Cosmological Models for Late Time Acceleration of the Universe

Banani Kashyap

Project Supervisor: Prof. T.R Seshadri

DISSERTATION REPORT

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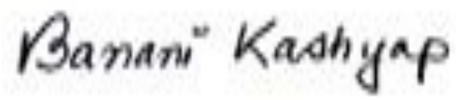
CERTIFICATE

This is to certify that the Dissertation entitled **Cosmological Models For Late Time Acceleration of The Universe**, is a bonafide record of independent work done by me under suprvison of **Prof. T.R. Seshadri** and is submitted to the **Department of Physics and Astrophysics, University of Delhi**, in partial fulfilment for the award of Degree **Master of Science in Physics**.



Signature of the Student

Signature of the Supervisor

Banani Kashyap

Examination Roll No:19025762011

College: Hansraj College

Date: June 30, 2021

Abstract

From observations over the past two decades and a half, it is fairly clear that the universe is at present undergoing an accelerated expansion and that it entered the late-time accelerated phase. While several theoretical approaches have been adopted to explain the precise origin of this accelerated expansion phenomenon, a satisfactory solution is still a question to the cosmologists. The observational evidence suggests a possible probe of explaining the source of cosmic acceleration is some kind of vacuum energy with negative pressure that makes up 70% of the total constituents of the universe, called dark energy. The pressure p and density ρ should be related by $p < -\rho/3$ in order to satisfy the acceleration condition. The standard ΛCDM Model is a well-accepted dark energy model where dark energy is considered as the result of cosmological constant Λ with the equation of sate $p = -\rho$. However, the tuning problem of the cosmological constant leads to the search for different approaches to explain cosmic expansion. Several generalizations have been studied in the literature of which the main ones are higher dimensional cosmology and the f (R) theories of gravity. My work focuses on the latter type of generalization. I tried to construct a generalized model with f(R) gravity that is consistent with ΛCDM model. I have found a class of f(R) that takes the value of cosmological constant at the epoch of decoupling($z \sim 1100$) and found their evolution to the present epoch($z = 0$). I have also constructed another family of curves for f(R) that take the value of cosmological constant as its final state at the present epoch($z = 0$)and showed their devolution to the epoch of decoupling($z \sim 1100$).

Contents

1 Introduction

Cosmology is the study of the origin, evolution, and existence of the universe. The universe as a whole consists of a vast range of structures from planets to stars to galaxies to galaxy clusters and superclusters. However, the subject line deals with the objects at very large distances, extremely bigger in size, and over a long time scale[17].

The understanding of the universe by mankind has evolved significantly over time with the advancement of scientific tools. In the early history of astronomy, Earth was regarded as the center of all things, with planets and stars orbiting it. In the 16th century, Polish scientist Nicolaus Copernicus suggested that Earth and the other planets in the solar system is orbiting the sun, creating a profound shift in the understanding of the cosmos. In the early 17th century Galileo Galilei developed the telescope and used it to explore the universe. In the same decade, Johannes Kepler gave the laws of planetary motion of planets based upon observations. In the late 17th century, Issac Newton formulated the laws of mechanics and given the universal law of gravitation that two massive bodies are gravitationally bound. He considered space to be an infinite Euclidean plane in which gravitationally bound structures are distributed uniformly and time to be absolute[22]. Thus, these structures remained in balance, although unstable making the universe as a whole static and infinite. However, Newton's foundation actually did not generate the solution for a static universe as one can intuitively argue: If we imagine a static universe consisting of a number of uniformly distributed galaxies, then since gravity is an attractive force, there will always be a collapse, and hence the universe cannot be static. But Newton's model of the universe is still considered the first scientific foundation of cosmology.

Approximately two centuries later, in 1917, Albert Einstein proposed the theory of general relativity in order to explain the dynamics of the universe more adequately. His theory is based on the description of space-time by Riemannian Geometry in four dimensions with three spatial and one temporal or time dimension. His infamous field equation relates the distribution of matter and energy with the curvature of space-time, which yields Newton's theory of gravitation as a limiting case for weak, stationary field and non-relativistic particles. He proposed the Perfect Cosmological Principle that at larger scale distribution of matter is homogeneous and isotropic in the universe. Einstein soon discovered that his general relativity field equations did not allow for a static solution as in the case of Newtonian cosmology. Einstein then introduced another force λr in his formulation that acts as a repulsive force to balance the gravity force where Λ being called "The Cosmological Constant". for Λ sufficiently small, the new force is unimportant at "small" distances relevant for our solar system. However, at larger distances of cosmological interest, this force can provide enough repulsion to stabilize the situation and solution of the field equation provides a static universe without any global movement or expansion.

Later, in 1922 Friedmann and in 1927 Lemaitre independently found that this static Universe wasn't the unique solution. They discovered that the cosmological constant was not sufficient to maintain the system in balance, because any disturbance and the Universe expands or contracts, depending of the density of its matter. This model supposes a closed universe, but similar solutions are possible involving an open universe (which expands infinitely) or a flat universe (in which expansion continues infinitely but gradually approaches a rate of zero). But at that time those models were strictly not admissible as Einstein had already formulated the idea of a static universe.

It was Hubble's discovery in 1929 about the linear relation between the redshift and distance of the galaxies that concludes that the universe is not static in nature rather it is expanding. Post-Hubble's discovery Einstein dropped the cosmological constant from his equation [17] and along with William de Sitter in 1932 proposed a cosmological model with Vanishing curvature and Cosmological Constant but matter only. Nevertheless, in the 1930s Eddington and Lemaitre also proposed a model that includes Λ term. But still, it was unknown till a few decades whether the universe is expanding in an accelerated manner to infinity or it first expands from a singularity then decelerates to a single point. All those questions generally gave rise to another field of study for cosmology for the upcoming years.

In 1998, two teams studying distant Type Ia supernovae presented independent evidence that the expansion of the Universe is speeding up [18; 20]. The discovery of cosmic acceleration is arguably one of the most important developments in modern cosmology[9]. Over years people have found different ideas of cosmological models to explain the acceleration of the universe, that includes Modified and Non-modified Gravity Models, but a highly acceptable model that is admissible by observations is still the Standard ΛCDM Model.

1.1 Cosmological Principle

Albert Einstein's theory of General Relativity permits many possible types of universes. In applying the theory to describe the dynamics of our Universe, Einstein made a central empirical assumption to limit the number of possible solutions to the equations. He assumed that on very large scales the distribution of matter in the Universe is constant, making the Universe appear smooth. This idea is a form of the modern cosmological principle.

