11/10/24

① : 16 T/F

② : Fill in Blanks $\nearrow 2$

; 2 Multiple choice

③ : 4 one word answer given figure

④ : Nominal sign of lateral stability derivatives

Gliding flight

Equilibrium Gliding Flight

$$C_D \frac{1}{2}\rho V^2 S = -W \sin\gamma$$

$$C_L \frac{1}{2}\rho V^2 S = W \cos\gamma$$

$$\dot{h} = V \sin\gamma$$

$$\dot{r} = V \cos\gamma$$

Max Range? or you want to minimize Δh ← minimize fuel burn rate

occurs at max $(L/D)_{max}$

Sink Rate

$\frac{dh}{dt} \leftarrow \frac{dh}{dt} = 0$

minimize

$(L/D)_{ME} \approx 0.86 (L/D)_{max}$

$V_{ME} = 0.76\, V_{L/D\,max}$

Climb Flight

$\frac{dh}{dt}$ = excess energy

max rate of climb $\Rightarrow \frac{dh}{dt} = 0$

Max steady Rate of Climb Jet, Prop.

Flight envelop

Service: 100 ft

Performance: 200

Maneuvering V-n diagram.

Turn rate $\frac{w\sqrt{n^2-1}}{mV}$ *

$R_{turn} = \frac{V}{\dot{\xi}}$ *

Translation position

interchangeable

Rotation

321

$\psi \ \theta \ \phi$ DCM

Angular momentum

Moments of Product of inertia

Euler angular Rates

remember what is left

Gliding Flight

$C_D \frac{1}{2}\rho V^2 S = -W\sin\gamma$ Trade PE to keep V the same

$C_L \frac{1}{2}\rho V^2 S = W\cos\gamma$

$\dot{h} = V\sin\gamma$ height ~ fuel in this case

$\dot{r} = V\cos\gamma$

- Maximize range, Δr, you minimize Δh per Δr:
  - this occurs at $\left(\frac{L}{D}\right)_{max}$
- Sink rate $\left(\frac{dh}{dt}\right)$:

$D = C_D \frac{1}{2}\rho V^2 S = -W\sin\gamma$ $\quad L = C_L \frac{1}{2}\rho V^2 S = W\cos\gamma$

$\sin\gamma = -\frac{D}{W}$ $\quad V = \sqrt{\frac{2W\cos\gamma}{C_L \rho S}}$

$\Rightarrow \dot{h} = V\sin\gamma$

$= -\sqrt{\frac{2W\cos\gamma}{C_L\rho S}}\cos\gamma\left(\frac{1}{\frac{L}{D}}\right)$

To minimize $\frac{dh}{dt}$, take its derivative w.r.t. V and set it to zero.

$\Rightarrow \left(\frac{L}{D}\right)_{MF} = 0.86\left(\frac{L}{D}\right)_{max}$

$V_{MF} = 0.76\, V_{L/D\,max}$

Climbing flight

$\frac{dh}{dt}$ = rate of climb = specific excess power

$\dot{h} = V\sin\gamma = V\frac{(T-D)}{W} = \frac{P_{thrust} - P_{drag}}{W} = \frac{\text{Excess Power}}{\text{unit weight}}$

- Steady rate of Climb

$\dot{h} = V\left(\frac{T(h)}{W}\right) - \frac{C_{D_0}\rho(h)V^3}{2(W/S)} - \frac{2\varepsilon(W/S)\cos^2\gamma}{\rho(h)V}$

- Maximum steady rate of climb. Remember to take dh/dV to find max steady rate.

