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ASE 367K FLIGHT DYNAMICS
Fall 2024

HOMEWORK 3
Due: 2024-09-20 at 11:59pm via Canvas

Problem 1

The vertical tail volume of an aircraft is defined by

$$V_F = \frac{S_f(x_{ac_f} - x_{cg})}{Sb},$$

where S_f and S are the fin and wing planform area, respectively, b is the wing span, and $x_{ac_f} - x_{cg}$ is the distance between the aerodynamic center of the fin and the CG of the aircraft. For an airplane with $C_{L_{\alpha_f}} = 5.5$, assuming negligible sidewash effects, what tail volume is required to achieve a weathercock stability derivative of $C_{n_\beta} = 0.16$?

Problem 2

For the airplane below, assume the right engine fails when the aircraft is in level flight at 130 knots, creating a yawing moment of 100000 [ft-lb]. Neglect the fuselage effect, determine:

a. The resulting sideslip angle in degrees if no rudder deflection is input.
b. The rudder input δ_r required to maintain the direction of flight prior to the engine failure.

Assume sea level, standard conditions: $\rho = 0.002378$ [slugs/ft^3]. Also, 1 [knot] $= 1.6878$ [ft/s].

Wing		Horizontal Tail		Vertical Fin	
b	$=$ 80 [ft]				
S	$=$ 1200 [ft^2]	S	$=$ 250 [ft^2]	S	$=$ 125 [ft^2]
$\bar{c}$	$=$ 10 [ft]	$\bar{c}$	$=$ 5 [ft]	$\bar{c}$	$=$ 5 [ft]
i	$=$ 0 [deg]	i	$=$ -1.6 [deg]		
C_{L_α}	$=$ 5.2 [rad^{-1}]	C_{L_α}	$=$ 4.5 [rad^{-1}]	C_{L_α}	$=$ 5.5 [rad^{-1}]
C_{L_0}	$=$ 0.1	C_{L_0}	$=$ 0.0	C_{L_0}	$=$ 0.0
$C_{M_{ac}}$	$=$ -0.1	$C_{M_{ac}}$	$=$ 0.0	$C_{M_{ac}}$	$=$ 0.0
		$C_{L_{\delta_e}}$	$=$ 1.5 [rad^{-1}]	$C_{L_{\delta_r}}$	$=$ 1.5 [rad^{-1}]
x_{ac}	$=$ 13 [ft]	x_{ac}	$=$ 40 [ft]	x_{ac}	$=$ 40 [ft]
		ϵ_0	$=$ 1.0 [deg]	σ_0	$=$ 0.0 [deg]
		ϵ_α	$=$ 0.3	σ_β	$=$ 0.0
		η	$=$ 0.95	η	$=$ 1.0

The cg of the aircraft is at $x_{cg} = 14$ [ft], the weight of the aircraft is $20,000$ [lb], and

$$C_{M_{0_p}} = 0.0\,, \quad C_{M_{0_f}} = 0.0\,, \quad C_{M_{\alpha_p}} = 0.0\ [\text{rad}^{-1}]\,, \quad \text{and} \quad C_{M_{\alpha_f}} = 0.0\ [\text{rad}^{-1}]\,.$$

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9/20/24

1) $V_V = \frac{S_F(x_{ac_f} - x_{cg})}{Sb}$

$C_{L\alpha_f} = 5.5$ $\quad C_{L\alpha_f} = \frac{dC_L}{d\alpha_f}$ $\quad \frac{\partial \sigma}{\partial \beta} = 0$

$C_{n\beta} = 0.16$ $\quad \alpha_F = -\beta + \cancel{\sigma}$ 0

$C_{n_F} = -V_V \left(\frac{V_F}{V}\right)^2 C_{L_F}$

Assume fin is not in propeller slipstream

$\frac{\partial C_{n_F}}{\partial \alpha_F} = -V_V \left(\frac{V_F}{V}\right)^2 \frac{\partial C_{L_F}}{\partial \alpha_F}$

