EE 242 Spr. 2024 Take Home HW 1 [48]

Due: Apr. 22, 12 noon

Directions for Submission: Scan/convert to pdf and submit on Canvas.
Be sure to write your name/student # LEGIBLY.
Check your scan version post-submission: any unreadable solution will NOT be graded.

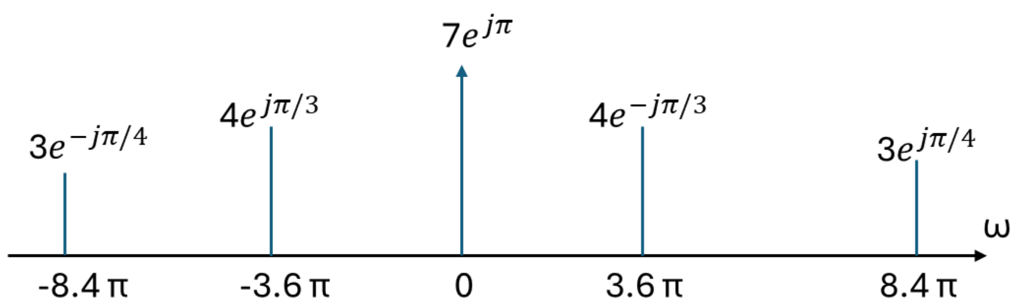
1. For each of the signals below, determine if they are i) causal/non-causal, ii) even/odd, iii) periodic/aperiodic, iv) energy/power signal:
   a. $3e^{j5t}$ b. t $u(t)$ [2+2=4]

2. Find the even and odd components of $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$
   Identify if this is Energy/Power signal and compute corr. Energy or Power.
   [2+2+2=6]

3. Consider x(t) = r(t) – r(t-2) + u(t-2) – 2u(t-3) + u(t-5); using the fact that Delta function $\delta(t) = \frac{d}{dt}$u(t), sketch y(t) = $\frac{d}{dt}$x(t) and evaluate $\int_{-\infty}^{\infty} y(t)dt$. [2+2=4]

4.The complex sinusoidal signal x(t) = $e^{j\omega_0 t}$ has a fundamental period T_0 = ${}^{2\pi}/_{\omega_0}$. Consider the discrete-time sequence x(n) obtained by sampling x(t) uniformly with interval T_s. Find the condition on T_s such that x(n) is periodic. [3]

5. The complex Fourier Spectrum of a periodic signal x(t) is shown below [2+3=5]



(a) Determine the fundamental period T_0 of x(t).
(b) List all the non-zero complex Fourier coefficients c_k and write a final expression for $x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_0 t}$.

6. You are given the following information about a signal x(t). [4]

i. Periodic with period T=1

ii. Real, odd

iii. Complex Fourier Series coefficients c_k = 0 for |k| > 1 & $\int_0^1 |x(t)|^2$ dt = 1.

Write a final expression for x(t) consistent with all the above.

7. Let x(t) be a periodic signal with period T and corr. complex Fourier Series coefficients c_k, i.e. $x(t) = \sum_{n=-\infty}^{\infty} c_n \ e^{jn\omega_0 t}$ $(\omega_0 = {}^{2\pi}/_{T})$.
   Let $y(t) = x(t-a) + x\ (a-t)$; show that y(t) is periodic (T) and find an expression for its Fourier Series coefficients d_n i.e. $y(t) = \sum_{n=-\infty}^{\infty} d_n \ e^{jn\omega_0 t}$, in terms of c_n, a. [4]

8. We've seen that any function x(t) can be decomposed into its even $x_e(t)$ and odd $x_o(t)$ components, i.e. $x(t) = x_e(t) + x_o(t)$. Let $x(t) \leftrightarrow X(\omega)$ [Fourier pair, i.e. $X(\omega)$ is the Fourier transform of *x(t)*.
   Prove that Fourier transform of $x_e(t)$ equals *Re* [$X(\omega)$] and that of $x_o(t)$ equals j *Im* [$X(\omega)$] [4]

9. By direct application of the Fourier Transform integral, evaluate the F.T of the following time functions: [2.5+2.5=5]

   (a) $x(t) = e^{-\propto|t-1|}$ $\forall\ t, \propto > 0$ (b) y(t) = $\cos(\omega_0 t)$, $-\frac{T}{2} \leq t \leq \frac{T}{2}$ & 0 else

10. Using the inverse Fourier Transform relation, evaluate the corresponding time signals whose Fourier transforms are: [2+3=5]
    (a) X $(j\omega) = 2\pi\ \delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$
    (b) X $(j\omega)$ = cos (2ω) sin (ω)

11. Refer to the derivation of Fourier Transform of $x(t) = sgn(t)$ in lecture, we now seek to derive the same result via a different route as follows.
    a) Write $sgn(t) = 2\ [u(t) - 1]$ and by using any of the properties of the Fourier Transform listed in the table (Lecture 5b), find the Fourier Transform $X(j\omega)$.
    b) Use the above result to write down the Fourier Transform of u(t). [3+1=4]