The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

24 SEPTEMBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

JOHN-PAUL CLARKE
Ernest Cockrell, Jr. Memorial Chair in Engineering, The University of Texas at Austin

Topics for Today

- Topic(s):
  - Straight and Level Flight
  - Cruise Range

The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

STRAIGHT AND LEVEL FLIGHT

JOHN-PAUL CLARKE
Ernest Cockrell, Jr. Memorial Chair in Engineering, The University of Texas at Austin

Power = Watts = Force x Velo.

increase speed cause reduce in α for propeller causes ↓ in Lift for propeller which is thrust

Typical Effects of Altitude and Velocity on Power and Thrust

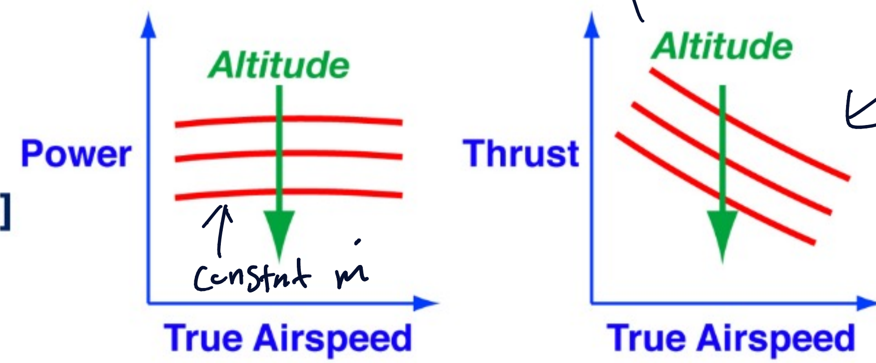
This means propeller: $\dot{m}$ related to power, P

Jet: $\dot{m}$ related to thrust.

* Propeller:
$\dot{m}$ effects how much power is available, not thrust. because fuel is supplied to an engine that rotates the propeller. It doesn't directly create thrust. Thrust depends on power.

Jet: The $\dot{m}$ directly influence thrust by bringing in more air. Power depends on thrust

- **Propeller**
  [Air-breathing engine]

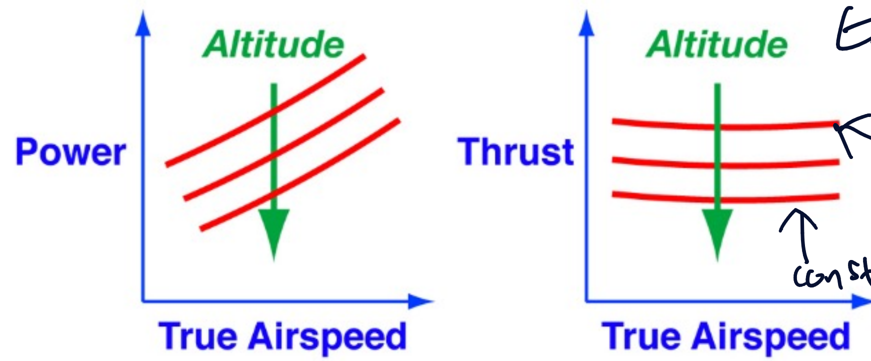


Propeller becomes less efficient

- **Turbofan**

[In between]

- **Turbojet**



I keep constant thrust and power just depends on thrust.

this line indicates max thrust it can exert.

- **Battery**

[Independent of altitude and airspeed]

Performance Parameters

L = lift
~ how much work lift is doing

Lift-to-Drag Ratio

$$\boxed{L/D = C_L/C_D}$$

Load Factor

$$\boxed{n = L/W = L/mg, \text{"}g\text{"}s}$$

Thrust-to-Weight Ratio

$$\boxed{T/W = T/mg, \text{"}g\text{"}s}$$

must be larger than 1 for rocket and fighters

how much

Wing Loading

$$\boxed{W/S, \quad N/m^2 \text{ or } lb/ft^2}$$

Trimmed Lift Coefficient, C_L

- **Trimmed lift coefficient, C_L**
  - Proportional to weight and wing loading factor, W/S
  - Decreases with V^2
  - At constant true airspeed, increases with altitude

$$\boxed{W = C_{L_{trim}}\left(\frac{1}{2}\rho V^2\right)S = C_{L_{trim}}\bar{q}S}$$

$$\boxed{C_{L_{trim}} = \frac{1}{\bar{q}}(W/S) = \frac{2}{\rho V^2}(W/S)}$$

maybe not

Not faster but higher?

