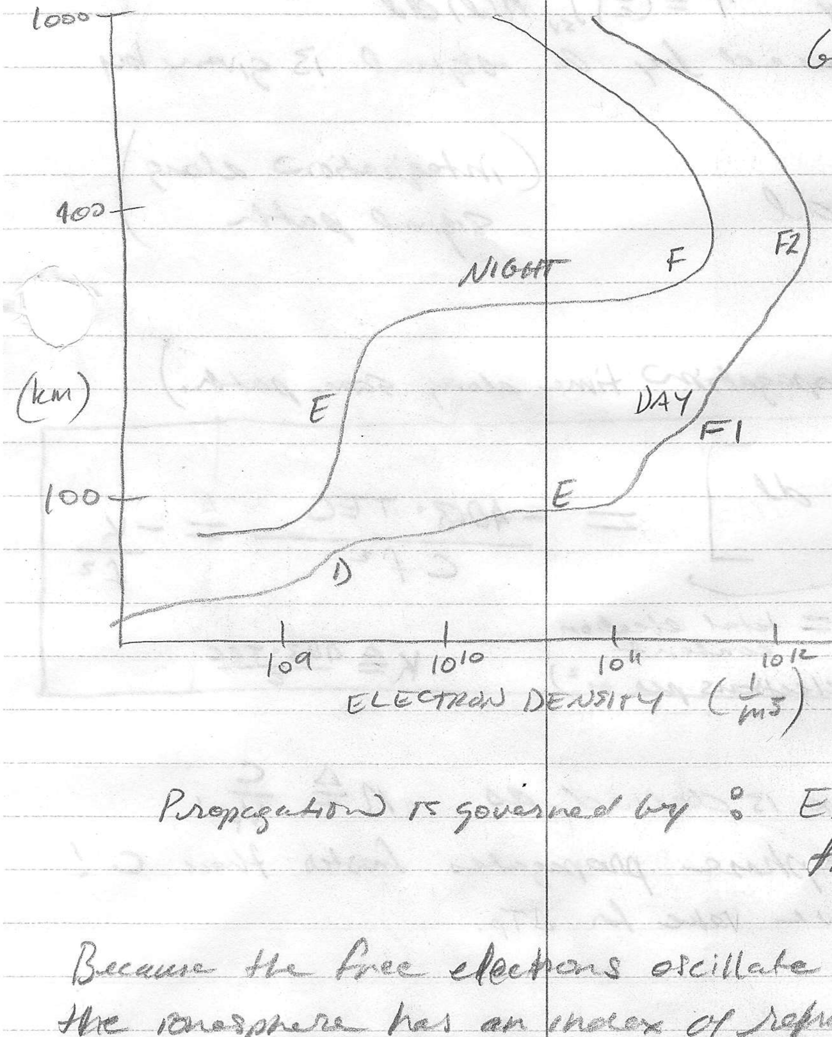
5.3 in M&E

Lecture 7: Propagation Effects

The neutral and ionized media lying between a GNSS SV and RX have a strong effect on the signals. We're equipped to study this effect now that we understand the signal structure.

Ionosphere

Ionization of upper atmosphere caused by ultraviolet solar radiation.



GPS SVs orbit at ~20,000 km alt.

thickest layer at ~350 km.

Density also varies with:
- time of year
- geomagnetic latitude
- sunspot cycle phase

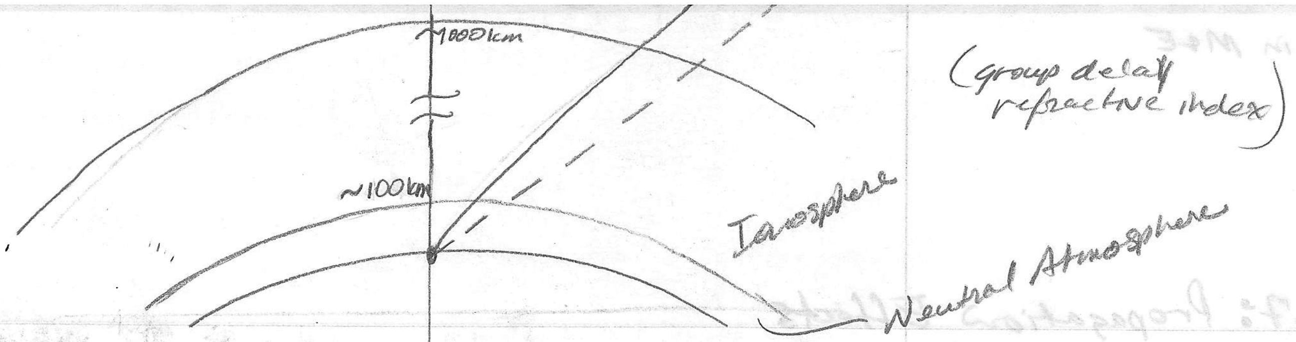
Main Ionospheric Effects:
1. Range errors (group delay) 2 – 30 m at zenith
2. carrier phase advance
3. Doppler shift, (range rate error)
4. Distortion of waveform / scintillation

