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Cockrell School of Engineering

19 NOVEMBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Term Project
  - Pre-Reading
  - Launch Vehicle Dynamics and Trajectory Computation
  - The Atmosphere
  - Re-Entry

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TERM PROJECT

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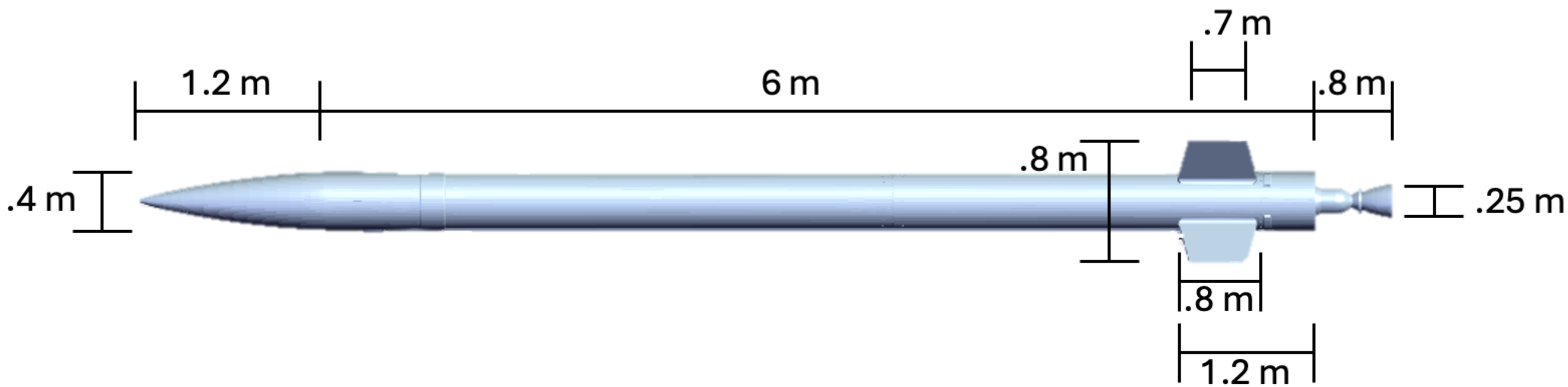
The Rocket

- Static Thrust: 15.5 kN
- Tolerances: +/- 2mm



The Challenge

- Develop a callable function that outputs the location of the center of gravity (CG) considering the effects of uncertainty in dimensions and mass.
- Develop a callable function that outputs the location of the center of pressure (CP) as a function of angle of attack (α) considering the effects of uncertainty in dimensions.
- Develop a callable function with an appropriate turbulence (e.g., Dryden Wind Turbulence Model) that outputs the winds at a given input altitude.
- Develop Monte Carlo simulation model for the rocket trajectory assuming no change in the location of the CG due to consumption of fuel.
- Use the Monte Carlo simulation model to determine whether the rocket would be stable during an ascent to 35 km from the surface of a round, non-rotating earth.

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PRE-READING

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Using the Barrowman Equation

- Barrowman Method of Calculating Normal Force
  - https://www.nakka-rocketry.net/RD_Appendix_B.html

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LAUNCH VEHICLE DYNAMICS

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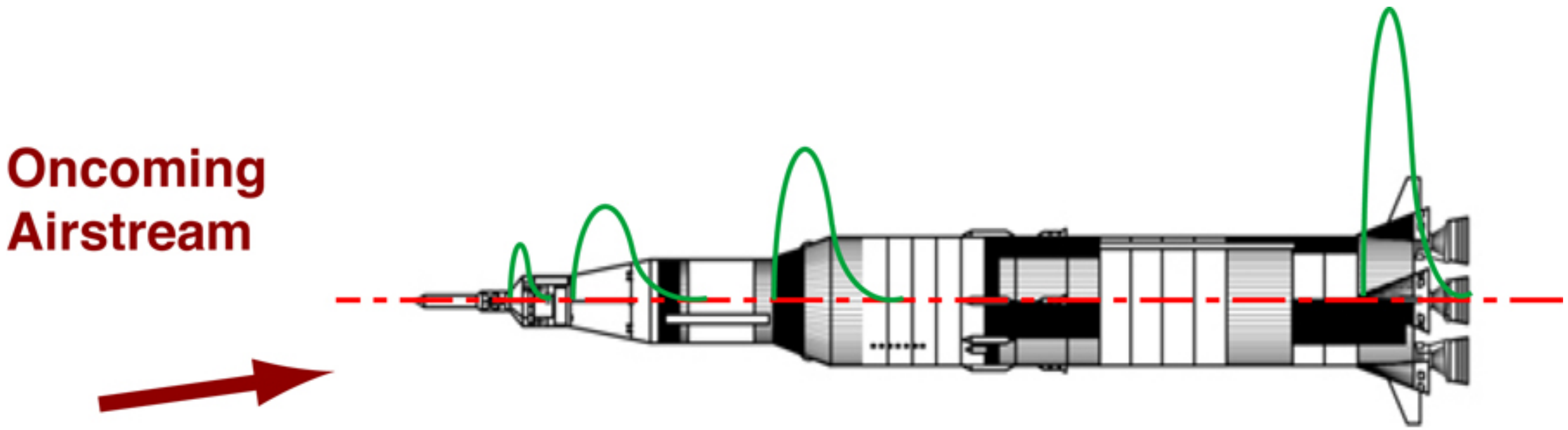
Normal Force about equal to Lift

$$Normal\ Force = C_N \frac{1}{2}\rho V^2 S \approx \frac{\partial C_N}{\partial \alpha}\alpha \frac{1}{2}\rho V^2 S$$

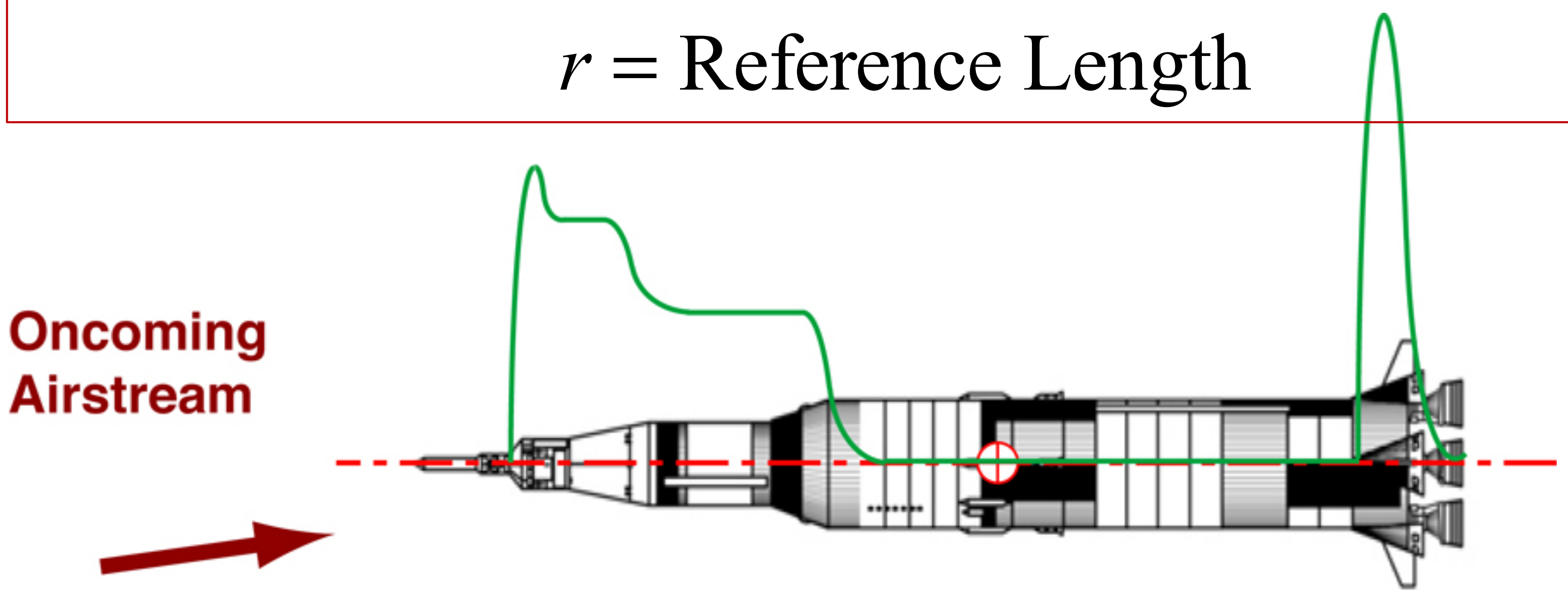


- Perpendicular to body centerline
- For small angle of attack, normal force is approximately the same as lift

