EE 242 Spr. 2024 Take Home HW 2 [52]

Due: May 13, 12 noon

Directions for Submission: Scan/convert to pdf and submit on Canvas.
Be sure to write your name/student # LEGIBLY.
Check your scan version post-submission: any unreadable solution will NOT be graded.

1. For the following systems, state whether if it satisfies the following properties
   i. Linear ii. Time-Invariance iii. Stable iv. Causal v. Invertible
   (in case it does, sketch the proof; if not, provide a counter-example)
   a) $y(t) = e^{-x(t)}$ b) $y(t) = \int_{-\infty}^{2t} x(u)du$ [5 =2.5+2.5]

2. Given the interconnections between input x(t) and output y(t) shown in Fig 1a, where the functions in the boxes denote the resp. impulse responses, write a final expression for the equivalent LTI system response H(t) in Fig 1b. [4]

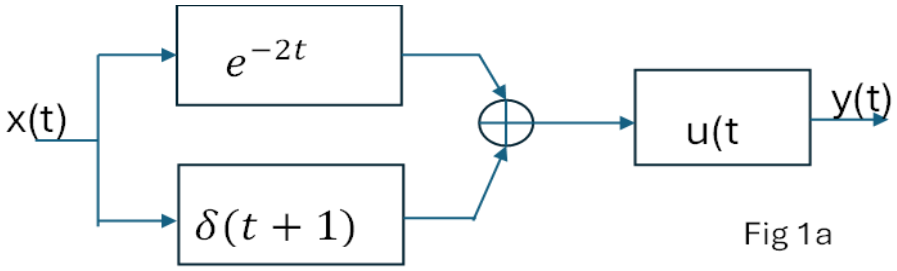


Fig 1a

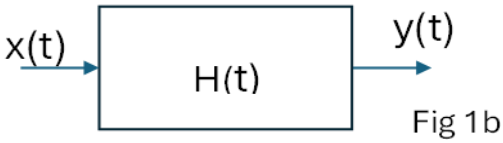
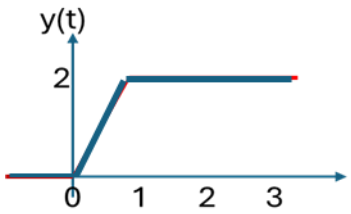


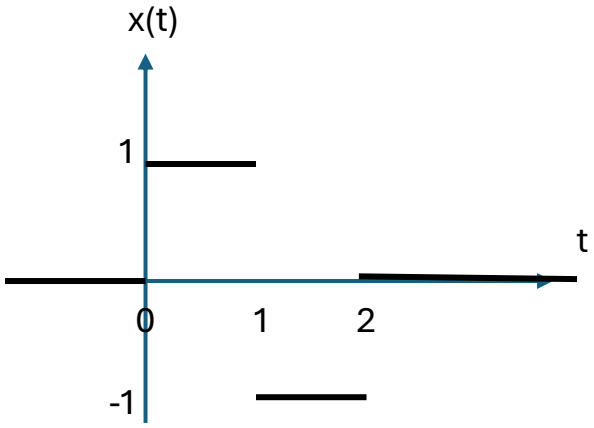
Fig 1b

3. The input sequence **x(n) = […., 0, 1, -1, 0 …]** (x(0)=1, x(1) =-1, and 0 everywhere else) is passed thru a causal LTI system with finite impulse response h(n) to yield the output **y(n) […0, 1, 0, 0, 0, -1, 0 ….]** (y(0)=1, y(1) =0 , y(2) =0, y(3)=0, y(4) =-1, & 0 else). Find h(n). [4]

4. The response to a unit step input x(t) = u(t) for a LTI system is shown below:



Sketch the output of the same LTI system to the following input x(t) using only properties of LTI system. [5]



5. The input x(t) into a LTI system with impulse response h(t) yields output y(t), find the output when $x(t - t_0)$ is applied to an LTI system with impulse response $h(t + t_1)$ [i.e. your answer should be expressed in terms of y(.)] [3]

6. The step response (i.e. output corresponding to a unit step input) s(n) of discrete-time LTI system is given by $s(n) = \alpha^n u(n)$, $0 < \alpha < 1$. Find the impulse response h(n) of the system. [5]

7. The input $x(t) = \cos(t)$ is applied to an LTI system with transfer function $H(j\omega) = \frac{1}{1+j\omega}$. Write down the output y(t) using the Eigenfunctions of LTI system. [4]

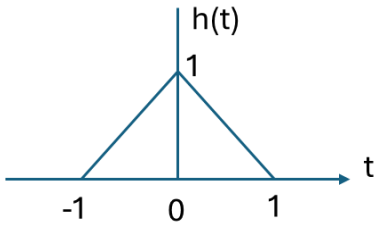
8. For the following LTI systems with given impulse responses, determine if they are
   i) Causal ii) stable [4=2+2+2]

   a) $h(t) = \cos(\omega_0 t)\, u(t)$ b) $h(n) = (0.8)^n\, u(n+2)$

9. Input x(t)= e^{-at} u(t), a > 0 is passed thru an LTI system with h(t)=e^{-bt} u(t), b > 0. Evaluate the output y(t). Using your result, find the output when b=a. [5]

10. Compute the convolution of the following pair $x(n) = \propto^n u(n)$ and $h(n) = \beta^n$ u(n) to obtain the output y(n). [4]

11. An LTI system has the impulse response h(t) shown below (triangular pulse)



The input is the periodic (T) impulse train $x(t) = \sum_k \delta\,(t - kT)$. Sketch the output y(t) for a) T=3 and b) T=1. [4]

12. An LTI system has the transfer function $H(j\omega) = 1 + \alpha e^{-j\omega t_0}, \alpha > 0.$
    a) Write down the impulse response h(t).
    b) Derive expressions for the magnitude $|H(j\omega)|$ and phase $\angle H(j\omega)$ in a final simplified form and sketch as functions of ω. **[5]**