This principle is not exact since much of the Universe's matter is found clustered together in planets, stars, and galaxies, but when considered at sufficient scales the distribution of galaxies and clusters is approximately even. The idea of Homogeneity and Isotropy is considered to be valid at a much larger scale.

Hubble's discovery, coupled with the modern interpretation of the cosmological principle, led to the development and eventual acceptance of the Big Bang model. Based on the theoretical work of Alexander Friedmann and Georges Lemaitre, the model describes the fiery origins of the Universe as a "primordial atom" and its subsequent evolutionary history of expansion and cooling.

1.2 The Robertson-Walker Line Element

For a maximally symmetric space obeying the cosmological principle, a homogeneous and isotropic metric was derived in the 1930s by H.P. Robertson and Walker independently often referred to as $Robertson-Walker\ Line\ Element$. Although we are not following the rigorous derivation[25], a partly intuitive and partly heuristic argument can be made as given as [17].

The metric tensor $g_{\mu\nu}$is introduced in four dimension($\mu, \nu = 0, 1, 2, 3$) as:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1.2.0.1}$$

If the distribution of matter is uniform, then the space is uniform and isotropic. This, in turn, means that one can define a universal time (or proper time) such that at any instant the three-dimensional spatial metric $dl^2 = g_{ij} dx^i dx^j$ and the space-time metric(in natural units where c = 1) must be of the form:

$$ds^2 = -dt^2 + dl^2 = -dt^2 + g_{ij} dx^i dx^j \tag{1.2.0.2}$$

The Robertson–Walker metric is obtained from (1.2.0.2), where the spatial part is simply the three-dimensional homogeneous and isotropic space. One finds that for the three-dimensional flat, positively curved, and negatively curved spaces one has

$$dl^2 = a^2(dr^2 + r^2 d\Omega^2) \tag{1.2.0.3}$$

$$dl^2 = a^2(d\chi^2 + sin^2\chi d\Omega^2) = a^2(\frac{dr^2}{1-r^2} + r^2 d\Omega^2) \tag{1.2.0.4}$$

$$dl^2 = a^2(d\chi^2 + sinh^2\chi d\Omega^2) = a^2(\frac{dr^2}{1+r^2} + r^2 d\Omega^2) \tag{1.2.0.5}$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$; $r = sin\chi$ with $0 \leq \chi \leq \pi$ in (1.2.0.4) and $r = sinh\chi$ with $0 \leq \chi < \infty$ in (1.2.0.5). If one combines all these equations for the value of $k = 0, 1, -1$ correspond, respectively, to the Hypersphere, Euclidean space and space of constant negative curvature to get the form of (1.2.0.2) as,

$$ds^2 = -dt^2 + a^2(t)(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)) \tag{1.2.0.6}$$

This is called the Robertson-Walker metric or commonly known as Friedmann-Lemaitre-Robertson-Walker Line Element metric. In the FLRW metric:

- r, θ, ϕ are comoving coordinates and are time-independent and t is the cosmic time.
- k is a constant and is a measure of intrinsic curvature. Without loss of generality, we can scale the coordinate r in such a way that k can take either of the three values +1, 0, -1 corresponding to a closed, flat and open universe respectively.
- a(t) is called the cosmic scale factor. It is a measure of the spatial size of the universe at time t. Although expansion rate is constant in all directions at a given time, it changes with time and is therefore given as a function of cosmic time as $H(t)$ known as Hubble Parameter.

1.3 Hubble's Law

The extraordinary evidence that we live in an expanding universe was first evident by Edwin Hubble in 1929. Although this landmark discovery was usually attributed to Edwin Hubble, certainly resulted from the joint efforts of astronomers such as Vesto Slipher [Slipher, 1917] and cosmologists such as George Lemaitre [Lemaitre, 1927].

Hubble discovered that the farther a galaxy is the faster it recedes from us that redshifts of galaxies increase roughly in proportions to the distances from us as given in Figure1.

$$v = H_0 r \tag{1.3.0.1}$$

where v is the recessional velocity, r is the distance and H_0 is a constant named after Hubble. The value of H_0 determined by Hubble himself was with huge error. A more precise estimate was made by Sandage in 1958: $H_0 = 75\ kms^{-1}Mpc^{-1}$ A recent measurement done by the BOSS collaboration [10] gives $H_0 = 67.6^{+0.7}_{-0.6}\ kms^{-1}Mpc^{-1}$ Roughly speaking, this number means that for each Mpc away a source recedes 67.6 km/s faster. At a certain radius, the receding velocity attains the velocity of light and therefore we are unable to see farther objects. This radius is called Hubble radius. The Hubble constant can be measured also with fair precision by using the time delay among variable signals coming from lensed distant sources [5] to be $H_0 = 71^{+2.4}_{-3.0}\ kms^{-1}Mpc^{-1}$ and via gravitational waves [1] to be $H_0 = 70^{+12.4}_{-8.0}\ kms^{-1}Mpc^{-1}$.

As already mentioned, Hubble's constant is strictly a constant only at a particular instant of time, and Hubble's Law can be given as $v = H(t)r$ it changes with time and is connected to the expansion factor as:

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{1.3.0.2}$$

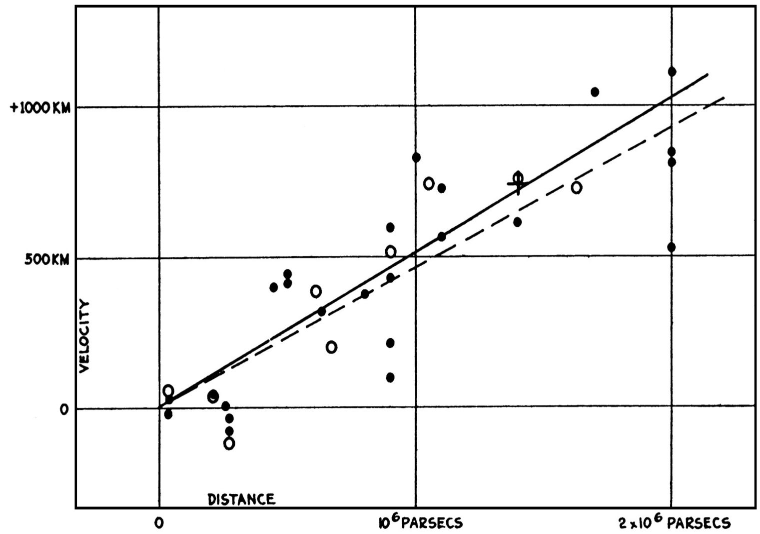


Figure 1: Edwin Hubble's original plot of the relation between redshift (vertical axis) and distance (horizontal axis)[Hubble,1929][13]

