The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

10 OCTOBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Rigid Body Equations of Motion
  - Aerodynamic Damping

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RIGID BODY EQUATIONS OF MOTION

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Rigid-Body Equations of Motion

- **Translational Position**

$$\mathbf{r}_I = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_I$$

- **Angular Position**

$$\mathbf{\Theta}_I = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_I$$

- **Translational Velocity**

$$\mathbf{v}_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_B$$

- **Angular Velocity**

$$\boldsymbol{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$$

- **Rate of change of Translational Position**

The Rotation Matrix

The three-angle rotation matrix is the product of 3 single-angle rotation matrices:

$$\boxed{\dot{\mathbf{r}}_I(t) = \mathbf{H}_B^I(t)\,\mathbf{v}_B(t)}$$

$$\boxed{\mathbf{H}_I^B(\phi,\theta,\psi) = \mathbf{H}_2^B(\phi)\,\mathbf{H}_1^2(\theta)\,\mathbf{H}_I^1(\psi)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Direction of cosine

$$= \left[\begin{array}{c|c|c} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \hline -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \hline \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{array}\right]$$

Rigid-Body Equations of Motion

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- **Angular Velocity**

$$\boldsymbol{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$$

- **Rate of change of Translational Position**

$$\boxed{\dot{\mathbf{r}}_I(t) = \mathbf{H}_B^I(t)\,\mathbf{v}_B(t)}$$

- **Rate of change of Angular Position**

Relationship Between Euler-Angle Rates and Body-Axis Rates

- $\dot{\psi}$ is measured in the Inertial Frame
- $\dot{\theta}$ is measured in Intermediate Frame #1
- $\dot{\phi}$ is measured in Intermediate Frame #2
- **... which is**

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{I}_3 \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \mathbf{H}_2^B \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \mathbf{H}_2^B \mathbf{H}_1^2 \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \left[\begin{array}{c|c|c} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{array}\right] \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \mathbf{L}_I^B \dot{\boldsymbol{\Theta}}$$

not orthonormal

Inverse transformation $[(.)^{-1} \neq (.)^T]$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{L}_B^I \boldsymbol{\omega}_B$$

Rigid-Body Equations of Motion

- **Translational Position**

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$$\boldsymbol{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$$

- **Rate of change of Translational Position**

$$\boxed{\dot{\mathbf{r}}_I(t) = \mathbf{H}_B^I(t)\,\mathbf{v}_B(t)}$$

- **Rate of change of Angular Position**

$$\boxed{\dot{\boldsymbol{\Theta}}_I(t) = \mathbf{L}_B^I(t)\,\boldsymbol{\omega}_B(t)}$$

- **Rate of change of Translational Velocity**

Point-Mass Dynamics

- **Inertial rate of change of translational position**

$$\boxed{\dot{\mathbf{r}}_I = \mathbf{v}_I = \mathbf{H}_B^I \mathbf{v}_B}$$

$$\mathbf{v}_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- **Body-axis rate of change of translational velocity**
  - Identical to angular-momentum transformation

$$\boxed{\dot{\mathbf{v}}_I = \frac{1}{m}\mathbf{F}_I}$$

acceleration w.r.t inertial

Centrepetal force

accel due to rotating inertial frame.

$$\dot{\mathbf{v}}_B = \mathbf{H}_I^B \dot{\mathbf{v}}_I - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B = \frac{1}{m}\mathbf{H}_I^B \mathbf{F}_I - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B$$

$$= \frac{1}{m}\mathbf{F}_B - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B$$

$$\mathbf{F}_B = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} C_X \bar{q} S \\ C_Y \bar{q} S \\ C_Z \bar{q} S \end{bmatrix}$$

Rigid-Body Equations of Motion

- **Translational Position**

$$\mathbf{r}_I = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_I$$

- **Rate of change of Translational Position**

$$\boxed{\dot{\mathbf{r}}_I(t) = \mathbf{H}_B^I(t)\,\mathbf{v}_B(t)}$$

- **Angular Position**

$$\boldsymbol{\Theta}_I = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_I$$

- **Rate of change of Angular Position**

$$\boxed{\dot{\boldsymbol{\Theta}}_I(t) = \mathbf{L}_B^I(t)\,\boldsymbol{\omega}_B(t)}$$