Aerodynamic Pitching Moment

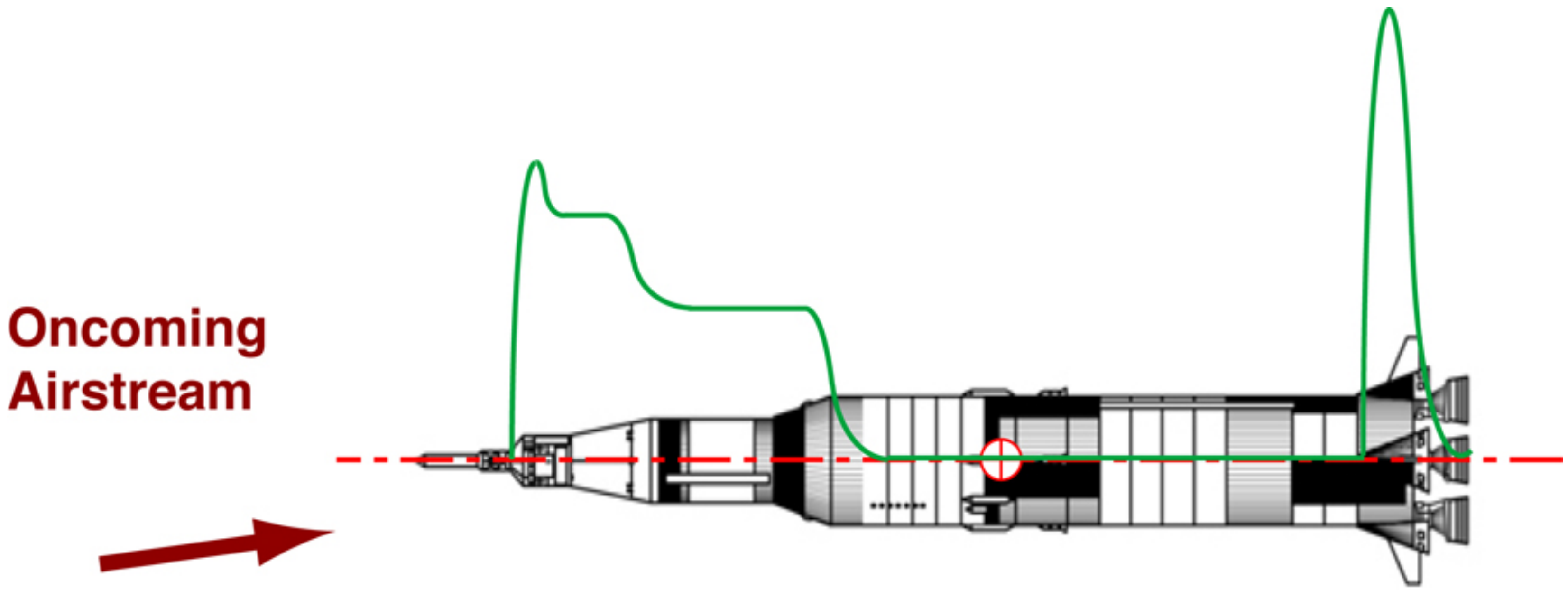
$$Pitching\ \ Moment = C_m \frac{1}{2}\rho V^2 Sr \approx \frac{\partial C_m}{\partial \alpha}\alpha \frac{1}{2}\rho V^2 Sr$$

$$r = \text{Reference Length}$$



- *Pitching moment components integrate over length to produce net pitching moment*
  - *Increase in cross-sectional area*
  - *Tail fins*
- *... plus pitching moment due to thrust vectoring for control*

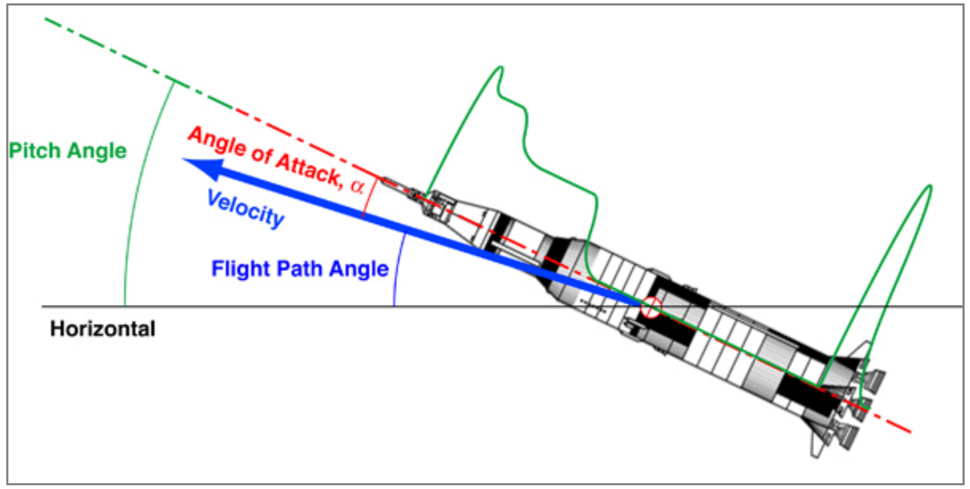
Pitching Moment Distribution Causes Large Bending Effects



Aerodynamic and thrust-vectoring effects <u>bend the vehicle</u>

Trajectory shaped to reduce structural loads

Angular Attitude Perturbations



- **Pitch-angle perturbation, Δθ, is about the same as angle-of-attack perturbation, Δα**

$$\Delta\ddot{\theta} \approx \Delta\ddot{\alpha} = \frac{Net\ Pitching\ Moment}{Pitching\ Moment\ of\ Inertia}$$

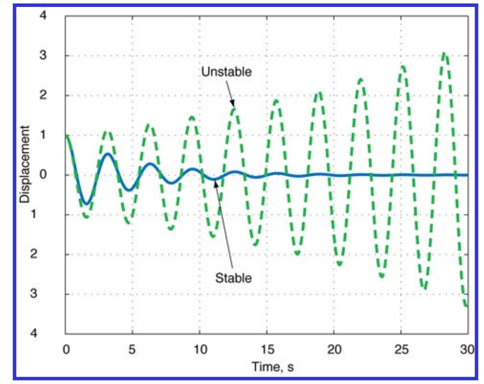
$$\Delta\ddot{\alpha} = \frac{M_{y_{aero}} + M_{y_{thrust}}}{I_{yy}} \approx \frac{1}{I_{yy}}\left[\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M_{y_{net}}}{\partial \alpha}\Delta\alpha\right]$$

Attitude Stability

$$\Delta\ddot{\alpha} = \frac{1}{I_{yy}}\left[\frac{\partial M_{y_{net}}}{\partial\dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M_{y_{net}}}{\partial\alpha}\Delta\alpha\right]$$

- **Attitude perturbations are stable if**

$$\frac{\partial M_{y_{net}}}{\partial\dot{\alpha}} < 0, \quad \frac{\partial M_{y_{net}}}{\partial\alpha} < 0$$



- **Oscillatory divergence if**

$$\frac{\partial M_{y_{net}}}{\partial\dot{\alpha}} > 0 \quad \textit{Dynamic Instability}$$

- **Non-oscillatory divergence if**

$$\frac{\partial M_{y_{net}}}{\partial\alpha} > 0 \quad \textit{Static Instability}$$

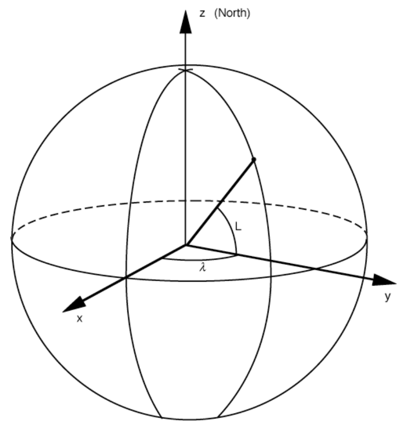
Thrust-vector feedback control normally required to provide static and dynamic stability

Combined Equations of Motion for a Point Mass

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix}_I = \begin{bmatrix} v_x \\ v_y \\ v_z \\ f_x/m \\ f_y/m \\ f_z/m \end{bmatrix}_I = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix}_I + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I$$

With

$$\boxed{\mathbf{F}_I = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I = \left[\mathbf{F}_{gravity} + \mathbf{F}_{aerodynamics} + \mathbf{F}_{thrust}\right]_I}$$



Math Models of Gravity

- ***Flat-earth approximation***
  - ***g is gravitational acceleration***
  - ***mg is gravitational force***
  - ***Independent of position***
- ***Round, rotating earth***
  - ***Inverse-square gravitation***
  - ***"Centripetal acceleration"***
  - ***Non-linear function of position***
  - $\mu = 3.986 \times 10^5\ km^3/s^2$
  - $\Omega = 7.29 \times 10^{-5}\ rad/s$

$$m\mathbf{g}_f = m\begin{bmatrix} 0 \\ 0 \\ g_o \end{bmatrix} \quad ; \quad g_o = 9.807\ m/s^2$$

$$\mathbf{g}_r = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \mathbf{g}_{gravity} \qquad [\text{non-rotating frame}]$$

$$\mathbf{g}_r = \mathbf{g}_{gravity} + \mathbf{g}_{rotation} \qquad [\text{rotating frame}]$$

$$= \frac{-\mu}{r^3}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \Omega^2\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \quad ; \quad r = \left[x^2 + y^2 + z^2\right]^{1/2}$$

Equations of Motion with Round-Earth Gravity Model
(Inertial, Non-Rotating Frame)

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{v}_x\\ \dot{v}_y\\ \dot{v}_z\end{bmatrix}_E = \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ -\mu/r^3 & 0 & 0 & 0 & 0 & 0\\ 0 & -\mu/r^3 & 0 & 0 & 0 & 0\\ 0 & 0 & -\mu/r^3 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ v_x\\ v_y\\ v_z\end{bmatrix}_I + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 1/m & 0 & 0\\ 0 & 1/m & 0\\ 0 & 0 & 1/m\end{bmatrix}\left[\begin{pmatrix}f_x\\ f_y\\ f_z\end{pmatrix}_{aero} + \begin{pmatrix}f_x\\ f_y\\ f_z\end{pmatrix}_{thrust}\right]_I$$

Position of the vehicle (in spherical coordinates)

$$\mathbf{r} = \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}\cos L\cos\lambda\\ \cos L\sin\lambda\\ \sin L\end{bmatrix}(R+h)$$

R : *Earth's radius*

h : *Altitude (height)*

L : *Latitude*

λ : *Longitude*

Detailed Equations for Trajectory Computation

- NRL Report 8237
  - chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://apps.dtic.mil/sti/tr/pdf/ADA069296.pdf

