The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

29 AUGUST 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Wing Types & Parameters
- Aircraft Notations
- Review of Statics
- Review of Dynamics

Wing : 2 g

T : 0.62

Rud : 0.21

Canopy : 0.21

Fuse : 2.08

Metal 1.75

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WING TYPES & PARAMETERS

From Stengel + Nelson + Etkin & Reid

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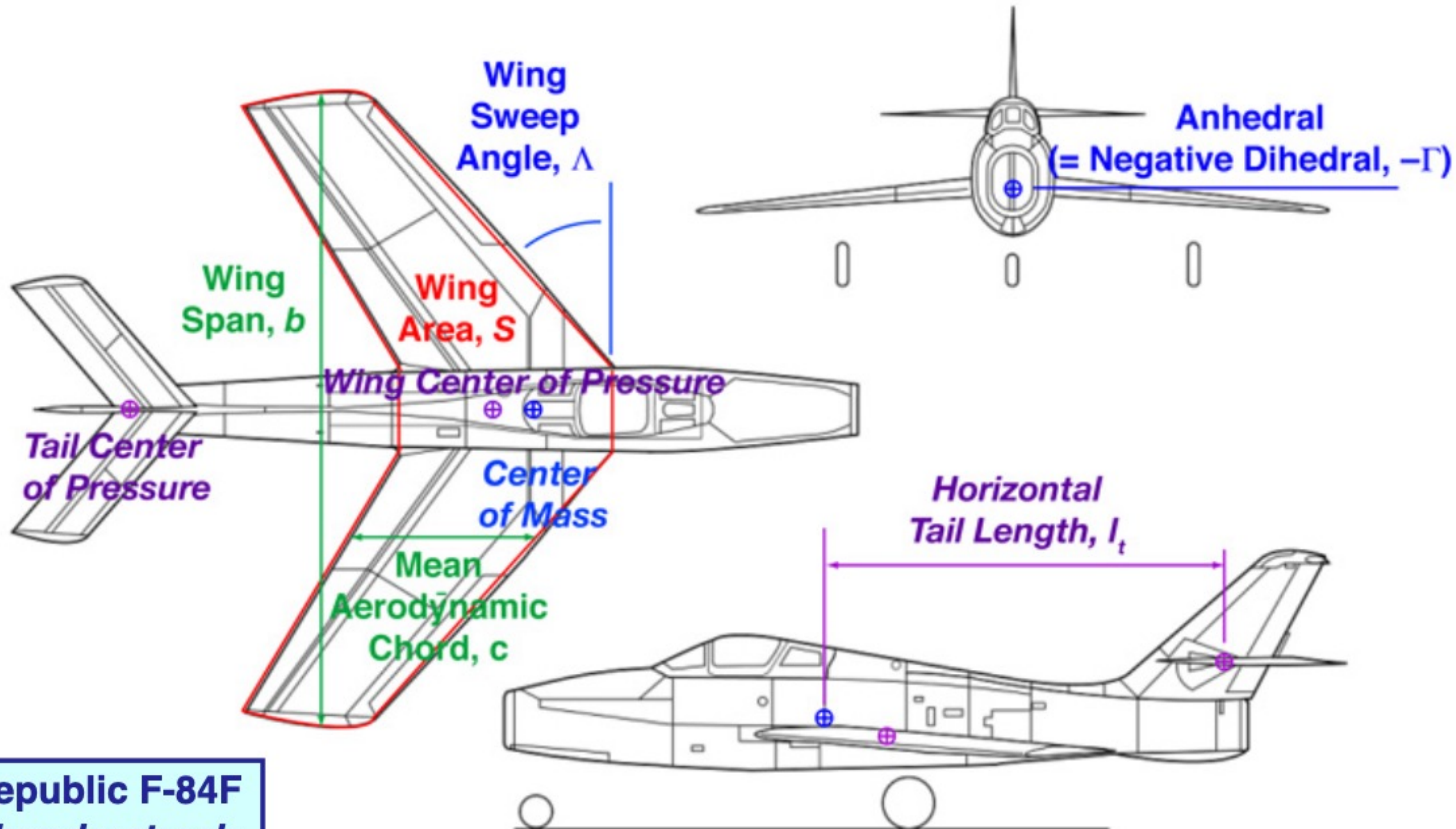
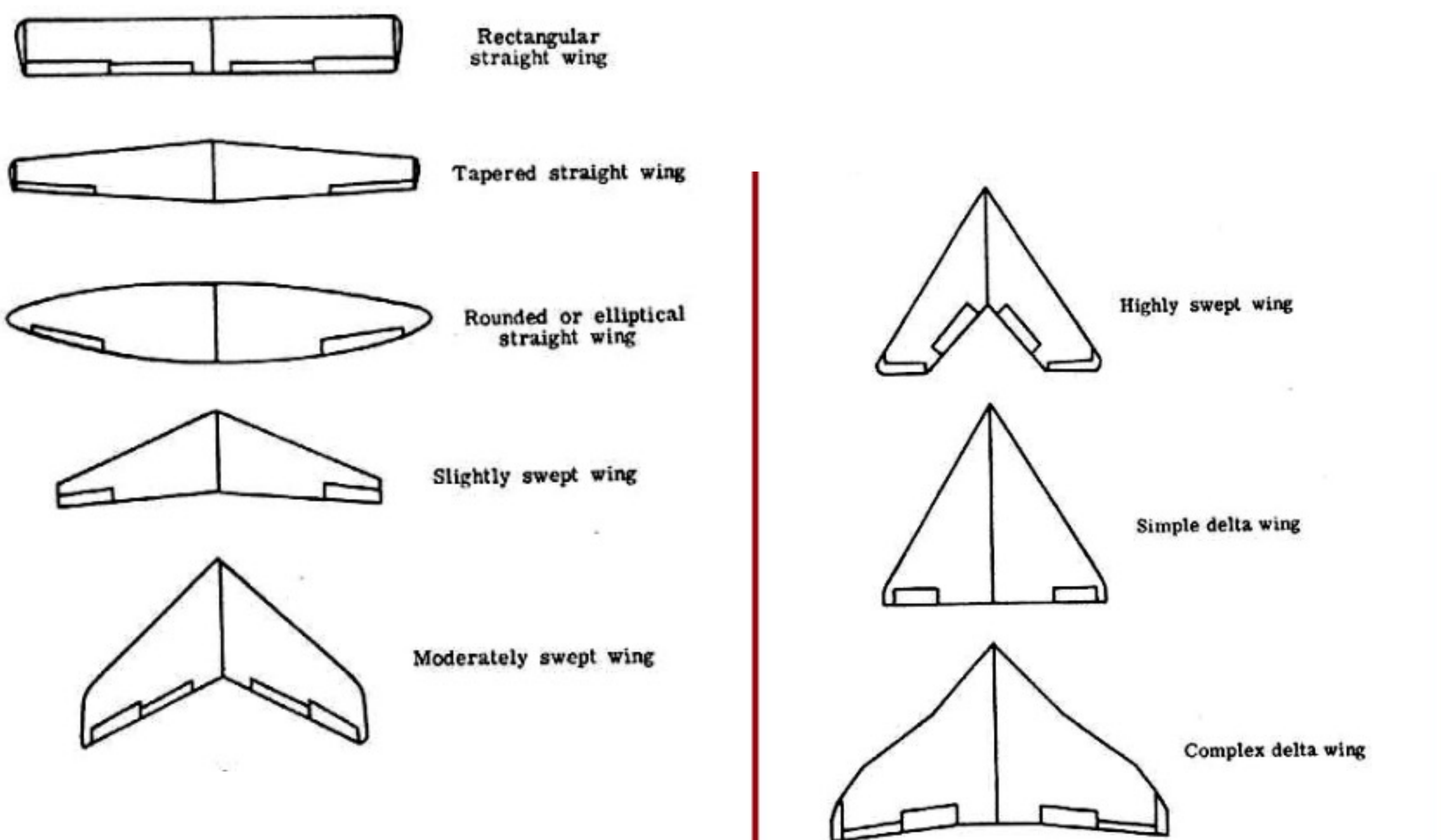
A Few Definitions



Republic F-84F
Thunderstreak



(b) Examples of wing planform.