Propagation is governed by: EM wave theory + magnetized plasma flow + collisions.

Because the free electrons oscillate with the incoming electric field, the ionosphere has an index of refraction different from 1 (that of free space). $n \triangleq \frac{c}{V_p}$ — phase velocity

$$n = \sqrt{1 - \underbrace{\frac{n_e e^2}{4\pi^2 f^2 \epsilon_0 m}}_{\ll 1}}$$

n_e = electron density
e = charge magnitude of electron
ϵ_0 = permittivity of free sp.
m = mass of electron



Approximate by series expansion:

$$n \cong 1 - \frac{b \cdot n_e}{f^2} \qquad b \cong 40.3 = \text{const}$$

$$\left[(1-\epsilon)^{1/2} \cong 1 - \frac{\epsilon}{2}, \quad \epsilon \ll 1 \right]$$

Because n varies with f, we call the ionosphere a dispersive medium (analogous to a prism). Note $n < 1$.

Total propagation delay is given by $\tau = \left(\frac{1}{c}\right)\int_{SV}^{RX} n(l)\, dl$

The excess phase delay (in seconds) experienced by a signal is given by

$$\Delta\tau_p = \frac{1}{c}\int_{SV}^{RX} [n(l) - 1]\, dl \qquad \text{(integration along signal path)}$$

(In excess of free-space propagation time along same path.)

$$\boxed{\Delta\tau_p = -\frac{b}{c f^2}\underbrace{\left[\int_{SV}^{RX} n_e(l)\, dl\right]}_{TEC} = \frac{-40.3 \cdot TEC}{c f^2} \triangleq \frac{-K}{f^2}}$$

TEC = total electron content (electrons per m^2)

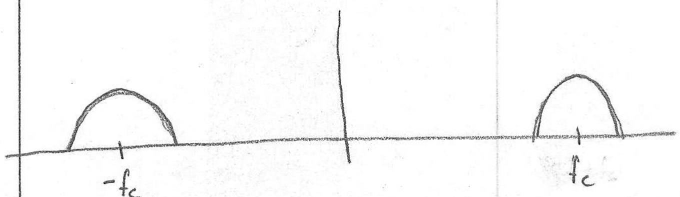
$$K \triangleq \frac{40.3 \cdot TEC}{c}$$

Recall that refractive index is defined as $n \triangleq \frac{c}{V_p}$. Then $n < 1$ suggests that the phase propagates faster than c! This is confirmed by a negative value for $\Delta\tau_p$.

Code-Carrier Divergence

Consider a simplified GNSS signal impinging on a GNSS RX:

$$r(t) = \underbrace{c(t)}_{\text{spreading code with } B_1 \cong 2\,MHz} \underbrace{\sin(2\pi f_c t)}_{\text{carrier}} \qquad \text{(ionosphere-free)}$$



$$c(t) = \int_{-\infty}^{\infty} \tilde{C}(f)\, e^{j2\pi f t}\, df \quad \text{where} \quad \tilde{C}(f) = \mathcal{F}[\,c(t)\,]$$

$$r(t) = c(t)\sin 2\pi f_c t = \int_{-\infty}^{\infty} \frac{\tilde{C}(f)}{2j}\left[e^{j2\pi(f+f_c)t} - e^{j2\pi(f-f_c)t} \right] df$$

As written, $r(t)$ is delay-free; now we include delays:

Each single-freq. term on RHS of eqn. experiences a delay. [$\tilde{C}(f)$ is only a complex weight.]

