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2015 Mathematical Contest in Modeling (MCM) Summary Sheet

Considering key factors of the sustainability of a country, we build an evaluation model to create a more sustainable world, and discuss some issues in this topic.

Firstly, by taking economy, environment, and society as the common index of sustainable development, we construct a multi-index comprehensive evaluation model to evaluate the sustainability of a country. Using the expectation of Gaussian distribution for the data, we get the critical value of each factor and give a clear definition of sustainable or unsustainable development. That is, if three indexes of a country are all beyond the critical value in consecutive three years, then this country is sustainable. Based on this measurement rule, we make a distinction between more sustainable countries and less ones. The result also provides some suggestions to ICM on those countries that need most support.

The second task is to create a 20-year plan. We choose Cambodia as a least developed country to test model in Task 1. It is shown that the index of economy is far lower than the critical value in Cambodia, while environment and society are in good condition. In view of this characteristic of Cambodia, our suggestion is that Cambodia should rapidly develop economy to become a more sustainable country.

Thirdly, we build a sustainability prediction model based on Markov chain. Considering the initial 3 indices of sustainable development and using state transition matrix of Markov chain, we obtain the expectation of those indices in the next years. Moreover, according to the effect of different policies on different index in the process of development, we find that the policy "Apply high-tech in basic industries" plays an important role for the sustainable development of a country.

Finally, we make a sensitivity analysis for our model and discuss the possibility of extending the model to other fields.

1 Introduction

1.1 Background

Sustainable development is a controversial disputed with a wide range of meanings. It is embraced by big business, governments, social reformers and environmental activists, all of which put their own interpretation on what sustainable development means, ever defined by the 1987 Brundtland Report as “development that meets the needs of the present without compromising the ability of future generations to meet their own needs.”. The concept of sustainable development is the result of growing awareness of the global links between mounting environmental problems, socio-economic issues to deal with poverty and inequality and concerns about a healthy future for humanity.

Among the principal omissions from the Brundtland report was detailed consideration of non-human species, and their ‘rights’, an area which had received considerable attention during the last ten years or so. The ‘Brundtland Report’ (after its Chairperson, the Norwegian Prime Minister at the time) also opened the way for Non Governmental Organisations (NGOs) to be considered a serious element in environment and development issues, a process that culminated, as we shall see, with the first Earth Summit in Rio de Janeiro in 1992.[1]

In 2012, the UN conference on sustainable development recognized that: that poverty eradication, changing unsustainable and promoting sustainable patterns of consumption and production and protecting and managing the natural resource base of economic and social development are the overarching objectives of and essential requirements for sustainable development. Decreasing personal poverty and vulnerability, encouraging economic development, and maintaining ecosystem health are the pillars of sustainable development.

We will develop a multi-index comprehensive evaluation model, this model is used to distinguish that those countries are more sustainable or less sustainable. A 20 years sustainable development plan is created based on the model in Task 1.

1.2 Our Work

On the basis of above discussion, to measure the sustainability of a country and to promote it to be applied in the real life, we may boil down the tasks to the following four questions:

- Develop a model to evaluate the sustainability of a country and to reach whether or not that a country is sustainable. Then advise the ICM which country need to support and invest.
- Select a country from UN list of LDC and create a 20 year sustainable development plan for the country to move it towards a more sustainable future.
- Create a 20-year sustainability plan based on the model in Task 1 and evaluate the effect of the plan on the country’s sustainability.

2 Assumptions

- The standard we build is based on the current world condition.
- The influence of factors like climate change, and natural disasters is limited and it will not interrupt or badly disturb the development of the country.

3 Symbol Description

In this section, the paper uses the following symbols for constructing the model as follows.