Talay, NASA SP-367

Wing Planform Variables

Aspect Ratio

$$AR = \frac{b}{c} \quad \text{rectangular wing}$$

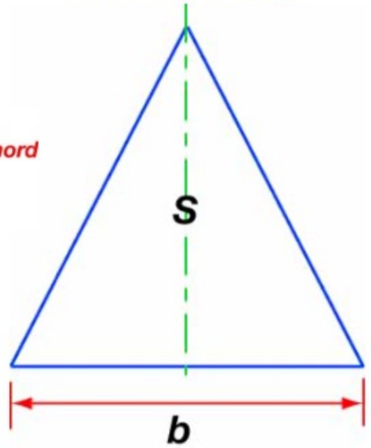
$$= \frac{b \times b}{c \times b} = \frac{b^2}{S} \quad \text{any wing}$$

Taper Ratio

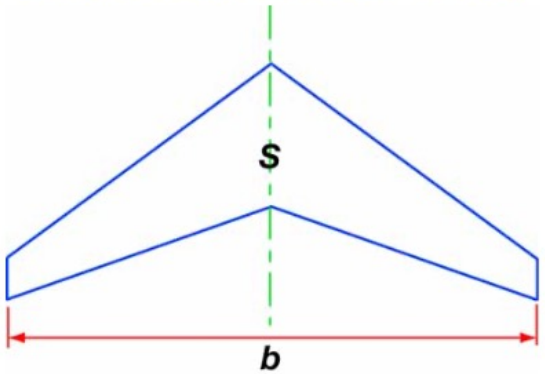
$$\lambda = \frac{c_{tip}}{c_{root}} = \frac{\text{tip chord}}{\text{root chord}}$$

Rectangular Wing

Delta Wing



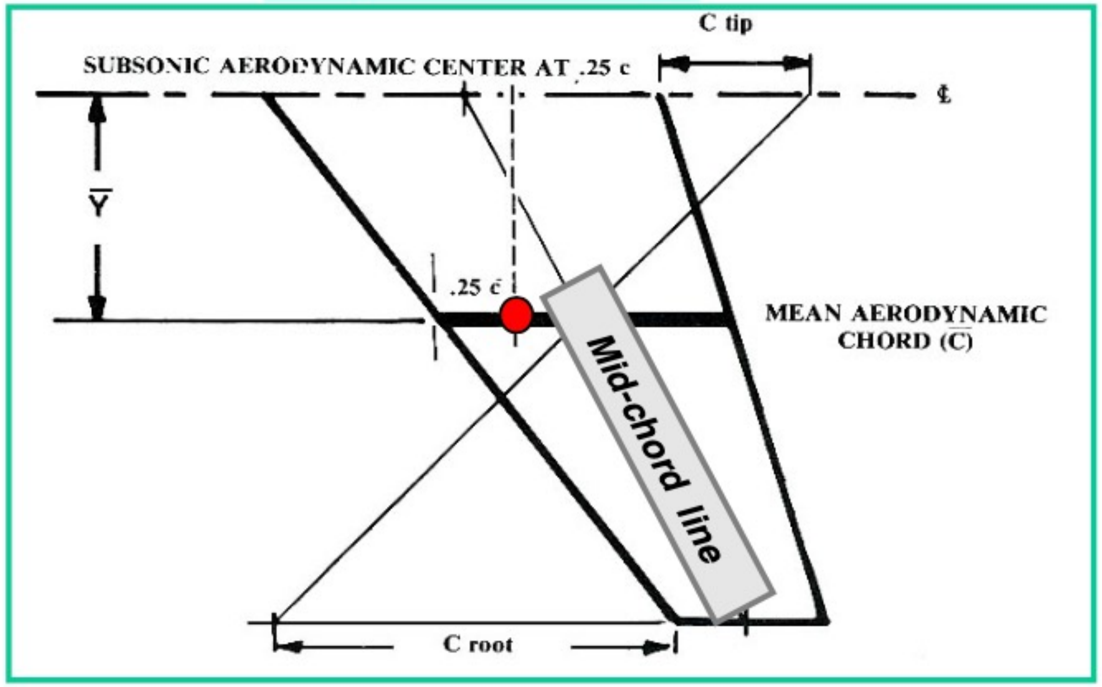
Swept Trapezoidal Wing



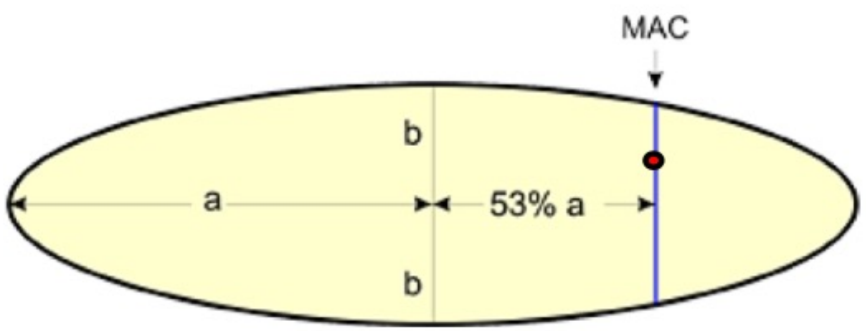
Location of Mean Aerodynamic Chord and Aerodynamic Center

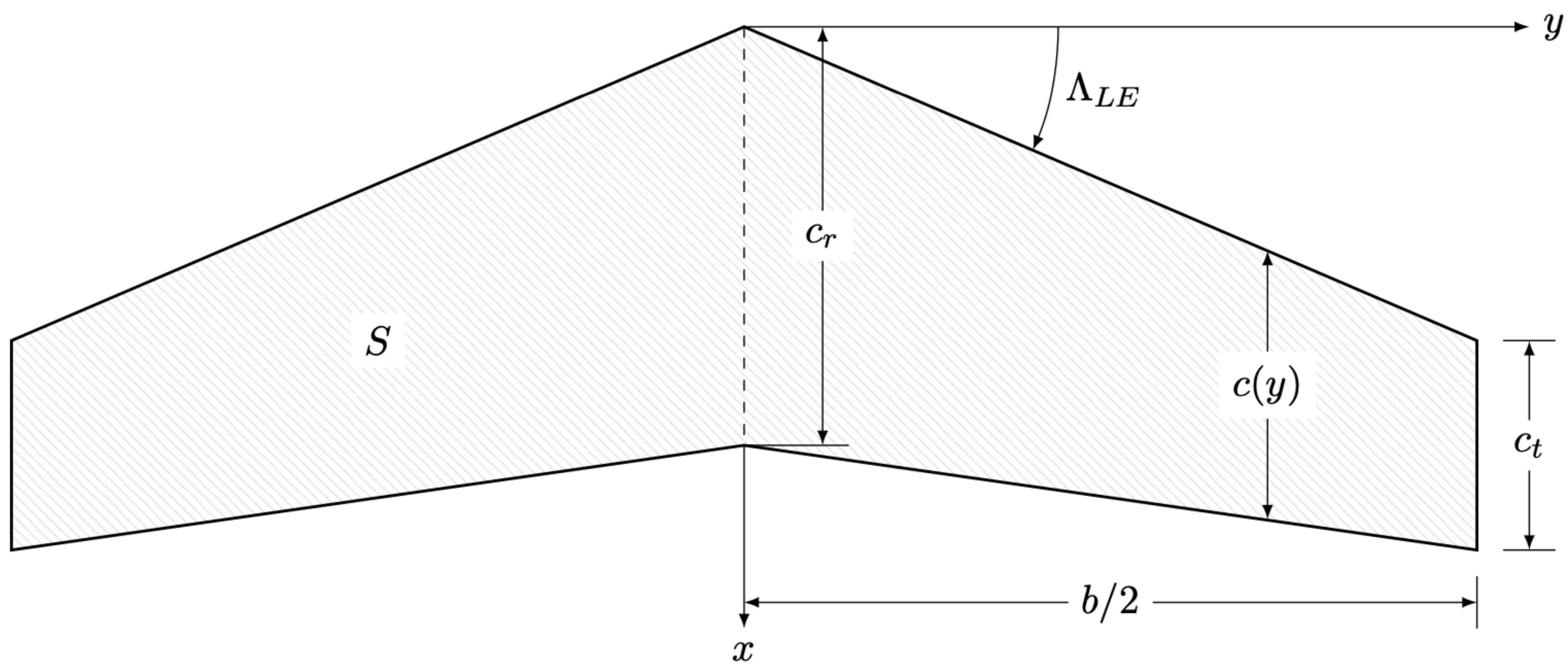
- Axial location of the wing's <u>subsonic</u> aerodynamic center (a.c.)
  - Determine spanwise location of m.a.c.
  - Assume that aerodynamic center is at 25% m.a.c.

