The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

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ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Lateral Modes
  - Useful Videos
  - Handling Qualities
  - Yaw Damper

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LATERAL MODES

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The linearized lateral EOM in Matrix Form is...

$$\begin{bmatrix} u_1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -(I_{xz}/I_{xx}) & 0 & 0 \\ 0 & -(I_{xz}/I_{zz}) & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \\ \Delta\dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_\beta & Y_p & (Y_r - u_1) & g\cos\theta_1 & 0 \\ L_\beta & L_p & L_r & 0 & 0 \\ (N_\beta + N_{T_\beta}) & N_p & (N_r + N_{T_r}) & 0 & 0 \\ 0 & 1 & \tan\theta_1 & 0 & 0 \\ 0 & 0 & \sec\theta_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \\ \Delta\psi \end{bmatrix} + \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

Therefore, the lateral dynamics are given by the linear matrix equation

$$\boldsymbol{M}\dot{\boldsymbol{x}} = \boldsymbol{R}\boldsymbol{x} + \boldsymbol{F}\boldsymbol{\delta}$$

where

$$\boldsymbol{M} = \begin{bmatrix} u_1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -(I_{xz}/I_{xx}) & 0 & 0 \\ 0 & -(I_{xz}/I_{zz}) & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} Y_\beta & Y_p & (Y_r - u_1) & g\cos\theta_1 & 0 \\ L_\beta & L_p & L_r & 0 & 0 \\ (N_\beta + N_{T_\beta}) & N_p & (N_r + N_{T_r}) & 0 & 0 \\ 0 & 1 & \tan\theta_1 & 0 & 0 \\ 0 & 0 & \sec\theta_1 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{F} = \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \\ \Delta\psi \end{bmatrix}, \quad \dot{\boldsymbol{x}} = \begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \\ \Delta\dot{\psi} \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\delta} = \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

In standard linear systems notation, this is $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$ where $\boldsymbol{A} = \boldsymbol{M}^{-1}\boldsymbol{R}$ and $\boldsymbol{B} = \boldsymbol{M}^{-1}\boldsymbol{F}$

A Boeing 747 airplane has the following characteristics

$$W = 636,636 \text{ [lb]}, \quad I^b_{xx} = 1.82 \times 10^7 \text{ [slugs}\cdot\text{ft}^2], \quad I^b_{zz} = 4.97 \times 10^7 \text{ [slugs}\cdot\text{ft}^2],$$
$$I^b_{xz} = 9.70 \times 10^5 \text{ [slugs}\cdot\text{ft}^2], \quad S = 5,500 \text{ [ft}^2], \quad \text{and} \quad b = 195.7 \text{ [ft]}$$

The aircraft is in trim at $u_1 = 399$ [knots], $\theta_1 = 2.4$[deg], and $\alpha_1 = 2.4$ [deg] when flying at an atmospheric density of $\rho = 1.2673 \times 10^{-3}$ [slugs/ft^3]. The aerodynamic coefficients relevant to lateral dynamic stability of the 747 are given by

$$\begin{aligned}
C_{y_\beta} &= -0.9000 & C_{\ell_p} &= -0.3400 & C_{n_p} &= -0.0260 \\
C_{y_p} &= 0.0000 & C_{\ell_r} &= 0.1300 & C_{n_r} &= -0.2800 \\
C_{y_r} &= 0.0000 & C_{\ell_{\delta_a}} &= 0.0130 & C_{n_{T_r}} &= 0.0000 \\
C_{y_{\delta_a}} &= 0.0000 & C_{\ell_{\delta_r}} &= 0.0080 & C_{n_{\delta_a}} &= 0.0018 \\
C_{y_{\delta_r}} &= 0.1200 & C_{n_\beta} &= 0.1600 & C_{n_{\delta_r}} &= -0.1000 \\
C_{\ell_\beta} &= -0.1600 & C_{n_{T_\beta}} &= 0.0000
\end{aligned}$$

The A and B matrices are:

$$\mathbf{A} = \begin{bmatrix} -0.1067 & 0 & -1.0000 & 0.0477 & 0 \\ -2.7427 & -0.8404 & 0.3264 & 0 & 0 \\ 1.0146 & -0.0176 & -0.2554 & 0 & 0 \\ 0 & 1.0000 & 0.0419 & 0 & 0 \\ 0 & 0 & 1.0009 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 0.0142 \\ 0.2211 & 0.1482 \\ 0.0096 & -0.6231 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

With the matrix $\boldsymbol{A}$ built, we can compute the eigenvalues and eigenvectors of the matrix in order to determine stability and some modal characteristics of the system. For this case, we find that the eigenvalues are given by

$$\lambda_1 = 0\,, \quad \lambda_2 = -0.9388\,, \quad \lambda_3 = -0.0171\,, \quad \lambda_{4,5} = -0.1234 \pm 1.0416i$$

and that the normalized non-dimensional eigenvector magnitudes are

$$\|\tilde{\boldsymbol{v}}\|_1 = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 1.0000 \end{bmatrix}, \quad \|\tilde{\boldsymbol{v}}\|_2 = \begin{bmatrix} 0.0311 \\ 0.1364 \\ 0.0032 \\ 1.0000 \\ 0.0234 \end{bmatrix}, \quad \|\tilde{\boldsymbol{v}}\|_3 = \begin{bmatrix} 0.0039 \\ 0.0009 \\ 0.0025 \\ 0.3648 \\ 1.0000 \end{bmatrix}, \quad \|\tilde{\boldsymbol{v}}\|_{4,5} = \begin{bmatrix} 0.4859 \\ 0.1524 \\ 0.0699 \\ 1.0000 \\ 0.4589 \end{bmatrix}$$

