THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Aerospace Engineering and Engineering Mechanics

ASE 367K FLIGHT DYNAMICS
Fall 2024

HOMEWORK 2
Due: Friday 2024-09-13 at 11:59pm via Canvas

Problem 1

Consider the glider you have been given:

a. Measure the dimensions and weight of its components (NOTE: measure the weight of the "metal ballast" separately).
b. Compute the location of the center of gravity as a function of the location of the metal ballast.
c. Estimate the location of the aerodynamic center of the aircraft by repeatedly moving the location of the metal ballast and observing the stability of the glider in flight.

Problem 2

Derive an expression for the horizontal tail incidence angle, i_t, required to trim (i.e., with the elevator deflection angle, $\delta_e = 0$) an aircraft with the geometry depicted below such that it maintains a flight path angle, γ. HINT: You need to add a vector for the weight before creating your equations for the sum of forces. You also need to consider the effects of drag in your equation for the sum of moments. You may use the derivation provided in class as a guide but remember that not all steps were shown and neither the weight, drag, nor flight path angle were considered.

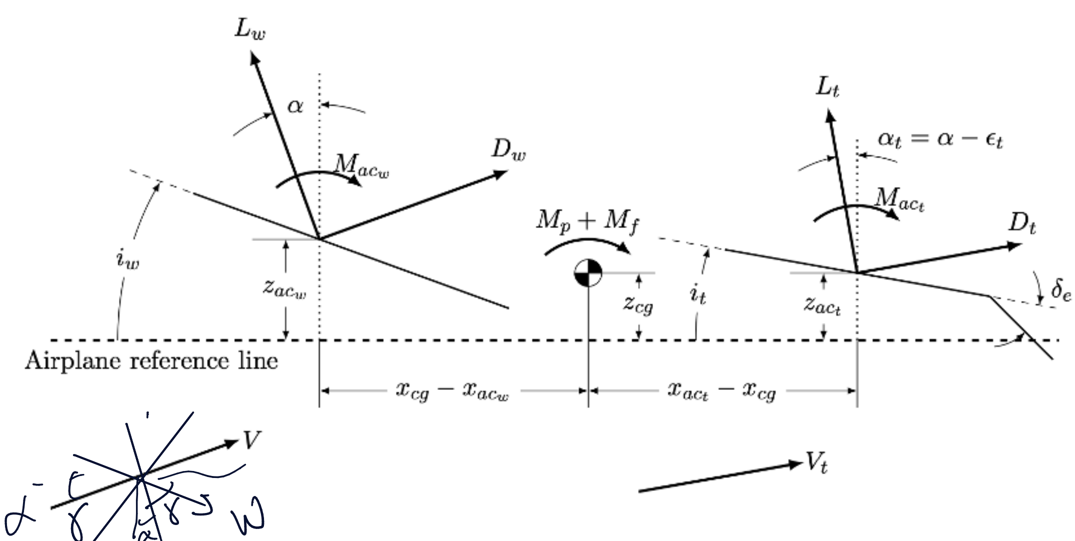

Figure 3.1: Simplified geometry of the wing and tail sections of an aircraft

α = V vs ref.

γ = V vs horizon

Problem 2

Consider an aircraft with the following characteristics…

Wing		Horizontal Stabilizer	
$S =$	232.00 [ft^2]	$S =$	54.00 [ft^2]
$\bar{c} =$	7.04 [ft]	$\bar{c} =$	3.83 [ft]
$x_{ac/le} =$	4.07 [ft]	$x_{ac/le} =$	2.79 [ft]
$x_{le} =$	16.40 [ft]	$x_{le} =$	36.90 [ft]
$i =$	1.00 [deg]		
$C_{L_0} =$	-0.0443	$C_{L_0} =$	0.0000
$C_{L\alpha} =$	5.0800	$C_{L\alpha} =$	4.2600
$C_{M_{ac}} =$	-0.0175	$C_{M_{ac}} =$	0.0000
		$C_{L_{\delta_{e_t}}} =$	1.8000
		$\epsilon_0 =$	0.642 [deg]
		$\epsilon_\alpha =$	0.426
		$\eta =$	0.9

$\cos(\alpha + 3) \times \alpha \cos 3$

$\cos \alpha \cos 3$

where…

- The wing and horizontal stabilizer are trapezoidal surfaces.
- The cg of the aircraft is at $x_{cg} = 10.56$ [ft].
- The weight of the aircraft is $W = 9,500$ [lb].
- The propulsive moment coefficients are $C_{M_{0_p}} = 0.0$ and $C_{M_{\alpha_p}} = 0.0$.
- The fuselage moment coefficients are $C_{M_{0_f}} = 0.0$ and $C_{M_{\alpha_f}} = 0.0$.
- The horizontal tail incidence can only be set between -0.5 and -7 [deg].

a. Determine the trim conditions when the aircraft is descending from 5,000 ft above sea level at a speed of 210 knots on a 3-degree glidepath when…

    i. the aircraft in its "clean" configuration, i.e., when the characteristics of the wing are as given above.

    ii. the flaps and landing gears of the aircraft are extended, i.e., when $C_{L_0} = 0.1000$.

b. Explain why, for this aircraft, we don't have to consider the increase in drag and the resulting moment due to increased thrust in our computations.

c. Explain how you would go about solving the problem for an aircraft where the thrust vector does not pass through the center of gravity.