- **Translational Velocity**

$$\mathbf{v}_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_B$$

- **Rate of change of Translational Velocity**

$$\boxed{\dot{\mathbf{v}}_B(t) = \frac{1}{m(t)}\mathbf{F}_B(t) + \mathbf{H}_I^B(t)\,\mathbf{g}_I - \tilde{\boldsymbol{\omega}}_B(t)\,\mathbf{v}_B(t)}$$

- **Angular Velocity**

$$\boldsymbol{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$$

- **Rate of change of Angular Velocity**

Rate of Change of Body-Referenced Angular Rate due to External Moment

In the body frame of reference, the angular momentum change is

$$\begin{aligned}\dot{\mathbf{h}}_B &= \mathbf{H}_I^B\dot{\mathbf{h}}_I + \dot{\mathbf{H}}_I^B\mathbf{h}_I = \mathbf{H}_I^B\dot{\mathbf{h}}_I - \boldsymbol{\omega}_B \times h_B \\ &= \mathbf{H}_I^B\dot{\mathbf{h}}_I - \tilde{\boldsymbol{\omega}}_B h_B = \mathbf{H}_I^B\mathbf{M}_I - \tilde{\boldsymbol{\omega}}_B\mathbb{I}_B\boldsymbol{\omega}_B \\ &= \mathbf{M}_B - \tilde{\boldsymbol{\omega}}_B\mathbb{I}_B\boldsymbol{\omega}_B\end{aligned}$$

For constant body-axis inertia matrix

$$\dot{\mathbf{h}}_B = \mathbb{I}_B\dot{\boldsymbol{\omega}}_B = \mathbf{M}_B - \tilde{\boldsymbol{\omega}}_B\mathbb{I}_B\boldsymbol{\omega}_B$$

Consequently, the differential equation for angular rate of change is

$$\dot{\boldsymbol{\omega}}_B = \mathbb{I}_B^{-1}\left(\mathbf{M}_B - \tilde{\boldsymbol{\omega}}_B\mathbb{I}_B\boldsymbol{\omega}_B\right)$$

Rigid-Body Equations of Motion

- **Translational Position**

$$\mathbf{r}_I = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_I$$

- **Rate of change of Translational Position**

$$\boxed{\dot{\mathbf{r}}_I(t) = \mathbf{H}_B^I(t)\mathbf{v}_B(t)}$$

- **Angular Position**

$$\boldsymbol{\Theta}_I = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_I$$

- **Rate of change of Angular Position**

$$\boxed{\dot{\boldsymbol{\Theta}}_I(t) = \mathbf{L}_B^I(t)\boldsymbol{\omega}_B(t)}$$

- **Translational Velocity**

$$\mathbf{v}_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_B$$

- **Rate of change of Translational Velocity**

$$\boxed{\dot{\mathbf{v}}_B(t) = \frac{1}{m(t)}\mathbf{F}_B(t) + \mathbf{H}_I^B(t)\mathbf{g}_I - \tilde{\boldsymbol{\omega}}_B(t)\mathbf{v}_B(t)}$$

- **Angular Velocity**

$$\boldsymbol{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$$

- **Rate of change of Angular Velocity**

$$\boxed{\dot{\boldsymbol{\omega}}_B(t) = \mathbb{I}_B^{-1}(t)\left[\mathbf{M}_B(t) - \tilde{\boldsymbol{\omega}}_B(t)\mathbb{I}_B(t)\boldsymbol{\omega}_B(t)\right]}$$

Aircraft Characteristics Expressed in Body Frame of Reference

Aerodynamic and thrust force

$$\mathbf{F}_B = \begin{bmatrix} X_{aero} + X_{thrust} \\ Y_{aero} + Y_{thrust} \\ Z_{aero} + Z_{thrust} \end{bmatrix}_B = \begin{bmatrix} C_{X_{aero}} + C_{X_{thrust}} \\ C_{Y_{aero}} + C_{Y_{thrust}} \\ C_{Z_{aero}} + C_{Z_{thrust}} \end{bmatrix}_B \frac{1}{2}\rho V^2 S = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_B \bar{q}\, S$$