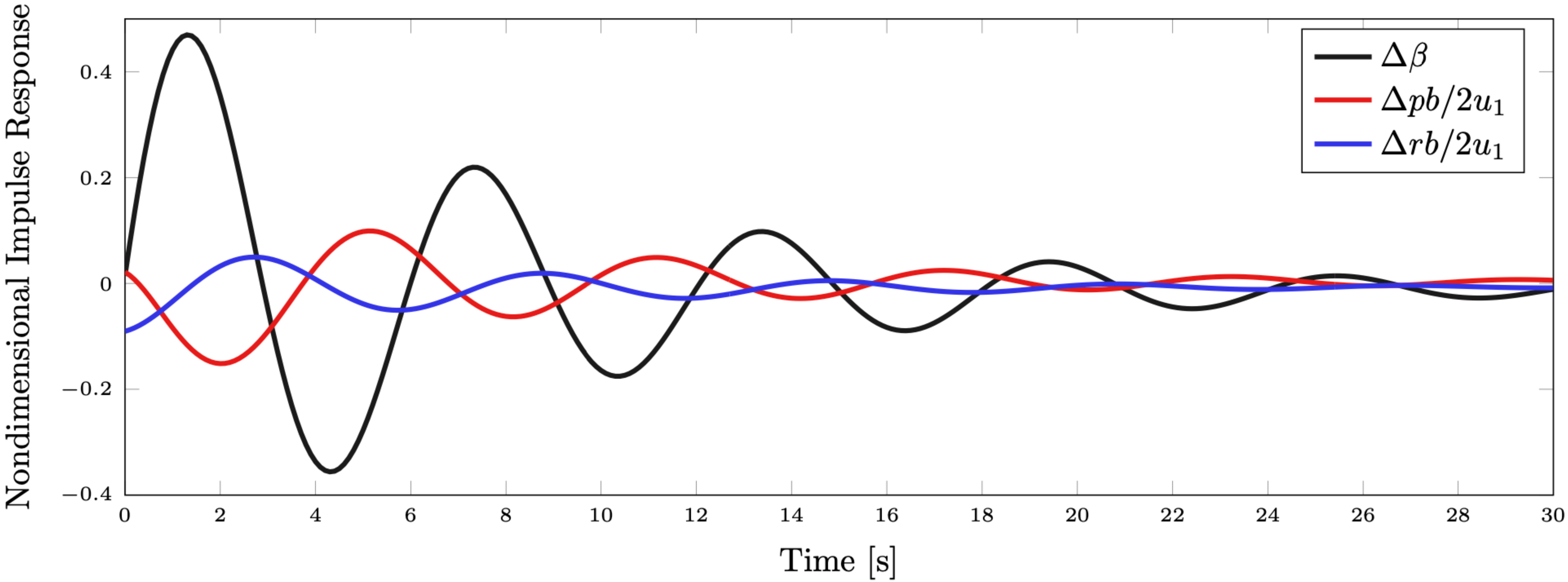
Remember the 5 components of the state vector:

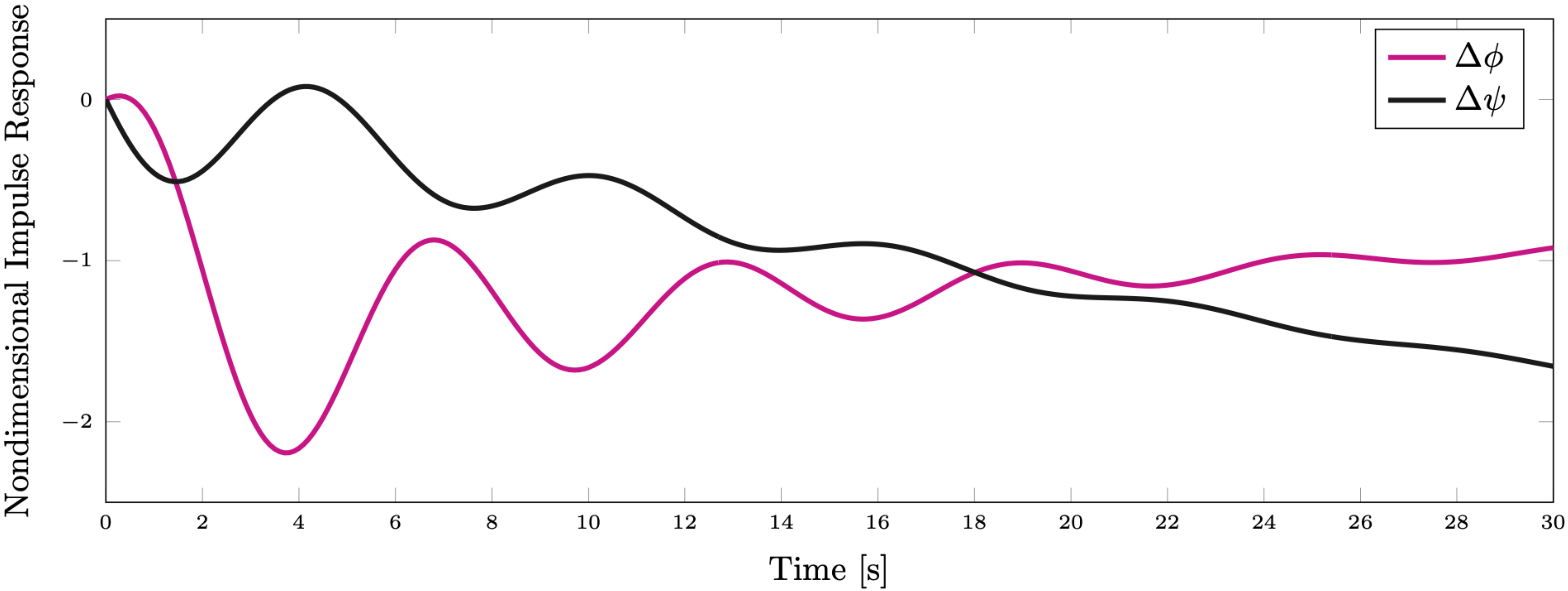
$$\boldsymbol{x} = \boxed{\begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \\ \Delta\psi \end{bmatrix}}$$

where normalization is done such that the largest element is one. Based on the above data, complete the following table:

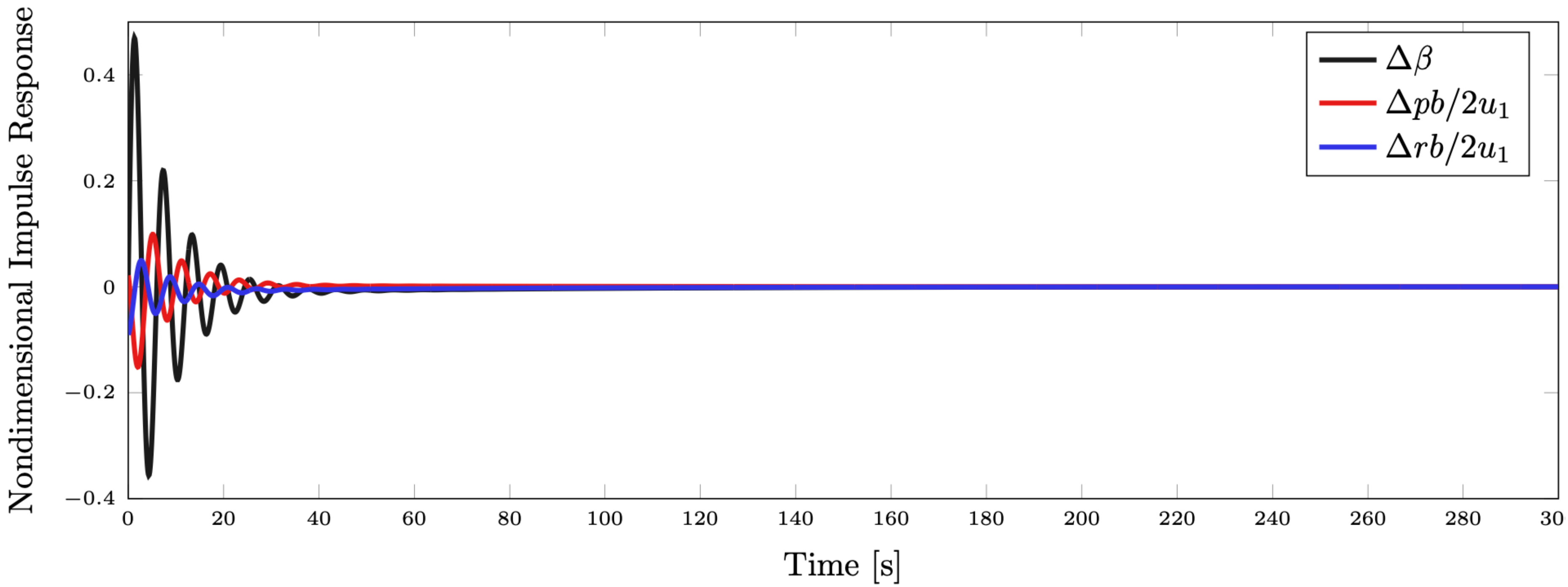
Eigenvalue	Stable?	Oscillatory?	Mode?	Dominant Motion?
λ_1				
λ_2				
λ_3				
$\lambda_{4,5}$				

Rudder Impulse Response of the Linear System:

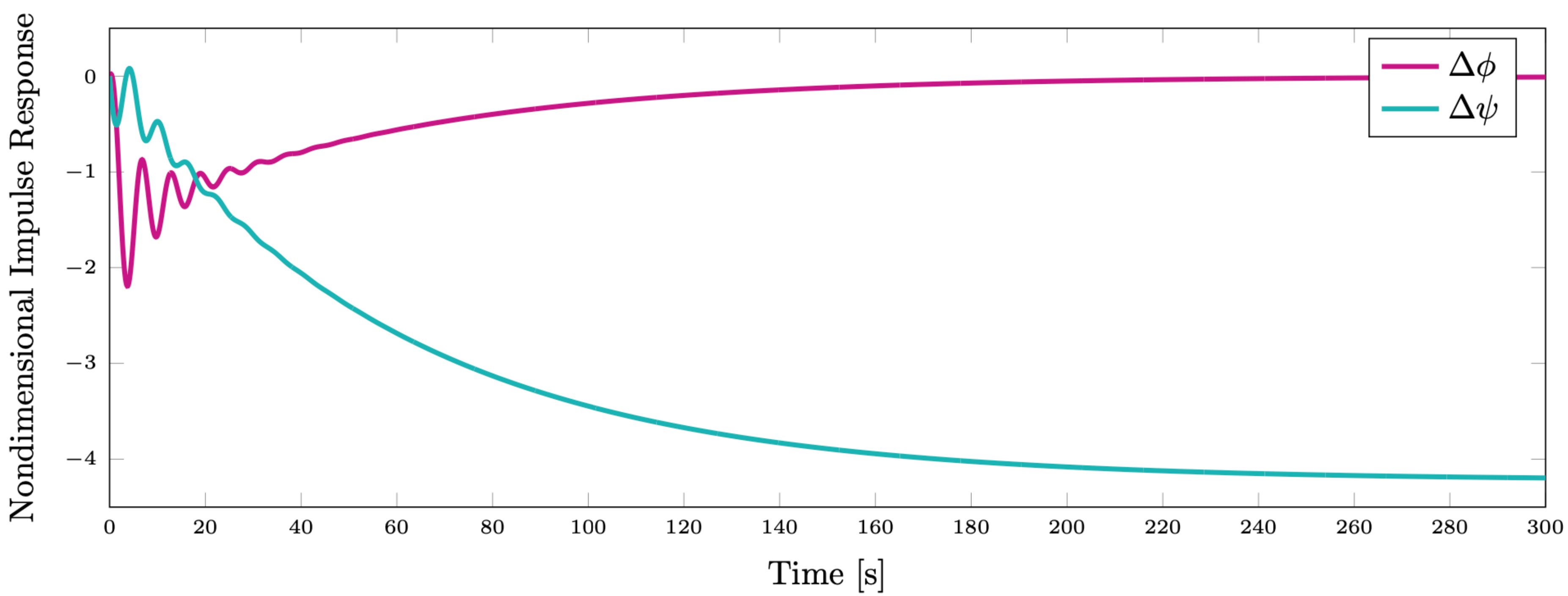




Nondimensional Impulse Response
0
−1
−2
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
Time [s]
$\Delta\phi$
$\Delta\psi$



Nondimensional Impulse Response
Time [s]
$\Delta\beta$
$\Delta pb/2u_1$
$\Delta rb/2u_1$



Nondimensional Impulse Response
Time [s]
$\Delta\phi$
$\Delta\psi$

Dutch-roll motion is primarily described by stability-axis yaw rate and sideslip angle