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THE ATMOSPHERE

Lecture Slides by Stengel

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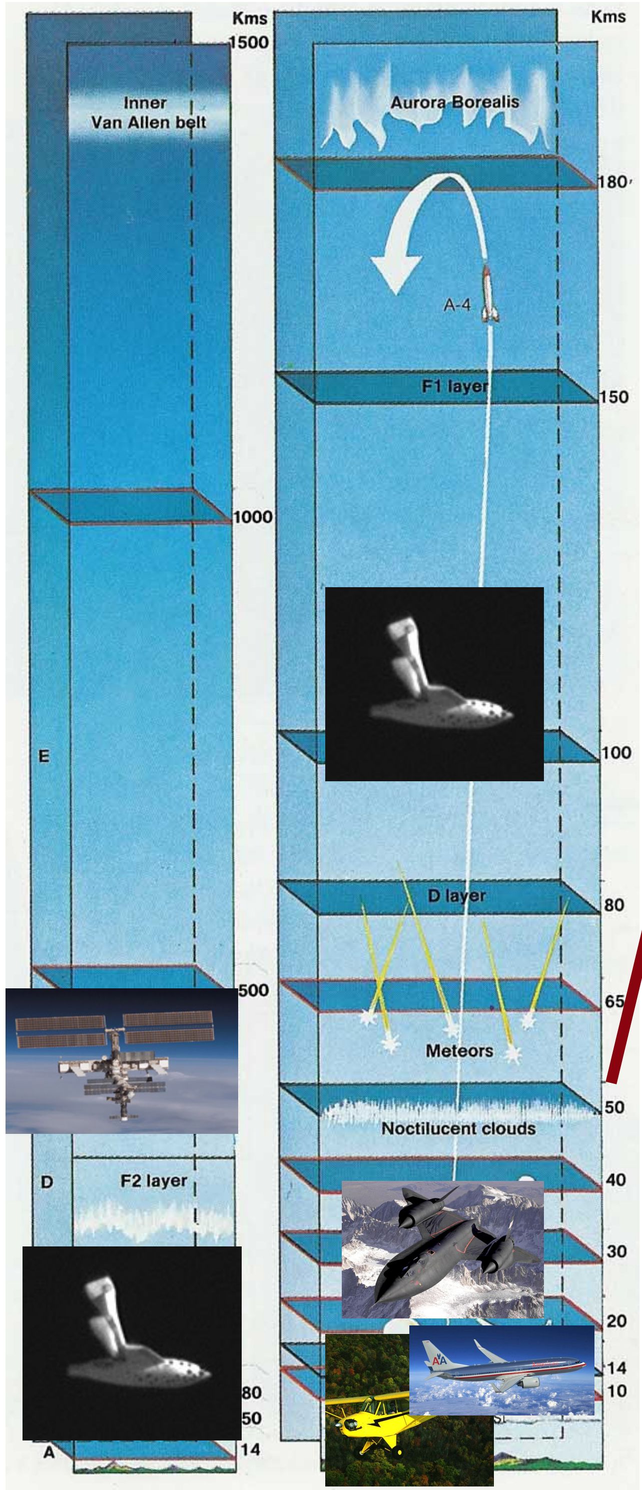
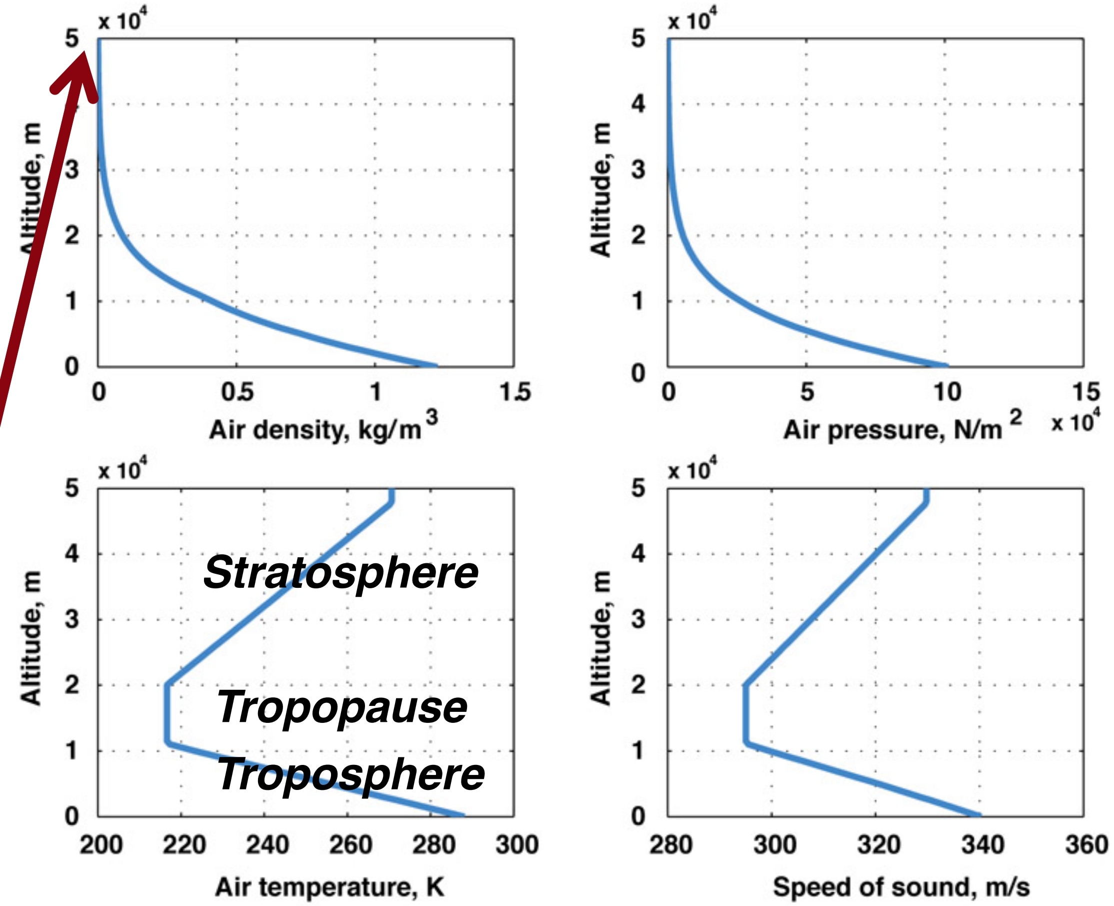
Properties of the Lower Atmosphere*





- Density and pressure decay exponentially with altitude
- Temperature and speed of sound are piecewise-linear functions of altitude

** 1976 US Standard Atmosphere*

Air Density, Dynamic Pressure, and Mach Number

$$\rho = \textit{Air density}, \text{ function of height}$$
$$= \rho_{sealevel} e^{-\beta h} = \rho_{sealevel} e^{\beta z}$$

$$\rho_{sealevel} = 1.225\,\text{kg/m}^3; \quad \beta = 1/9{,}042\,\text{m}$$

$$V_{air} = \left[v_x^2 + v_y^2 + v_z^2 \right]_{air}^{1/2} = \left[\mathbf{v}^T \mathbf{v} \right]_{air}^{1/2} = \textit{Airspeed}$$

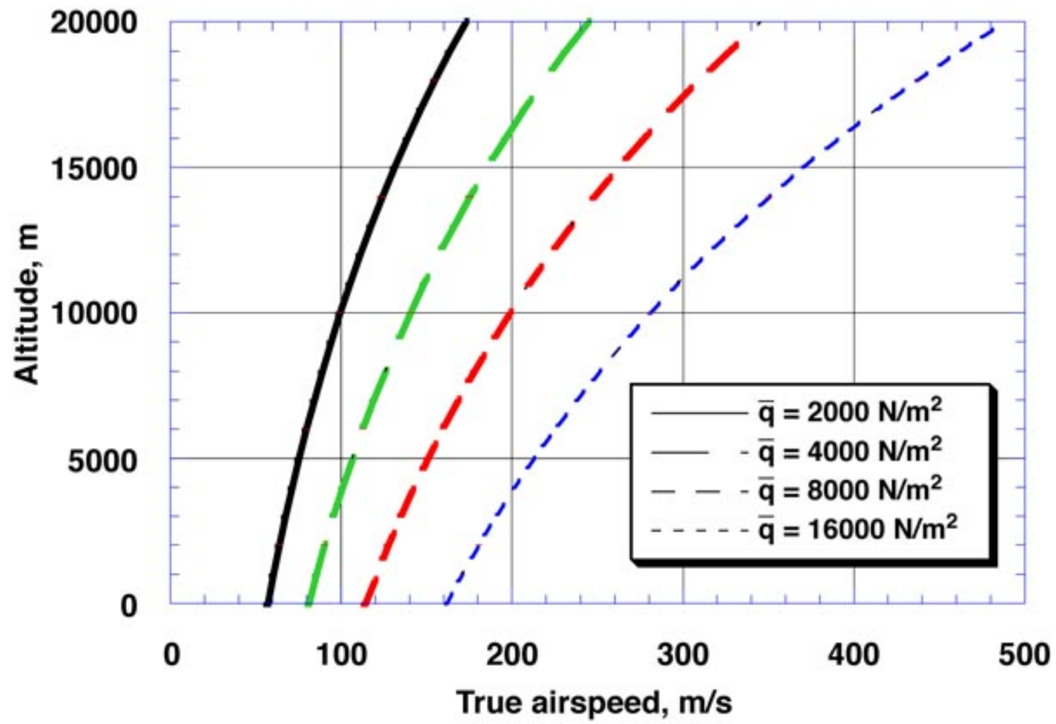
$$\textit{Dynamic pressure} = \bar{q} = \frac{1}{2} \rho(h) V_{air}^2 \triangleq \text{"qbar"}$$

$$\textit{Mach number} = \frac{V_{air}}{a(h)}; \quad a = \text{speed of sound, m/s}$$

