The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

24 SEPTEMBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

JOHN-PAUL CLARKE
Ernest Cockrell, Jr. Memorial Chair in Engineering, The University of Texas at Austin

Topics for Today

- Topic(s):
  - Straight and Level Flight
  - Cruise Range

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STRAIGHT AND LEVEL FLIGHT

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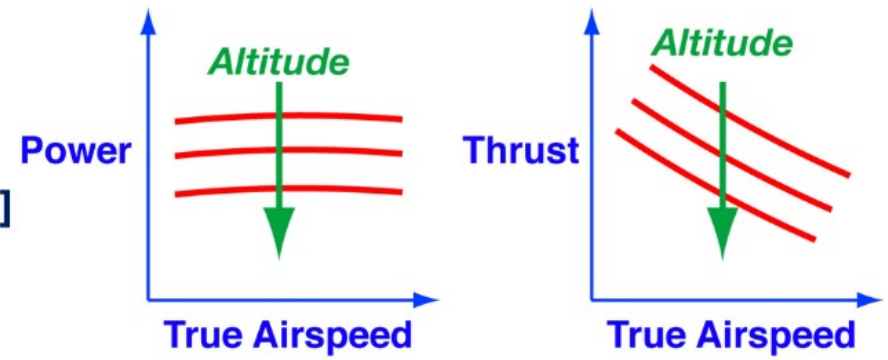
Typical Effects of Altitude and Velocity on Power and Thrust

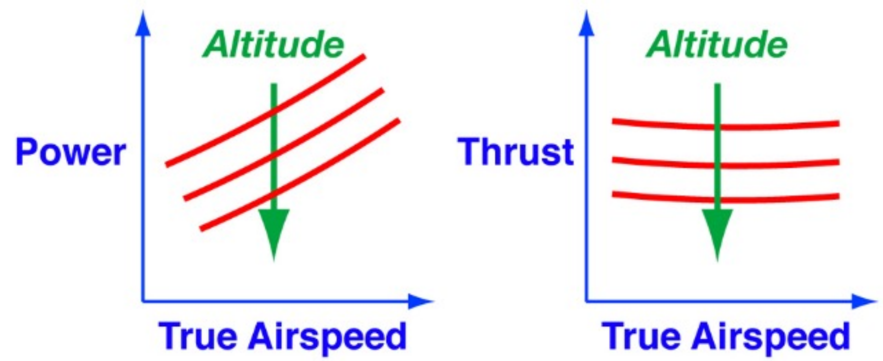
- **Propeller**
  [Air-breathing engine]



- **Turbofan**

[In between]

- **Turbojet**



- **Battery**

[Independent of altitude and airspeed]

Performance Parameters

Lift-to-Drag Ratio

$$\boxed{L/D = C_L/C_D}$$

Load Factor

$$\boxed{n = L/W = L/mg, \text{"}g\text{"}s}$$

Thrust-to-Weight Ratio

$$\boxed{T/W = T/mg, \text{"}g\text{"}s}$$

Wing Loading

$$\boxed{W/S, \quad N/m^2 \text{ or } lb/ft^2}$$

Trimmed Lift Coefficient, C_L

- **Trimmed lift coefficient, C_L**
  - Proportional to weight and wing loading factor
  - Decreases with V^2
  - At constant true airspeed, increases with altitude

$$\boxed{W = C_{L_{trim}}\left(\frac{1}{2}\rho V^2\right)S = C_{L_{trim}}\bar{q}S}$$

