THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Aerospace Engineering and Engineering Mechanics

ASE 367K FLIGHT DYNAMICS
Fall 2024

HOMEWORK 4
Due: 2024-09-27 at 11:59pm via Canvas

Problem 1

Answer the following questions related to yaw stability and control:

a. Determine the side-wash angle and the maximum deflection angle of the rudder of the Boeing 747-200 if the aircraft is designed to land (at any weight) in crosswind of up to 40 knots and the area of the rudder is 30% of the area of the vertical stabilizer.

b. Determine the sizes of the rudder and the vertical stabilizer assuming this is the limiting operating condition for the rudder.

N.B. You may assume that the minimum landing speed of the 747-200 is 120 knots; the derivative coefficients C_{n_β} and $C_{n_{\delta_r}}$ that are provided in the table below for M = 0.25 at sea level are valid for all low-speed, near sea-level operations; $C_{n_{\beta_f}}$ and C_{n_β} are approximately equal; l_f is approximately 150 ft.; $C_{L_{\alpha_f}}$ is approximately 1 per radian; the wing sweep angle is 35 degrees; $\frac{S_f}{S} = 0.156$; $z_w/d = 0.5$; and $AR_w = 7.5$.

Transport aircraft: Boeing 747

Longitudinal	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
M = 0.25 Sea level	1.11	0.102	5.70	0.66	−1.26	6.7	−3.2	5.4	−20.8	−0.81	0.0	0.27	0.338	−1.34
M = 0.90 40,000 ft	0.5	0.042	5.5	0.47	−1.6	0.006	−9.0	6.58	−25.0	0.2	0.25	−0.10	0.3	−1.2

Lateral	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
M = 0.25 Sea level	−0.96	−0.221	0.150	−0.45	−0.121	0.101	−0.30	0.0461	0.0064	0.175	0.007	−0.109
M = 0.90 40,000 ft	−0.85	−0.10	0.20	−0.30	0.20	0.20	−0.325	0.014	0.003	0.075	0.005	−0.09

Note: All derivatives are per radian.

You may also assume that $\eta_f\left(1+\frac{\partial\sigma}{\partial\beta}\right)$, the combined effects of the tail efficiency and sidewash in the expression $C_{n_{\beta_f}} = V_V \eta_f C_{L_{\alpha_f}}\left(1+\frac{\partial\sigma}{\partial\beta}\right)$ can be estimated using:

$$\eta_f\left(1+\frac{\partial\sigma}{\partial\beta}\right) = 0.724 + 3.06\frac{S_f/S}{1+\cos\Lambda_{c_w/4}} + 0.4\frac{z_w}{d} + 0.009AR_w$$

Problem 2

Answer the following questions related to roll stability and control:

a. Which of the following two statements is true?

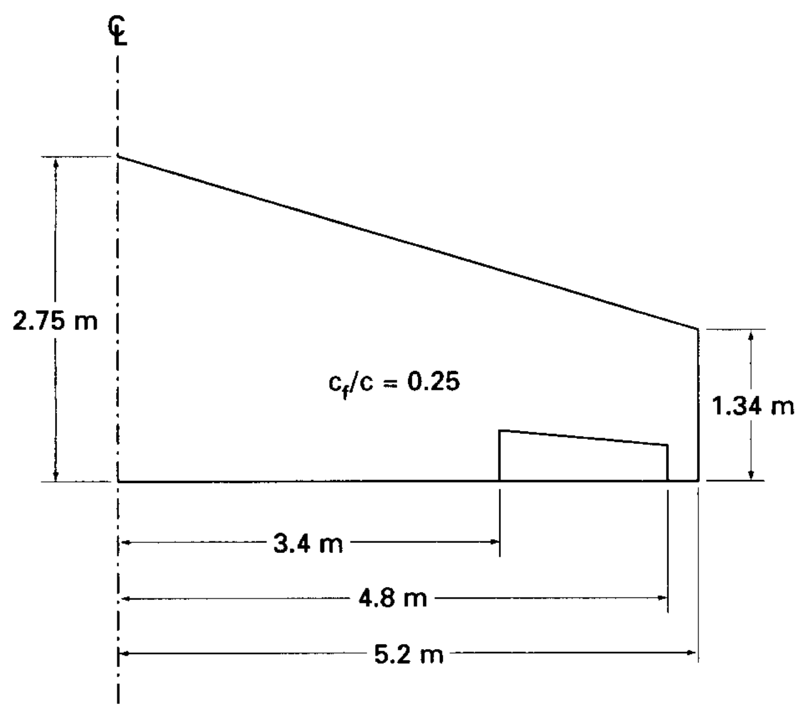
   i. When an aircraft is perturbed such that one wing is lowered relative to the other, dihedral causes the lower wing to increase its surface area relative to the airflow, thus increasing its lift. This acts to oppose the original roll motion.

   ii. When a disturbance causes an aircraft to roll away from its nominal straight and level position, the aircraft will sideslip in the direction of the wing that is going down. This creates an airflow component along the length of the wing from tip to root. The dihedral angle can be seen as presenting a positive angle of attack to this lateral flow, hence generating some additional lift. It is this lift, which restores the aircraft to its normal attitude

b. Explain using appropriate figures how wing sweep contributes to roll stability.

c. Derive an approximate expression (as a function of wing sweep angle) for the contribution of wing sweep to roll stability.

d. Suppose the wing planform below is incorporated into a low-wing aircraft design. Determine the wing dihedral angle necessary to produce a dihedral effect of $C_{l_\beta} = -0.1$ rad^{-1}. Neglect the fuselage interference on the wing dihedral contribution.