?

The faster I go the lower I can get α.

this goes down as you go up in altitude

Heavier → you need more lift by increasing α.

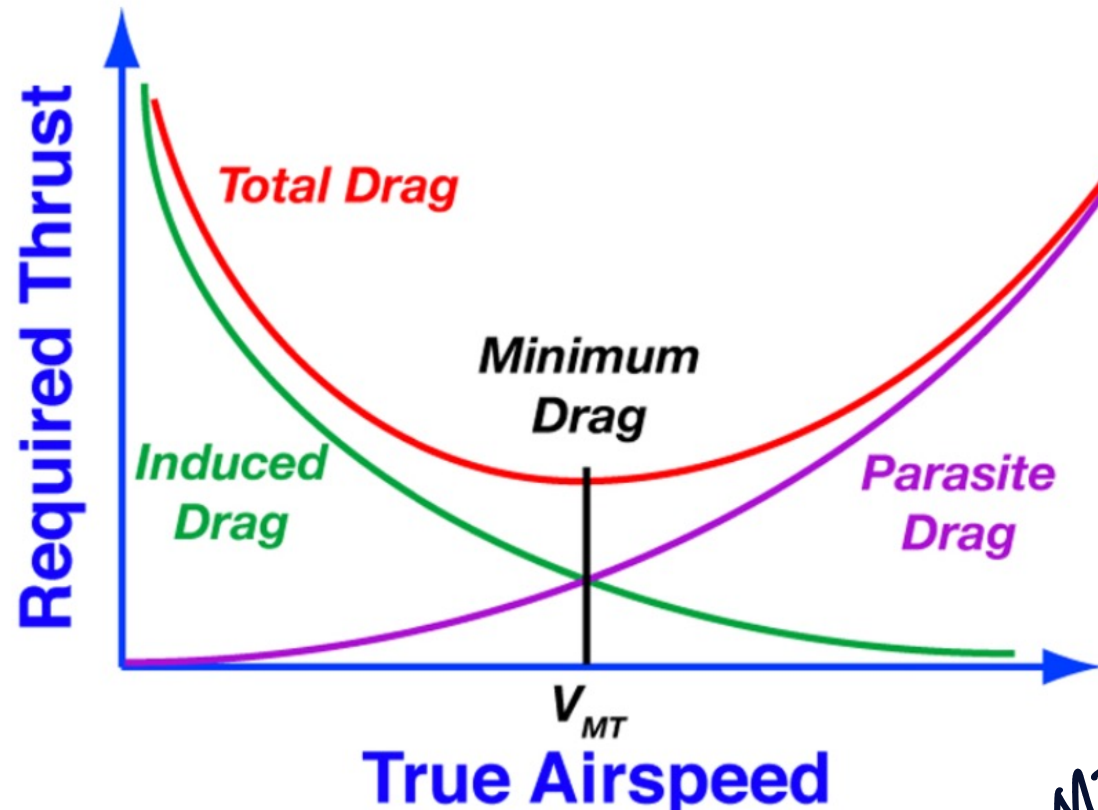
Trimmed Angle of Attack, α

- **Trimmed angle of attack, α**
  - Constant if dynamic pressure and weight are constant
  - If dynamic pressure decreases, angle of attack must increase

$$\boxed{\alpha_{trim} = \frac{2W/\rho V^2 S - C_{L_o}}{C_{L_\alpha}} = \frac{\frac{1}{\bar{q}}(W/S) - C_{L_o}}{C_{L_\alpha}}}$$