At the higher frequency $f+f_c$, we have a delay

$$\Delta T_{p,H}(f) = \frac{-K}{(f+f_c)^2}$$

[Might help to consider $\tilde{C}(f) = \begin{cases} 1 & |f| < f_g \\ 0 & \text{else} \end{cases}$ Then two components are obvious. Think in time domain. $\tilde{C}(f)$ is just a complex weight. Not time var.]

and at the lower frequency component $f - f_c$ we have

$$\Delta T_{p,L}(f) = \frac{-K}{(f-f_c)^2}$$

Thus
$$r(t) = \int_{-\infty}^{\infty} \frac{\tilde{C}(f)}{2j}\left[e^{j2\pi(f+f_c)(t-\Delta T_{p,H}(f))} - e^{j2\pi(f-f_c)(t-\Delta T_{p,L}(f))} \right] df$$

$$= c(t-\Delta T_g)\sin[2\pi f_c(t-\Delta T_p)] \qquad \text{(show in HW)}$$

where $\Delta T_g = \frac{K}{f_c^2} = \frac{40.3 \cdot TEC}{c \cdot f_c^2}$ (group delay)

$$\boxed{\Delta T_p = \frac{-K}{f_c^2} = -\frac{40.3 \cdot TEC}{c \cdot f_c^2}} \quad \text{(phase delay)}$$

$$\Delta T_g = -\Delta T_p \quad \text{(equal in magnitude, opposite in sign)}$$

In meters, a group delay at $f_c = f_{L1} = 1575.42$ MHz amounts to $c\Delta T_g| = 16.24$ cm per TECU, where 1 TECU $= 10^{16}$ electrons/m^2 (TEC is commonly measured in TECU).

The different signs on ΔT_g and ΔT_p suggest that the group and phase velocities are different in a dispersive medium like the ionosphere. The index of refraction given by $n = 1 - \frac{40.3 \cdot n_e}{f^2} < 1$ applies to the phase and is defined by $n = n_p = \frac{c}{V_p}$, where V_p is the phase velocity (speed). But the modulation $C(t)$ travels at a slower speed through the ionosphere, called the group velocity $V_g < V_p$. Thus $n_g = \frac{c}{V_g} = 1 + \frac{40.3 \cdot n_e}{f^2}$.

The phenomenon of the modulation and the carrier traveling at different speeds through a dispersive medium is called code-carrier divergence.

HW: Show that $|V_g - c| \cong |V_p - c|$ and that

$V_g = c\,n_p$. (Under small velocity departures from c.)

* Discuss "reflection" as a result of $n \to 0$, critical frequency.

* Hint: $V_g \cong c \cdot \left(\frac{1}{1+\epsilon}\right) \cong c[1-\epsilon]$

Q: What is the implication of code-carrier divergence on GNSS signal processing?

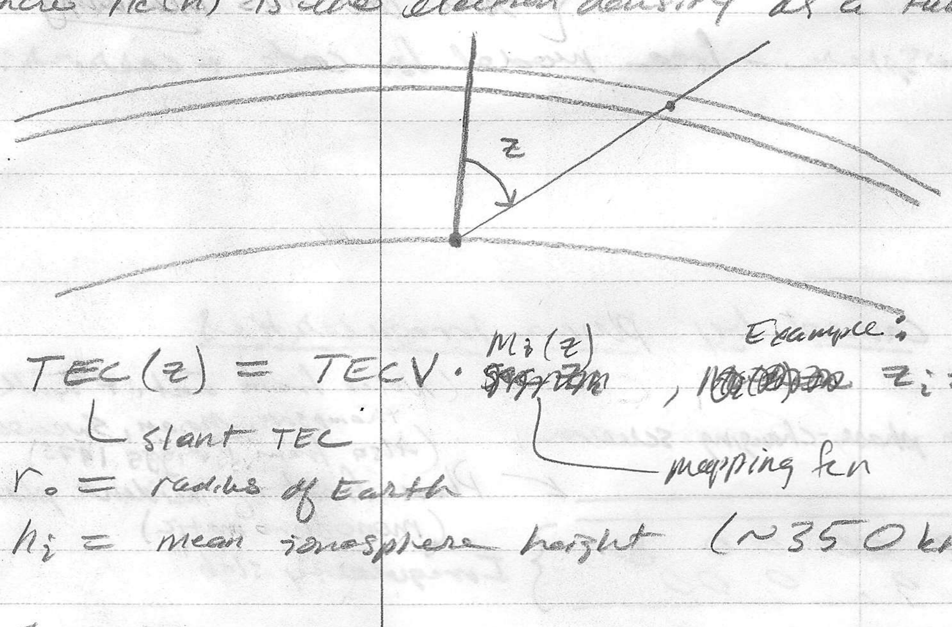
A: We typically have separate but linked feedback loops for tracking the carrier phase and the code phase. These loops had better not assume a perfect synchronous relationship between the code and the carrier. //