Oscillating mode

$\lambda = 0$

Roll and spiral motions are primarily described by stability-axis roll rate and roll angle

Roll Approximation

Torque and Moment → application vs result

We start from the equation that contains the Roll torque

$$\Delta\dot{p} - (I_{xz}/I_{xx})\Delta\dot{r} = L_\beta \Delta\beta + L_p \Delta p + L_r \Delta r + L_{\delta_a}\Delta\delta_a + L_{\delta_r}\Delta\delta_r$$

The roll mode occurs at approximately constant (and zero) sideslip and yaw, such that $\Delta r = \Delta\beta = 0$ and $\Delta\dot{r} = \Delta\dot{\beta} = 0$, which gives

$$\Delta\dot{p} = L_p \Delta p + L_{\delta_a}\Delta\delta_a + L_{\delta_r}\Delta\delta_r$$

The primary control action is through the aileron deflection,

$$\Delta\dot{p} = L_p \Delta p + L_{\delta_a}\Delta\delta_a$$

negative

let the control term be set to zero, it can be shown that

$$\Delta p(t) = \Delta p_0 e^{L_p t}, \quad t \geq 0$$

exponential decay

Spiral Approximation

The spiral mode is characterized by changes in the bank angle, ϕ, and the heading angle, ψ. The sideslip angle is usually quite small, but cannot be fully neglected. Typically, the spiral mode is very slow to develop following a disturbance, so it is usually assumed that $\Delta\beta$, Δp, and Δr are quasi-steady relative to the time scale of the mode, hence $\Delta\dot{\beta} = \Delta\dot{p} = \Delta\dot{r} = 0$. Making these substitutions

$$0 = -u_1\Delta r + g\Delta\phi + Y_{\delta_a}\Delta\delta_a \tag{11.24a}$$

$$0 = L_\beta\Delta\beta + L_p\Delta p + L_r\Delta r + L_{\delta_a}\Delta\delta_a \tag{11.24b}$$

$$0 = N_\beta\Delta\beta + N_p\Delta p + N_r\Delta r + N_{\delta_a}\Delta\delta_a \tag{11.24c}$$

$$\Delta\dot{\phi} = \Delta p \tag{11.24d}$$

Note that the yaw angle does not influence the above equations, so the last equation of Eqs. (11.23) has been omitted in Eqs. (11.24). Multiplying Eq. (11.24b) by N_β and Eq. (11.24c) by L_β, subtracting the resulting equations, and solving for Δr gives

$$\Delta r = \frac{L_\beta N_p - N_\beta L_p}{N_\beta L_r - L_\beta N_r}\Delta p + \frac{L_\beta N_{\delta_a} - N_\beta L_{\delta_a}}{N_\beta L_r - L_\beta N_r}\Delta\delta_a \tag{11.25}$$

Substituting Eq. (11.25) into Eq. (11.24a) and clearing the fractions yields

$$0 = -u_1(L_\beta N_p - N_\beta L_p)\Delta p + g(N_\beta L_r - L_\beta N_r)\Delta\phi + (Y_{\delta_a} - u_1(L_\beta N_{\delta_a} - N_\beta L_{\delta_a}))\Delta\delta_a$$

Finally, from Eq. (11.24d), $\Delta\dot{\phi} = \Delta p$, such that

$$0 = -u_1(L_\beta N_p - N_\beta L_p)\Delta\dot{\phi} + g(N_\beta L_r - L_\beta N_r)\Delta\phi + (Y_{\delta_a} - u_1(L_\beta N_{\delta_a} - N_\beta L_{\delta_a}))\Delta\delta_a$$

or

$$u_1(L_\beta N_p - N_\beta L_p)\Delta\dot{\phi} + g(L_\beta N_r - N_\beta L_r)\Delta\phi = (Y_{\delta_a} - u_1(L_\beta N_{\delta_a} - N_\beta L_{\delta_a}))\Delta\delta_a$$

In the presence of no control action

$$\Delta\dot{\phi} + \frac{g(L_\beta N_r - N_\beta L_r)}{u_1(L_\beta N_p - N_\beta L_p)}\Delta\phi = 0$$

Applying this solution to the zero-input governing equation for the spiral approximation yields

$$\Delta\phi(t) = \Delta\phi_0 e^{-(g/u_1)[(L_\beta N_r - N_\beta L_r)/(L_\beta N_p - N_\beta L_p)]t}, \quad t \geq 0$$

It is therefore clear that $\Delta\phi(t) \to \infty$ as $t \to \infty$ if

$$\frac{g(L_\beta N_r - N_\beta L_r)}{u_1(L_\beta N_p - N_\beta L_p)} < 0$$

In the case that this term is identically zero, the roll angle remains fixed at its initial value, but the response does not decay to zero. Thus, to obtain a stable spiral model, we require

$$\frac{g(L_\beta N_r - N_\beta L_r)}{u_1(L_\beta N_p - N_\beta L_p)} > 0 \tag{11.27}$$

Typically, $(L_\beta N_p - N_\beta L_p) > 0$, which means that we get the classical requirement that

$$L_\beta N_r > N_\beta L_r$$

Spiral Approximation

- The dihedral effect L_β is negative for static roll stability
- The directional stability N_β is positive for static yaw stability
- The yaw rate damping N_r is usually negative (if a positive yaw rate caused a torque that increased the yaw torque and hence further increased the yaw rate we would be in trouble)
- The roll moment L_r due to yaw rate is generally positive.