Thrust Required for Steady, Level Flight

L = W
D = Thrust
for
Trim



MT = Minimum
thrust

Thrust Required for Steady, Level Flight

Trimmed thrust

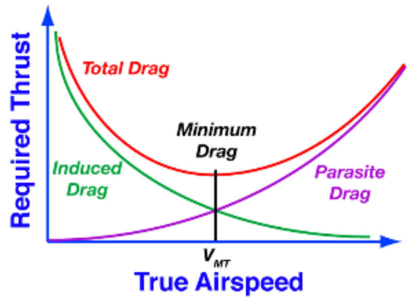
Parasitic Drag *Induced Drag*

$$T_{trim} = D_{cruise} = C_{D_o}\left(\frac{1}{2}\rho V^2 S\right) + \varepsilon \frac{2W^2}{\rho V^2 S}$$

Minimum required thrust conditions

$$\frac{\partial T_{trim}}{\partial V} = C_{D_o}(\rho V S) - \frac{4\varepsilon W^2}{\rho V^3 S} = 0$$

Necessary Condition:
Slope = 0



Necessary and Sufficient Conditions for Minimum Required Thrust

Necessary Condition = Zero Slope

$$C_{D_o}(\rho V S) = \frac{4\varepsilon W^2}{\rho V^3 S}$$

$$\Rightarrow V^4 = \frac{4\varepsilon W^2}{\rho^2 S^2 C_{D_o}}$$

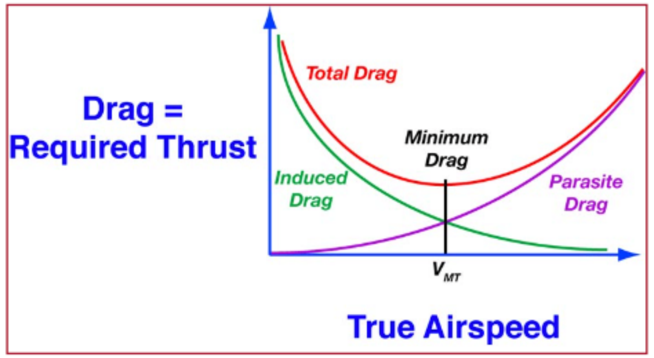
$$= \frac{4\varepsilon}{\rho^2 C_{D_o}} (W/S)^2$$

Sufficient Condition for a Minimum = Positive Curvature when slope = 0

$$\frac{\partial^2 T_{trim}}{\partial V^2} = \underset{(+)}{C_{D_o}(\rho S)} + \underset{(+)}{\frac{12\varepsilon W^2}{\rho V^4 S}} > 0$$

✓

Airspeed for Minimum Thrust in Steady, Level Flight



$\frac{1}{2}\rho V^3 S$

Continued from $\frac{\partial T_{trim}}{\partial V} = 0$

Satisfy necessary condition

$$\boxed{V^4 = \left(\frac{4\varepsilon}{C_{D_o}\rho^2}\right)(W/S)^2}$$

Fourth-order equation for velocity

Choose the positive root

$$\boxed{V_{MT} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{\varepsilon}{C_{D_o}}}}}$$