Trapezoidal Wing

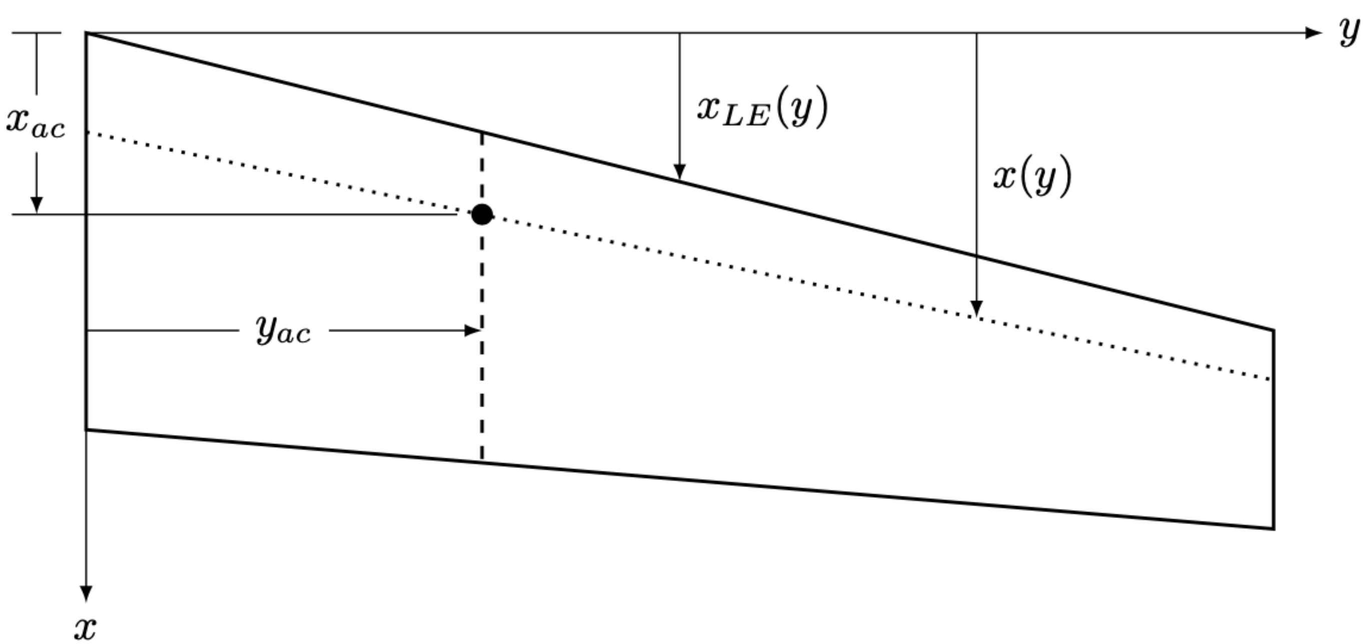


Elliptical Wing





y
Λ_{LE}
c_r
S
$c(y)$
c_t
$b/2$
x

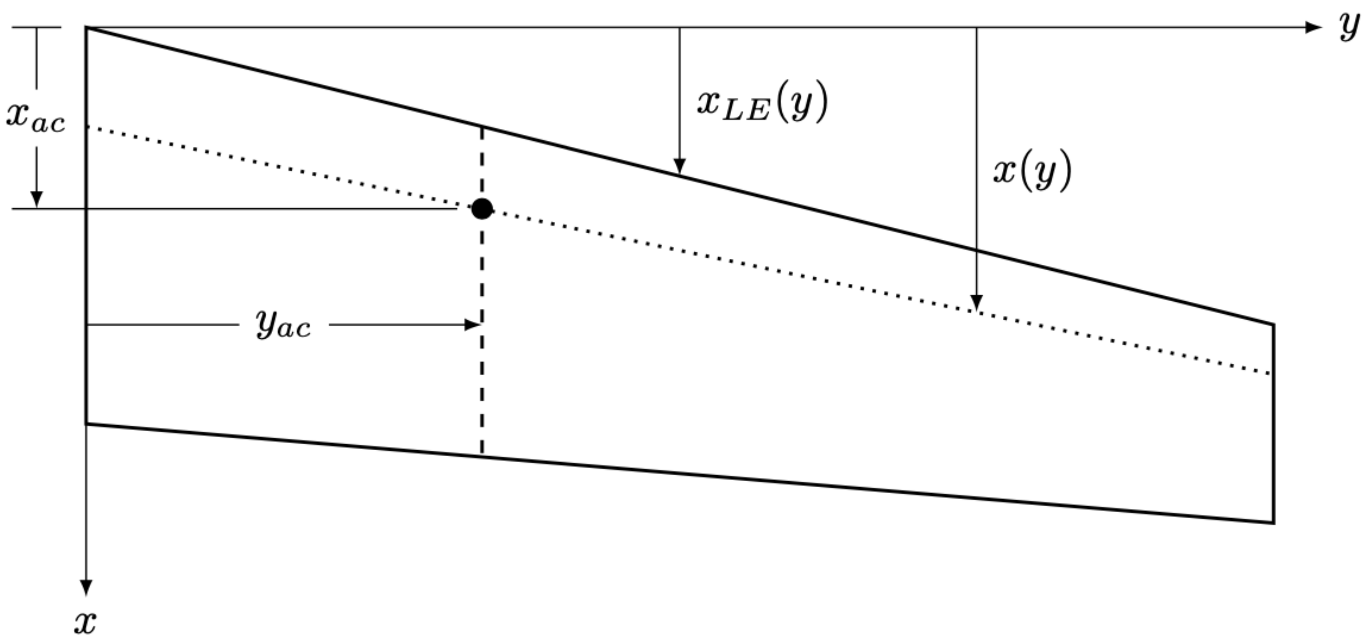


$$S = 2\int_0^{b/2} c(y)\,\mathrm{d}y$$

$$x_{ac} = \frac{2}{S}\int_0^{b/2} x(y)c(y)\,\mathrm{d}y$$

$$y_{ac} = \frac{2}{S}\int_0^{b/2} yc(y)\,\mathrm{d}y$$

$$\bar{c} = \frac{2}{S}\int_0^{b/2} c^2(y)\,\mathrm{d}y$$



The Boeing 737-900 has a wing span of 112.60 ft, a root chord of 25.85 ft, a tip chord of 4.10 ft, and a quarter-chord wing sweep of 25.02 deg. Assuming that the planform is trapezoidal, you are to:

(a) Determine the planform area.
(b) Determine the aspect ratio.
(c) Determine the leading edge wing sweep.
(d) Determine the trailing edge wing sweep.
(e) Determine the mean aerodynamic chord.
(f) Determine the lateral location of the aerodynamic center.
(g) Determine the longitudinal location of the aerodynamic center.

(a) $S = \frac{b}{2} c_r (1+\lambda) = \frac{112.6}{2} \times 25.85 \times (1 + 0.1586) = 1686.2\ ft^2$

where $\lambda = \frac{c_t}{c_r} = 0.1586$.

(b) $AR = \frac{b^2}{S} = \frac{112.6^2}{1686.2} = 7.52$

(c) $\Lambda_{LE} = \arctan\left(tan\left(\Lambda_{\frac{1}{4}}\right) + \frac{4p(1-\lambda)}{AR(1+\lambda)}\right) = 29.39°$

Where $\Lambda_{\frac{1}{4}} = 25.02°$ and $p = \frac{1}{4} = 0.25$.

(d) $\Lambda_{TE} = \arctan\left(\tan\left(\Lambda_{\frac{1}{4}}\right) - \frac{4(1-p)(1-\lambda)}{AR(1+\lambda)}\right) = 10.04°$

Where $\Lambda_{\frac{1}{4}} = 25.02°$ and $p = \frac{1}{4} = 0.25$.

