GPS Exam 3

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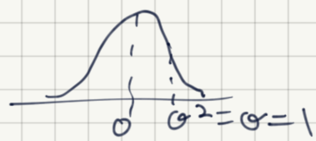
December 2024

1 Problem 1

(1)

Chi squared distribution:

Normal dist: $X \sim N(0,1)$ $\quad E[X] = 0 \quad Var(x) = 1$



Chi^2 dist: $Q = X^2$

$Q \sim \chi_1^2 \leftarrow$ degree of freedom (sample once in this case)

$Q = x_1^2 + x_2^2$

$Q \sim \chi_2^2$ (sample twice)

these are chi, different from x

//

$$Z = \sum_{k=1}^{N} |S_k|^2$$

$$S_k = \rho_k \exp[i \Delta \theta (z_{jk})] + n_k$$

For χ_0, $\Rightarrow$ $S_k^2 = n_k^2$

only noise exists

$$= (n_{Ik} + j n_{Qk})(n_{Ik} - j n_{Qk})$$

$$= n_{Ik}^2 + n_{Qk}^2$$

$$E[S_k]^2 = E[n_{Ik}^2] + E[n_{Qk}^2]$$

$$\boxed{\begin{aligned} \text{Since} \quad Var[X] &= E[X^2] - [EX]^2 \\ E[X^2] &= Var[X] + [EX]^2 \end{aligned}}$$

$$E[n_{Ik}^2] = 1 + 0$$

$$E[n_{Qk}^2] = 1 + 0$$

$$E[S_k^2] = 1 + 1 = 2$$

$$E[Z \mid \theta \in \chi_0] = E[Z] = E[\sum_{k=1}^{N} |S_k|^2] = \underline{\underline{2N}}$$

$$Var[S_k^2] = Var[n_{Ik}^2] + Var[n_{Qk}^2]$$

$$Var[n_{Ik}^2] = E[n_{Ik}^4] - [E[n_{Ik}^2]]^2$$

From Gaussian moments:

$$\boxed{E[x^4] = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4}$$

$$Var[n_{Ik}^2] = Var[n_{Qk}^2] = 0 + 0 + 3 - 1$$
$$= 2$$

$$Var[S_k^2] = 2 + 2 = 4$$

$$Var[Z \mid \theta \in \chi_0] = Var[Z] = Var[\sum_{k=1}^{N} |S_k|^2]$$
$$= \underline{\underline{4N}}$$

for χ_0, no signal only noise: //

- $S_k^2 = n_{Ik}^2 + n_{Qk}^2$

$$Z = \sum |S_k|^2 = \sum_{k=1}^{N} |n_{Ik}^2 + n_{Qk}^2|$$

We have N number of n_{Ik} and n_{Qk} that are normaly distributed individually

this looks identical to Chi^2 distribution

$$Q = x_1^2 + x_2^2 + x_3^2 \dots$$

$$\therefore p(Z \mid \theta \in \chi_0) = Z = \chi^2_{2N}$$

for χ_1 //

$$S_k = \rho + n_k$$

$$S_k^2 = (\rho + n_{Ik} + jn_{Qk})(\rho + n_{Ik} - jn_{Qk})$$

$$= \rho^2 + \cancel{\rho n_{Ik}} - \cancel{j\rho n_{Qk}} + \cancel{\rho n_{Ik}} + \cancel{n_{Ik}^2} - \cancel{jn_{Ik}n_{Qk}}$$
$$\cancel{+ jn_{Qk}\rho} + \cancel{jn_{Qk}n_{Ik}} \cancel{+ n_{Qk}^2}$$

$$= \rho^2 + n_{Ik}^2 + n_{Qk}^2 + 2\rho n_{Ik}$$

$$Z=\sum|S_K|^2 = \sum_{K=1}^{N} |\rho^2 + n_{IK}^2 + n_{QK}^2 + 2\rho n_{IK}|$$

$$= N\rho^2 + \underbrace{\sum_{K=1}^{N}(n_{IK}^2 + n_{QK}^2)}_{\chi^2_{2N}} + 2\rho\sum n_{IK}$$

$$\boxed{= \chi^2_{2N} + \underbrace{\lambda^2 + 2\rho\sum n_{IK}}_{\text{decentralizing term}}}$$

already defined above

$$E[Z|\theta\in X_1] = \sum E[\rho^2 + n_{IK}^2 + n_{QK}^2 + 2\rho n_{IK}]$$

$$= N\rho^2 + 2N$$

$$\boxed{= \lambda + 2N}$$

$$Var[Z|\theta\in X_1] = \sum Var[\rho^2 + n_{IK}^2 + n_{QK}^2 + 2\rho n_{IK}]$$

$$= \sum[4 + Var[\rho^2 + 2\rho n_{IK}]]$$

$$= 4N + 0 \quad + \sum Var[2\rho n_{IK}]$$

$$= 4N + 0 \quad + \sum Var[2\rho n_{IK}]$$

$$Var[2\rho n_{IK}] = E[2\rho n_{IK}]^2]$$

$$E[2\rho n_{IK}]^2$$

$$= 2^2\rho^2 E[n_{IK}^2]$$

$$2^2\rho^2 E[n_{IK}^2]$$

$$= 2^2\rho^2 Var[n_{IK}]$$

$$= 4\rho^2$$

$$\mathrm{Var}[\tau \mid \theta \in \mathcal{X}_1] = 4N + \sum_i 4\rho^2$$
$$= 4N + 4N\rho^2$$
$$= 4N + 4\lambda$$
$$= 4(N+\lambda)$$

2 Problem 2

2.1 a)

[redacted] From D.V. Hinkley,

a) $P_W(\omega) = \frac{b(\omega)\, d(\omega)}{\sqrt{2\pi}\, a^3(\omega)} \left[\Phi\left\{\frac{b(\omega)}{a(\omega)}\right\} - \Phi\left\{-\frac{b(\omega)}{a(\omega)}\right\}\right]$

$+ \frac{1}{\pi\, a^2(\omega)} \exp\left\{-\frac{c}{2}\right\}$

- $\sigma_1 = \sigma_I = 1,\ \sigma_2 = \sigma_Q = 1,\ \rho = 0$

$a(\omega) = \left(\frac{\omega^2}{\sigma_1^2} - \frac{2\rho\omega}{\sigma_1\sigma_2} + \frac{1}{\sigma_2^2}\right)^{\frac{1}{2}} = \left(\omega^2 + 1\right)^{\frac{1}{2}}$

$b(\omega) = \frac{\theta_1\omega}{\sigma_1^2} - \frac{\rho(\theta_1 + \theta_2\omega)}{\sigma_1\sigma_2} + \frac{\theta_2}{\sigma_2^2} = \theta_1 w + \theta_2 = \bar{\rho} w \sin\theta + \bar{\rho}\cos\theta$

$c = \frac{\theta_1^2}{\sigma_1^2} - \frac{2\rho\theta_1\theta_2}{\sigma_1\sigma_2} + \frac{\theta_2^2}{\sigma_2^2} = \theta_1^2 + \theta_2^2 = \bar{\rho}^2$

- $\theta_1 = \mu_I = \rho_k\cos\theta,\ \theta_2 = \mu_Q = \rho_k\sin\theta = 0,\ \rho_k \neq 0$ = Signal componet; Not a correlation coefficient

$d(\omega) = \exp\left\{\frac{b^2(\omega) - c a^2(\omega)}{2(1-\rho^2)a^2(\omega)}\right\} = \exp\left\{\frac{b^2 - ca^2}{2a^2}\right\}$

$\Phi(y) = \int_{-\infty}^{y} \phi(u)\,du$, where $\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}$

P.D.F. of normal standard distribution.

