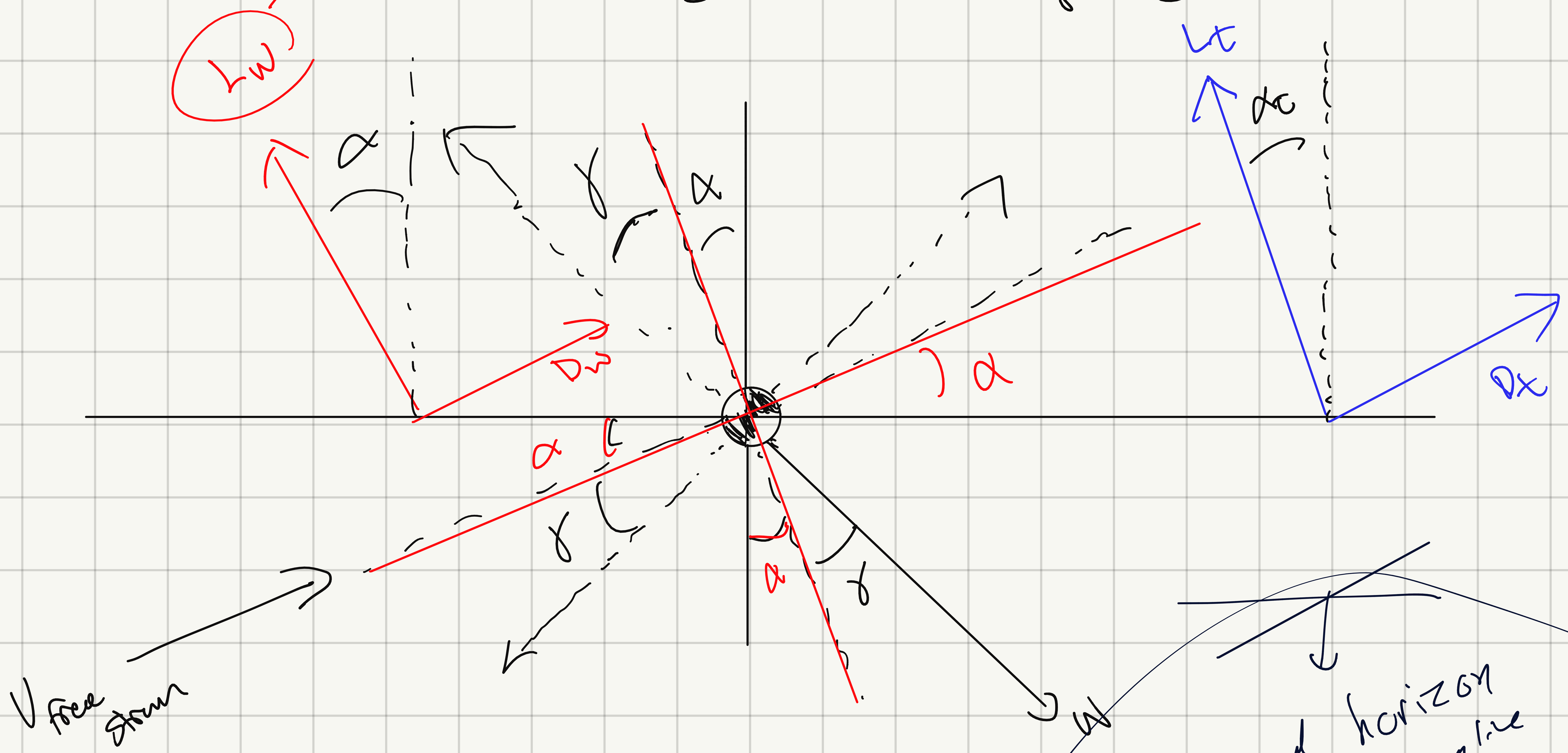
Bonsuck Koo
9/13/24

2

$\delta_e = 0$ $\qquad q = \frac{1}{2}\rho V^2$



$\gamma \Rightarrow$ V and horizon
$\alpha \Rightarrow$ V and reference line

Forces in lift direction

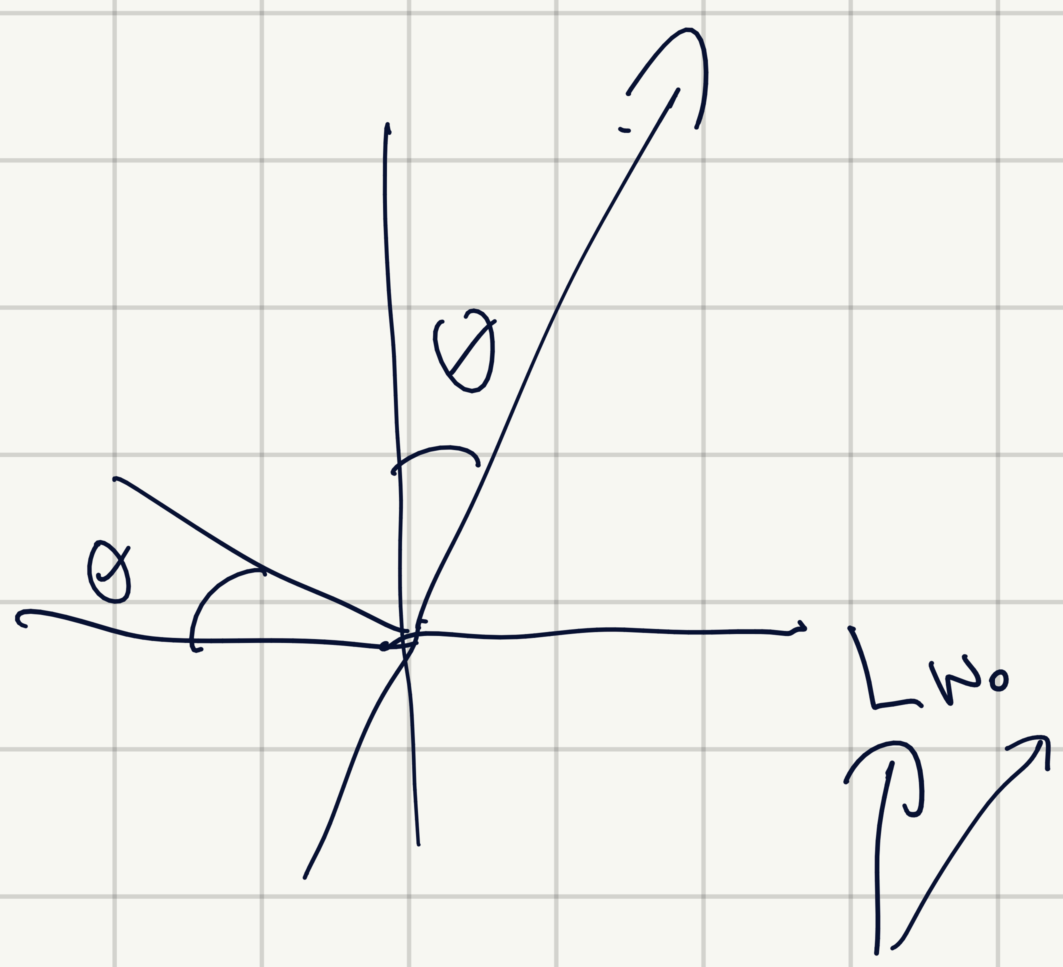
$$L - W\cos(\gamma) = 0$$

$$W\cos(\gamma) = L$$

$$L = L_w + L_t$$

$$L_w = qSC_{Lw}$$

$$= qS\left[C_{L0_w} + C_{L\alpha_w}(\alpha + i_w)\right]$$



$$L_t = q_t S_t\left[C_{L0_t} + C_{L\alpha_t}(\alpha_t + i_t) + \underbrace{\cancel{C_{L\delta_e}\delta_e}}_{0}\right]$$

$$= q_t S_t\left[C_{L0_t} + C_{L\alpha_t}(\alpha - \varepsilon_t + i_t)\right]$$

$$= q_t S_t\left[C_{L0_t} + C_{L\alpha_t}(\alpha - \varepsilon_{0_t} - \varepsilon_{\alpha_t}\alpha + i_t)\right]$$

$$= q_t S_t\left[C_{L0_t} + C_{L\alpha_t}\left[i_t - \varepsilon_{0_t} + (1 - \varepsilon_{\alpha_t})\alpha\right]\right]$$