(e) $\bar{c} = \frac{2}{S}\int_0^{\frac{b}{2}} c^2 dy = \frac{2}{3} c_r \left(\frac{\lambda^2+\lambda+1}{\lambda+1}\right) = 17.61\ ft$

(f) $y_{ac} = \frac{b}{6} \cdot \frac{1+2\lambda}{1+\lambda} = 21.34$ ft

(g) $x_{ac} = p \cdot \bar{c} + \tan(\Lambda_{LE}) \cdot y_{ac} = 16.42\ ft$

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AIRCRAFT NOTATIONS

From Stengel + Etkin & Reid

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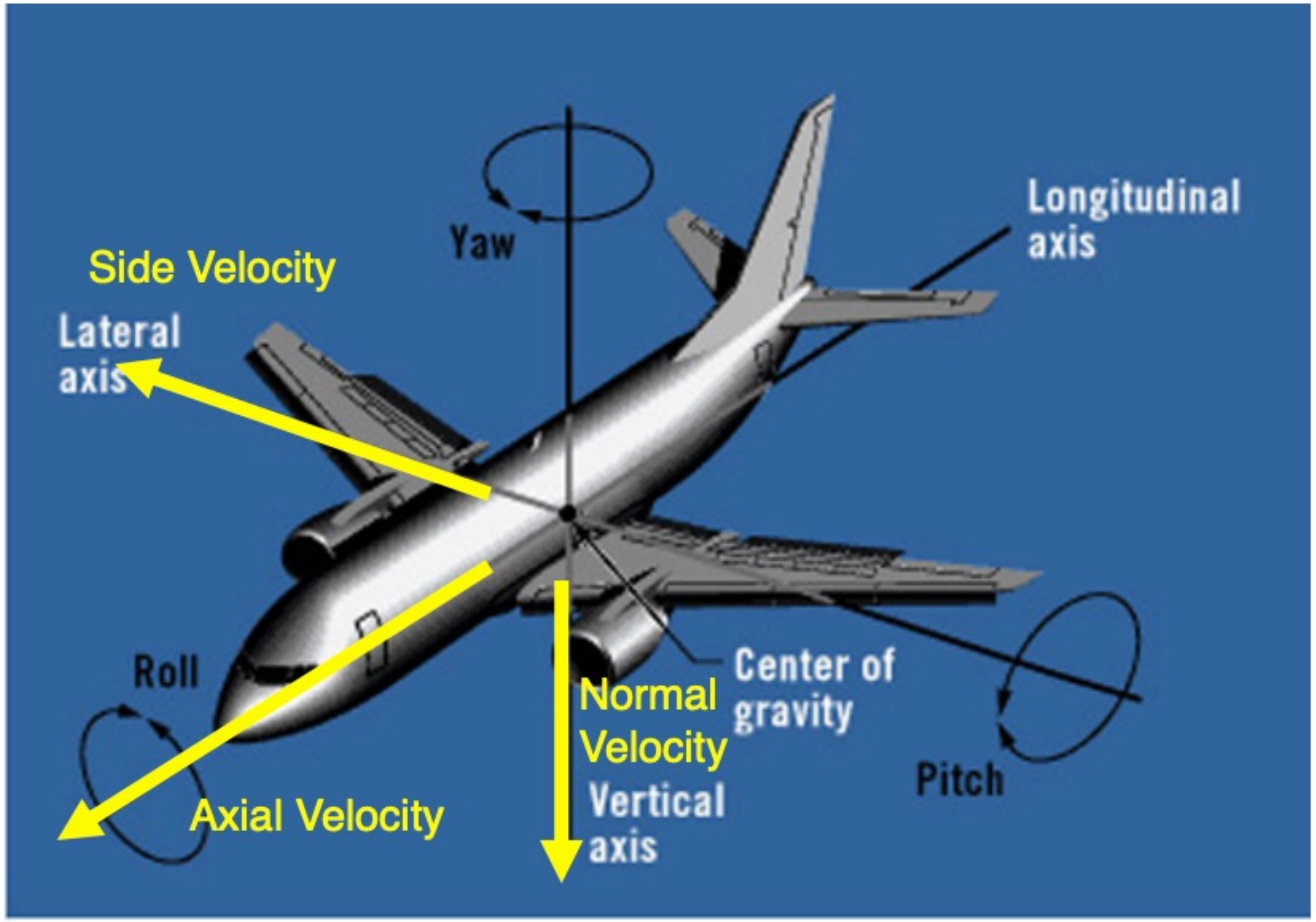
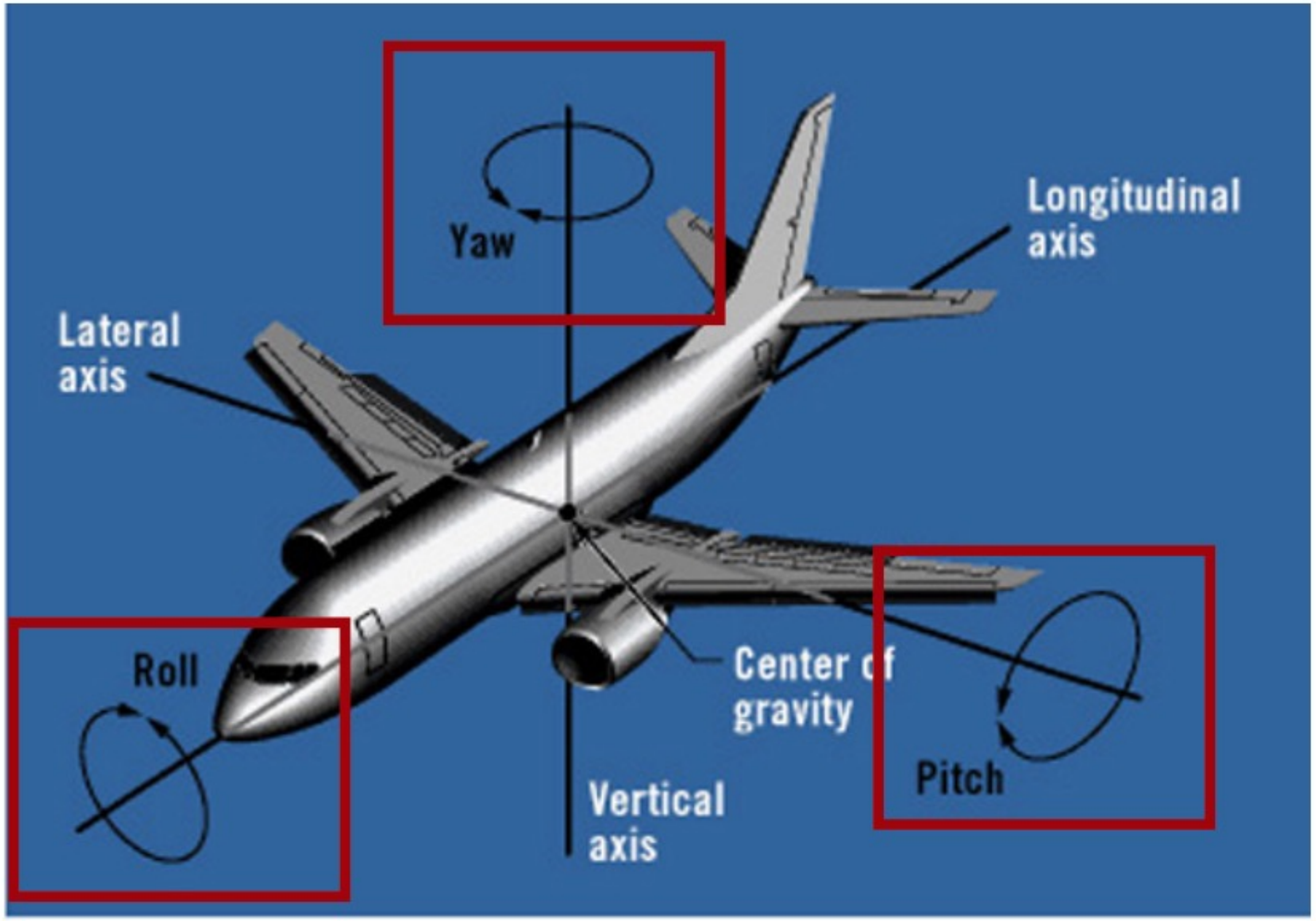
Airplane Translational Degrees of Freedom