Obliquity Factor

For simple modeling we approximate the ionosphere as a thin shell and we define a quantity called Vertical TEC, or TECV.

$$TECV = \int_0^\infty n_e(h)\, dh$$

where $n_e(h)$ is the electron density as a function of height.



$$TEC(z) = TECV \cdot \underbrace{M_i(z)}_{\text{mapping fcn}} \quad \text{Example: } M_i(z) = \sec z_i, \text{ with } z_i = \sin^{-1}\left[\left(\frac{r_0}{r_0 + h_i}\right)\sin z\right]$$

$TEC(z)$ = slant TEC

r_0 = radius of Earth

h_i = mean ionosphere height (~350 km)

Thus, the delay at an obliquity angle z can be related to the vertical delay by

$$\underbrace{\Delta T_g(z)}_{\text{slant delay}} = M_i(z)\, \underbrace{\Delta T_{g_0}}_{\text{vertical group delay}}$$

~~mapping function (could be $M_i(z) = \sec z_i$)~~

Power-law model is the one I use.

Modeling / Estimating Ionospheric Delay

1. Use a model of TECV (lat, lon, t_k) and some mapping function.
2. Ray trace through model ionosphere
3. Take advantage of the dispersive nature of ionosphere to estimate TEC for each signal path:

$$\Delta T_g(f_{L_1}) = \frac{K}{f_{L_1}^2}, \quad \Delta T_g(f_{L_2}) = \frac{K}{f_{L_2}^2}$$

$$K = \frac{\left[\Delta T_g(f_{L_2}) - \Delta T_g(f_{L_1})\right]}{\left(\frac{1}{f_{L_2}^2} - \frac{1}{f_{L_1}^2}\right)}, \quad TEC = \frac{K \cdot c}{40.3}$$

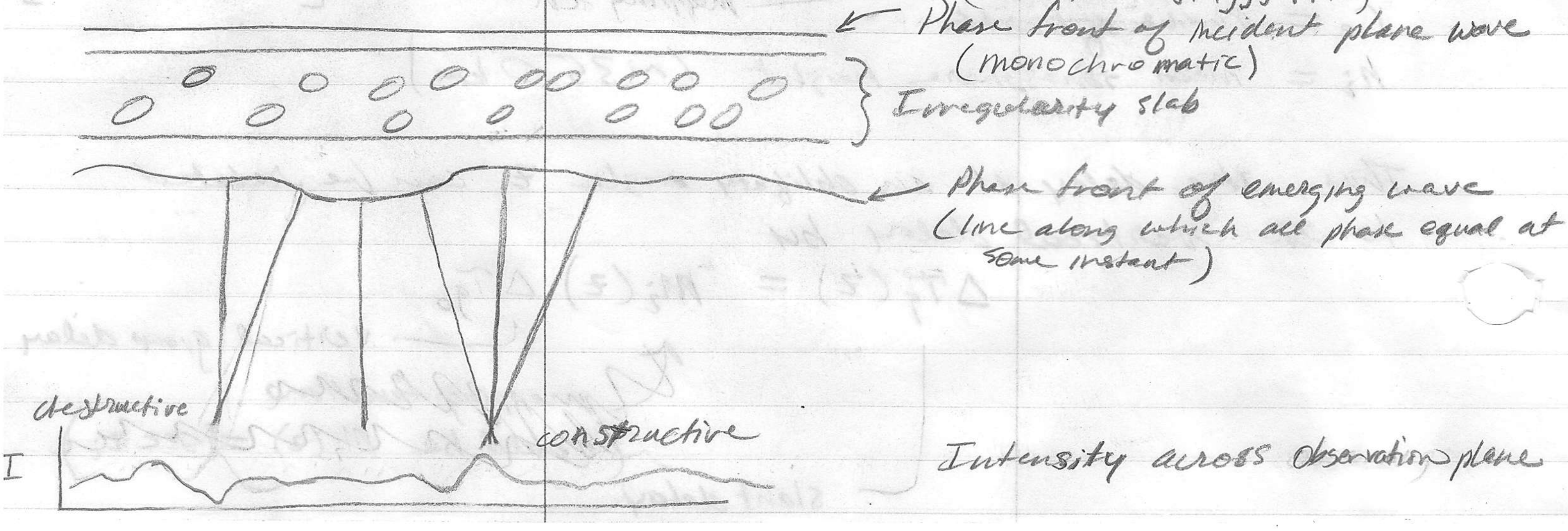
[Some call this measuring.]