- $P_W(\omega)$ is P.D.F. of $W = \frac{Q}{I}$

  $\hat{\theta}_{ML} = \arctan(W)$

P.D.F $\hat{\theta}_{ML}$

- $\arctan(x)$ is a monotone function.

  Let $Y = g(x) = \arctan(W)$ and $y = \hat{\theta}_{ML}$
  Y is monotonic. Then pdf for Y is

  $f_Y(y) = f_x(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|$ for all $y = g(x)$ such that $f_x(x) > 0$

  $f_x(x) = P_W(\omega)$

  $Y = \arctan(W)$
  $W = \tan(Y)$
  $g^{-1}(y) = \tan Y = \tan(\hat{\theta}_{ML})$

  $\frac{d}{dy} g^{-1}(y) = \sec^2(y) = \sec^2(\hat{\theta}_{ML})$

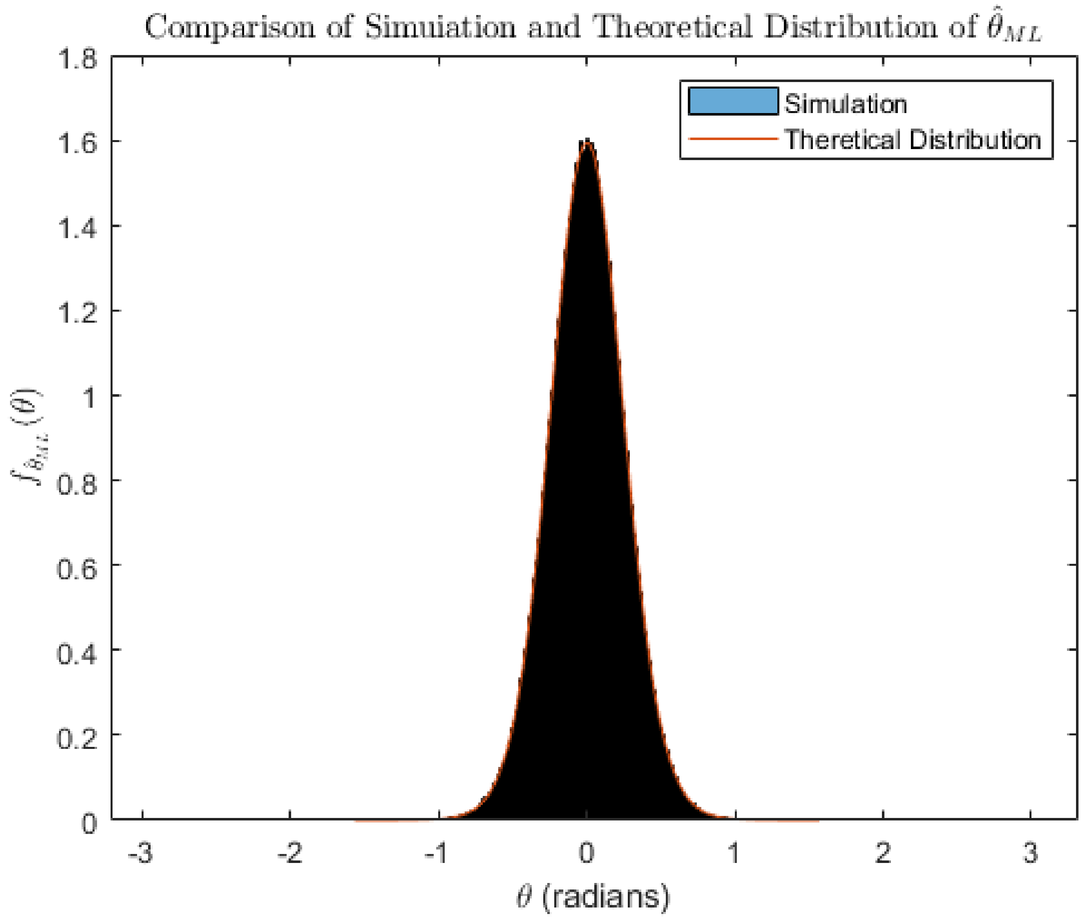
$f_Y(y) = P_W(\tan\hat{\theta}_{ML}) \quad |\sec^2(\hat{\theta}_{ML})|$

$f_{\hat{\theta}_{ML}}(\hat{\theta}) = P_W(\tan\hat{\theta}_{ML}) |1 + \tan^2\hat{\theta}_{ML}|$ for $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$= P_W(W) \, |1 + \tan^2(\operatorname{atan}(W))|$

or $P_W(W) \, |1 + W^2|$

2.2 b)



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Variance from the Simulation: 0.067438
CRLB : 0.062500
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c)

$$p(z|\theta) = \frac{1}{2\pi} \exp\left[-\frac{1}{2}[\underline{z}-\mu(\rho,\theta)]^T[z-\mu(\rho,\theta)]\right]$$

$$\mu(\rho,\theta) = \begin{bmatrix}\rho\cos\theta \\ \rho\sin\theta\end{bmatrix}$$

$$p(z|\theta) = \frac{1}{2\pi} \exp\left[-\frac{1}{2}[\underline{z}-\underline{\mu}]^2\right]$$

$$\ln(p(z|\theta)) = -\frac{1}{2}[\underline{z}-\underline{\mu}]^2 - \ln 2\pi$$

$$\frac{d}{d\theta}\ln(p(z|\theta)) = \frac{2}{2}[z-\underline{\mu}]^T\begin{bmatrix}\rho\sin\theta \\ -\rho\cos\theta\end{bmatrix}$$

$$= [I-\rho\cos\theta \quad Q-\rho\sin\theta]\begin{bmatrix}\rho\sin\theta \\ -\rho\cos\theta\end{bmatrix}$$

$$= I\rho\sin\theta - \rho^2\cos\theta\sin\theta - Q\rho\cos\theta + \rho^2\sin\theta\cos\theta$$

$$H_\theta = I\rho\sin\theta - Q\rho\cos\theta$$

$$H_\theta^2 = (I\rho\sin\theta - Q\rho\cos\theta)(I\rho\sin\theta - Q\rho\cos\theta)$$

$$= I^2\rho^2\sin^2\theta - 2IQ\rho^2\cos\theta\sin\theta + Q^2\rho^2\cos^2\theta$$

$$= E[\quad '' \quad]$$

$$= \rho^2 E[I^2\sin^2\theta - 2IQ\cos\theta\sin\theta + Q^2\cos^2\theta]$$

$$E[I^2] = E[(\rho\cos\theta + n_I)^2]$$

$$= E[\rho^2\cos^2\theta + 2\rho\cos\theta\, n_I + n_I^2]$$

$$= E[\rho^2\cos^2\theta] + 0 + 1$$

$$E[Q^2] = E[\rho^2\sin^2\theta] + 1$$

$$E[I] = E[\rho\cos\theta + n_I] = E[\rho\cos\theta]$$

$$E[Q] = E[\rho\sin\theta]$$

$$J = E[H_\theta]^2 = \rho^2 E[\rho^2\cos^2\theta\sin^2\theta + \cancel{\sin^2\theta} - 2\rho^2\sin^2\theta\cos^2\theta + \rho^2\sin^2\theta\cos^2\theta + \cos^2\theta]$$

$$= \rho^2 E[1 + \cancel{2\rho^2\cos^2\theta\sin^2\theta} - \cancel{2\rho^2\sin^2\theta\cos^2\theta}]$$

$$= \rho^2$$

$$\boxed{CRLB = \frac{1}{J} = \frac{1}{\rho^2}}$$

I expect the variance from part b to be greater than $\frac{1}{16} = 0.0625$ and my simulation obeys this bound.