Problem 3

Starting with the equation for the thrust required T_R in level flight (as a function of mass, altitude, and true airspeed):

a. Derive (separately for jet and propeller aircraft) an expression for the instantaneous speed and to achieve maximum endurance.

b. Derive (separately for jet and propeller aircraft) an expression for the instantaneous speed to achieve maximum range.

c. Explain why the pitch of the aircraft must be reduced as a function of time, i.e., as fuel is consumed, to maintain a constant altitude and Mach number during cruise.

d. Explain why a cruise climb at a constant Mach number is better (from a fuel consumed per distance traveled) than cruising at a constant altitude and Mach number.

① a) θ , δ_r max?

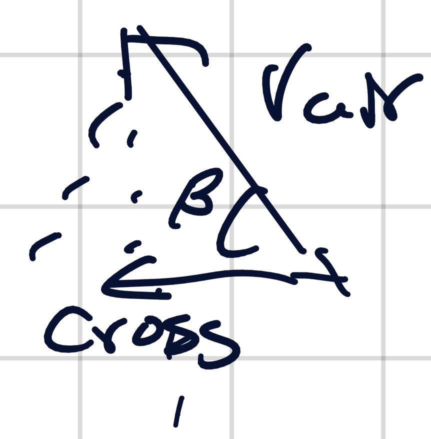
Given: $V_{crosswind} = 40$ knots ← maintain this

$\delta_r = 0.3\,\delta_f$

$V_{airspeed} = 120$ knots

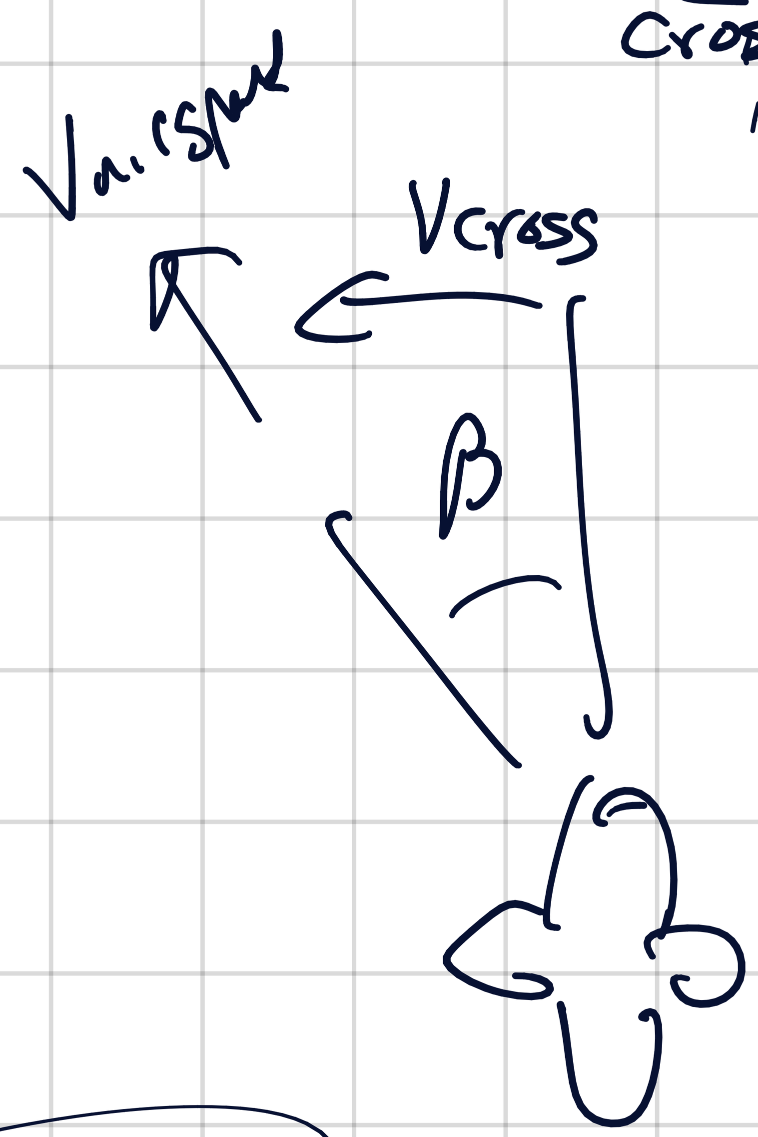
$$C_n = C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r$$

$$\beta = \sin^{-1}\left(\frac{V_{crosswind}}{V_{airspeed}}\right)$$



$C_n = 0$

$$\delta_{r,max} = \left| \frac{-C_{n_\beta}\beta}{C_{n_{\delta_r}}} \right|$$



$$\beta = \sin^{-1}\left(\frac{40}{120}\right) = 19.47°$$

$$\delta_{r,max} = \left| \frac{-0.15(0.3398)}{-0.109} \right| = 0.4676 = 26.79°$$

$$\eta_f\left(1+\frac{\partial\sigma}{\partial\beta}\right) = 0.724 + 3.06\,\frac{0.156}{1+\cos(35°)} + 0.4(0.5) + 0.009(7.5) = 1.2539$$