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1.4 Friedmann equations

Einstein's Field Equation with the Λ term is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu} \tag{1.4.0.1}$$

where $\kappa^2 = 8\pi G$ and $T_{\mu\nu}$ is the energy momentum tensor. Energy-Momentum Tensor for a perfect fluid is given as

$$T_{\mu\nu} = -p_m g_{\mu\nu} + (p_m + \rho_m) u_\mu u_\nu$$

$p = \sum p_i$ and $\rho = \sum \rho_i$ is the total pressure and energy density of the fluid components.But one can take Λ inside the $T_{\mu\nu}$ and modified field equation can be given as:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 \tilde{T}_{\mu\nu} \tag{1.4.0.2}$$

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{\Lambda}{8\pi G} g_{\mu\nu} = -p g_{\mu\nu} + (p + \rho) u_\mu u_\nu \tag{1.4.0.3}$$

The knowledge of the cosmological principle and hence the FLRW metric alone cannot determine a(t) and k unless the FLRW metric with dynamical equations for a(t) is considered. Einstein's general theory of relativity is that theory of gravitation which will be used to impose constraints on FLRW metric to get fundamental equations of dynamical cosmology commonly known as the Friedmann equations. Using the FLRW line element given in equation (1.2.0.6), one can find the solutions for the field equation as:

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{1.4.0.4}$$

$$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p) \tag{1.4.0.5}$$

Equation (1.4.0.4) can also written in terms of density parameter $\Omega = \frac{\rho}{\rho_c}$ using critical density $\rho_c(t) = \frac{3H^2(t)}{8\pi G}$ as:

$$\frac{k}{a^2} = (\Omega - 1)H^2 \tag{1.4.0.6}$$

If we consider different constituent of the universe separately so that $\Omega = \Omega_{m0} + \Omega_{k0} + \Omega_{\Lambda 0} + \Omega_{r0}$ with Ω_{m0}, Ω_{k0}, $\Omega_{\Lambda 0}$ and Ω_{r0} are the density parameters for matter, curvature, Cosmological Constant and radiation respectively then the equation (1.4.0.6) takes the form[21]:

$$\frac{H^2}{H_0^2} = \Omega_{\Lambda 0} + \frac{\Omega_{k0}}{a^2} + \frac{\Omega_{m0}}{a^3} + \frac{\Omega r0}{a^4} \tag{1.4.0.7}$$

For different constituents, the solution of these equations generate a solution of the scale factor $a(t)$ and for a given equation of state, $p = w\rho$ (w is the equation of state parameter) we can calculate the energy density ρ as a function of scale-factor and a(t) as a function of time for different constituents of the universe as given in Table 1.

Constituent	w	p	a(t)	ρ
Dust	0	0	$a \propto t^{\frac{2}{3}}$	$\rho_m \propto a^{-3}$
Radiation	1/3	$\rho_r/3$	$a \propto \sqrt{t}$	$\rho_r \propto a^{-4}$
Cosmological Constant	-1	$-\rho$	$a \propto e^{Ht}$	$\rho_\Lambda = constant$

Table 1: Relations between parameters for a single component universe

1.5 The Deceleration Parameter

The Hubble parameter H(t) measures the expansion rate at any particular time t for any model obeying the Cosmological Principle. It does, however, depending upon the contents of the Universe H(t) vary with time. One can express this by expanding the cosmic scale factor for times t close to t_0 in a power series:

$$a(t) = a(t_0) + (t - t_0)\left.\frac{\dot{a}}{a}\right|_{t=t_0} + (t - t_0)^2\left.\frac{\ddot{a}}{a}\right|_{t=t_0} + ...$$

Introducing a parameter

$$q(t) = -\frac{\ddot{a}(t)}{a(t)a^2(t)} \tag{1.5.0.1}$$

using (1.3.0.2) and $a(t_0) = 1$ and keeping upto second order we get the scale factor as:

$$a(t) = 1 + (t - t_0)H_0[1 - \frac{q_0}{2}(t - t_0)H_0] \tag{1.5.0.2}$$

here $q(t)$ is called the deceleration parameter; the suffix '0', as always, refers to the fact that $q_0 = q(t_0)$, the present value of the deceleration parameter. In [Riess et al., 1998] and [Perlmutter et al., 1999] analysis based on type Ia supernovae observation have shown that $q_0 < 0$ i.e the deceleration parameter is negative, and therefore the universe is in a state of accelerated expansion.

1.6 Friedmann Models

In 1922, Friedmann had set the appropriate framework for a GR cosmology by introducing its most general metric and the "Friedmann equations," which describe the evolution of a perfect-fluid cosmos of uniform mass density ρ. And he elucidated all three major scenarios for a nonstatic universe consistent with GR. In fact, he introduced the expression "expanding universe"[4].

Considering the universe to be constituting of dust only, rewriting (1.4.0.4) as

$$\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{1.6.0.1}$$

so at present epoch,

$$H_0{}^2 = \frac{8\pi G}{3}\rho_0 - k \implies \rho_0 = \frac{3}{8\pi G}(H_0{}^2 + k) \tag{1.6.0.2}$$

now for p=0, (1.4.0.5)-(1.4.0.4), using (1.5.0.1), one can relate k and q_0 as

$$k = (2q_0 - 1)H_0{}^2 \tag{1.6.0.3}$$

using (1.6.0.3) we can write (1.4.0.4) as

$$\dot{a}^2 = H_0^2[1 - 2q_0(1 - a^{-1})] \tag{1.6.0.4}$$

Boundary Conditions $a(0) = 0$ and $a(t_0) = 1$

1.6.1 Flat Model with k=0:

often called Einstein de Sitter Model from (1.6.0.3) $q_0 = \frac{1}{2}$ so equation (1.6.0.4) becomes

$$\dot{a}^2 = H_0^2 a^{-1}$$

On integrating, solution for the scale factor comes out to be

$$a(t) = (t/t_0)^{2/3} \tag{1.6.1.1}$$

where $t_0 = \frac{2}{3H_0^2} \implies$ present age of the universe

1.6.2 Closed Model with k=1:

For k=1, from (1.6.0.3) $q_0 = \frac{1}{2}(1 + H_0^{-2}) > \frac{1}{2}$
Therefore Equation (1.6.0.4) gives

$$\dot{a}^2 = (H_0^2 + 1)a^{-1} - 1$$

The parametric solution to this equations are

$$a = \frac{q_0}{2q_0 - 1}(1 - \cos\chi) \tag{1.6.2.1a}$$

$$t = \frac{q_0(\chi - \sin\chi)}{H_0(2q_0 - 1)^{3/2}} \tag{1.6.2.1b}$$

Applying the boundary conditions the age of the universe is found to be

$$t_0 = \frac{q_0}{H_0(2q_0 - 1)^{3/2}}[\cos^{-1}(\frac{1 - q_0}{q_0}) - \frac{(2q_0 - 1)^{1/2}}{q_0}] \tag{1.6.2.1c}$$

