The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

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ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Longitudinal Static Stability (from Etkin and Reid)
  - Lateral Forces
  - Lateral Static Stability

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LONGITUDINAL STABILITY

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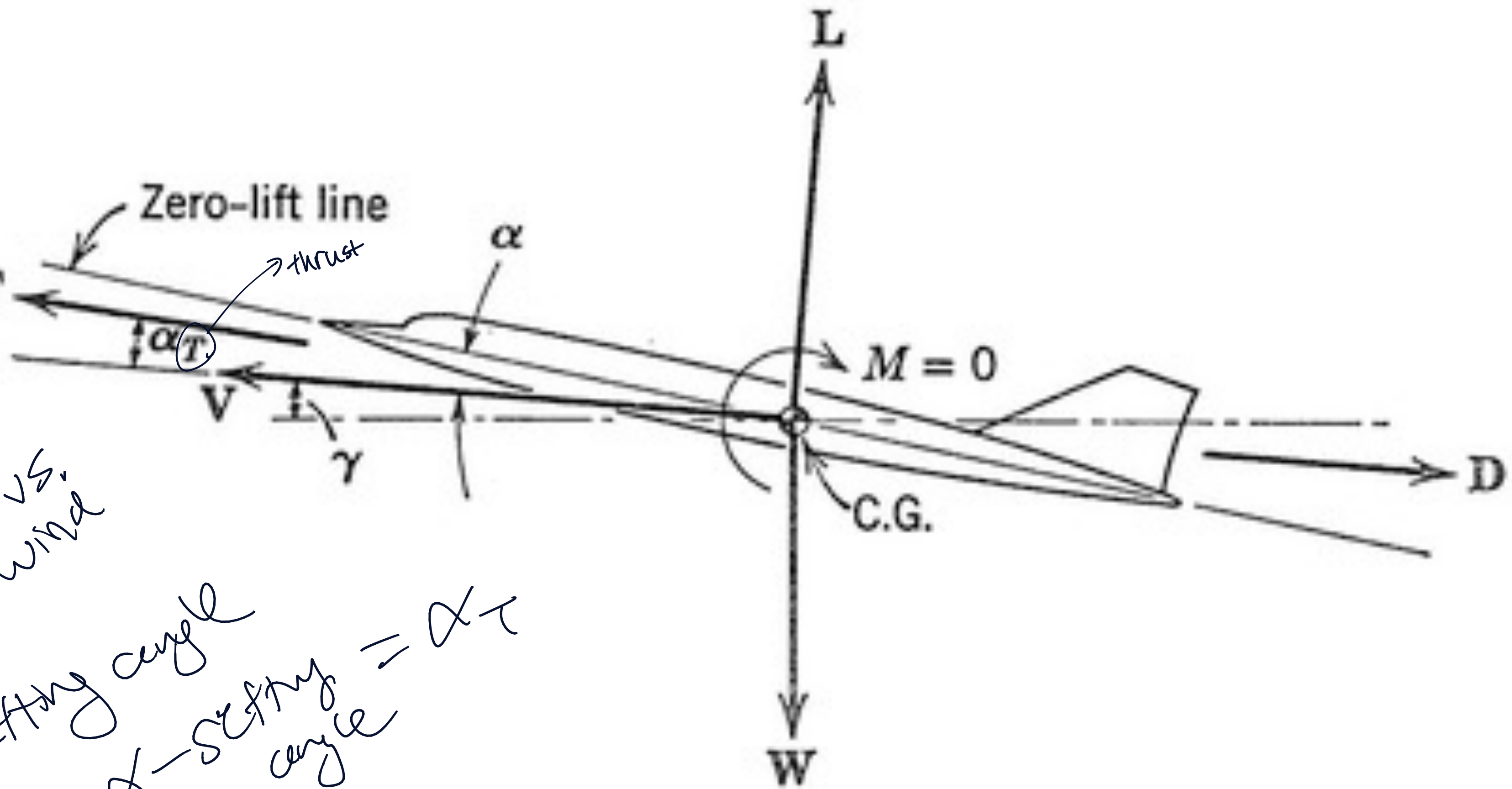
Basic Longitudinal Forces (1/3)

- The forces and moments on an aircraft in “steady flight” are…



ref line
lift = 0

axis of engine vs. wind
↓
setting angle

α − setting angle = α_T

Basic Longitudinal Forces (2/3)

- Points to note are…
  - The angle of attack is defined as the angle between the “zero-lift line” and the velocity vector.
    - i.e. the lift is equal to zero when there is no side-slip and the zero-lift line is aligned with the velocity vector.
  - The line of action of the thrust is not necessarily aligned with the zero-lift line and has its own angle of attack
    - i.e. the axis of the engines may be oriented at an angle relative to the zero-lift line.

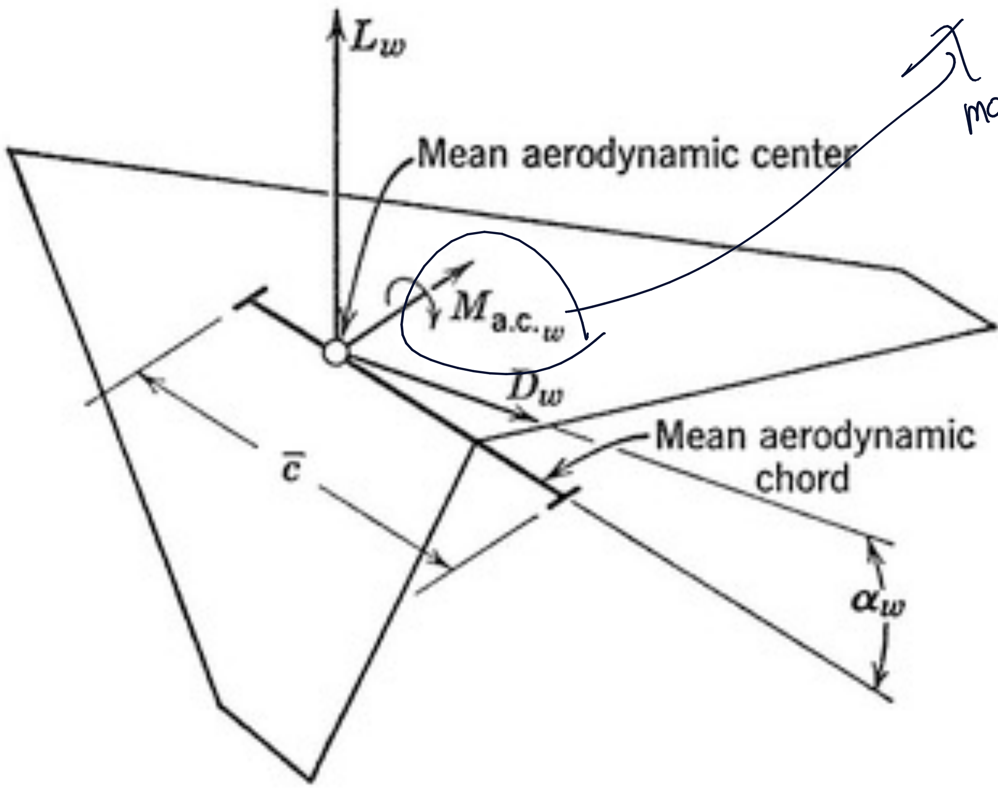
Basic Longitudinal Forces (3/3)