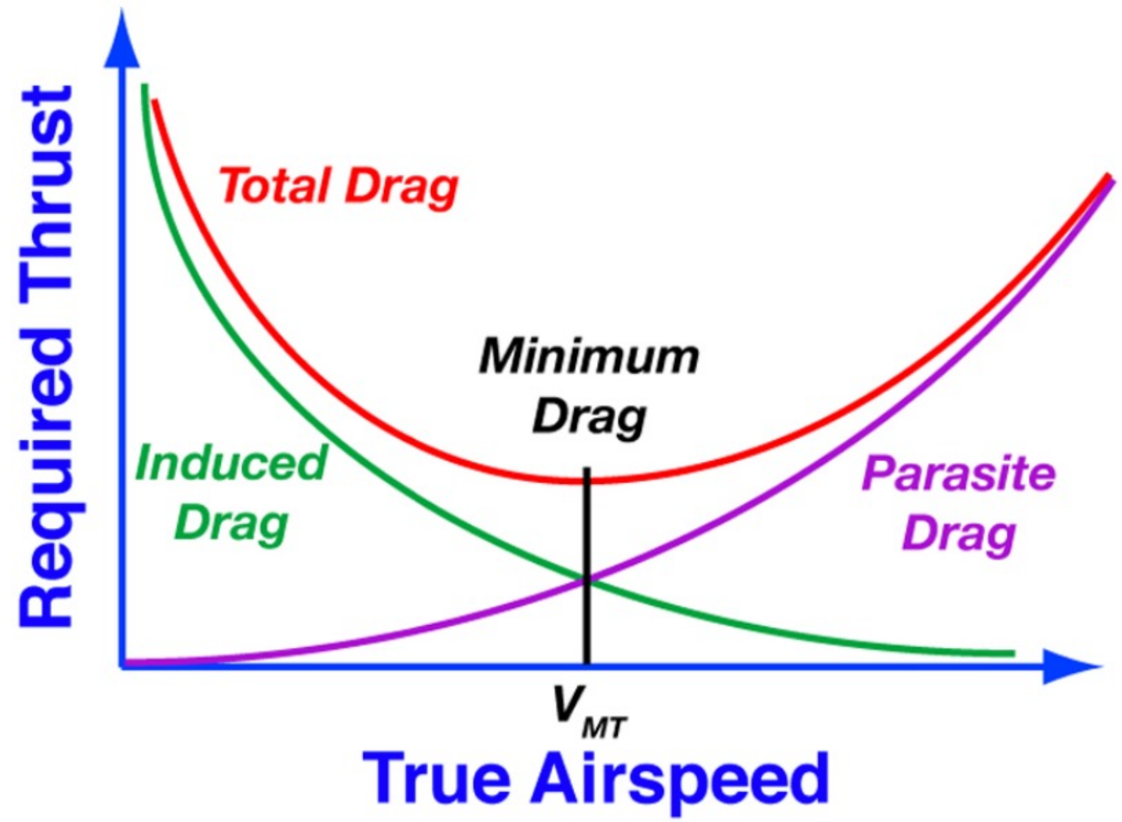
$$\boxed{C_{L_{trim}} = \frac{1}{\bar{q}}(W/S) = \frac{2}{\rho V^2}(W/S)}$$

Trimmed Angle of Attack, α

- **Trimmed angle of attack, α**
  - Constant if dynamic pressure and weight are constant
  - If dynamic pressure decreases, angle of attack must increase

$$\boxed{\alpha_{trim} = \frac{2W/\rho V^2 S - C_{L_o}}{C_{L_\alpha}} = \frac{\frac{1}{\bar{q}}(W/S) - C_{L_o}}{C_{L_\alpha}}}$$

Thrust Required for Steady, Level Flight



Thrust Required for Steady, Level Flight

Trimmed thrust

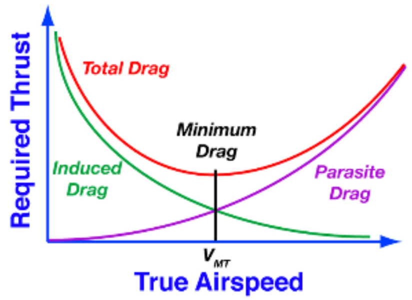
Parasitic Drag *Induced Drag*

$$\boxed{T_{trim} = D_{cruise} = C_{D_o}\left(\frac{1}{2}\rho V^2 S\right) + \varepsilon\frac{2W^2}{\rho V^2 S}}$$

Minimum required thrust conditions

$$\boxed{\frac{\partial T_{trim}}{\partial V} = C_{D_o}\left(\rho V S\right) - \frac{4\varepsilon W^2}{\rho V^3 S} = 0}$$