Airplane Rotational Degrees of Freedom



Body Axes Forces, Moments, Velocities, Rates

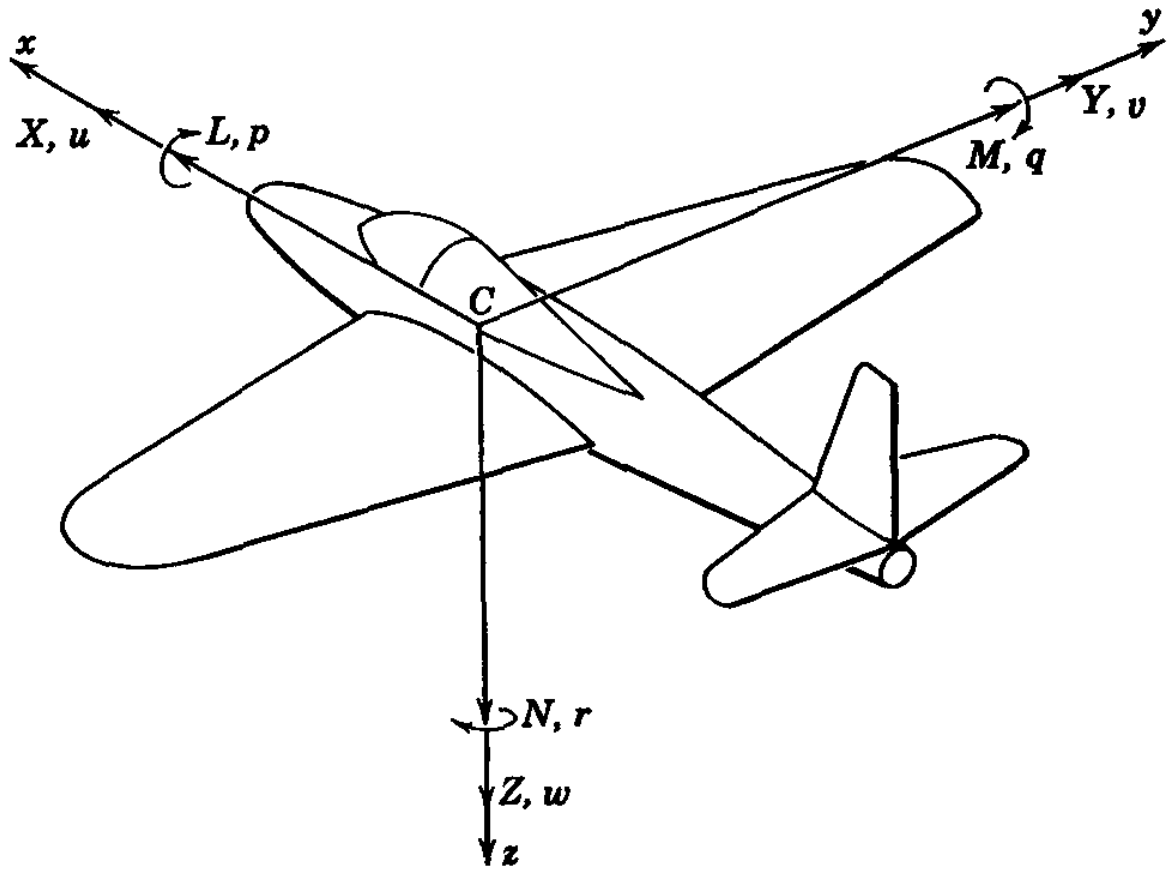


Figure 1.6 Notation for body axes. L = rolling moment, M = pitching moment, N = yawing moment, p = rate of roll, q = rate of pitch, r = rate of yaw. $[X, Y, Z]$ = components of resultant aerodynamic force. $[u, v, w]$ = components of velocity of C relative to atmosphere.

Angle of Attack and Sideslip

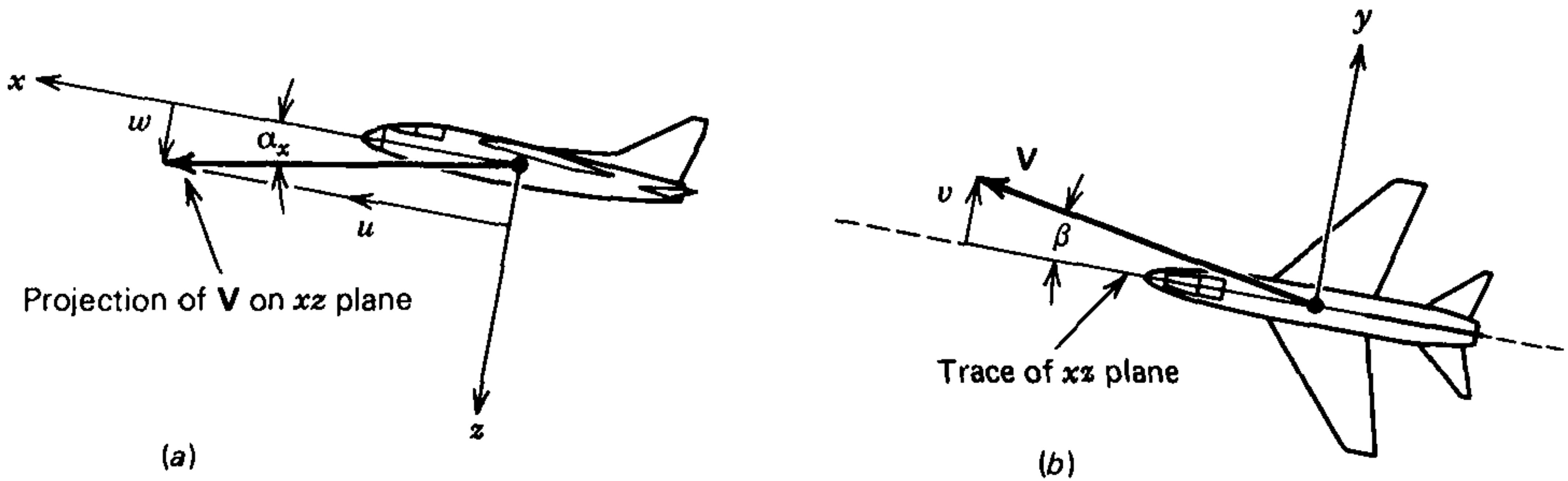


Figure 1.7 (*a*) Definition of α_x. (*b*) View in plane of y and $\mathbf{V}$, definition of β.

Angle of attack, $\alpha_x = \tan^{-1} \dfrac{w}{u}$

Angle of sideslip, $\beta = \sin^{-1} \dfrac{v}{V}$

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REVIEW OF STATICS

From Clarke

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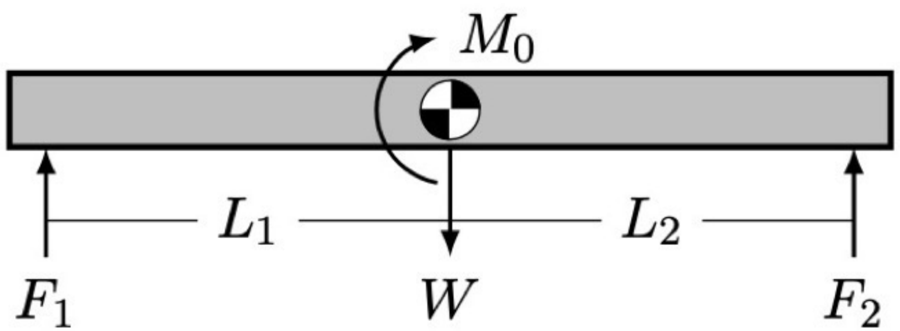
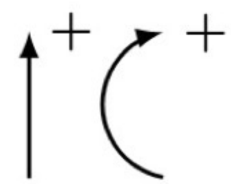




Question: What values of F_1 and F_2 will achieve equilibrium?