- and…
  - The net moment about the center of gravity is zero.
    - i.e. there is no tendency for the nose of the aircraft to rotate either up or down.
  - This steady state is usually achieved by deflecting the elevators and using trim tabs to keep them in the required position…
    - i.e. by “trimming” the aircraft.

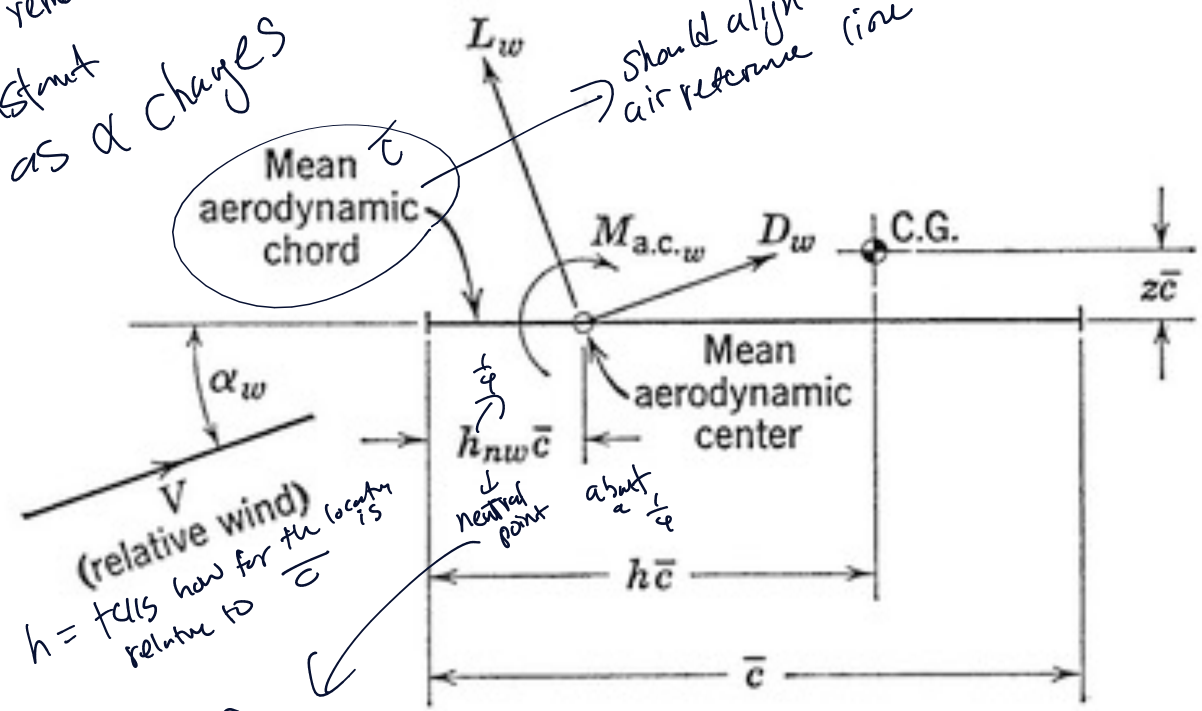
Pitching Moment of Wing (1/3)

- Aerodynamic forces on wing

Not equal to zero, moment remains constant as α changes



- Forces and moments about C.G. in the plane of symmetry



Should align with air reference line

$\bar{c}$

$\frac{1}{4}$

neutral point

about a $\frac{1}{4}$

h = tells how far the location is relative to $\bar{c}$

Same as aerodynamic center.

Pitching Moment of Wing (2/3)

dynamic pres. · $\bar{c}$

nondimensionalize

- The expression for the moment about the C.G. in the plane of symmetry is therefore…

$$M_w = M_{a.c._w} + (L_w \cos \alpha_w + D_w \sin \alpha_w)(h - h_{n_w})\bar{c} + (L_w \sin \alpha_w - D_w \cos \alpha_w)z\bar{c}$$

- and the corresponding expression for the coefficient of moment is…

small angle approx.

$$C_{m_w} = C_{m_{a.c._w}} + (C_{L_w} + C_{D_w}\alpha_w)(h - h_{n_w}) + (C_{L_w}\alpha_w - C_{D_w})z$$

Pitching Moment of Wing (3/3)

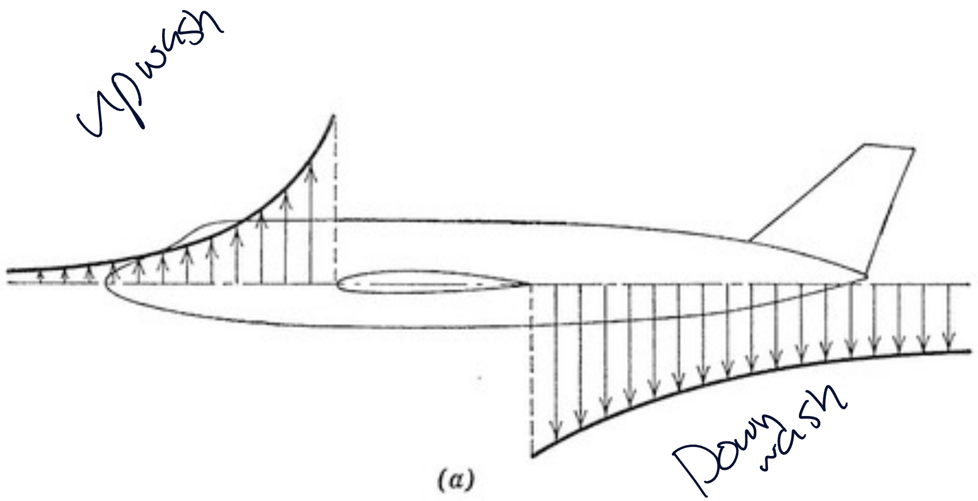
- In most cases…
  - $(C_{L_w}\alpha_w - C_{D_w})z$ is negligible