Necessary Condition:
Slope = 0



Necessary and Sufficient Conditions for Minimum Required Thrust

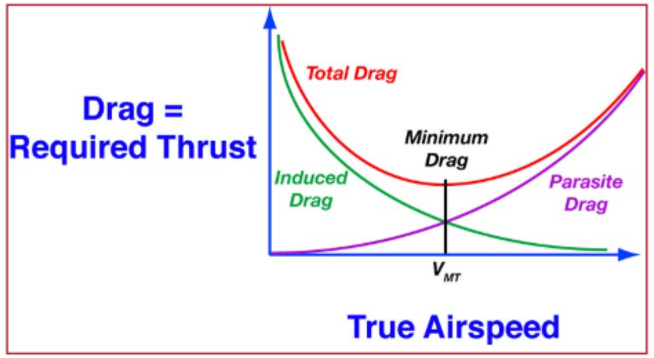
Necessary Condition = Zero Slope

$$C_{D_o}(\rho VS) = \frac{4\varepsilon W^2}{\rho V^3 S}$$

Sufficient Condition for a Minimum = Positive Curvature when slope = 0

$$\frac{\partial^2 T_{trim}}{\partial V^2} = \underset{(+)}{C_{D_o}(\rho S)} + \underset{(+)}{\frac{12\varepsilon W^2}{\rho V^4 S}} > 0$$

Airspeed for Minimum Thrust in Steady, Level Flight



Satisfy necessary condition

$$V^4 = \left(\frac{4\varepsilon}{C_{D_o}\rho^2}\right)(W/S)^2$$

Fourth-order equation for velocity

Choose the positive root

$$V_{MT} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{\varepsilon}{C_{D_o}}}}$$

Lift, Drag, and Thrust Coefficients in Minimum-Thrust Cruising Flight

Lift coefficient

$$C_{L_{MT}} = \frac{2}{\rho V_{MT}^2}\left(\frac{W}{S}\right)$$

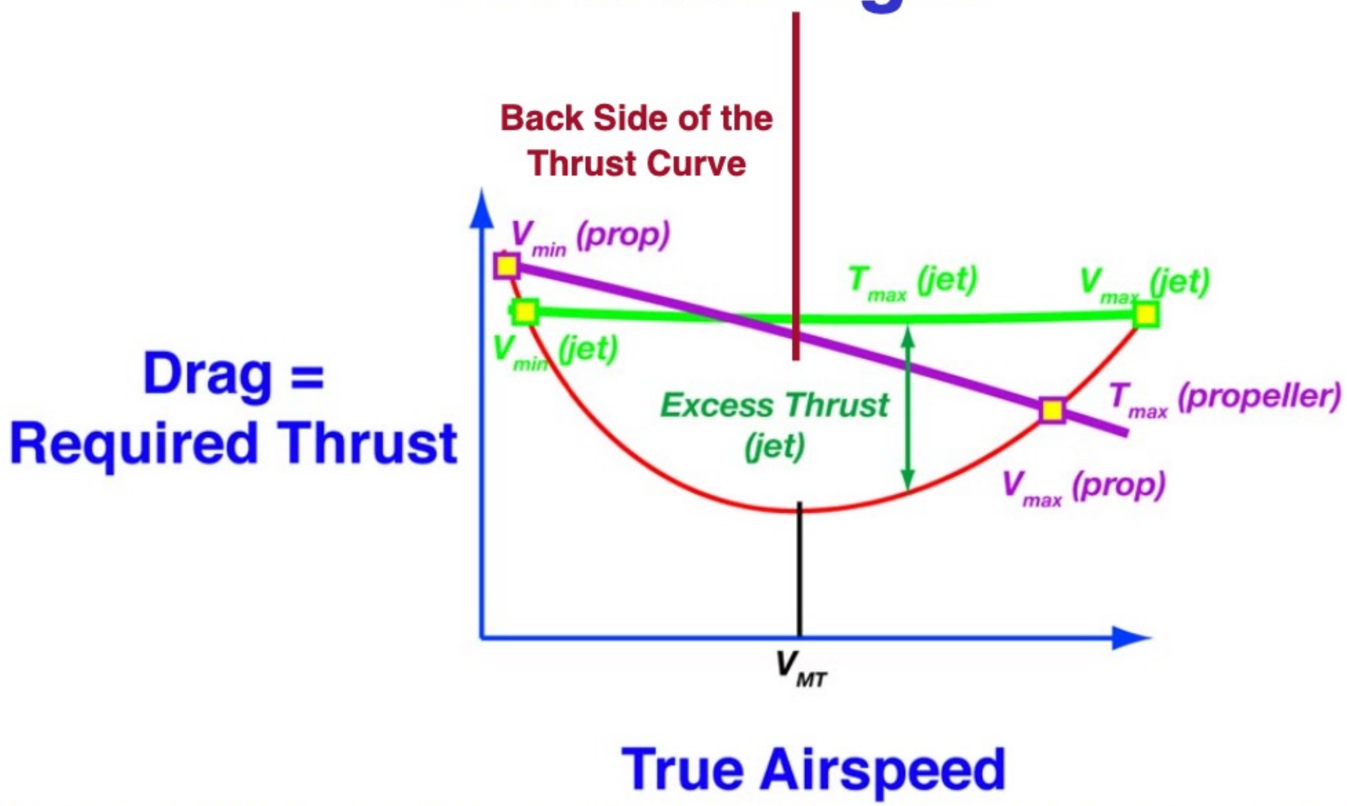
$$= \sqrt{\frac{C_{D_o}}{\varepsilon}} = \left(C_L\right)_{(L/D)_{\max}}$$