$\eta_f = 1$

$$1\left(1+\frac{\partial\sigma}{\partial\beta}\right) = 1.2539$$

$$\frac{\partial\sigma}{\partial\beta} = 0.25$$

$$\sigma = \frac{\partial\sigma}{\partial\beta}\beta = 0.25\left(\frac{19.47}{360}2\pi\right) = 0.08495 = 4.8675°$$

b) Find S_r and S_f

Given: $C_{n_{\beta_f}} \approx C_{n_\beta} = 0.150 \text{ rad}^{-1}$

$C_{L_{\alpha_f}} = 3 \text{ rad}^{-1}$

$l_f = 150 \text{ ft}$ $\quad \Lambda = 35°$

$\frac{S_f}{S} = 0.156$ $\quad \frac{z_w}{d} = 0.5$

$AR_w = 7.5$

$S = 5500 \text{ ft}^2$

$S_f = 5500(0.156) = 858 \text{ ft}^2$ Vertical stabilizer

$S_r = 858(0.3) = 257.4 \text{ ft}^2$ Rudder

if $\frac{S}{S_f}$ was not given...

$$C_{n_{\beta_f}} = V_v \eta_f C_{L_{\alpha_f}} \left(1 + \frac{d\sigma}{d\beta}\right)$$

$b = 195.56$

$$V_v = \frac{S_f l_f}{S b} \qquad \frac{S_f}{S} = x$$

$$= x \frac{150}{195.56} = 0.767x$$

$C_{L_{\alpha_f}} = 1$

$$\eta_f \left(1 + \frac{d\sigma}{d\beta}\right) = 0.724 + 3.06 \frac{x}{1 + \cos\Lambda} + 0.4 \frac{z_w}{d} + 0.009 AR_w$$