    small
  - $C_{D_w}\alpha_w \ll C_{L_w}$
- and the expression for the coefficient of the moment becomes…

$$C_{m_w} = C_{m_{a.c._w}} + C_{L_w}(h - h_{n_w})$$

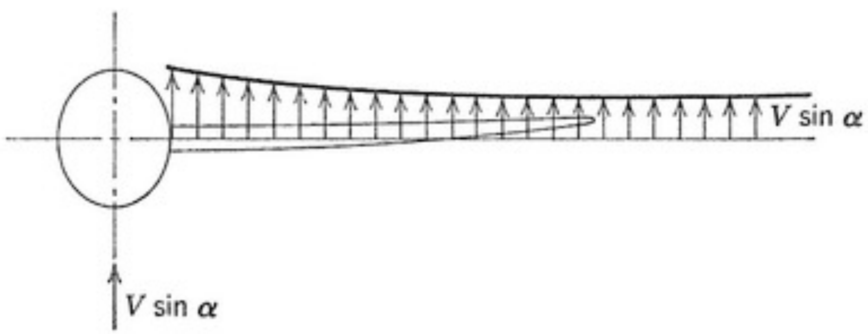
Pitching Moment of Body and Nacelles (1/2)

- The wing affects the flow that the body experiences…



(a)

- The body also affects the flow that the wing experiences…



(b)

Pitching Moment of Body and Nacelles (2/2)

- The effects of adding a body or nacelles are…
  - Shift forward of the mean aerodynamic center;
  - Increase in the slope of the lift curve;
  - Negative increment in $C_{m_{\mathrm{a.c.}}}$
- The net effect is given by….

$\neq h_{nw}$

$$C_{m_{wb}} = C_{m_{\mathrm{a.c.}_{wb}}} + C_{L_{wb}}(h - h_{n_{wb}})$$

h_{nwb}

wing body combined

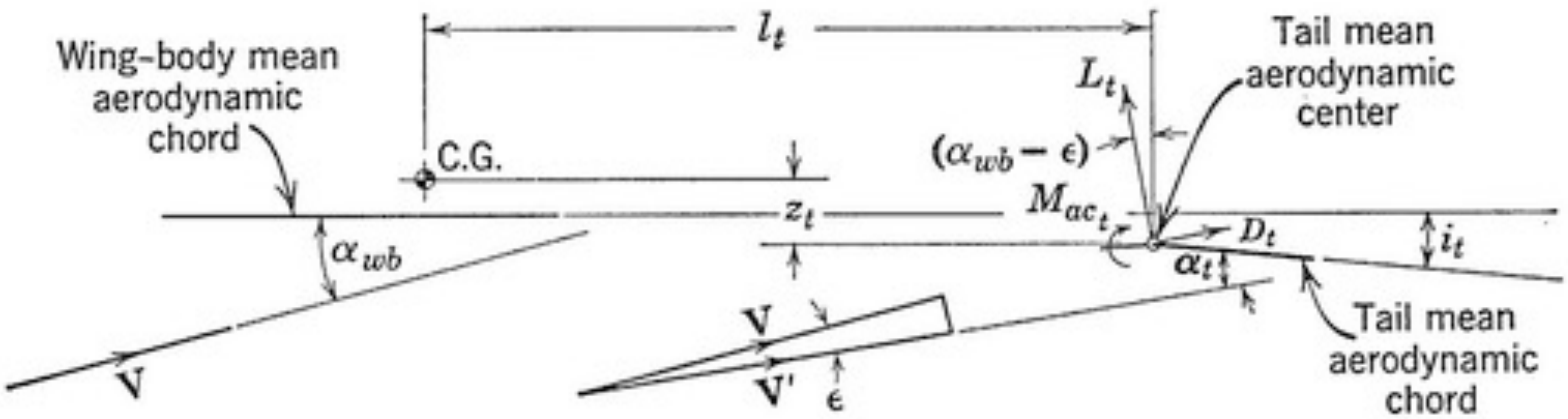
$\bar{l}_t \frac{S_t}{S} = \frac{l_t S_t}{\bar{c} S} - \frac{S_t}{S}(\bar{c})$

$\frac{S_t}{S}\left(\frac{l_t}{\bar{c}} - \bar{c}\right)$

$\left(\frac{l_t}{\bar{c}} - \frac{\bar{c}}{\bar{c}}\right)$

Pitching Moment of Tail (1/3)

- Forces and moments about the C.G. in the plane of symmetry are…



Pitching Moment of Tail (2/3)

- The expression for the moment about the C.G. in the plane of symmetry is therefore…

$$M_t = -l_t[L_t \cos(\alpha_{wb} - \epsilon) + D_t \sin(\alpha_{wb} - \epsilon)] - z_t[D_t \cos(\alpha_{wb} - \epsilon) - L_t \sin(\alpha_{wb} - \epsilon)] + M_{ac_t}$$

Distance to the tail

- which due to the first term being dominant and small angle approximations reduces in most cases to…

$$M_t = -l_t L_t = -l_t C_{L_t} \tfrac{1}{2} \rho V^2 S_t$$

how big is my tail
how big is wing
chord St

Similar for most commercial plane.

$l_t - (l_b - \bar{c}(h - h_{n_{wb}}))\frac{S_t}{S}\frac{1}{\bar{c}}$

Pitching Moment of Tail (3/3)

- The corresponding expression for the coefficient of moment is…

$$C_{m_t} = \frac{M_t}{\frac{1}{2}\rho V^2 S\bar{c}} = -\frac{l_t}{\bar{c}}\frac{S_t}{S}C_{L_t}$$

horizontal Tail Volume Ratio

horizontal Volume ratio

- which can also be written as…

$$C_{m_t} = -V_H C_{L_t}$$

design stuff with this

- where $V_H = \overline{V}_H - \frac{S_t}{S}(h - h_{n_{wb}})$ and $\overline{V}_H = \frac{\bar{l}_t S_t}{\bar{c} S}$

CG vs a.c. wing body tu

Pitching Moment of Propulsion System

- Moments due to…
  - Forces acting on the propulsive system itself;
  - Interaction of propulsive slipstream with other parts of the aircraft.