3 Problem 3

②

a) $\alpha = [f, \rho, \theta]^T$

$\alpha_1 = f = \frac{\omega}{2\pi} \quad \alpha_2 = \rho = b \quad \alpha_3 = \theta$

$$f(z \mid \alpha) = \left(\frac{1}{\sigma^2 2\pi}\right)^N \exp\left[-\frac{1}{2\sigma^2} \sum_{k=0}^{N-1} (X_k - \mu_k)^2 + (Y_k - V_k)^2\right]$$

$$X_n = I = \rho\cos(2\pi f k T_a + \theta_0) + n_{I_k}(j)$$

$$Y_n = Q = \rho\sin(2\pi f k T_a + \theta_0) + n_{Q_k}(j)$$

$$\mu_k = \rho\cos(2\pi f k T_a + \theta)$$
$$V_k = \rho\sin(2\pi f k T_a + \theta)$$

$$\frac{\partial \mu_k}{\partial f} = -\rho\sin(2\pi f k T_a + \theta) \cdot 2\pi k T_a$$

$$\frac{\partial V_k}{\partial f} = \rho\cos(2\pi f k T_a + \theta) \cdot 2\pi k T_a$$

$$\frac{\partial \mu_k}{\partial \rho} = \cos('')$$

$$\frac{\partial V_k}{\partial \rho} = \sin('')$$

$$\frac{\partial \mu_k}{\partial \theta} = -\rho\sin(2\pi f k T_a)$$

$$\frac{\partial V_k}{\partial \theta} = \rho\cos(2\pi f k T_a)$$

$$J_{11} = \frac{1}{\sigma^2} \sum_{k=0}^{N-1} \left[\frac{\partial \mu_k}{\partial f} \frac{\partial \mu_k}{\partial f} + \frac{\partial V_k}{\partial f} \frac{\partial V_k}{\partial f}\right]$$

$$= \frac{1}{\sigma^2} \sum \left[\rho^2 4\pi^2 k^2 T_a^2 (\sin^2('') + \cos^2(''))\right)$$

$$= \frac{1}{\sigma^2} \sum \left[\rho^2 4\pi^2 k^2 T_a^2\right]$$

$$= \frac{\rho^2 4\pi^2 T_a^2}{\sigma^2} \sum_{k=0}^{N-1} k^2$$

eq. 15

$$= \frac{\rho^2 4\pi^2 T_a^2}{\sigma^2} \frac{N(N-1)(2N-1)}{6}$$

$$= \frac{\rho^2 4\pi^2 T_a^2 \, Q}{\sigma^2}$$

$$J_{12} = \frac{1}{\sigma^2}\Sigma\left[\frac{\partial \mu_k}{\partial f}\frac{\partial \mu_k}{\partial \rho} + \frac{\partial \nu_k}{\partial f}\frac{\partial \nu_k}{\partial \rho}\right]$$

$$= \frac{1}{\sigma^2}\Sigma\left[-\rho\sin('')\,2\pi k T_a \cos('') + \rho\cos('')\,2\pi k T_a \sin('')\right]$$

$$= 0$$

$$J_{13} = \frac{1}{\sigma^2}\Sigma\left[\frac{\partial \mu_k}{\partial f}\frac{\partial \mu_k}{\partial \theta} + \frac{\partial \nu_k}{\partial f}\frac{\partial \nu_k}{\partial \theta}\right]$$

$$= \frac{1}{\sigma^2}\Sigma\left[-\rho\sin('')\,2\pi k T_a(-\rho\sin('')) + \rho\cos('')\,2\pi k T_a\,\rho\cos('')\right]$$

$$= \frac{1}{\sigma^2}\Sigma\left[2\pi k T_a\,\rho^2\sin^2('') + 2\pi k T_a\,\rho^2\cos^2('')\right]$$

$$= \frac{1}{\sigma^2}2\pi T_a\rho^2\,\Sigma[k]$$

$$= \frac{2\pi\rho^2 T_a}{\sigma^2}\,\frac{N(N-1)}{2}$$

$$= \frac{2\pi\rho^2 T_a}{\sigma^2}P$$

$$J_{21} = \frac{1}{\sigma^2}\Sigma\left[\frac{\partial \mu_k}{\partial \rho}\frac{\partial \mu_k}{\partial f} + \frac{\partial \nu_k}{\partial \rho}\frac{\partial \nu_k}{\partial f}\right]$$

$$= J_{12} = 0$$

$$J_{22} = \frac{1}{\sigma^2}\Sigma\left[\frac{\partial \mu_k}{\partial \rho}\frac{\partial \mu_k}{\partial \rho} + \frac{\partial \nu_k}{\partial \rho}\frac{\partial \nu_k}{\partial \rho}\right]$$

$$= \frac{1}{\sigma^2}\Sigma\left[\cos^2('') + \sin^2('')\right]$$

$$= \frac{1}{\sigma^2}N$$

$$J_{23} = \frac{1}{\sigma^2}\Sigma\left[\frac{\partial \mu_k}{\partial \rho}\frac{\partial \mu_k}{\partial \theta} + \frac{\partial \nu_k}{\partial \rho}\frac{\partial \nu_k}{\partial \theta}\right]$$

$$= \frac{1}{\sigma^2}\Sigma\left[\cos('')(-\rho\sin('')) + \sin('')(\rho\cos(''))\right]$$

$$= 0$$

$$J_{31} = J_{13}$$

$$= \frac{2\pi\rho^2 T_a}{\sigma^2}P$$

$$J_{32} = J_{23} = 0$$

$$J_{33} = \frac{1}{\sigma^2}\Sigma\left[\frac{\partial \mu_k}{\partial \theta}\frac{\partial \mu_k}{\partial \theta} + \frac{\partial \nu_k}{\partial \theta}\frac{\partial \nu_k}{\partial \theta}\right]$$

$$= \frac{1}{\sigma^2}\Sigma\left[\rho^2\sin^2('') + \rho^2\cos('')\right]$$

$$= \frac{\rho^2 N}{\sigma^2}$$

$$J = \frac{1}{\cancel{\sigma^2}\,1}\begin{bmatrix} \rho^2 4\pi^2 T_a^2 Q & 0 & 2\pi\rho^2 T_a P \\ 0 & N & 0 \\ 2\pi\rho^2 T_a P & 0 & \rho^2 N \end{bmatrix}$$

b) ML estimator of $\underline{\alpha}$

$$f(\underline{Z}\,|\,\underline{\alpha}) = \left(\frac{1}{\sigma^2 2\pi}\right)^N \exp\left[-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}(x_k - \mu_k)^2 + (Y_k - \nu_k)^2\right]$$

$$\ln(f) = \ln\left(\tfrac{1}{\sigma^2 2\pi}\right)^N - \frac{1}{2\sigma^2}\sum\left[(X_k-\mu_k)^2 + (Y_k - \nu_k)^2\right]\Big)$$

$$= N\ln\left(\tfrac{1}{\sigma^2 2\pi}\right) - \;''$$

$$= -N\ln(\sigma^2 2\pi) - \frac{1}{2\sigma^2}\sum\left[(X_k-\mu_k)^2 + (Y_k-\nu_k)^2\right]$$

drop any constants since we are only interested in finding α that relatively maximizes L.