$$= 0.9915 + 1.6821x$$

$$C_{n_{\beta_f}} = 0.767x(0.9915 + 1.6821x)(1)$$

$$0.15 = 0.7605x + 1.2902x^2$$

$$x = -0.7454 \text{ or } 0.1560$$

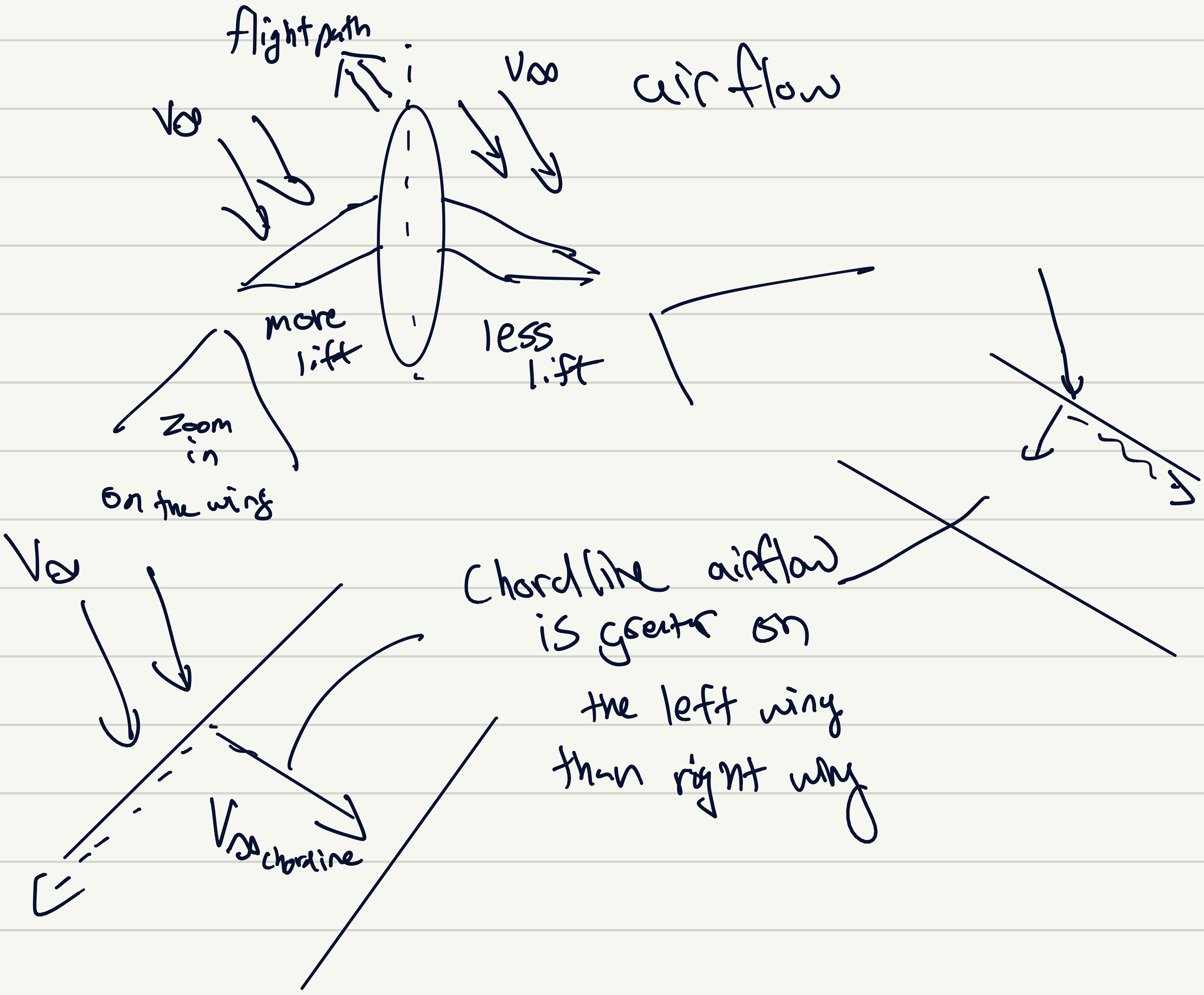
$\frac{S_f}{S} = 0.1560$

given $S = 5500 \Rightarrow S_f = 858 \text{ ft}^2$

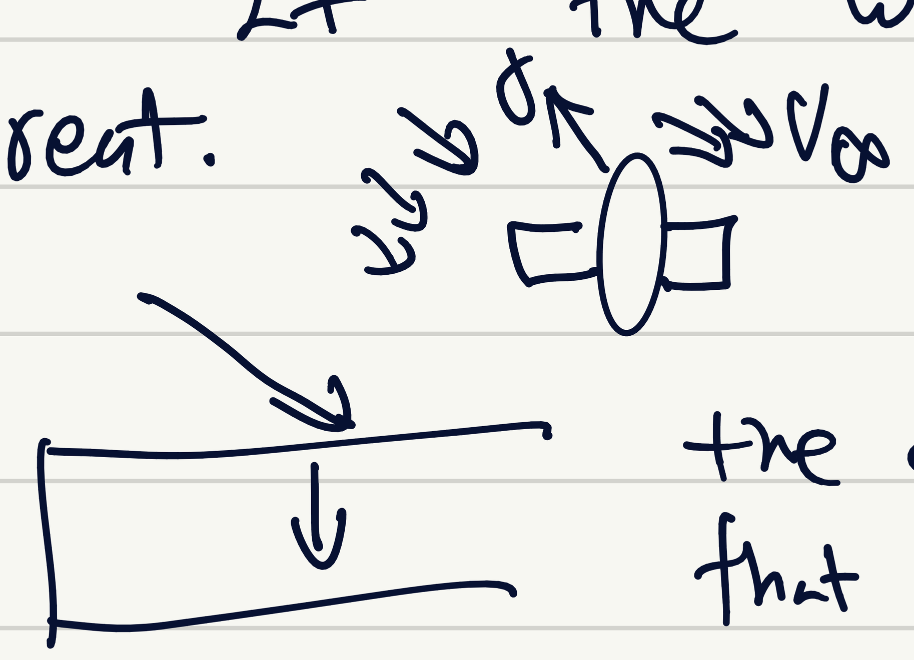
$S_r = 257.4 \text{ ft}^2$

② a) ii is true

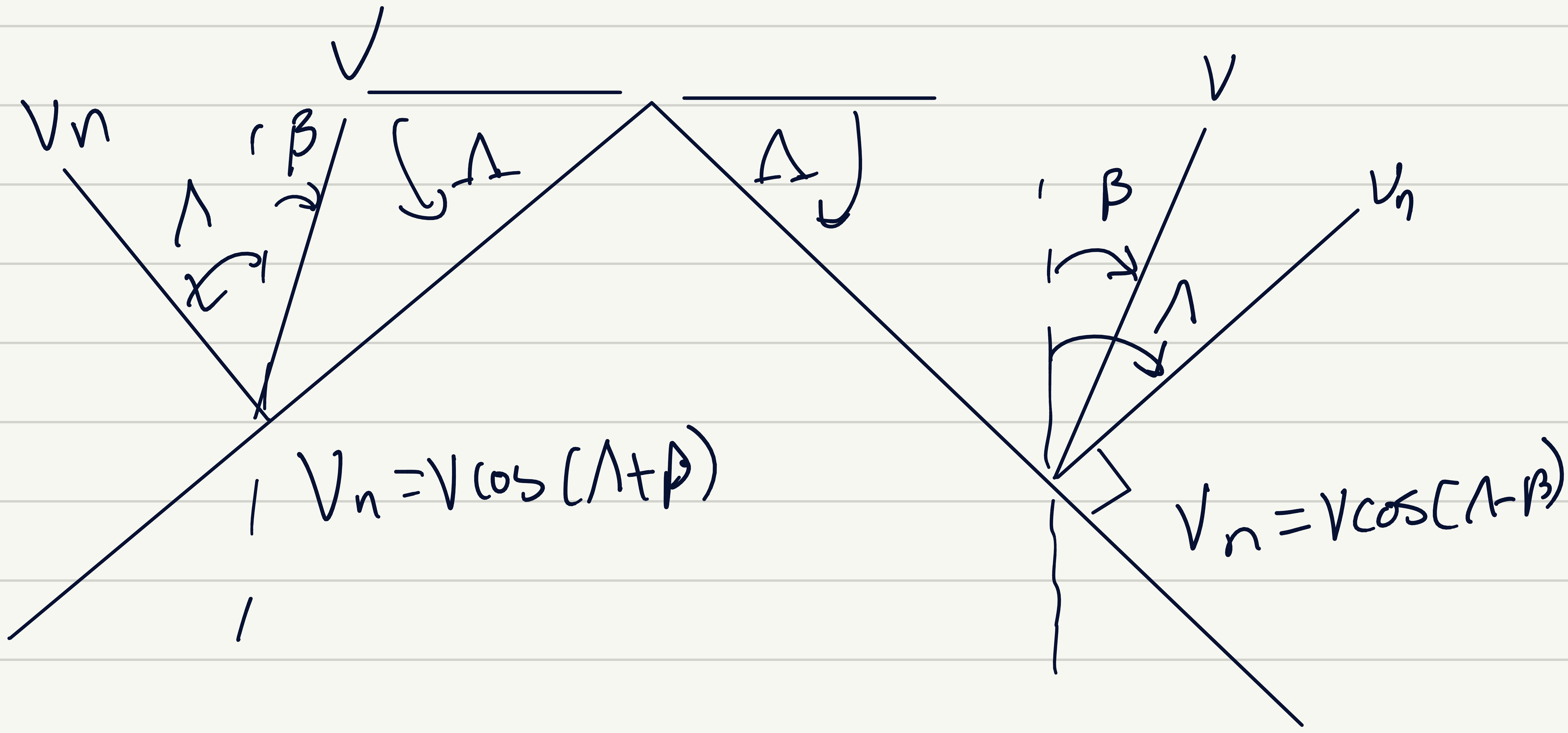
b) When an aircraft rolls, it sideslips to. The sideslip causes airflows that are angled against the aircraft's longitudinal line. The wing sweep helps the vehicle's roll stability by exposing the wing towards the sidslip direction to a greater chordline airflow than the one further away. This relatively greater chordline airflow on the wing that is close to the sideslip direction causes a greater lift to be generated on itself than on the the further wing wing, so the airplane rolls back to its original position.



If the wing was rectangular, this effect isn't as great.