Lift, Drag, and Thrust Coefficients in Minimum-Thrust Cruising Flight

Lift coefficient

$$C_{L_{MT}} = \frac{2}{\rho V_{MT}^2}\left(\frac{W}{S}\right)$$

$$= \sqrt{\frac{C_{D_o}}{\varepsilon}} = \left(C_L\right)_{(L/D)_{\max}}$$

Plug in V_{MT} from previous slide

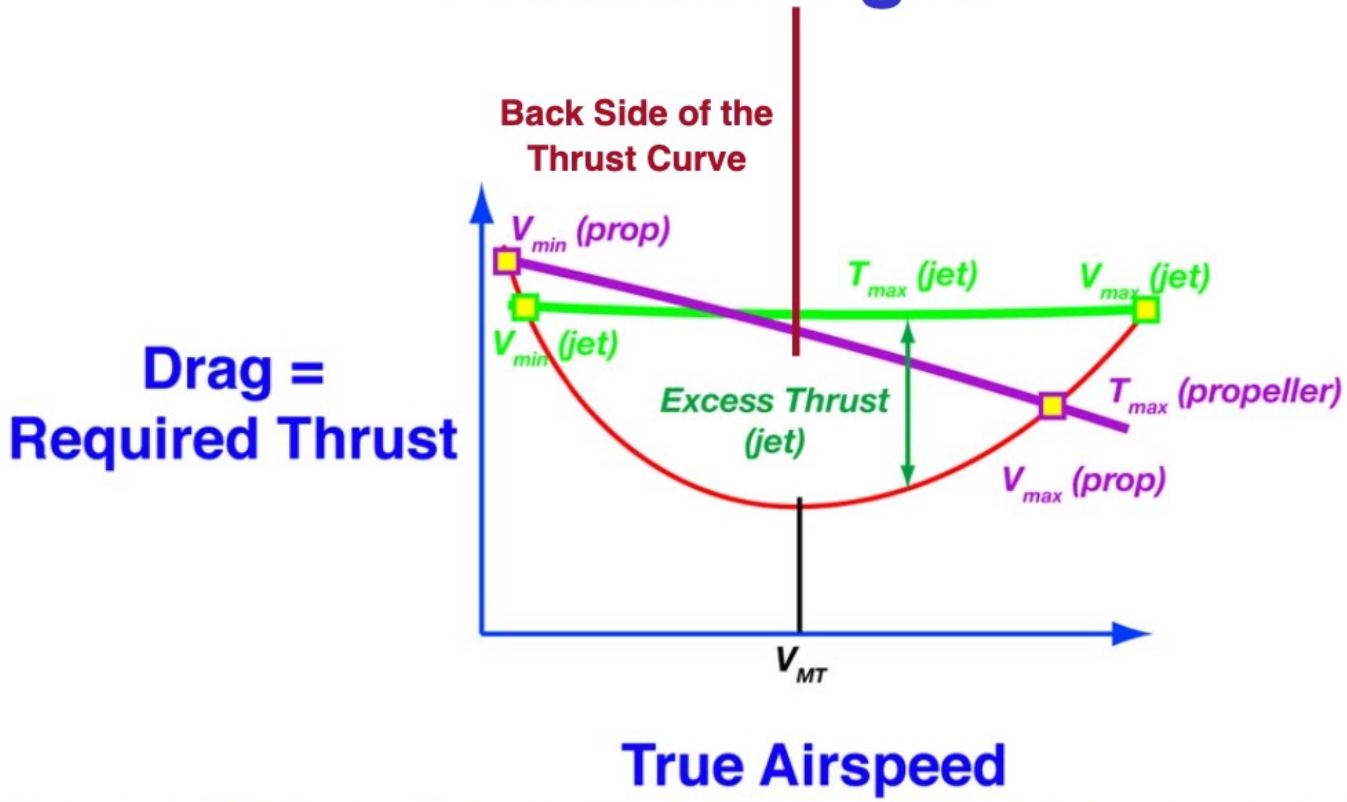
Drag and thrust coefficients

$$C_{D_{MT}} = C_{D_o} + \varepsilon C_{L_{MT}}^2 = C_{D_o} + \varepsilon \frac{C_{D_o}}{\varepsilon}$$

