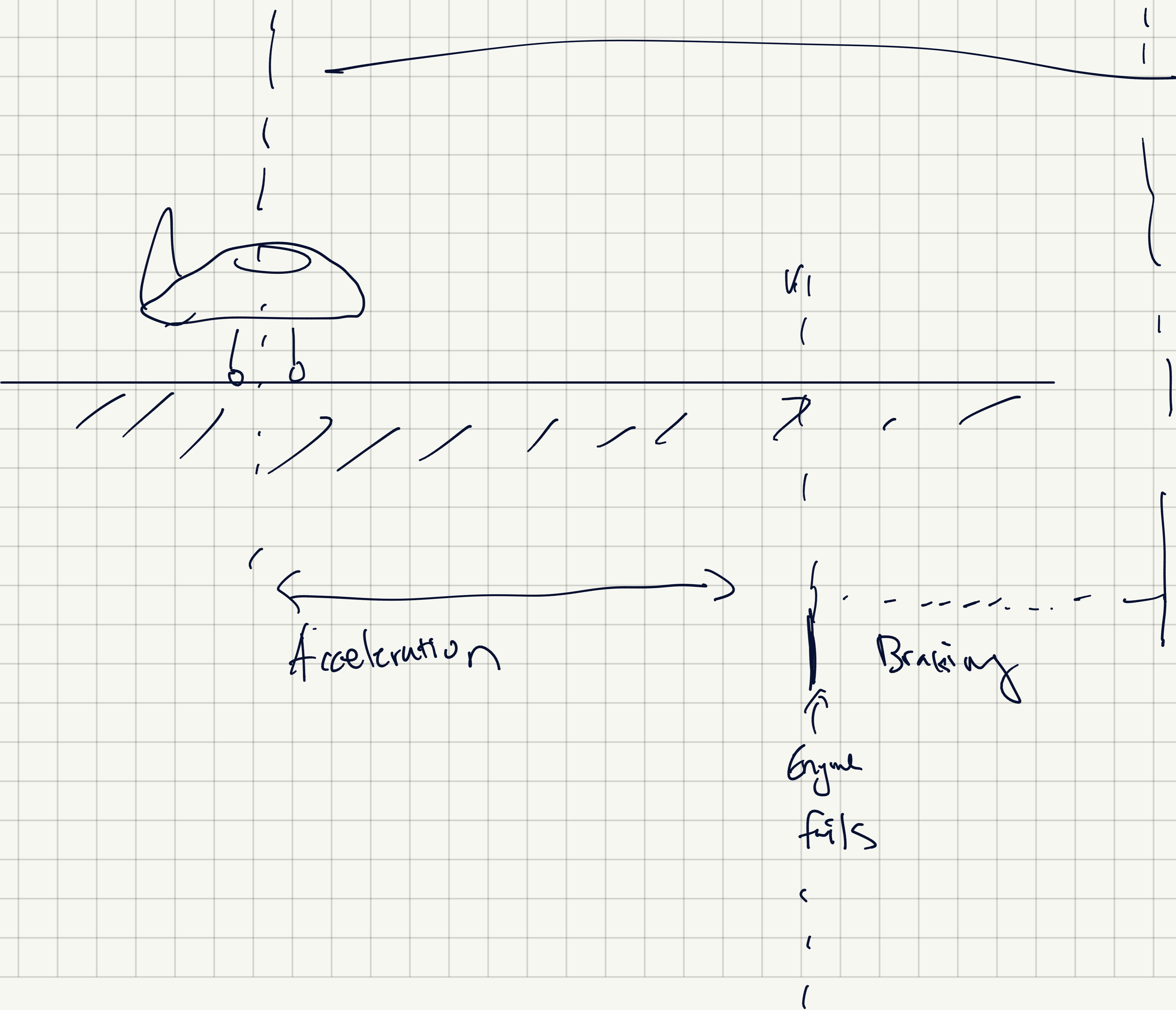
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Cockrell School of Engineering

