The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

5 SEPTEMBER 2024

ASE 367K: FLIGHT DYNAMICS

TTH 09:30-11:00
CMA 2.306

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Topics for Today

- Topic(s):
  - Lift of Aircraft + Moment about CG
  - Trim
  - Upside Down Flying Wing

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LIFT OF AIRCRAFT + MOMENT ABOUT CG

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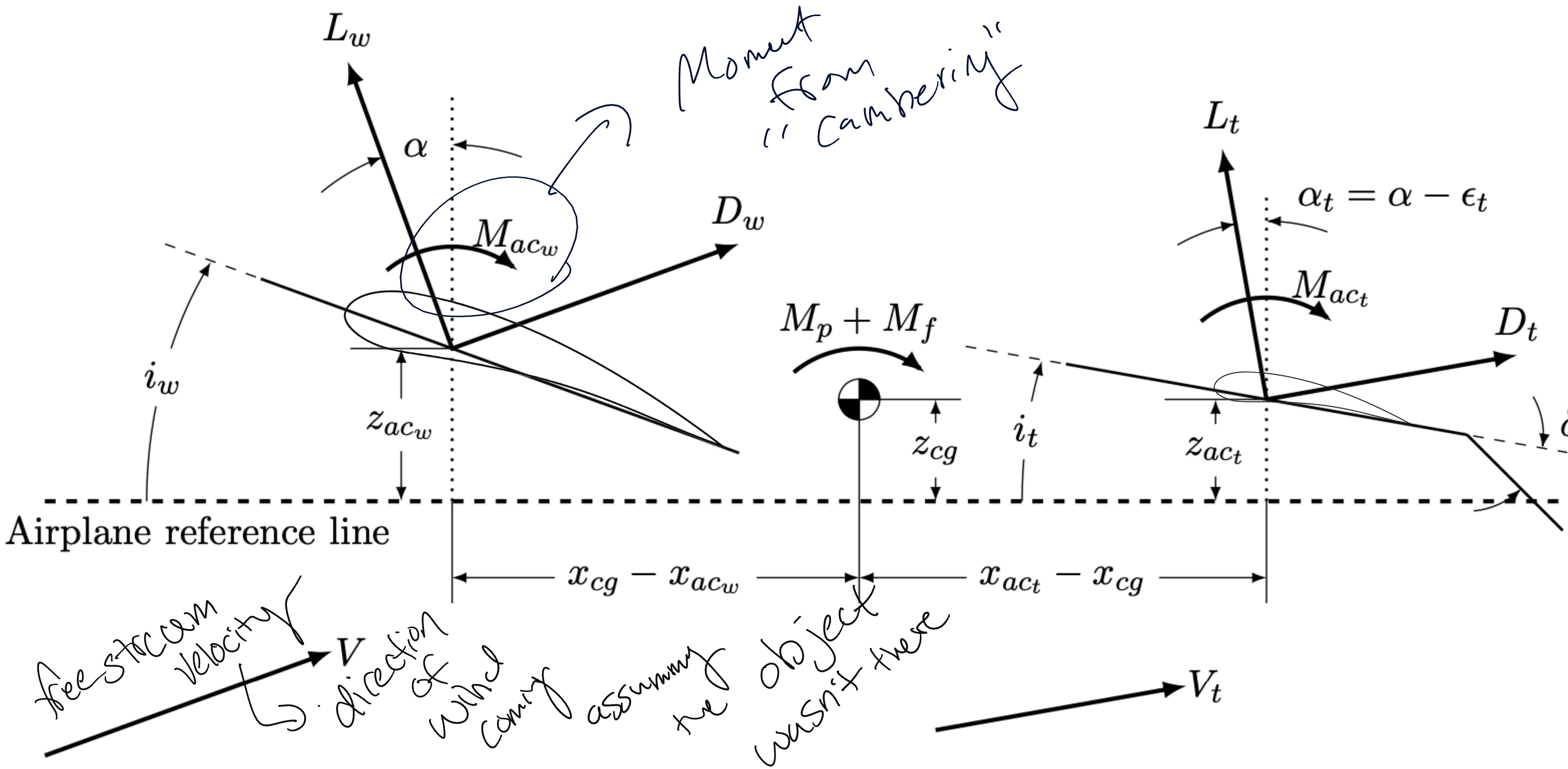
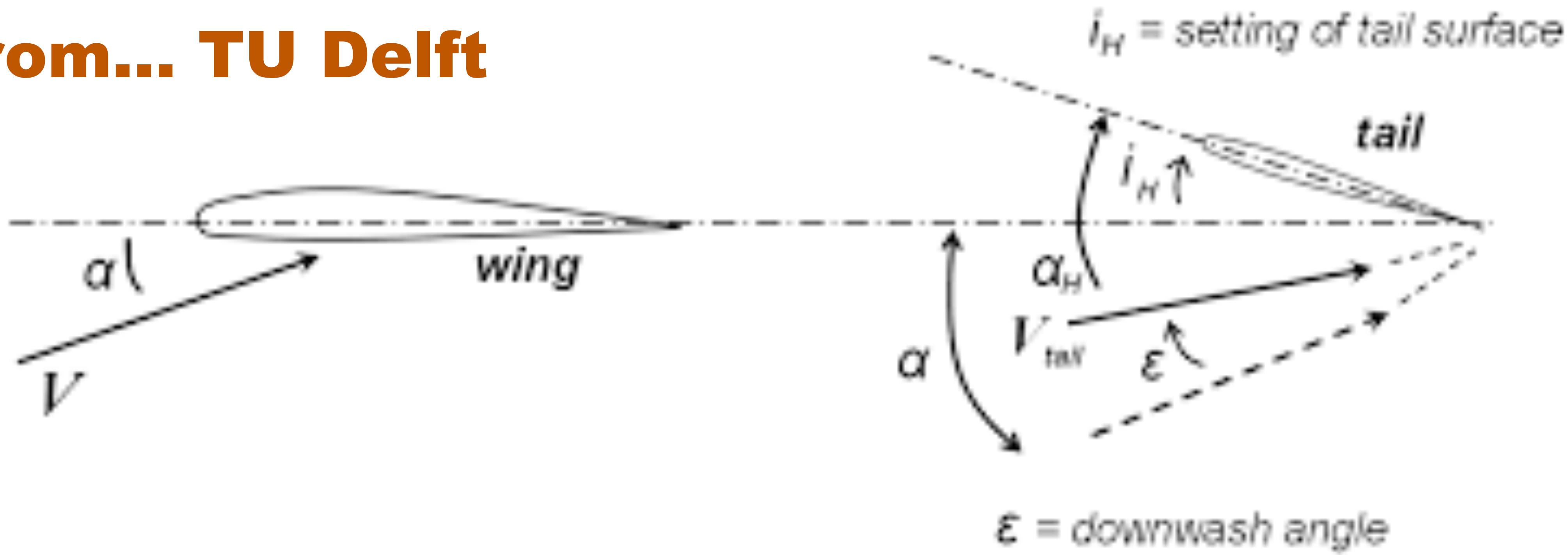


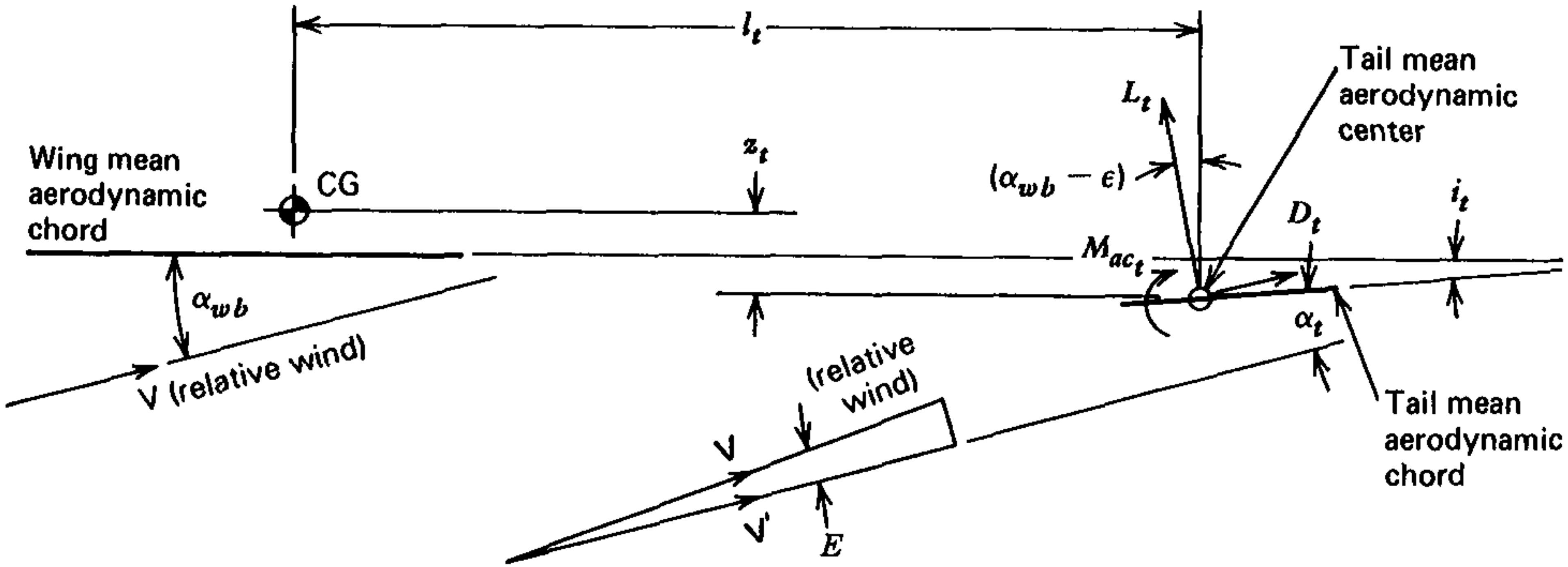
NOTE: (a) the angles of attack of the wing and horizontal tail are defined relative to the airplane reference line.
(b) the incidence angles of the wing is the angle between the chord of the wing and the airplane reference line.
(b) the incidence angles of the horizontal tail is the angle between the chord of the horizontal tail and the airplane reference line.