Contours of Constant Dynamic Pressure, $\bar{q}$

- **In steady, cruising flight,**

$$Weight = Lift = C_L \frac{1}{2}\rho V_{air}^2 S = C_L \bar{q} S$$



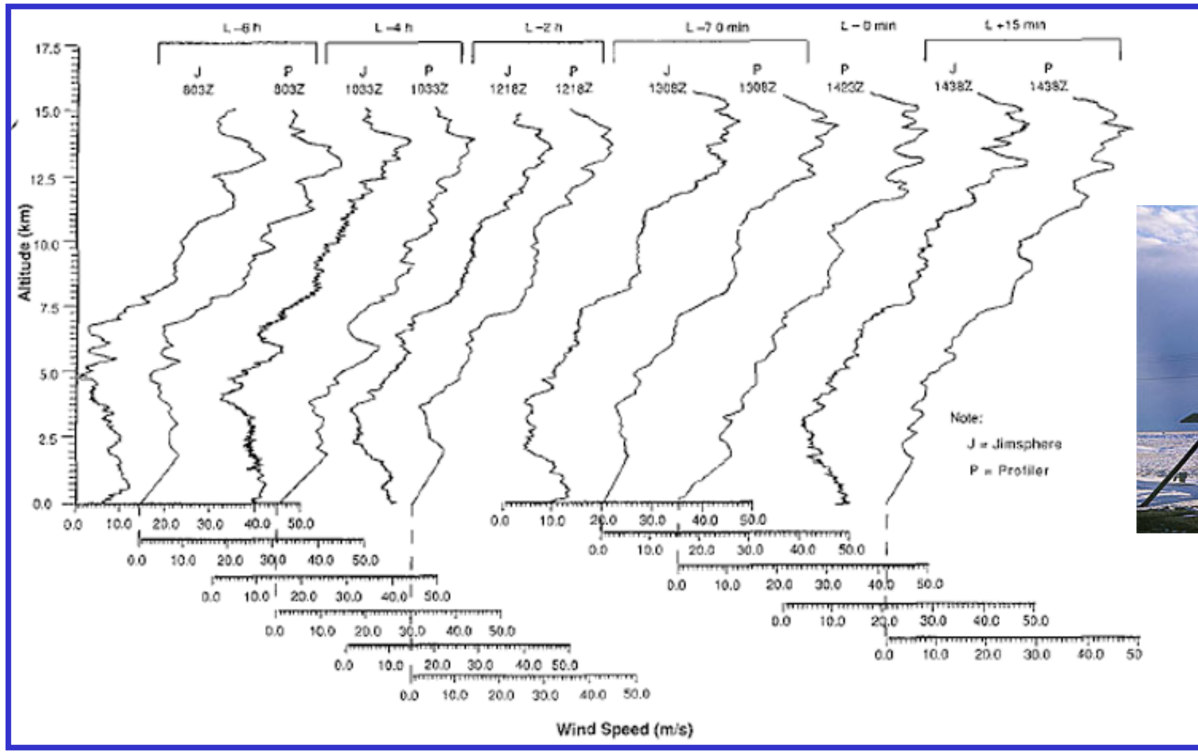
True airspeed must increase as altitude increases to maintain constant dynamic pressure

Wind: Motion of the Atmosphere

Zero wind at Earth's surface = Rotating air mass

Wind measured with respect to Earth's <u>rotating</u> surface

Wind Velocity Profiles vary over Time



Airspeed = Airplane's speed with respect to air mass

Earth-relative velocity = Wind velocity ± True airspeed ***[vector]***

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RE-ENTRY

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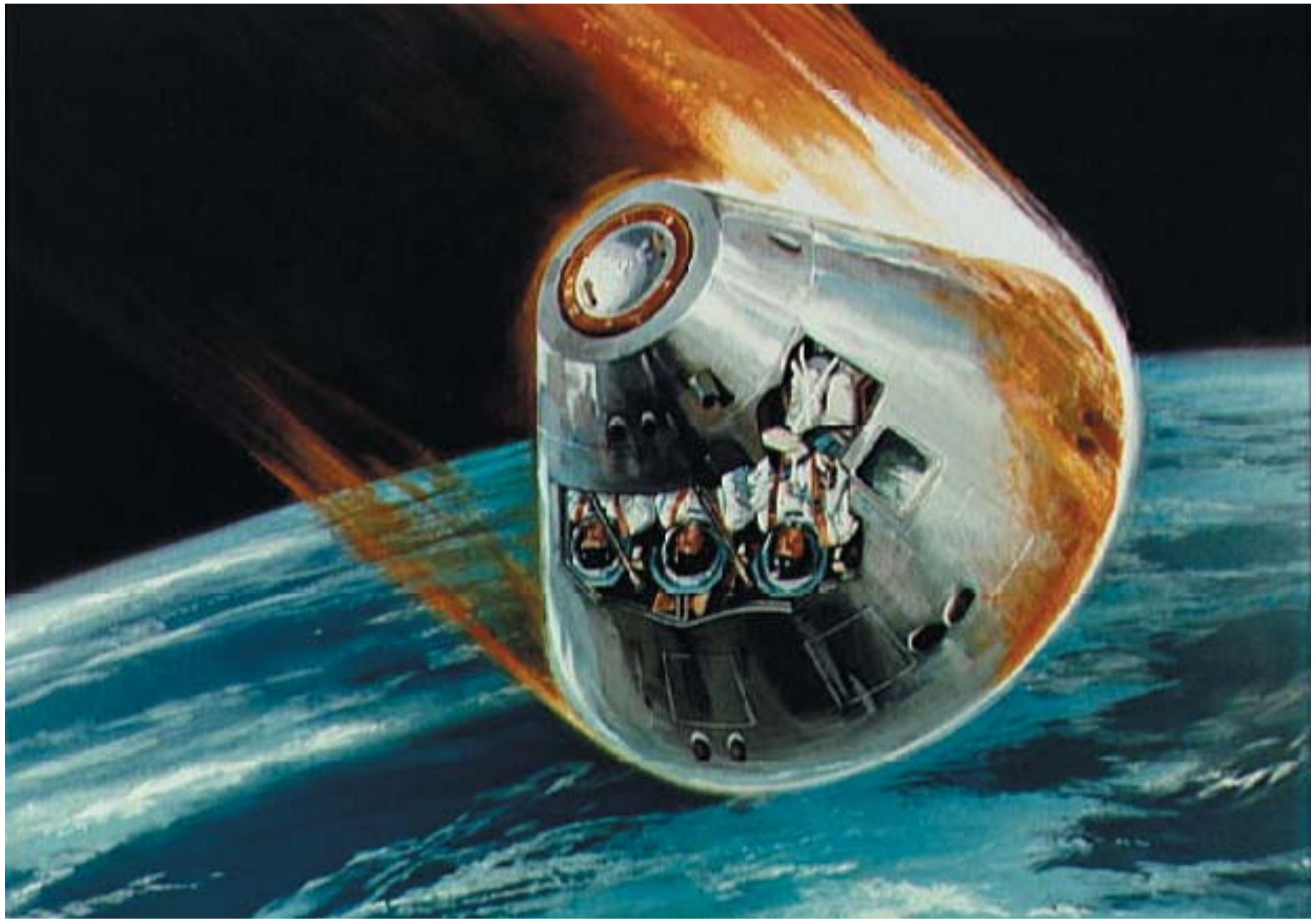
Apollo capsule re-entry



The tradeoff

- Like all phases of a space mission, during the re-entry phase we must delicately balance three, often competing, requirements:
  - Deceleration
  - Heating
  - Accuracy of landing or impact

Deceleration

- The vehicle's structure and payload limit the maximum deceleration or "g's" it can withstand.
- When subjected to enough g's, even steel and aluminum can crumple like paper.
  - Fortunately, the structural g limits for a well-designed vehicle can be quite high, perhaps hundreds of g's.
- But a fragile human payload would be crushed to death long before reaching that level.
  - Humans can withstand a maximum deceleration of about 12 g's (about 12 times their weight) for only a few minutes at a time.
- Too little deceleration can also cause serious problems.
  - Similar to a rock skipping off a pond, a vehicle that doesn't slow down enough may literally bounce off the atmosphere and back into the cold reaches of space.