$$= 2C_{D_o} \equiv C_{T_{MT}}$$

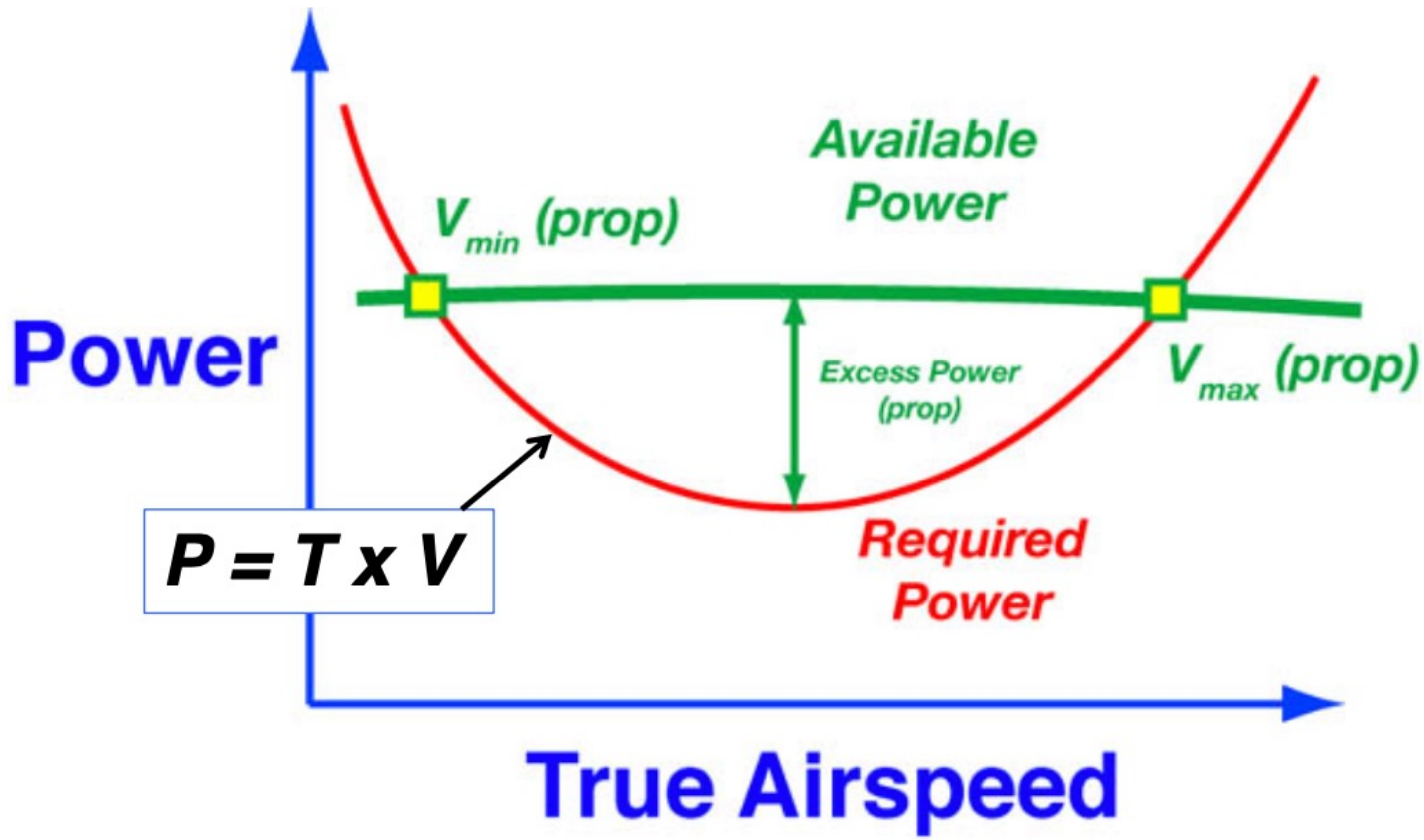
thrust, minimum thrust coeff.

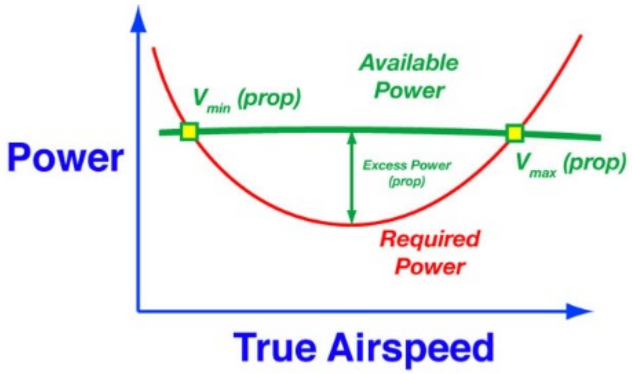
Achievable Airspeeds in Constant-Altitude Flight