For the spiral mode to be stable, (for negative L_β) the spiral mode approximation requires:

$$L_\beta N_r > N_\beta L_r$$

Want roll stab better than yaw stability

- We need static yaw stability and static roll stability
- Remember, yaw static stability puts the nose towards relative air velocity
- If we have "too" much yaw static stability but not enough roll static stability, the roll angle not being compensated will cause sideslip to increase and the yaw will follow sideslip causing the airplane to fly in a tighter and tighter spiral

Dutch Roll Approximation

In the prior two modes we focused on the roll angle and roll rate. We study the Dutch Roll approximation by focusing the other two: sideslip angle and yaw rate. Neglecting roll in Dutch roll is clearly contradictory, but it is based on the fa that the mode is first a yawing oscillation and aerodynamic coupling causes rolling motion as a secondary effect.
It is generally true that, for most aircraft, the roll to yaw ratio in Dutch rolling motion is less that one; in some cases, it may be much less than one, lending the assumption some small credibility. Given that the Dutch roll mode consists o primarily sideslipping and yawing motions, the approximation is:

$$u_1 \Delta\dot{\beta} = Y_\beta \Delta\beta + Y_p \Delta p + (Y_r - u_1)\Delta r + g\cos\theta_1 \Delta\phi + Y_{\delta_a}\Delta\delta_a + Y_{\delta_r}\Delta\delta_r$$
$$\Delta\dot{r} - (I_{xz}/I_{zz})\Delta\dot{p} = \left(N_\beta + N_{T_\beta}\right)\Delta\beta + N_p \Delta p + \left(N_r + N_{T_r}\right)\Delta r + N_{\delta_a}\Delta\delta_a + N_{\delta_r}\Delta\delta_r$$

From the assumption that no rolling motion is involved, setting Δp, $\Delta\dot{p}$, $\Delta\phi$, and $\Delta\dot{\phi}$ to zero leads to

$$u_1 \Delta\dot{\beta} = Y_\beta \Delta\beta + (Y_r - u_1)\Delta r + Y_{\delta_a}\Delta\delta_a + Y_{\delta_r}\Delta\delta_r$$
$$\Delta\dot{r} = \left(N_\beta + N_{T_\beta}\right)\Delta\beta + \left(N_r + N_{T_r}\right)\Delta r + N_{\delta_a}\Delta\delta_a + N_{\delta_r}\Delta\delta_r$$

Finally, to arrive at the Dutch roll approximation, neglect the propulsive effects and note that the primary control action occurs through the rudder deflection, such that

$$u_1 \Delta\dot{\beta} = Y_\beta \Delta\beta + (Y_r - u_1)\Delta r + Y_{\delta_r}\Delta\delta_r$$
$$\Delta\dot{r} = N_\beta \Delta\beta + N_r \Delta r + N_{\delta_r}\Delta\delta_r$$

or, in standard matrix form

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} Y_\beta/u_1 & Y_r/u_1 - 1 \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} Y_{\delta_r}/u_1 \\ N_{\delta_r} \end{bmatrix} \Delta\delta_r$$

$$\begin{vmatrix} Y_\beta/u_1 - \lambda & Y_r/u_1 - 1 \\ N_\beta & N_r - \lambda \end{vmatrix} = 0$$

or

$$(Y_\beta/u_1 - \lambda)(N_r - \lambda) - (Y_r/u_1 - 1)N_\beta = 0$$

Reducing this to a standard degree two polynomial in λ gives the characteristic equation

$$\lambda^2 - \frac{Y_\beta + N_r u_1}{u_1}\lambda + \frac{N_r Y_\beta - N_\beta Y_r + N_\beta u_1}{u_1} = 0$$

$$\omega_n = \sqrt{\frac{N_r Y_\beta - N_\beta Y_r + N_\beta u_1}{u_1}} \qquad \zeta = -\frac{1}{2\omega_n}\frac{Y_\beta + N_r u_1}{u_1}$$

Dutch Roll Approximation

- The approximated damping ratio is $\zeta = -\frac{1}{2\omega_n}\frac{Y_\beta + N_r u_1}{u_1}$

- The yaw rate damping Nr is usually negative
- Yb is negative (sideslip is the negative of the "a" seen by the fin)

- If you want to damp the oscillations faster… You can increase the damping ratio by increasing the vertical fin area.
- If you increase the vertical fin area… You will also increase ~~Nb~~ $N\beta$ and hurt the spiral mode
- For most transport and fighter aircraft, Nr is usually increased using an autopilot, the so-called Yaw Damper

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USEFUL VIDEOS

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Useful Videos

- https://www.youtube.com/watch?v=rFWfrmjAQxY

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HANDLING QUALITIES

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MIL-F-8785C Aircraft Types

I. Small, light airplanes, e.g., utility aircraft and primary trainers
II. Medium-weight, low-to-medium maneuverability airplanes, e.g., small transports or tactical bombers
III. Large, heavy, low-to-medium maneuverability airplanes, e.g., heavy transports, tankers, or bombers
IV. Highly maneuverable aircraft, e.g., fighter and attack airplanes

MIL-F-8785C Flight Phase

A. **Non-terminal flight requiring rapid maneuvering precise tracking, or precise flight path control**
   - air-to-air combat
   - ground attack
   - in-flight refueling (receiver)
   - close reconnaissance
   - terrain following
   - close formation flying

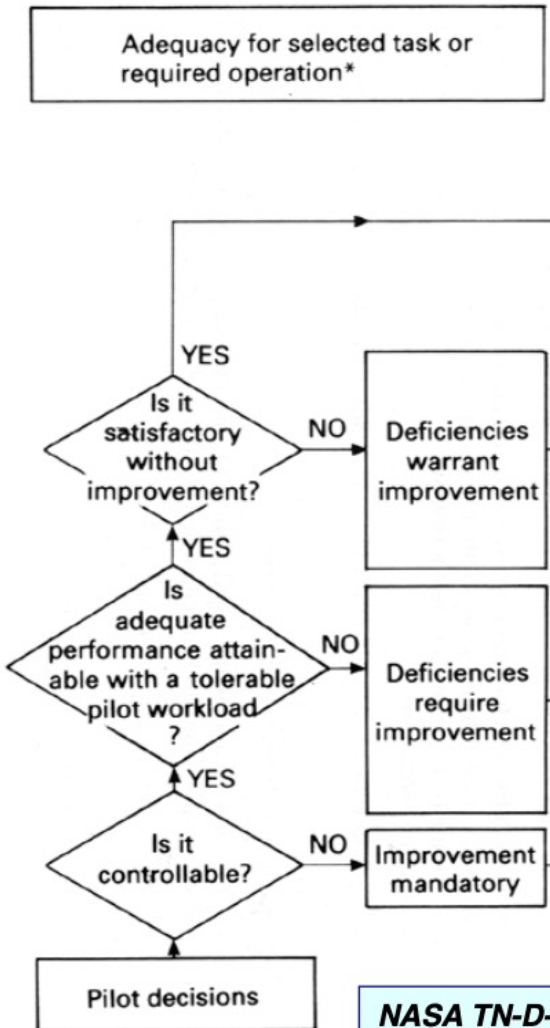
B. **Non-terminal flight requiring gradual maneuvering**
   - climb, cruise
   - in-flight refueling (tanker)
   - descent