Symbol	Description
F	Economical index of sustainable development ability
E	Environmental index of sustainable development ability
S	Social index of sustainable development ability
f_i	Economic sustainable development ability of i th index
e_i	Environmental sustainable development ability of i th index
s_i	Social sustainable development ability of i th index
p	The property value of index
C_{f_i}	The state transition matrix of index
$I_{f_i}(n)$	The state probability vector of index after n years
$h_{f_i}(n)$	The multiplier of index after n years
$f_i(n)$	The numeral value of index after n years
$F - i$	the i th economic factor
$E - i$	the i th environmental factor
$S - i$	the i th social factor

Other symbols instructions will be given in the text .

4 Mult-index comprehensive evaluation model

Before modelling, to avoid ambiguity, we distinguish all the factors into two types.

Positive factors :the value of factors are the higher the better

Negative factors :the value of factors are the lower the better

4.1 Analysis of the problem

Considering all the factors we found, we conclude that there are 3 main aspects: Environment, Economy and Society.

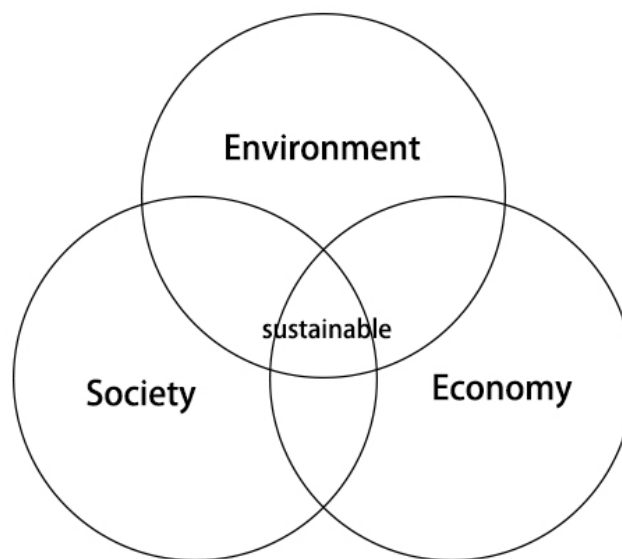


Figure 1 Common three-ring sector view of sustainable development

Based on the factor-select principles [3] consisted of scientificity, feasibility, simplicity, etc. (you can notice it in Fig.2), the factors we chose are as follow.

Economy: per capita GDP, total stockage (including gold, which counted with current dollars), foreign exchange reserves, human development index, percentage of listed companies' market capitalization in GDP, percentage of total taxation in GDP, Gini coefficient.

Environment: percentage of forest area in land area, alternative energy and nuclear energy (percentage of total energy use), energy use(per kg of oil equivalent), per capita renewable inland freshwater resources (M3), arable percentage of land area, carbon dioxide emissions (per capita in metric tons), carbon monoxide emissions.

Society: public health spending accounts for the proportion of medical expenditure, life expectancy at birth (overall), population growth rate (annual percentage), the urban population (the proportion of the total population), the literacy rate (Youth overall), total public expenditure on Education (share of government expenditure), total unemployment accounted for the proportion of the total labor force.

Finally, processing data can divided into four steps.

Step 1: Determine the standard value of each factors.

Step 2: Standardize these factors in order to compare them

Step 3: Give weights to these indexes in order to evaluate three aspects totally and respectively, by using principal analytic hierarchy process.

Step 4: Compound these indexes and analyze three aspects respectively.

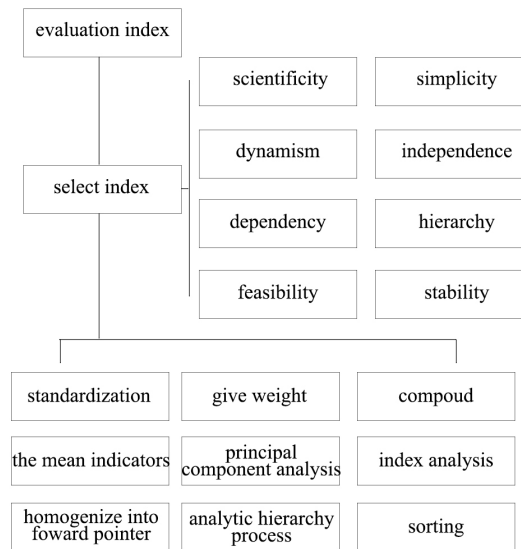


Figure 2 Procedures and methodologies of building assessment index of sustainable development