$\frac{d\dot{h}}{dV} = 0 = \left[\left(\frac{T}{W}\right) + V\left(\frac{\partial T/\partial V}{W}\right)\right] - \frac{3C_{D_0}\rho V^2}{2(W/S)} + \frac{2\varepsilon(W/S)\cos^2\gamma}{\rho V^2}$

Propeller

constant power $\frac{\partial P_{thrust}}{\partial V} = 0 = \frac{T}{W} + V\left(\frac{\partial T}{\partial V}\frac{1}{W}\right)$

$\frac{\partial \dot{h}}{\partial V} = -\frac{3}{2}\frac{C_{D_0}\rho V^2}{(W/S)} + \frac{2\varepsilon(W/S)}{\rho V^2}$

$\Rightarrow$ Airspeed for maximum rate of climb at maximum Power

$V_{MF} = \sqrt{\frac{2(W/S)}{\rho}}\sqrt{\frac{\varepsilon}{3C_{D_0}}}$

Jet

Airspeed for maximum rate of climb at max thrust, T_{max}

$\frac{\partial \dot{h}}{\partial V} = 0 = \frac{3C_{D_0}\rho}{2(W/S)}(V^2)^2 + \left(\frac{T}{W}\right)V^2 + \frac{2\varepsilon(W/S)}{\rho}$

Flight Envelope

Allowed Alt. & TAS

Available climb rate:

- Absolute: 0 ft/min
- Service: 100 ft/min
- Performance: 200 ft/min

2 thing thrust available vs thrust required.

Other consideration:

Max Mach, dynamic pressure

wing stall → buffeting

Coffin corner: only one speed you can operate.

U-2. Operates little corner between Mach limit and stall limit.

Turning rate and radius

Turning Rate limited by $C_{L\,max}$ or T_{max} or n_{max}

$\dot{\xi} = \frac{W\sqrt{n^2-1}}{mV} = \frac{\sqrt{(T_{req}-D_0)\rho V^2 S/2\varepsilon - W^2}}{mV}$ Weight is a big factor.

Turning Radius

$R_{turn} = \frac{V}{\dot{\xi}} = \frac{V^2}{g\sqrt{n^2-1}}$

$DCM = \begin{bmatrix}\cos\delta_x \\ \cos\delta_y \\ \cos\delta_z\end{bmatrix}$ $\quad r \cdot DCM = r = \begin{bmatrix}x \\ y \\ z\end{bmatrix}$

321 for airplane

Angular Momentum

$d\underline{h} = \underline{r} \times \underbrace{(V_0 + \omega \times r)}_{V_m} dm$

$\tilde{r} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$

$h = r \times V = I\omega$

$\dot{h}_I = \dot{I}\omega + I\dot{\omega} = H_B^I \dot{h}_B + \dot{H}_B^I h_B = H_B^I \dot{h}_B + \tilde{\omega}_I h_I$ ($\omega_I \times h_I$)

$\dot{h}_B = I_B\dot{\omega}_B = M_B - \tilde{\omega}_B I_B \omega_B$ ← energy to rotate the body

Euler angles

Not orthonormal, so Inverse ≠ Transpose.

Rigid Body dynamics

$\dot{r}_I(t) = H_B^I(t)\, V_B(t)$

$\dot{\Theta}_I(t) = L_B^I(t)\,\omega_B(t)$

$\dot{V}_B(t) = \frac{1}{m(t)}F_B(t) + H_I^B(t)\, g_I - \tilde{\omega}_B(t)\, V_B(t)$ — velocity due to rotation of body frame.

$\dot{\omega}_B(t) = I_B^{-1}(t)\left[M_B(t) - \underbrace{\tilde{\omega}_B(t) I_B(t)\,\omega_B(t)}_{\text{effects of rotation in one axis to another}}\right]$

$F_B = \begin{bmatrix} X_{aero} + X_{thrust} \\ \vdots \quad \vdots \end{bmatrix}_B = \begin{bmatrix} C_{X_{aero}} + C_{X_{thrust}} \\ \vdots \quad \vdots \end{bmatrix}_B \frac{1}{2}\rho V^2 S = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_B \bar{q} S$