1.6.3 Open Model with k=-1:

For k=-1, from (1.6.0.3) $q_0 = \frac{1}{2}(1 - H_0^{-2}) < \frac{1}{2}$
Therefore Equation (1.6.0.4) gives

$$\dot{a}^2 = (H_0^2 - 1)a^{-1} + 1$$

The parametric solution to this equations are

$$a = \frac{q_0}{1-2q_0}(\cosh\xi - 1) \tag{1.6.3.1a}$$

$$t = \frac{q_0(\sinh\xi - \xi)}{H_0(1-2q_0)^{3/2}} \tag{1.6.3.1b}$$

Applying the boundary conditions the age of the universe is found to be

$$t_0 = \frac{q_0}{H_0(1-2q_0)^{3/2}}[\frac{(1-2q_0)^{1/2}}{q_0} - \cosh^{-1}(\frac{1-q_0}{q_0})] \tag{1.6.3.1c}$$

The fate of the universe that can be obtained from these solutions are illustrated in the Figure 2

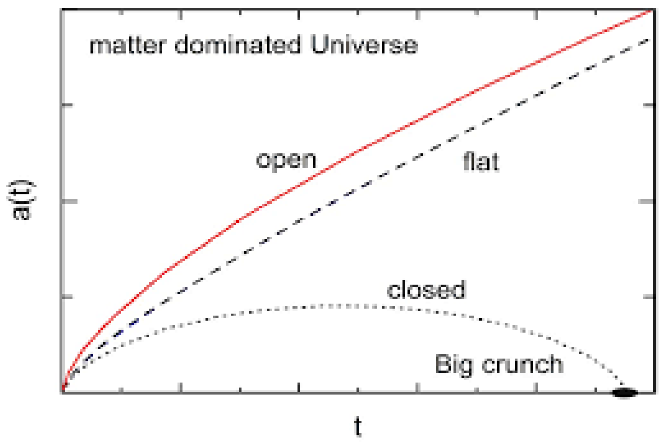

Figure 2: Figure showing how the size of the universe as a whole depends on time for a dust-only universe. From the plot, it is evident that the universe forever expands, "just barely" expands forever, expands to a maximum radius, and then re collapses for $k = -1, 0, 1$ cases respectively. Image Source:

2 Acceleration of The Universe

Post-Hubble's discovery, most scientists believed that there was no cosmological constant for the next few decades. The matter-dominated universe is expected to have a slow down the expansion and the rate of slowing down is solely dependent on the amount of matter present. In the mid-1990s, two competing teams initially intended to observe the slowing down rate by observing supernova found out just the opposite that the universe is not slowing rather is expanding at an accelerated rate.

Two breakthroughs enabled the discovery of cosmic acceleration. Taking type Ia supernovae (SNe Ia) are standard candles, The High-z SN Search team[20] used the SN Hubble diagram to lager distance possible. Both the teams found out distant SNe($0.16 \leq z \leq 0.97$) are 0.25 to 0.28 mag dimmer than they are expected to be appeared in a decelerating universe indicating a positive value for cosmological constant and $\Omega_{\Lambda 0} > 0$ with 99% confidence. The distance modulus and redshifts of SNe obtained from the spectroscopic analysis show that the universe is currently undergoing an accelerated expansion($q_0 < 0$) and the dynamical age of the universe is obtained as 14.2 ± 1.5 Gyr. The Supernova Cosmology Project[18] reported the measurements of the mass density, Ω_{m0} and cosmological-constant energy density, $\Omega_{\Lambda 0}$, of the universe based on the analysis of 42 Type Ia supernovae discovered by the Supernova Cosmology Project. For a flat universe($\Omega_\Lambda + \Omega_m = 1$) with cosmological constant they found $\Omega_{m0} = 0.28^{+0.09}_{-0.08}$.

2.1 Observational evidences for Dark Energy

From observations over the past two decades and a half, the universe has entered the late-time accelerated phase in recent times. While several theoretical approaches have been adopted to explain this phenomenon, a satisfactory solution has till now eluded cosmologists. One of the challenging problems of modern cosmology is to identify the cause of this late time acceleration. Within the framework in which gravity is described by the Einstein-Hilbert action, any normal (i.e, with positive pressure) kind of matter as the source term for expansion will produce a decelerated cosmological expansion. Within the framework of this action, the late time accelerated cosmological expansion can be explained only by assuming the existence of a strange kind of matter with negative pressure. In fact, negative pressure is a necessary condition but not the sufficient one. In equation (1.4.0.5) in order to get $\ddot{a} > 0$, The pressure p and density ρ should be related by $p < -\rho/3$. Such kind of matter is collectively called Dark Energy. The simplest candidate for dark energy is the cosmological constant. The cosmological constant can be brought to the right-hand side of Einstein's Equations and can be interpreted as a constant energy density. The pressure turns out to be just the negative of density. Hence for this $p = -\rho$ and thus satisfies the condition for dark energy namely $p < -\rho/3$. Such a model is called ΛCDM. This model reproduces all the main observations, Age of the universe, the dimming of type Ia Supernovae, Cosmic Microwave Background Radiation(CMBR) anisotropies, Large Scale Structure formation, Baryon oscillations and Weak Lensing.[26].

2.1.1 The age of the Universe

The age of the Universe, t_0 is a rough measure in order of H_0^{-1}. One can calculate the age by considering different models and comparing it to the age of some oldest globular clusters present today. One can neglect the contribution of the redshift to be zero at smaller redshift($\Omega_{r0} \to 0$, and the equation (1.4.0.7) can be used to measure the presence of the universe. In absence of dark energy ($\Omega_{\Lambda 0} = 0$), for a flat universe($k = 0$) the age comes out to be in a rage from 8.2 Gyr to 10.2 Gyr[3]. However, some of the oldest globular clusters present show a lifetime larger than 11 Gyr[6][14][11]. In order to get cosmic age larger than 11 Gyr, one may need $0 < \Omega_{m0} < 0.55$. So the only two possible models remains are : An open Universe($\Omega_{m0} + \Omega_{k0} = 1$) without the cosmological constant($\Omega_{\Lambda 0} = 0$) or a flat universe($\Omega_{k0} = 1$) with the cosmological constant($\Omega_{m0} + \Omega_{\Lambda 0} = 1$). An open universe without Λ also fulfills the condition $t_0 > 11 Gyr$ but since the curvature, $\Omega_k 0$ has been constrained to be much smaller than unity, this model does not satisfy the Cosmic age bound of $t_0 = 13.73 \pm 0.12$ Gyr given by the WMAP measurements[15]. The only possible model that can satisfy the observational bound is a flat universe with a *Dark Energy* ($\Omega_{\Lambda 0} > 0$) and matter-energy density $0.271 < \Omega_{m0} < 0.289$.