From... TU Delft



NOTE: **(a) the chord of the wing and the aircraft reference line are colinear, thus the angle of attack of the wing is defined relative to the chord of the wing.**
(b) the wing has no incidence angle.
(c) the angle of attack of the horizontal tail is defined relative to the aircraft reference line (or extended wing chord).
(d) the incidence angle of the tail is the angle between the chord of the horizontal tail and the aircraft reference line.

From... Etkin and Reid



NOTE: (a) the chord of the wing and the aircraft reference line are colinear, thus the angle of attack of the wing is defined relative to the chord of the wing.
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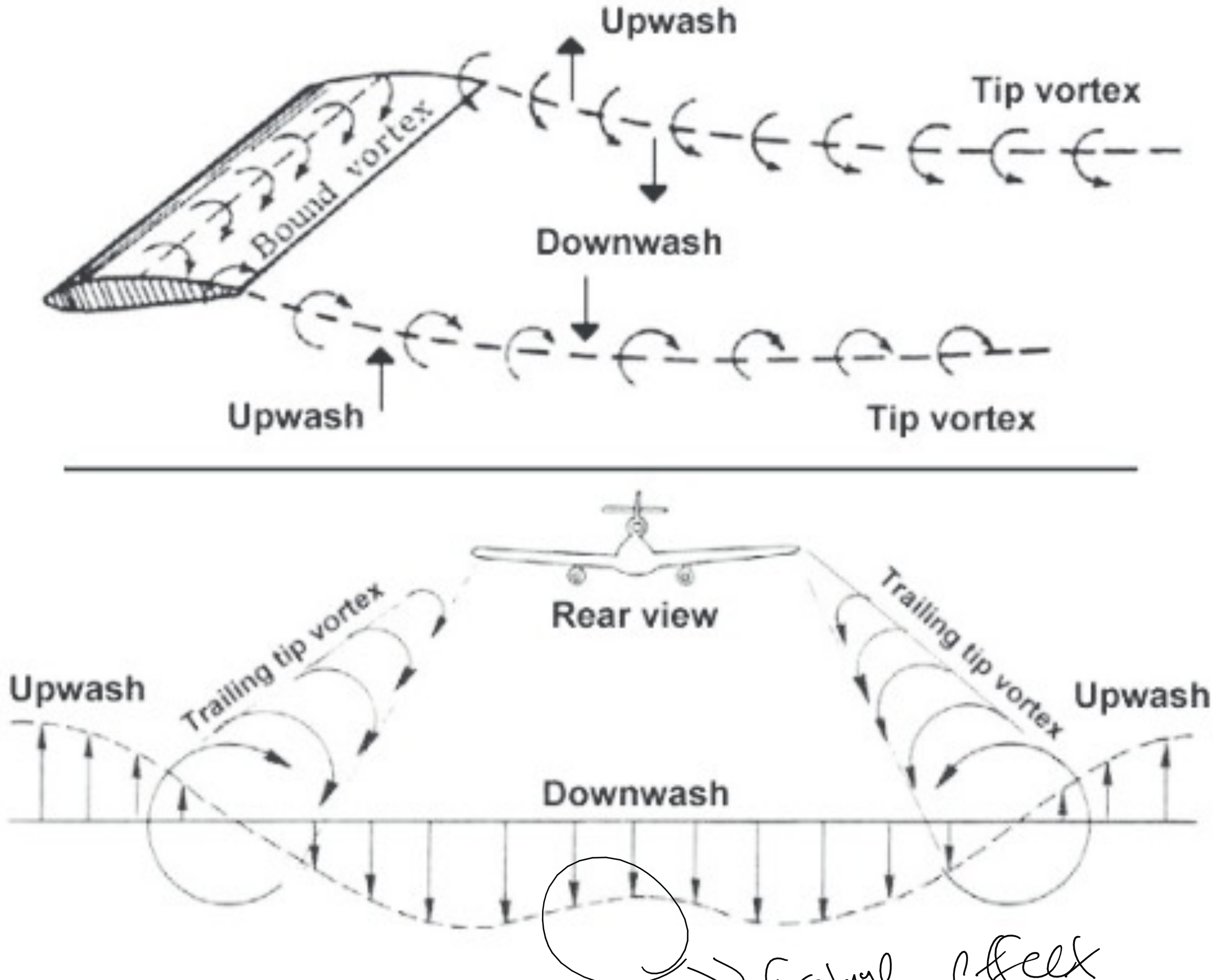


fuselage effect

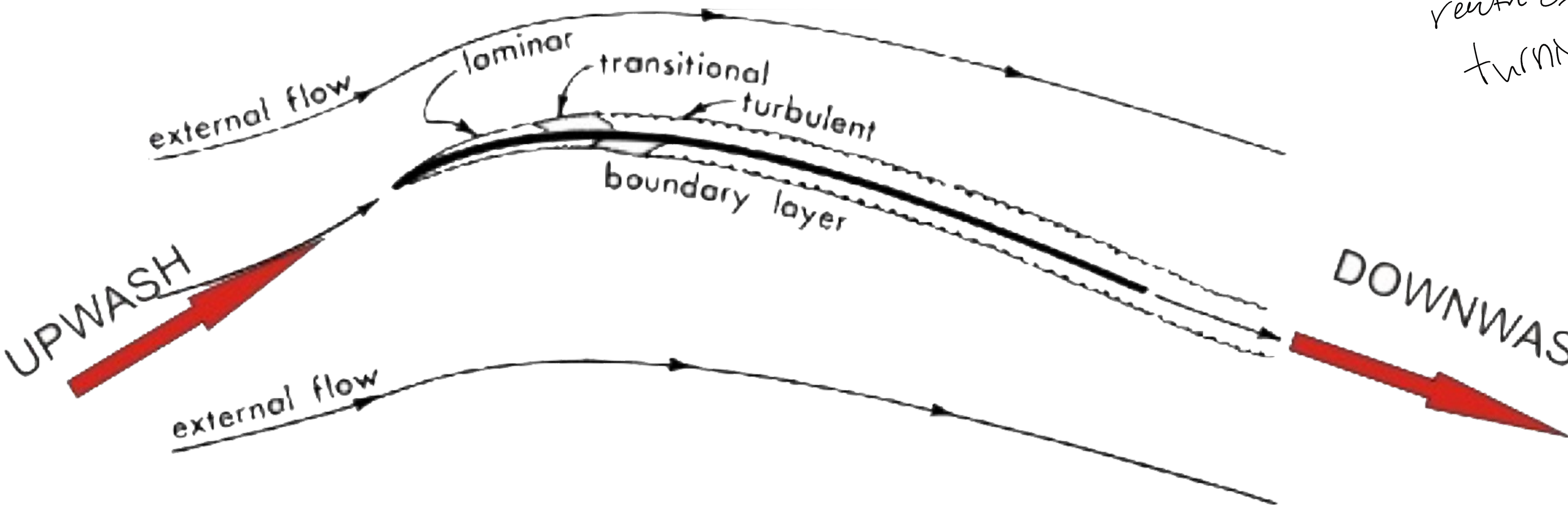
increasing angle of attack brings up the separation point, more down wash.