Heating

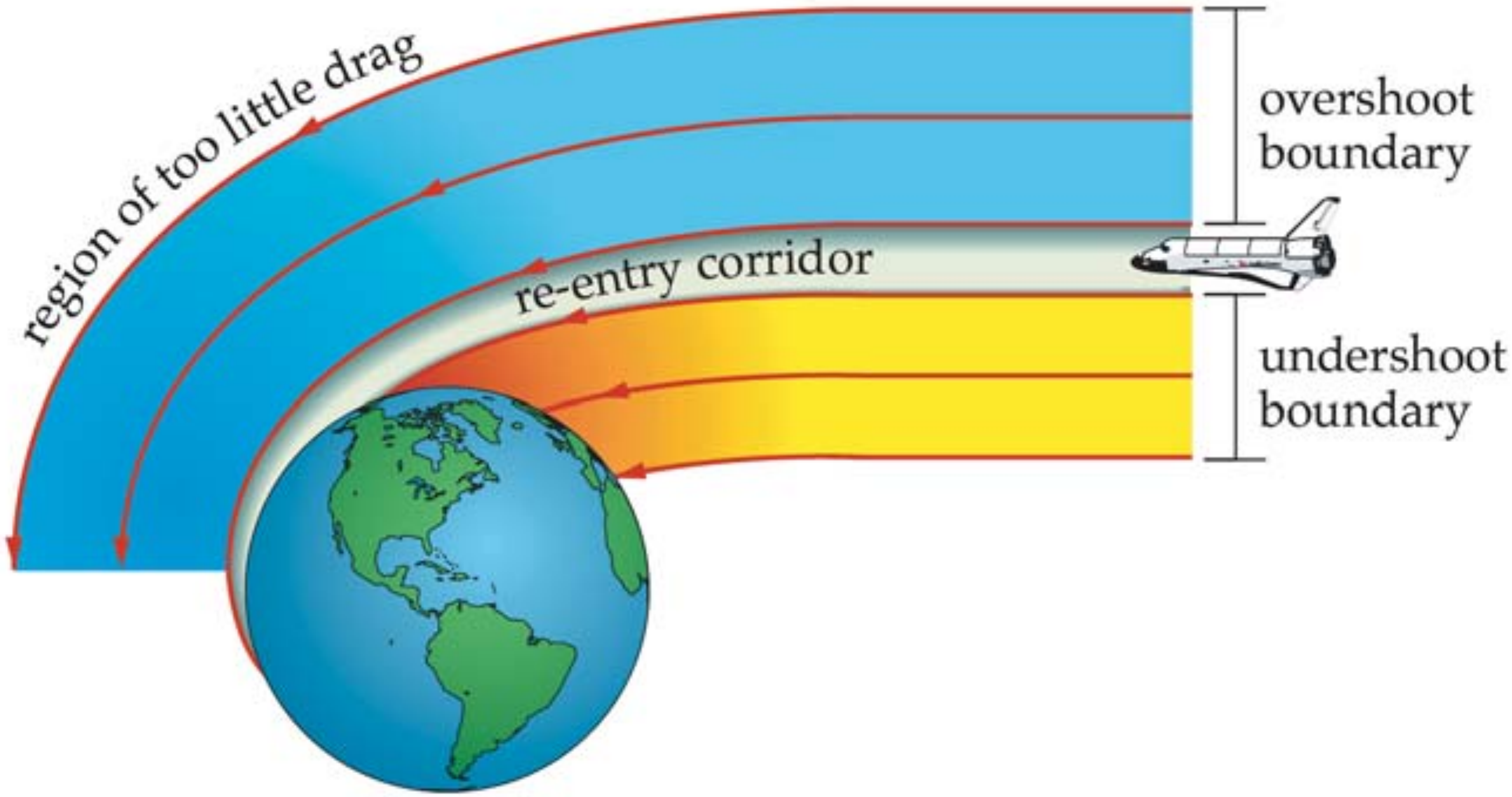
- The fiery trail of a meteor streaking across the night sky shows that re-entry can get hot!
  - This intense heat is a result of friction between the speeding meteor and the air.
- How hot can something get during re-entry?
  - The Space Shuttle in orbit has a mass of 100,000 kg (220,000 lb.), an orbital velocity of 7700 m/s (17,225 m.p.h.), and an altitude of 300 km (186 mi.).
  - An object's total mechanical energy depends on its kinetic energy (energy of motion) and its potential energy (energy of position).
  - Its total mechanical energy is E = 3.23 × 1012 joules = 3.06 × 109 Btu, which is enough energy to heat the average home in Colorado for 41 years!
  - It must “lose” all this energy in only about one-half hour to come to a full stop on the runway (at Earth’s surface.
  - Energy is conserved, so it is all converted to heat (from friction).
  - This heat makes the Shuttle’s surfaces reach temperatures of up to 1477° C (2691° F).

Accuracy of landing or impact

- Vehicle trajectory and design are dependent on landing constraints.
  - Beginning its descent from more than 6440 km (4000 mi.) away, the Space Shuttle must land on a runway only 91 m (300 ft.) wide.
  - The re-entry vehicle (RV) of an Intercontinental Ballistic Missile (ICBM) has even tighter accuracy requirements.
- If a vehicle can land in a larger area, the accuracy constraint becomes less important.
  - For example, the Apollo missions required the capsules to land in large areas in the Pacific Ocean, a much larger landing zones than for an ICBM's RV payload.
  - Thus, the Apollo capsule was less streamlined and used a trajectory with a shallower re- entry angle.

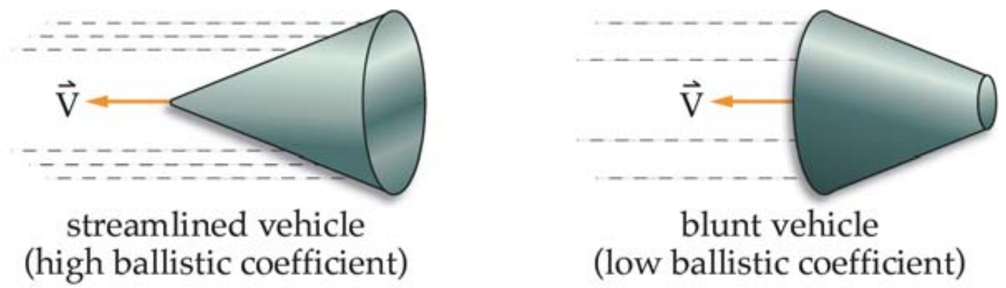
Re-entry corridor

- The re-entry corridor is a narrow region in space that a re-entering vehicle must fly through.
- If the vehicle strays above the corridor, it may skip out.
- If it strays below the corridor, it may burn up.



Ballistic Coefficient

- The amount of deceleration an object experiences while traveling through the atmosphere is inversely related to the object's ballistic coefficient.



$$BC = \frac{m}{C_D A}$$

$$\vec{a} = \frac{1}{2}\rho V^2 \frac{C_D A}{m}$$

where

BC = vehicle's ballistic coefficient (kg/m^2)

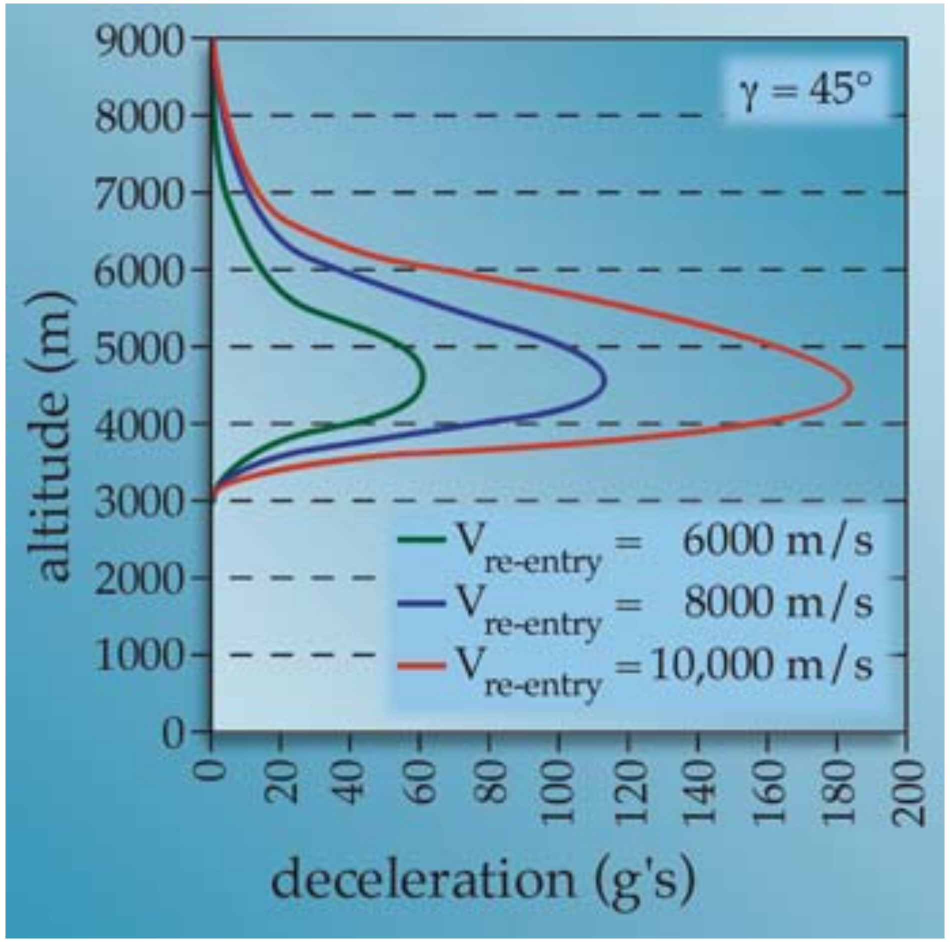
m = vehicle's mass (kg)

C_D = vehicle's drag coefficient (unitless)

