Appendix III

Comprehensive solution wedge stability

III.1 Introduction

This appendix presents the equations and procedure to calculate the factor of safety for a wedge failure as discussed in Chapter 7. This comprehensive solution includes the wedge geometry defined by five surfaces, including a sloped upper surface and a tension crack, water pressures, different shear strengths on each slide plane, and up to two external forces (Figure III.1). External forces that may act on a wedge include tensioned anchor support, foundation loads and earthquake motion. The forces are vectors defined by their magnitude, and their plunge and trend. If necessary, several force vectors can be combined to meet the two force limit. It is assumed that all forces act through the center of gravity of the wedge so no moments are generated, and there is no rotational slip or toppling.

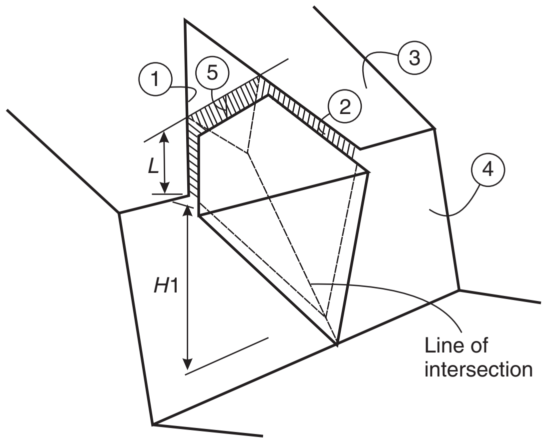

Figure III.1 Dimensions and surfaces defining size and shape of wedge.

III.2 Analysis methods

The equations presented in this appendix are identical to those in appendix 2 of *Rock Slope Engineering,* third edition (Hoek and Bray, 1981). These equations have been found to be versatile and capable of calculating the stability of a wide range of geometric and geotechnical conditions. The equations form the basis of the wedge stability analysis programs SWEDGE (Rocscience, 2001) and ROCKPACK III (Watts, 2001). However, two limitations to the analysis are discussed in Section III.3.

As an alternative to the comprehensive analysis presented in this appendix, there are two shorter analyses that can be used for a more limited set of input parameters. In Section 7.3, a calculation procedure is presented for a wedge formed by planes 1, 2, 3 and 4 shown in Figure III.1, but with no tension crack. The shear strength is defined by different cohesions and friction angles on planes 1 and 2, and the water pressure condition assumed is that the slope is saturated. However, no external forces can be incorporated in the analysis.

A second rapid calculation method is presented in the first part of appendix 2 in *Rock Slope Engineering*, third edition. This analysis also does not incorporate a tension crack or external forces, but does include two sets of shear strength parameters and water pressure.

III.3 Analysis limitations

For the comprehensive stability analysis presented in this appendix there is one geometric limitation related to the relative inclinations of plane 3 and the line of intersection, and a specific procedure for modifying water pressures. The following is a discussion of these two limitations.

Wedge geometry. For wedges with steep upper slopes (plane 3), and a line of intersection that has a shallower dip than the upper slope (i.e. $\psi_3 > \psi_i$), there is no intersection between the plane and the line; the program will terminate with the error message "Tension crack invalid" (see equations (III.50) to (III.53)). The reason for this error message is that the calculation procedure is to first calculate the dimensions of the overall wedge from the slope face to the apex (intersection of the line of intersection with plane 3). Then the dimensions of a wedge between the tension crack and the apex are calculated. Finally, the dimensions of the wedge between the face and the tension crack are found by subtracting the overall wedge from the upper wedge (see equations (III.54) to (III.57).

However, for the wedge geometry where ($\psi_3 > \psi_i$), a wedge can still be formed if a tension crack (plane 5) is present, and it is possible to calculate a factor of safety using a different set of equations. Programs that can investigate the stability wedges with this geometry include YAWC (Kielhorn, 1998) and (PanTechnica, 2002).