directly depent on α

$$L = qS\left[C_{L0_w} + C_{L\alpha_w}(\alpha + i_w) + \frac{q_t S_t}{qS}\left(C_{L0_t} + C_{L\alpha_t}(i_t - \varepsilon_{0_t} + (1 - \varepsilon_{\alpha_t})\alpha)\right)\right]$$

$$L = qS\,C_L$$

$$C_L = C_{L0} + C_{L\alpha}\alpha + C_{L i_t} i_t$$

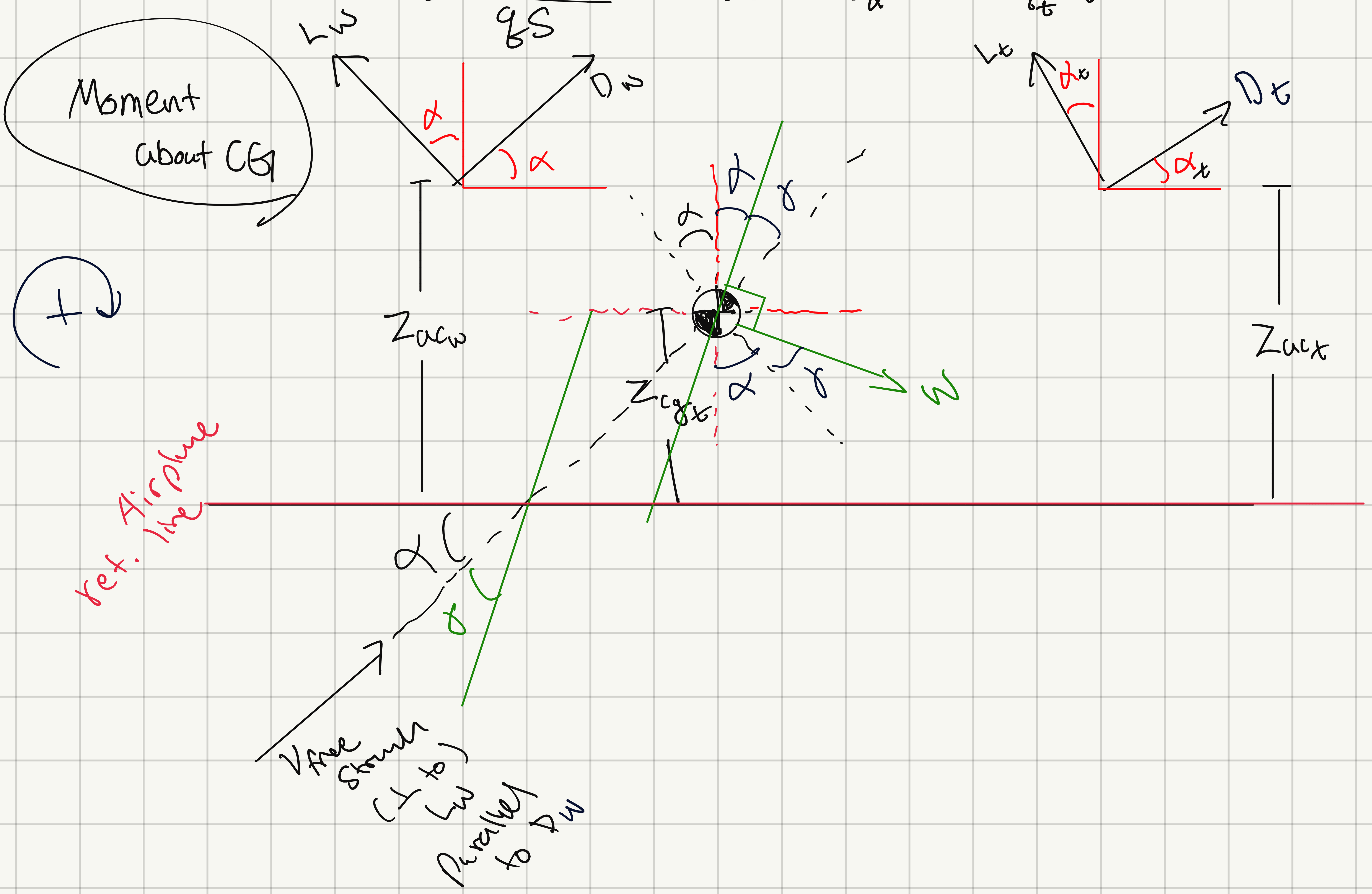
$$C_{L0} = C_{L0_w} + C_{L\alpha_w} i_w + \frac{q_t S_t}{qS}\left(C_{L0_t} - C_{L\alpha_t}\varepsilon_{0_t}\right)$$