To give a better understanding, Fig.2 shows the procedures and methodologies of building assessment index of sustainable development. In order to determine the critical value of each factors, we collect the data of each factors from varies of countries. We find that almost all the data obey normal distribution. So we fit those data to normal distribution, and get the exception of the normal distribution as the critical value.

The following definition can be defined by according to *A summary of the ten Bellagio Principles for gauging progress towards sustainable development* [5]

4.2 Model Set up

For the index f_i , e_i and s_i , the equation you can see in the following (taking an example of f_i)

$$f_i = 1 + p_{f_i} \times \frac{d_{f_i} - d_{f_*}}{d_{f_*}}$$

where p_{f_i} represents the property of factors (with the value of 1) or negative attribute (with the value of -1) of the factor; d_{f_i} equals to the value of the factor; d_{f_*} equals to the boundary of each factor. Those values are showed in the Table 1.

By using the data of indexes from internet [4], the expectation after fitting by normal distribution can be given in Table 1, 2, and 3.

If f_i (or e_i , s_i) is greater than 1, it means that the specific factor is better than the critical value.

Then we combine those factors of economic, environment and society with the weights determined

Table 1 critical value of Economy

Indexes	critical value	The property of factors
Per capita GDP	14309.34	1
Total stockage	71541337862.30	1
Foreign exchange reserves	81784312662.72	1
Human development index	0.811	1
Percentage of listed companies' market capitalization in GDP	10.60	1
Percentage of total taxation in GDP	17.07	1
Gini coefficient	0.40	-1

Table 2 critical value of Environment

Indexes	critical value	The property of factors
Percentage of forest area in land area	31.56	1
Alternative energy and nuclear	8.43	1
Energy use	2522.6	1
Per capita renewable inland freshwater resources	15355.89	1
Arable percentage of land area	13.94	-1
Carbon dioxide emissions	4.943	-1
Carbon monoxide emissions	120020.3	-1

Table 3 critical value of Society

Indexes	critical value	The property of factors
PHS accounts for the proportion of medical expenditure	58.78	1
Life expectancy at birth	70.58	1
Population growth rate	1.328	1
The urban population	57.85	1
The literacy rate	86.49	1
Total public expenditure on Education	14.02	1
The unemployment accounted for the proportion of the total	8.55	-1

by AHP.

The economic index of sustainable development ability F

$$F = \sum_{i=1}^7 w_i f_i$$

where w_i is the weight of factors of the field of economic.

The environment index of sustainable development ability E

$$E = \sum_{i=1}^7 x_i e_i$$

where x_i is the weight of factors of the field of environment.

The social index of sustainable development ability S

$$S = \sum_{i=1}^7 y_i s_i$$

where y_i is the weight of factors of the field of society.

The determination of the weights by using AHP are:

$$w = (0.125, 0.125, 0.125, 0.1875, 0.125, 0.125, 0.1875)$$

$$x = (0.15, 0.15, 0.125, 0.15, 0.125, 0.15, 0.15)$$

$$y = (0.125, 0.125, 0.125, 0.125, 0.1875, 0.1875, 0.125) .$$

. The index of economic, environment and society, seen as F, E and S are fluctuate around 1. All the f_i (or e_i, s_i) are divided by the boundary 1 so F, E and S also has the boundary as 1.