Water pressure. The analysis incorporates the average values of the water pressure on the sliding planes (u_1 and u_2), and on the tension crack (u_5). These values are calculated assuming that the wedge is fully saturated. That is, the water table is coincident with the upper surface of the slope (plane 3), and that the pressure drops to zero where planes 1 and 2 intersect the slope face (plane 4). These pressure distributions are simulated as follows. Where no tension crack exists, the water pressures on planes 1 and 2 are given by $u_1 = u_2 = \gamma_w H_w/6$, where H_w is the vertical height of the wedge defined by the two ends of the line of intersection. The second method allows for the presence of a tension crack and gives $u_1 = u_2 = u_5 = \gamma_w H_{5w}/3$, where H_{5w} is the depth of the bottom vertex of the tension crack below the upper ground surface. The water forces are then calculated as the product of these pressures and the areas of the respective planes.

To calculate stability of a partially saturated wedge, the reduced pressures are simulated by reducing the unit weight of the water, γ_w. That is, if it is estimated that the tension crack is one-third filled with water, then a unit weight of $\gamma_w/3$ is used as the input parameter. It is considered that this approach is adequate for most purposes because water levels in slopes are variable and difficult to determine precisely.

III.4 Scope of solution

This solution is for computation of the factor of safety for translational slip of a tetrahedral wedge formed in a rock slope by two intersecting discontinuities (planes 1 and 2), the upper ground surface (plane 3), the slope face (plane 4), and a tension crack (plane 5 (Figure III.1)). The solution allows for water pressures on the two slide planes and in the tension crack, and for different strength parameters on the two slide planes. Plane 3 may have a different dip direction to that of plane 4. The influence of an external load E and a cable tension T are included in the analysis, and supplementary sections are provided for the examination of the minimum factor of safety for a given external load, and for minimizing the anchoring force required for a given factor of safety.

The solution allows for the following conditions:

(a) interchange of planes 1 and 2;
(b) the possibility of one of the planes overlying the other;
(c) the situation where the crest overhangs the toe of the slope (in which case $\eta = -1$); and
(d) the possibility of contact being lost on either plane.

III.5 Notation

The wedge geometry is illustrated in Figure III.1; the following input data are required:

$\psi, \alpha =$ dip and dip direction of plane, or plunge and trend of force
$H1 =$ slope height referred to plane 1
$L =$ distance of tension crack from crest, measured along the trace of plane 1
$u =$ average water pressure on planes 1 and 2
$c =$ cohesion of each slide plane
$\phi =$ angle of friction of each slide plane
$\gamma =$ unit weight of rock
$\gamma_w =$ unit weight of water
$T =$ anchor tension
$E =$ external load
$\eta = -1$ if face is overhanging, and $+1$ if face does not overhang

Other terms used in the solution are as follows:

FS $=$ factor of safety against sliding along the line of intersection, or on plane 1 or plane 2
$A =$ area of sliding plane or tension crack
$W =$ weight of wedge
$V =$ water thrust on tension crack (plane 5)

$$\left.\begin{array}{ll} N_a = & \text{total normal force of plane 1} \\ S_a = & \text{shear force on plane 1} \\ Q_a = & \text{shear resistance on plane 1} \\ FS_1 = & \text{factor of safety} \end{array}\right\} \text{when contact is maintained on plane 1 only}$$

$$\left.\begin{array}{ll} N_b = & \text{total normal force on plane 2} \\ S_b = & \text{shear force on plane 2} \\ Q_b = & \text{shear resistance on plane 2} \\ FS_2 = & \text{factor of safety} \end{array}\right\} \text{when contact is maintained on plane 2 only}$$

$$\left.\begin{array}{ll} N_1, N_2 = & \text{effective normal reactions} \\ S = & \text{total shear force on planes 1 and 2} \\ Q = & \text{total shear resistance on planes 1 and 2} \\ FS_3 = & \text{factor of safety} \end{array}\right\} \text{when contact is maintained on both planes 1 and 2}$$