$$C_{L\alpha} = C_{L\alpha_w} + \frac{q_t}{q}\frac{S_t}{S}C_{L\alpha_t}(1 - \varepsilon_{\alpha_t})$$

$$C_{L i_t} = \frac{q_t S_t}{qS}C_{L\alpha_t}$$

$$W\cos(\gamma) = qSC_L$$

$$\frac{W\cos(\gamma)}{qS} = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_t}} i_t$$



$$M_r = M_{ac_w} + L_w\cancel{\cos\alpha(x_r - x_{ac_w})} + \cancel{L_w\sin\alpha(Z_r - Z_{ac_w})} + \cancel{M_{ac_t}} + \cancel{L_t\cos\alpha_t(x_r - x_{ac_t})}$$
$$+ \cancel{L_t\sin\alpha_t(Z_r - Z_{ac_t})} + D_w\cancel{\cos\alpha}(Z_r - Z_{ac_w}) + \cancel{D_w\sin\alpha(x_r - x_{ac_w})}$$
$$\cancel{+D_t\cos\alpha_t}(Z_r - Z_{ac_t}) + \cancel{D_t\sin\alpha_t(x_r - x_{ac_t})} + M_f + M_p$$
$$+ W\cos(\alpha+\gamma)(x_r - x_{cg}) + W\sin(\alpha+\gamma)(Z_r - Z_{cg})$$

$$M_{ac_w} = qS\bar{c}\,C_{M_{ac_w}} \qquad M_{ac_t} = q_t S_t \bar{c}_t C_{M_{ac_t}}$$

$$M_p = qS\bar{c}\,C_{M_p} \qquad M_f = qS\bar{c}\,C_{M_f}$$

$$M_r = M_{ac_w} + (L_w\cos\alpha + D_w\sin\alpha)(x_r - x_{ac_w}) + (L_w\sin\alpha + D_w\cos\alpha)(Z_r - Z_{ac_w})$$
$$+ M_{ac_t} + (L_t\cos\alpha_t + D_t\sin\alpha_t)(x_r - x_{ac_t}) + (L_t\sin\alpha_t + D_t\cos\alpha_t)(Z_r - Z_{ac_t})$$
$$+ M_f + M_p + W\cos(\alpha+\gamma)(x_r - x_{cg}) + W\sin(\alpha+\gamma)(Z_r - Z_{cg})$$

$$C_{M_r} = C_{M_{ac_w}} + \left[(C_{L_{0w}} + C_{L_{\alpha_w}}(\alpha + i_w))\cos\alpha + (C_{D_{0w}} + C_{D_{\alpha_w}}(\alpha + i_w))\sin\alpha\right](\bar{x}_r - \bar{x}_{ac_w})$$
$$+ \left[(C_{L_{0w}} + C_{L_{\alpha_w}}(\alpha + i_w))\sin\alpha + (C_{D_{0w}} + C_{D_{\alpha_w}}(\alpha + i_w))\cos\alpha\right](\bar{Z}_r - \bar{Z}_{ac_w})$$
$$+ \frac{q_t S_t \bar{c}_t C_{M_{ac_t}}}{qS\bar{c}} + \left[(C_{L_{0t}} + C_{L_{\alpha_t}}(i_t - \epsilon_{0t} + (1-\epsilon_{\alpha_t})\alpha_t))\cos\alpha_t + (C_{D_{0t}} + C_{D_{\alpha_t}}(i_t - \epsilon_{0t} + (1-\epsilon_{\alpha_t})\alpha))\sin\alpha_t\right](\bar{x}_r - \bar{x}_{ac_t})$$

$$\frac{q_t S_t}{qS}$$

...