$M_B = \begin{bmatrix} L_{aero} + L_{thrust} \\ \\ \end{bmatrix}_B = \begin{bmatrix} (C_{L_{aero}} + C_{L_{thrust}})b \\ (\quad '' \quad)\bar{c} \\ (\quad\quad)b \end{bmatrix} \frac{1}{2}\rho V^2 S = \begin{bmatrix} C_l b \\ C_m \bar{c} \\ C_n b \end{bmatrix} \bar{q} S$

$\dot{x}_I = (\;)u + (\;)v + (\;)w$

$\dot{y}_I = (\;)u \quad ''$

$\dot{z}_I =$

$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta$

$\dot{\theta} = q\cos\phi - r\sin\phi$

$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta$

$\dot{u} = \frac{X}{m} - g\sin\theta + rv - qw$

$\dot{v} = \frac{Y}{m} + g\sin\phi\cos\theta - ru + pw$

$\dot{w} = \frac{Z}{m} + g\cos\phi\cos\theta + qu - pv$

Aero damping

$M_B = C_m \bar{q} S\bar{c} \approx (C_{m_0} + C_{m_q}\hat{q} + C_{m_\alpha}\alpha)\bar{q}S\bar{c}$

$\Delta\alpha = \frac{-q\Delta x}{V}$

Roll damping

$\hat{p} = \frac{pb}{2V}$

2nd order dynamics

$t_r = \frac{\pi - \beta}{\omega_d}$ $\quad t_p = \frac{\pi}{\omega_d}$ $\quad M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

$t_s = \frac{4}{\zeta\omega_n}$

Step: $x(t) = \frac{1}{m\omega_n^2}\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi_0)\right]$

$\phi_0 = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

$x(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t$

Linearized Longitudinal EOM

$\Delta u\ \Delta\alpha\ \Delta q\ \Delta\theta$

Short Period: Δ in θ and α $\quad \Delta u = 0$

$\Delta\alpha$, Δq

Long Period: $\Delta\alpha = 0$

Δu, $\Delta\theta$ left

Phugoid much more pronounced at lower speed

$\zeta = \frac{1}{\sqrt{2}} \frac{C_D}{C_L}$

Lateral

$\Delta\beta$, Δp, Δr, $\Delta\phi$, $\Delta\psi$

3 modes: Dutch Roll:

Roll

Spiral: stable => Roll stability is more dominant than yaw stability.

Phased out Δp, Δr means trading off roll and yaw.

$L_\beta N_r > N_\beta L_r$ for spiral stability

Dutch roll: $\Delta\beta$ Δr

$C_{l_\beta} < 0$

$C_{n_\beta} > 0$

$C_{m_\alpha} < 0$

Handling qualities

MIL-F-8785C: I → IV gets larger and sophisticated

Dutch Roll flying qualities: ζ, $\omega_n\zeta$, ω_n

devices

- flaps, slats ← increase camber of the wing ↑ drag too. used for take off at low speed.
- Spoilers reduce lift and increase drag used for landing.

Take off Roll EOM

$L = \frac{1}{2}\rho V^2 S C_L$

$D = \frac{1}{2}\rho V^2 S C_D$

$F = k_F f_1(h) f_2(T) f_3(M(V(t)), T)$

$W = W_0 - \int_0^t [TSFC \cdot F(\tau)] d\tau$

$N = W(t) - L(t)$

$f = \mu N$

Ground effect most noticeable when airplane is above the ground less than half of its wing span. it gets stronger as you get closer.

On liftoff, it increases lift but decreases α stall angle.

Rate

Rate of change of Translational Velocity

$$\dot{u} = X/m - g\sin\theta + rv - qw$$
$$\dot{v} = Y/m + g\sin\phi\cos\theta - ru + pw$$
$$\dot{w} = Z/m + g\cos\phi\cos\theta + qu - pv$$