- Assuming interference effects are captured in the wing, body and tail moments, the remaining moments are lumped into…

$$C_{m_p}$$

Total Pitching Moment (1/4)

- The total pitching moment is given by…

$$C_m = C_{m_{\mathrm{a.c.}_{wb}}} + \left(C_{L_{wb}} + C_{L_t}\frac{S_t}{S}\right)(h - h_{n_{wb}}) - \bar{V}_H C_{L_t} + C_{m_p}$$

- which reduces to…

$$C_m = C_{m_{\mathrm{a.c.}wb}} + C_L(h - h_{n_{wb}}) - \bar{V}_H C_{L_t} + C_{m_p}$$

- because…

$$C_L = C_{L_{wb}} + C_{L_t}\frac{S_t}{S}$$

Total Pitching Moment (2/4)

- The equation for the gradient of the moment coefficient with respect to the angle of attack is therefore…

0 at a.c.

$$C_{m_\alpha} = \frac{\partial C_{m_{a.c._{wb}}}}{\partial \alpha} + C_{L_\alpha}(h - h_{n_{wb}}) - \overline{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}$$

- which when a classic “true aerodynamic center” exists reduces to…

$$C_{m_\alpha} = C_{L_\alpha}(h - h_{n_{wb}}) - \overline{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}$$

Total Pitching Moment (3/4)

- Defining the neutral point as the C.G. position where the derivative of the moment coefficient with respect to the angle of attack is equal to zero, results in the following expression for the location of the neutral point…

$$h_n = h_{n_{wb}} - \frac{1}{C_{L_\alpha}}\left(\frac{\partial C_{m_{a.c._{wb}}}}{\partial \alpha} - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}\right)$$

Total Pitching Moment (4/4)

- The derivative of the moment coefficient with respect to the angle of attack can therefore be re-written as…

$$C_{m_\alpha} = C_{L_\alpha}(h - h_n)$$

- and the difference between the location of the C.G. and the location of the neutral point defined as the static margin…

$$K_n = (h_n - h)$$

CG

neutral point
= a.c.
$\frac{\partial C_{m\alpha}}{\partial \alpha} = 0$

Linear Lift and Moment (1/7)

- When all the forces and moments are linear functions of the angle of attack, the component lift coefficients become…

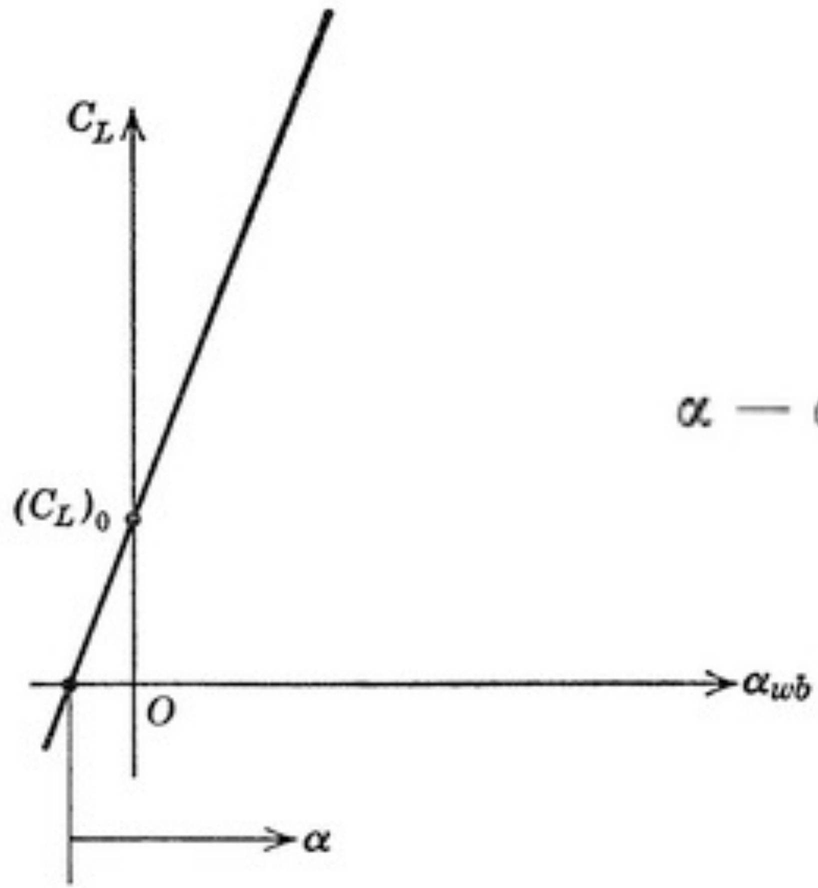
$$C_{L_{wb}} = a_{wb}\alpha_{wb}$$

$$C_{L_t} = a_t\alpha_t$$

$$C_{m_p} = C_{m_{0_p}} + \frac{\partial C_{m_p}}{\partial \alpha} \cdot \alpha$$

Linear Lift and Moment (2/7)

- The difference between the angles of attack is given by…



$$\alpha - \alpha_{wb} = \frac{a_t}{a}\frac{S_t}{S}(i_t - \epsilon_0)$$

Linear Lift and Moment (3/7)

- The resulting expressions for the moment coefficient are...

$$C_m = C_{m_0} + C_{m_\alpha}\alpha \qquad (a)$$

$$C_m = \bar{C}_{m_0} + C_{m_\alpha}\alpha_{wb} \qquad (b)$$

- where…

$$C_{m_\alpha} = a(h - h_{n_{wb}}) - a_t\bar{V}_H\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) + \frac{\partial C_{m_p}}{\partial \alpha} \qquad (a)$$

$$C_{m_\alpha} = a_{wb}(h - h_{n_{wb}}) - a_t V_H\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) + \frac{\partial C_{m_p}}{\partial \alpha} \qquad (b)$$

Linear Lift and Moment (4/7)

- and…

$$C_{m_0} = C_{m_{\text{a.c.}_{wb}}} + C_{m_{0_p}} + a_t\bar{V}_H(\epsilon_0 - i_t) \times \left[1 - \frac{a_t}{a}\frac{S_t}{S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right] \qquad (a)$$

$$\bar{C}_{m_0} = C_{m_{\text{a.c.}_{wb}}} + \bar{C}_{m_{0_p}} + a_t V_H(\epsilon_0 - i_t) \qquad (b)$$

$$\bar{C}_{m_{0_p}} = C_{m_{0_p}} + (\alpha - \alpha_{wb})\frac{\partial C_{m_p}}{\partial\alpha} \qquad (c)$$

Linear Lift and Moment (5/7)

- Further, the overall neutral point is...

$$h_n = h_{n_{wb}} + \frac{a_t}{a}\overline{V}_H\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - \frac{1}{a}\frac{\partial C_{m_p}}{\partial \alpha}$$