$\Downarrow$

$$+\frac{q_t S_t}{q S}\left[\left(C_{L_{0t}} + C_{L_{\alpha_t}}\left(i_t - \epsilon_{0t} + (1-\epsilon_{\alpha_t})\alpha_t\right)\right)\sin\alpha_t + \left(C_{D_{0t}} + C_{D_{\alpha_t}}\left(i_t - \epsilon_{0t} + (1-\epsilon_{\alpha_t})\alpha\right)\right)\cos\alpha_t\right](\bar{z}_r - \bar{z}_{ac_t})$$

$$+ C_{M_{0f}} + C_{M_{\alpha_f}}\alpha + C_{M_{0p}} + C_{M_{\alpha p}}$$

$$\bar{x}_r = \frac{x_r}{\bar{c}}, \quad \bar{x}_{ac_w} = \frac{x_{ac_w}}{\bar{c}} \quad \bar{x}_{ac_t} = \frac{x_{ac_t}}{\bar{c}}$$

$$\bar{z}_r = \frac{z_r}{\bar{c}}, \quad \bar{z}_{ac_w} = \frac{z_{ac_w}}{\bar{c}} \quad \bar{z}_{ac_t} = \frac{z_{ac_t}}{\bar{c}}$$

$$C_{M_r} = C_{M_{0r}} + C_{M_{\alpha_r}}\alpha + C_{M_{i_{t_r}}} i_t$$

$$C_{M_{0r}} = C_{M_{ac_w}} + \frac{q_t S_t \bar{c}_t}{q S \bar{c}} C_{M_{ac_t}} + C_{M_{0p}} + C_{M_{0f}}$$
$$+ (\bar{x}_r - \bar{x}_{ac_w})\left[(C_{L_{0w}} + C_{L_{\alpha w}} i_w)\cos\alpha + (C_{D_{0w}} + C_{D_{\alpha w}} i_w)\sin\alpha\right]$$
$$+ (\bar{z}_r - \bar{z}_{ac_w})\left[(C_{L_{0w}} + C_{L_{\alpha w}} i_w)\sin\alpha + (C_{D_{0w}} + C_{D_{\alpha w}} i_w)\cos\alpha\right]$$
$$+ \frac{q_t S_t}{q S}(\bar{x}_r - \bar{x}_{ac_t})\left[(C_{L_{0t}} - C_{L_{\alpha_t}}\epsilon_{0t})\cos\alpha_t + (C_{D_{0t}} - C_{D_{\alpha_t}}\epsilon_{0t})\sin\alpha\right]$$

$$C_{M_{\alpha_r}} = C_{M_{\alpha p}} + C_{M_{\alpha f}} + (\bar{x}_r - \bar{x}_{ac_w})\left[C_{L_{\alpha w}}\cos\alpha + C_{D_{\alpha w}}\sin\alpha\right]$$
$$(\bar{z}_r - \bar{z}_{ac_w})\left[C_{L_{\alpha w}}\sin\alpha + C_{D_{\alpha w}}\cos\alpha\right]$$
$$+ \frac{q_t S_t}{q S}(\bar{x}_r - \bar{x}_{ac_t})\left[C_{L_{\alpha_t}}(1-\epsilon_{\alpha_t})\cos\alpha_t + C_{D_{\alpha_t}}(1-\epsilon_{\alpha_t})\sin\alpha_t\right]$$
$$+ \frac{q_t S_t}{q S}(\bar{z}_r - \bar{z}_{ac_t})\left[C_{L_{\alpha_t}}(1-\epsilon_{\alpha_t})\sin\alpha_t + C_{D_{\alpha_t}}(1-\epsilon_{\alpha_t})\cos\alpha_t\right]$$

$$C_{M_{i_{t_r}}} = \frac{q_t S_t}{q S}\left[(\bar{x}_r - \bar{x}_{ac_t})(C_{L_{\alpha_t}}\cos\alpha_t + C_{D_{\alpha_t}}\sin\alpha_t) + (\bar{z}_r - \bar{z}_{ac_t})(C_{L_{\alpha_t}}\sin\alpha_t + C_{D_{\alpha_t}}\cos\alpha_t)\right]$$

$$\frac{W\cos\gamma}{\bar{q}S} = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_t}} i_t$$