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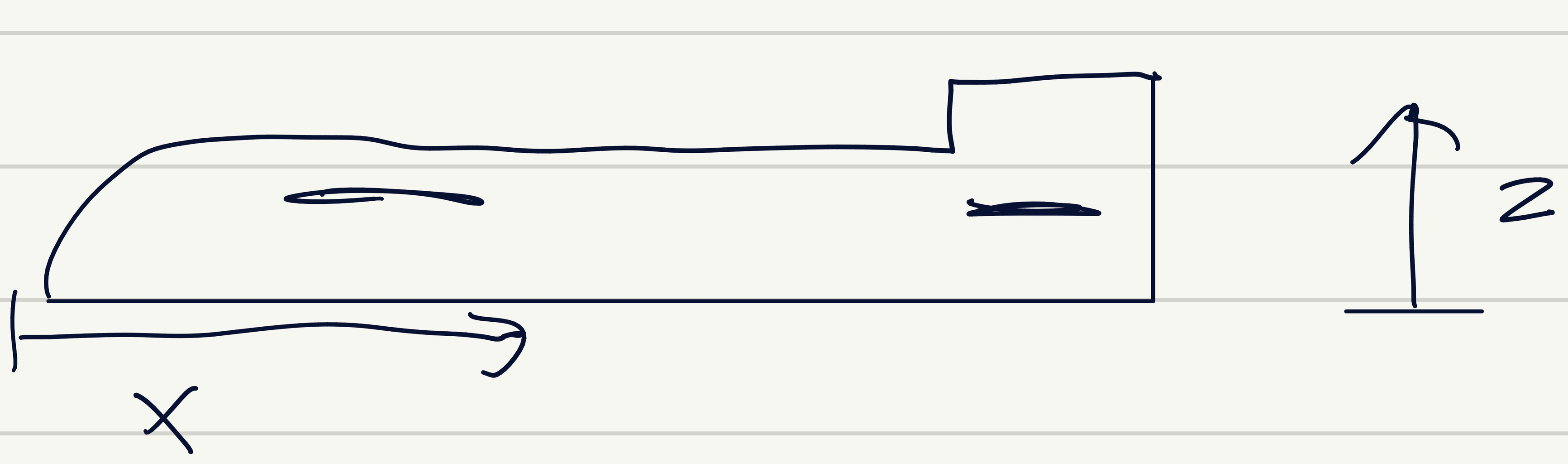
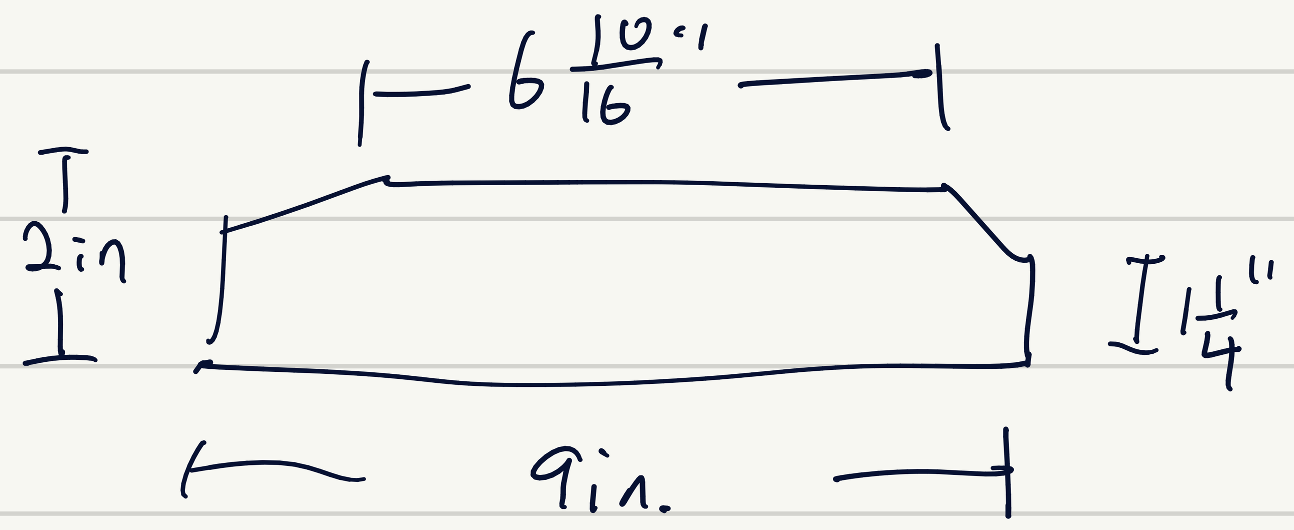
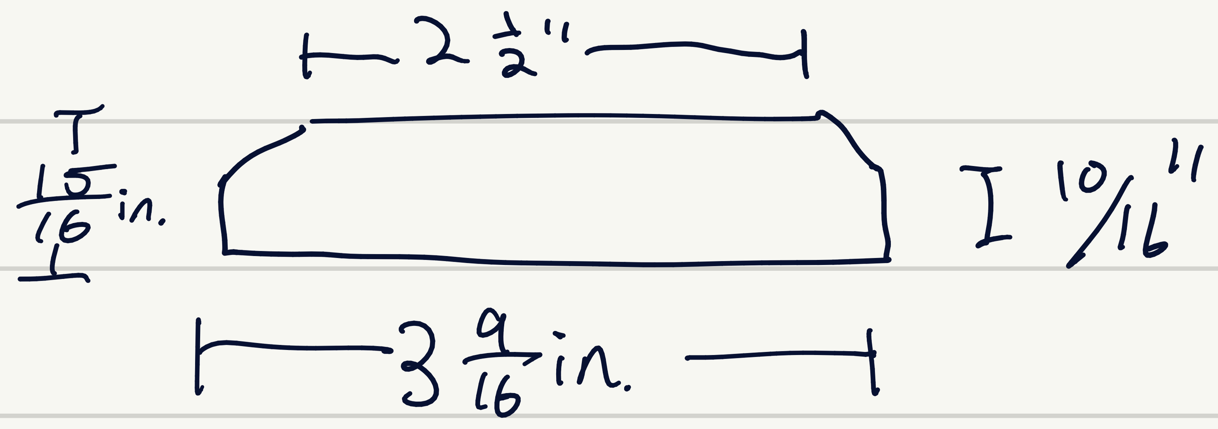