$$\sum_i F_i = 0$$

$$\Rightarrow F_1 + F_2 - W = 0$$

$$\sum_i M_i = 0$$

$$\Rightarrow M_0 + L_1 F_1 - L_2 F_2 = 0$$

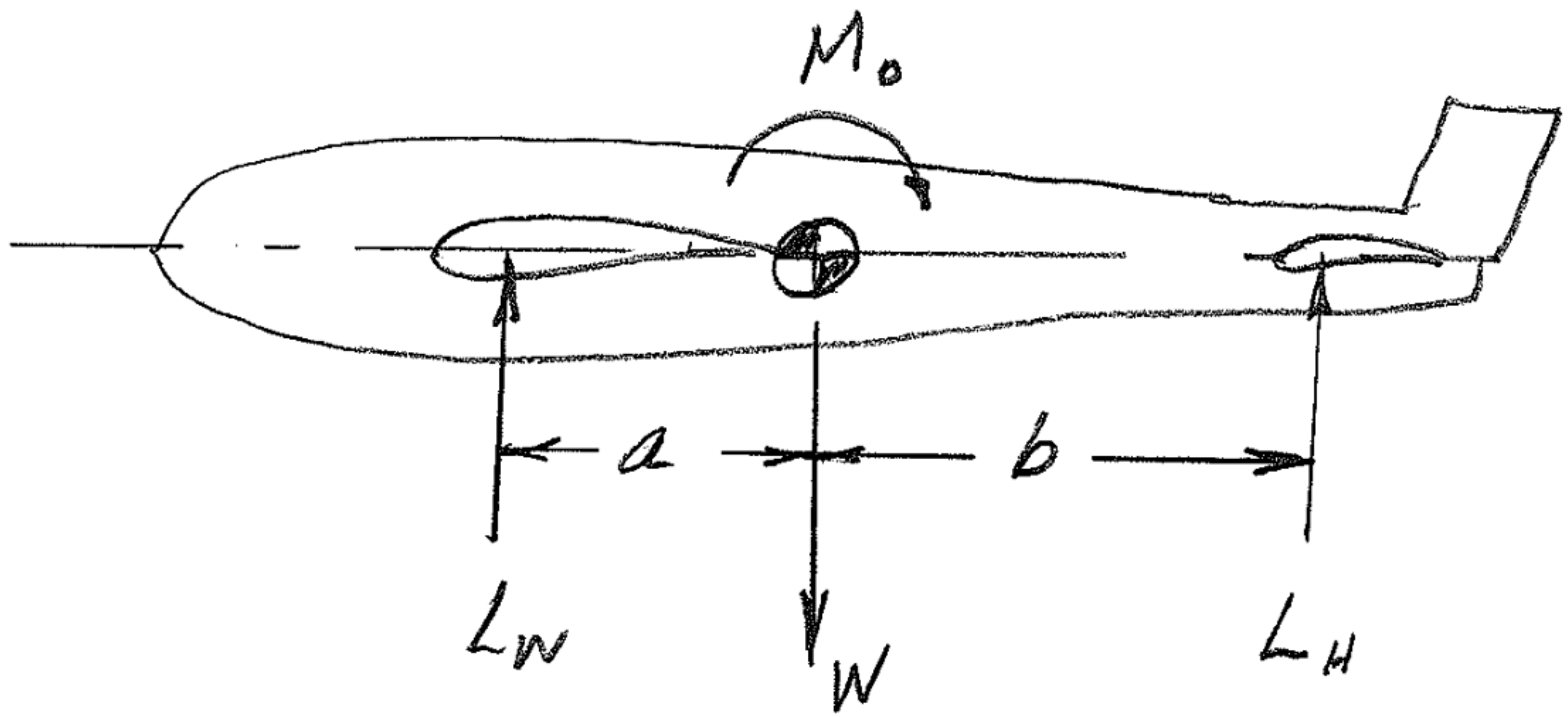
$$\underbrace{\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ L_1 & -L_2 \end{bmatrix}^{-1} \begin{bmatrix} W \\ -M_0 \end{bmatrix}}$$

$$\Rightarrow \boldsymbol{x} = A^{-1} \boldsymbol{b}$$

$$A \quad \boldsymbol{x} = \boldsymbol{b}$$

$$\overbrace{\begin{bmatrix} 1 & 1 \\ L_1 & -L_2 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} W \\ -M_0 \end{bmatrix}}$$

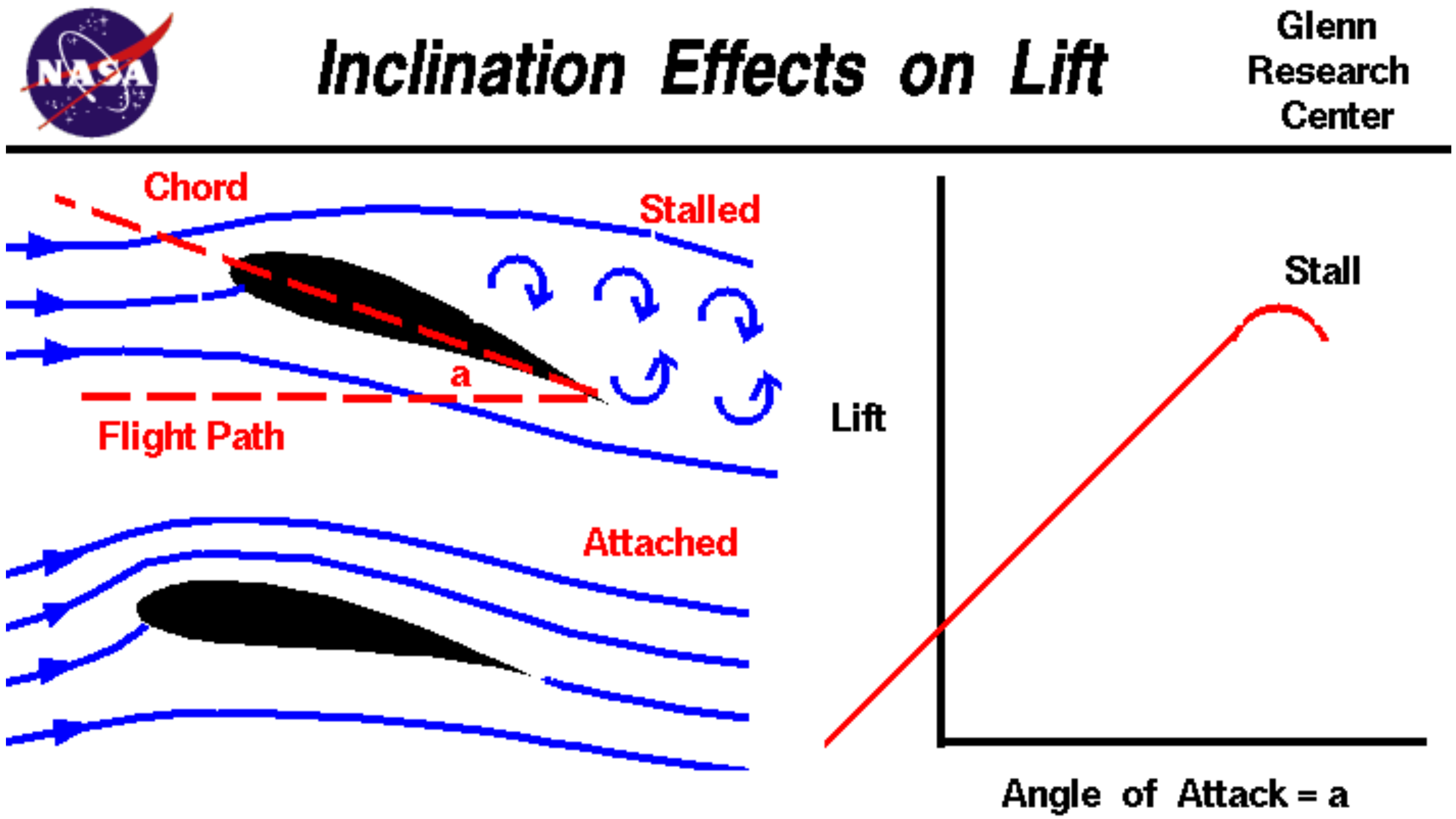
Question: How does statics related to flight dynamics?



$$L_W = \tfrac{1}{2}\rho V^2 S C_{L_W} \quad \text{where} \quad C_{L_W} = C_{L_{W_0}} + C_{L_{W_\alpha}}\alpha$$

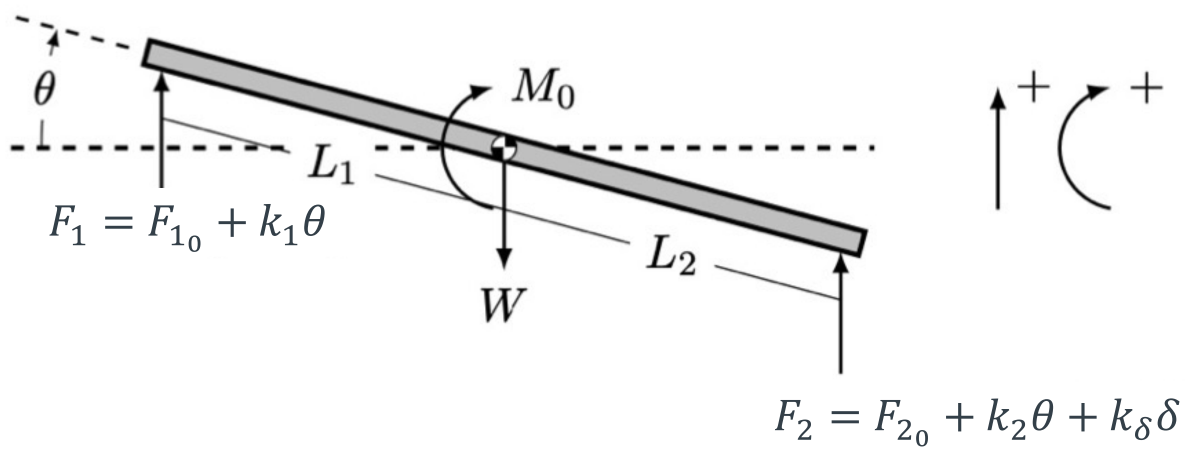
$$L_H = \tfrac{1}{2}\rho V^2 S_H C_{L_H} \quad \text{where} \quad C_{L_H} = C_{L_{H_0}} + C_{L_{H_\alpha}}\alpha + C_{L_{H_\delta}}\delta$$

Question: Is it reasonable to assume that lift is a linear function of α?