Aerodynamic and thrust moment

$$\mathbf{M}_B = \begin{bmatrix} L_{aero} + L_{thrust} \\ M_{aero} + M_{thrust} \\ N_{aero} + N_{thrust} \end{bmatrix}_B = \begin{bmatrix} \left(C_{l_{aero}} + C_{l_{thrust}}\right) b \\ \left(C_{m_{aero}} + C_{m_{thrust}}\right) \bar{c} \\ \left(C_{n_{aero}} + C_{n_{thrust}}\right) b \end{bmatrix}_B \frac{1}{2}\rho V^2 S = \begin{bmatrix} C_l b \\ C_m \bar{c} \\ C_n b \end{bmatrix}_B \bar{q}\, S$$

Inertia matrix

$$\mathbb{I}_B = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}_B$$

Reference Lengths

b = *wing span*

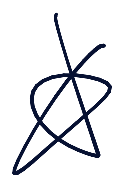
$\bar{c}$ = *mean aerodynamic chord*

Rigid-Body Equations of Motion: Position

Rate of change of Translational Position

$$\dot{x}_I = \left(\cos\theta\cos\psi\right)u + \left(-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi\right)v + \left(\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi\right)w$$
$$\dot{y}_I = \left(\cos\theta\sin\psi\right)u + \left(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi\right)v + \left(-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi\right)w$$
$$\dot{z}_I = \left(-\sin\theta\right)u + \left(\sin\phi\cos\theta\right)v + \left(\cos\phi\cos\theta\right)w$$

Rate of change of Angular Position



$$\dot{\phi} = p + \left(q\sin\phi + r\cos\phi\right)\tan\theta$$
$$\dot{\theta} = q\cos\phi - r\sin\phi$$
$$\dot{\psi} = \left(q\sin\phi + r\cos\phi\right)\sec\theta$$

Put it on cheat sheet

Rigid-Body Equations of Motion: Rate

angular velocity in z-axis (yaw rate)

Rate of change of Translational Velocity

What 2 feel

$$\dot{u} = X/m - g\sin\theta + rv - qw$$

$$\dot{v} = Y/m + g\sin\phi\cos\theta - ru + pw$$

$$\dot{w} = Z/m + g\cos\phi\cos\theta + qu - pv$$

in static reds go all way

velocity (rotation) induce rotation in third direction.

P u
Q v
R w

Rate of change of Angular Velocity

$$\dot{p} = \left(I_{zz}L + I_{xz}N - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)p + \left[I_{xz}^2 + I_{zz}\left(I_{zz} - I_{yy}\right)\right]r\right\}q\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

$$\dot{q} = \left[M - \left(I_{xx} - I_{zz}\right)pr - I_{xz}\left(p^2 - r^2\right)\right] \Big/ I_{yy}$$

$$\dot{r} = \left(I_{xz}L + I_{xx}N - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)r + \left[I_{xz}^2 + I_{xx}\left(I_{xx} - I_{yy}\right)\right]p\right\}q\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

Mirror symmetry, $I_{xz} \neq 0$

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AERODYNAMIC DAMPING

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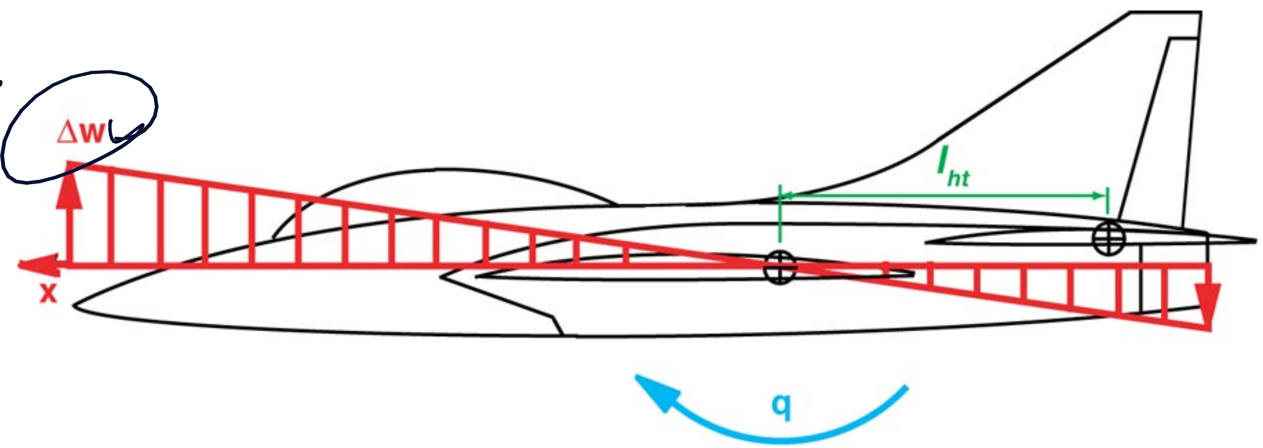
Pitching Moment due to Pitch Rate