$$0 = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{i_t}} i_t$$

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{i_t}} \\ C_{M_\alpha} & C_{M_i} \end{bmatrix} \begin{bmatrix} \alpha_{trim} \\ i_{trim} \end{bmatrix} = \begin{bmatrix} \frac{W\cos\gamma}{\bar{q}S} - C_{L_0} \\ -C_{M_0} \end{bmatrix}$$

$$\alpha = \left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0} - C_{L_i} i_t\right)\frac{1}{C_{L_\alpha}}$$

$$0 = C_{M_0} + C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0} - C_{L_i} i_t\right)\frac{1}{C_{L_\alpha}} + C_{M_{i_t}} i_t$$

$$0 = C_{M_0} + \frac{C_{M_\alpha}}{C_{L_\alpha}}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) - \frac{C_{M_\alpha} C_{L_{i_t}} i_t}{C_{L_\alpha}} + C_{M_{i_t}} i_t$$

$$= \quad '' \quad + \frac{-C_{M_\alpha} C_{L_i} i_t + C_{L_\alpha} C_{M_{i_t}} i_t}{C_{L_\alpha}}$$

$$\frac{C_{M_\alpha} C_{L_{i_t}} - C_{L_\alpha} C_{M_{i_t}}}{C_{L_\alpha}} i_t = C_{M_0} + \frac{C_{M_\alpha}}{C_{L_\alpha}}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right)$$

$$i_{t_{trim}} = \frac{C_{L_\alpha} C_{M_0} + C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right)}{C_{M_\alpha} C_{L_{i_t}} - C_{L_\alpha} C_{M_{i_t}}}$$

$$\alpha_{trim_{\delta_e=0}} = -\frac{C_{M_{i_t}}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_{i_t}} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

3 a) 210 knots = 354.44 ft/s

assume $\delta_e = 0$

$$\alpha_{trim_{\delta_e=0}} = -\frac{C_{M_{i_t}}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_{i_t}} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

$$i_{trim} = \frac{C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0} + C_{L_\alpha} C_{M_0}\right)}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

$\alpha = 3°$

$\alpha_t = \alpha - \epsilon_t$

$= 3 - (\epsilon_0 + \epsilon_{\alpha_t}\alpha)$

$= 3 - (0.642 + 0.426(3))$

$= 1.08°$

$$\bar{q} = \frac{\rho V^2}{2} = \frac{0.002378 \text{ slugs/ft}^3}{2}(354.44)^2$$