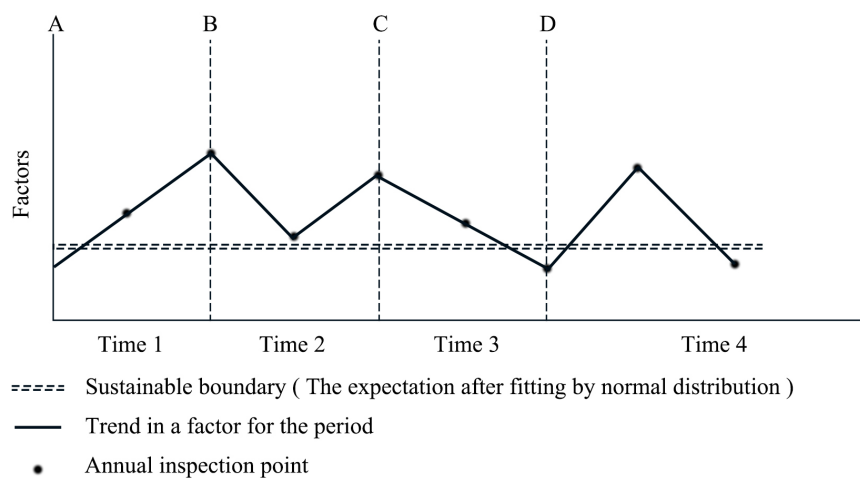
4.3 Identifying sustainable countries and unsustainable ones

We defined, if a country is sustainable, the following two conditions will be met:

- 1) F, E and S are all greater than 1;
- 2) Each of the indexes is greater than 1 in three consecutive years.

For a particular explanation, the Fig.3 presents the trend of country's factors in a period, judging whether the country is sustainable or not.

Obviously, Fig.4 shows the three indexes of Cambodia, Fig.5 shows those of the Korea. Every point represent a specific factor, the distance between the center of the circle and the point is proportional to the value of the factor. The coloured circle has the radius of 1. So if a point appear in the circle, it means that the corresponding factors are under the boundary.



Time period	Reference point	Likely interpretation
1	A	sustainable trend
2	B	sustainable trend
3	C	unsustainable trend
4	D	unsustainable trend
1 to 4	A	sustainable trend (overall trend in quality is level)

Figure 3 Trend in factors for periods

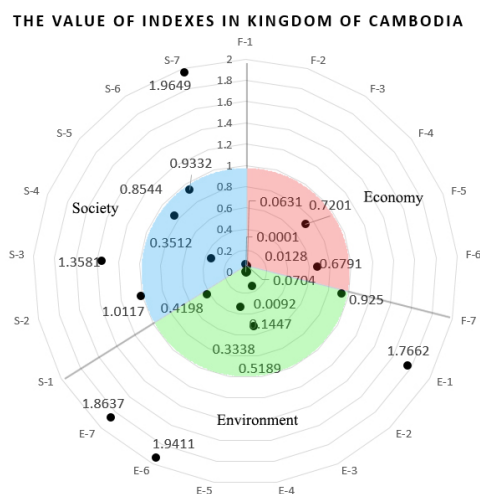


Figure 4 the value of indexes in Kingdom of Cambodia

Then the final result F , E and S for the US, Korea, Kingdom of Cambodia, and Rwanda can be seen in Table.2.

According to the analysis above, Korea is the most sustainable development country, and the unsustainable countries include Kingdom of Cambodia, Rwanda, and the US. The result conforms to the reality and the reports can be found to verify the result. Like Ministry of Commerce of the People's Republic of China [6] ever reported that World Competitiveness Center published *The global sustainable development report*, it gave the ranking of sustainable countries, the result of this passage is closed to the recognized results.

By statistics, Cambodia is weak in economic, environment and society, but the environment and society are both close to the standard level. It can also be seen that some developed countries such as the America also wake in environment and society.

So we conclude that countries like Cambodia that wake in all area, especially in economic, should be supported and intervention in priority.

From the results of the four countries, we can conclude that most countries in the world are develop along a imbalance way. There are only a few countries that operated in a sustainable way. Developed countries always have better economic condition, but there environment may be bad. Some poor country may has less pollution and abundant nature resources, however, it can't been seen as sustainable. So balance development is a important part of sustainable development.