Downwash

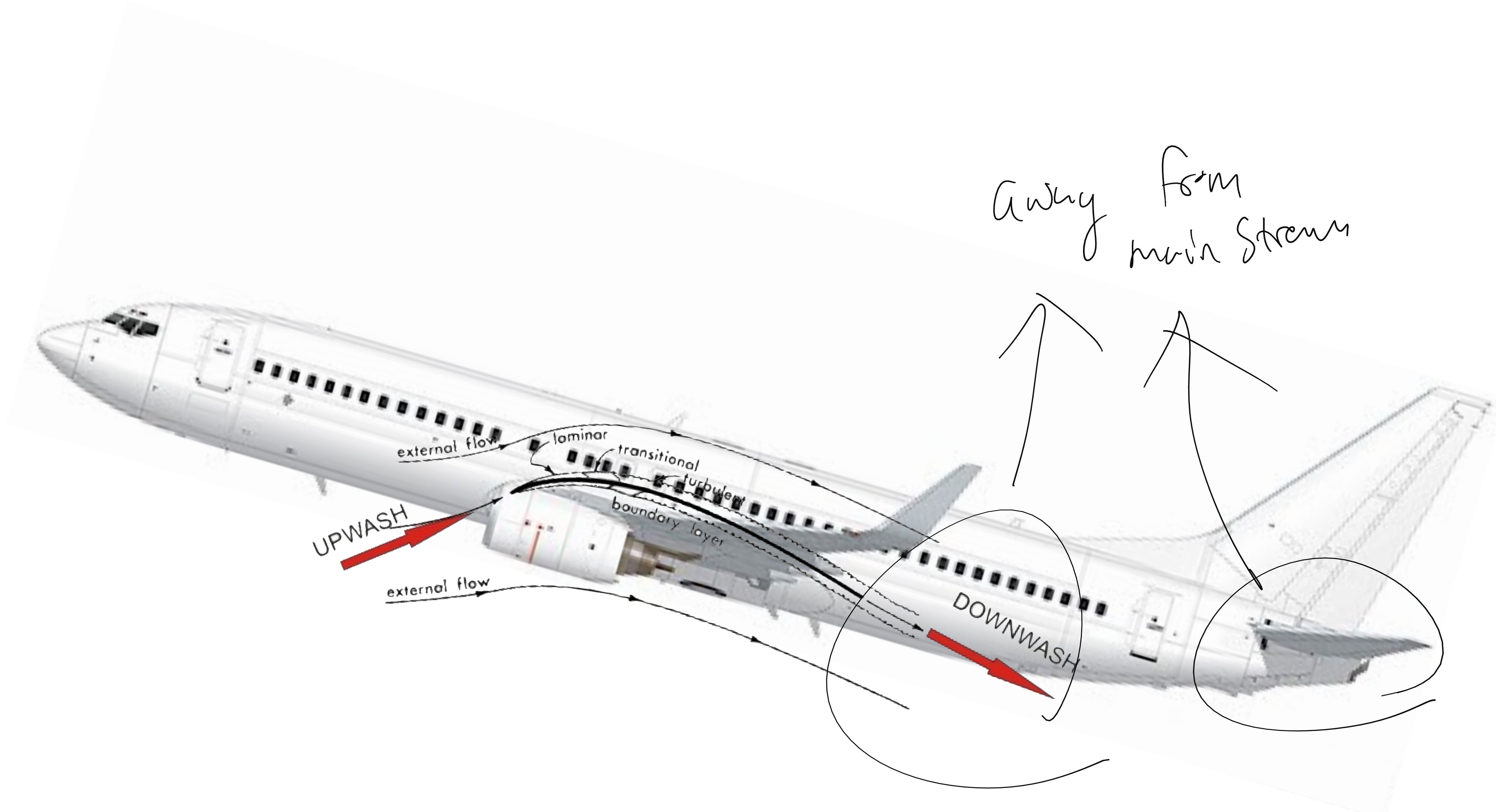
linear function

$$\epsilon_t = \epsilon_{0_t} + \epsilon_{\alpha_t}\alpha$$

Lift ↑
induced ↑
Downwash
result of
turning

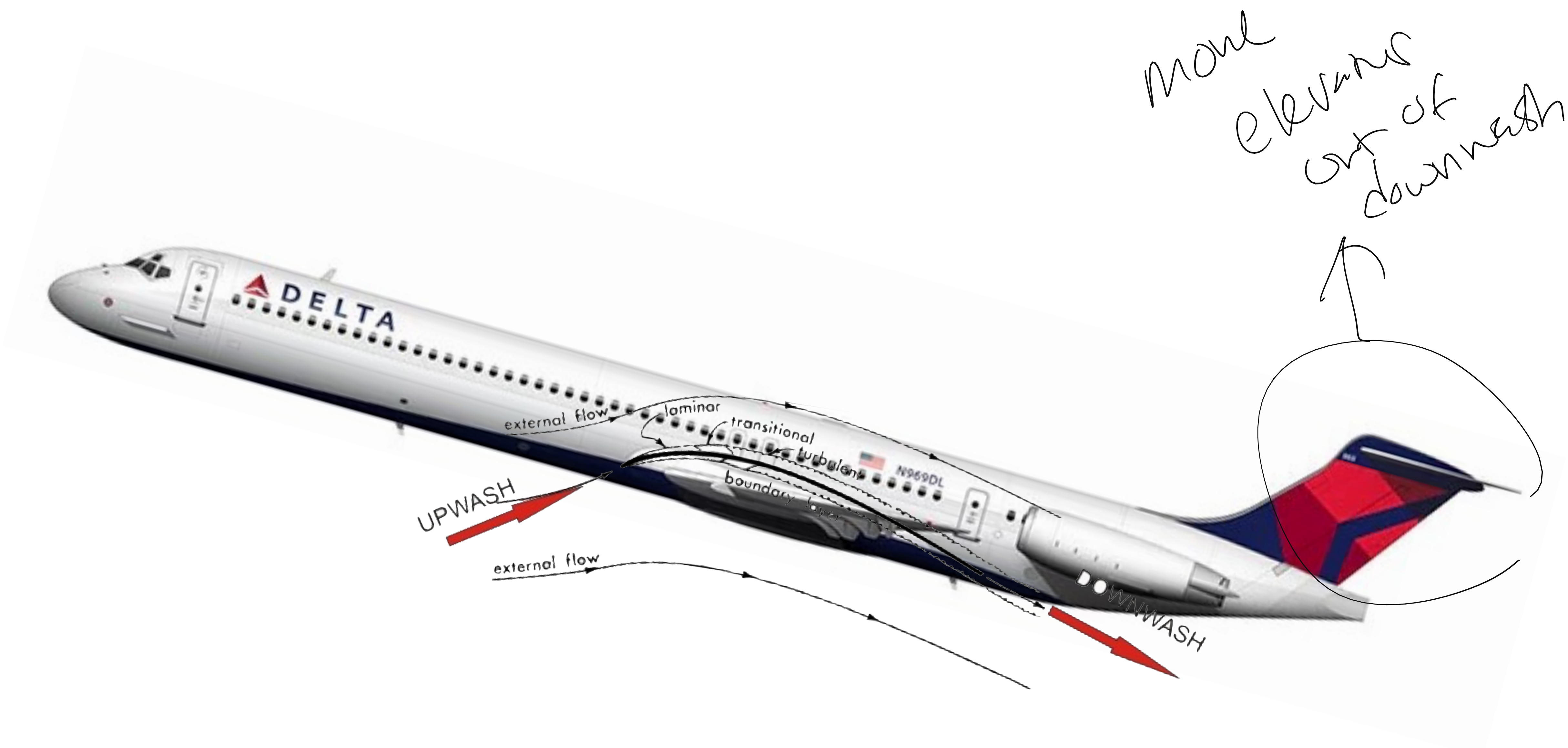


The downwash ϵ_0 at $\alpha_{wb} = 0$ results from the induced velocity field of the body and from wing twist; the latter produces a vortex wake and downwash field even at zero total lift. The constant derivative $\partial\epsilon/\partial\alpha$ occurs because the main contribution to the downwash at the tail comes from the wing trailing vortex wake, the strength of which is, in the linear case, proportional to C_L.



Away from
main Stream
external flow
laminar
transitional
turbulent
boundary layer
UPWASH
external flow
DOWNWASH

Put center
of
pressure behind
to counteract
engine weight



DELTA
N969DL
external flow
laminar
transitional
turbulent
boundary layer
UPWASH
external flow
DOWNWASH
move elevator out of downwash

constant

partial derivative wrt. α

wrt i_t

Total Lift Summary

wrt δ_e

doesn't change

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_t}} i_t + C_{L_{\delta_e}}\delta_e$$

intercept at $\alpha = 0$

where

$$C_{L_0} = C_{L_{0_w}} + C_{L_{\alpha_w}} i_w + \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}\left(C_{L_{0_t}} - C_{L_{\alpha_t}}\epsilon_{0_t}\right)$$

Gradients

$$C_{L_\alpha} = C_{L_{\alpha_w}} + \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S} C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t})$$

Not constant

$$C_{L_{i_t}} = \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S} C_{L_{\alpha_t}}$$

$$C_{L_{\delta_e}} = \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S} C_{L_{\delta_{e_t}}}$$

Pitch Moment Summary

moment arm

baseline lift

$$C_{M_r} = C_{M_{0_r}} + C_{M_{\alpha_r}}\alpha + C_{M_{i_{t_r}}} i_t + C_{M_{\delta_{e_r}}}\delta_e$$

about arbitrary ref point

$$C_{M_{0_r}} = C_{M_{ac_w}} + \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}\frac{\bar{c}_t}{\bar{c}}C_{M_{ac_t}} + C_{M_{0_p}} + C_{M_{0_f}} + (\bar{x}_r - \bar{x}_{ac_w})(C_{L_{0_w}} + C_{L_{\alpha_w}} i_w)$$
$$+ \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_r - \bar{x}_{ac_t})(C_{L_{0_t}} - C_{L_{\alpha_t}}\epsilon_{0_t})$$

baseline lift tail

$$C_{M_{\alpha_r}} = C_{M_{\alpha_p}} + C_{M_{\alpha_f}} + (\bar{x}_r - \bar{x}_{ac_w})C_{L_{\alpha_w}} + \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_r - \bar{x}_{ac_t})C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t})$$

propulsion

fuselage

change in wing lift

Δ in tail Lift

$$C_{M_{i_{t_r}}} = \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_r - \bar{x}_{ac_t})C_{L_{\alpha_t}}$$

$$C_{M_{\delta_{e_r}}} = \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_r - \bar{x}_{ac_t})C_{L_{\delta_{e_t}}}$$

Aerodynamic Center of Aircraft

- What is so unique about the aerodynamic center?

@ a.c. CM is 0.

$$C_{M_{\alpha_{ac}}} = 0$$

What if CG was on A.C.