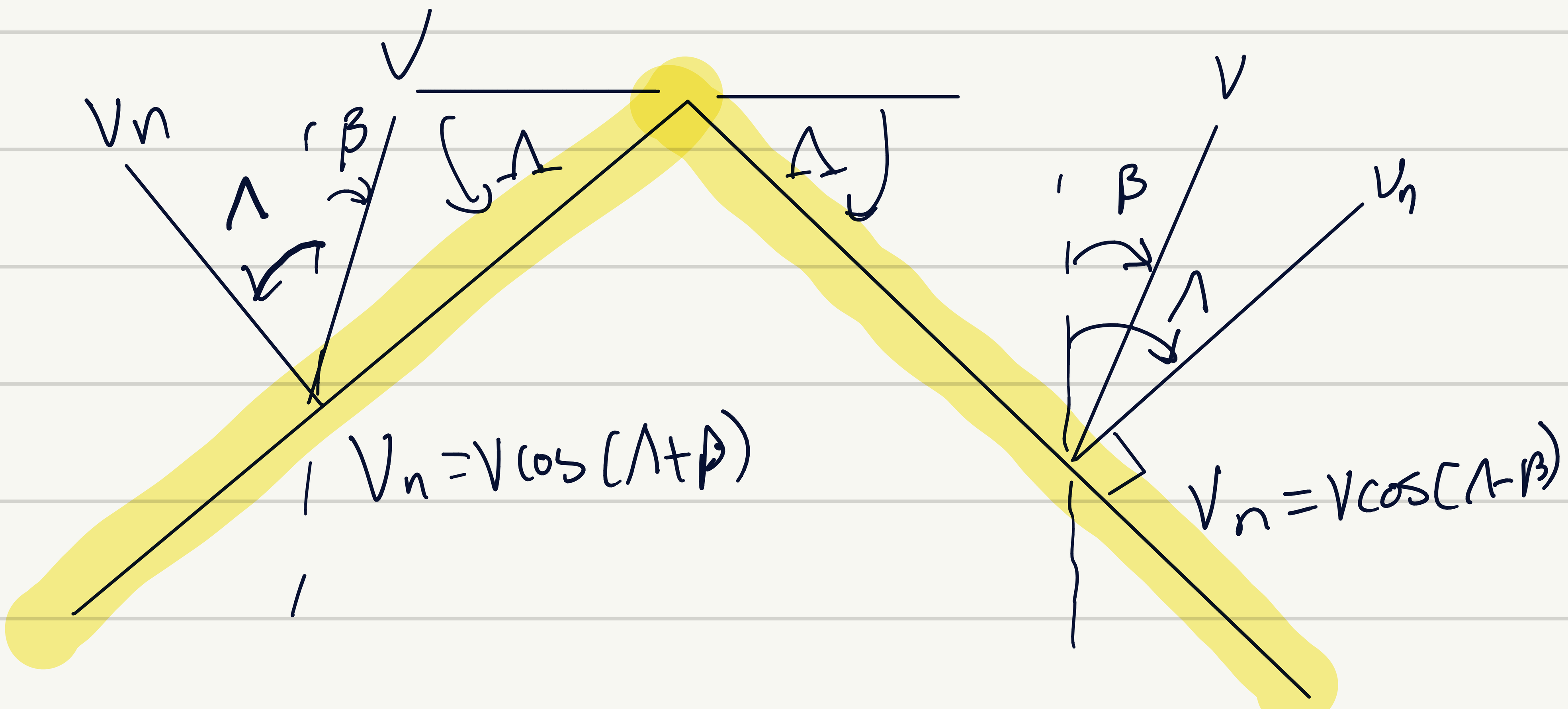


the chordline airflow is smaller than that of swept wing.



One wing will always have a greater V_n; thus more lift on one wing than the other and cause a moment to roll back to stability. Wing sweep aids stability

C) Roll stability $C_{\ell_\beta} < 0$. C_L not affected by β



$V_{n_L} = V\cos(\Lambda+\beta)$ $\quad$ $V_{n_R} = V\cos(\Lambda-\beta)$

$\text{Lift}_L = q\frac{S}{2} C_L \cos^2(\Lambda+\beta)$ $\quad$ $\text{Lift}_R = q\frac{S}{2} C_L \cos^2(\Lambda-\beta)$

Moment $L = -y_{acw}\, q\frac{S}{2} C_L \left[\cos^2(\Lambda-\beta) - \cos^2(\Lambda+\beta)\right]$

$$\frac{1}{2}\left[1+\cos(2\Lambda-2\beta) - 1 - \cos(2\Lambda+2\beta)\right]$$

$$= \frac{1}{2}\left[\cos(2\Lambda-2\beta) - \cos(2\Lambda+2\beta)\right]$$

$$= \frac{2}{2}\left[-\sin\left(\frac{2\Lambda-2\beta+2\Lambda+2\beta}{2}\right)\sin\left(\frac{2\Lambda-2\beta-2\Lambda-2\beta}{2}\right)\right.$$

$$= -\sin(2\Lambda)\sin(-2\beta)$$

$$L = -y_{acw}\, q \frac{S}{2} C_L \left[\sin(2\Lambda)\, 2\beta \right]$$
$$= -y_{acw}\, q S\, C_L \left[\sin(2\Lambda) \right] \beta$$

$$\frac{L}{qSb} = C_l = \frac{-y_{acw}\, qS\, C_L}{qSb} \left[\sin(2\Lambda) \right] \beta$$

$$C_{l_\beta} = \frac{-y_{acw}}{b} C_L \left[\sin 2\Lambda \right] \beta$$, since $C_l \propto C_{l_\beta}$

also $C_{l_\beta} < 0$ when $\Lambda > 0$.

d) $$C_{l_\beta} = -0.1$$

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta} = -\frac{y_{acw}}{b} C_{L_{\alpha w}} \Gamma$$

$$b = 5.2 \times 2 = 10.4 \text{ m}$$

$$y_{acw} = \frac{b}{6} \frac{1+2\lambda}{1+\lambda}$$

$$\lambda = \frac{C_t}{C_r} = \frac{1.34}{2.75} = 0.4873$$

$$y_{acw} = \frac{10.4}{6} \frac{1+2(0.4873)}{1+0.4873}$$
$$= 2.3012 \text{m}$$

$$-0.1 = \frac{-2.3012}{10.4} C_{L\alpha} \Gamma$$

$$\frac{0.4519}{C_{L\alpha}} = \Gamma$$

(3) minimum thrust required to counter Drag.