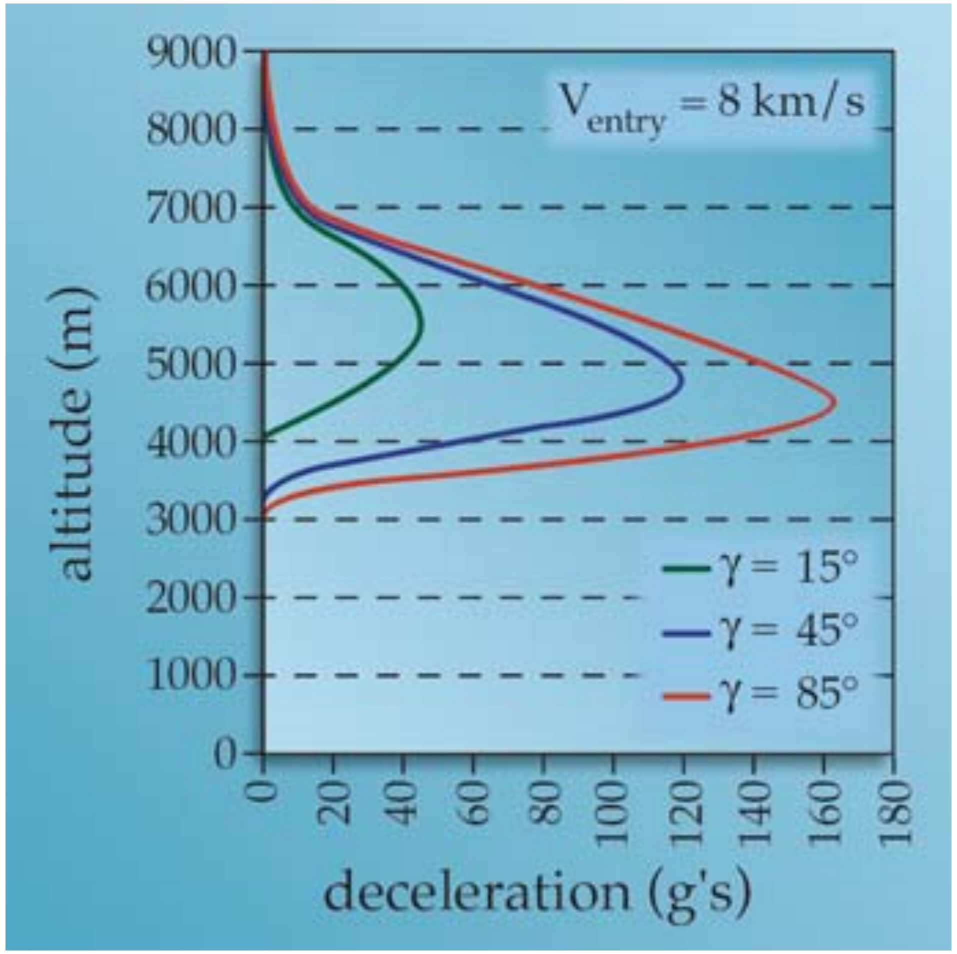
A = vehicle's cross-sectional area (m^2)

Deceleration

- Varying re-entry velocities…



- Varying re-entry angles…

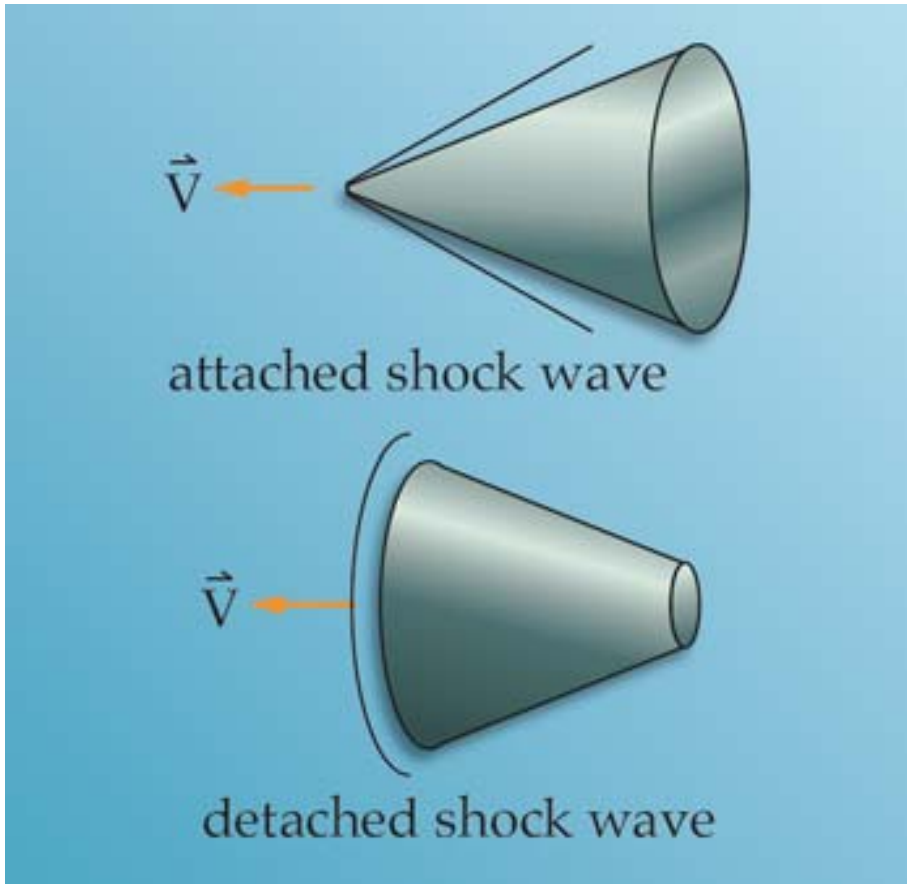


Heating

- **Radiation or radiative heat transfer** involves the transfer of energy from one point to another through electromagnetic waves.
  - If you've ever held your hand in front of a glowing space heater, you've felt radiative heat transfer.
- **Conduction or conductive heat transfer** moves heat energy from one point to another through some physical medium.
  - For example, try holding one end of a metal rod and sticking the other end in a hot fire. Before too long the end you're holding will get HOT (ouch)! The heat "conducts" along the metal rod.
- **Convection or convective heat transfer** occurs when a fluid flows past an object and transfers energy to it or absorbs energy from it (depending on which object is hotter).

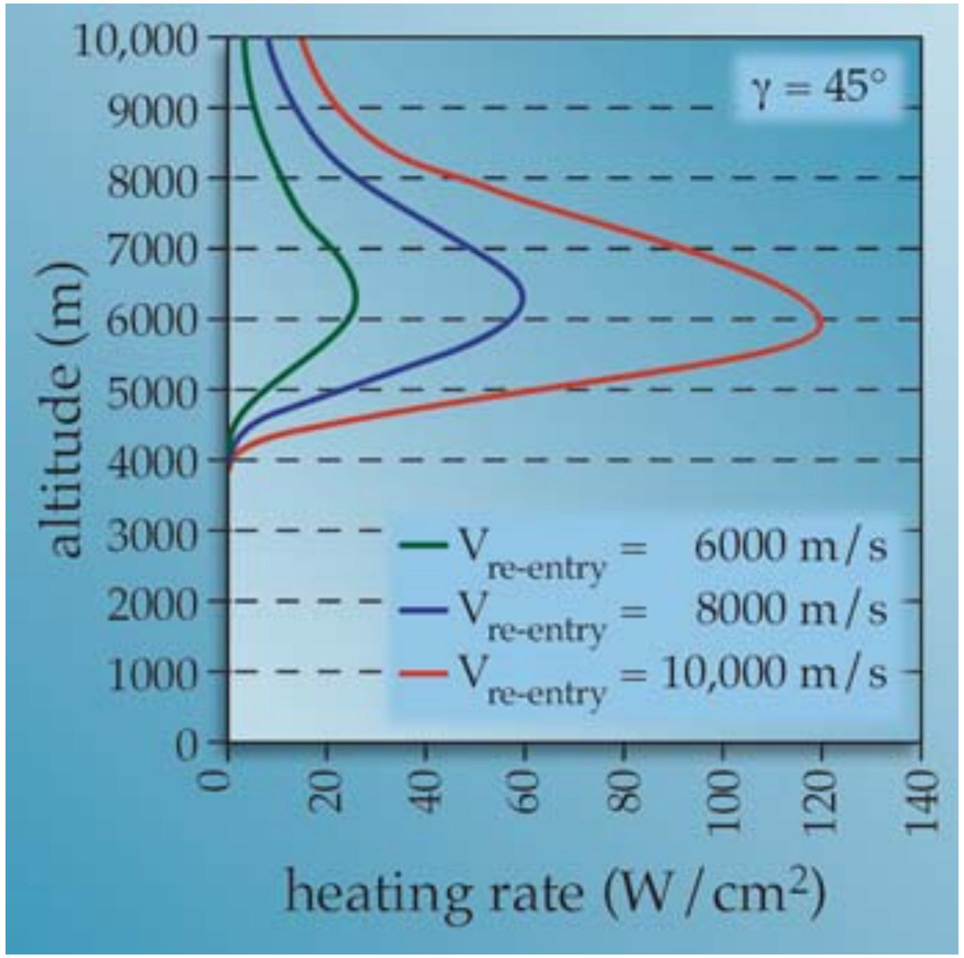
Heating

- The shock wave around the vehicle can either be attached or detached.
  - If the vehicle is streamlined (high BC, like a cone), the shock wave may attach to the tip and transfer a lot of heat, causing localized heating at the attachment point.
  - If the vehicle is blunt (low BC, like a rock), the shock wave will detach and curve in front of the vehicle, leaving a boundary of air between the shock wave and the vehicle's surface.
- The hot air molecules around the vehicle transfer some of their heat to the vehicle by convection – the primary means of heat transfer to a vehicle entering Earth's atmosphere at speeds under about 15,000 m/s.
  - Above this speed, the air molecules get so hot they begin to transfer more of their energy to the vehicle by radiation.