HW [Derive ionosphere-free model for code + carrier.

Scintillation caused by plasma irregularities

(Notes from Int. + Synth. by Thompson, Moran, Swenson. (Also from Briggs 1975))

Physical model: Gaussian phase-changing screen



Intuitive model of effects of irregularities:

Recall our earlier expression for the (phase) index of refraction of a plasma:

$$n = \sqrt{1 - \frac{n_e e^2}{4\pi^2 f^2 \epsilon_0 m}}$$

By invoking $\sqrt{1-\epsilon} \simeq 1 - \frac{\epsilon}{2}$ for small ϵ, we rewrite as

$$n \simeq 1 - \frac{r_e n_e \lambda^2}{2\pi} \qquad \text{with } \lambda = c/f \qquad r_e = \frac{e^2}{4\pi\epsilon_0 m c^2} \text{ the "classical electron radius"}$$

Let's assume for simplicity that the irregularities ("blobs") within the slab are all about the same size, a, and that Δn_e is the excess density within a blob compared to the average iono. density. Then the excess phase shift across one blob for a monochromatic signal is

derive from ΔT_p. HW

$$\Delta\phi_0 = a r_e \lambda \Delta n_e$$

Note: proportional to wavelength λ of signal

If the thickness of the slab is L, then our wave will encounter about $N_e = L/a$ blobs. Let's model the phase change across i^{th} blob as an independent random variable $\Delta\phi_i \sim N(0, \sigma^2)$. Then the total phase variance of the emerging wave will be

$$\sigma_{\Delta\phi}^2 = \mathrm{Var}\left[\sum_{i=1}^{N_e} \Delta\phi_i\right] = N_e\, \sigma^2, \text{ where } \sigma = a r_e \lambda\, \sigma_{\Delta n_e}$$

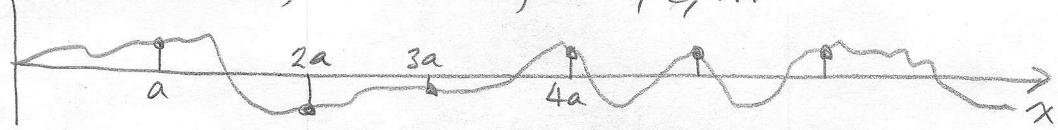
$$\Rightarrow \boxed{\sigma_{\Delta\phi} = \sigma\sqrt{\frac{L}{a}} = r_e \lambda\, \sigma_{\Delta n_e} \sqrt{La}}$$

rms deviation in Δn_e

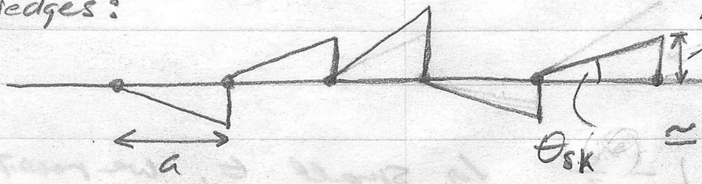
Let $\Delta\phi(x)$ be the excess phase of the wave emerging from the slab, then $\sigma_{\Delta\phi}$ ~~tells us how "crinkled"~~ is the std of $\Delta\phi(x)$:

Note: In this model we assume that only the phase of the wave is altered by passage through the slab; amplitude remains unchanged.