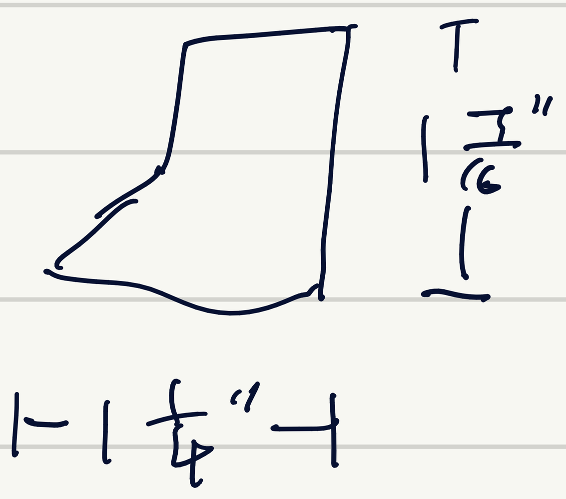
Wing : 2g



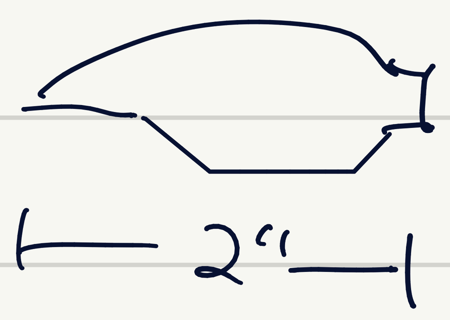
Stabilizer : 0.62 g



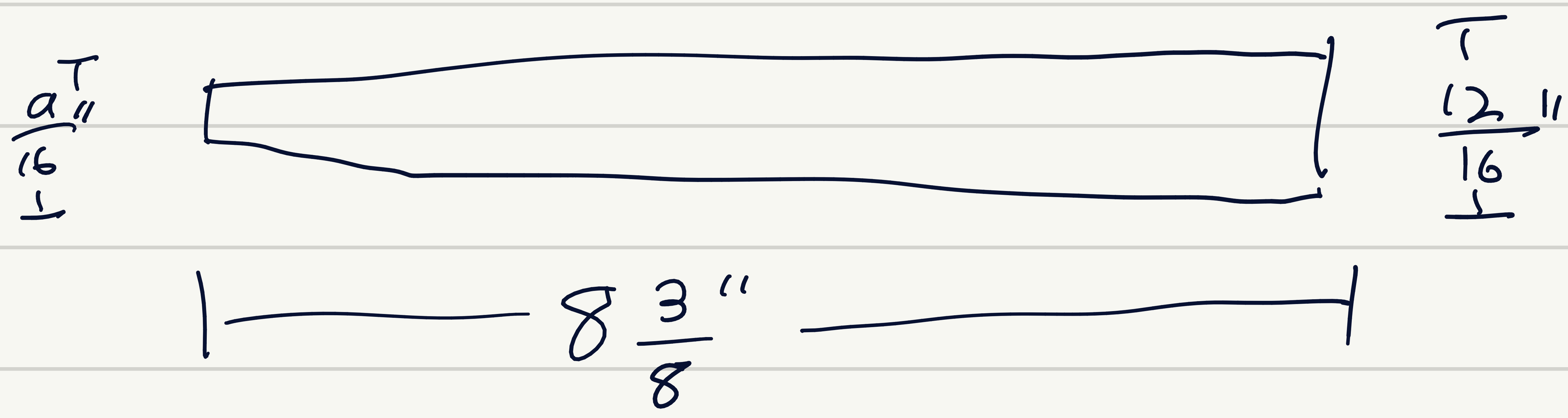
Rudder : 0.21g



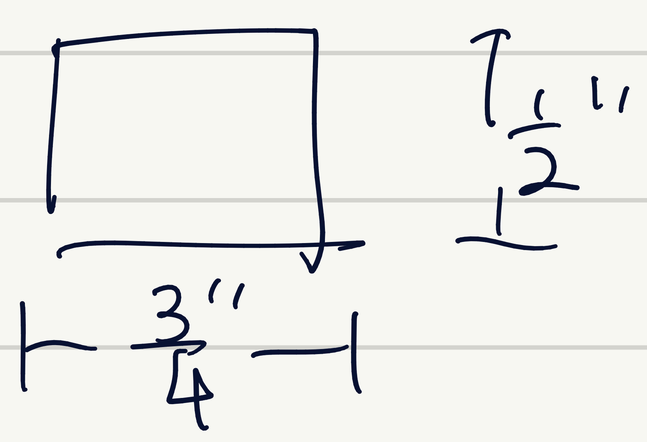
Canopy : 0.21 g



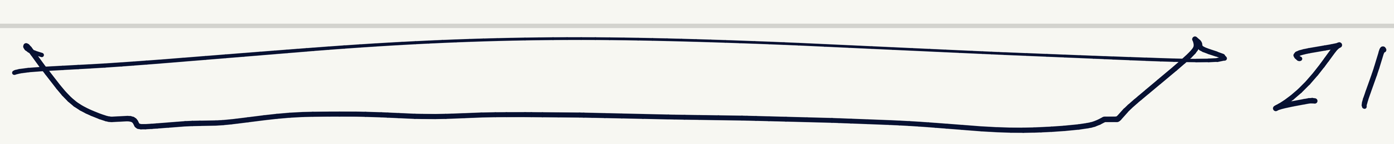
Fuselage : 2.08g



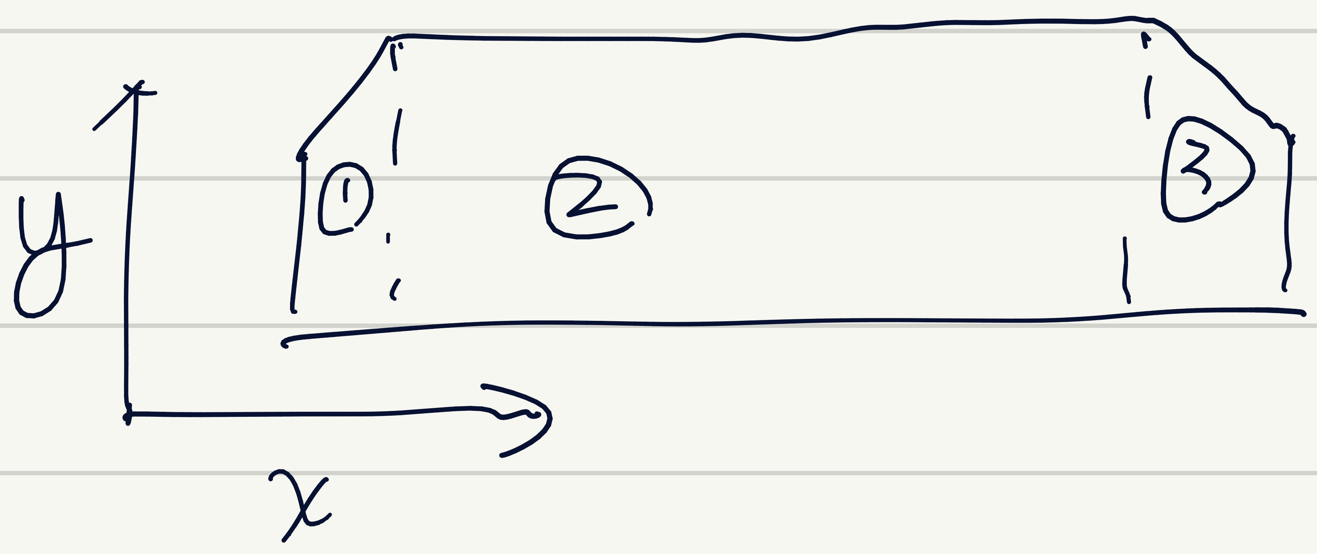
Metal : 1.75 g



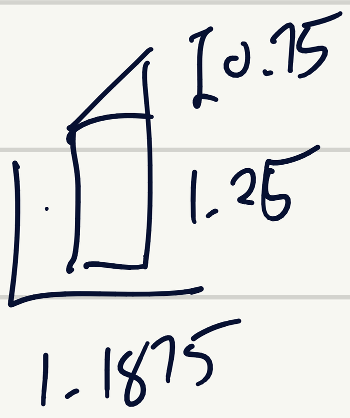
Metal 1.75



Wing C.g.



$$\bar{x}_w = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_3 A_2}{A_1 + A_2 + A_3}$$



$1.4844 = A_1$

$A_2 = 0.4453$

$\bar{x}_1 = 0.6394$

$\bar{y}_1 = 0.8269$

$$\bar{x}_1 = \frac{1.25 + 2(2)}{3(2+1.25)} 1.1875$$

$$= 0.6394'' \rightarrow \text{from base "a"}$$

$\bar{y}_1 = 0.8269$

$$A_1 = \frac{a+b}{2} h = \frac{1.25+2}{2}(1.1875) = 1.9297 \text{ in}^2$$

$\bar{x}_2 = 4.5''$ $\quad A_2 = 13.25 \text{ in}^2$ $\quad \bar{y}_2 = 1''$

$\bar{x}_3 = 9 - 0.6394 = 8.3606''$ $\quad \bar{y}_3 = \bar{y}_1$

$A_3 = A_3$

$$\bar{x}_w = \frac{0.6394(1.9297) + 4.5(13.25) + 8.3606(1.9297)}{2(1.9297) + 13.25}$$

$$= 4.5''$$

$$\bar{y}_w = \frac{2(0.8269)(1.9297) + 1(13.25)}{2(1.9297) + 13.25}$$

$$= 0.96095''$$

$$W_{c.g.} = (4.5'', 0.96095'')$$

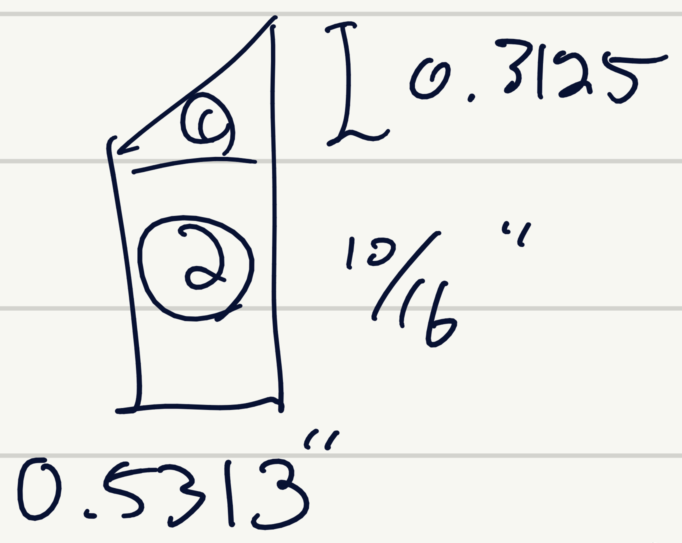
$A = 1.4844$ $\quad y = 0.625$

$A = 0.4453$ $\quad y = 1.5$

$\bar{y} = 0.8269$

Horizontal stabilizer

$$\bar{x}_t = 1.7813''$$



$A_1 = 0.083$ $\quad \bar{y}_1 = 0.7292$

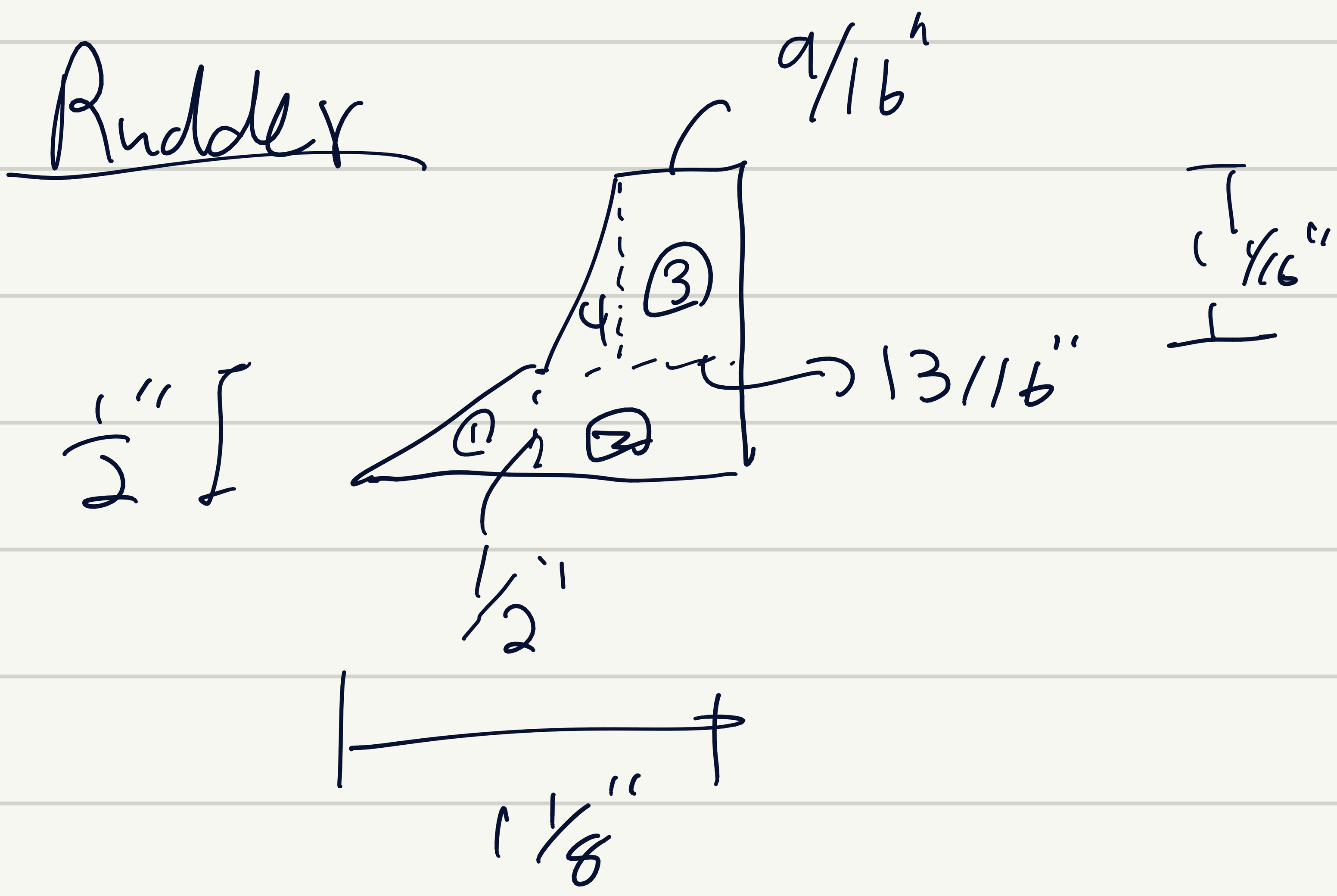
$A_2 = 0.3321$ $\quad \bar{y}_2 = 0.625$