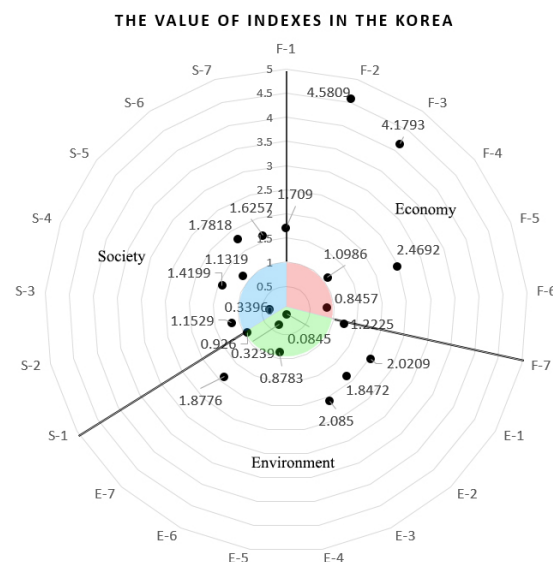


Figure 5 the value of indexes in the Korea

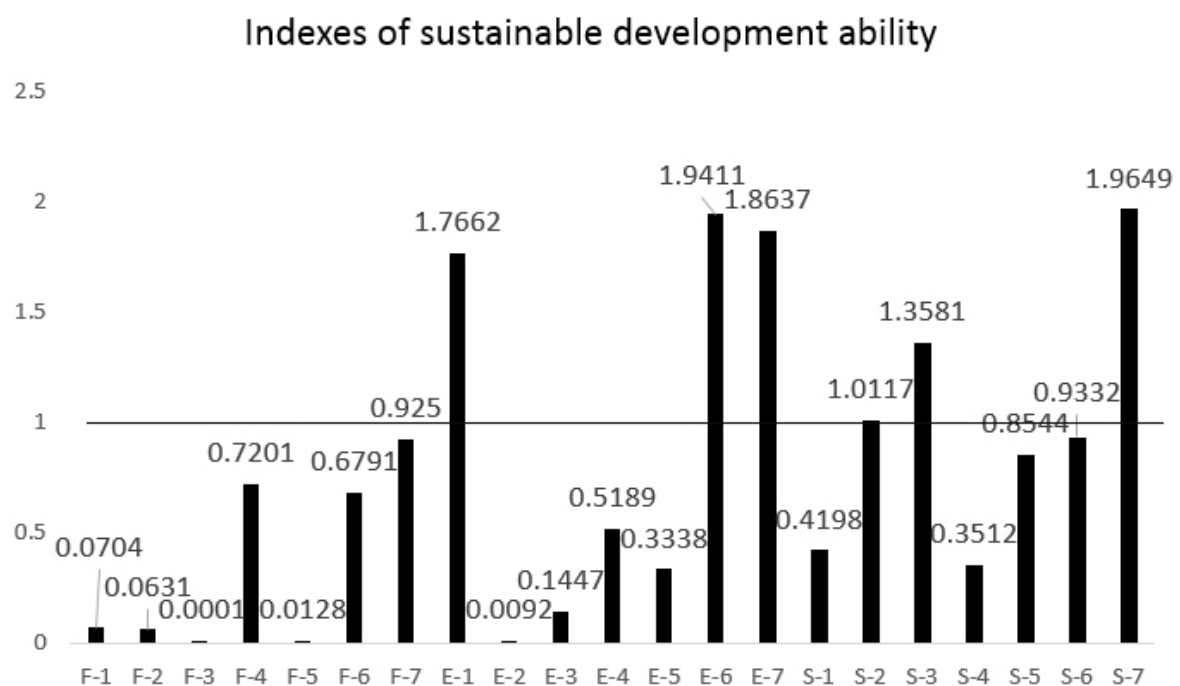
Table 2 The index of sustainable development ability in different countries