tangential Velocity

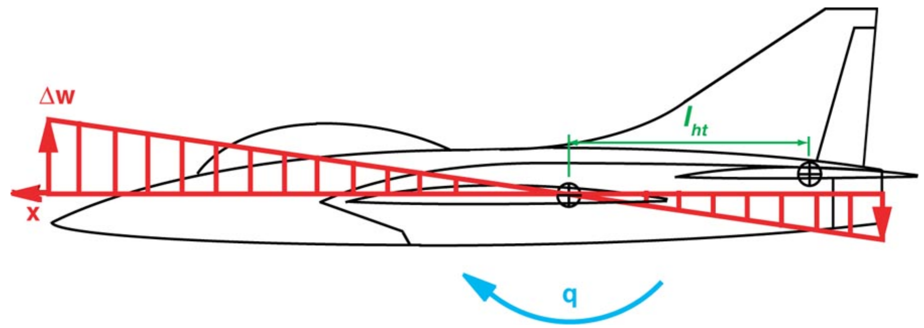


$$M_B = C_m \bar{q} S \bar{c} \approx \left(C_{m_o} + C_{m_q} q + C_{m_\alpha} \alpha \right) \bar{q} S \bar{c}$$

$$\approx \left(C_{m_o} + \frac{\partial C_m}{\partial q} q + C_{m_\alpha} \alpha \right) \bar{q} S \bar{c}$$

add this now damping effect

Angle of Attack Distribution Due to Pitch Rate



Aircraft pitching at a constant rate, *q* rad/s, produces a normal velocity distribution along *x*

$$\boxed{\Delta w = -q\Delta x}$$

Corresponding angle of attack distribution

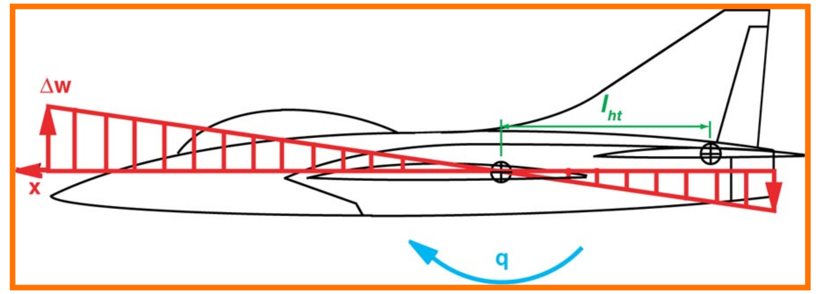
$$\boxed{\Delta\alpha = \frac{\Delta w}{V} = \frac{-q\Delta x}{V}}$$