$$\bar{y}_t = \frac{2 \cdot 0.7292(0.083) + 2 \cdot 0.3321(0.625) + 2.3438(0.4688)}{2(0.7292) + 2 \cdot 0.3321 + 2.3438}$$

$$= 0.3661''$$

$$t_{cg} = (1.7813, 0.3661)$$

Rudder



$\bar{x}_1 = 0.2083$

$\bar{y}_1 = \frac{1}{6}$ $\quad A_1 = 0.07813$

$\bar{x}_2 = 0.71875$ $\quad A_2 = 0.4063$

$\bar{y}_2 = 0.25$

$\bar{x}_3 = 0.84375$

$\bar{y}_3 = 1.0313$ $\quad A_3 = 0.5977$

$\bar{x}_4 = 0.4792$ $\quad A_4 = 0.1328$

$\bar{y}_4 = 0.8542$

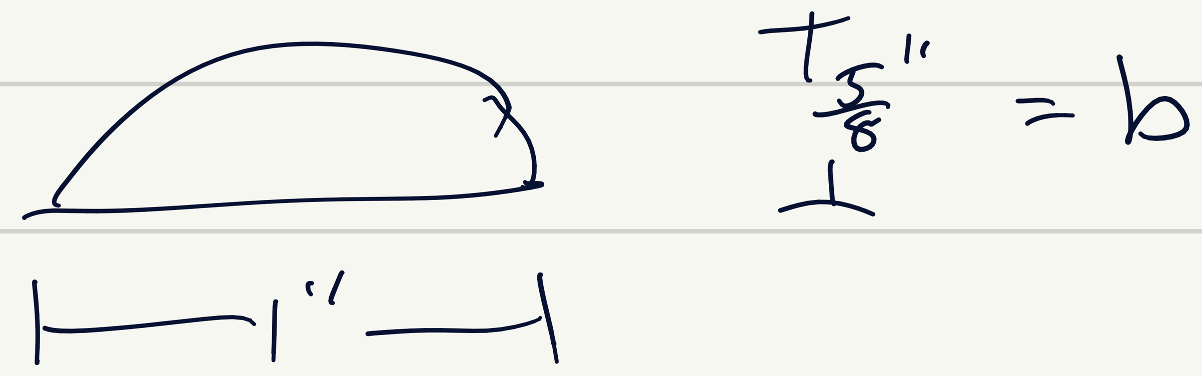
$$\bar{x}_R = \frac{0.2083(0.07813) + 0.7188(0.4063) + 0.9438(0.5977) + 0.4792(0.1328)}{0.07813 + 0.4063 + 0.5977 + 0.1328}$$

$$= 0.7213$$

$$\bar{y}_R = \frac{\frac{1}{6}(0.07813) + 0.25(0.4063) + 1.0313(0.5977) + 0.8542(0.1328)}{0.07813 + 0.4063 + 0.5977 + 0.1328}$$

$$= 0.6951$$

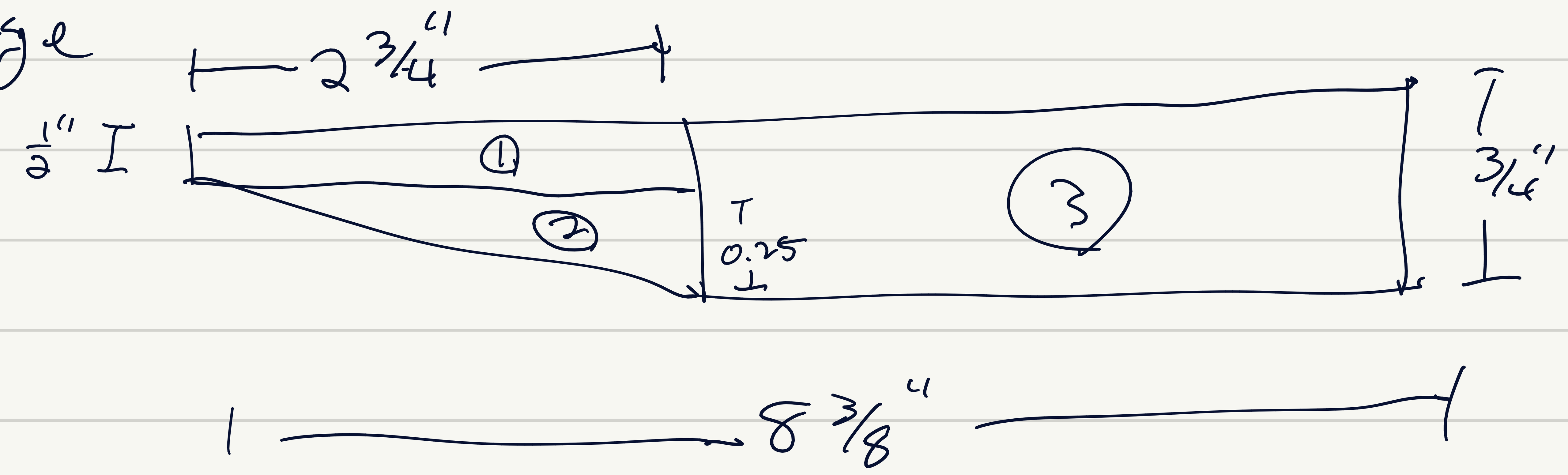
Canopy



$\bar{x}_c = 0.5''$

$\bar{y}_c = \frac{4b}{3\pi} = 0.2653''$

fuselage



$\bar{x}_1 = 1.375$ $\quad A_1 = 1.375$

$\bar{y}_1 = 0.25$

$\bar{x}_2 = 1.8\bar{3}$ $\quad A_2 = 0.3438$

$\bar{y}_2 = 0.58\bar{3}$

$\bar{x}_3 = 5.5625$ $\quad A_3 = 4.2188$

$\bar{y}_3 = 0.375$

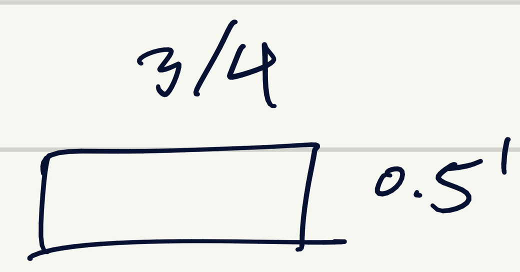
$$\bar{x}_f = \frac{1.375(1.375) + 1.8\bar{3}(0.3438) + 5.5625(4.2188)}{1.375 + 0.3438 + 4.2188}$$

$$= 4.3769$$

$$\bar{y}_f = \frac{0.25(1.375) + 0.58\bar{3}(0.3438) + 0.375(4.2188)}{1.375 + 0.3438 + 4.2188}$$

$$= 0.3581$$

Metal



$\bar{x}_m = 0.375$

$\bar{y}_m = 0.25$

Total

$$\bar{X} = \frac{2(3\tfrac{1}{16}+1.0391) + 0.62(7.5+0.5714) + 0.21(7+0.7213) + 0.21(2.25+0.5) + 2.08(3.9981) + 1.75(x+0.375)}{\cancel{2+0.62+}2\cdot 0.21 + 2.08 + 1.75}$$

$$\bar{X}(x_{metal}) = \frac{24.3787 + 1.75\,x_{metal}}{6.87}$$

$$\bar{y} = \frac{2(0.375) + 0.62(0.4375) + 0.21(0.75+0.6951) + 0.21(0.625+0.2653) + 2.08(0.3581) + 1.75(y_{metal}+0.25)}{2+0.62+2\cdot 0.21 + 2.08 + 1.75}$$