$$L_0 = -\frac{1}{N}\sum_k\left[(x_k-\mu_k)^2 + (Y_k-\nu_k)^2\right]$$

constants

$$= -\frac{1}{N}\sum_k\left[\cancel{x_k^2} - 2x_k\mu_k + \mu_k^2 + \cancel{Y_k^2} - 2Y_k\nu_k + \nu_k^2\right]$$

$$L = \frac{2}{N}\sum\left[x_k\mu_k + Y_k\mu_k\right] - \frac{1}{N}\sum(\mu_k^2 + \nu_k^2)$$

$$= \frac{2}{N}\sum\left[x_k\rho\cos(2\pi f k T_a + \theta) + Y_k\rho\sin(2\pi f k T_a + \theta)\right]$$

$$-\frac{1}{N}\sum\left[\rho^2\cos^2('') + \rho^2\sin('')\right]$$

$$= \frac{2}{N}\sum\left[X_k\rho\cos('') + Y_k\rho\sin('')\right] - \rho^2$$

$$= \frac{2\rho}{N}\sum\left[X_k\cos('') + Y_k\sin('')\right] - \rho^2$$

Next Page

- $Z_K = X_K + iY_K$
- $Z_K \exp(-i(2\pi f K T_a + \theta))$

$$= Z_K \,(\cos(2\pi f_K T_a + \theta) - j\sin(2\pi f_K T_a + \theta))$$

$$= X_K \cos('') + iY_K \cos('') - iX_K \sin('') + Y_K \sin('')$$

$$= X_K \cos('') + Y_K \sin('') - i[X_K \sin('') - Y_K \cos('')]$$

$$Re\{Z_K \exp('')\} = X_K \cos('') + Y_K \sin('')$$

$$L = \frac{2\rho}{N} \Sigma [X_K \cos('') + Y_K \sin('')] - \rho^2$$

$$= \frac{2\rho}{N} \Sigma Re\{Z_K \exp(-(i2\pi f K T_a + \theta))\} - \rho^2$$

$$A(f) = \frac{1}{N} \Sigma Z_K \exp(-i2\pi f K T_a)$$

- $L = 2\rho \, Re\{A(f) \exp(-i\theta)\}$ ← identical to eq. 24 from rife

$$\Sigma$$

$$\hat{f}_{ML} = \text{argmax}[A(f)]$$

$$\hat{\rho}_{ML} = \text{argmax}[A(\hat{f})]$$

$$\hat{\theta}_{ML} = \text{argmax}[\cancel{e^{-j\hat{a}st_0}}\,1\, A(\hat{f})]$$