C. **Terminal flight**
   - takeoff (normal and catapult)
   - approach
   - wave-off/go-around
   - landing

MIL-F-8785C Levels of Performance

1. Flying qualities clearly adequate for the mission flight phase
2. Flying qualities adequate to accomplish the mission flight phase, with some increase in pilot workload or degradation of mission effectiveness
3. Flying qualities such that the aircraft can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate

Cooper-Harper Handling Qualities Rating Scale



Aircraft characteristics	Demands on the pilot in selected task or required operation*	Pilot rating
Excellent Highly desirable	Pilot compensation not a factor for desired performance	1
Good Negligible deficiencies	Pilot compensation not a factor for desired performance	2
Fair – some mildly unpleasant deficiencies	Minimal pilot compensation required for desired performance	3
Minor but annoying deficiencies	Desired performance requires moderate pilot compensation	4
Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	5
Very objectionable but tolerable deficiencies	Adequate performance requires extensive pilot compensation	6
Major deficiencies	Adequate performance not attainable with maximum tolerable pilot compensation. Controllability not in question	7
Major deficiencies	Considerable pilot compensation is required for control	8
Major deficiencies	Intense pilot compensation is required to retain control	9
Major deficiencies	Control will be lost during some portion of required operation	10

*Definition of required operation involves designation of flight phase and/or subphases with accompanying conditions

NASA TN-D-5153,1969

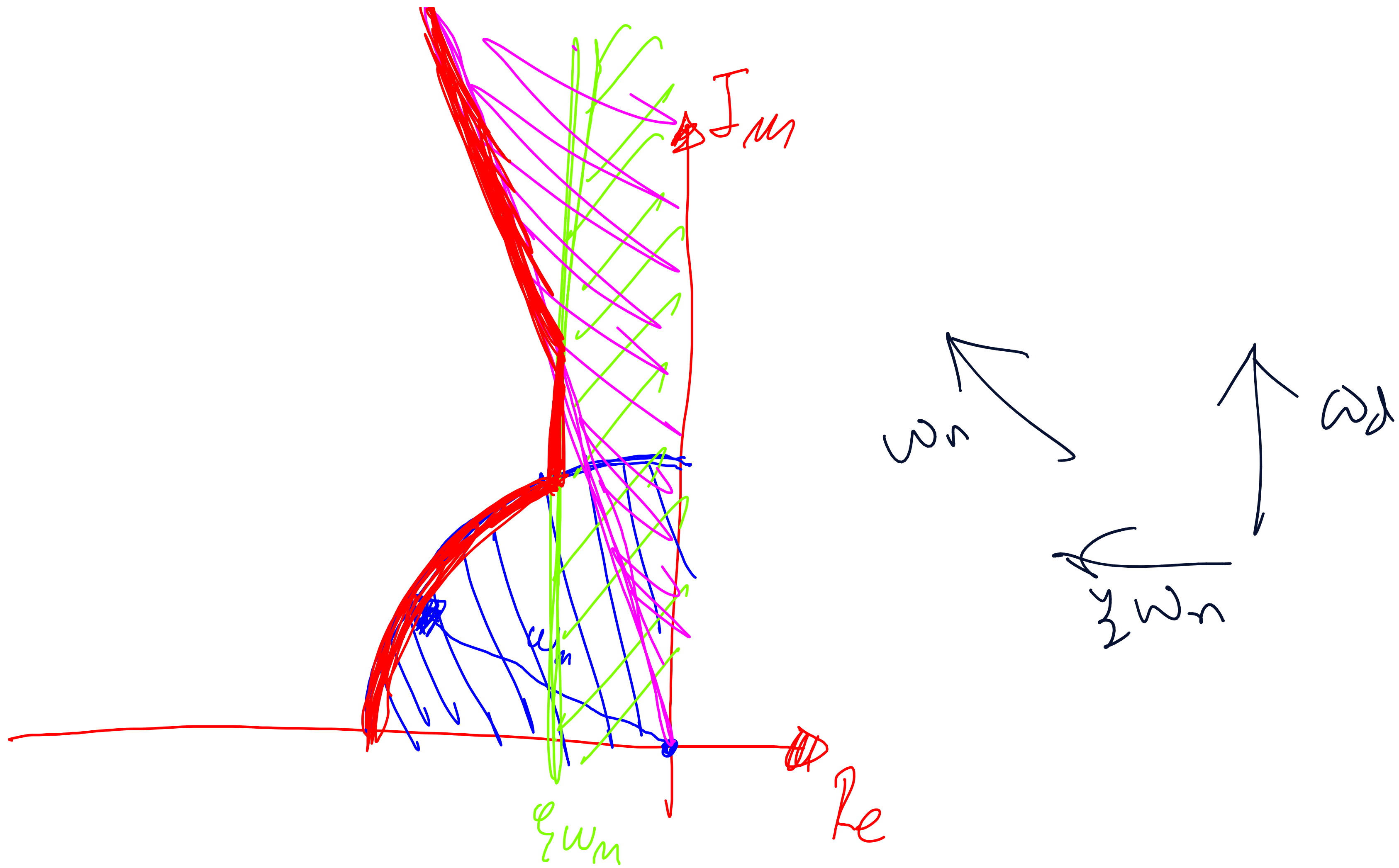
Dutch Roll Flying Qualities

Level	Category	Class	Min ζ*	Min $\omega_n\zeta$* Rad/s	Min ω_n Rad/s
1	A	I, IV	0.19	0.35	1.0
		II, III	0.19	0.35	0.4
1	B	All	0.08	0.15	0.4
1	C	I, II-C, IV	0.08	0.15	1.0
		II-L, III	0.08	0.15	0.4
2	All	All	0.02	0.05	0.4
3	All	All	0.02	-	0.4