For small angles, lift is related to angle.
Greater Angle = Greater Lift

For larger angles, the lift relation is complex.
Included in Lift Coefficient

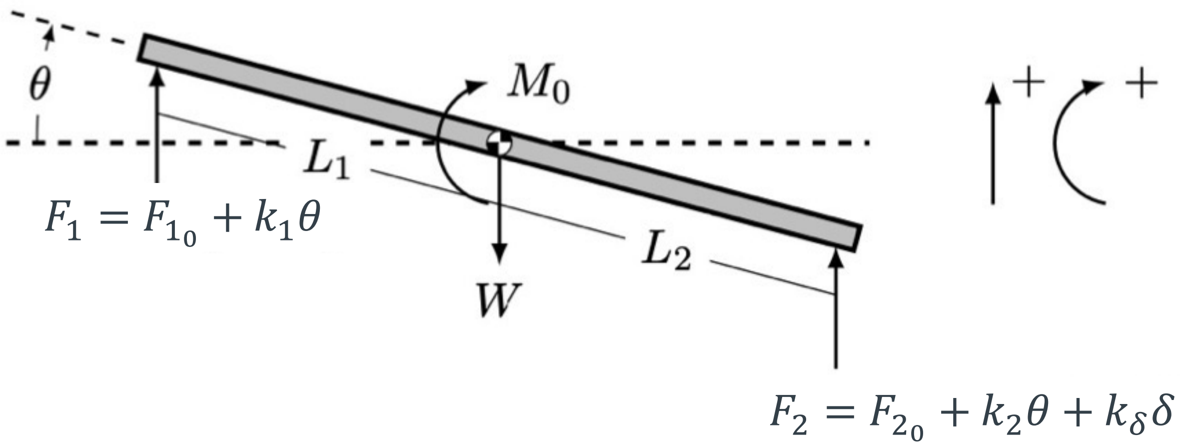


Thought: Let's model a beam with a pivot at it's CG

Assume: α and θ are equal, and θ and δ are small…

Question: What values of θ_0 and δ_0 will achieve trim?

$$\left.\begin{aligned}
&\sum_i F_i = 0 \\
&\qquad \Rightarrow F_{1_0} + k_1\theta + F_{2_0} + k_2\theta + k_\delta\delta - W = 0 \\
&\sum_i M_i = 0 \\
&\qquad \Rightarrow M_0 + L_1\left(F_{1_0} + k_1\theta\right) - L_2\left(F_{2_0} + k_2\theta + k_\delta\delta\right) = 0
\end{aligned}\right\} \cdots$$



Thought: Let's model a beam with a pivot at it's CG

Assume: α and θ are equal, and θ and δ are small…

Question: What values of θ_0 and δ_0 will achieve trim?

$$\begin{bmatrix} F_{1_0} + F_{2_0} \\ L_1 F_{1_0} - L_2 F_{2_0} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & k_\delta \\ L_1 k_1 - L_2 k_2 & -L_2 k_\delta \end{bmatrix} \begin{bmatrix} \theta \\ \delta \end{bmatrix} = \begin{bmatrix} W \\ -M_0 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} \theta_0 \\ \delta_0 \end{bmatrix} = \begin{bmatrix} k_1 + k_2 & k_\delta \\ L_1 k_1 - L_2 k_2 & -L_2 k_\delta \end{bmatrix}^{-1} \left(\begin{bmatrix} W \\ -M_0 \end{bmatrix} - \begin{bmatrix} F_{1_0} + F_{2_0} \\ L_1 F_{1_0} - L_2 F_{2_0} \end{bmatrix} \right)$$

Question: Given that an aircraft in trim condition is in equilibrium…
Can we simplify our model?

If:

$$\begin{bmatrix}\theta\\ \delta\end{bmatrix} = \begin{bmatrix}\theta_0\\ \delta_0\end{bmatrix} + \begin{bmatrix}\Delta\theta\\ \Delta\delta\end{bmatrix}$$

Then:

$$\begin{bmatrix}F_{1_0}+F_{2_0}\\ L_1F_{1_0}-L_2F_{2_0}\end{bmatrix} + \begin{bmatrix}k_1+k_2 & k_\delta\\ L_1k_1-L_2k_2 & -L_2k_\delta\end{bmatrix}\left(\begin{bmatrix}\theta_0\\ \delta_0\end{bmatrix} + \begin{bmatrix}\Delta\theta\\ \Delta\delta\end{bmatrix}\right) = \begin{bmatrix}W\\ -M_0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} F_{1_0} + F_{2_0} \\ L_1 F_{1_0} - L_2 F_{2_0} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & k_\delta \\ L_1 k_1 - L_2 k_2 & -L_2 k_\delta \end{bmatrix} \begin{bmatrix} \theta_0 \\ \delta_0 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & k_\delta \\ L_1 k_1 - L_2 k_2 & -L_2 k_\delta \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\delta \end{bmatrix} = \begin{bmatrix} W \\ -M_0 \end{bmatrix}$$

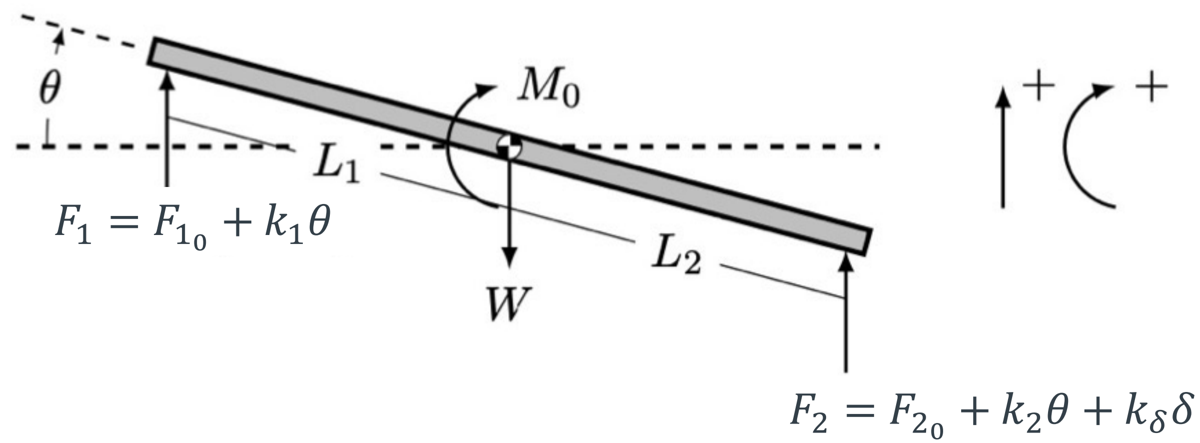
$$\Rightarrow \cancel{\begin{bmatrix} F_{1_0} + F_{2_0} \\ L_1 F_{1_0} - L_2 F_{2_0} \end{bmatrix}} + \cancel{\begin{bmatrix} W \\ -M_0 \end{bmatrix}} - \cancel{\begin{bmatrix} F_{1_0} + F_{2_0} \\ L_1 F_{1_0} - L_2 F_{2_0} \end{bmatrix}} + \begin{bmatrix} k_1 + k_2 & k_\delta \\ L_1 k_1 - L_2 k_2 & -L_2 k_\delta \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\delta \end{bmatrix} = \cancel{\begin{bmatrix} W \\ -M_0 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} k_1 + k_2 & k_\delta \\ L_1 k_1 - L_2 k_2 & -L_2 k_\delta \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Question: Given that an aircraft in trim condition is in equilibrium…
Can we simplify our model?