- and the moment coefficient is…

distance CG – N.P

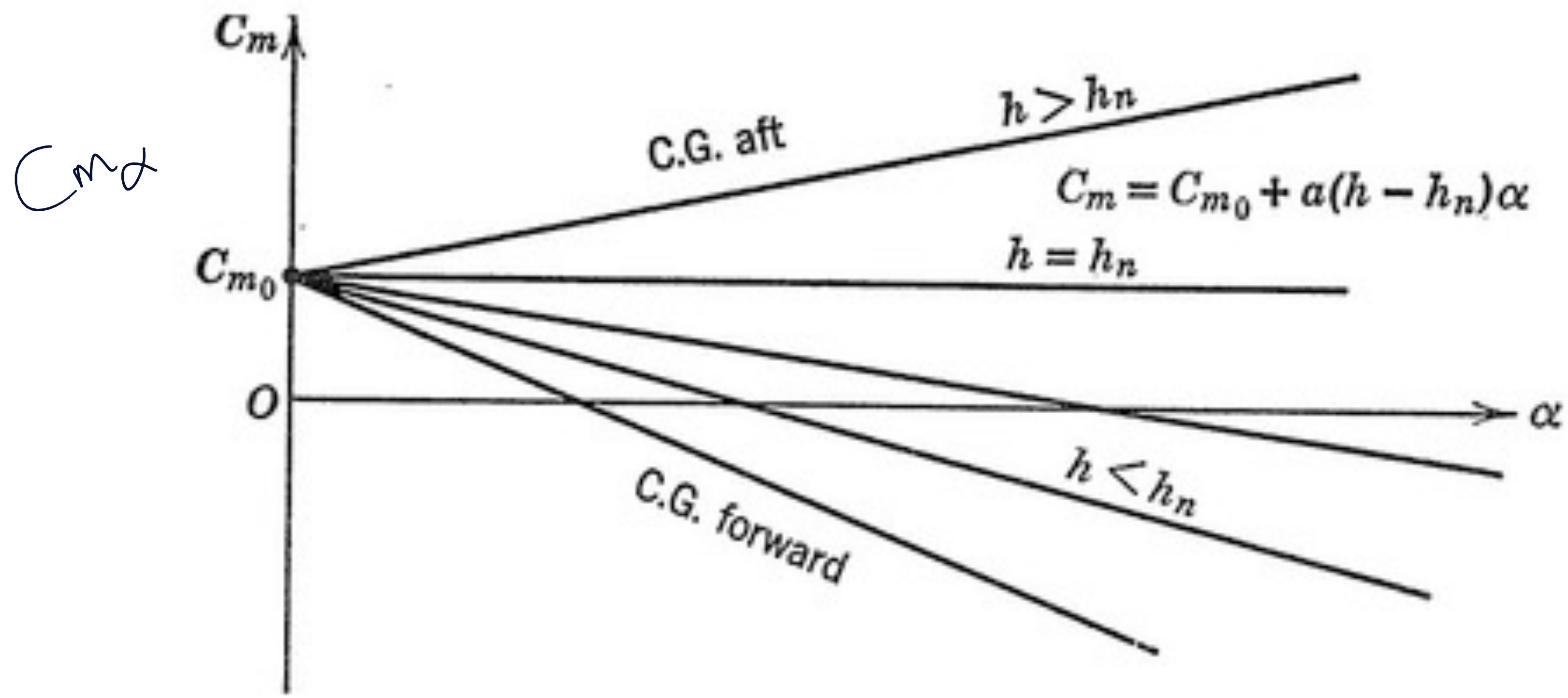
static margin

$$C_m = C_{m_0} + C_L(h - h_n) \qquad (a)$$

$$C_m = C_{m_0} + a\alpha(h - h_n) \qquad (b)$$

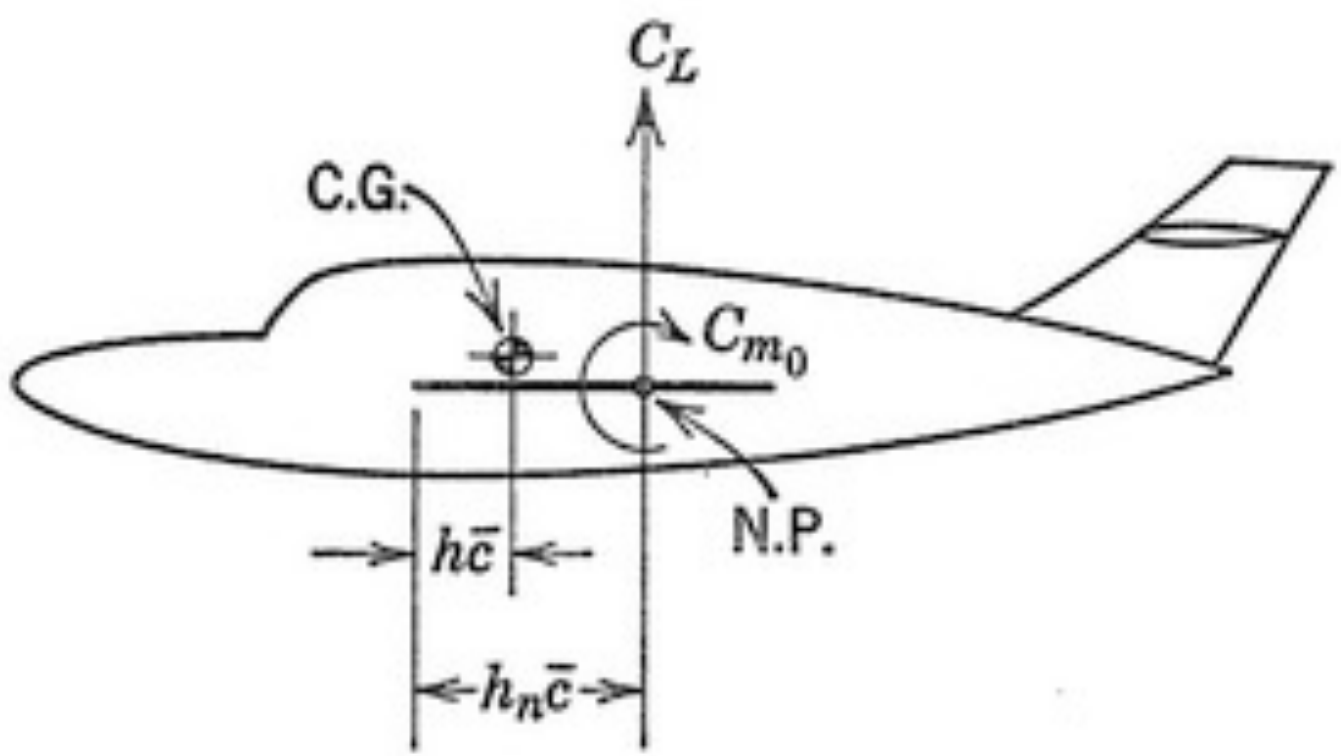
Linear Lift and Moment (6/7)

- The effect of the C.G. position on the moment versus angle of attack curve and thus on pitch stiffness is...