$-\frac{\partial C_{n_F}}{\partial \beta} = -V_V \left(\frac{V_F}{V}\right)^2 C_{L\alpha F}$

$C_{n\beta} = 0.16 = -V_V \left(\frac{V_F}{V}\right)^2 (5.5)$

$-\frac{0.16}{5.5} = -V_V (1)^2$

$V_V = +0.029$

$C_{n\beta} = \cancel{\frac{q_F}{q}} V_V C_{L\alpha_F}(1 - \sigma_\beta)$

$C_{n_F} = -V_V \left(\frac{V_F}{V}\right)^2 C_{L_F}$

$\frac{\partial C_{n_F}}{\partial \beta} = -V_V \left(\frac{V_F}{V}\right)^2 \frac{C_{L_F}}{\partial \beta}$

$C_{L_F} = a_F(-\beta + \cancel{\sigma}) + a_r \delta_r$ $\quad a_F = C_{L\alpha_F}$

$C_{L_F} = -\beta a_F + a_F \alpha$

$\frac{\partial C_{L_F}}{\partial \beta} = -a_F$

$\frac{\partial C_{n_F}}{\partial \beta} = a_F V_V \left(\frac{V_F}{V}\right)^2$

$0.16 = 5.5 V_V \left(\frac{V_F}{V}\right)^2$

$V_V = 0.029$

2)

130 Knots $= 219.415$ ft/s

$N_{engine} = 100{,}000$ ft-lb.

a) $C_n = C_{n_\beta}\beta + \cancel{C_{n_{\delta_r}}\delta_r}$ 0, since $\delta_r = 0$

$C_{L_F} = a_F(-\beta + \sigma) + a_F\delta_r$

$\frac{\partial C_{L_F}}{\partial \beta} = -a_F + a_F \frac{\partial \sigma}{\partial \beta}$

$$C_n = \frac{N}{\frac{1}{2}\rho V^2 S b}$$

$$C_{n_\beta}\beta = \frac{N}{\frac{1}{2}\rho V^2 S b}$$

$$\beta = \frac{2N}{\rho V^2 S b}\frac{1}{C_{n_\beta}}$$

assume $\frac{V_F}{V} = 1$ for free stream.

$$C_{n_\beta} = -V_v\left(\cancel{\frac{V_F}{V}}\right)^2 \frac{\partial C_{L_F}}{\partial \beta} = a_F V_v \left(\frac{V_F}{V}\right)^2\left(1 - \cancel{\frac{\partial \sigma}{\partial \beta}}\right)$$

0, no sidewash

$$\boxed{a_F = C_{L_{\alpha_F}}}$$

$$V_v = \frac{S_F l_F}{S b} = \frac{125(40-14)}{1200(80)} = 0.03385$$

$$C_{n_\beta} = 5.5(0.03385)(1)(1) = 0.1862$$

$$\beta = \frac{2(100{,}000)}{0.002378(219.415)^2 \, 1200 \cdot 80 \cdot 0.1862}$$

$= 0.09773$ rad $\approx 5.60°$

b) $$C_n = \cancelto{0}{C_{n_\beta}\beta} + C_{n_{\delta_r}}\delta_r$$

$$= C_{n_{\delta_r}}\delta_r$$

$$C_{n_{\delta_r}} = -a_r V_v \left(\cancelto{1}{\frac{V_F}{V}}\right)^2$$

$$a_r = C_{L_{\delta_{r_F}}} = 1.5$$

$$V_v = 0.03385$$

$$C_{n_{\delta_r}} = -1.5(0.03385) = -0.050775$$

$$C_n = \frac{N}{\frac{1}{2}\rho V^2 S b}$$

$$= \frac{100,000}{\frac{1}{2}(0.002378)(219.415)^2(1200)(80)}$$

$$= 0.0181976$$

$$0.0181976 = -0.050775\,\delta_r$$

$$\delta_r = -0.3584 = -20.5346^\circ$$