3.1 c) ESTIMATE α

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Maximum likelihood estimation of frequency: 292.4 Hz
Maximum likelihood estimation of rho: 10.1
Maximum likelihood estimation of theta: 0.3 radians
```

4 Problem 4

4.1 a) Acquire and Track PRN 18

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----------------------------------------------------------
PRN :18
Apparent Doppler Frequency: -677 Hz
Approximate Start Time from first sample: 244.8958 microseconds
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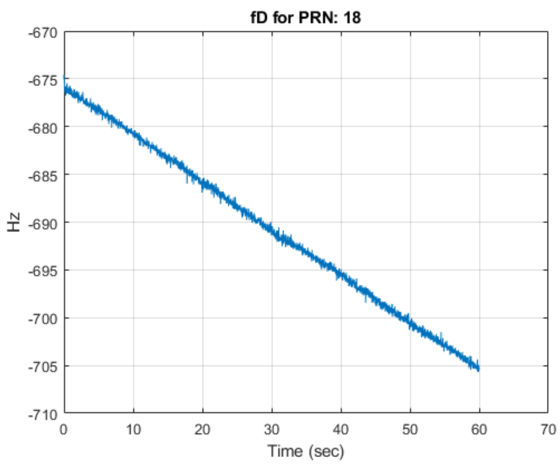

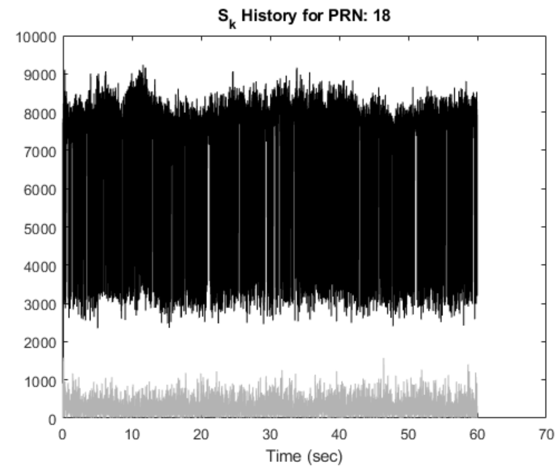

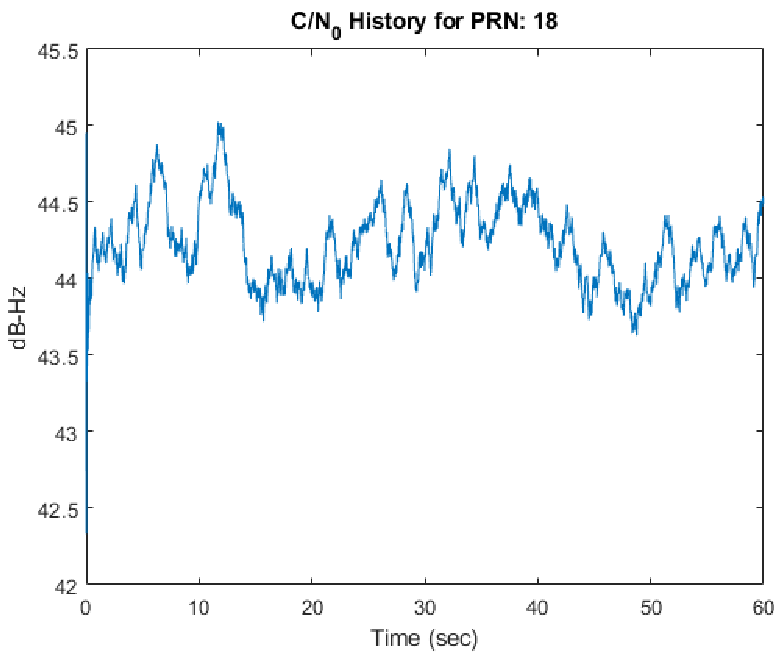

4.2 b) Acquire and Track 6 Strongest Signals

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-----------------------------------------------------------
PRN :10
Apparent Doppler Frequency: 2320 Hz
Approximate Start Time from first sample: 435.5208 microseconds
C/N0: 43.1884
SigmaIQ^2: 9361.8287
-----------------------------------------------------------
```
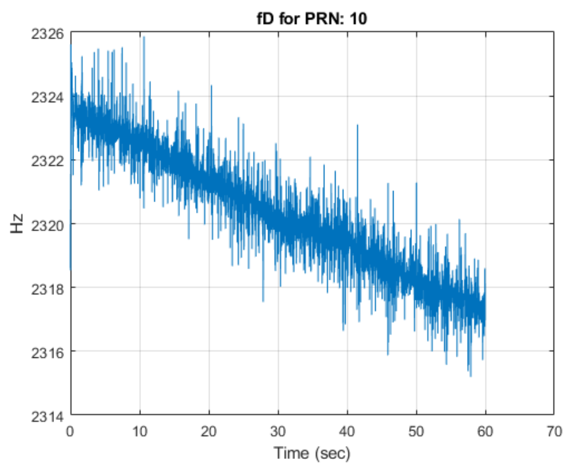
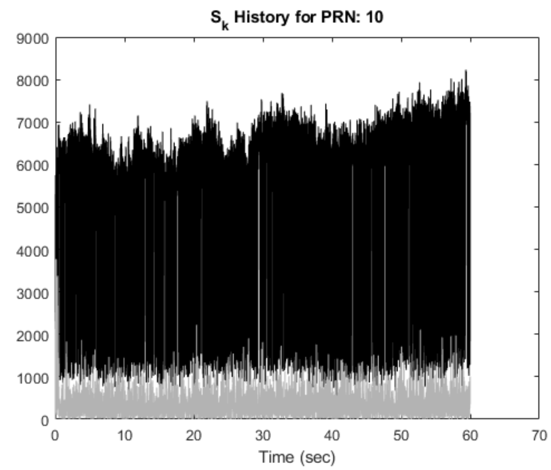
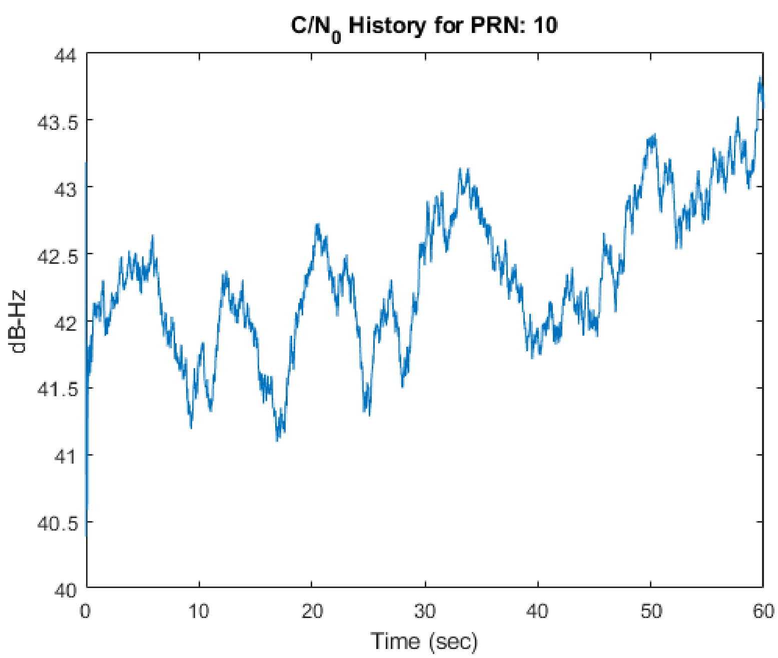
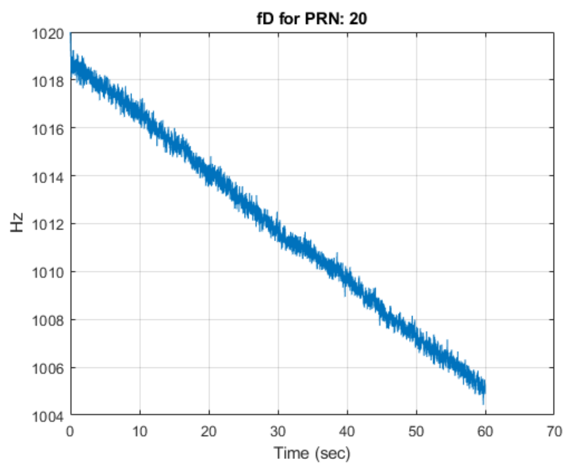
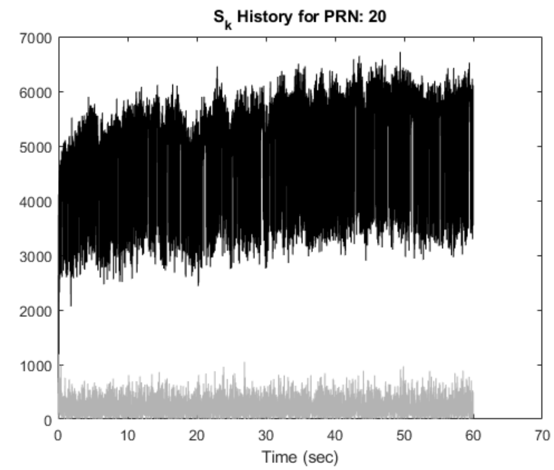



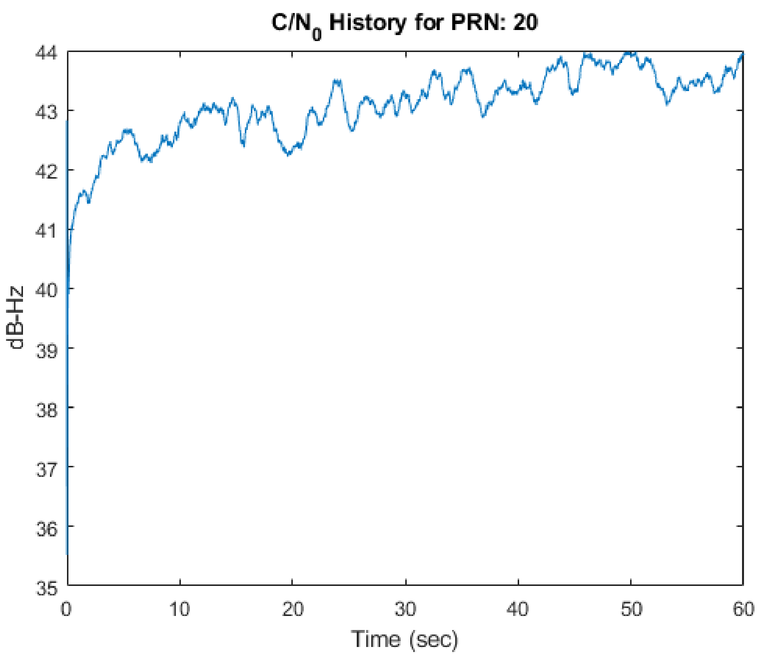




```
---------------------------------------------------------
PRN :20
Apparent Doppler Frequency: 1020 Hz
Approximate Start Time from first sample: 223.125 microseconds
C/N0: 42.821
SigmaIQ^2: 6396.4763
---------------------------------------------------------
```