- Two equilibrium airspeeds for a given thrust or power setting
  - Low speed, high C_L, high α
  - High speed, low C_L, low α
- Achievable airspeeds between minimum and maximum values with maximum thrust or power

Power Required for Steady, Level Flight





Airspeed for Minimum Power in Steady, Level Flight

We looked at minimum thrust in the previous slides

- Satisfy necessary condition

$$C_{D_o}\frac{3}{2}\left(\rho V^2 S\right)=\frac{2\varepsilon W^2}{\rho V^2 S}$$

- Fourth-order equation for velocity
  - Choose the positive root

$$V_{MP}=\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{\varepsilon}{3C_{D_o}}}}$$

- Corresponding lift and drag coefficients

$$C_{L_{MP}}=\sqrt{\frac{3C_{D_o}}{\varepsilon}}$$

$$C_{D_{MP}}=4C_{D_o}$$

Achievable Airspeeds for Jet in Cruising Flight

Thrust = constant

$$\boxed{T_{avail} = C_D \bar{q} S = C_{D_o}\left(\frac{1}{2}\rho V^2 S\right) + \frac{2\varepsilon W^2}{\rho V^2 S}}$$

$$\boxed{C_{D_o}\left(\frac{1}{2}\rho V^4 S\right) - T_{avail} V^2 + \frac{2\varepsilon W^2}{\rho S} = 0}$$

$$\boxed{V^4 - \frac{2T_{avail}}{C_{D_o}\rho S} V^2 + \frac{4\varepsilon W^2}{C_{D_o}(\rho S)^2} = 0}$$

4^{th}-order algebraic equation for V

Achievable Airspeeds for Jet in Cruising Flight

Solutions for *V^2* can be put in quadratic form and solved easily

$$\boxed{V^2 \triangleq x; \quad V = \pm\sqrt{x}}$$

$$V^4 - \frac{2T_{avail}}{C_{D_o}\rho S}V^2 + \frac{4\varepsilon W^2}{C_{D_o}(\rho S)^2} = 0$$

$$x^2 + bx + c = 0$$

$$\boxed{x = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c} = V^2}$$

The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

CRUISE RANGE

JOHN-PAUL CLARKE
Ernest Cockrell, Jr. Memorial Chair in Engineering, The University of Texas at Austin

Cruising Range and Specific Fuel Consumption

- Thrust = Drag
- Lift = Weight

$$\boxed{\begin{aligned}0 &= \left(C_T - C_D\right)\frac{1}{2}\rho V^2 S \Big/ m \\ 0 &= \left(C_L \frac{1}{2}\rho V^2 S - mg\right)\Big/ mV\end{aligned}}$$

- Level flight

$$\boxed{\begin{aligned}\dot{h} &= 0 \\ \dot{r} &= V\end{aligned}}$$

- **Thrust specific fuel consumption, *TSFC* = c_T**
  - **Fuel mass burned per sec per unit of thrust**

$$\boxed{c_T : \frac{kg/s}{kN}} \qquad \boxed{\dot{m}_f = -c_T T}$$

- **Power specific fuel consumption, *PSFC* = c_P**
  - **Fuel mass burned per sec per unit of power**

$$\boxed{c_P : \frac{kg/s}{kW}} \qquad \boxed{\dot{m}_f = -c_P P}$$

increase fuel eff.

By L/D

Breguet Range Equation for Jet Aircraft

Rate of change of range with respect to weight of fuel burned

$$\frac{dr}{dm} = \frac{dr/dt}{dm/dt} = \frac{\dot{r}}{\dot{m}} = \frac{V}{(-c_T T)} = -\frac{V}{c_T D} = -\left(\frac{L}{D}\right)\frac{V}{c_T m g}$$

$$dr = -\left(\frac{L}{D}\right)\frac{V}{c_T m g}dm$$

Range traveled

$$Range = R = \int_0^R dr = -\int_{W_i}^{W_f}\left(\frac{L}{D}\right)\left(\frac{V}{c_T g}\right)\frac{dm}{m}$$