the US	2010	2011	2012	Korea	2010	2011	2012
<i>F</i>	2.0213	2.1662	2.2638	<i>F</i>	2.1119	2.114	2.1582
<i>E</i>	0.5894	0.5877	0.5823	<i>E</i>	1.1913	1.2014	1.1964
<i>S</i>	0.985	0.9866	0.9964	<i>S</i>	1.2256	1.255	1.2293
Kingdom of Cambodia	2010	2011	2012	Rwanda	2010	2011	2012
<i>F</i>	0.3931	0.4139	0.4116	<i>F</i>	0.4417	0.4174	0.4109
<i>E</i>	0.9964	0.9849	0.9747	<i>E</i>	0.5105	0.5121	0.5279
<i>S</i>	0.9437	0.946	0.9734	<i>S</i>	1.2155	1.1824	1.1721

5 The development plan for Cambodia

5.1 Analysis of Cambodia

According to the Task 2, Cambodia is selected and analysed through the 48 least developed countries by using the model of this paper. The results of the analysis we can see, in economic terms, Cambodia's economic development level is far below the boundary of evaluation given by last chapter, but the gap between the rich and the poor people is not so great. As for the environment, although the reason for geographic location leads cultivated land area to be less and freshwater resources to be not enough, the whole natural ecological environment is protected very well. But the whole countries on the use of natural resources is still at a relatively low level. In the social aspect, the national average life expectancy



is at a high level and employment rate is also very high. In addition, the country relatively pay attention to the education, but it still has a long distance of the process of cities and public health expenditure.

Overall, Cambodia can be able to become a sustainable country, while there are some problems in its economic and social development. One reason that its economic level is far lower than the standard of evaluation belongs to a big problem, secondly the social aspects of these problems that the main city of the process speed is too slow and the public health expenditure is mainly related with the economy, however, the biggest problem is the economy.

5.2 Create the plan

In this regard, a 20 year plan for the case will be designed in order to develop Cambodia and make it become more sustainable.

Economy:

a) Take the natural advantage of national geographic location, in the premise of not destroying its natural ecological environment, the plan will greatly stimulate the rapid growth of its tourism and service industry.

b) Encourage and support the development of private enterprises;

c) Attract foreign investment and stimulate domestic economic growth.

d) Import high technology, update the industry structure, reform economic system, and rapidly develop light industry and high added value industries.

e) Apply high-tech in basic industries such as agriculture, increasing the productivity and efficiency of these industries.

f) Promote economic exchanges with other countries, and learn economic system advantages from other countries.

Environment:

a) Develop exploration and natural resources boldly and get maximum efficiency in the premise of not damaging the ecological environment.

b) Put an end to large amount of damaging to the environment resolutely at the expense the mode of economic growth.

c) Establish the ecological protection zone, which can not only protect the ecological environment but also the development of the tourism industry.

d) Educate citizens from the habit of saving water and cherish the water resource. Social:

a) Increase public spending on health care to give better physical health and greater security of citizens.

b) Increase funds of education and scientific research, education and science are the important motive force of a country.

c) Build large cities suitably for places which have abundant tourism resources to attract more visitors.

d) Increase the process of urbanization.

Aid:

ICM can provide a certain amount of financial support, such for supporting above factors.

6 Sustainability prediction model based on Markov chain

6.1 Model Set up

Through the analysis of the second question which reveals developing the planned policy's effect on the 21 indicators, and the combination of the effects of climate change and foreign investment on it, we give the following list analysis.