Let:

$$C_{L_\theta} = k_1 + k_2$$
$$C_{L_\delta} = k_\delta$$
$$C_{M_\theta} = L_1 k_1 - L_2 k_2$$
$$C_{M_\delta} = -L_2 k_\delta$$



Then:

$$\begin{bmatrix} C_{L_\theta} & C_{L_\delta} \\ C_{M_\theta} & C_{M_\delta} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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REVIEW OF DYNAMICS

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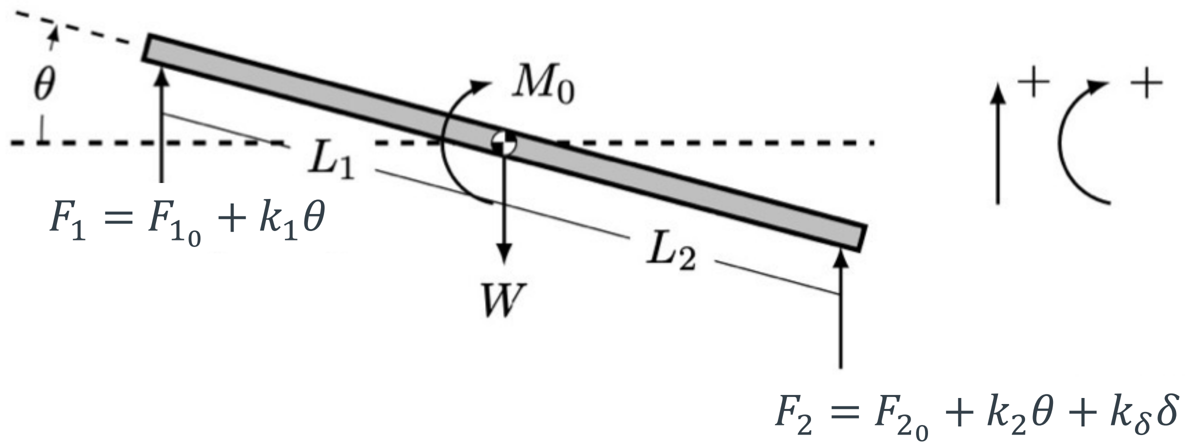


Question: What happens when the body is in motion?

$$\sum F_V = m\ddot{h} \qquad \text{and} \qquad \sum M_{CG} = I\ddot{\theta}$$

$$\begin{bmatrix} F_{1_0} + F_{2_0} \\ L_1 F_{1_0} - L_2 F_{2_0} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & k_\delta \\ L_1 k_1 - L_2 k_2 & -L_2 k_\delta \end{bmatrix} \begin{bmatrix} \theta \\ \delta \end{bmatrix} + \begin{bmatrix} -W \\ M_0 \end{bmatrix} = \begin{bmatrix} m\ddot{h} \\ I\ddot{\theta} \end{bmatrix}$$

$$\Downarrow$$

System of Differential Equations!

Question: Given that an aircraft in trim condition is in equilibrium…
Can we model the dynamic behavior in a simpler way?

If:

$$\begin{bmatrix} \theta \\ \delta \end{bmatrix} = \begin{bmatrix} \theta_0 \\ \delta_0 \end{bmatrix} + \begin{bmatrix} \Delta\theta \\ \Delta\delta \end{bmatrix}$$

Then:

$$\begin{bmatrix} C_{L_\theta} & C_{L_\delta} \\ C_{M_\theta} & C_{M_\delta} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\delta \end{bmatrix} = \begin{bmatrix} m\ddot{h} \\ I\ddot{\theta} \end{bmatrix}$$

Further: Taking the mass and the moment of inertia to the "other side"…

$$\begin{bmatrix} \ddot{h} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{gC_{L_\theta}}{W} & \frac{gC_{L_\delta}}{W} \\ \frac{C_{M_\theta}}{I} & \frac{C_{M_\delta}}{I} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\delta \end{bmatrix}$$

$$\Downarrow$$

Much Simpler!

Question: What happens if we hold the elevator at its trim angle, i.e., $\Delta\delta = 0$

$$\begin{bmatrix} \ddot{h} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{gC_{L_\theta}}{W} & \frac{gC_{L_\delta}}{W} \\ \frac{C_{M_\theta}}{I} & \frac{C_{M_\delta}}{I} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$$\ddot{h} = \frac{gC_{L_\theta}}{W}\Delta\theta \quad \text{and} \quad \ddot{\theta} = \frac{C_{M_\theta}}{I}\Delta\theta$$

$$\Downarrow$$

PENDULUM

Question: What happens if $C_{M_\theta} > 0\ldots$

Recall: $C_{M_\theta} = L_1 k_1 - L_2 k_2$

$$\Delta\theta(t) = \mathrm{A}e^{-\lambda t} + \mathrm{B}e^{\lambda t}$$

$$\Rightarrow \quad \mathrm{A} = \tfrac{1}{2}\left(\Delta\theta(0) - \frac{\dot{\theta}(0)}{\lambda}\right)$$

$$\mathrm{B} = \tfrac{1}{2}\left(\Delta\theta(0) + \frac{\dot{\theta}(0)}{\lambda}\right)$$

$$\lambda^2 = \frac{C_{M_\theta}}{I} \quad \Rightarrow \quad \text{UNSTABLE}$$

Question: What happens if $C_{M_\theta} < 0$

Recall: $C_{M_\theta} = L_1 k_1 - L_2 k_2$

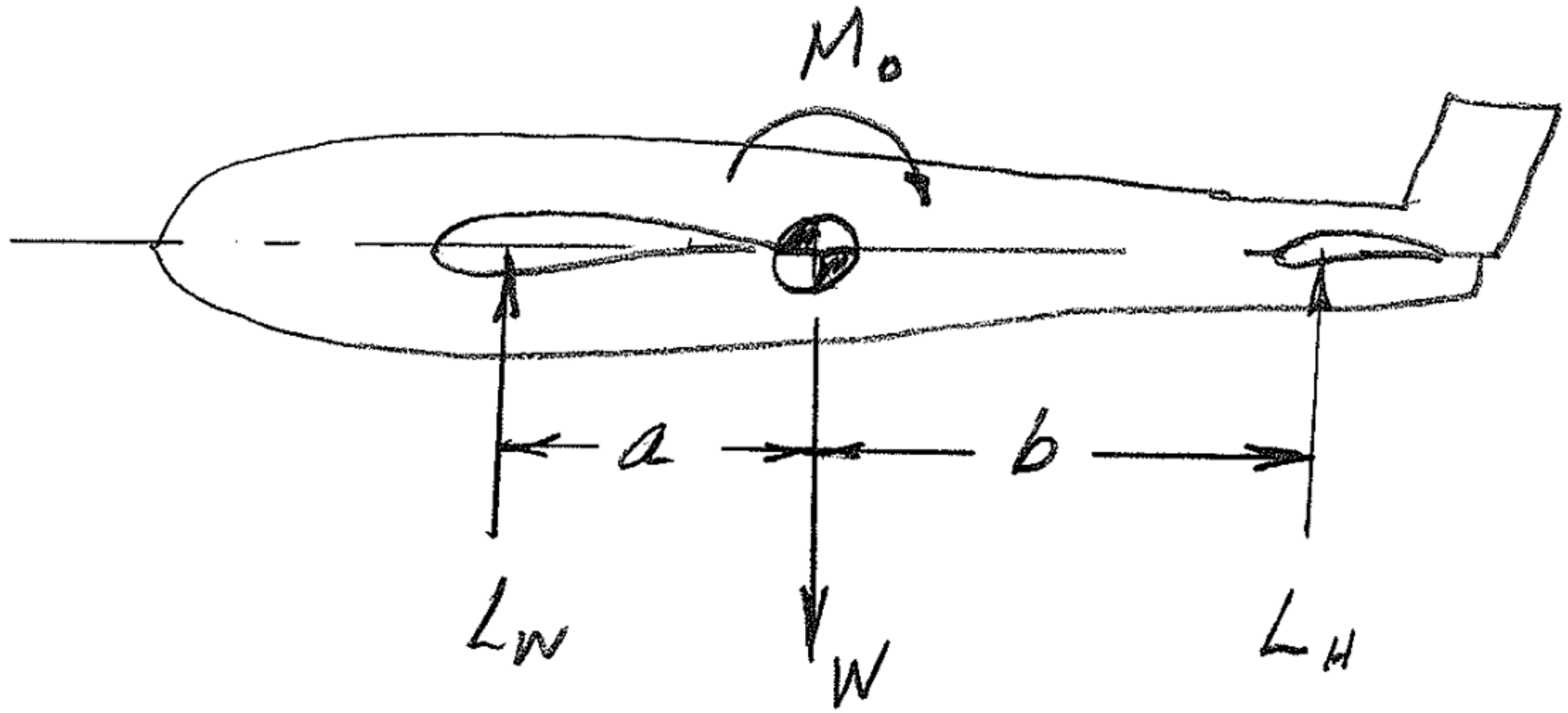
$$\Delta\theta(t) = \mathrm{A}\cos\omega t + \mathrm{B}\sin\omega t$$

➡ $$\mathrm{A} = \Delta\theta(0)$$

$$\mathrm{B} = \frac{\dot{\theta}(0)}{\omega}$$

$$\omega^2 = -\frac{C_{M_\theta}}{I}$$ ➡ OSCILLATORY

Question: How does all this mean?



Answer: The moment from the tail must dominate the moment from the wing and…
If CG is forward of the wing lift vector (i.e., the aerodynamic center of the wing) then the tail must provide a downward force.

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