Moment won't change

A.C. = the Neutral point.

but No control authority

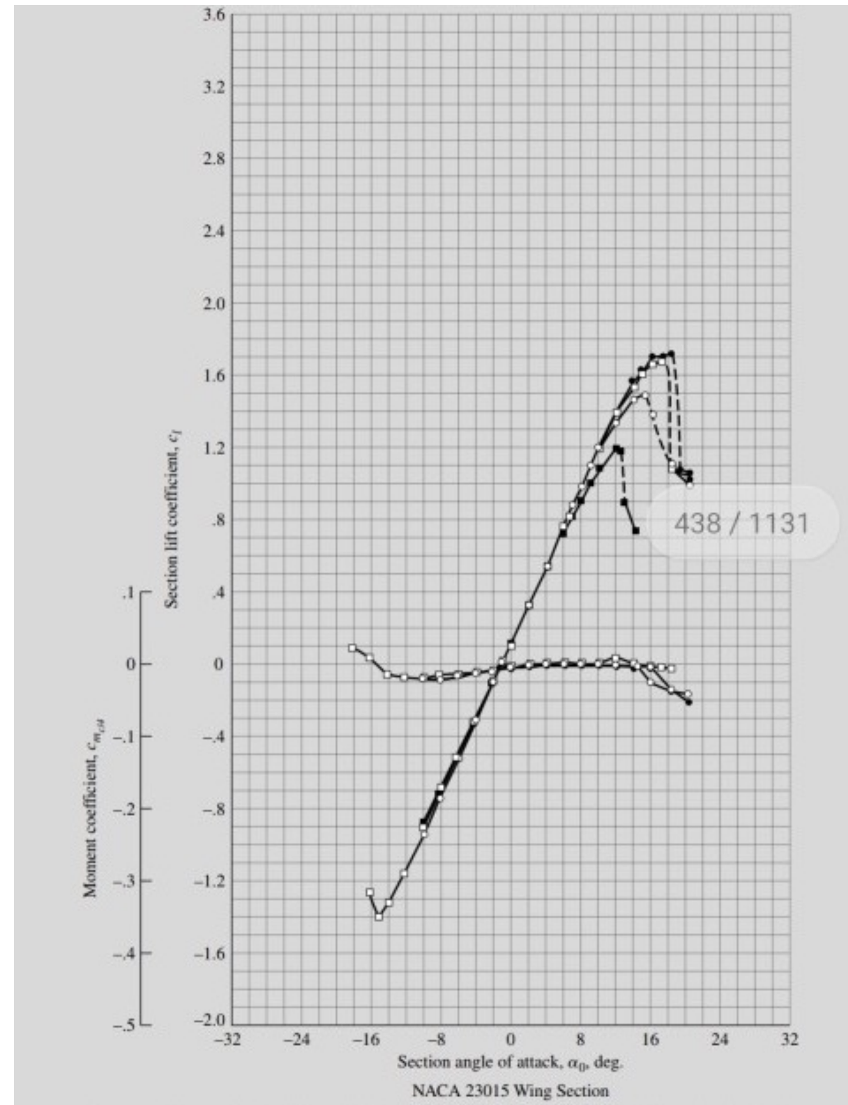


Figure 5.2a Lift coefficient and moment coefficient about the quarter-chord for the NACA 23015 airfoil.

Aerodynamic Center of Aircraft

- How do we determine the aerodynamic center?

a.c where gradients of moments cancel out and is constant.

$$\bar{x}_r = \bar{x}_{ac}$$

$$C_{M_{\alpha_{ac}}} = C_{M_{\alpha_p}} + C_{M_{\alpha_f}} + (\bar{x}_{ac} - \bar{x}_{ac_w})C_{L_{\alpha_w}} + \frac{q_t S_t}{qS}(\bar{x}_{ac} - \bar{x}_{ac_t})C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t}) = 0$$

$$\Rightarrow \quad \bar{x}_{ac}\left[C_{L_{\alpha_W}} + \frac{q_t S_t}{qS}C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t})\right] = \bar{x}_{ac_w}C_{L_{\alpha_w}} + \frac{q_t S_t}{qS}\bar{x}_{ac_t}C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t}) - C_{M_{\alpha_p}} - C_{M_{\alpha_f}}$$

$$\Rightarrow \quad \bar{x}_{ac}C_{L_\alpha} = \bar{x}_{ac_w}C_{L_{\alpha_w}} + \frac{q_t S_t}{qS}\bar{x}_{ac_t}C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t}) - C_{M_{\alpha_p}} - C_{M_{\alpha_f}}$$

$$\Rightarrow \quad \bar{x}_{ac} = \frac{1}{C_{L_\alpha}}\left[\underbrace{\bar{x}_{ac_w}}_{\text{distance from nose to ac w}}C_{L_{\alpha_w}} + \frac{q_t S_t}{qS}\underbrace{\bar{x}_{ac_t}}_{\text{Nose to ac t}}C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t}) - C_{M_{\alpha_p}} - C_{M_{\alpha_f}}\right]$$

a.c. point on the airplane where gradient carefully balance out.

Pitch Moment about the CG

$C_M = C_L l_{ht}$

$$C_M = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{i_t}} i_t + C_{M_{\delta_e}}\delta_e$$

$$\begin{aligned}
C_{M_0} &= C_{M_{ac_w}} + \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}\frac{\bar{c}_t}{\bar{c}} C_{M_{ac_t}} + C_{M_{0_p}} + C_{M_{0_f}} + (\bar{x}_{cg} - \bar{x}_{ac_w})\left(C_{L_{0_w}} + C_{L_{\alpha_w}} i_w\right) \\
&\quad + \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_{cg} - \bar{x}_{ac_t})\left(C_{L_{0_t}} - C_{L_{\alpha_t}} \epsilon_{0_t}\right) \\
C_{M_\alpha} &= C_{M_{\alpha_p}} + C_{M_{\alpha_f}} + (\bar{x}_{cg} - \bar{x}_{ac_w}) C_{L_{\alpha_w}} + \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_{cg} - \bar{x}_{ac_t}) C_{L_{\alpha_t}} (1 - \epsilon_{\alpha_t}) \\
C_{M_{i_t}} &= \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_{cg} - \bar{x}_{ac_t}) C_{L_{\alpha_t}} \\
C_{M_{\delta_e}} &= \frac{\bar{q}_t}{\bar{q}}\frac{S_t}{S}(\bar{x}_{cg} - \bar{x}_{ac_t}) C_{L_{\delta_{e_t}}}
\end{aligned}$$

Static Margin

Natural tendency to go back to equilibrium.
i.e. α changes, will the airplane go back to equilibrium? to its natural state

- Can we simplify the equation for C_{M_α} ?

$$C_{M_\alpha} = C_{M_{\alpha_p}} + C_{M_{\alpha_f}} + (\bar{x}_{cg} - \bar{x}_{ac_w})C_{L_{\alpha_w}} + \frac{q_t S_t}{qS}(\bar{x}_{cg} - \bar{x}_{ac_t})C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t})$$

$$= \bar{x}_{cg}\left[C_{L_{\alpha_W}} + \frac{q_t S_t}{qS}C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t})\right] - \left[\bar{x}_{ac_w}C_{L_{\alpha_W}} + \frac{q_t S_t}{qS}\bar{x}_{ac_t}C_{L_{\alpha_t}}(1 - \epsilon_{\alpha_t}) - C_{M_{\alpha_p}} - C_{M_{\alpha_f}}\right]$$

$$= \bar{x}_{cg}C_{L_\alpha} - \bar{x}_{ac}C_{L_\alpha}$$

$$= -(\bar{x}_{ac} - \bar{x}_{cg})C_{L_\alpha}$$

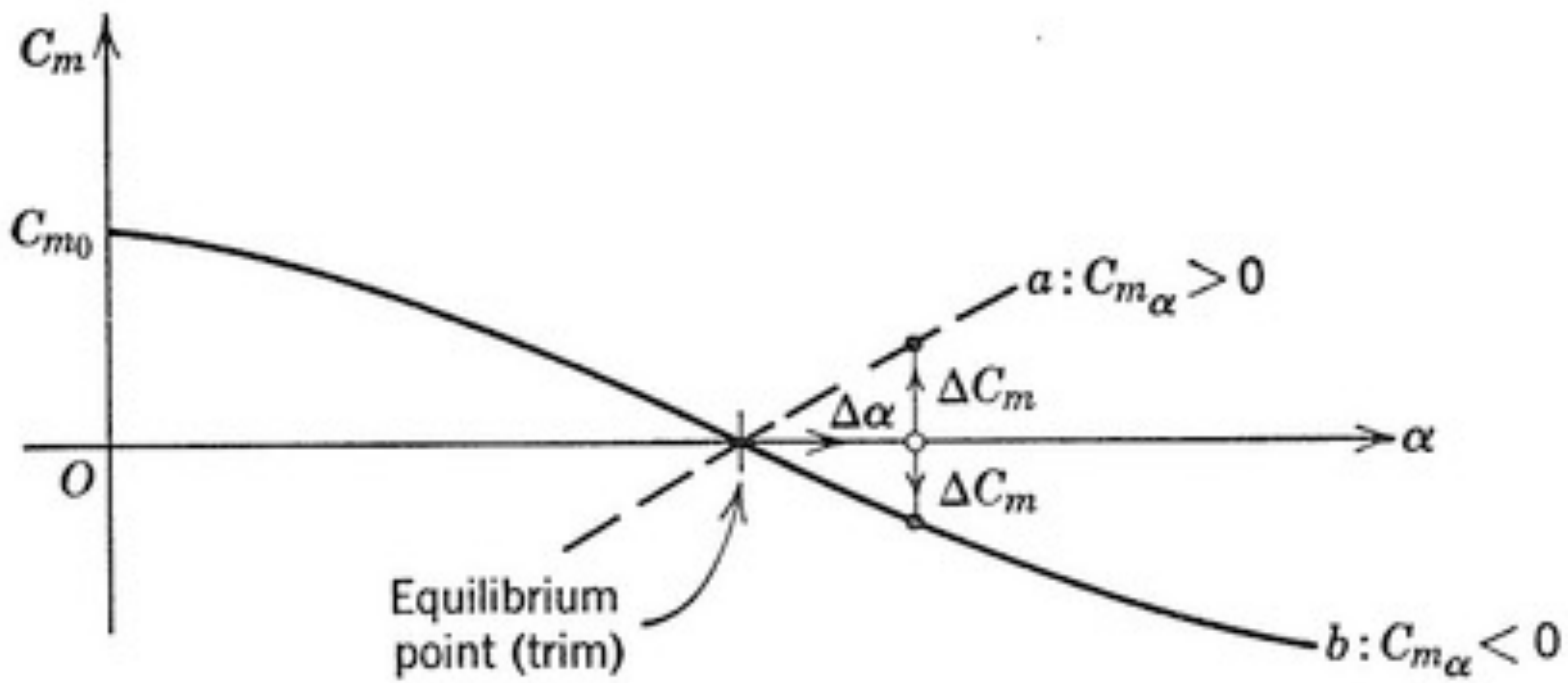
need to be positive a.c further back than c.g.