Therefore, the existence of dark energy has been considered as one of the crucial conclusions in the case of the observed constraint to the age of the universe.

2.1.2 Supernovae type Ia

Type Ia Supernova observations are one significant piece of evidence for cosmic acceleration [20][18]. The explosion of Type Ia occurs when the mass of a white dwarf in a binary system exceeds the Chandrasekhar limit by absorbing gas from another star. At the peak of the explosion, SN Ia always has the same luminosity and almost constant brightness so that it can be used as a standard candle. Knowing the absolute magnitude and measuring the apparent magnitude, we can estimate the luminosity distance. The redshift z of the corresponding SN Ia can be found by measuring the wavelength Λ of light. The observations of many SN Ia provide the dependence of the luminosity distance d_L in terms of z. Comparing observational data with the theoretical distance, it is possible to know the expansion history of the Universe for a redshift of the order less than 1. the observational data in the high-redshift regime favor the luminosity distance larger than the one predicted by the CDM model($\Omega_{m0} = 1$ and $\Omega_{\Lambda 0} = 0$). This changes the whole idea of an expanding universe.

Since its detection there had been a lot of questions on the cause of the dimming of SN. Many scientists had a doubt that the distant supernovae could appear fainter just due to extinction. Absorption by intervening dust could also lead to characteristic reddening[26]. But recent observation of the Hubble space telescope shows the observations for redshift greater than 1 from ground-based observations resulting in the conclusion of cosmic acceleration as the reason for the dimming of a distant supernova.

2.1.3 Cosmic Microwave Background Radiation

The observation in the CMB anisotropy is another important aspect for probing dark energy. At the surface of the last scattering surface or before the recombination epoch photons are tightly coupled to baryons and electrons before the decoupling epoch at $z \sim 1100$, but they could freely move to us after that. Penzias and Wilson in 1963 first detected CMB photons thermalized to almost uniform temperature across the sky but in 1992, the COBE satellite first measured the temperature anisotropies of the CMB at large angular separations. The angular power spectrum of CMB temperature anisotropies, measured by WMAP and by ground-based experiments that probe to smaller angular scales, is dominated by acoustic peaks that arise from gravity-driven sound waves in the photon-baryon fluid[26]. The positions and amplitudes of the acoustic peaks encode a wealth of cosmological information. The combination of CMB, SN Ia observations, Larger Scale Structure Measurements indicates that our Universe is spatially flat and the matter filled with the $\Omega_{m0} \sim 0.3$ that implies the need for missing components as dark energy with $\Omega_{\Lambda 0} = 1 - \Omega_{m0} \sim 0.7$.

2.1.4 Baryon Acoustic Oscillation

Studying BAO has been significant evidence for dark energy cosmology. As mentioned above baryons are tightly coupled to photons before the recombination epoch. After decoupling baryons stay at the distance of the sound horizon whereas dark matter stays at the center of overdensity. They attract matter and eventually form galaxies. Therefore it is expected that a number of galaxies are separated by the sound horizon. By observing the large-scale structure (LSS) of galaxies, one can measure the sound horizon scales and compare them with theoretical predictions. The oscillations of sound waves should be imprinted in the baryon perturbations as well as the CMB temperature anisotropies[3]. Eisenstein[8] found a peak of baryon acoustic oscillations measured from a spectroscopic sample of 46,748 luminous red galaxies in the large-scale correlation function peak at $100h^{-1}Mpc$ separation from the Sloan Digital Sky Survey (SDSS). From the overall shape of the correlation function and independent of the constraints provided by the CMB acoustic scale, we found $\Omega_{m0} = 0.273 \pm 0.025 + 0.123(1+w_0) + 0.137\Omega_{k0}$. Including the CMB acoustic scale, we find that the spatial curvature is $\Omega_{k0} = -0.010 \pm 0.009$ if the dark energy is a cosmological constant This detection of baryon oscillations provided another independent test for constraining the property of dark energy.

The precise origin of this accelerated expansion is as yet not fully understood. One of the challenging problems of modern cosmology is to identify the cause of this late time acceleration. Within the framework in which gravity is described by the Einstein-Hilbert action, any normal (i.e., with positive pressure) kind of matter as the source term for expansion will produce a decelerated cosmological expansion. Within the framework of this action, the late time accelerated cosmological expansion can be explained only by assuming the existence of a strange kind of matter with negative pressure.

2.2 The Standard ΛCDM Model

The non-zero positive value of the cosmological constant strongly indicates that one construct a cosmological model to obtain the solution for the cosmic scale factor a(t) for the expanding universe with Ω_{m0} $\Omega_{\Lambda 0}$, Einstein Hilbert-Action have been reconstructed with the cosmological constant as as

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R-2\Lambda] + \int d^4x\sqrt{-g}L_m \tag{2.2.0.1}$$

Varying this action (2.2.0.1) with respect to $g_{\mu\nu}$ gives the Einstein's equations can be expressed as as given in equation (1.4.0.2), and so Friedmann equations are modified to the form (1.4.0.4) with $\rho = \rho_m + \rho_\Lambda$. For the flat universe k=0 Friedmann Equations are now modified to the form:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \tag{2.2.0.2}$$

Here, $\rho = \rho_m + \rho_\Lambda$, ρ_m is the energy density due to matter and ρ_Λ is the energy density due to the cosmological constant.