Angle of attack perturbation at tail center of pressure

$$\boxed{\Delta\alpha_{ht} = \frac{q l_{ht}}{V}}$$

$$\boxed{l_{ht} = horizontal\ tail\ distance\ from\ c.m.}$$

Horizontal Tail Lift Due to Pitch Rate



Incremental tail lift due to pitch rate, referenced to tail area, S_{ht}

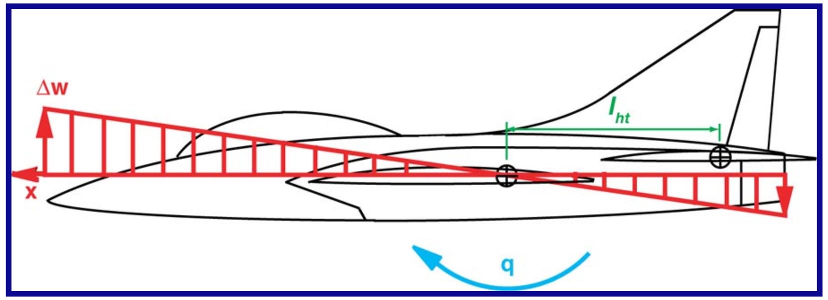
$$\Delta L_{ht} = \left(\Delta C_{L_{ht}}\right)_{ht} \frac{1}{2}\rho V^2 S_{ht}$$

Incremental tail lift coefficient due to pitch rate, referenced to wing area, S

$$\left(\Delta C_{L_{ht}}\right)_{aircraft} = \left(\Delta C_{L_{ht}}\right)_{ht}\left(\frac{S_{ht}}{S}\right) = \left[\left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft} \Delta\alpha\right] = \left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft}\left(\frac{q l_{ht}}{V}\right)$$

Lift coefficient sensitivity to pitch rate referenced to wing area

$$C_{L_{q_{ht}}} \equiv \frac{\partial\left(\Delta C_{L_{ht}}\right)_{aircraft}}{\partial q} = \left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft}\left(\frac{l_{ht}}{V}\right)$$



Moment Coefficient Sensitivity to Pitch Rate of the Horizontal Tail

Differential pitch moment due to pitch rate

$$\frac{\partial \Delta M_{ht}}{\partial q} = C_{m_{q_{ht}}} \frac{1}{2}\rho V^2 S\bar{c} = -C_{L_{q_{ht}}} \left(\frac{l_{ht}}{V}\right)\frac{1}{2}\rho V^2 S\bar{c}$$

$$= -\left[\left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft} \left(\frac{l_{ht}}{V}\right)\right]\left(\frac{l_{ht}}{\bar{c}}\right)\frac{1}{2}\rho V^2 S\bar{c}$$

Coefficient derivative with respect to pitch rate

$$C_{m_{q_{ht}}} = -\frac{\partial C_{L_{ht}}}{\partial \alpha}\left(\frac{l_{ht}}{V}\right)\left(\frac{l_{ht}}{\bar{c}}\right) = -\frac{\partial C_{L_{ht}}}{\partial \alpha}\left(\frac{l_{ht}}{\bar{c}}\right)^2\left(\frac{\bar{c}}{V}\right)$$

Pitch-Rate Derivative Definitions

- Pitch-rate derivatives are often expressed in terms of a normalized pitch rate

$$\hat{q} \triangleq \frac{q\bar{c}}{2V}$$

$$C_{m_{\hat{q}}} = \frac{\partial C_m}{\partial \hat{q}} = \frac{\partial C_m}{\partial \left(q\bar{c}/2V \right)} = \left(\frac{2V}{\bar{c}} \right) C_{m_q}$$

Pitching moment sensitivity to pitch rate

$$C_{m_q} = \frac{\partial C_m}{\partial q} = \left(\frac{\bar{c}}{2V} \right) C_{m_{\hat{q}}}$$

$$\frac{\partial M}{\partial q} = C_{m_q} \left(\rho V^2 /2 \right) S\bar{c} = C_{m_{\hat{q}}} \left(\frac{\bar{c}}{2V} \right) \left(\frac{\rho V^2}{2} \right) S\bar{c} = C_{m_{\hat{q}}} \left(\frac{\rho V S \bar{c}^2}{4} \right)$$