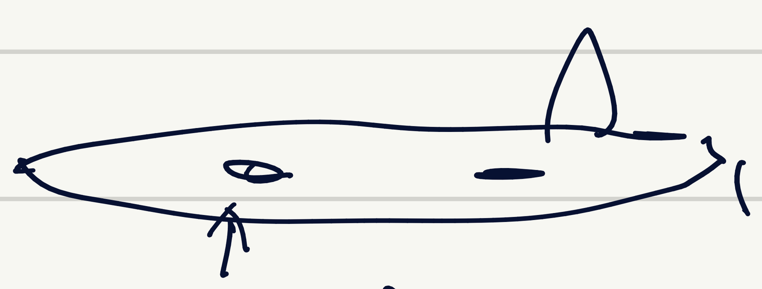
$$\bar{y}_{metal} = \frac{2.6940 + 1.75\,y_{metal}}{6.87}$$

(c) estimated a.c.: $\hat{x}_{a.c.} = \dfrac{23.72 + 1.75(0.75+0.25)}{6.87}$

$= 3.71''$

$\hat{y}_{a.c.} = \dfrac{2.6940 + 1.75(0+0.375)}{6.87}$

$= 0.49''$



Metal location (0.75, 0)

When I put the metal at (0.75, 0) in reference to airplane nose and bottom, the airplane began flipping over, pitching up. However, when I put the metal a bit forward, the plane flew stably for awhile, so (0.75, 0) must be close to the aerodynamic center.

Force eqs

② $\perp \sum F = 0$

$$= L_w \cos\alpha + D_w \sin\alpha + L_t \cos\alpha_t + D_t \sin\alpha_t - W\cos(\alpha+\gamma)$$

$$= L_w + D_w\alpha + L_t + D_t\alpha_t - W$$

$\parallel \sum F = 0$

$$= -L_w \sin\alpha + D_w \cos\alpha - L_t \sin\alpha_t + D_t \cos\alpha_t + W\sin(\alpha+\gamma)$$

$$= -L_w\alpha + D_w - L_t\alpha_t + D_t + W(\alpha+\gamma) + T$$

Define coefficients

$$0 = L_w + D_w\alpha + L_t + D_t\alpha_t - W$$

$$L_w = qSC_{L_w} = qS\left[C_{L_{0w}} + C_{L_{\alpha w}}(\alpha + i_w)\right]$$

$$L_t = q_t S_t C_{L_t} = q_t S_t\left[C_{L_{0t}} + C_{L_{\alpha t}}(\alpha_t + i_t) + C_{L_{\delta e}}\delta_e\right]$$

$$= q_t S_t\left[C_{L_{0t}} + \cancel{C_{L_{\alpha t}}(\alpha - \epsilon_t + i_t)} + C_{L_{\delta e}}\delta_e\right]$$

$$= q_t S_t\left[C_{L_{0t}} + C_{L_{\alpha t}}(i_t - \epsilon_{0t} + (1-\epsilon_{\alpha t})\alpha) + C_{L_{\delta e}}\delta_e\right]$$

$$L = qS\left[C_{L_{0w}} + C_{L_{\alpha w}}(\alpha + i_w) + \frac{q_t S_t}{qS}\left[C_{L_{0t}} + C_{L_{\alpha t}}[i_t - \epsilon_{0t} + (1-\epsilon_{\alpha t})\alpha] + C_{L_{\delta e}}\delta_e\right]\right]$$

$$C_L = C_{L_{0w}} + C_{L_{\alpha w}}(\alpha + i_w) + \frac{q_t S_t}{qS}\left[C_{L_{0t}} + C_{L_{\alpha t}}[i_t - \epsilon_{0t} + (1-\epsilon_{\alpha t})\alpha] + C_{L_{\delta e}}\delta_e\right]$$

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_t}} i_t + \cancel{C_{L_{\delta e}}\delta_e}^{\,0}$$

$$C_{L_0} = C_{L_{0w}} + C_{L_{\alpha w}} i_w + \frac{q_t}{q}\frac{S_t}{S}(C_{L_{0t}} - C_{L_{\alpha t}}\epsilon_{0t})$$

$$C_{L_\alpha} = C_{L_{\alpha w}} + \frac{q_t}{q}\frac{S_t}{S}C_{L_{\alpha t}}(1 - \epsilon_{\alpha t})$$

$$C_{L_{i_t}} = \frac{q_t}{q}\frac{S_t}{S}C_{L_{\alpha t}}$$

$$C_{L_{\delta e}} = \frac{q_t S_t}{qS}C_{L_{\delta e t}}$$

$D_w = qSC_{D_W}$
$= qS[\,C_{D_{0w}} + KC_{L_w}^2\,]$
$= qS[\,C_{D_{0w}} + K[\,C_{L_{0w}} + C_{L\alpha w}\alpha\,]^2\,]$ HOT
$= qS[\,C_{D_{0w}} + K[\,C_{L_{0w}}^2 + 2C_{L_{0w}}C_{L_{0w}}\alpha + \cancel{C_{L_{\alpha_w}}^2\alpha^2}\,]\,]$

$\approx qS[\,C_{D_{0w}} + K[\,[\,C_{L_{0w}}]^2 + 2C_{L_{0w}}C_{L\alpha_w}\alpha\,]$
$\approx qS[\,C'_{D_{0w}} + 2KC_{L_{0w}}C_{L\alpha_w}\alpha\,]$
where $C'_{D_{0w}} = C_{D_{0w}} + K[\,C_{L_{0w}}]^2$

$D_t = q_tS_tC_{D_t}$
$= q_tS_t[\,C_{D_{0t}} + KC_{L_t}^2\,]$
$= q_tS_t[\,C_{D_{0t}} + K[\,C_{L_{0t}} + C_{L\alpha_t}\alpha_t\,]^2\,]$ HOT
$= q_tS_t[\,C_{D_{0t}} + K[\,C_{L_{0t}}^2 + 2C_{L_{0t}}C_{L_{0t}}\alpha_t + \cancel{C_{L_{\alpha_t}}^2\alpha_t^2}\,]$

$\approx q_tS_t[\,C'_{D_{0t}} + 2KC_{L_{0t}}C_{L\alpha_t}\alpha_t\,]$; $C'_{D_{0t}} = C_{D_{0t}} + K[\,C_{L_{0t}}]^2$
$\approx q_tS_t[\,C'_{D_{0t}} + 2KC_{L_{0t}}C_{L\alpha_t}\,(\alpha - \epsilon_t)\,]$
$\approx q_tS_t[\,C'_{D_{0t}} + 2KC_{L_{0t}}C_{L\alpha_t}\alpha - 2KC_{L_{0t}}C_{L\alpha_t}(\epsilon_{0t}) - 2KC_{L_{0t}}C_{L\alpha_t}(\epsilon_{\alpha_t}\alpha)]$

Plug in coeff.
and solve for α

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$W = L_w + D_w\alpha + L_t + D_t\alpha_t$ HOT
$= qS\,[\,C_{L_{0w}} + C_{L\alpha w}(\alpha + i_w)\,] + qS\,[\,C'_{D_{0w}} \cancel{+ 2KC_{L_{0w}}C_{L\alpha_w}\alpha}\,]\alpha$
$+ \; q_tS_t[\,C_{L_{0t}} + C_{L\alpha_t}(i_t - \epsilon_{0_t} + (1-\epsilon_{\alpha_t})\alpha) + C_{L_{\delta_e}}\delta_e\,]$
$+ \; q_tS_t[\,C'_{D_{0t}} + 2KC_{L_{0t}}\cancel{C_{L\alpha_t}\alpha_t}\,]\,\alpha_t$ HOT