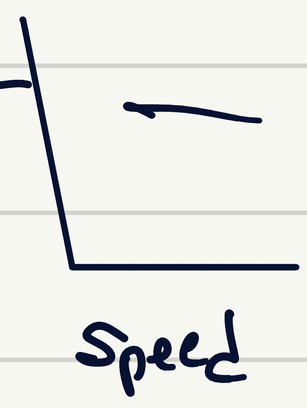
$$T_{trim,\ Required\ (R)} = D_{cruise} = C_{D_0}\left(\tfrac{1}{2}\rho V^2 S\right) + \varepsilon \frac{2W^2}{\rho V^2 S}$$

$$P_{trim,\ Required\ (R)} = C_{D_0}\left(\tfrac{1}{2}\rho V^3 S\right) + \varepsilon \frac{2W^2}{\rho V S}$$

a) $\boxed{\text{Max Endurance} = \frac{dm}{dt} = 0}$

Prop. Power

Jet thrust



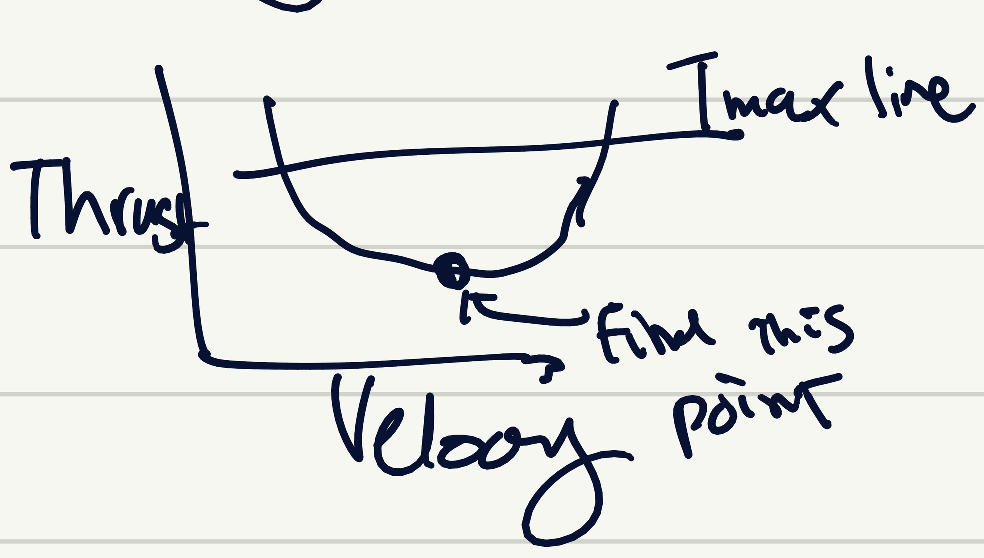
Jet:

$\frac{dm}{dt} \sim$ minimize fuel consumption rate, $\dot{m}$, to maximize time. We can relate $\dot{m}$ to thrust for jet, since you inject fuel to increase thrust directly and power just depends on thrust.

$$\frac{dm}{dt} \sim \frac{dT}{dt}$$

We take derivative of Thrust = Drag and find its min point respect to velocity to find the point at which I can use least amt of thrust but still counter Drag force.

⇓



$$T_R = C_{D_0}\left(\tfrac{1}{2}\rho V^2 S\right) + \varepsilon \frac{2W^2}{\rho V^2 S}$$

$$\frac{d}{dt}(T_R) = \frac{d}{dt}(\text{''})$$

$$0 = \frac{dT_R}{dt} = C_{D_0}\rho S V + \frac{2\varepsilon W^2}{\rho S} \cdot -2\frac{1}{V^3}$$

$$\frac{4\varepsilon W^2}{\rho S V^3} = C_{D_0}\rho S V$$

$$\frac{4\varepsilon W^2}{\rho^2 S^2 C_{D_0}} = V^4$$

$$V^2 = \frac{2W}{\rho S}\sqrt{\frac{\varepsilon}{C_{D_0}}}$$

$$V_{MT} = \sqrt{\frac{2W}{\rho S}}\sqrt{\frac{\varepsilon}{C_{D_0}}}$$

Propeller:

$$P = C_{D_0}\left(\tfrac{1}{2}\rho V^3 S\right) + \varepsilon \frac{2W^2}{\rho V S}$$

$$\frac{dP}{dt} = \frac{d}{dt}\left(\quad " \quad \right)$$

$$0 = C_{D_0}\tfrac{3}{2}\rho V^2 S - \varepsilon \frac{2W^2}{\rho V^2 S}$$

$$\frac{2W^2\varepsilon}{\rho S V^2} = \tfrac{3}{2} C_{D_0} \rho S V^2$$

$$\frac{4W^2\varepsilon}{3\rho^2 S^2 C_{D_0}} = V^4$$

$$V^2 = \frac{2W}{\rho S}\sqrt{\frac{\varepsilon}{3C_{D_0}}}$$

$$V_{MP} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{\varepsilon}{3C_{D_0}}}}$$

b) $\boxed{\text{Range} = \frac{dm}{dr} = 0}$

- minimize dm per distance

Jet:

$$\frac{dr}{dm} = \frac{V}{-C_T T} = \frac{L}{mg}\left(-\frac{V}{C_T T}\right) \quad \text{, since } L = W = mg$$

$$\frac{L}{mg} = 1$$

and $T = D$

$$\int_{r_i}^{r_f} dr = \int_{m_i}^{m_f} -\left(\frac{L}{D}\right)\left(\frac{V}{C_T m g}\right) dm$$

$$\text{Range} = -\frac{L}{D}\frac{V_{cruise}}{C_T g}\ln\left(\frac{m_i}{m_f}\right) = -\frac{C_L}{C_D}\frac{V_{cruise}}{C_T g}\ln\left(\frac{m_i}{m_f}\right)$$

$$\frac{\partial R}{\partial C_L} \propto \frac{\partial\left(V_{cruise}\, C_L/C_D\right)}{\partial C_L} = 0$$