Drag and thrust coefficients

$$C_{D_{MT}} = C_{D_o} + \varepsilon C_{L_{MT}}^2 = C_{D_o} + \varepsilon \frac{C_{D_o}}{\varepsilon}$$

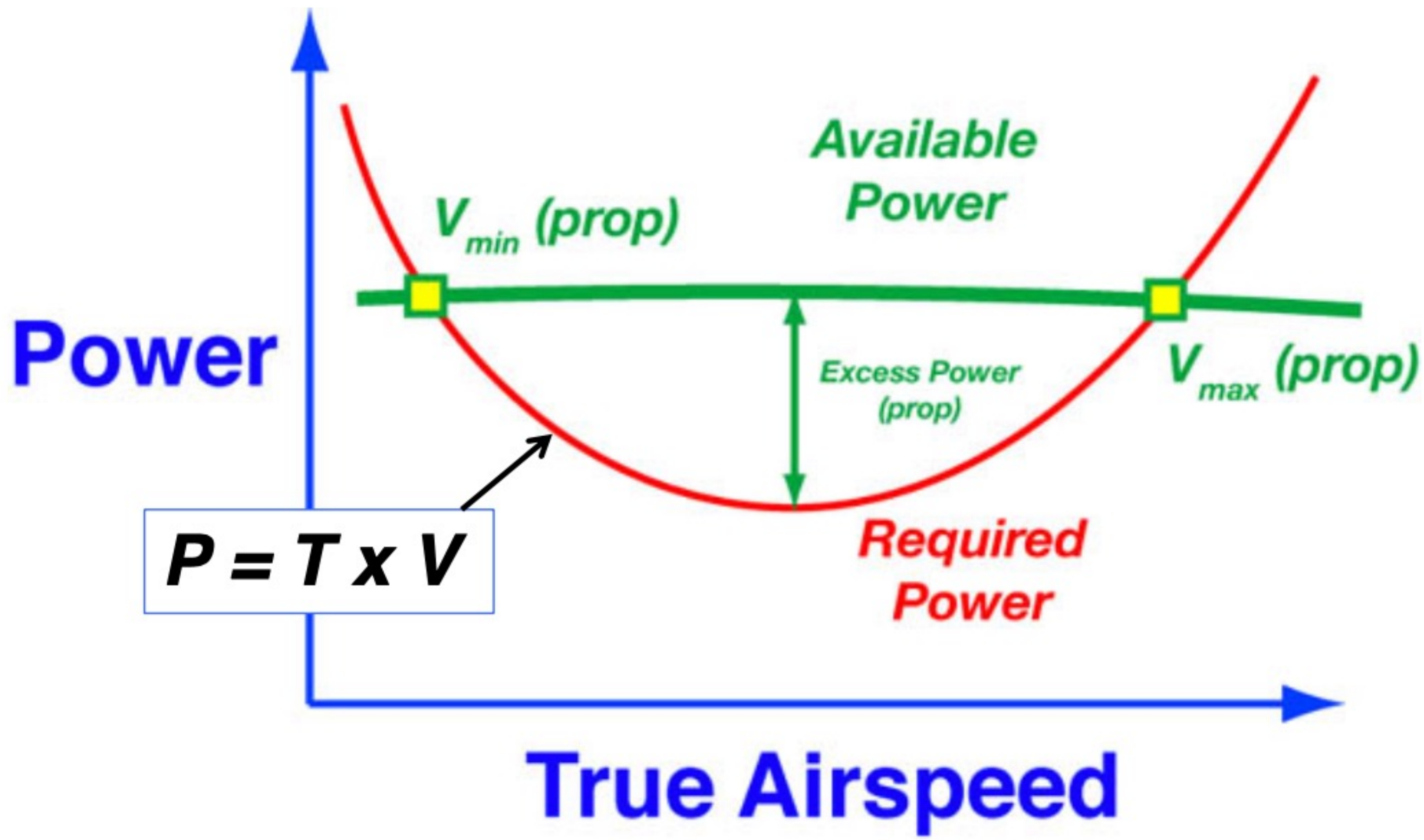
$$= 2C_{D_o} \equiv C_{T_{MT}}$$

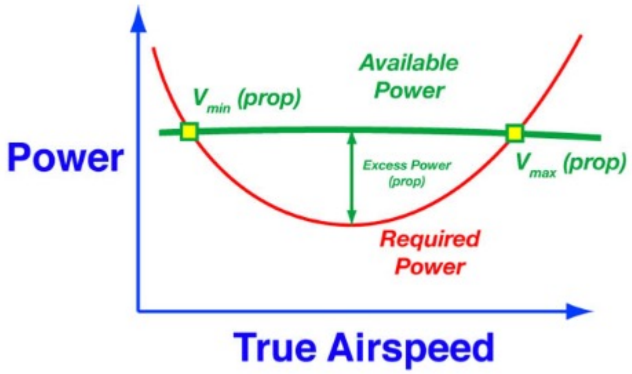
Achievable Airspeeds in Constant-Altitude Flight



- **Two equilibrium airspeeds for a given thrust or power setting**
  - **Low speed, high C_L, high α**
  - **High speed, low C_L, low α**
- **Achievable airspeeds between minimum and maximum values with maximum thrust or power**

Power Required for Steady, Level Flight





Airspeed for Minimum Power in Steady, Level Flight

- Satisfy necessary condition

$$C_{D_o}\frac{3}{2}\left(\rho V^2 S\right) = \frac{2\varepsilon W^2}{\rho V^2 S}$$

- Fourth-order equation for velocity
  - Choose the positive root

$$V_{MP} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{\varepsilon}{3C_{D_o}}}}$$

- Corresponding lift and drag coefficients

$$C_{L_{MP}} = \sqrt{\frac{3C_{D_o}}{\varepsilon}}$$