Linear Lift and Moment (7/7)

- The total lift and moment can be depicted as...



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LATERAL FORCES

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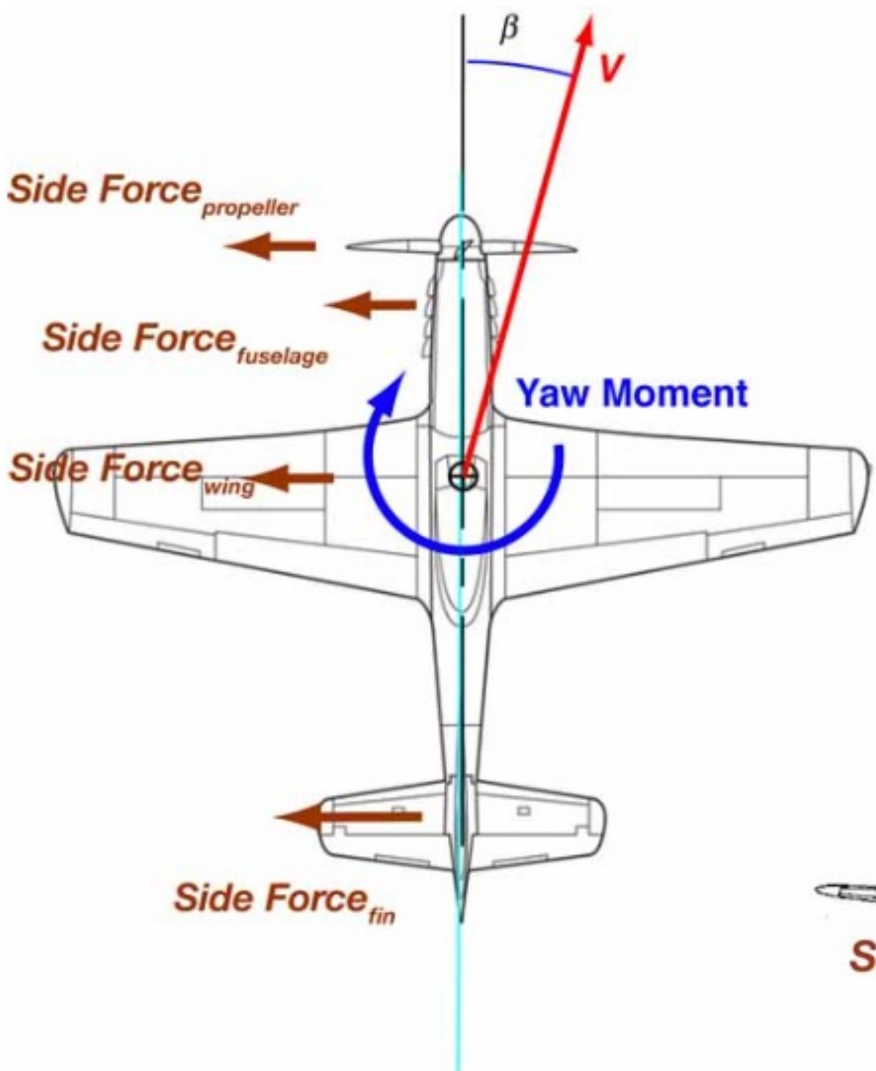
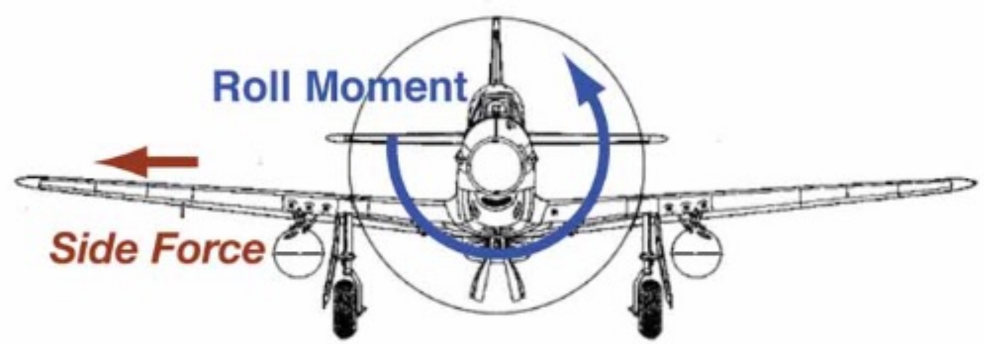
Sideslip Angle Produces Side Force, Yawing Moment, and Rolling Moment



- Sideslip usually a small angle (±5 deg)
- Side force generally not a significant effect
- Yawing and rolling moments are principal effects



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LATERAL MOMENTS & STATIC STABILITY

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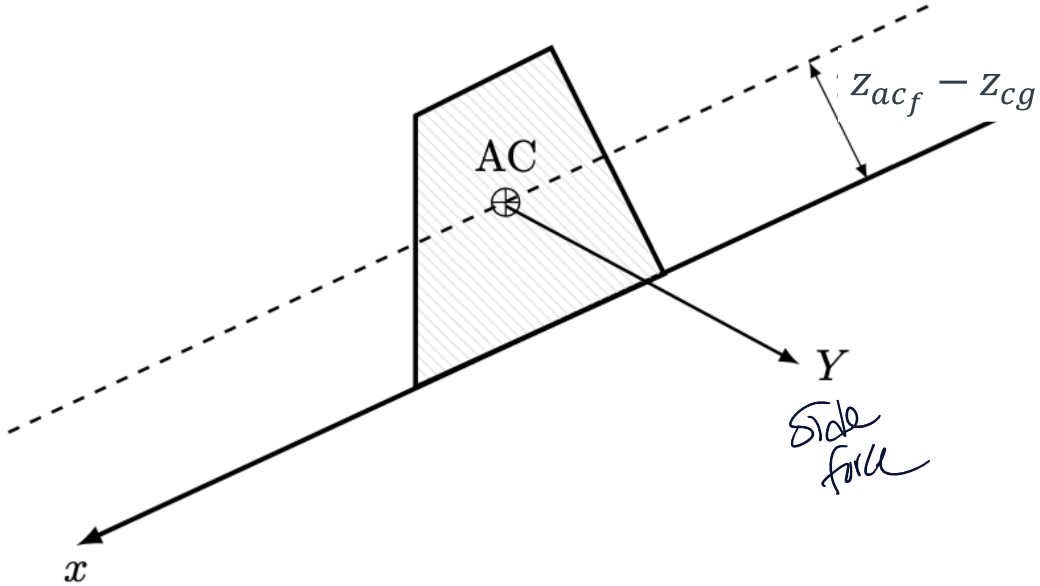
What do these two things have in common?