Maximum Range of a Jet Aircraft Flying at Constant Altitude

At constant altitude and *SFC*

$$\boxed{V_{cruise}(t) = \sqrt{2W(t)/C_L\rho\left(h_{fixed}\right)S}}$$

→ Minimum thrust or power

$$Range = -\int_{W_i}^{W_f}\left(\frac{C_L}{C_D}\right)\left(\frac{1}{c_T g}\right)\sqrt{\frac{2}{C_L \rho S}}\frac{dm}{m^{1/2}}$$

$$= \left(\frac{\sqrt{C_L}}{C_D}\right)\left(\frac{2}{c_T g}\right)\sqrt{\frac{2}{\rho S}}\left(m_i^{1/2} - m_f^{1/2}\right)$$

Range is maximized when $\boxed{\left(\frac{\sqrt{C_L}}{C_D}\right) = \text{maximum}}$

Can't specify in this equation b/c V_{cruise} can be selected arbitrarily

C_L depends on V_{cruise}

Breguet Range Equation for Jet Aircraft at Constant Airspeed

For constant true airspeed, $V = V_{cruise}$, and *SFC*

$$R = -\left(\frac{L}{D}\right)\left(\frac{V_{cruise}}{c_T g}\right)\ln(m)\Big|_{m_i}^{m_f}$$

$$= \left(\frac{L}{D}\right)\left(\frac{V_{cruise}}{c_T g}\right)\ln\left(\frac{m_i}{m_f}\right)$$

$$= \left(V_{cruise}\frac{C_L}{C_D}\right)\left(\frac{1}{c_T g}\right)\ln\left(\frac{m_i}{m_f}\right)$$

- $V_{cruise}(C_L/C_D)$ as large as possible
- $M \rightarrow M_{crit}$
- ρ as small as possible
- h as high as possible

Keep it constant

Maximize Jet Aircraft Range Using Optimal Cruise-Climb

$$\frac{\partial R}{\partial C_L} \propto \frac{\partial\left(V_{cruise} \, C_L / C_D\right)}{\partial C_L} = \frac{\partial\left[V_{cruise} \, C_L / \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = 0$$

One thing we can change

- read ↑
- As we fly fuel is spent, then less lift is required

$$V_{cruise} = \sqrt{2W / C_L \rho S}$$

Assume $\sqrt{2W(t)/\rho(h)S} = \text{constant}$

i.e., airplane **climbs at constant *TAS*** as fuel is burned

What does cruising flight mean? How diff from trimmed flight

why did we assume this?

Is this ① just saying W(t) and ρ(h) are changing at the same rate?

② Is there a difference btwn TAS and Vcruise?

Description

Cruise climb is the most fuel efficient cruising technique. It allows the aircraft to constantly operate at its optimal performance.

As fuel is burnt, the aircraft gradually becomes lighter. Therefore, less lift is required to balance the weight. This means that either speed will increase, or altitude will increase, or thrust will be reduced. Increasing the speed will also increase drag, hence fuel consumption. Reducing thrust means the engine would run in a sub-optimal mode. Increasing the altitude, on the other hand, will keep the engine setting and reduce drag due to air density reduction.

The downside of cruise climb is that it is often incompatible with ATS procedures and traffic demand. In busier airspaces (e.g. Europe, USA, etc.) traffic levels are such that clearing a flight to perform a cruise climb will deny several others the opportunity to fly at or near their optimal levels.

Another issue with clearing an aircraft to perform a cruise climb is that the vertical speed is much lower than the usual 1000-2000 feet per minute. This results in a situation where the

Maximize Jet Aircraft Range Using Optimal Cruise-Climb

$$\frac{\partial\left[V_{cruise} C_L \Big/ \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = \sqrt{\frac{2w}{\rho S}} \frac{\partial\left[C_L^{1/2} \Big/ \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = 0$$

$$\sqrt{\frac{2w}{\rho S}} = \text{Constant; let } C_L^{1/2} = x, \quad C_L = x^2$$

$$\frac{\partial}{\partial x}\left[\frac{x}{\left(C_{D_o} + \varepsilon x^4\right)}\right] = \frac{\left(C_{D_o} + \varepsilon x^4\right) - x\left(4\varepsilon x^3\right)}{\left(C_{D_o} + \varepsilon x^4\right)^2} = \frac{\left(C_{D_o} - 3\varepsilon x^4\right)}{\left(C_{D_o} + \varepsilon x^4\right)^2}$$