$$C_{D_{MP}} = 4C_{D_o}$$

Achievable Airspeeds for Jet in Cruising Flight

Thrust = constant

$$\boxed{T_{avail} = C_D \bar{q} S = C_{D_o}\left(\frac{1}{2}\rho V^2 S\right) + \frac{2\varepsilon W^2}{\rho V^2 S}}$$

$$\boxed{C_{D_o}\left(\frac{1}{2}\rho V^4 S\right) - T_{avail} V^2 + \frac{2\varepsilon W^2}{\rho S} = 0}$$

$$\boxed{V^4 - \frac{2T_{avail}}{C_{D_o}\rho S} V^2 + \frac{4\varepsilon W^2}{C_{D_o}(\rho S)^2} = 0}$$

4th-order algebraic equation for *V*

Achievable Airspeeds for Jet in Cruising Flight

Solutions for V^2 can be put in quadratic form and solved easily

$$\boxed{V^2 \triangleq x; \quad V = \pm\sqrt{x}}$$

$$V^4 - \frac{2T_{avail}}{C_{D_o}\rho S}V^2 + \frac{4\varepsilon W^2}{C_{D_o}(\rho S)^2} = 0$$

$$x^2 + bx + c = 0$$

$$\boxed{x = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c} = V^2}$$

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CRUISE RANGE

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Cruising Range and Specific Fuel Consumption