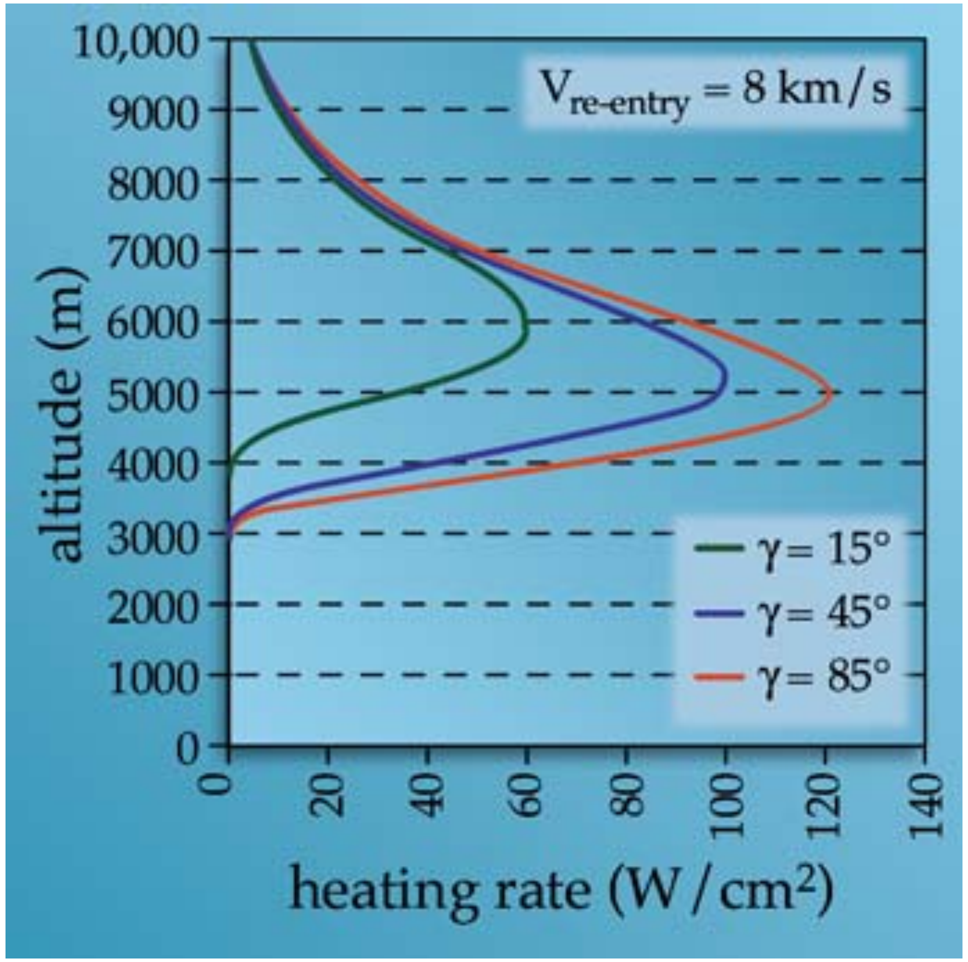


Heating

- Varying re-entry velocities…



- Varying re-entry angles…



Accuracy of landing or impact

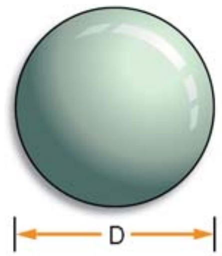
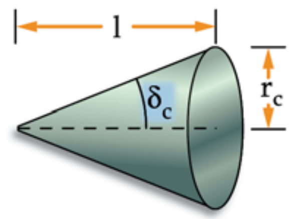
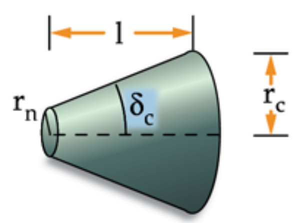
- Drag and lift forces perturb the trajectory of the vehicle from the path it would follow under gravity alone.
  - Whether we're modeling the density, ρ, or the drag coefficient, CD, the values we use are, at best, only close to the real values and, at worst, mere approximations.
  - Thus, the actual trajectory path will be somewhat different, so when we try to aim at a particular target we might miss!
- To reduce these atmospheric effects, and improve our accuracy, we want a trajectory that spends the least time in the atmosphere.
- So for the greatest accuracy we would choose a high re-entry velocity and a steep re-entry angle, but as we've just seen, this increases the severity of deceleration and heating.
  - To achieve highly accurate re-entry for ICBMs, we build these vehicles to withstand extremely high g forces and peak heating.
  - Manned vehicles, on the other hand, accept lower accuracy to get much lower peak deceleration and heating.

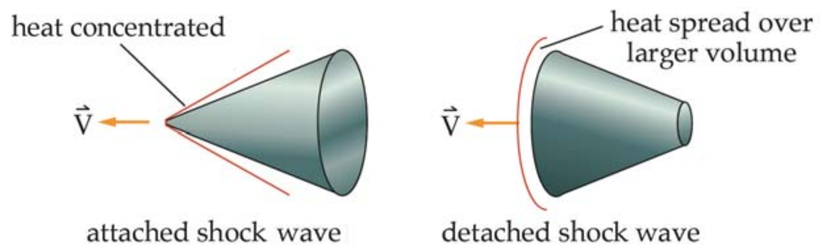
Trajectory tradeoff for re-entry design

Parameter	Maximum Deceleration	Altitude of Maximum Deceleration	Maximum Heating Rate	Altitude of Maximum Heating Rate	Accuracy	Corridor Width
Re-entry velocity, $V_{re\text{-}entry}$ (constant γ)						
High	High	Same	High	Same	High	Narrow
Low	Low	Same	Low	Same	Low	Wide
Re-entry flight-path angle, γ (constant $V_{re\text{-}entry}$)						
Steep	High	Low	High	Low	High	Narrow
Shallow	Low	High	Low	High	Low	Wide

Effect of vehicle shape

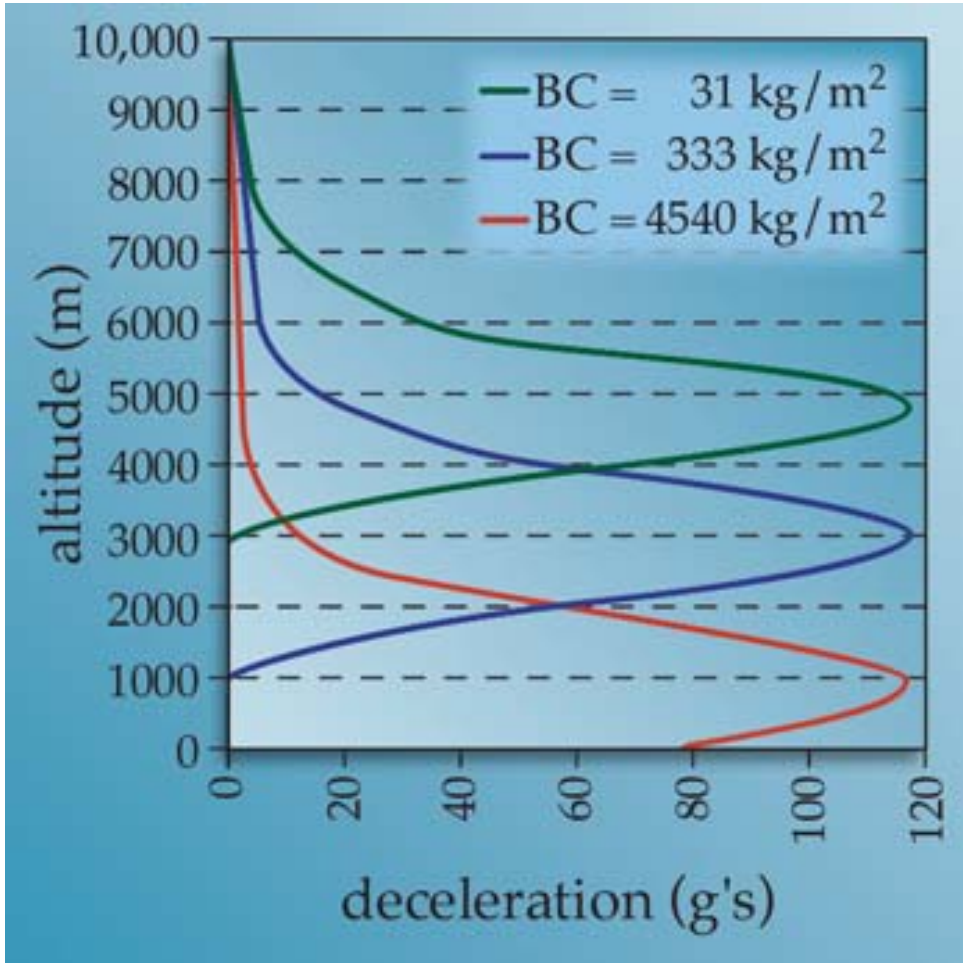
- On ballistic coefficient…

Shape	Example Values	Estimated Ballistic Coefficient
Sphere 	$D = 2$ m $C_D = 2.0$ $m = 2094$ kg (Assumes density = 500 kg/m^3)	BC $\cong$ 333 kg/m^2
Cone 	$l = 3.73$ m $\delta_c = 15°$ = cone half angle $r_c = 1$ m = cone radius $C_D \cong 2\,\delta_c^2 = 0.137$ $m = 1954$ kg (Assumes density = 500 kg/m^3)	BC $\cong$ 4543 kg/m^2
Blunted cone 	$l = 3.04$ m $\delta_c = 15°$ = cone half angle $r_c = 1$ m = cone radius $r_n = 0.304$ m $m = 1932$ kg (Assumes density = 500 kg/m^3) $C_D = (1 - \sin^4\delta_c)\left(\frac{r_n}{r_c}\right)^2 + 2\sin^2\delta_c\left[1 - \left(\frac{r_n}{r_c}\right)^2\cos^2\delta_c\right]$ $C_D \cong 0.188$	BC $\cong$ 3266 kg/m^2