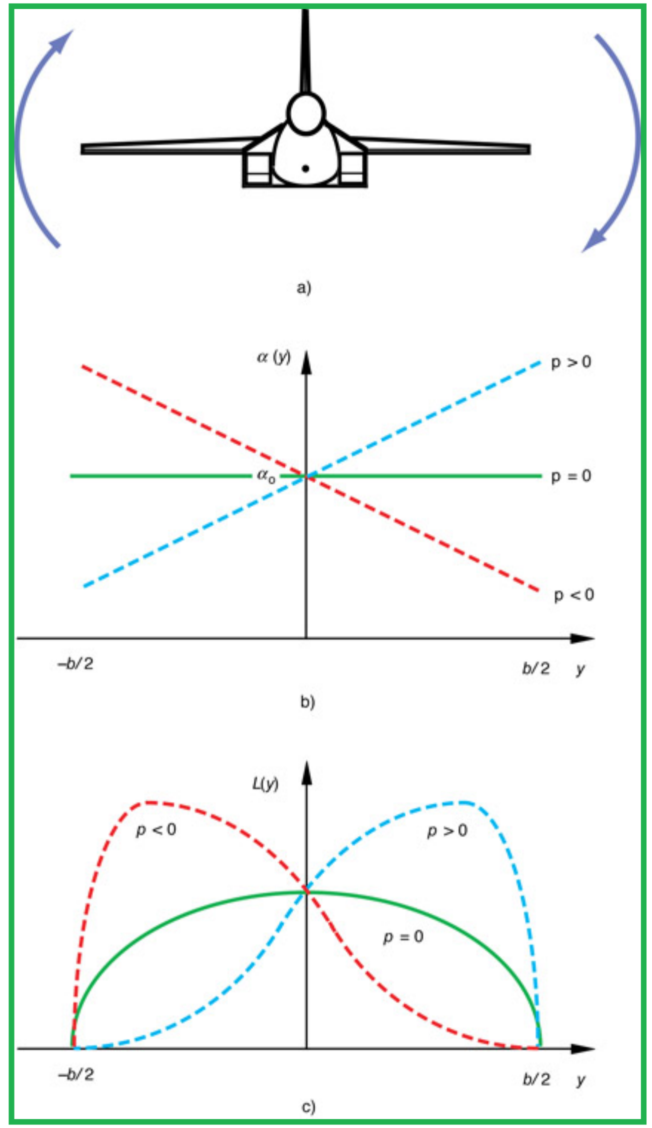
Roll Damping Due to Roll Rate

$$C_{l_p}\left(\frac{\rho V^2}{2}\right)Sb = C_{l_{\hat{p}}}\left(\frac{b}{2V}\right)\left(\frac{\rho V^2}{2}\right)Sb$$

$$= C_{l_{\hat{p}}}\left(\frac{\rho V}{4}\right)Sb^2$$

< 0 for stability

$$\hat{p} = \frac{pb}{2V}$$



- **Vertical tail, horizontal tail, and wing are principal contributors**

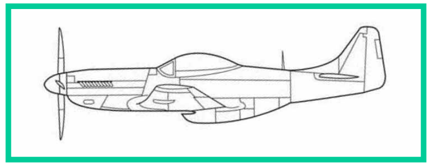
- **Roll damping of wing with taper**

$$\left(C_{l_{\hat{p}}}\right)_{Wing} = \frac{\partial\left(\Delta C_l\right)_{Wing}}{\partial\hat{p}} = -\frac{C_{L_\alpha}}{12}\left(\frac{1+3\lambda}{1+\lambda}\right)$$