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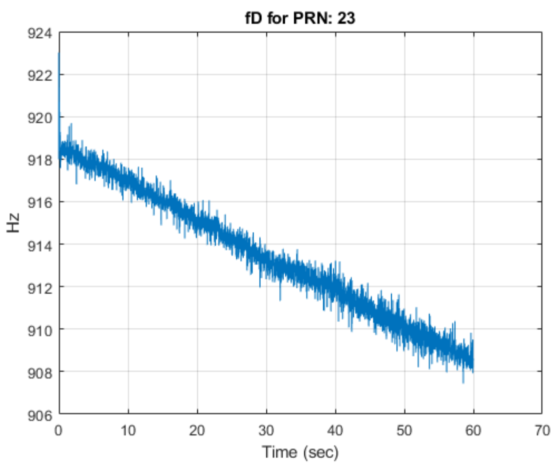
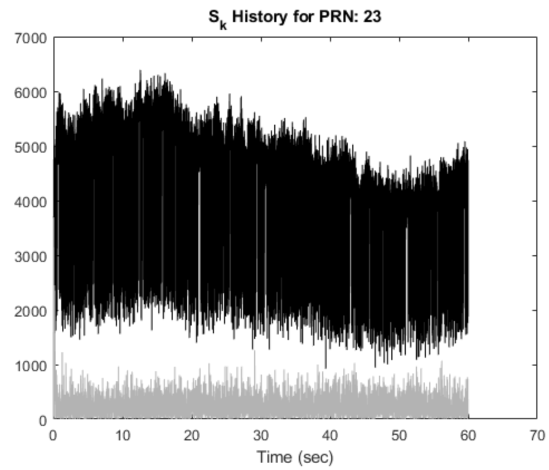
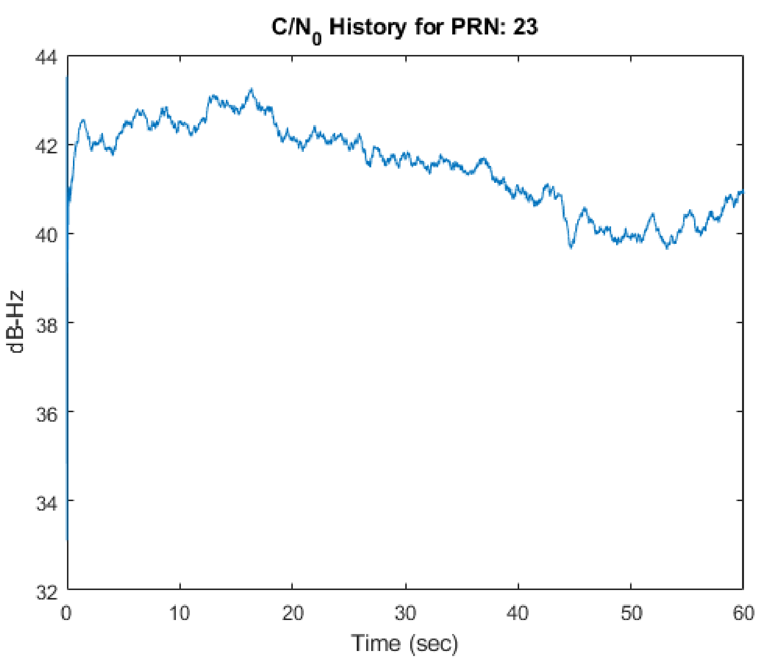
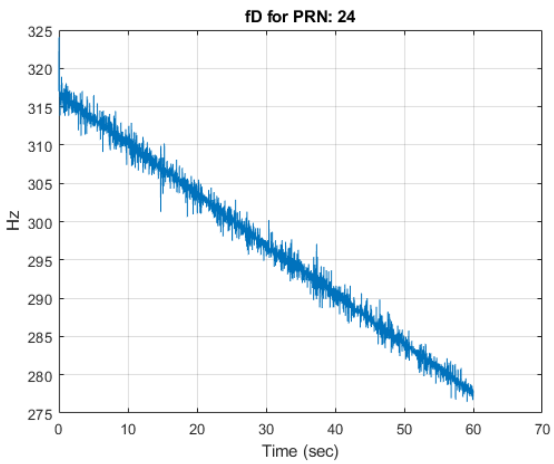
```
----------------------------------------------------------
PRN :23
Apparent Doppler Frequency: 918 Hz
Approximate Start Time from first sample: 352.2917 microseconds
C/N0: 43.5232
SigmaIQ^2: 6763.3873
----------------------------------------------------------
```

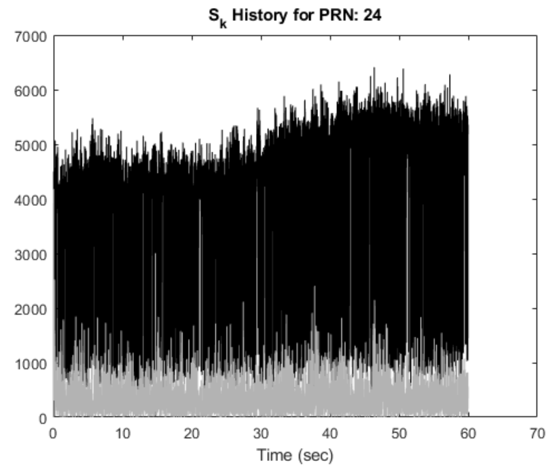


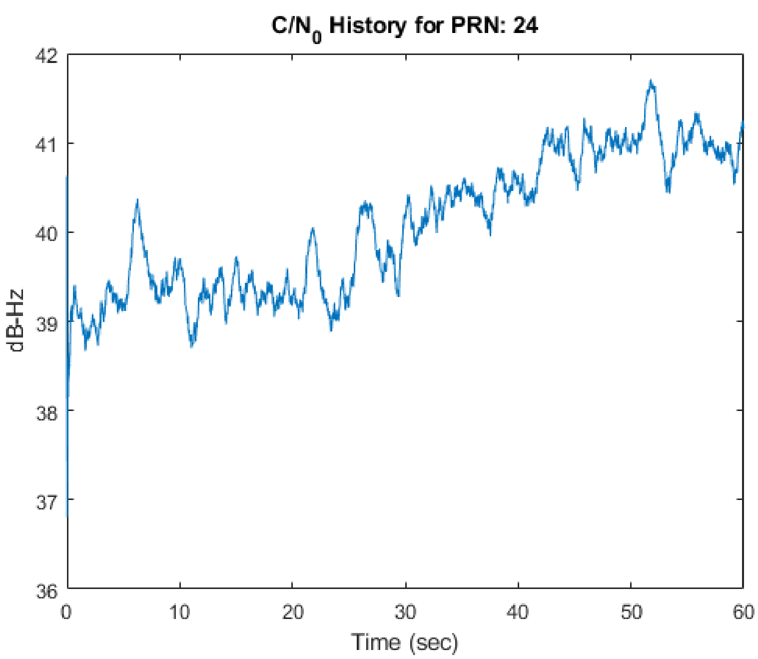




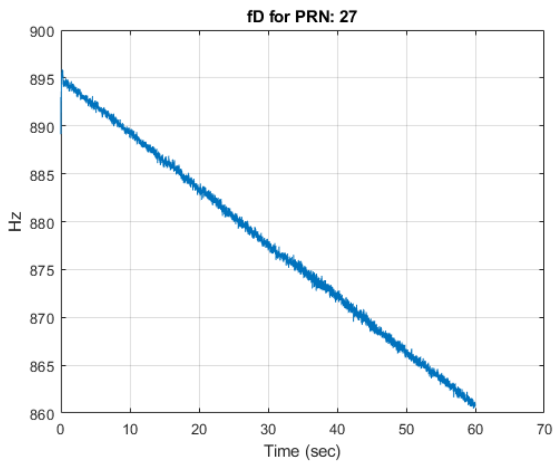
```
--------------------------------------------------------------
PRN :24
Apparent Doppler Frequency: 317 Hz
Approximate Start Time from first sample: 867.5 microseconds
C/N0: 40.6287
SigmaIQ^2: 8752.2037
--------------------------------------------------------------
```