$W = \cancel{qS}\,[\,\cancel{C_{L_{0w}}} + C_{L\alpha_w}\alpha + \cancel{C_{L\alpha_w}i_w} + C'_{D_{0w}}\alpha\,]$
$+ q_tS_t\,[\,\cancel{C_{L_{0t}}} + \cancel{C_{L\alpha_t}i_t} - \cancel{C_{L\alpha_t}\epsilon_{0_t}} + C_{L\alpha_t}\alpha - C_{L\alpha_t}\epsilon_{\alpha_t}\alpha + \cancel{C_{L_\delta}\delta_e}$
$\cancel{+ C'_{D_{0t}}} + 0\,]$

$\frac{W}{qS} - C_{L_{0w}} - C_{L\alpha_w}i_w = C_{L\alpha_w}\alpha + C'_{D_{0w}}\alpha + \frac{q_tS_t}{qS}[\;//\;]$

$\frac{W}{qS} - C_{L_{0w}} - C_{L\alpha_w}i_w - \frac{q_tS_t}{qS}[\,C_{L_{0t}} + C_{L\alpha_t}i_t + C_{L_\delta}\delta_e + C'_{D_{0t}}\,] = C_{L\alpha_w}\alpha + C'_{D_{0w}}\alpha$
$+ \frac{q_tS_t}{qS}[\,C_{L\alpha_t}\alpha - C_{L\alpha_t}\epsilon_{\alpha_t}\alpha\,]$

$$\frac{W - qS[C_{L0w} - C_{L\alpha w} i_w] - q_t S_t [C_{L0t} + C_{L\alpha t} i_t + C_{L\delta}\delta_e + C'_{D0t}]}{qS}$$

$$=$$

$$\frac{\alpha[qS[C_{L\alpha w} + C'_{D0w}] + q_t S_t [C_{L\alpha t} - C_{L\alpha t}\epsilon_{\alpha t}]]}{qS}$$

$$\alpha = \frac{W - qS[C_{L0w} - C_{L\alpha w} i_w] - q_t S_t [C_{L0t} + C_{L\alpha t} i_t + C_{L\delta}\delta_e + C'_{D0t}]}{qS[C_{L\alpha w} + C'_{D0w}] + q_t S_t [C_{L\alpha t} - C_{L\alpha t}\epsilon_{\alpha t}]}$$

Moment about CG

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@ Trim

$$\Sigma M_{cg} = 0 = M_p + M_f + M_{ac_w} + M_{ac_t}$$
$$+ L_w \cos\alpha\,(x_{cg} - x_{ac_w}) + L_w \sin\alpha\,(z_{ac_w} - z_{cg})$$
$$+ D_w \sin\alpha\,(x_{cg} - x_{ac_w}) + D_w \cos\alpha\,(z_{ac_w} - z_{cg})$$
$$+ L_t \cos\alpha_t\,(x_{cg} - x_{ac_t}) + L_t \sin\alpha_t\,(z_{ac_t} - z_{cg})$$
$$+ D_t \sin\alpha_t\,(x_{cg} - x_{ac_t}) + D_t \cos\alpha_t\,(z_{ac_t} - z_{cg})$$

$$0 = M_p + M_f + M_{ac_w} + M_{ac_t}$$
$$+ (x_{cg} - x_{ac_w})(L_w + D_w\alpha) + (z_{ac_w} - z_{cg})(L_w\alpha + D_w)$$
$$+ (x_{cg} - x_{ac_t})(L_t + D_t\alpha_t) + (z_{ac_t} - z_{cg})(L_t\alpha + D_t)$$

$$M_{ac_w} = qS\bar{c}\,C_{M_{ac_w}} \qquad M_{ac_t} = q_t S_t \bar{c}_t C_{M_{ac_t}}$$

$$M_p = qS\bar{c}\,C_{M_p} \qquad M_f = qS\bar{c}\,C_{M_f}$$

Plug in coeff.

$$0 = M_p + M_f + M_{ac_w} + M_{ac_t} + (x_{cg} - x_{ac_w})\Big[qS[C_{L0w} + C_{L\alpha w}(\alpha + i_w)]$$
$$+ qS[C'_{D0w} + 2k\,\cancel{C_{L0w}}\,C_{L\alpha w}\,\alpha]\alpha$$
$$+ (z_{ac_w} - z_{cg})\Big[qS[C_{L0w} + C_{L\alpha w}(\cancel{\alpha} + i_w)]\alpha$$
$$+ qS[C'_{D0w} + 2k\,C_{L0w} C_{L\alpha w}\,\alpha]\Big]$$
$$+ (x_{cg} - x_{ac_t})\Big[q_t S_t[C_{L0t} + C_{L\alpha t}(i_t - \epsilon_{0t} + (1 - \epsilon_{\alpha t})\alpha) + C_{L\delta e}\delta_e]$$
$$+ q_t S_t[C'_{D0t} + 2k\,C_{L0t} C_{L\alpha t}\,\alpha_t]\alpha_t\Big]$$

$$+ (z_{ac_t} - z_{cg})\Big[q_t S_t[C_{L0t} + C_{L\alpha t}(i_t - \epsilon_{0t} + (1 - \epsilon_{\alpha t})\alpha) + C_{L\delta e}\delta_e]\alpha_t$$
$$+ q_t S_t[C'_{D0t} + 2k\,C_{L0t} C_{L\alpha t}\,\alpha_t]\Big]$$

$$-M_p - M_f - M_{ac_w} - M_{ac_t} = (x_{cg} - x_{ac_w})\, qS\left[\cancel{C_{L0w}} + \cancel{C_{L\alpha w}\alpha} + \cancel{C_{L\alpha w} i_w} + \cancel{C'_{D0w}\alpha}\right]$$

$$+ [z_{ac_w} - z_{cg}]\, qS\left[\cancel{C_{L0w}\alpha} + \cancel{C_{L\alpha w}\underset{0}{i_w}\alpha} + \cancel{C'_{D0w}}\right.$$

$$\left. + 2k\,\cancel{C_{L0w}C_{L\alpha w}\alpha}\right]$$

$$+ (x_{cg} - x_{ac_t})\, q_t S_t\left[\cancel{C_{L0t}} + \cancel{C_{L\alpha_t} i_t} - \cancel{C_{L\alpha_t}\epsilon_{0t}}\right.$$

$$\left. + \cancel{C_{L\alpha_t}\alpha} - \cancel{C_{L\alpha_t}\overset{0}{\epsilon_{\alpha_t}\alpha}} + \cancel{C_{L\delta_e}\delta_e} + \cancel{C'_{D0t}\alpha_t}\right] \quad 0$$

$$+ (z_{cg} - z_{ac_t})\, q_t S_t\left[\cancel{C_{L0t}\alpha_t} + \cancel{C_{L\alpha_t}\overset{0}{i_t}\alpha_t} - \cancel{C_{L\alpha_t}\epsilon_{0t}\alpha_t}\right.$$

$$0 \quad + \cancel{C_{L\alpha_t}\alpha\alpha_t} - \cancel{C_{L\alpha_t}\epsilon_{\alpha_t}\alpha\alpha_t} + \cancel{C_{\delta_e}\delta_e\alpha_t}$$

$$\left. + \cancel{C'_{D0t}} + 2k\, C_{L0t} C_{L\alpha_t}\alpha_t\right]$$

$$-M_p - M_f - M_{ac_w} - M_{ac_t} = (x_{cg} - x_{ac_w})\, qS\left[\cancel{C_{L_{0w}}} + \cancel{C_{L_{\alpha w}}\alpha} + \cancel{C_{L_{\alpha w}} i_w} + \cancel{C'_{D_{0w}}\alpha}\right]$$

$$+ [z_{ac_w} - z_{cg}]\, qS\left[\cancel{C_{L_{0w}}\alpha} + \cancel{C'_{D_{0w}}} + \cancel{2k C_{L_{0w}} C_{L_\alpha}\alpha}\right]$$

$$+ (x_{cg} - x_{ac_t})\, q_t S_t \left(\cancel{C_{L_{0t}}} + \cancel{C_{L_{\alpha t}} i_t} - \cancel{C_{L_{\alpha t}}\epsilon_{0t}} + \cancel{C_{L_{\alpha t}}\alpha}\right.$$