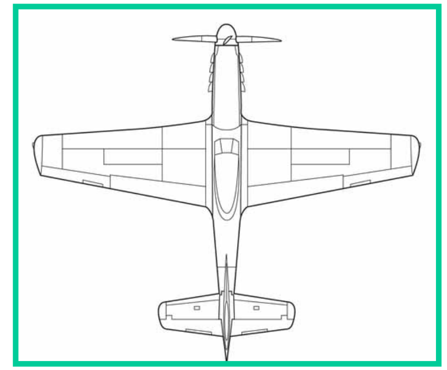
- **For thin triangular wing**

$$\left(C_{l_{\hat{p}}}\right)_{Wing} = -\frac{\pi AR}{32}$$

NACA-TR-1098, 1952
NACA-TR-1052, 1951



Roll Damping Due to Roll Rate



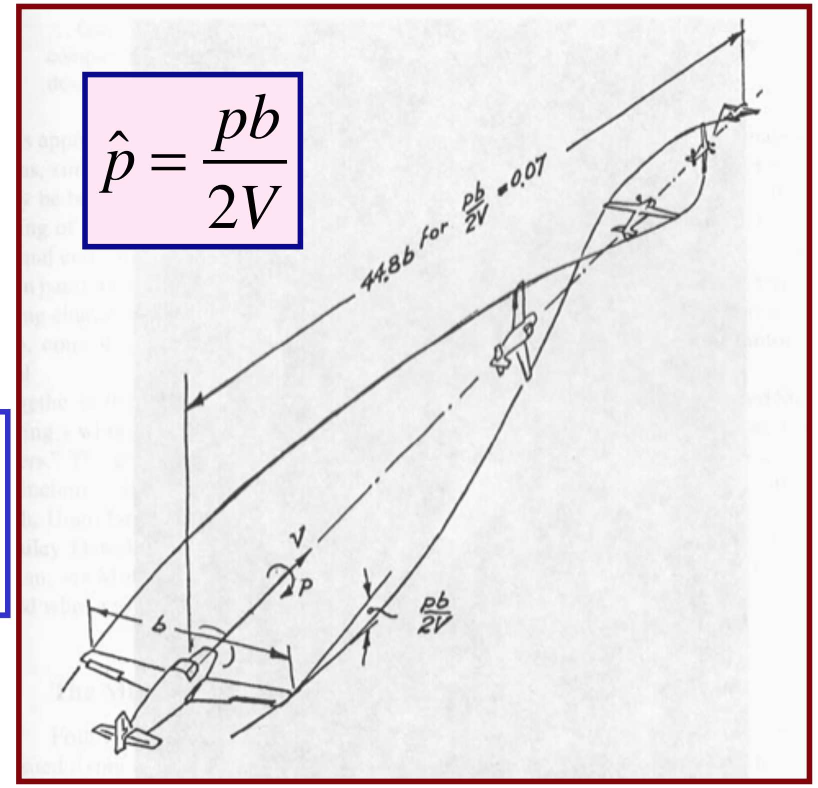
Tapered vertical tail

$$\left(C_{l_{\hat{p}}}\right)_{vt} = \frac{\partial\left(\Delta C_l\right)_{vt}}{\partial \hat{p}} = -\frac{C_{Y_{\beta_{vt}}}}{12}\left(\frac{S_{vt}}{S}\right)\left(\frac{1+3\lambda}{1+\lambda}\right)$$

Tapered horizontal tail

$$\left(C_{l_{\hat{p}}}\right)_{ht} = \frac{\partial\left(\Delta C_l\right)_{ht}}{\partial \hat{p}} = -\frac{C_{L_{\alpha_{ht}}}}{12}\left(\frac{S_{ht}}{S}\right)\left(\frac{1+3\lambda}{1+\lambda}\right)$$

_pb/2V_ describes helix angle for a steady roll



$$\hat{p} = \frac{pb}{2V}$$

Yaw Damping Due to Yaw Rate

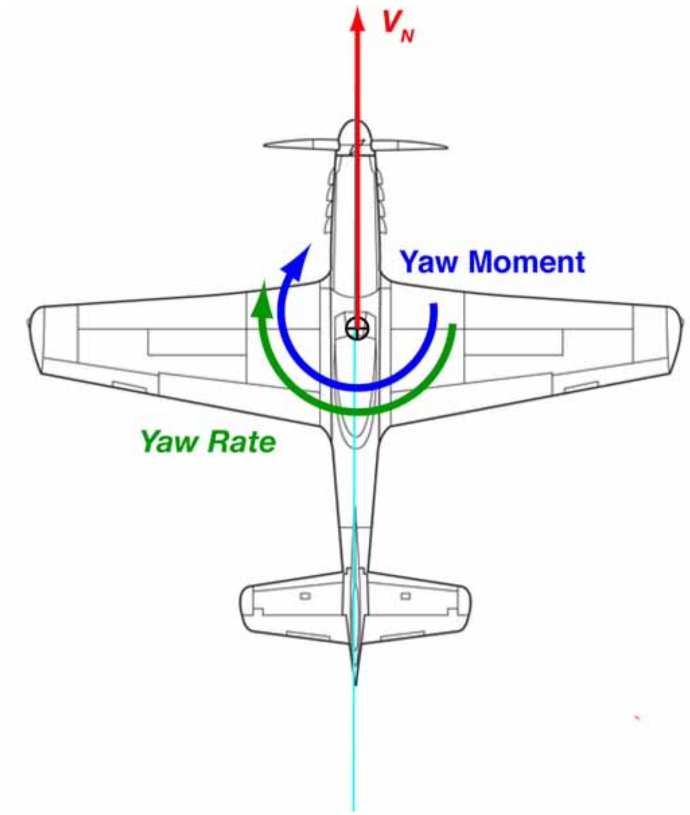
$$C_{n_r}\left(\frac{\rho V^2}{2}\right)Sb = C_{n_{\hat{r}}}\left(\frac{b}{2V}\right)\left(\frac{\rho V^2}{2}\right)Sb$$