in red

velocity (no include rotation th

Rate of change of Angular Velocity

$$\dot{p} = \left(\mathbb{I}_{zz}L + \mathbb{I}_{xz}N - \left\{\mathbb{I}_{xz}\left(\mathbb{I}_{yy} - \mathbb{I}_{xx} - \mathbb{I}_{zz}\right)p + \left[\mathbb{I}_{xz}^2 + \mathbb{I}_{zz}\left(\mathbb{I}_{zz} - \mathbb{I}_{yy}\right)\right]r\right\}q\right)/\left(\mathbb{I}_{xx}\mathbb{I}_{zz} - \mathbb{I}_{xz}^2\right)$$
$$\dot{q} = \left[M - \left(\mathbb{I}_{xx} - \mathbb{I}_{zz}\right)pr - \mathbb{I}_{xz}\left(p^2 - r^2\right)\right]/\mathbb{I}_{yy}$$
$$\dot{r} = \left(\mathbb{I}_{xz}L + \mathbb{I}_{xx}N - \left\{I_{xz}\left(\mathbb{I}_{yy} - \mathbb{I}_{xx} - \mathbb{I}_{zz}\right)r + \left[\mathbb{I}_{xz}^2 + \mathbb{I}_{xx}\left(\mathbb{I}_{xx} - \mathbb{I}_{yy}\right)\right]p\right\}q\right)/\left(\mathbb{I}_{xx}\mathbb{I}_{zz} - \mathbb{I}_{xz}^2\right)$$

Mirror symmetry, $I_{xz} \neq 0$

Rate of change of Translational Position

$$\dot{x}_I = (\cos\theta\cos\psi)u + (-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi)v + (\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)w$$
$$\dot{y}_I = (\cos\theta\sin\psi)u + (\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi)v + (-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi)w$$
$$\dot{z}_I = (-\sin\theta)u + (\sin\phi\cos\theta)v + (\cos\phi\cos\theta)w$$

Rate of change of Angular Position

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta$$
$$\dot{\theta} = q\cos\phi - r\sin\phi$$
$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta$$

Propeller

constant Power $\frac{\delta P_{thrust}}{\delta V} = 0 = \frac{T}{W} + V\left(\frac{\delta T}{\delta V}\frac{1}{W}\right)$

$$\frac{\delta \dot{h}}{\delta V} = -\frac{3}{2}\frac{C_{D_0}\rho V^2}{(W/S)} + \frac{2\varepsilon(W/S)}{\rho V^2}$$

$\Rightarrow$ Airspeed for maximum rate of climb at maximum Power

$$V_{ME} = \sqrt{\frac{2(W/S)}{\rho}}\sqrt{\frac{\varepsilon}{3C_{D_0}}}$$

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Airspeed for maximum rate of climb at max thrust, T_{max}

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weight is a big factor.

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321 for airplane

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$$\underline{h} = r \times v = I\underline{\omega}$$

$$\dot{h}_I = \dot{I}\omega + I\dot{\omega} = H^I_B \dot{h}_B + \cancel{\dot{H}^I_B} h_B = H^I_B \dot{h}_B + \tilde{\omega}_I h_I$$

$\omega_I \times h_I$

$$\dot{h}_B = I_B \dot{\omega}_B = M_B - \tilde{\omega}_B I_B \omega_B$$

energy to rotate the body

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Not orthonormal, so inverse $\neq$ Transpose.

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velocity due to rotation of Body frame.

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effects of rotation in one axis to another

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$$\dot{x}_I = (\quad)u + (\quad)v + (\quad)w$$
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$\Delta u \; \Delta\alpha \; \Delta q \; \Delta\theta$

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$\Delta\alpha, \Delta q$

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$\Delta\theta$ left

Phugoid much more pronounced at lower speed

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Lateral

$\Delta\beta, \Delta p, \Delta r, \Delta\phi, \Delta\psi$

3 modes Dutch Roll :
Roll
Spiral : stable => Roll stability is more prominent than yaw stability.

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Dutch roll : $\Delta\beta$ Δr

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$$f = \mu N$$

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On liftoff, it increases lift but decreases α stall angle.

Stability derivatives

$$C_{l_\beta} = \frac{\delta C_l}{\delta \beta} < 0$$

$$C_{n_\beta} > 0$$