Observations indicate that the energy-momentum tensor in the universe is predominantly due to pressure-less matter and that due to cosmological constant. The contribution of radiation density is negligible today. The equation (2.2.0.2) can alternatively be written in terms of the density parameters of matter and cosmological constant, Ω_{m0} and $\Omega_{\Lambda 0}$ as:

$$H^2 = H_0^2(\frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda 0}) \tag{2.2.0.3}$$

if we consider a constant $\alpha = \frac{\Omega_{m0}}{\Omega_{\Lambda 0}}$ the solution of the (2.2.0.3) can be given in terms of integration as:

$$H_0 t = \frac{1}{\sqrt{\Omega_{\Lambda 0}}}\int \frac{\sqrt{a}da}{\sqrt{\alpha + a^3}} \tag{2.2.0.4}$$

just by putting $a = \alpha^{1/3}sinh^{2/3}\theta$, equation (2.2.0.4) yields the solution for scale factor as

$$a(t) = (\frac{\Omega_{\Lambda 0}}{\Omega_{m0}})^{1/3}\sinh^{2/3}(\frac{3}{2}H_0\sqrt{\Omega_{\Lambda 0}t})$$

$$a(t) = \frac{\sinh^{2/3}(At)}{\sinh^{2/3}B} \tag{2.2.0.5}$$

where $A = \frac{3}{2}H_0\sqrt{\Omega_{\Lambda_0}}$ and $B = \sinh^{-1}\sqrt{\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}}$ Since the evolution of the scale factor is an observed fact, even if gravity is not described by Einstein-Hilbert action, the time dependence of the scale factor would still remain the same, The physical interpretation of the parameters A and B will of course be different in that case. This form of the scale factor is valid for the later epochs to the epoch of matter-radiation equality (z 1100). Hence this scale factor is a significant tool to explain the late-time acceleration of the universe.

2.3 Fine Tuning Problem

The origin of cosmic acceleration, the cosmological constant Λ is considered as the strongest candidate that exerts vacuum energy, called dark energy. From the observational evidences, Λ can be calculate from H_0^2 and we can calculate the vacuum energy as $\rho_\Lambda^{obs} \leq (10^{-12}GeV)^4$ [3]. But there is a serious problem of its this energy scale. The vacuum means a state of minimum energy. For example, the minimum energy of a harmonic oscillating particle is a particle sitting in the minimum of the potential $V(x) = 1/2m\omega^2x^2$ i.e $x = 0$. But the uncertainty in position and momentum in quantum mechanics leads to the lowest possible energy state has energy $E_0 = 1/2\hbar\omega$. If we consider this zero-point energy in Planck scale M_{pl}, from dimensional consideration can find the vacuum energy as

$$\rho_\Lambda \sim \hbar M_{pl}^4$$

For the Planck scale $M_{pl} = 1/\sqrt{8\pi G} \sim 10^{18}GeV$, the gives the vacuum energy as $\rho_\Lambda \sim (10^{18}GeV)^4$, which is around 10^{120} times bigger than the observational value ρ_Λ^{obs}. This discrepancy has been called 'the worst theoretical prediction in the history of physics![26]. This problem persisted even at the time of the discovery of dark energy in 1998 when the cosmological constant was expected to have zero value.

The second problem of the cosmological constant as dark energy not only ρ_Λ is small but of the same order of the present mass density of the universe. Although this problem is not specific to the cosmological constant, all the dark energy models invoked this issue.

It is worth reminding oneself that the necessity to invoke Dark Energy (even though one does not understand its properties) arises because one confines oneself to the Einstein Hilbert action to describe gravity. This crisis in our understanding of this unusual type of matter may be a signal that Einstein Hilbert's action may only be at best an approximation and that one needs to explore generalizations of this as the fundamental theory of gravity.

Several generalizations have been studied in the literature of which the main ones are higher dimensional cosmology and the f(R) theories of gravity.

2.4 Modified Gravity Models

After the discovery of the non-zero nature of the cosmological constant,[20],[18] only possible explanation that was concluded as a reason for cosmic acceleration is the dark energy contribution as vacuum energy in addition to the matter-energy density. However, the failure of understanding the mystery and underlying physical properties of dark energy allowed physicists to explore alternative ways. As GR is based on the idea of the geometry of space-time, the effect of gravity on the matter fields can only be through interactions with the rank-2 tensor that explains the curvature of the space-time. The term 'gravitational theory' can then be functionally defined by the set of field equations obeyed by the rank-2 tensor, and any other non-matter fields it interacts with. If these equations are anything other than Einstein's equations, then we consider it to be a 'modified theory of gravity[19]. There are two manners how to extend gravity in order to solve these problems.

There are two possible approaches to modify the gravity- The first one is the Modification of the Einstein-Hilbert Action by adding another term. F(R) one of the ways to modify gravity consists in introducing additional terms to the gravitational sector. Such terms are given by scalars constructed on the base of the metric tensor, i.e. these scalars are functions of the Riemann tensor, the Ricci tensor, possibly, their covariant derivatives, and the scalar curvature. Instead of using the Functions of scalar curvature only, one can also use the functions of other scalars. some of the common examples of such formulations are f(R,Q) gravity, The Lovelock Gravity, and the Gauss-Bonnet gravity. The second approach, which involves the recovery of Einstein's Gravity besides using the metric field, the addition of some extra scalar or vector fields which must not be confused with the matter being treated as ingredients, there are other interesting manners to introduce new scalar fields in the gravity, moreover, while the quintessence field is treated as a matter, the scalar fields introduced within these approaches are interpreted as ingredients of the complete description of the gravity rather than the matter. One of these manners is the Brans-Dicke gravity where the gravitational constant whose negative dimension is responsible for a non-renormalizability of the gravity. is suggested to be not a fundamental constant but a function of some slowly varying fundamental scalar field. Another one is the four-dimensional Chern-Simons modified gravity where the pseudoscalar field allows to implementation of the CPT (and in certain cases Lorentz) symmetry breaking in the gravity context.

However, our present work is only limited to the consideration of only the first type of generalization i.e f(R) Gravity[7][23] Formulation to explore the possible cosmological models in order to explain the late-time cosmic acceleration.

3 f(R) Theory of Gravity

The simplest family of modified gravity models is obtained by replacing the Ricci scalar R in the usual Hilbert Einstein Lagrangian density for some function f(R)[2]. In these models, one assumes that the invariant Lagrangian density need not necessarily be the curvature scalar but can be any function of the curvature scalar. Such a generalization is perfectly consistent with the requirement that the invariant Lagrangian density is a scalar quantity. In the Einstein Hilbert action, one chooses the simplest form of this scalar constructed from geometric parameters, namely R or a linear combination of R and the cosmological constant Λ. In this generalization, one considers a general function of R. The modified action can be expressed as,

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R+f(R)] \tag{3.0.0.1}$$

The additional term, $f(R)$, in the Lagrangian density, should have a functional form such that it can explain late time acceleration. Some models with this generalization have been suggested in the literature. Starobinsky has suggested a model in which $f(R)$ is given by, $\lambda R_0[(1+R^2/R_0^2)^{-n}-1$ here n and λ are the parameters[24]. Hu-Sawicki have suggested a model in which $f(R) == m^2 c\frac{(R/m^2)^n}{(1+c_1(R/m^2))^n}$ where $m^2 = \frac{8\pi G}{3}\rho_0, c_1$ and n are parameters in this Model[12]. Linder model has a form $f(R) == -cR_0[1-e^{\frac{R}{R_0}}]$ with c as parameter[16] . Almost, if not all models that have been studied have a serious problem of curvature singularity. Here we follow an empirical approach. Starting with the expansion history of the universe as suggested by observations, we reconstruct an f (R) model that produces the same form of the expansion as indicated from observations.