$= -SM \cdot C_{L_\alpha}$ where SM is the "Static Margin"

C_M to be 0.

contour acts $C_{L\alpha}$

Why is the Static Margin important?



Why is the Static Margin important?

- Which of the following aircraft is stable?

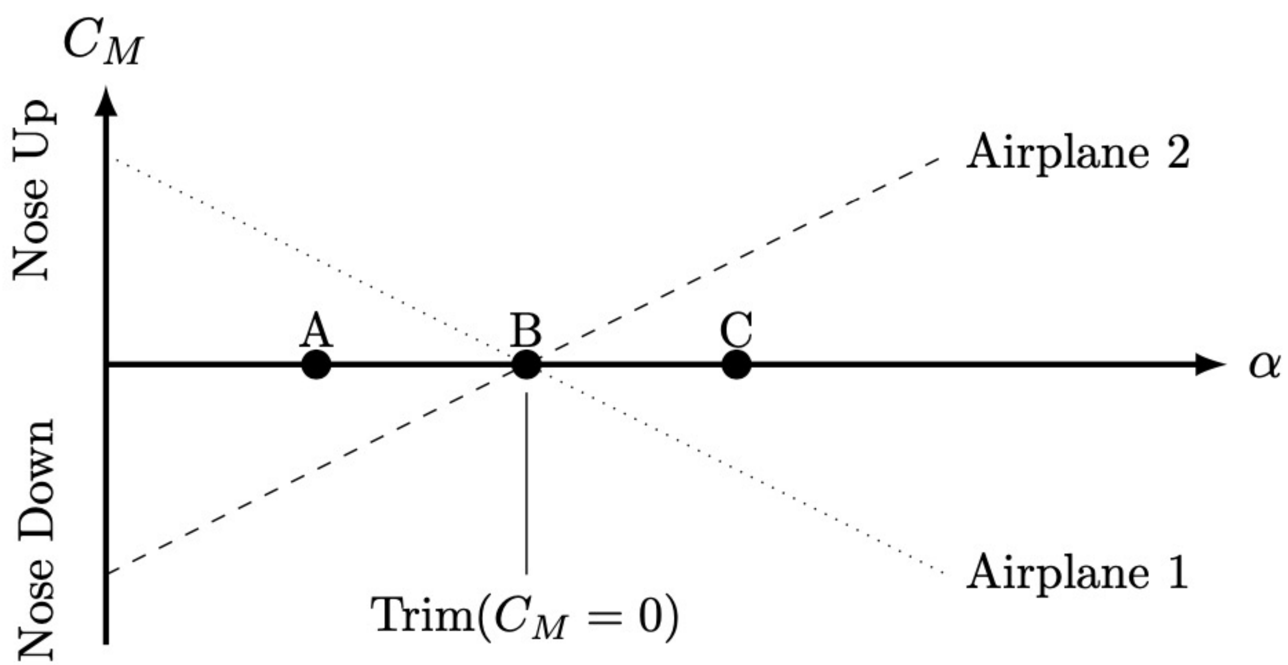


Figure 3.4: Pitch moment coefficient vs. angle of attack

For a trimmed aircraft…

1. If C_{M_α} (the slope of the line) is positive, then an increase in pitch (and thus in the angle of attack) results in a nose up pitching moment.
2. If C_{M_α} is negative, then an increase in pitch (and thus in the angle of attack) results in a nose down pitching moment.
3. Thus, for the aircraft have longitudinal static stability we need C_{M_α} to be negative.
4. Because C_{L_α} is positive…
5. The SM must also be positive, and…
6. The center of gravity must be ahead, i.e., closer to the nose than the aerodynamic center of the aircraft.

the static margin should be positive to achieve longitudinal static stability:

this also means that the center of gravity should be ahead of the aerodynamic center to achieve longitudinal static stability:

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TRIM

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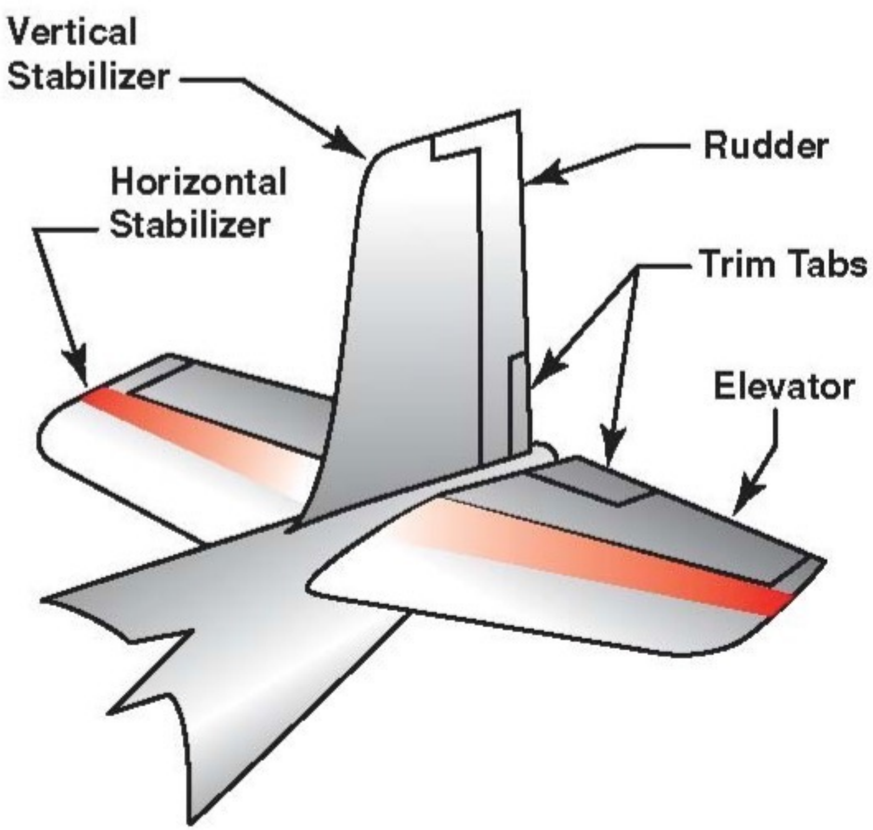
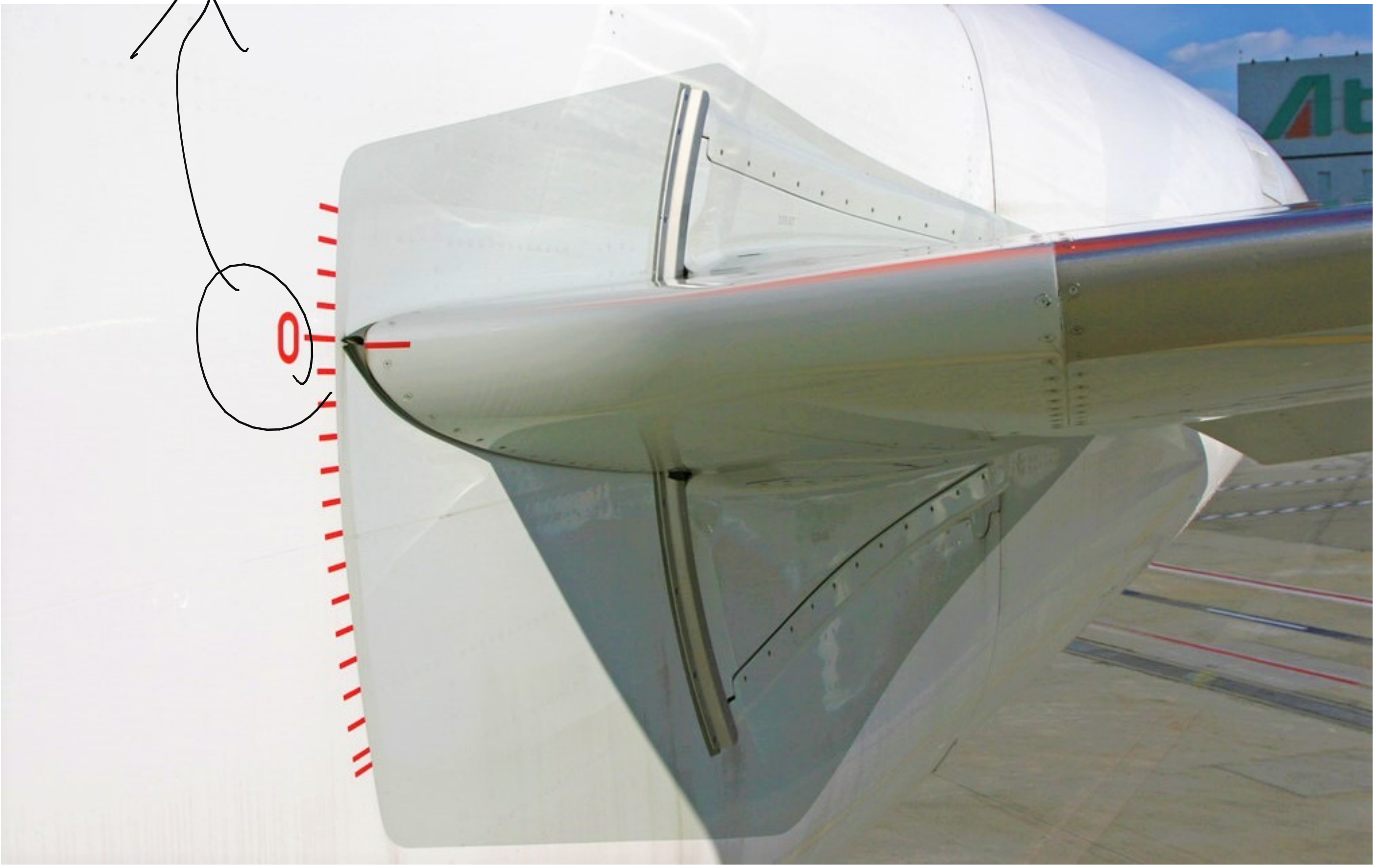
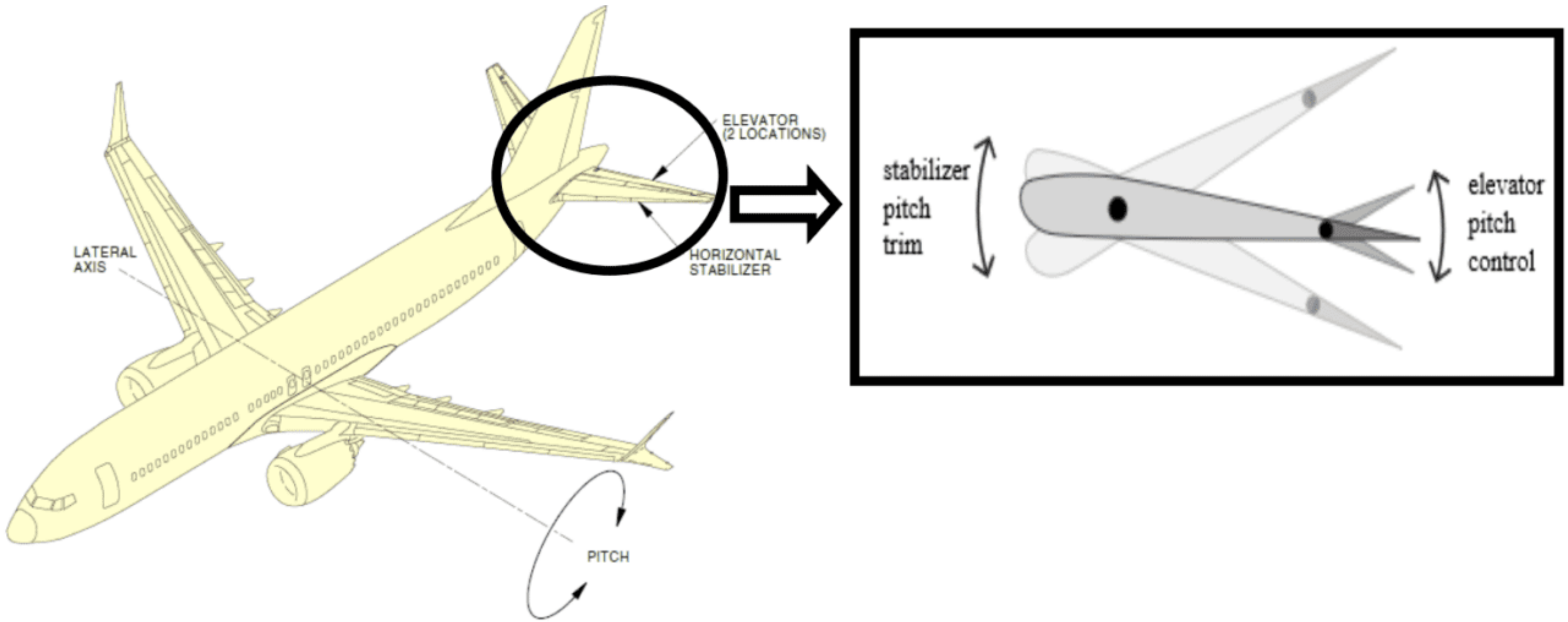
Fixed v. Variable-Position Horizontal Stabilizer