Sample $\Delta\phi(x)$ at $x = ka$, $k = 1, 2, \ldots$



Because of the "crinkeling" in $\Delta\phi(x)$,
the wavefront that exits the slab propagates with a variety of angles. Let's model tilt by a set of refracting wedges:



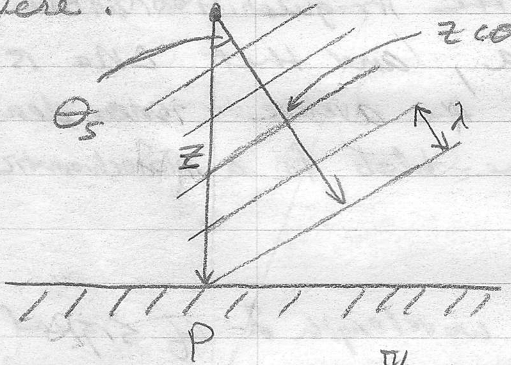
$\Delta\phi(ka)\cdot\left(\frac{\lambda}{2\pi}\right)$ (units of meters) (assume θ_s small)

$\theta_{sk} \simeq \left(\frac{\lambda}{2\pi a}\right)\cdot\Delta\phi(ka)$, $k = 1, 2, \ldots$

rms value of θ_{sk} is: $\boxed{\sigma_{\theta_s} = \left(\frac{\lambda}{2\pi a}\right)\cdot\sigma_{\Delta\phi} = \frac{1}{2\pi} r_e \lambda^2 \sigma_{\Delta n_e}\sqrt{\frac{L}{a}}}$

Q: How far downrange of the slab do the waves begin to interfere?

A:



The phase difference between the undeviated and angled beams at point P is $\frac{2\pi}{\lambda}(z - z\cos\theta_s)$. Approx $\cos\theta_s \simeq (1 - \theta_s^2/2)$ (small θ_s) and the phase difference becomes $\pi z\,\theta_s^2/\lambda$. When this reaches $\pi/2$ we get significant interference. Happens at a downrange distance of $z_0 = \frac{\lambda}{2\theta_s^2}$†. If $\sigma_{\Delta\phi} < 1$ then $\sigma_{\theta_s} \ll 1$ and we don't get much interference (note: $\lambda \ll a$ typically). If $\sigma_{\Delta\phi} = 2\pi$ we get significant interference at an expected distance

$$E[z_0] \simeq \frac{\lambda}{2E[\theta_s^2]} = \frac{\lambda}{2\sigma_{\theta_s}^2} = \frac{a^2}{2\lambda}.\ *\ \text{(See note below)}$$

We can turn this argument around to say that at a certain distance z from the ionosphere, we get maximum intensity variation from blobs of a <u>certain</u> <u>size</u>:

~~If $\sigma_{\Delta\phi} = 2\pi$ then $\sigma_{\theta_s} = \frac{\lambda}{a}$ and $z = \frac{a^2}{2\lambda}$.~~

$a_F = \sqrt{2z\lambda}$ is the size we're sensitive to at a downrange distance z.

☆ See Briggs paper for more rigor.

* In this case the size of the blob is approximately the size of the "first Fresnel zone".

† Note that if θ_s small, z_0 large.

Receiver Effects Model of Scintillation

(see my papers)

Due to scintillation, the amplitude and phase of an incoming signal get changed by the complex channel response function $z(t)$:

$$z(t) = \alpha(t) e^{j\theta(t)} = \underbrace{\bar{z}}_{\substack{\text{direct component} \\ \text{(complex constant)}}} + \underbrace{\xi(t)}_{\text{time-varying multipath component.}}$$

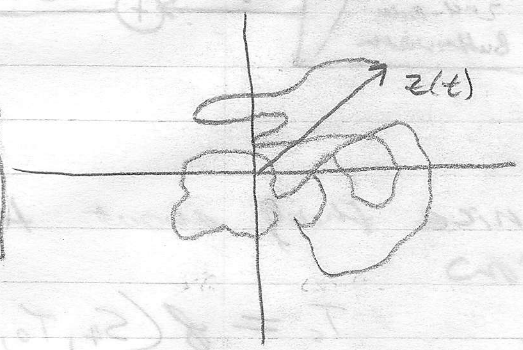
Received bandpass signal is then:

$$r(t) = Re\{ z(t) x_\ell(t) e^{j2\pi f_c t} \}$$

For purposes of predicting cycle slips we need to know the amplitude distribution $p(\alpha)$ and the autocorrelation function

$$R_\xi(\tau) = \left(\tfrac{1}{2}\right) E\left[\xi^*(t)\xi(t+\tau)\right]$$

Think of $z(t)$ as a phasor:



$$\boxed{\text{When } z(t) \text{ is small and moving fast, cycle slips occur.}}$$

$\alpha(t)$ measures size

$R_\xi(\tau)$ measures speed

As it turns out, $\alpha(t)$ is well modeled by the Rice dist.