3.1 f(R) Reconstruction

As mentioned earlier we can use the modified form of the Einstein-Hilbert action as (3.0.0.1).

$$S = \frac{1}{16\pi G}\int d^x\sqrt{-g}[R+f(R)]$$

We can explore the possible forms of f(R) that will produce the same time dependence of scale factor as given in the previous section by equation (2.2.0.5)

$$a(t) = \frac{\sinh^{2/3}(At)}{\sinh^{2/3}B}$$

Clearly ΛCDM Model is a special case of this action with $f(R) = -2\Lambda$.if we allow $f(R)$ not to be a constant, there will be a class of functions that will be allowed.

3.2 Modified Field equations

Rewriting the action (3.0.0.1) as

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R + f(R)] + \int d^4x\sqrt{-g}L_m \tag{3.2.0.1}$$

where L_m is the Lagrangian of pressure-less matter.
Varying this action (3.0.0.1) with respect to $g_{\mu\nu}$ gives the modified field, by variational principle we can write:

$$\delta S = 0 = \delta(S_g + S_f + S_m) \tag{3.2.0.2}$$

where, s_g, S_f, S_m is the action due to gravity, $f(R)$ and Matter respectively. we can now check for the individual variation:

- **Matter Action S_m Variation:**

  we know the matter energy-momentum can be written as

  $$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g^{\mu\nu}}$$

  using the above equation, we can write matter action variation as:

  $$\delta S_m = -\frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu} \tag{3.2.0.3}$$

- **Variation of Gravity Action, S_g:** variation of the gravity Lagrangian $L_g = \sqrt{-g}R$ gives:

  $$\delta S_g = \frac{1}{16\pi G}\int \sqrt{-g}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)\delta g^{\mu\nu} \tag{3.2.0.4}$$

- **Variation of the f(R) Action, S_f:**

  Varying the $f(R)$ action with respect to $g_{\mu\nu}$ we get

  $$\delta s_f = \frac{1}{16\pi G}\int d^4x\delta(\sqrt{-g}f(R)) = \int d^4x(f(R)\delta\sqrt{-g} + \sqrt{-g}\delta f(R))$$

  $$\implies \delta S_f = \frac{1}{16\pi G}[-\frac{1}{2}f(R)g_{\mu\nu}\delta g^{\mu\nu} + \Box f_R g_{\mu\nu} - \Delta_\mu\Delta_\nu f_R + f_R R_{\mu\nu}]\delta g^{\mu\nu} \tag{3.2.0.5}$$

  Here, $\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$. $f_R = df/dR$ and $f_{RR} = d^2/dR^2$.

Using equation (3.2.0.3),(3.2.0.4), (3.2.0.5) in the action principle, we get the modified field equation in the form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + [\Box f_R - \frac{1}{2}f(R)]g_{\mu\nu} + f_R R_{\mu\nu} - \Delta_\mu \Delta_\nu f_R = 8\pi G T_{\mu\nu} \tag{3.2.0.6}$$

For a Homogeneous and Isotropic Flat(k=0) universe, (1.2.0.6) becomes

$$ds^2 = -dt^2 + a(t)(dx^2 + dy^2 + dz^2) \tag{3.2.0.7}$$

Using this line element we can find out 0-0th component of the Modified Field Equation which comes out to be:

$$H^2 + \frac{f}{6} - f_R(HH' + H^2) + H^2 f_{RR} R' = \frac{\Omega_{m0} H_0^2}{a^3} \tag{3.2.0.8}$$

where, $\prime$ is the derivative with respect to $ln(a)$. Ω_{m0} is the density parameter for matter today and H_0 is the Hubble parameter today.

3.3 Consistency with ΛCDM Model

Instead of the above variables, it will be more convenient to work with appropriately constructed dimensionless quantities. Accordingly, we define the dimensionless Hubble parameter h as the ratio of the hubble parameter at any epoch to the Hubble parameter today. As per our discussion at the beginning of this section, we demand that the scale factor evolves as in equation(8). Hence, the dimensionless quantity, h corresponding to the Hubble parameter(from (1.4.0.6)) is

$$h = \sqrt{\frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda 0}} \tag{3.3.0.1}$$

The function f has the dimensions of R. Dividing f by the curvature scalar today, R_0 , we define the corresponding dimensionless function $\bar{f}$ as

$$\bar{f} = \frac{f}{R_0} \tag{3.3.0.2}$$

Ricci tensor for the flat FRW metric (1.2.0.6) and field equations (1.4.0.2) is given by:

$$R = 6(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}) \tag{3.3.0.3a}$$

in turns, R can be expressed in terms of the Hubble Parameter and the Deceleration parameter as:

$$R = 6H^2(1-q) \tag{3.3.0.3b}$$

If we introduce a scalar parameter α as

$$\alpha = \frac{R_0}{H_0^2} \tag{3.3.0.4}$$

Using (2.2.0.5) in (3.3.0.3a) for present epoch we get the parameter as,

$$\alpha = 3(1+3\Omega_{\Lambda 0}) \tag{3.3.0.5}$$

on introducing the parameter h, α in Equation (3.2.0.8), we get

$$h^2 + \frac{\alpha \bar{f}}{6} - \bar{f}_x(hh' + h^2) + h^2 \bar{f}_{xx} x' = \frac{\Omega_{m0}}{a^3} \tag{3.3.0.6}$$

where $x = \frac{R}{R_0}$, x can also be found by R and R_0 from equation (3.3.0.3a) using the scale factor $a(t)$ as

$$x = \frac{3\Omega_{\Lambda 0}}{\alpha}(\frac{1}{a^3 \sinh^2 B} + 2) \tag{3.3.0.7}$$

using x we can express $\bar{f}_x$ and $\bar{f}_{xx}$ in terms of the derivative with respect to a,$\bar{f}_a$ and $\bar{f}_{aa}$

$$\bar{f}_x = \frac{-\alpha a^4}{9\Omega_{m0}} \bar{f}_a \tag{3.3.0.8a}$$

$$\bar{f}_{xx} = (\frac{\alpha a^4}{9\Omega_{m0}})^2 (\frac{4}{a}\bar{f}_a + \bar{f}_{aa}) \tag{3.3.0.8b}$$

rearranging equation (3.3.0.6) we get

$$\bar{f}_{aa} = -\frac{4}{a}\bar{f}_a + \frac{1}{\alpha a h^2}[\frac{9\Omega_{m0}}{a^4}(\Omega_{\Lambda 0} + \frac{\alpha \bar{f}}{6}) + \alpha \bar{f}_a(\Omega_{\Lambda 0} - \frac{\Omega_{m0}}{2a^3})] \tag{3.3.0.9}$$

this is a second-order differential equation. To solve it two initial conditions are required.