$$\alpha_t = \alpha - \epsilon_t \qquad + \left.\cancel{C'_{D_{0t}}(\alpha)} - \cancel{C'_{D_{0t}}(\epsilon_{0t})}\right)]$$

$$= \alpha - \epsilon_{0t} - \epsilon_{\alpha t}\alpha \qquad + (z_{cg} - z_{ac_t})\, q_t S_t\left[\cancel{C_{L_{0t}}\alpha_t} + \cancel{C'_{D_{0t}}} + \overset{0}{\cancel{2k C_{L_{0t}} C_{L_{\alpha t}}[\alpha - \epsilon_{0t} - \epsilon_{\alpha t}\alpha]}}\right]$$

$$C_{M_0} = C_{m_{ac_w}} + \frac{q_t S_t \bar{c}_t}{q S \bar{c}} C_{m_{ac_t}} + C_{M_{0p}} + C_{M_{0f}} + (\bar{x}_{cg} - \bar{x}_{ac_w})(C_{L_{0w}} + C_{L_{\alpha w}} i_w + C'_{D_{0w}\alpha})$$

$$+ (\bar{z}_{ac_w} - \bar{z}_{cg})\, qS\,[C'_{D_{0w}}] + (\bar{x}_{cg} - x_{ac_t})\frac{q_t S_t}{qS}\left[C_{L_{0t}} - C_{L_{\alpha t}}\epsilon_{0t} - C'_{D_{0t}}\epsilon_{0t}\right]$$

$$+ (z_{cg} - z_{ac_t})\frac{q_t S_t}{qS}\left(C'_{D_{0t}} - 2k C_{L_{0t}} C_{L_{\alpha t}}\epsilon_{0t}\right)$$

$$C_{M_\alpha} = C_{M_{\alpha p}} + C_{M_{\alpha f}} + (\bar{x}_{cg} - \bar{x}_{ac_w}) C_{L_{\alpha w}} + (\bar{z}_{ac_w} - \bar{z}_{cg})\left[C_{L_{0w}} + 2k C_{L_{0w}} C_{L_{\alpha w}}\right]$$

$$+ (\bar{x}_{cg} - \bar{x}_{ac_t})\frac{q_t S_t}{qS}\left[C_{L_{\alpha t}} + C'_{D_{0t}}\right] + (\bar{z}_{cg} - \bar{z}_{ac_t})\frac{q_t S_t}{qS}\left[C_{L_{0t}} + 2k C_{L_{0t}} C_{L_{\alpha t}}\right]$$

$$C_{M_i} = \frac{q_t S_t}{qS}(\bar{x}_{cg} - \bar{x}_{ac_w}) C_{L_{\alpha t}}$$

Solve fo $i_{t_{trim}}$ and α_{trim} $\delta_e = 0$

$$0 = L_w + D_w\alpha + L_t + D_t\alpha_t - W$$

$$\frac{W}{qS} = C_{L_0} + C_{L_\alpha}\alpha + C_{L_i} i_t + \left(C'_{D_{0w}} + \overset{0}{\cancel{2k C_{L_{0w}} C_{L_{\alpha w}}\alpha}}\right)\alpha$$

$$\frac{q_t S_t}{qS}\left[C'_{D_{0t}} + \underset{0}{\cancel{2k C_{L_{0t}} C_{L_{\alpha t}}\alpha_t}}\right]\alpha_t$$

- $$\frac{W}{qS} = C_{L_0} + C_{L_\alpha}\alpha + C_{L_i} i_t + C'_{D_{0w}}\alpha + \frac{q_t S_t}{qS}\left[C'_{D_{0t}}\alpha - C'_{D_{0t}}\epsilon_{0t}\right]$$
- $$0 = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{i_t}} i_t$$

$$\alpha = \frac{-C_{M_0} - C_{M_{i_t}} i_t}{C_{M_\alpha}}$$

$$\frac{W}{qS} = C_{L_0} + C_{L_i} i_t + \frac{q_t S_t}{qS} C'_{D_{0t}} \epsilon_{0t} + \alpha \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0t}} \right]$$

$$\frac{W}{qS} - C_{L_0} - \frac{q_t S_t}{qS} C'_{D_{0t}} \epsilon_{0t} = C_{L_{i_t}} i_t + \alpha \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0t}} \right]$$

$$\frac{W}{qS} - C_{L_0} - \frac{q_t S_t}{qS} C'_{D_{0t}} \epsilon_{0t} = C_{L_{i_t}} i_t + \frac{-C_{M_0} - C_{M_{i_t}} i_t}{C_{M_\alpha}} \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0t}} \right]$$

$$'' = \frac{C_{M_\alpha} C_{L_i} i_t}{C_{M_\alpha}} + \frac{-C_{M_0} - C_{M_{i_t}} i_t}{C_{M_\alpha}} \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0w}} \right]$$

$$'' + \frac{C_{M_0}}{C_{M_\alpha}} \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0w}} \right] = \frac{C_{M_\alpha} C_{L_i} i_t - C_{M_{i_t}} i_t \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0w}} \right]}{C_{M_\alpha}}$$

$$\frac{C_{M_\alpha} \left[\frac{W}{qS} - C_{L_0} - \frac{q_t S_t}{qS} C'_{D_{0t}} \epsilon_{0t} \right] + C_{M_0} \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0w}} \right]}{C_{M_\alpha} C_{L_i} - C_{M_{i_t}} \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0w}} \right]} = i_{t_{trim}}$$

$$i_t = \frac{-C_{M_0} - C_{M_\alpha} \alpha}{C_{M_{i_t}}}$$

$$\frac{W}{qS} = C_{L_0} + C_{L_\alpha} \alpha + C_{L_i} i_t + C'_{D_{0w}} \alpha + \frac{q_t S_t}{qS} \left[C'_{D_{0t}} \alpha - C'_{D_{0t}} \epsilon_{0t} \right]$$

$$\frac{W}{qS} - C_{L_0} - \frac{q_t S_t}{qS} C'_{D_{0t}} \epsilon_{0t} = C_{L_{i_t}} i_t + \alpha \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0t}} \right]$$

$$'' = -\frac{C_{L_{i_t}} C_{M_0}}{C_{M_{i_t}}} - \frac{C_{L_{i_t}} C_{M_\alpha}}{C_{M_{i_t}}} \alpha + \alpha \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0t}} \right]$$

$$\frac{W}{qS} - C_{L_0} - \frac{q_t S_t}{qS} C'_{D_{0t}} \epsilon_{0t} + \frac{C_{L_{i_t}} C_{M_0}}{C_{M_{i_t}}} = \alpha \left[\frac{-C_{L_{i_t}} C_{M_\alpha} + C_{M_{i_t}} \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0w}} \right]}{C_{M_{i_t}}} \right]$$

$$\boxed{\frac{C_{M_{i_t}} \left[\frac{W}{qS} - C_{L_0} - \frac{q_t S_t}{qS} C'_{D_{0t}} \epsilon_{0t} \right] + C_{L_{i_t}} C_{M_0}}{-C_{L_{i_t}} C_{M_\alpha} + C_{M_{i_t}} \left[C_{L_\alpha} + C'_{D_{0w}} + \frac{q_t S_t}{qS} C'_{D_{0w}} \right]} = \alpha_{t_{trim}}}$$