$$= 149.37 \text{ lb/ft}^2$$

$\eta = \frac{q_t}{\bar{q}}$ $\quad 0.9 = \frac{q_t}{149.37}$

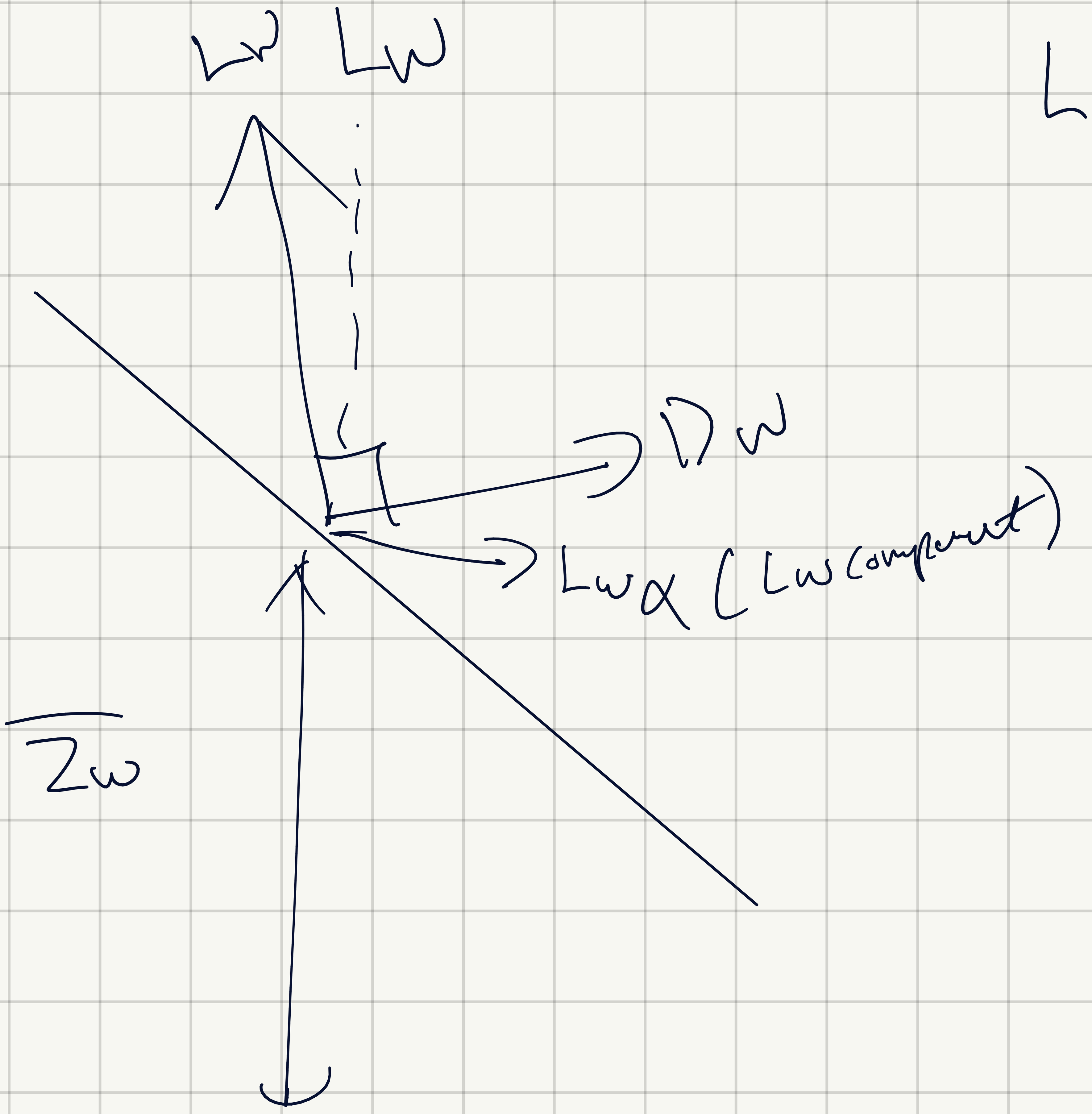
$$q_t = 134.43$$

$$C_{L_{i_t}} = \frac{q_t S_t}{\bar{q}S} C_{L_{\alpha_t}} = \frac{134.43(54)}{149.37(232)} 4.26 = 0.8924$$

$$C_{M_{i_t}} = \frac{q_t S_t}{\bar{q}S}\left[\left(\cancel{\bar{x}_r - \bar{x}_{ac_t}}\right)\left(C_{L_{\alpha_t}}\cos\alpha_t + C_{D_{\alpha_t}}\sin\alpha_t\right) + 0\left(\cancel{\bar{z}_r - \bar{z}_{ac_t}}\right)\left(C_{L_{\alpha_t}}\sin\alpha_t + C_{D_{\alpha_t}}\cos\alpha_t\right)\right]$$

$\frac{x_{ac/le}}{c}$ — small angle approx.

$= 0.8924\left[\frac{4.07}{7.04}(4.26 + \right.$



$L_w + D\alpha + L_t + D_t\alpha_t - W = 0$

$D = D_0 + kC_L^2$

$\boxed{C_D = C_{D0} + kC_L^2}$

$K = \frac{1}{\pi A R \varepsilon}$?

oswald efficiency

$(L_w + D_w\alpha)(X_{cg} - X_w) + (L_w\alpha + D_w)(Z_w - Z_{cg}) = M_w$

$\perp$ TA

$0 = W\cos(\theta) - L_w\cos(\alpha) - D_w\sin(\alpha) - L_t\cos(\alpha_t)$
$- D_t\sin(\alpha_t)$

$\parallel$

$F_{\parallel} = 0 = T - W\sin\theta + L_w\sin(\alpha) - D_w\cos(\alpha)$
$+ L_t\sin\alpha_t - D_t\cos(\alpha_t)$

$M = M_p + M_f + M_{ac_w} + M_{ac_t} + (X_{cg} - X_{cg_w})$