Planned Policy	Influential Index
economic policy 1	F-1,F-2,F-4,F-6,F-7,E-1,E-2,S-4,S-7
economic policy 2	F-1,F-2,F-4,F-6,F-7,S-7
economic policy 3	F-1,F-2,F-3,F-4,F-6,F-7,E-3,E-6,S-4,S-5,S-7
economic policy 4	F-1,F-2,F-3,F-4,F-5,F-6,F-7,E-2,E-3,E-6,E-7,S-7
economic policy 5	F-1,F-2,F-4,F-6,F-7,E-2,E-5,E-6
economic policy 6	F-1,F-2,F-3,F-4,F-5
environmental policy 1	F-1,F-2,F-6,E-2,E-3,3-6,E-7,S-7
environmental policy 2	E-1,E-2,E-3,E-4,E-5,E-6,E-7
environmental policy 3	E-1,E-4,E-5,E-6,E-7
environmental policy 4	F-1,E-4
social policy 1	F-1,F-2,F-4,F-7,
social policy 2	F-1,F-2,,F-4,F-5,F-7,E-2,E-3,E-4,E-6,S-5,S-6
social policy 3	F-1,F-2,F-4,F-7,E-1,E-5,S-4,S-6
social policy 4	F-1,F-2,F-4,F-7,E-1,E-5,S-4,S-6
aid	F-1,F-2,F-3,F-4,F-6,F-7
climate change and natural disasters	E-1,E-3,E-4,E-5,S-1
foreign investment and aid	F-1,F-2,F-3,F-4,F-6,F-7,S-1,S-6,S-7

where $F - i$ is the No.i factor of the Economic area.

According to the table, we can obtain the state transition matrix and the change of state vector for 21 factors (seen in the accessory) .

The initial vector of the initial state vector(the first year state of index)is

$$I_{f_i}(0) = I_{e_i}(0) = I_{s_i}(0) = (0, 0, 1, 0)^T$$

Thus, the state vectors for indexes after n years are:

$$I_{f_i}(n) = C_{f_i}^n I_{f_i}(0)$$

$$I_{e_i}(n) = C_{e_i}^n I_{e_i}(0)$$

$$I_{s_i}(n) = C_{s_i}^n I_{s_i}(0)$$

let $n = 20$, then we can obtain the state probability vectors after twenty years are $I_{f_i}(20)$, $I_{e_i}(20)$ and $I_{s_i}(20)$, where $i = 1, 2, \dots, 7$ We can get 21 vectors in total.

The state of the n years later can be predicted by the expectation of state transition, then we can get the growth rate of indicators:

$$h_{f_i}(n) = (I_{f_i}(n))^T \cdot a_{f_i}$$

$$h_{e_i}(n) = (I_{e_i}(n))^T \cdot a_{e_i}$$

$$h_{s_i}(n) = (I_{s_i}(n))^T \cdot a_{s_i}$$

Where h_i is the predicted growth rate of the three sustainability indexes, while $n = 1, 2, 3, \dots, 20$

Then the sustainability indexes of economy, environment and society after n years can be calculated:

$$f_i(n) = \prod_{j=1}^n h_{f_i}(j) \times f_i$$

$$e_i(n) = \prod_{j=1}^n h_{e_i}(j) \times e_i$$

$$s_i(n) = \prod_{j=1}^n h_{s_i}(j) \times s_i$$

6.2 Prediction for Cambodia

According to the prediction model based on Markov chain above, we predict the 20-year development of Cambodia. Figure.7 showed the development tendency of the three indexes during the 20 years. It can be seen that the three indexes are going high all the time.

The result can conclude that our plan are efficient, with the consideration of varies of factors.And it can also prove that our model in Task 1 is credible and useful.