* The requirement on ζ is the larger of the two

+ -C and –L denote carrier-based and land-based aircrafts

Dutch Roll Flying Qualities



Example Problem

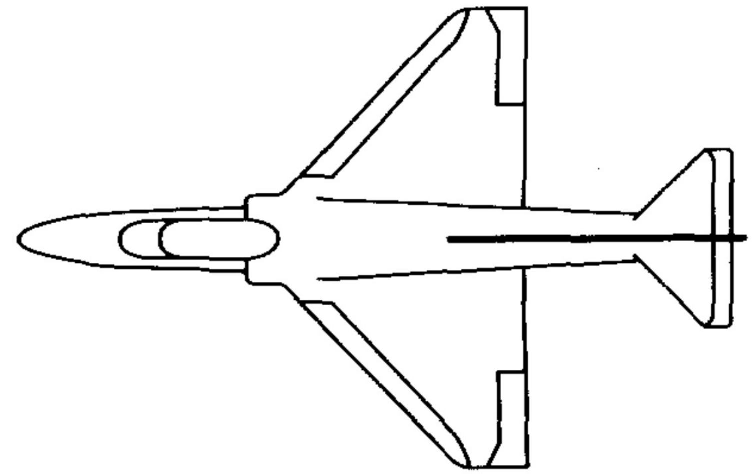
Use the Dutch roll approximation to analyze the handling qualities of the following fighter aircraft flying at M=0.8 at an altitude of 35,000 ft.

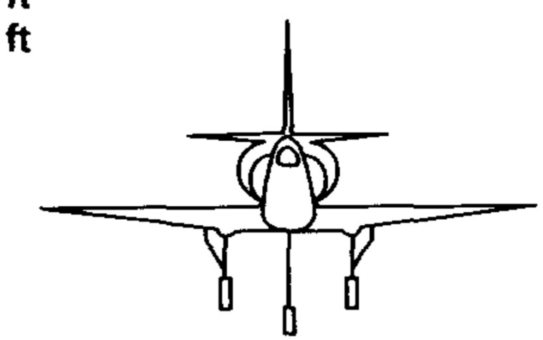
Center of gravity and mass characteristics

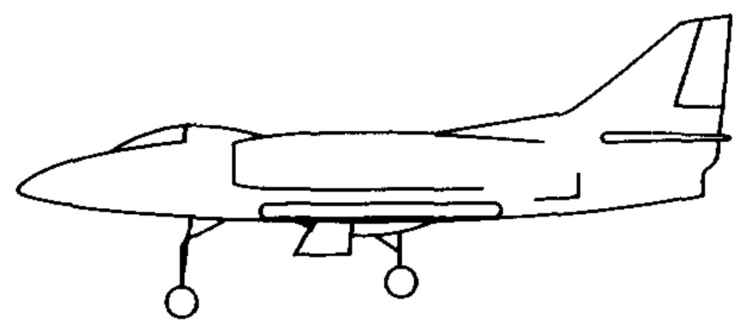
$W = 17{,}578$ lb
CG at 25% MAC
$I_x = 8090$ Slug·ft²
$I_y = 25{,}900$ Slug·ft²
$I_z = 29{,}200$ Slug·ft²
$I_{xz} = 1300$ Slug·ft²

Reference geometry

$S = 260$ ft²
$b = 27.5$ ft
$\bar{c} = 10.8$ ft







$$\begin{bmatrix}\Delta\dot{\beta}\\ \Delta\dot{r}\end{bmatrix}=\begin{bmatrix}Y_\beta/u_1 & Y_r/u_1-1\\ N_\beta & N_r\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta r\end{bmatrix}+\begin{bmatrix}Y_{\delta_r}/u_1\\ N_{\delta_r}\end{bmatrix}\Delta\delta_r$$

Longitudinal	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
M = 0.4 Sea level	0.28	0.03	3.45	0.30	−0.38	0.72	−1.1	0.0	−3.6	0.0	0.0	0.0	0.36	−0.50
M = 0.8 35,000 ft	0.30	0.038	4.0	0.56	−0.41	1.12	−1.65	0.0	−4.3	0.15	0.03	−0.05	0.4	−0.60

Lateral	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
M = 0.4 Sea level	−0.98	−0.12	0.25	−0.26	0.022	0.14	−0.35	0.08	0.06	0.17	−0.105	0.032
M = 0.8 35,000 ft	−1.04	−0.14	0.27	−0.24	0.029	0.17	−0.39	0.072	0.04	0.17	−0.105	0.032

At 35,000 ft, $a = 973.1$ ft/s

$\rho = 7.382 \times 10^{-4}$ slugs/ft^3

$u_1 = Ma$, $\bar{q} = \frac{1}{2}\rho u_1^2$, $g = 32.2$ ft/s^2

$m = \dfrac{W}{g} = 545.9$ slugs

$$Y_\beta = \frac{\bar{q}S}{m} C_{y_\beta} = -110.8 \ ft/s^2$$

$$Y_r = \frac{\bar{q}Sb}{2mu_1} C_{y_r} = 0$$

$$N_\beta = \frac{\bar{q}Sb}{I_{zz}} C_{n_\beta} = 14.79 \ s^{-1}$$

$$N_r = \frac{\bar{q}Sb^2}{2I_{zz}u_1} C_{n_r} = -0.3773 \ s^{-1}$$

$$\Rightarrow \quad \omega_n = 3.85 \text{ rad/sec}$$

$$\zeta\omega_n = 0.260 \text{ rad/sec}$$

$$\zeta = 0.0674$$

Dutch Roll Flying Qualities

Level	Category	Class	Min ζ*	Min $\omega_n\zeta$* Rad/s	Min ω_n Rad/s
1	A	I, IV	0.19	0.35	1.0
		II, III	0.19	0.35	0.4
1	B	All	0.08	0.15	0.4
1	C	I, II-C, IV	0.08	0.15	1.0
		II-L, III	0.08	0.15	0.4
2	All	All	0.02	0.05	0.4
3	All	All	0.02	-	0.4

* The requirement on ζ is the larger of the two
+ -C and –L denote carrier-based and land-based aircrafts

The University of Texas at Austin
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YAW DAMPER