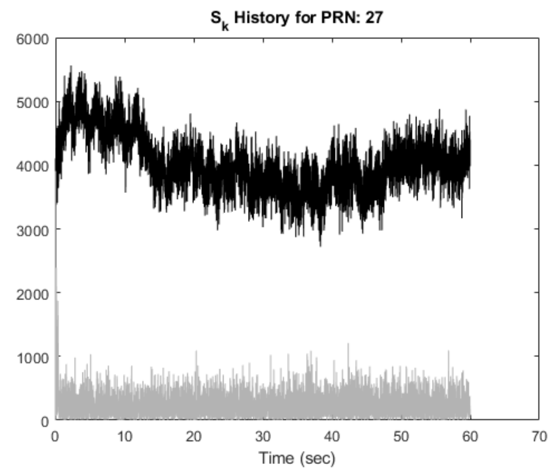


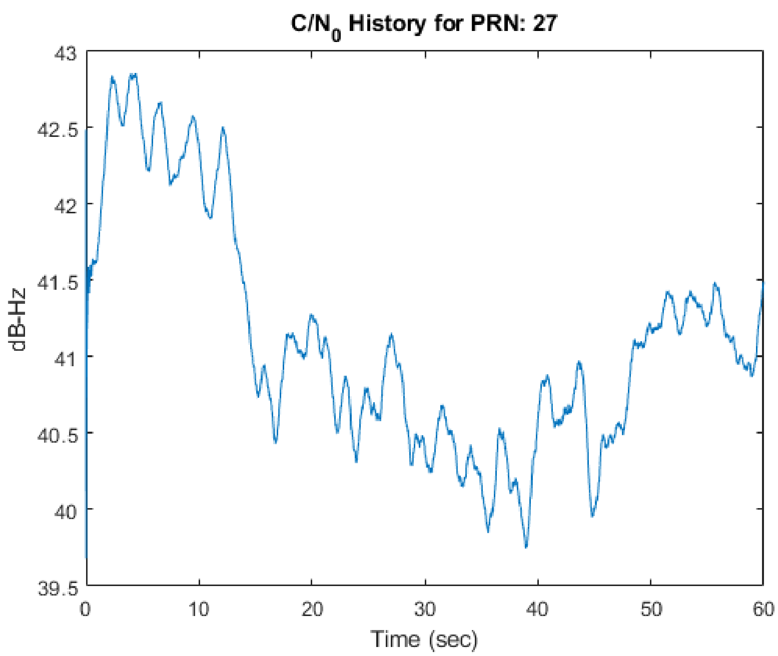




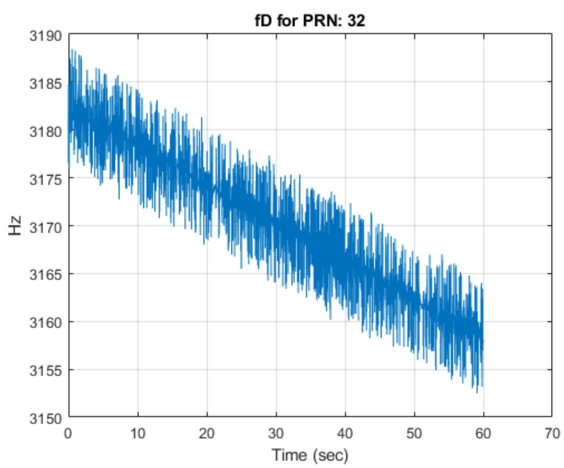
```
------------------------------------------------------------
PRN :27
Apparent Doppler Frequency: 893 Hz
Approximate Start Time from first sample: 225.3125 microseconds
C/N0: 42.4872
SigmaIQ^2: 6268.0666
------------------------------------------------------------
```

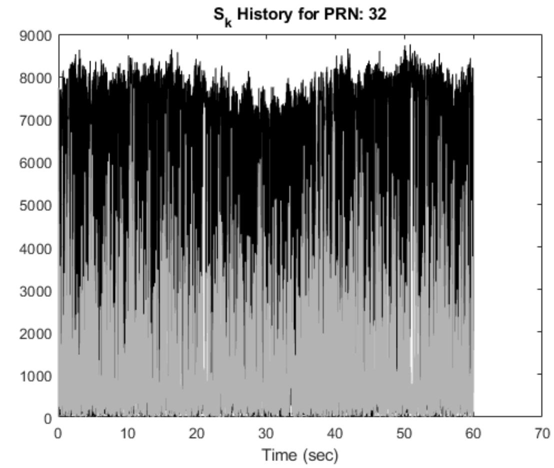


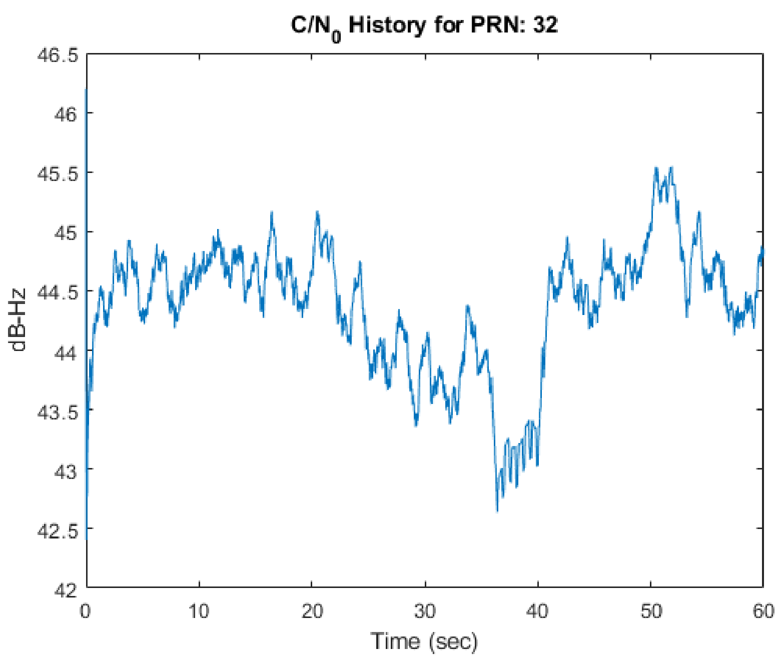




```
-----------------------------------------------------------
PRN :32
Apparent Doppler Frequency: 3182 Hz
Approximate Start Time from first sample: 299.0625 microseconds
C/N0: 46.1993
SigmaIQ^2: 8218.6548
```