- Thrust = Drag
- Lift = Weight

$$\boxed{\begin{aligned} 0 &= (C_T - C_D)\frac{1}{2}\rho V^2 S \Big/ m \\ 0 &= \left(C_L \frac{1}{2}\rho V^2 S - mg\right)\Big/ mV \end{aligned}}$$

- Level flight

$$\boxed{\begin{aligned} \dot{h} &= 0 \\ \dot{r} &= V \end{aligned}}$$

- **Thrust specific fuel consumption, *TSFC* = c_T**
  - Fuel mass burned per sec per unit of thrust

$$\boxed{c_T : \frac{kg/s}{kN}} \qquad \boxed{\dot{m}_f = -c_T T}$$

- **Power specific fuel consumption, *PSFC* = c_P**
  - Fuel mass burned per sec per unit of power

$$\boxed{c_P : \frac{kg/s}{kW}} \qquad \boxed{\dot{m}_f = -c_P P}$$

Breguet Range Equation for Jet Aircraft

Rate of change of range with respect to weight of fuel burned

$$\frac{dr}{dm} = \frac{dr/dt}{dm/dt} = \frac{\dot{r}}{\dot{m}} = \frac{V}{(-c_T T)} = -\frac{V}{c_T D} = -\left(\frac{L}{D}\right)\frac{V}{c_T m g}$$

$$dr = -\left(\frac{L}{D}\right)\frac{V}{c_T m g}dm$$

Range traveled

$$Range = R = \int_0^R dr = -\int_{W_i}^{W_f}\left(\frac{L}{D}\right)\left(\frac{V}{c_T g}\right)\frac{dm}{m}$$

Maximum Range of a Jet Aircraft Flying at Constant Altitude

At constant altitude and *SFC*

$$\boxed{V_{cruise}(t) = \sqrt{2W(t)/C_L\rho\left(h_{fixed}\right)S}}$$

$$Range = -\int_{W_i}^{W_f}\left(\frac{C_L}{C_D}\right)\left(\frac{1}{c_T g}\right)\sqrt{\frac{2}{C_L\rho S}}\frac{dm}{m^{1/2}}$$

$$= \left(\frac{\sqrt{C_L}}{C_D}\right)\left(\frac{2}{c_T g}\right)\sqrt{\frac{2}{\rho S}}\left(m_i^{1/2} - m_f^{1/2}\right)$$

Range is maximized when $\boxed{\left(\frac{\sqrt{C_L}}{C_D}\right) = \text{maximum}}$

Breguet Range Equation for Jet Aircraft at Constant Airspeed

For constant true airspeed, $V = V_{cruise}$, and SFC

$$R = -\left(\frac{L}{D}\right)\left(\frac{V_{cruise}}{c_T g}\right)\ln(m)\Big|_{m_i}^{m_f}$$
$$= \left(\frac{L}{D}\right)\left(\frac{V_{cruise}}{c_T g}\right)\ln\left(\frac{m_i}{m_f}\right)$$

$$= \left(V_{cruise}\frac{C_L}{C_D}\right)\left(\frac{1}{c_T g}\right)\ln\left(\frac{m_i}{m_f}\right)$$

- $V_{cruise}(C_L/C_D)$ as large as possible
- $M \rightarrow M_{crit}$
- ρ as small as possible
- h as high as possible

Maximize Jet Aircraft Range Using Optimal Cruise-Climb

$$\frac{\partial R}{\partial C_L} \propto \frac{\partial\left(V_{cruise} \, C_L \big/ C_D\right)}{\partial C_L} = \frac{\partial\left[V_{cruise} \, C_L \big/ \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = 0$$

$$V_{cruise} = \sqrt{2W / C_L \rho S}$$

Assume $\sqrt{2W(t)/\rho(h)S} = \text{constant}$

i.e., airplane **climbs at constant *TAS*** as fuel is burned

Maximize Jet Aircraft Range Using Optimal Cruise-Climb

$$\frac{\partial\left[V_{cruise} C_L \Big/ \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = \sqrt{\frac{2w}{\rho S}} \frac{\partial\left[C_L^{1/2} \Big/ \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = 0$$

$$\sqrt{\frac{2w}{\rho S}} = \text{Constant}; \;\; \text{let } C_L^{1/2} = x, \quad C_L = x^2$$

$$\frac{\partial}{\partial x}\left[\frac{x}{\left(C_{D_o} + \varepsilon x^4\right)}\right] = \frac{\left(C_{D_o} + \varepsilon x^4\right) - x\left(4\varepsilon x^3\right)}{\left(C_{D_o} + \varepsilon x^4\right)^2} = \frac{\left(C_{D_o} - 3\varepsilon x^4\right)}{\left(C_{D_o} + \varepsilon x^4\right)^2}$$