Assume $V_{cruise} = \sqrt{2W/C_L \rho S}$

$\boxed{\tfrac{1}{2}\rho V^2 S C_L = W \text{ for trim}}$

Assume $\sqrt{\frac{2W}{\rho S}} = \text{constant}$

$$C_L^{1/2} = x \qquad C_L = x^2$$

$$0 = \frac{\partial\left(V_{cruise}\frac{C_L}{C_D}\right)}{\partial x^2} = \sqrt{\frac{2W}{\rho S}}\,\frac{\partial\left(\frac{x}{(C_{D_0}+\varepsilon x^4)}\right)}{\partial x} = \sqrt{\frac{2W}{\rho S}}\,\frac{(C_{D_0}+\varepsilon x^4) - x(4\varepsilon x^3)}{(C_{D_0}+\varepsilon x^4)^2}$$

$$0 = \sqrt{\frac{2W}{\rho S}}\,\frac{C_{D_0} - 3\varepsilon x^4}{(C_{D_0}+\varepsilon x^4)^2}$$

$$C_{D_0} = 3\varepsilon x^4$$

$$\sqrt{\frac{C_{D_0}}{3\varepsilon}} = C_{L_{MR}}$$

$$C_{D_{MR}} = C_{D_0} + \frac{C_{D_0}}{3} = \frac{4}{3}C_{D_0}$$

$$V_{cruise-crimb} = \sqrt{\frac{2W}{\rho S C_{L_{MR}}}}$$

Propeller:

$$\frac{dm}{dr} \sim \frac{dP}{dr} = \frac{dTV}{V} = \frac{dT}{dt}$$

$$T_{available} = C_{D_0}\left(\frac{1}{2}\rho V^2 S\right) + \frac{2\varepsilon W^2}{\rho V^2 S}$$

$$\frac{\partial T_{available}}{\partial t} = C_{D_0}\rho V S - \frac{4\varepsilon W^2}{\rho V^3 S}$$

$$\frac{4\varepsilon W^2}{\rho V^3 S} = C_{D_0}\rho V S$$

$$V_{MR}^4 = \frac{4\varepsilon W^2}{\rho^2 S^2 C_{D_0}}$$

$$V_{MR} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{\varepsilon}{C_{D_0}}}}$$

c) The pitch has to be reduced as a function of time because fuel is consumed and the weight is reduced. Less weight means The weight no longer balances the lift, so the airplane will want to rise up the altitude. To maintain the altitude, we must reduce α to decrease the lift force or C_L, lift coefficient that is dependent on α.

Basically, with weight reduced, α_{trim} and $C_{L_{trim}}$ that are required α and C_L to maintain certain altitude decreases. Thus adjust the airplane to achieve α_{trim} and $C_{L_{trim}}$.

$\alpha_{trim} \sim W$ proportional to... $C_{L_{trim}} \sim W$ Proportional to...

Maintaining altitude helps to maintain Mach number, since the speed of sound is proportional to altitude, $a_{\text{speed of sound}} \sim h_{\text{altitude}}$, and constant altitude keeps speed of sound constant. $Mach = \frac{V}{a_{sound}}$

d) Cruise climb allows the airplane to naturally increase its altitude as fuel is consumed and weight is lost. This allows the airplane to fly at the optimal speed to maintain Mach and also C_L to C_D ratio. In contrast, for constant altitude and Mach flight, the airplane is prevented from increasing its altitude even though the weight is lost by fuel consumption and it wants to rise. It maintains altitude by reducing alpha and C_L. Thus the airplane can't always maintain the maximum C_L to C_D ratio for minimum $\dot{m}$.

Optimal as in the max $\frac{C_L}{C_D}$ ratio to minimize $\dot{m}$.

Also increasing altitude allows airplane to feel less drag due to thinner air or lower ρ.

- C_L/C_D is almost constant for level flight
- better → C_L/C_D increases for cruise climb.

- Also, As aircraft climbs you need less thrust for cruise climb. but for step thrust is constant.

~~Cruise:~~

$$\downarrow T = \frac{1}{2}\rho V^2 \left(\overset{\downarrow}{C_{D_0}} + \varepsilon \overset{\uparrow}{C_L^2}\right)$$

$$\downarrow W = \frac{1}{2}\rho V^2 C_L \uparrow$$

~~Step:~~

$$\text{const. } T = \frac{1}{2}\rho V^2 \left(\overset{\uparrow}{C_{D_0}} + \varepsilon \overset{\downarrow}{C_L^2}\right)$$

$$\downarrow W = \frac{1}{2}\rho V^2 C_L \downarrow$$