$$= C_{n_{\hat{r}}}\left(\frac{\rho V}{4}\right)Sb^2$$

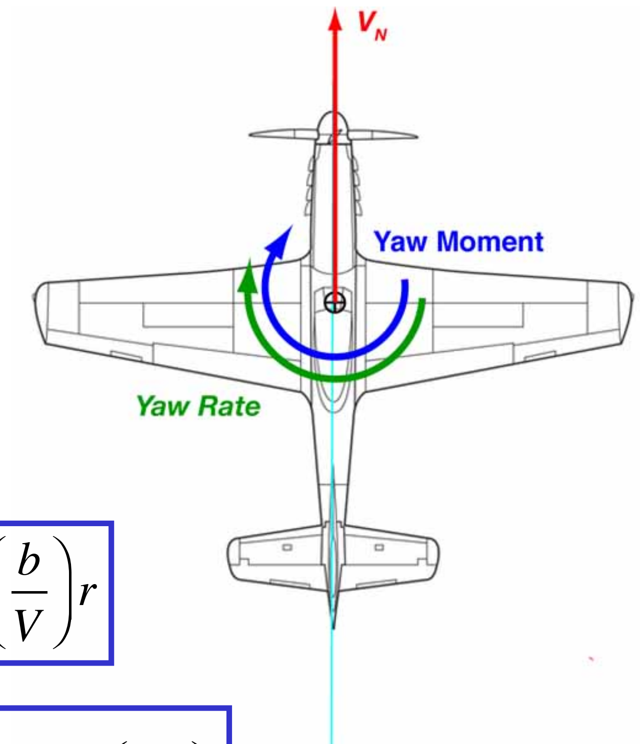
< 0 for stability

Normalized yaw rate

$$\hat{r} = \frac{rb}{2V}$$



Yaw Damping Due to Yaw Rate



Vertical tail contribution

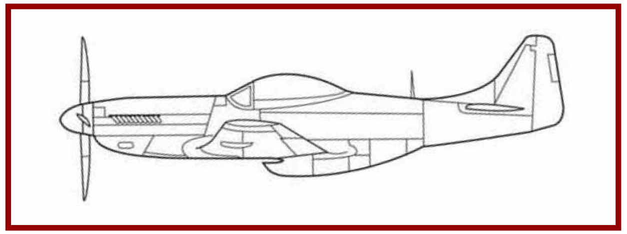
$$\Delta\left(C_n\right)_{Vertical\,Tail} = -\left(C_{n_\beta}\right)_{Vertical\,Tail}\left(rl_{vt}/V\right) = -\left(C_{n_\beta}\right)_{Vertical\,Tail}\left(\frac{l_{vt}}{b}\right)\left(\frac{b}{V}\right)r$$

$$\left(C_{n_{\hat{r}}}\right)_{vt} = \frac{\partial\Delta\left(C_n\right)_{Vertical\,Tail}}{\partial\left(rb/2V\right)} = \frac{\partial\Delta\left(C_n\right)_{Vertical\,Tail}}{\partial\hat{r}} = -2\left(C_{n_\beta}\right)_{Vertical\,Tail}\left(\frac{l_{vt}}{b}\right)$$

Wing contribution

$$\left(C_{n_{\hat{r}}}\right)_{Wing} = k_0 C_L^2 + k_1 C_{D_{Parasite,\,Wing}}$$

***k_0* and *k_1* are functions of aspect ratio and sweep angle**



NACA-TR-1098, 1952
NACA-TR-1052, 1951

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