Optimal values:

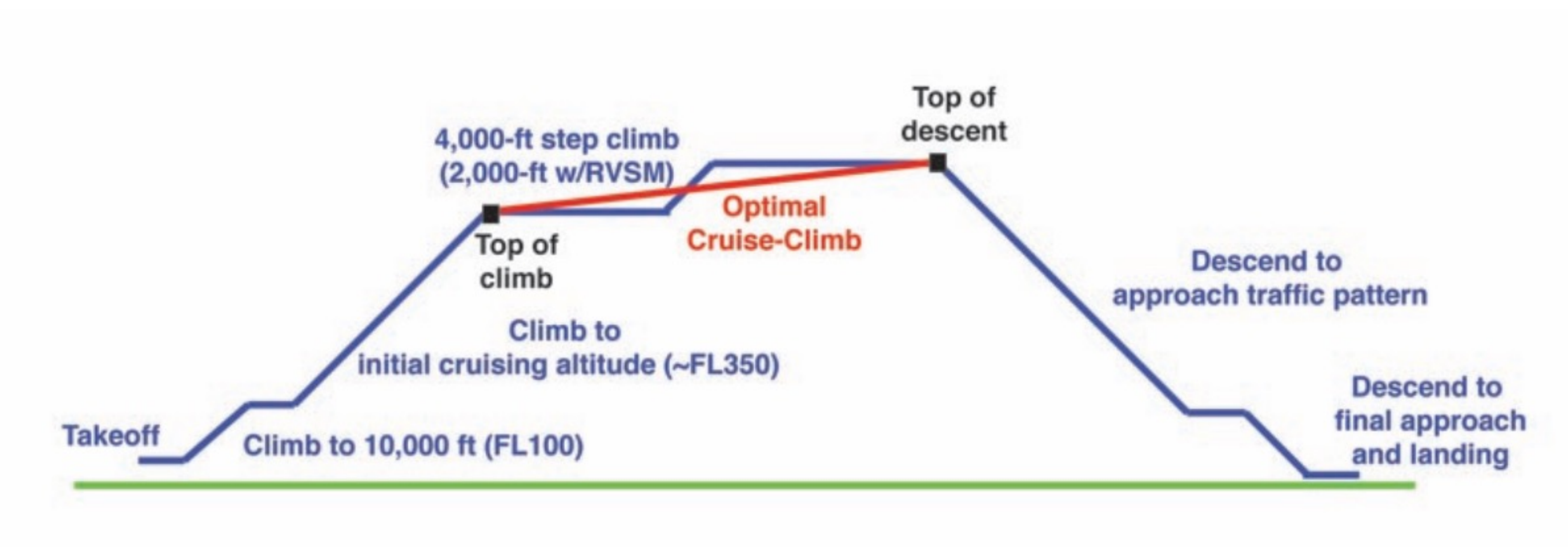
$$C_{L_{MR}} = \sqrt{\frac{C_{D_o}}{3\varepsilon}} : \;\; C_{D_{MR}} = C_{D_o} + \frac{C_{D_o}}{3} = \frac{4}{3} C_{D_o}$$

$$V_{cruise-climb} = \sqrt{2W(t) / C_{L_{MR}} \rho(h) S} = a(h) M_{cruise\text{-}climb}$$

$a(h)$: Speed of sound; $M_{cruise\text{-}climb}$: Mach number

Step-Climb Approximates Optimal Cruise-Climb

- Cruise-climb usually violates air traffic control rules
- Constant-altitude cruise does not
- <u>Compromise</u>: Step climb from one allowed altitude to the next as fuel is burned



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