Weathercock Stability

Rudder design considerations:

- Adverse yaw
- Crosswind landing
- Asymmetric power
- Spin recovery



Roll Moment

$$L = \left(z_{ac_f} - z_{cg}\right) Y$$

$$= -q_f S_f C_{L_{\alpha_f}}\left[\left(1 - \sigma_\beta\right)\beta - \sigma_0\right]\left(z_{ac_f} - z_{cg}\right)$$

sideslip

side wash

usually 0, b/c fuselage is normally symmetric

coefficients of moment L not lift

around x axis

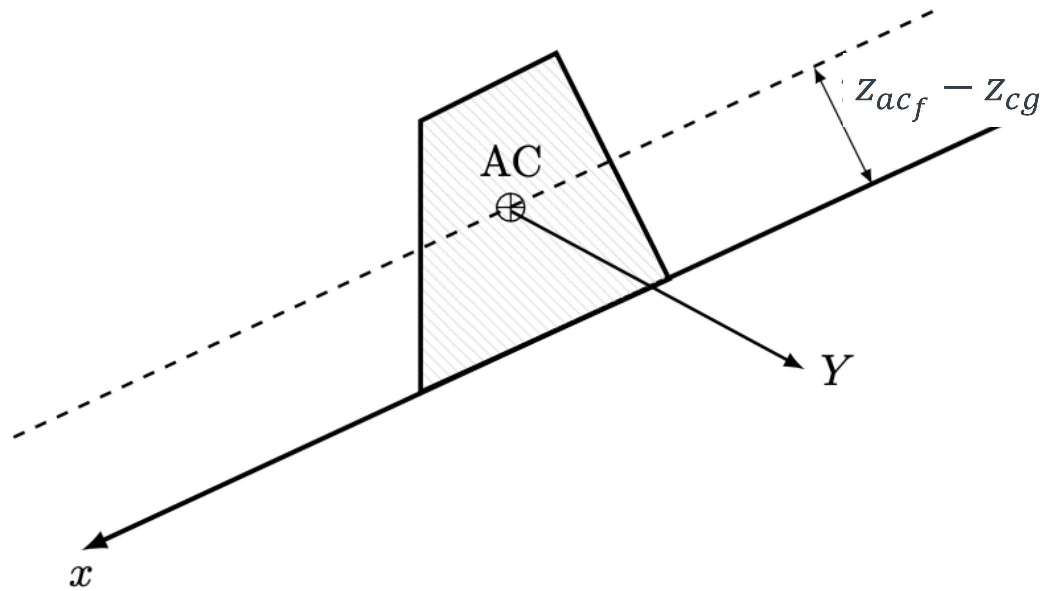
Normalizing L ...

$$C_\ell = \frac{L}{qSb} = C_{\ell_0} + C_{\ell_\beta}\beta$$

$$= \frac{q_f S_f}{qS}\left(\frac{z_{ac_f} - z_{cg}}{b}\right) C_{L_{\alpha_f}} \sigma_0$$

$$- \frac{q_f S_f}{qS}\left(\frac{z_{ac_f} - z_{cg}}{b}\right) C_{L_{\alpha_f}} \left(1 - \sigma_\beta\right)\beta$$

Roll Moment



Thus ...

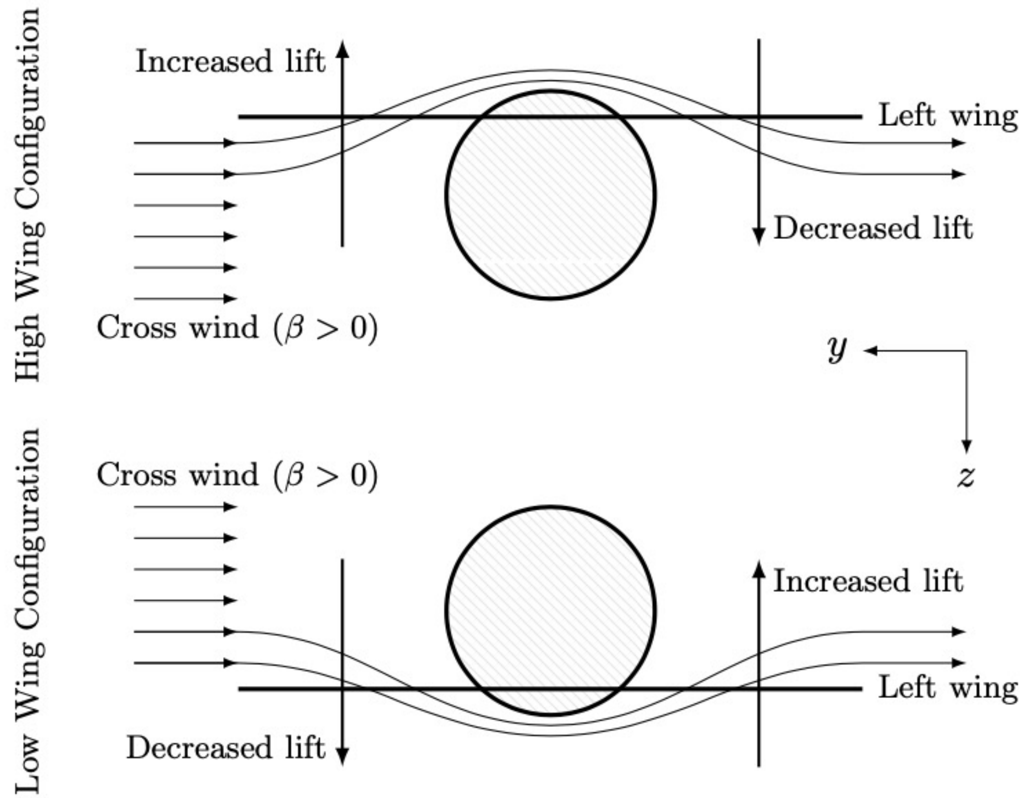
$$C_{\ell_\beta} = -\frac{q_f S_f}{qS}\left(\frac{z_{ac_f} - z_{cg}}{b}\right) C_{L_{\alpha_f}}\left(1 - \sigma_\beta\right)$$

... and for roll stability we want ...