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Controlling Pure Yaw Motion

$$\left.\begin{array}{l}\text{No Rolling}\\ \text{No Pitching}\end{array}\right\} \Rightarrow \quad \sum \text{Yawing torques} = I_{zz}\,\ddot{\psi}$$

$$N = I_{zz}\,\ddot{\psi}$$

Linearizing $\quad \Delta N = I_{zz}\,\Delta\ddot{\psi}$

$$\Delta N = \frac{\partial N}{\partial \beta}\Delta\beta + \cancelto{\approx 0}{\frac{\partial N}{\partial \dot{\beta}}}\Delta\dot{\beta} + \frac{\partial N}{\partial r}\Delta r + \cancelto{\approx 0}{\frac{\partial N}{\partial p}}\Delta p$$

$$+ \frac{\partial N}{\partial \delta_r}\Delta\delta_r + \cancelto{\approx 0}{\frac{\partial N}{\partial \delta_a}}\Delta\delta_a$$

Controlling Pure Yaw Motion

$$\Delta\psi = -\Delta\beta \qquad \Delta\dot{\psi} = -\Delta\dot{\beta} = \Delta r$$

$$\Delta\ddot{\psi} = \frac{\Delta N}{I_{zz}} \Rightarrow \underbrace{\Delta\ddot{\psi}}_{rad/s^2} - N_r \underbrace{\Delta\dot{\psi}}_{rad/s} + N_\beta \Delta\psi = N_{\delta_r} \Delta\delta_r$$

$$\Rightarrow \lambda^2 - N_r\lambda + N_\beta = 0 \quad \text{or} \quad \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 = 0$$

$$\omega_n = \sqrt{N_\beta}$$

$$\zeta = \frac{-N_r}{2\sqrt{N_\beta}}$$

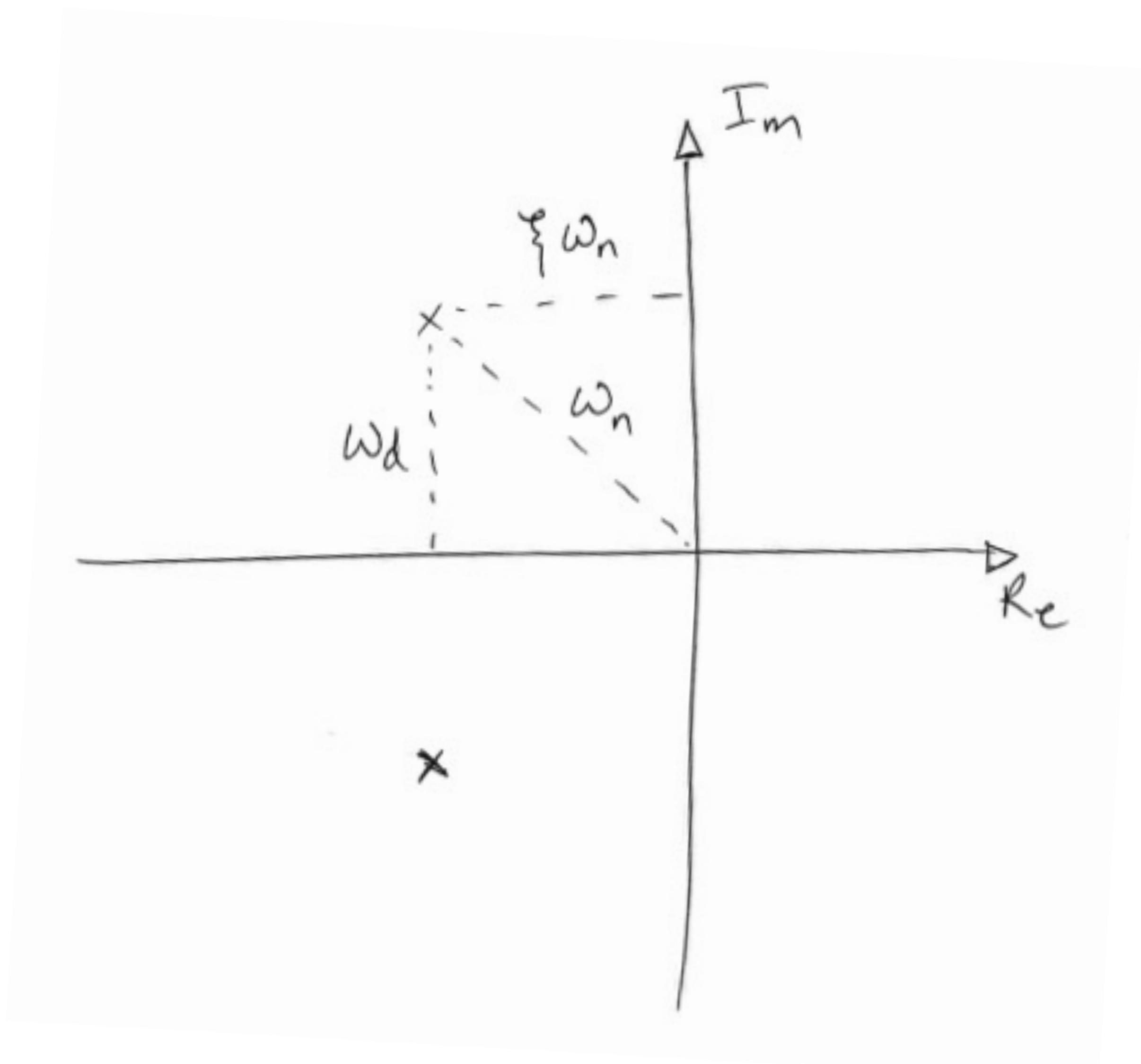
Controlling Pure Yaw Motion

Now let's put in some controls.... a PD controller where

$$\Delta\delta_r = -K_d\,\Delta\dot{\psi} - K_p\,\Delta\psi$$

$$\Rightarrow\ \Delta\ddot{\psi} - \underbrace{\left(N_r - K_d N_{\delta_r}\right)}_{\substack{\text{control damping}\\ \text{ratio with } K_d\\ \text{(given a value}\\ \text{of } K_p\text{)}}}\Delta\dot{\psi} + \underbrace{\left(N_\beta + K_p N_{\delta_r}\right)}_{\substack{\text{control}\\ \text{natural}\\ \text{frequency}\\ \text{with } K_p}}\Delta\psi = 0$$

Controlling Pure Yaw Motion



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