EE 242 Spr. 2025 Take Home HW 3 [50]

Due: Thur. Jun 5, 12 noon.
Directions for Submission: Scan/convert to pdf and submit on Canvas.

1. [0.5+2+3=5.5]

An LTI system has impulse response $h(t) = 2\,\frac{\sin(\pi t)}{\pi t}\cos(4\pi t)$.

Sketch H(ω) & determine the output y(t) corresponding to the following inputs:

(a) $x(t) = 1 + \cos(\pi t) + \sin(4\pi t)$ (b) $x(t) = \sum_m \delta(t-m)$

2. [2+3=5]

The frequency response of an LTI filter is given by $H(\omega) = \begin{cases} j\omega, & |\omega| < 3\pi \\ 0, & else \end{cases}$

i. Identify the corresponding time-domain operation (consult your table of Fourier transform properties if needed).
ii. Determine the output y(t) of the filter for the following inputs:
   1. x(t) = A cos (2πt +θ)   2. x(t) = A cos (4πt +θ)

3. [2.5+3=5.5] Consider the system shown in Fig. 1a

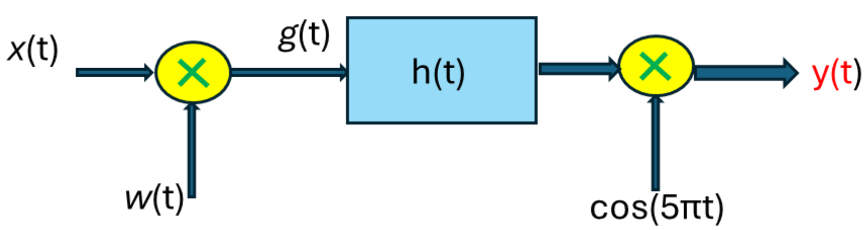


Fig. 1a

where the F.T. of the input x(t) is given by Fig. 1b.

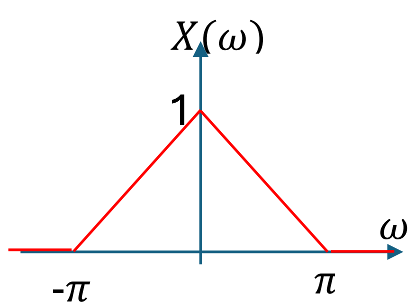


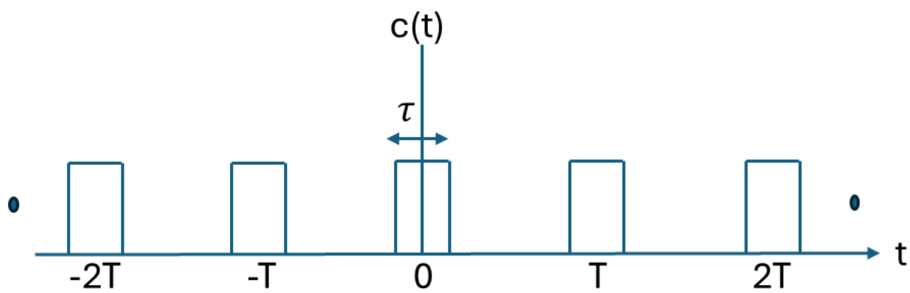
Fig. 1b

Sketch the F.T $G(\omega), Y(\omega)$ of the signals $g(t)$, *y(t)* resp. for the following case:

$$w(t) = \cos(5\pi t)\,,\ \ h(t) = \frac{\sin(6\pi t)}{\pi t}$$

[4] [2+3=5]

A signal x(t) is multiplied by a periodic (T) rectangular pulse train c(t) shown below to produce the signal y(t) = x(t) c(t).



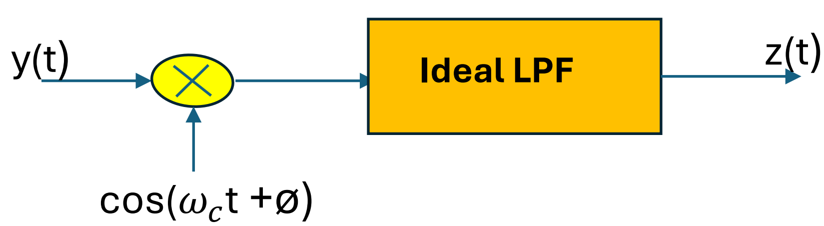
(a) What constraints should be placed on the input signal X(ω) such that x(t) can be recovered from y(t) via an ideal lowpass (reconstruction) filter [express any condition in terms of τ, T].

(b) Specify the cut-off frequency ω_c and the passband gain of the ideal LPF needed for the above recovery (assuming the condition in (a) are satisfied).