$i_t = 0$





Variable-Position Horizontal Stabilizer (1)



Variable-Position Horizontal Stabilizer (2)

1. Set desired speed, V, and fix $\bar{q} = \rho V^2/2$. Assume that $\delta_e = 0$ at this speed.

2. Determine α_{trim} and $i_{t_{\text{trim}}}$ from

$$\frac{W\cos\gamma}{\bar{q}S} = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_t}} i_t$$
$$0 = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{i_t}} i_t$$

which is equivalent to solving the linear system of equations

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{i_t}} \\ C_{M_\alpha} & C_{M_{i_t}} \end{bmatrix} \begin{bmatrix} \alpha_{\text{trim}} \\ i_{t_{\text{trim}}} \end{bmatrix} = \begin{bmatrix} \frac{W\cos\gamma}{\bar{q}S} - C_{L_0} \\ -C_{M_0} \end{bmatrix}$$

Variable-Position Horizontal Stabilizer (3)

and has the solution

$$\alpha_{\text{trim}_{\delta_e=0}} = -\frac{C_{M_{i_t}}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_{i_t}} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

$$i_{t_{\text{trim}}} = \frac{C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_\alpha} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

3. With $i_{t_{\text{trim}}}$ determined for the specified $\bar{q}$, define C'_{L_0} and C'_{M_0} as

$$C'_{L_0} = C_{L_0} + C_{L_{i_t}} i_{t_{\text{trim}}}$$

$$C'_{M_0} = C_{M_0} + C_{M_{i_t}} i_{t_{\text{trim}}}$$

Variable-Position Horizontal Stabilizer (4)

4. Determine δ_e to trim at other $\bar{q}$'s. That is, solve

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_e}} \\ C_{M_\alpha} & C_{M_{\delta_e}} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} \frac{W\cos\gamma}{\bar{q}S} - C'_{L_0} \\ -C'_{M_0} \end{bmatrix}$$

to find α_{trim} and $\delta_{e_{\text{trim}}}$, which gives

$$\alpha_{\text{trim}} = -\frac{C_{M_{\delta_e}}\left(\frac{W\cos\gamma}{\bar{q}S} - C'_{L_0}\right) + C_{L_{\delta_e}} C'_{M_0}}{C_{M_\alpha} C_{L_{\delta_e}} - C_{M_{\delta_e}} C_{L_\alpha}}$$

$$\delta_{e_{\text{trim}}} = \frac{C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C'_{L_0}\right) + C_{L_\alpha} C'_{M_0}}{C_{M_\alpha} C_{L_{\delta_e}} - C_{M_{\delta_e}} C_{L_\alpha}}$$

how much I need to deflect

Let's do some number crunching…

leading edge

Wing		Horizontal Stabilizer	
$S =$	232.00 [ft^2]	$S =$	54.00 [ft^2]
$\bar{c} =$	7.04 [ft]	$\bar{c} =$	3.83 [ft]
$x_{ac/le} =$	4.07 [ft]	$x_{ac/le} =$	2.79 [ft]
$x_{le} =$	16.40 [ft]	$x_{le} =$	36.90 [ft]
$i =$	1.00 [deg]		
$C_{L_0} =$	-0.0443	$C_{L_0} =$	0.0000
$C_{L_\alpha} =$	5.0800	$C_{L_\alpha} =$	4.2600
$C_{M_{ac}} =$	-0.0175	$C_{M_{ac}} =$	0.0000
		$C_{L_{\delta_{e_t}}} =$	1.8000
		$\epsilon_0 =$	0.642 [deg]
		$\epsilon_\alpha =$	0.426
		$\eta =$	0.9

- The aircraft is in level flight.
- The gravitational acceleration is 32.174 [ft/s^2].
- The wing and horizontal stabilizer are trapezoidal surfaces.
- The cg of the aircraft is at $x_{cg} = 10.56$ [ft].
- The weight of the aircraft is $W = 9,500$ [lb].
- The propulsive moment coefficients are $C_{M_{0_p}} = 0.0$ and $C_{M_{\alpha_p}} = 0.0$.
- The fuselage moment coefficients are $C_{M_{0_f}} = 0.0$ and $C_{M_{\alpha_f}} = 0.0$.
- The atmospheric density is 0.002378 [slugs/ft^3].
- The horizontal tail incidence can only be set between -0.5 and -7 [deg].

coincident with cg

half way on fuselage

$$\eta = \frac{q_t}{q}$$

Number Crunching (1)

How to determine CG?