$\parallel \Sigma F = 0$

$$0 = -L_w\alpha + D_w - L_t\alpha_t + D_t + W(\alpha+\gamma) - T$$

$$0 = -(C_{L_0} + \cancel{C_{L_\alpha}\alpha} + \cancel{C_{L_i} i_t})\,\alpha + C'_{D_{0w}} + 2k C_{L_{0w}} C_{L_{\alpha w}}\alpha$$
$$- \frac{q_t S_t}{qS}\left[C_{L_{0t}} + C_{L_{\alpha t}}(i_t - \epsilon_{0t} + (1-\epsilon_{\alpha t})\alpha) + \cancel{C_{L_{\delta e}}\delta_e}\right]\alpha_t$$
$$+ \frac{q_t S_t}{qS}\left[C'_{D_{0t}} + 2k C_{L_{0t}} C_{L_{\alpha t}}\alpha_t\right] + W(\alpha+\gamma)^{0} - T$$

$$0 = \cancel{-C_{L_0}\alpha} + \cancel{C'_{D_{0w}}} + 2\cancel{k C_{L_{0w}}} C_{L_{\alpha w}}\alpha$$
$$- \frac{q_t S_t}{qS}\left[C_{L_{0t}}\alpha_t + C'_{D_{0t}} + 2k C_{L_{0t}} C_{L_{\alpha t}}(\cancel{\alpha} - \epsilon_{0t}) + W(\alpha+\gamma) - \cancel{T}\right.$$
$$\Downarrow$$
$$\cancel{C_{L_{0t}}(\alpha)} + \cancel{C_{L_{0t}}\epsilon_{0t}}$$

$$\boxed{\begin{aligned} T = {} & \alpha_{trim}\left[-C_{L_0} + 2k C_{L_{0w}} C_{L_{\alpha w}} - \frac{q_t S_t}{qS}\left[C_{L_{0t}} + 2k C_{L_{0t}} C_{L_{\alpha t}}\right]\right] \\ & + C'_{D_{0w}} - \frac{q_t S_t}{qS}\left[C_{L_{0t}}\epsilon_{0t} + C'_{D_{0t}} - 2k C_{L_{0t}} C_{L_{\alpha t}}\epsilon_{0t}\right] \\ & - W(\alpha+\gamma) \end{aligned}}$$

③

216 knots $= 354.44$ ft/s $\qquad \gamma = 3°$

$\rho = 0.002378$ slugs/ft^3 $\qquad \delta_e = 0°$

i) first assume $\delta_e = 0$ at this speed

$$\alpha_{trim_{\delta_e=0}} = -\frac{C_{M_{i_t}}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_{i_t}} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

$$i_{trim} = \frac{C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_\alpha} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

$q = \frac{1}{2}\rho V^2 = 149.37$ lb/ft^2 $\qquad q_t = \eta q = 134.43$ lb/ft^2

$$C_{M_\alpha} = \cancel{C_{M_{\alpha_P}}}^{0} + \cancel{C_{M_f}}^{0} + (\bar{x}_{cg} - \bar{x}_{ac_w}) C_{L_{\alpha_w}} + \frac{q_t S_t}{\bar{q} S}(\bar{x}_{cg} - \bar{x}_{ac_t}) C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t})$$

$$\bar{x}_{cg} = \frac{x_{cg}}{c} = \frac{10.56}{7.04} = 1.5 \qquad \bar{x}_{ac_w} = \frac{x_{ac_w}}{c} = \frac{16.4 - 4.07}{7.04} = 1.75$$

$$\bar{x}_{ac_t} = \frac{x_{ac_t}}{c} = \frac{36.9 - 2.79}{7.04} = 4.85$$

$$C_{M_\alpha} = (1.5 - 1.75)\,4.26 + \frac{134.43(54)}{149.37(232)}(1.5 - 4.85)\,4.26(1 - 0.426)$$

$$= \underline{-2.78}$$

$$C_{L_0} = C_{L_{0_w}} + C_{L_{\alpha_w}} i_w + \frac{q_t}{q}\frac{S_t}{S}(C_{L_{0_t}} - C_{L_{\alpha_t}}\epsilon_{0_t})$$

$$= -0.0443 + 5.08(0.0174) + \frac{134.43(54)}{149.37(232)}(-4.26 \cdot 0.0112)$$

$$= \underline{0.034}$$

$$C_{L_{i_t}} = \frac{134.43(54)}{149.37(232)}\,4.26 = \underline{0.8924}$$

$$C_{M_0} = C_{M_{ac_w}} + \frac{q_t S_t}{q S}\frac{\bar{c}_t}{\bar{c}} C_{M_{ac_t}} + C_{M_{0_P}} + C_{M_{0_f}} + (\bar{x}_{cg} - \bar{x}_{ac_w})(C_{L_{0_w}} + C_{L_{\alpha_w}} i_w)$$

$$+ \frac{q_t S_t}{q S}(\bar{x}_{cg} - \bar{x}_{ac_t})(C_{L_{0_t}} - C_{L_{\alpha_t}}\epsilon_{0_t})$$

$$= -0.0175 + \frac{134.43(54)}{149.37(232)}\frac{3.83}{7.04}(0) + 0 + 0 + (1.5 - 1.75)(-0.0443 + 5.08(0.0174))$$

$$+ \frac{134.43(54)}{149.37(232)}(1.5 - 4.85)[0 - 4.26(0.0112)]$$

$$= \underline{0.0055}$$

$$C_{M_{i_t}} = \frac{q_t S_t}{q S}(\bar{x}_{cg} - \bar{x}_{ac_t}) C_{L\alpha_t}$$

$$= \frac{134.43(54)}{149.37(232)}(1.5 - 4.85)4.26 = \underline{-2.99}$$

$$C_{L\alpha} = C_{L\alpha_w} + \frac{q_t S_t}{q S} C_{L\alpha_t}(1 - \epsilon_{\alpha_t})$$

$$= 5.08 + \frac{134.43(54)}{149.37(232)}(4.26)(1 - 0.426)$$

$$= \underline{5.59}$$

$$i_{t_{trim}} = \frac{-2.78\left(\frac{9500\cos 3^\circ}{149.37(232)} - 0.034\right) + 5.59(0.0055)}{(-2.78)(0.8924) - (-2.99)(5.59)}$$

$$= -0.04467 = 2.56^\circ$$

$$C_{L_0}' = C_{L_0} + C_{L_{i_t}} i_{t_{trim}} = 0.034 + 0.8924(-0.04467)$$

$$= \underline{-0.00586}$$

$$C_{M_0}' = C_{M_0} + C_{M_{i_t}} i_{t_{trim}} = 0.0055 + (-2.99)(-0.04467)$$

$$= \underline{0.1391}$$

$$C_{L_{\delta_e}} = \frac{q_t S_t}{q S} C_{L_{\delta_{e_t}}} = \frac{134.43(54)}{149.37(232)}(1.8) = \underline{0.377}$$

$$C_{M_{\delta_e}} = \frac{q_t S_t}{q S}(\bar{x}_{cg} - \bar{x}_{ac_t}) C_{L_{\delta_{e_t}}} = \frac{134.43(54)}{149.37(232)}(1.5 - 4.85)0.377$$

$$= \underline{-0.2646}$$

$$\alpha_{trim} = -\frac{-0.2646\left(\frac{9500\cos(3)}{149.37(232)} + 0.00586\right) + 0.377(0.1391)}{-2.78(0.377) + 0.2646(5.59)}$$

$$= 0.0499 = 2.86^\circ$$

$$\delta_{e_{trim}} = \frac{-2.78\left(\frac{9500\cos(3^\circ)}{149.37(232)} + 0.00586\right) + 5.59(0.1391)}{-2.78(0.377) + 0.2646(5.59)}$$

$$= 0.0005 = 0.02^\circ$$

ii) $C_{L_0} = 0.1$

$$i_{trim} = \frac{-2.78\left(\frac{9500 \cos 3^\circ}{149.37(232)} - 0.1\right) + 5.59(0.0055)}{(-2.78)(0.8924) - (-2.99)(5.59)}$$

$$= -0.0318 = 1.822^\circ$$

$$C_{L_0}' = 0.034 + 0.8924(-0.0318) = \underline{0.00562}$$

$$C_{M_0}' = 0.0055 + (-2.99)(-0.0318) = \underline{0.100582}$$

$$\alpha_{trim} = -\frac{-0.2646\left(\frac{9500 \cos(3)}{149.37(232)} - 0.00562\right) + 0.377(0.100582)}{-2.78(0.377) + 0.2646(5.59)}$$

$$= 0.0766 = 4.39^\circ$$

$$\delta_{e_{trim}} = \frac{-2.78\left(\frac{9500 \cos(3^\circ)}{149.37(232)} - 0.00562\right) + 5.59(0.100582)}{-2.78(0.377) + 0.2646(5.59)}$$

$$= -0.4250 = 24.35^\circ$$

b) We don't need to consider increase in drag b/c at trim conditions, airplane speed is held constant so drag is constant as well.

The resulting moment from increased thrust is not considered because $C_{m_{0_p}} = 0$ indicates the thrust vector is in line with and pass through the center of gravity.

c) There would be $C_{m_{0_p}}$, and I would rewrite the moment equation that includes moment caused by thrust. Also, I would rewrite the force equations, resolving Thrust into x and y components.