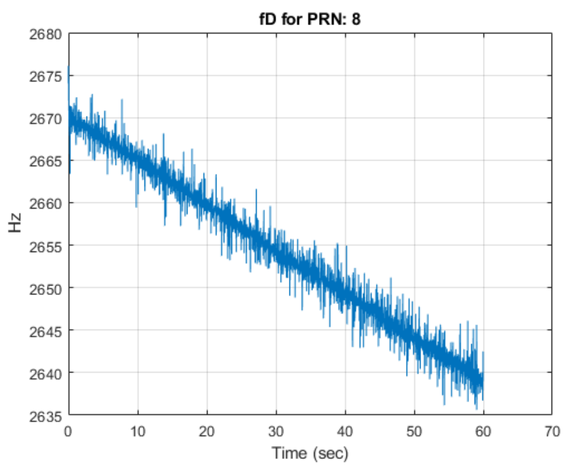


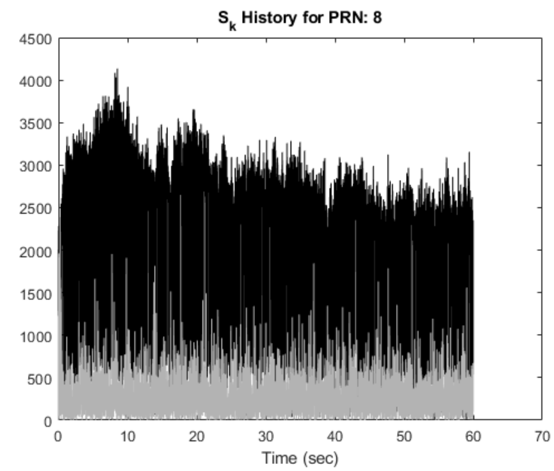


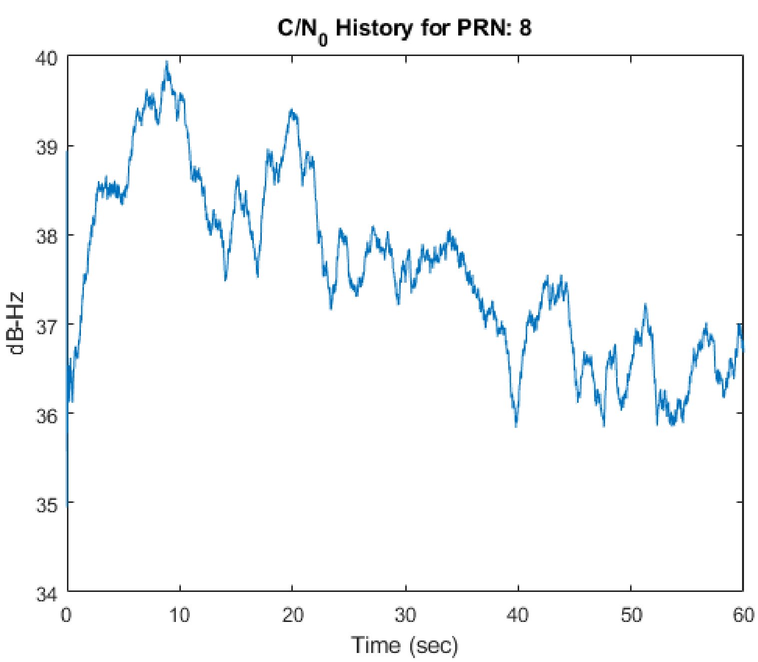


4.3 c) Acquire and Track the 2 Weakest signals

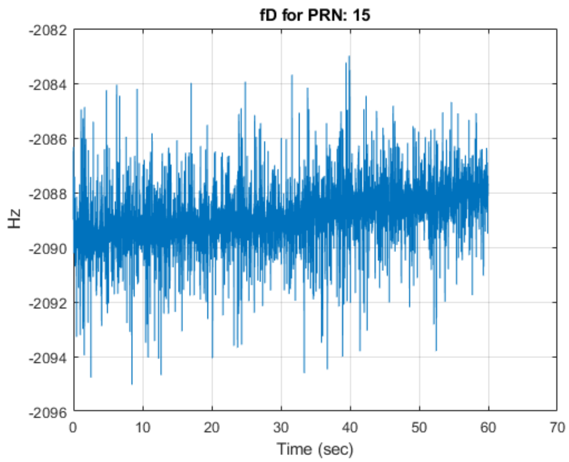
```
----------------------------------------------------------
PRN :8
Apparent Doppler Frequency: 2674 Hz
Approximate Start Time from first sample: 60.9375 microseconds
C/N0: 38.9348
SigmaIQ^2: 4871.8628
```

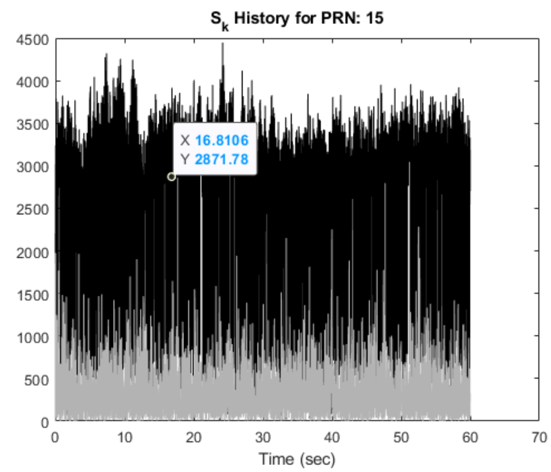


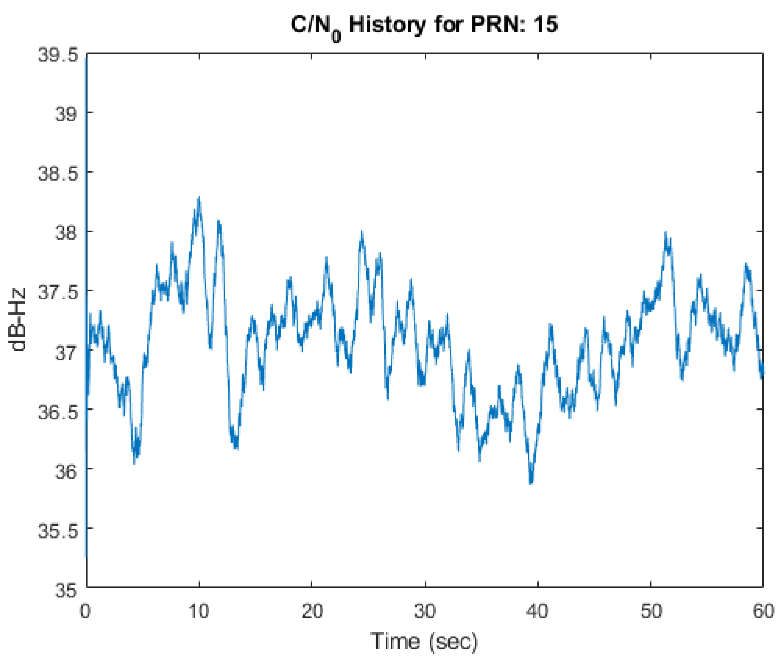




```
---------------------------------------------------------
PRN :15
Apparent Doppler Frequency: -2089 Hz
Approximate Start Time from first sample: 560.8333 microseconds
C/N0: 39.4561
SigmaIQ^2: 7851.8812
```







5 Problem 5

The correct location of the ECEF is F with $\delta t_R = 0.031416$ second.

```
-----------------------------------------------------------
Pseudoranges
 PRN 10: 30703522.193 m
 PRN 18: 30403729.738 m
 PRN 24: 31602899.566 m
 PRN 32: 31003314.648 m
-----------------------------------------------------------
Reciever ECEF Postion estimate
  x: -755431.157  m
  y: -5464585.448 m
  z: 3191025.861  m
δt R : 3.141600e-02 seconds
```