3.4 Evolution

This is a second-order differential equation. To solve it, two initial conditions are required. Observations of the CMBR anisotropy demands that the two initial conditions are not arbitrary. In order that our model is consistent with the observed CMBR anisotropy, the value of f_{RR} at the epoch of decoupling should be small. Hence, we set the initial conditions at the epoch of decoupling and demand that $\bar{f}_{RRi} = 0$. This initial condition ensures that f(R) does not affect CMB anisotropy observed today. From this initial condition. However upon setting this initial condition $\bar{f}$ is now constrained by the choice of the initial condition of $\bar{f}_{Ri}$ as we get the following relation,

$$\bar{f}_i = \frac{\bar{f}_{ai}}{\Omega_{m0}}[3\Omega_{m0}a + 2\Omega_{\Lambda 0}a^4] - \frac{6\Omega_{\Lambda 0}}{9} \tag{3.4.0.1}$$

In that case at the epoch of decoupling $\bar{f}_{ai}$ is kept as a parameter. With these initial conditions in the above differential equation, when solved produces a class of f(R) models as shown in Figure 3 . Each model corresponds to different values of the parameter $\bar{f}_{ai}$ at the time of decoupling. This class of f (R) models produces the same expansion history as non-relativistic dust-like matter and the cosmological constant.

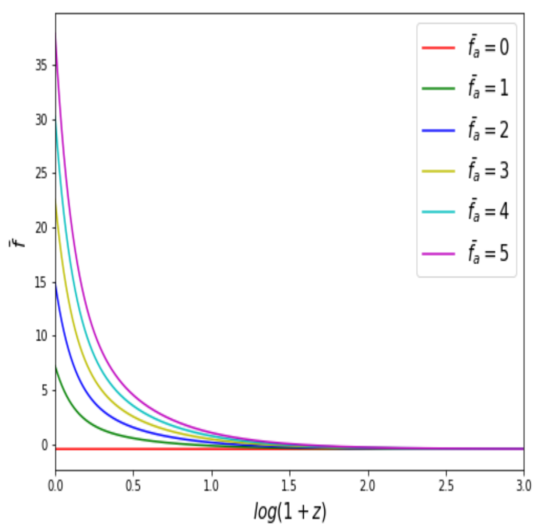

Figure 3: Behaviour of $\bar{f}$ as a function of $\log(1+z)$ for different values of parameter $\bar{f}_{ai}$

The second possible set of $f(R)$ can be obtained by considering the fact that at the final epoch that is at today's epoch the function takes the value $f(R) = -2\Lambda$ i.e generates the standard ΛCDM Model. Hence setting the final condition, and taking different final conditions for $\bar{f}_a$ the numerical solution to the second-order differential equation gives the class of $\bar{f}$ as shown in the Figure 4.

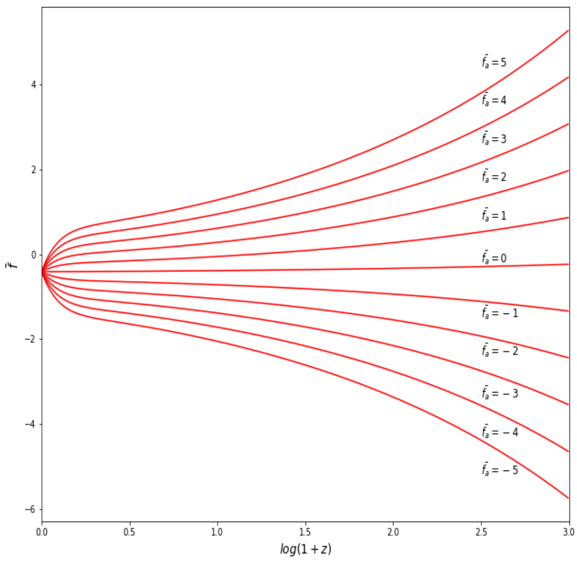

Figure 4: Class of $\bar{f}$ obtained Numerical Solution of the 00-th component of dimensionless Modified field Equation for different slopes $\bar{f}_a$

In this f(R) universe, there is no cosmological constant, the form of f(R) is reconstructed such that it produces the same expansion as ΛCDM model at the present epoch($z \sim 0$). In this set we haven't kept the $\bar{f}_{aa} = 0$ throughout the evolution of the function, rather we have set the final conditions as $f(R) = -2\Lambda$ and different final conditions for the value if $\bar{f}_a$ as $z = 0$ and take the evolution of the function back up to the epoch of decoupling $z \sim 1000$.

4 Summary and discussions

$f(R)$ models are quite popular models to explain the late-time acceleration. However, a large fraction of these models corresponds to the definite choice of the function $f(R)$. In this course work, we have first found out a modified version of the field equation that involves cosmological constant as the modification of geometric nature of space-time. On imposing the Roberson-walker line element for the flat universe, we achieved the time-time component of the equation. Then we have explored the possible numerical models for $f(R)$ that lead to the acceleration with the scale factor $a(t)$ as given by equation (2.2.0.5). The choice of these models has been constrained by choice of the initial conditions on f_R and f_{RR}. As to be consistent with ΛCDM Model, we have checked that on setting $f_{RRi} = f_{Ri} = 0$ at the epoch of $z \sim 1100$ the Modified action takes the form of Einstein-Hilbert action of ΛCDM Model with $f(R) = -2\Lambda$. Furthermore if we take into account the present observational evidences towards dark energy with equation of state $p = -\rho$, and set the final fate to the function to be $f(R) = -2\Lambda$ with this condition we checked for the growth of the function backward and that generates a class of $f(R)$ as given in figure 4.

The possible numerical solution of the function f(R) has been achieved upon consideration of the scale factor a(t) in (2.2.0.5) regardless of the presence of the Λ, only with the A and B parameter actually produces the acceleration. For a the value of $\Omega_{m0} = 0.3$ as approximately predicted by different observational result, the f(R) has been constructed along with the value of Hubble parameter today $H_0 = 70.0\ kms^{-1}Mpc^{-1}$. The discrepancy in these values will generally result in the functional discrepancy in the f(R).

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