0th-order modified Bessel fcn. (first kind)

(5) $$p(\alpha) = \frac{2\alpha(1+K)}{\Omega} I_0\left(2\alpha\sqrt{\frac{K+K^2}{\Omega}}\right)$$

(3) $$\Omega = \langle \alpha^2(t) \rangle$$

(2) $$S_4^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}$$ "scintillation index"

(4) $$K = \frac{\sqrt{1 - S_4^2}}{1 - \sqrt{1 - S_4^2}}$$

(1) $$I = \alpha^2$$

$\langle \cdot \rangle$ denotes time average.

[Proceed through quantities as shown.]

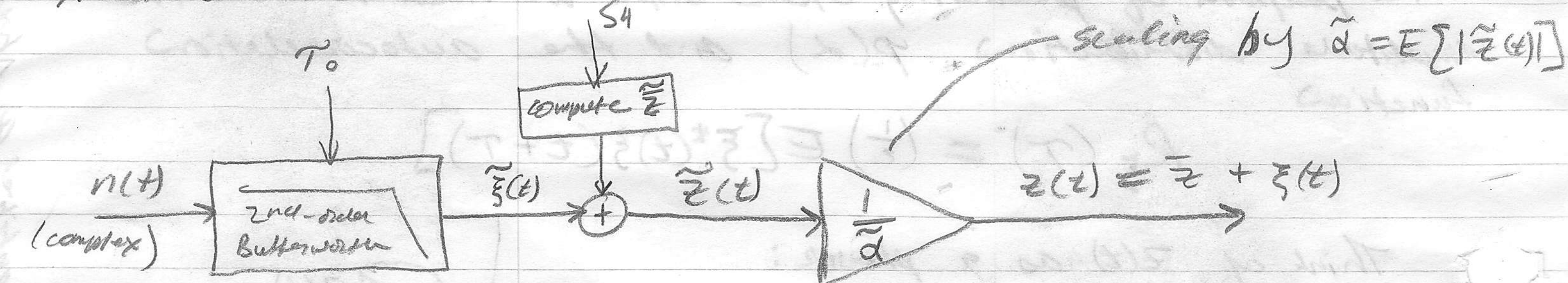
$R_\xi(\tau)$ is best modeled as white noise passing through a 2^{nd}-order Butterworth filter:

$$R_\xi(\tau) = \sigma_\xi^2 \, e^{(-\beta|\tau|/T_0)} \left[\cos\left(\frac{\beta \tau}{T_0}\right) + \sin\left(\frac{\beta |\tau|}{T_0}\right) \right]$$

where $\beta \simeq 1.24$ ensures that $\dfrac{R_\xi(T_0)}{R_\xi(0)} = e^{-1}$

T_0 is called the decorrelation time.

Model:

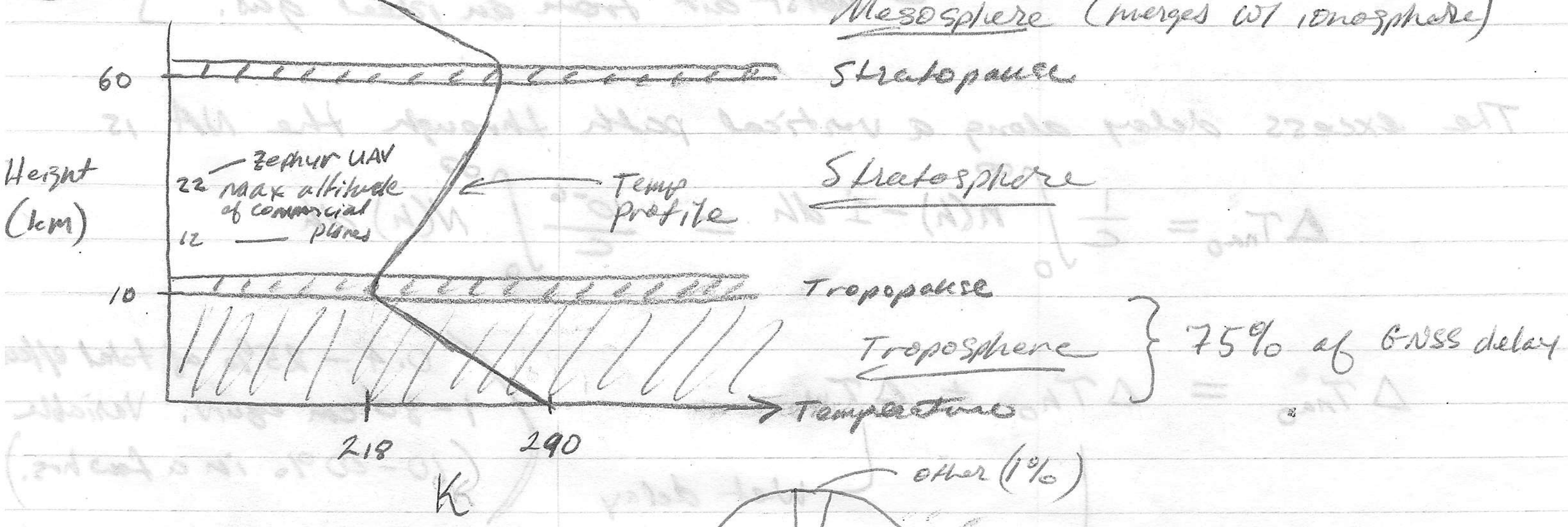


The nice thing about this model is that there exists a function

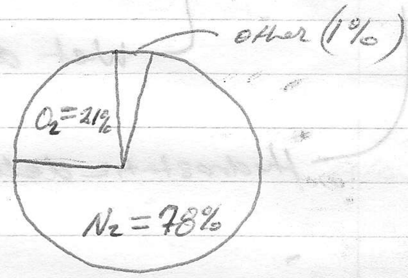
$$T_s = f(S_4, T_0, C/N_0)$$

that predicts the mean time between cycle slips T_s.

Propagation Effects: Neutral Atmosphere (NA)



Chemical Composition:



Main neutral atmosphere effects:

1. range + carrier phase delay ~2.4m at zenith
2. attenuation
3. scintillation

} negligible except for occultation experiments

In the NA, the index of refraction departs from unity less than in the ionosphere at GNSS freqs. Also, the refractive index is not significantly dependent on frequency in the NA. Hence, the NA is non-dispersive at GNSS freqs.

For convenience, we define refractivity
$N = (n-1) \times 10^6 \geq 0$, where n is the refractive index.

By experiment, we know that

$$N = 222.76\rho + (17 \pm 10)\frac{e}{T} Z_w^{-1} + 377600 \frac{e}{T^2} Z_w^{-1}$$

ρ = mass density of moist air ($kg\,m^{-3}$)
T = temperature (K)
e = partial pressure of water vapor (mb)

Z_w = factor near unity [accounts for small departure of moist air from an ideal gas.]

The excess delay along a vertical path through the NA is

$$\Delta T_{na_0} = \frac{1}{c}\int_0^\infty n(h)-1\, dh = \frac{10^{-6}}{c}\int_0^\infty N(h)\, dh$$

$$\Delta T_{na_0} = \Delta T_{h_0} + \Delta T_{w_0}$$

Wet delay (0.4 – 25% of total effect
1–80 cm equiv. Variable
(10–20% in a few hrs.))

Hydrostatic delay (~90% of total, 2.3 m equiv.
Highly predictable
mostly dry gases)

We map the zenith delay to a slant delay with separate hydrostatic and wet mapping functions:

$$\Delta T_{na}(z) = M_h(z)\,\Delta T_{h_0} + M_w(z)\,\Delta T_{w_0}$$

The best mapping functions are based on <u>weather</u> <u>forecasts</u>, as are the best models for ΔT_{h_0} and ΔT_{w_0}.

[Mention atmospheric tides.]

An even better approach: ray trace through a refractivity field $N(lat, lon, h)$ based on P, e, T from numerical weather models.