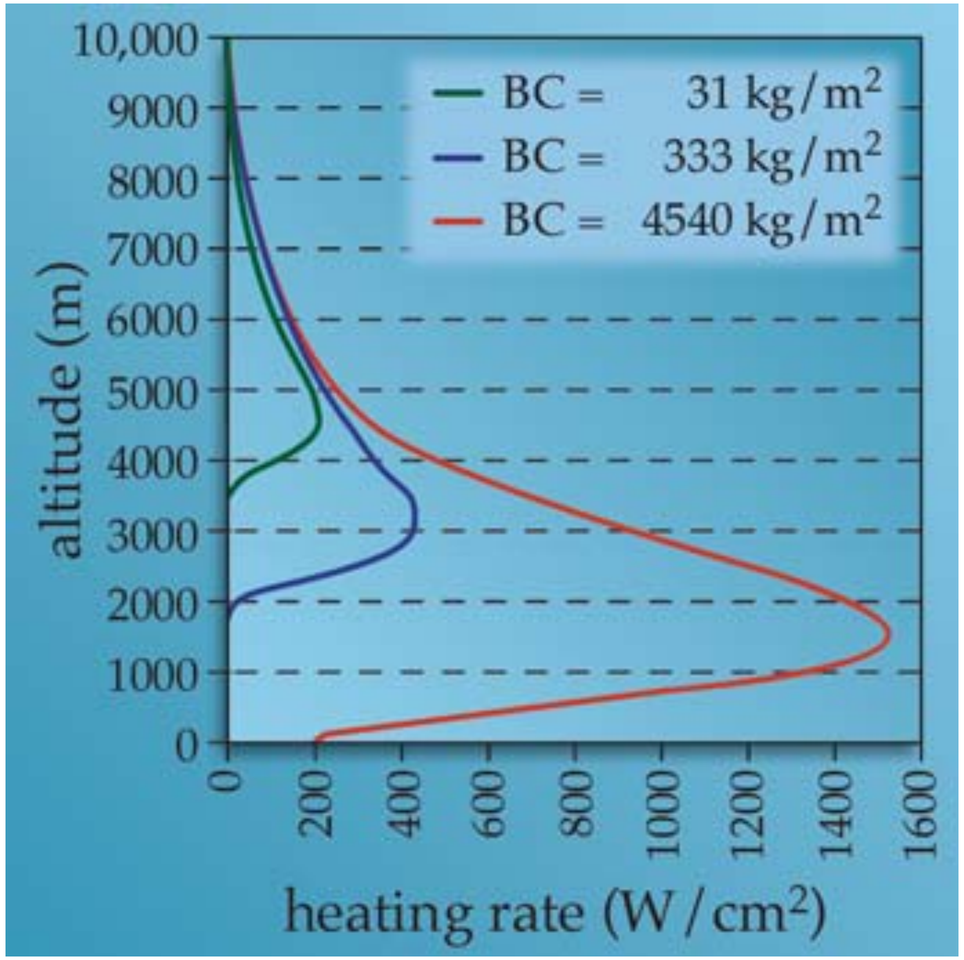


Effect of vehicle shape

- On deceleration…



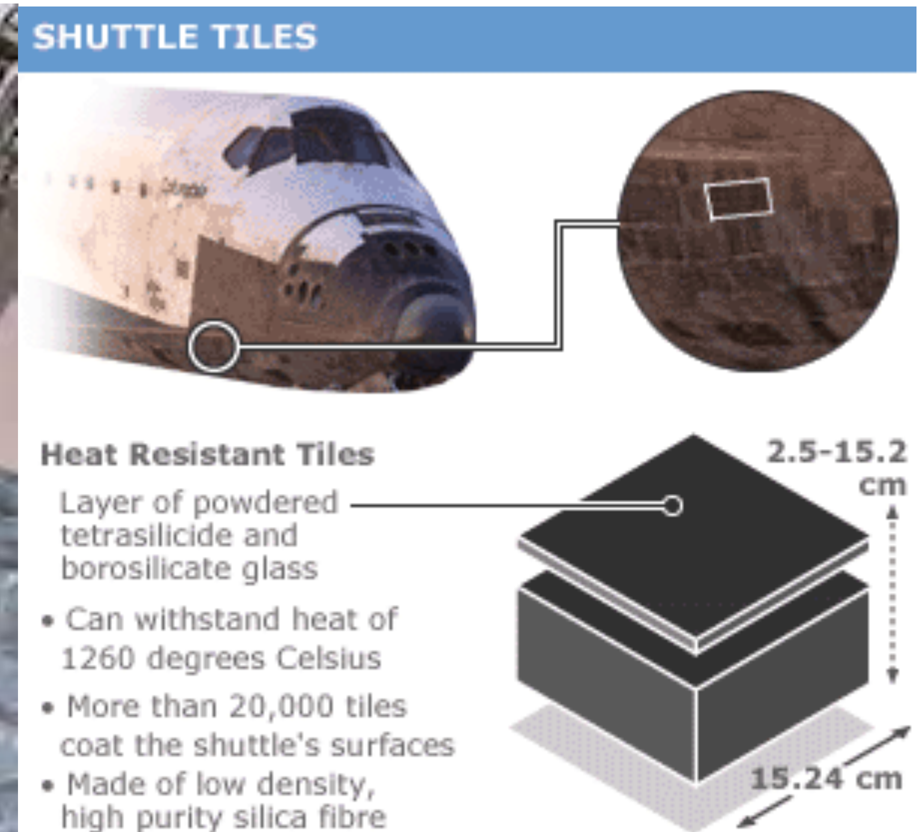
- On heating…



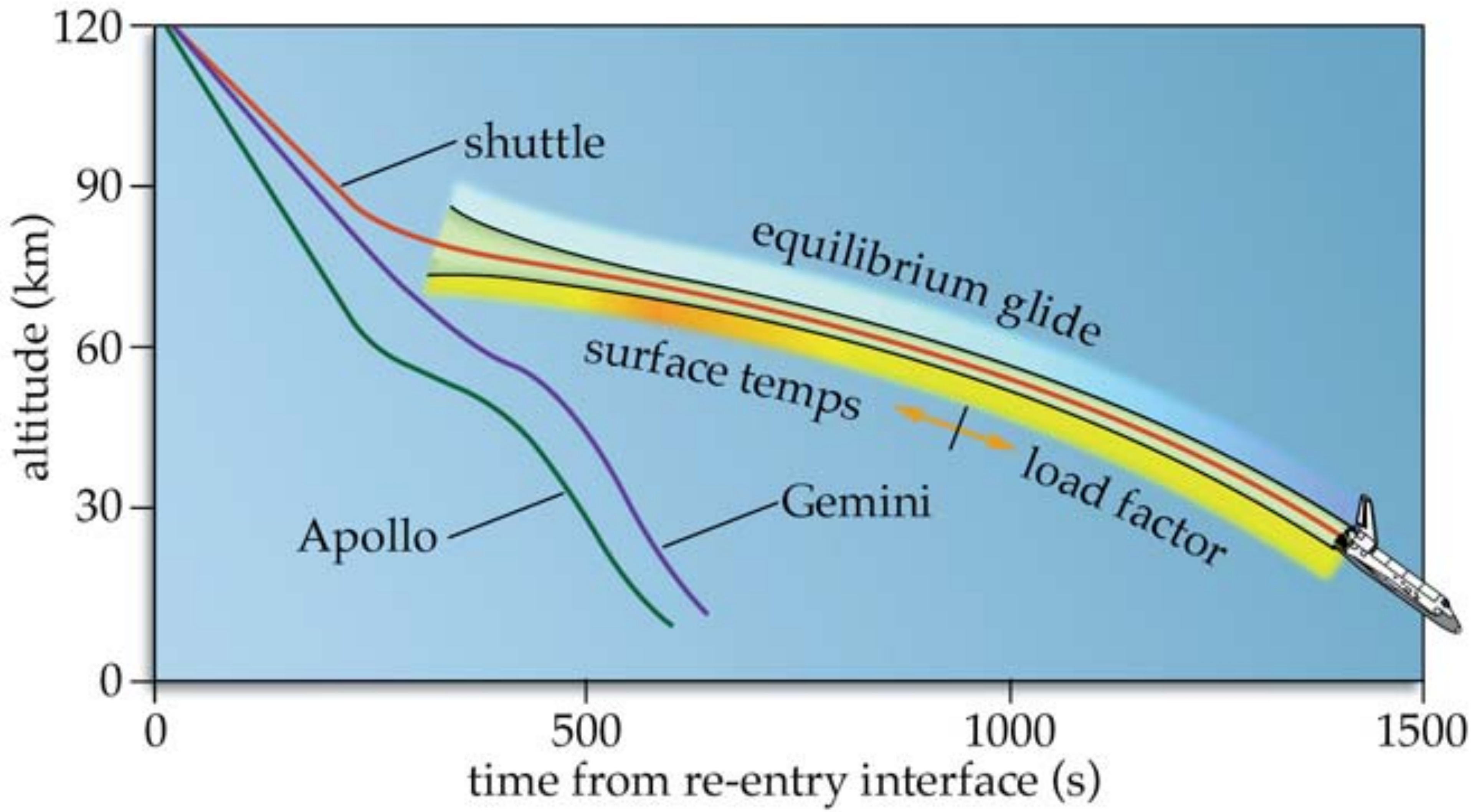
Thermal protection systems

- Heat sinks
  - ICBMs, in the 1950s, they used extra material to absorb the heat, keeping the peak temperature lower.
  - Unfortunately, for a given launch vehicle, as designers increased a heat sink's mass, they had to drastically limit the available payload mass.
- Ablation
  - In the Apollo era, they coated the vehicle's surface with a material having a very high latent heat of fusion, such as carbon or ceramics.
  - As this material melts or vaporizes, it soaks up large amounts of heat energy and protects the vehicle.
- Radiative cooling
  - When you apply heat to an object, it will do three things—transmit the heat (like light through a pane of glass), reflect it (like light on a mirror), or absorb it (like a rock in the Sun).
  - If an object absorbs enough heat, it warms up and, at the same time, radiates some of the heat through emission.
  - If heat energy continues to strike the object, it heats until the energy emitted balances the energy absorbed. At this point, it's in thermal equilibrium, where its temperature levels off and stays constant.

Space shuttle tiles



Lifting re-entry



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