R = max @ $\frac{dR}{dC_L} = 0$ @

Optimal values:

$$C_{L_{MR}} = \sqrt{\frac{C_{D_o}}{3\varepsilon}} : \quad C_{D_{MR}} = C_{D_o} + \frac{C_{D_o}}{3} = \frac{4}{3} C_{D_o}$$

Max Range

You stay at a certain mach #. constant

$$V_{cruise-climb} = \sqrt{2W(t) / C_{L_{MR}} \rho(h) S} = a(h) M_{cruise-climb}$$

$a(h)$: Speed of sound; $M_{cruise-climb}$: Mach number

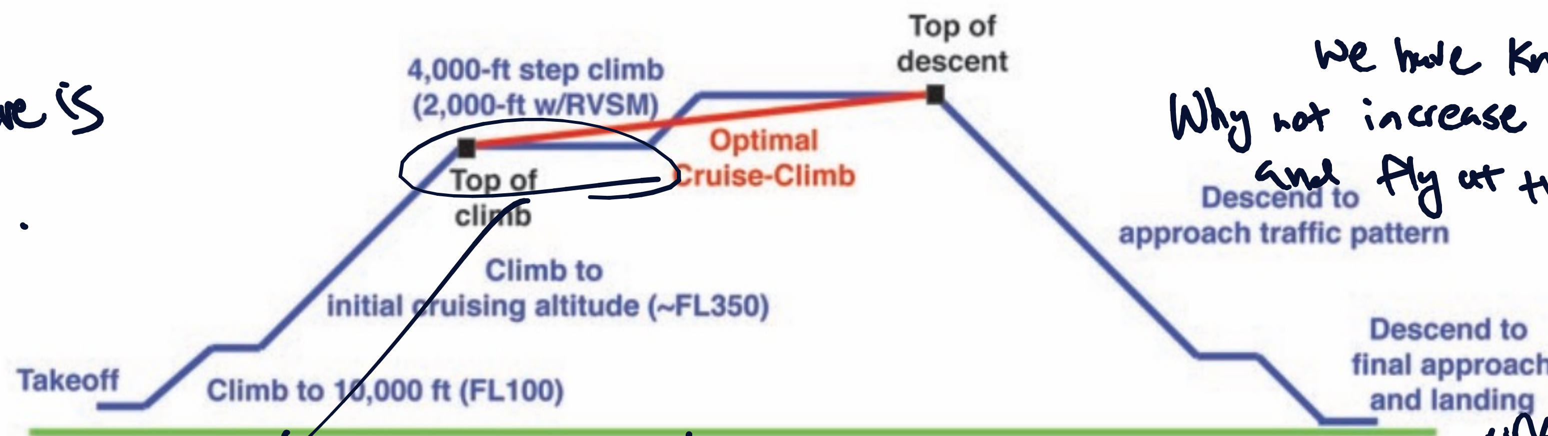
Step-Climb Approximates Optimal Cruise-Climb

- Cruise-climb usually violates air traffic control rules
- Constant-altitude cruise does not
- Compromise: Step climb from one allowed altitude to the next as fuel is burned

① We learned we can maximize range by cruise-climbing. To do that we set $\sqrt{\frac{2W(t)}{\rho(h)S}}$ constant

. How would we have known to do that?. Why not increase alt. in the first place and fly at the altitude?

✱ We want to climb so there is less drag. less thrust required



You pitch down as you burn fuel but keep altitude. At some point you get light enough to reach an equilibrium at a higher altitude

✱ How does airplane work optimal vs step. Does the pilot put the thrust at constant thrust for optimal? what about step?.

The University of Texas at Austin
Aerospace Engineering and Engineering Mechanics
Cockrell School of Engineering