$C_{\ell_\beta} < 0$ ➡ ac_f above the cg

we don't want vehicle to roll over and not drop the right wing

What about the fuselage? Does it help or hurt? It depends!



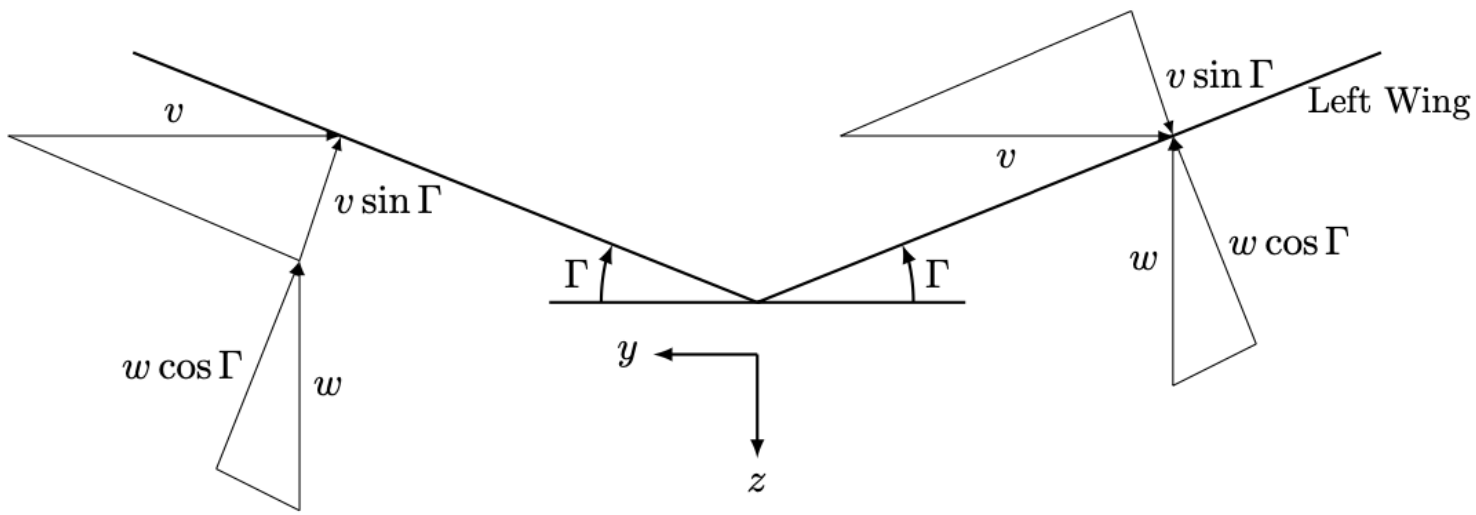
High (straight) wing configuration is statically roll stabilizing ...

$$C_{\ell_\beta} < 0$$

High wing, slight dihedral → better roll Stab.

Low wing, large dihedral

Does wing dihedral help roll stability?



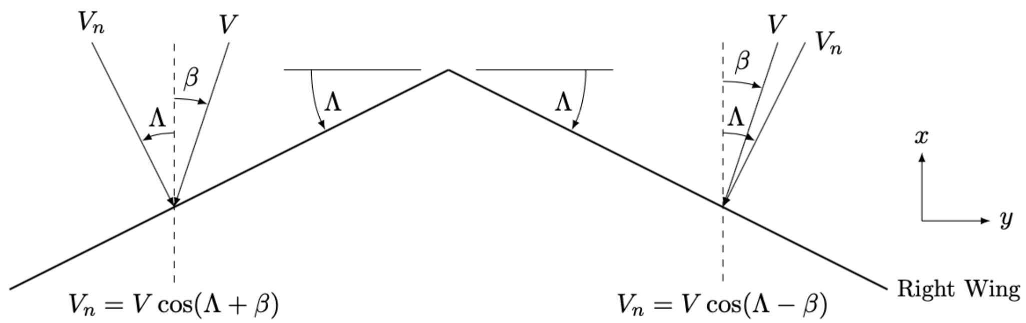
For $C_{\ell_\beta} < 0$

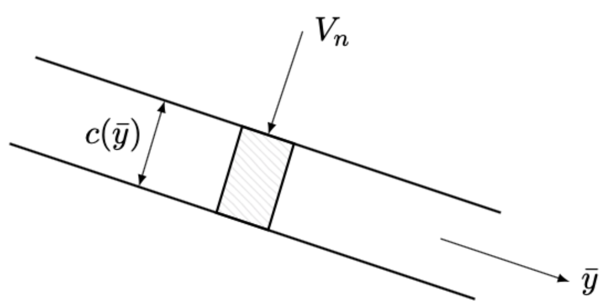
➡ $\Gamma > 0$

$$
\begin{aligned}
L &= -\frac{1}{2}\bar{q}SC_{L_{\alpha_w}}(\alpha + \Gamma\beta)y_{ac_w} + \frac{1}{2}\bar{q}SC_{L_{\alpha_w}}(\alpha - \Gamma\beta)y_{ac_w} \\
&= -\bar{q}SC_{L_{\alpha_w}}\Gamma y_{ac_w}\beta
\end{aligned}
$$

➡ $C_\ell = -\dfrac{y_{ac_w}}{b}C_{L_{\alpha_w}}\Gamma\beta$ and … $C_{\ell_\beta} = \dfrac{\partial C_\ell}{\partial \beta} = -\dfrac{y_{ac_w}}{b}C_{L_{\alpha_w}}\Gamma$

What about wing sweep? Does it help or hurt? **It helps!**





$$V_{n_L} = V\cos(\Lambda + \beta)$$

$$V_{n_R} = V\cos(\Lambda - \beta)$$

Thus …

$$Lift_L = q\frac{S}{2}C_L\cos^2(\Lambda + \beta)$$

$$Lift_R = q\frac{S}{2}C_L\cos^2(\Lambda - \beta)$$

… the net moment $L = -y_{ac_w} q\frac{S}{2}C_L\left[\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right]$

… and $C_{\ell_\beta} = -\frac{y_{ac_w}}{b}C_L\sin(2\Lambda)\beta < 0$ when $\Lambda > 0$

Summary of effects …

- Fin (vertical tail) in the back -> yaw stabilizing
- Fin (vertical tail) pointing upwards -> roll stabilizing
- Dihedral angle…

  $\Gamma > 0$ -> roll stabilizing

  $\Gamma < 0$ -> roll destabilizing
- Wing configuration…

  high wing -> roll stabilizing

  low wing -> roll destabilizing
- Wing sweep -> roll stabilizing

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