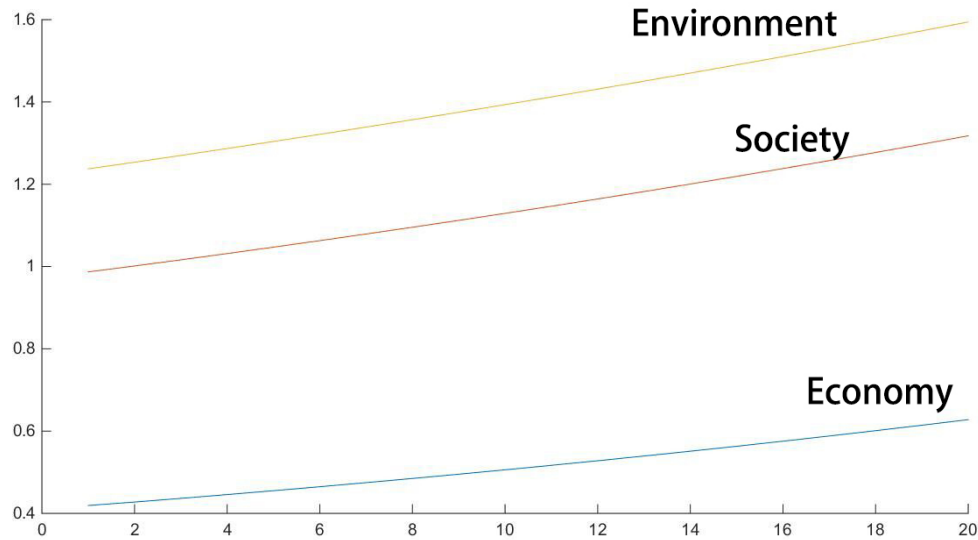


Figure 6 The development tendency of the three indexes during 20 years

6.3 Highly effective strategies

The strategy to Identify high influence:

Firstly we define the comprehensive sustainability indexes, and the index of economy, environment and society are given the weight of 0.4, 0.4 and 0.3.

Therefore, the strategy with high influence can be defined as the strategy to provide the sustainability index with most improvement. $\max \left\{ \frac{0.4w_i(f_i^n - f_i)}{0.4F + 0.3E + 0.3S}, \frac{0.3x_i(e_i^n - e_i)}{0.4F + 0.3E + 0.3S}, \frac{0.3y_i(s_i^n - s_i)}{0.4F + 0.3E + 0.3S} \right\}$

This formula is to find the factor that increase the sustainability most.

7 Strengths and weaknesses

Strengths:

a) It comprehensively and objectively reflects the degree of a country's sustainable development ability, with strong practicability.

b) The needed data and statistics of the calculation can be obtained easily, and the calculation is not complicated

Weaknesses:

a) The weightings between subordinate indexes are lack of authority, it is difficult to measure the real important contrastive relationship between these indicators

b) the correlation between subordinate indexes is strong, mutual influence between them is very

difficult to describe

8 Model extension

This model uses comprehensive evaluation methods, and quantifies sustainable or unsustainable critical value. It can be widely used to quantify the fuzzy concept in a variety of large number of factors. For example, in judging whether a person is a moral model, or judging Miyoshi students can use this model. Can be widely used in the field of fuzzy mathematics. Under this title for sustainable development, the model is also for the details, such as index selection, fitting normal distribution, predicting state of different period.

The need to increase forest area, we can increase its cover area by reverting the farmland to forest, which improves the air condition. Besides, Compared with sustainable nation, the developing one should pay more attention to the development of alternative energy sources, such as renewable fuel research. While the oil use should be reduced as far as possible. Using the new available fuel [8] with high-tech or improving the efficiency of fuel to reduce emissions of greenhouse gases, aiming to improve the environment. The reduction of PM10 is necessary, which has important significance for the human health. It's vital to reduce the number of heavily polluting factories and improve output efficiency to reduce pollution or purchase new sewage purification system. And in the improvement of the human survival environment, we can see the importance of improving science and technology, such as the construction of Green house to the environmental progress. Rather than the premise of the whole housing construction, IOT intervention can reduce building energy consumption, such as smart watch detection control air-conditioning temperature to save energy.

The total public expenditure on education has great influence to the development of a country, and the development of the Internet is quite good today, internet education has become a high efficiency, strong pertinence, anywhere with network is studying available. Therefore, this model may wish to join a major influencing factors stupendously, such as a country's new materials discovery, or major disasters. And a new model for aid or rapidly developing will be created after adding an important indicator.

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