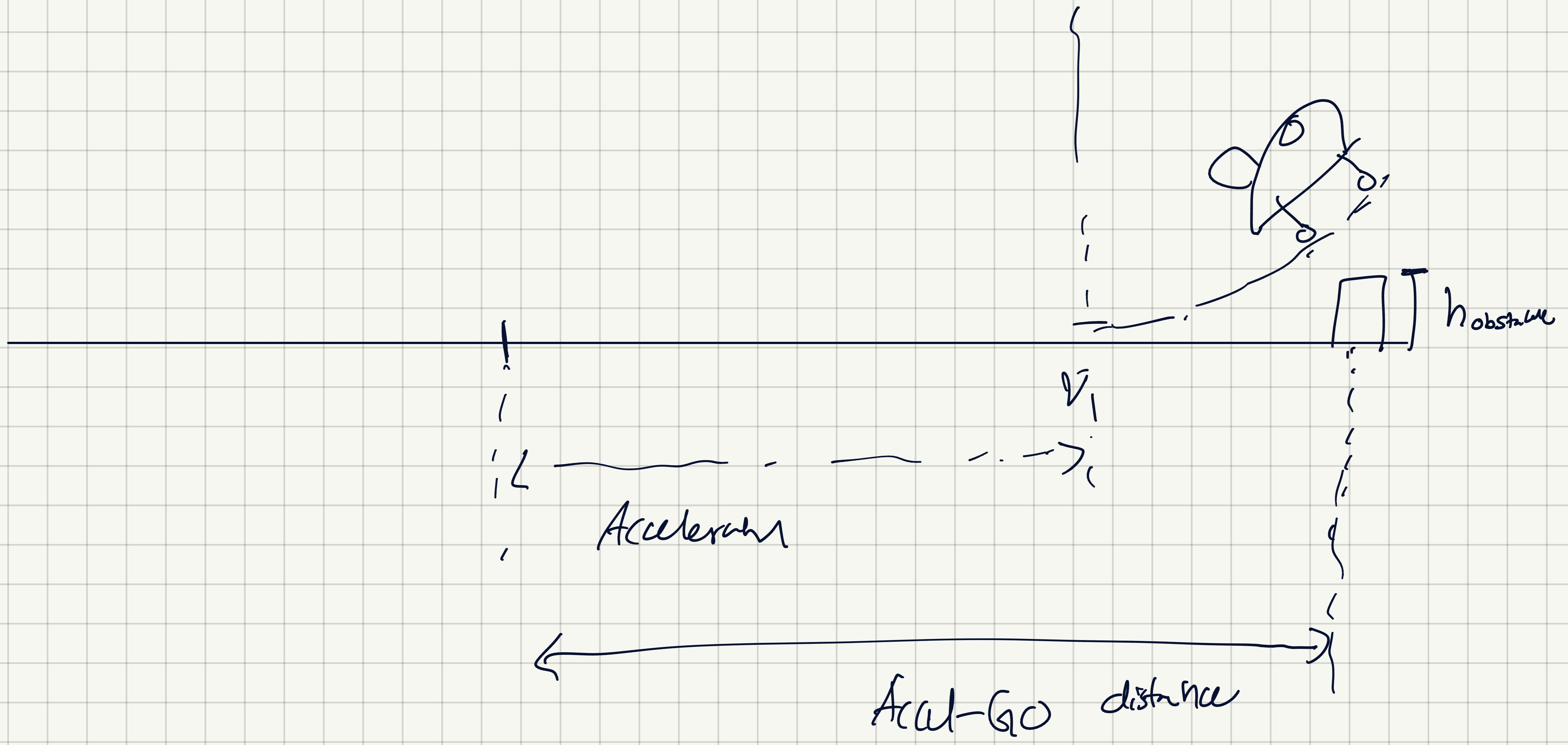
Balanced

Field

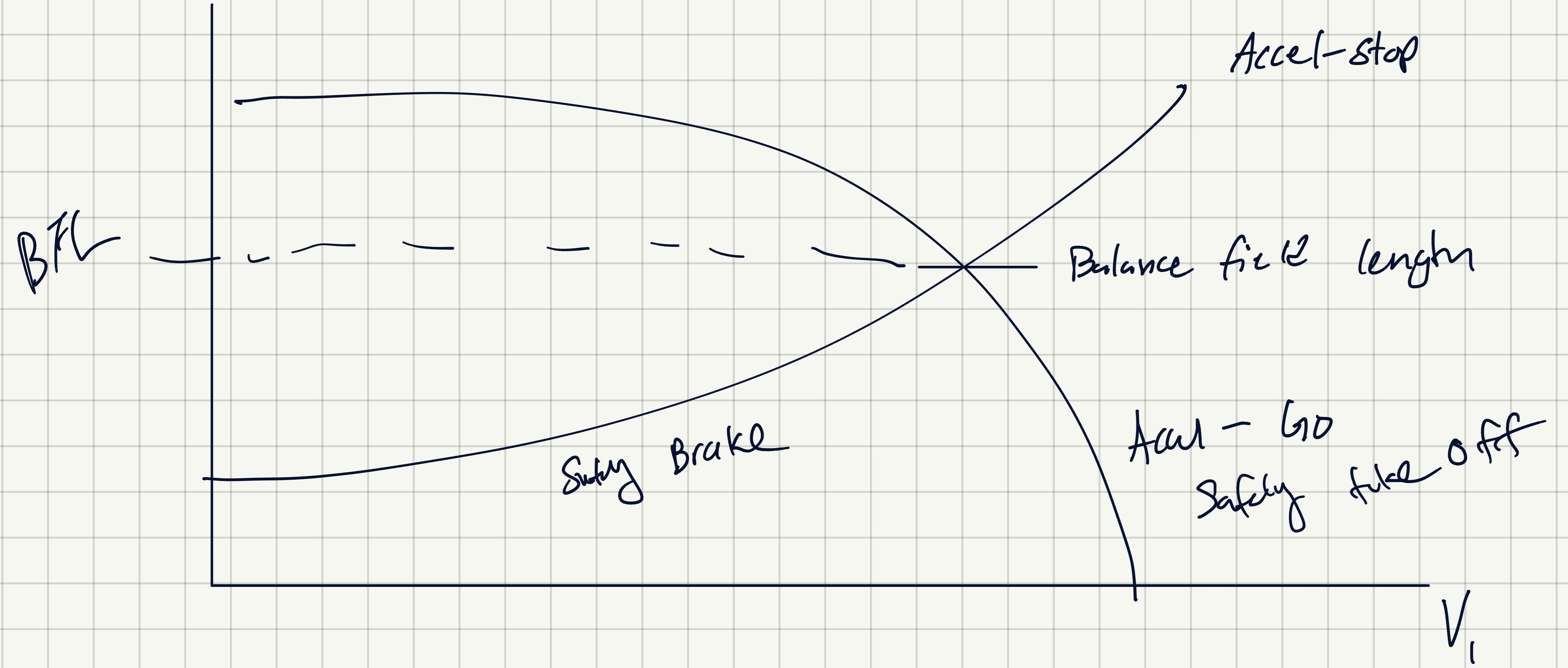
length (BFL)

accel-stop
distance





Distance Required



Practical Problem

Inclined ground roll

Landing on up hill slope with angle η

Known: C_{L0} , C_{D0} , μ , S , const weight, air density ρ



Power off

Find ODE



$$\begin{aligned} L - W \cos \eta + N &= 0 \\ -W \sin \eta - \mu N - D &= m \ddot{v} \end{aligned}$$

$$\frac{W}{g} \frac{dV}{dt}$$

$$L = \frac{1}{2} \rho V^2 S C_{L0}, \quad D = \frac{1}{2} \rho V^2 S C_{D0}$$

$$\frac{dV}{dt} = \frac{\rho S g}{2W} (\mu C_{L0} - C_{D0}) V^2 - g(\mu \cos \eta + \sin \eta)$$

$$\frac{dV}{dt} = AV^2 + B$$