[5] [5]

The signal y(t) = x(t) cos(ω_ct) is the amplitude modulated signal corresponding to the message signal x(t) that is *BL*($-\omega_m$, ω_m). We want to explore the impact of demodulation on y(t) with a (local) carrier that is *not* synchronous (phase-locked) with the transmitted carrier, i.e. consider the demodulator below (Fig. 2)

Fig. 2



where the ideal LPF is BL ($-\omega_m$, ω_m) and has a passband gain = 2.

Write down an expression for the output z(t) and comment on the impact of phase offset.

[6] [2.5+2.5+2=7]

The signals $x_1(t), x_2(t)$ are input to the respective LTI filters $H_1(\omega), H_2(\omega)$ as shown in Fig. 3 below and their respective outputs $y_1(t), y_2(t)$ multiplied to produce $y(t) = y_1(t)y_2(t)$.

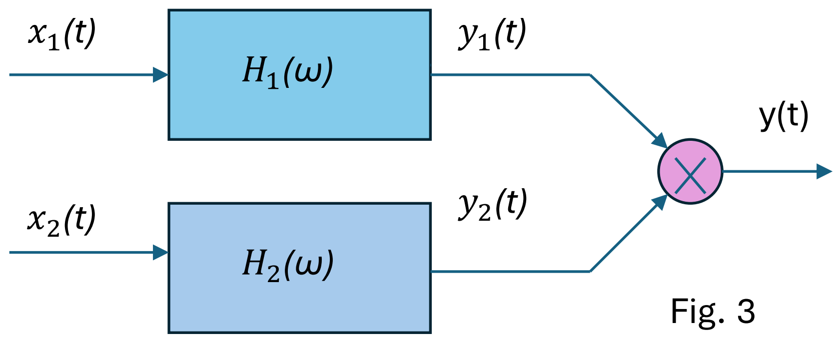


Fig. 3

For $x_1(t) = 10^4\, rect(10^4 t)\,, x_2(t) = \delta(t)$ and $H_1(\omega) = rect(\frac{\omega}{40{,}000\pi})\,, H_2(\omega) \;= rect(\frac{\omega}{20{,}000\pi})$

(a) Sketch (with proper labeling of axes) $Y_1(\omega), Y_2(\omega)$.

(b) Determine the bandwidth of the signal y(t).

[7] [(1.5+1.5)+1.5+2.5=7]

[i] $x(t)$ is a BL $(-\omega_B, \omega_B)$ signal. Determine the Nyquist sampling rate for each of the following derived signals:

(a) $x^2(t)$ (b) $\frac{d\,x(t)}{dt}$

[ii] x_1(t), $x_2(t)$ are two BL signals, restricted to $(-\omega_1, \omega_1)$ & $(-\omega_2, \omega_2)$ resp. Determine the Nyquist sampling rate for y(t) = x_1(t) $x_2(t)$.

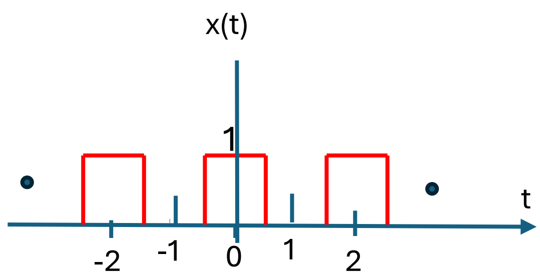
[iii] The sequence x(n) = $(-1)^n$ is obtained by sampling the continuous time signal x(t)=cos($\omega_0 t$) @ $T_s = 10^{-3}$sec. Identify atleast 2 distinct values for ω_0 that yield the same x(n).

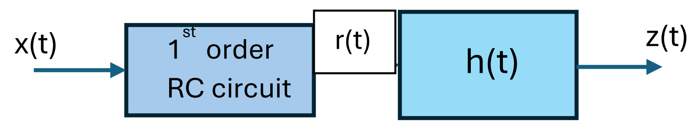
[8] (2.5+3.5=6)

The periodic signal x(t) [50% duty cycle on-off pulse train with period =2] shown is input to the series cascade of LTI systems shown below. The 1st order RC circuit is described by the input-output relation $r(t) + RC\,\frac{d\,r(t)}{dt} = x(t)$. The output r(t) is then filtered by $h(t) = 2\,sinc(2t)$ = 2 $\frac{sin(2t)}{2t}$.

(a) Write down the frequency response $H_{RC}(\omega)$ of the 1st order RC circuit.

(b) Write an expression for the output z(t).





[9] (4)

The message signal to be sent is given by $m(t) = \frac{\sin(1000\,\pi t)}{\pi t}$ via AM that creates the modulated signal for transmission $s(t) = (m(t) + A)\cos(10{,}000\,\pi t)$. Assuming that asynchronous demodulation (envelope detection) is used, determine a) the min. A necessary to enable envelope detection and b) corresponding modulation index *m* for this choice of A.

Note: modulation index $m = \frac{\max|m(t)|}{A}$