1. Set desired speed, V, and fix $\bar{q} = \rho V^2/2$. Assume that $\delta_e = 0$ at this speed.

Let's assume aircraft is flying at … $500\ knots = 843.9\ ft/s$

given $\rho = 0.002378\ slugs/ft^3$ then $q = 846.8\ lb/ft^2$

Number Crunching (2)

2. Determine α_{trim} and $i_{t_{\text{trim}}}$ from

$$\alpha_{\text{trim}_{\delta_e=0}} = -\frac{C_{M_{i_t}}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_{i_t}} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

$$i_{t_{\text{trim}}} = \frac{C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C_{L_0}\right) + C_{L_\alpha} C_{M_0}}{C_{M_\alpha} C_{L_{i_t}} - C_{M_{i_t}} C_{L_\alpha}}$$

First we compute the things we don′t have … $C_{L_{i_t}}$, $C_{M_{i_t}}$, C_{M_0}

… then we "*plug and chug*"

Number Crunching (3)

3. With $i_{t_{\text{trim}}}$ determined for the specified $\bar{q}$, define C'_{L_0} and C'_{M_0} as

$$C'_{L_0} = C_{L_0} + C_{L_{i_t}} i_{t_{\text{trim}}}$$
$$C'_{M_0} = C_{M_0} + C_{M_{i_t}} i_{t_{\text{trim}}}$$

Number Crunching (4)

4. Determine δ_e to trim at other $\bar{q}$'s. That is, solve

$$\alpha_{\text{trim}} = -\frac{C_{M_{\delta_e}}\left(\frac{W\cos\gamma}{\bar{q}S} - C'_{L_0}\right) + C_{L_{\delta_e}}C'_{M_0}}{C_{M_\alpha}C_{L_{\delta_e}} - C_{M_{\delta_e}}C_{L_\alpha}}$$

$$\delta_{e_{\text{trim}}} = \frac{C_{M_\alpha}\left(\frac{W\cos\gamma}{\bar{q}S} - C'_{L_0}\right) + C_{L_\alpha}C'_{M_0}}{C_{M_\alpha}C_{L_{\delta_e}} - C_{M_{\delta_e}}C_{L_\alpha}}$$

Why is this all important?

- Air Midwest Flight 5481
  - https://www.youtube.com/watch?v=MMsbpLjfWlo
  - https://www.youtube.com/watch?v=CHj9Lmjo2Ng

- Crash at Bagram Air Base
  - https://www.youtube.com/watch?v=5fpxm0D46iQ
  - https://www.youtube.com/watch?v=wXJ_MfAnjgQ

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UPSIDE DOWN FLYING WING

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Moment of Wing Alone

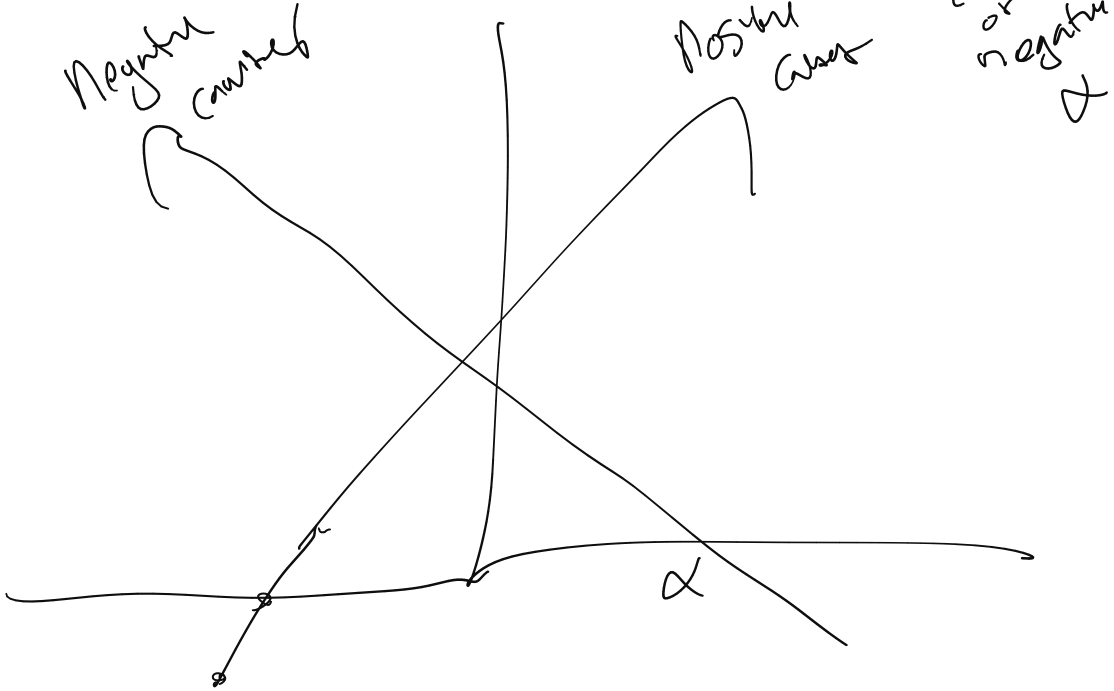
when it's upside down

Positive camber
C_{m_0} negative

when it is pitched down or negative α

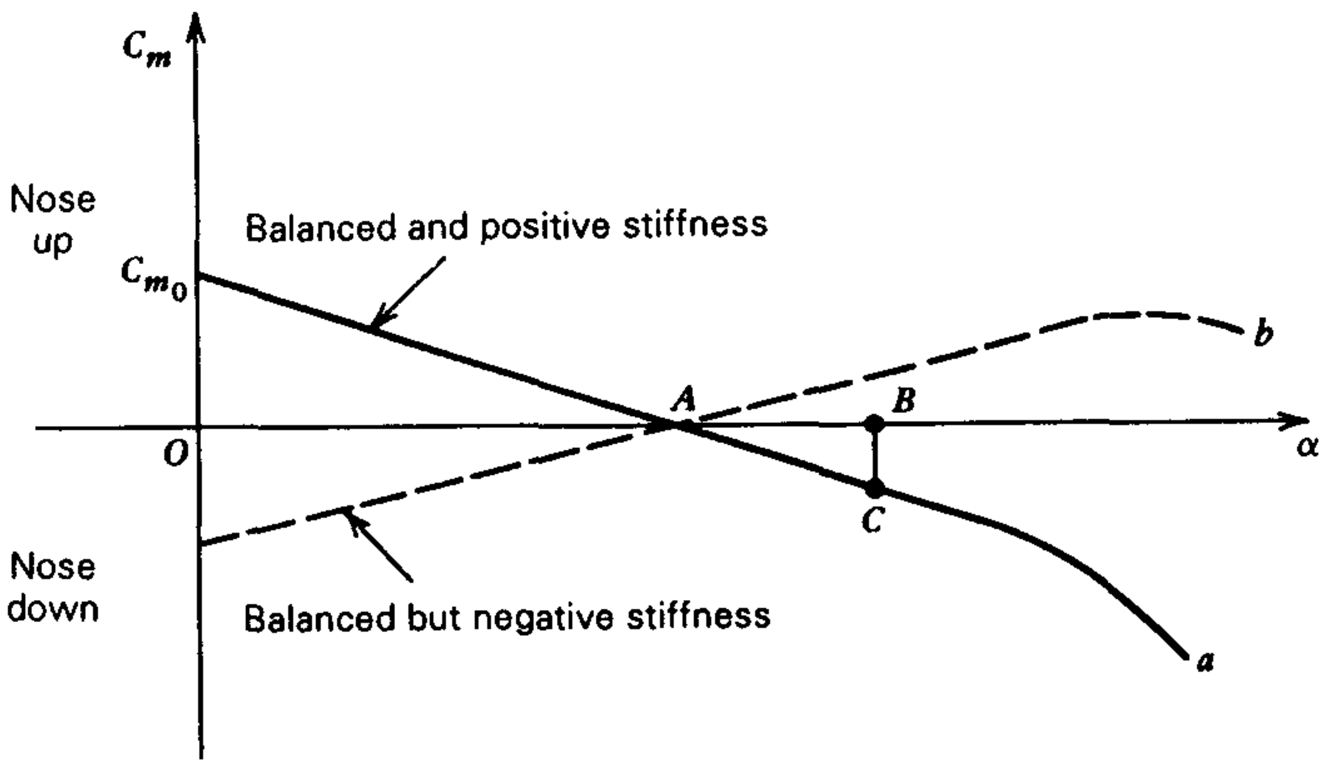
Zero camber
$C_{m_0} = 0$

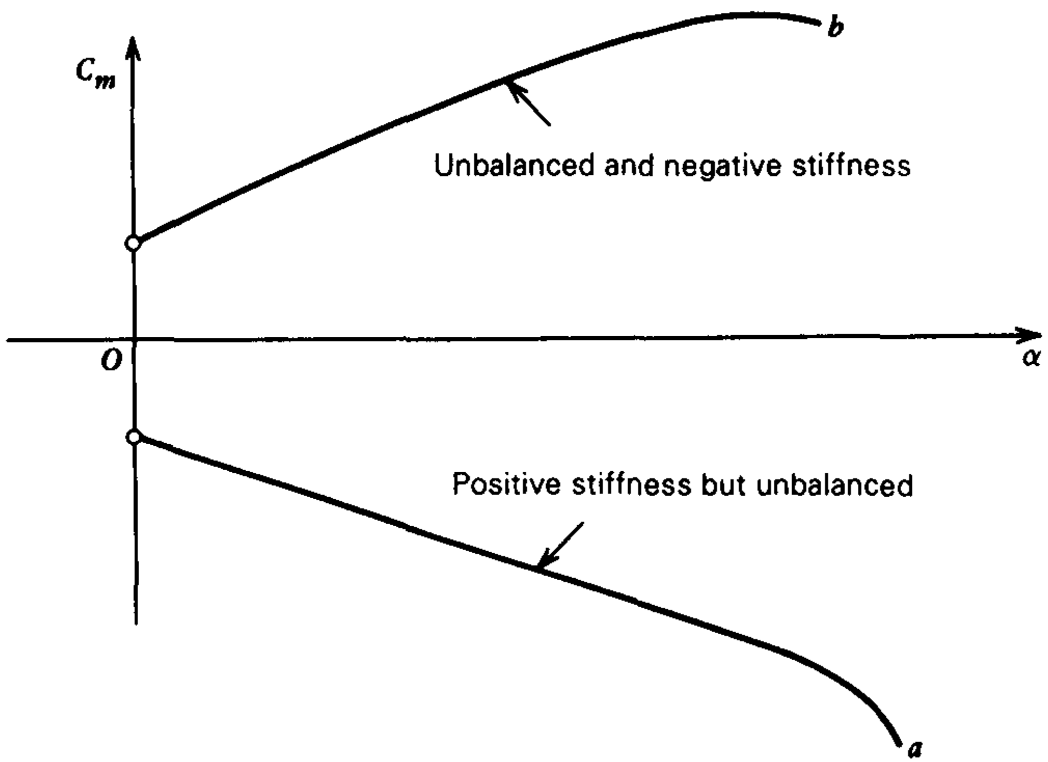
Negative camber
C_{m_0} positive



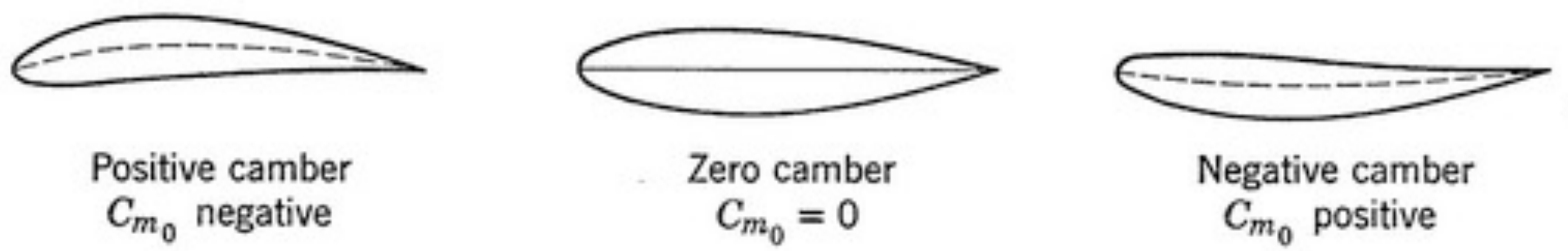
C_{m_α} = negative

Balance and Stiffness





Moment of Wing Alone

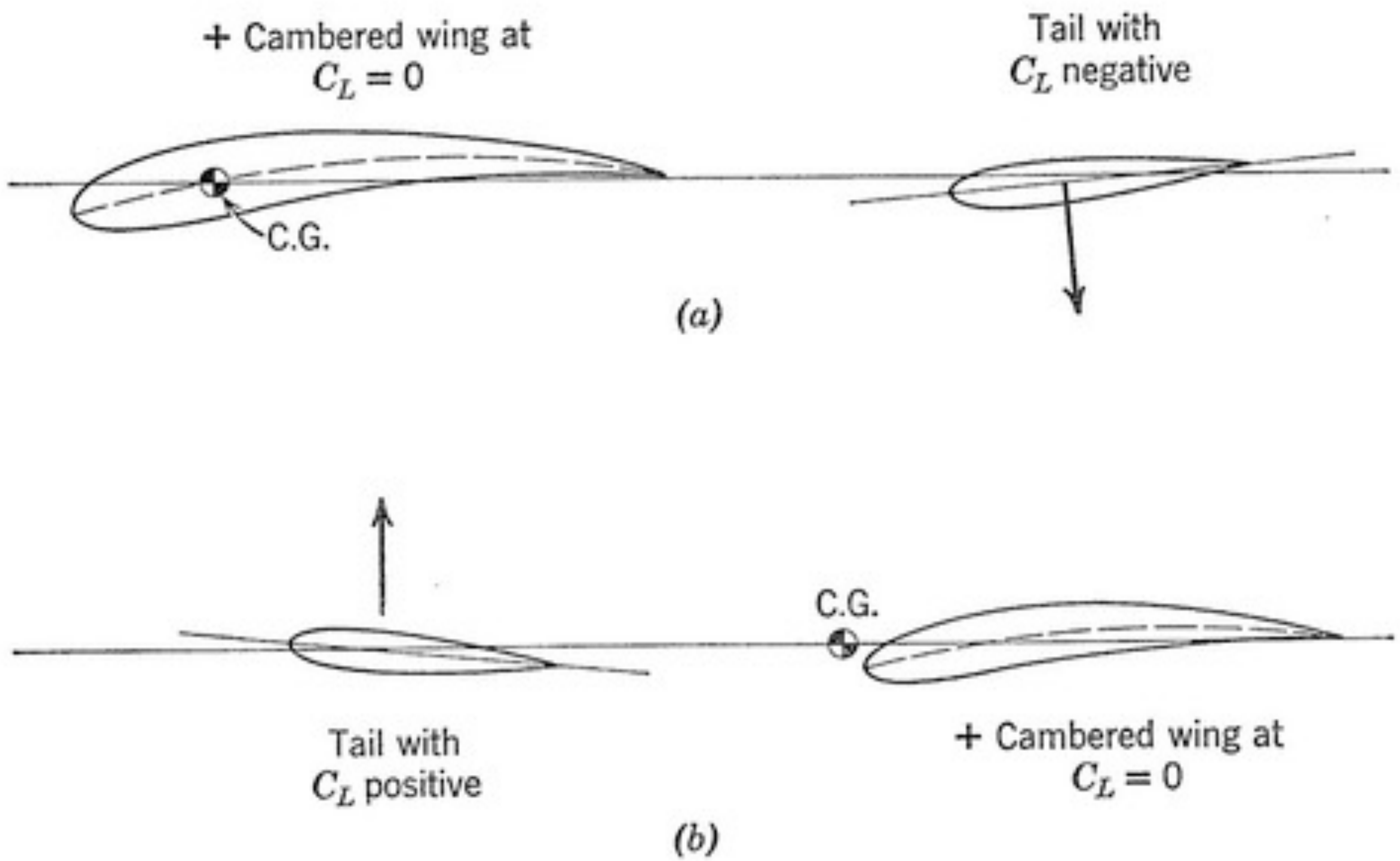


Negative camber—flight possible at $\alpha > 0$; i.e. $C_L > 0$.

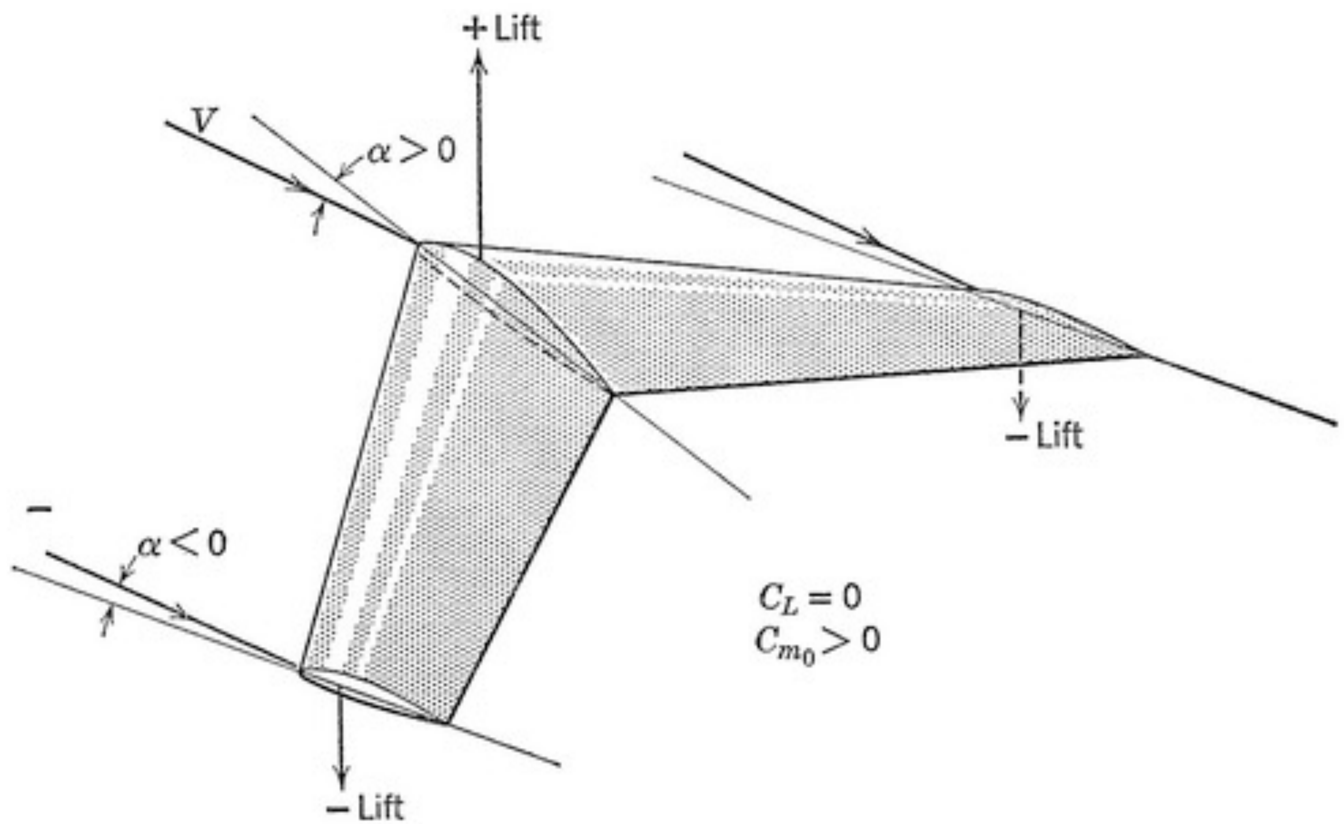
Zero camber—flight possible only at $\alpha = 0$, or $C_L = 0$.

Positive camber—flight not possible at any positive α or C_L.

Positive Pitch Stiffness - Tails + Canards



Positive Pitch Stiffness – Sweep + Twisted Tips



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