N_1', N_2', S', etc. $=$ values of N_1, N_2, S etc. when $T = 0$
$N_1'', N_2'' S''$, etc. $=$ values of N_1, N_2, S etc. when $E = 0$
$\vec{a} =$ unit normal vector for plane 1
$\vec{b} =$ unit normal vector for plane 2
$\vec{d} =$ unit normal vector for plane 3
$\vec{f} =$ unit normal vector for plane 4
$\vec{f_5} =$ unit normal vector for plane 5
$\vec{g} =$ vector in the direction of intersection line of 1, 4
$\vec{g_5} =$ vector in the direction of intersection line of 1, 5
$\vec{i} =$ vector in the direction of intersection line of 1, 2
$\vec{j} =$ vector in the direction of intersection line of 3, 4
$\vec{j_5} =$ vector in the direction of intersection line of 3, 5
$\vec{k} =$ vector in plane 2 normal to $\vec{i}$
$\vec{l} =$ vector in plane 1 normal to $\vec{i}$
$R =$ magnitude of vector $\vec{i}$
$G =$ square of magnitude of vector $\vec{g}$
$G_5 =$ square of magnitude of vector $\vec{g_5}$

Note: The computed value of V is negative when the tension crack dips away from the toe of the slope, but this does not indicate a tensile force.

III.6 Sequence of calculations

1 *Calculation of factor of safety when the forces T and E are either zero or completely specified in magnitude and direction.*

(a) Components of unit vectors in directions of normals to planes 1–5, and of forces T and E.

$$(a_x, a_y, a_z) = (\sin\psi_1 \sin\alpha_1, \sin\psi_1 \cos\alpha_1, \cos\psi_1) \quad \text{(III.1)}$$

$$(b_x, b_y, b_z) = (\sin\psi_2 \sin\alpha_2, \sin\psi_2 \cos\alpha_2, \cos\psi_2) \quad \text{(III.2)}$$

$$(d_x, d_y, d_z) = (\sin\psi_3 \sin\alpha_3, \sin\psi_3 \cos\alpha_3, \cos\psi_3) \quad \text{(III.3)}$$

$$(f_x, f_y, f_z) = (\sin\psi_4 \sin\alpha_4, \sin\psi_4 \cos\alpha_4, \cos\psi_4) \quad \text{(III.4)}$$

$$(f_{5x}, f_{5y}, f_{5z}) = (\sin\psi_5 \sin\alpha_5, \sin\psi_5 \cos\alpha_5, \cos\psi_5) \quad \text{(III.5)}$$

$$(t_x, t_y, t_z) = (\cos\psi_t \sin\alpha_t, \cos\psi_t \cos\alpha_t, -\sin\psi_t) \quad \text{(III.6)}$$

$$(e_x, e_y, e_z) = (\cos\psi_e \sin\alpha_e, \cos\psi_e \cos\alpha_e, -\sin\psi_e) \quad \text{(III.7)}$$

(b) Components of vectors in the direction of the lines of intersection of various planes.

$$(g_x, g_y, g_z) = (f_y a_z - f_z a_y), (f_z a_x - f_x a_z), (f_x a_y - f_y a_x) \quad \text{(III.8)}$$

$$(g_{5x}, g_{5y}, g_{5z}) = (f_{5y} a_z - f_{5z} a_y), (f_{5z} a_x - f_{5x} a_z), (f_{5x} a_y - f_{5y} a_x) \quad \text{(III.9)}$$

$$(i_x, i_y, i_z) = (b_y a_z - b_z a_y), (b_z a_x - b_x a_z), (b_x a_y - b_y a_x) \quad \text{(III.10)}$$

$$(j_x, j_y, j_z) = (f_y d_z - f_z d_y), (f_z d_x - f_x d_z), (f_x d_y - f_y d_x) \quad \text{(III.11)}$$

$$(j_{5x}, j_{5y}, j_{5z}) = (f_{5y} d_z - f_{5z} d_y), (f_{5z} d_x - f_{5x} d_z), (f_{5x} d_y - f_{5y} d_x) \quad \text{(III.12)}$$

$$(k_x, k_y, k_z) = (i_y b_z - i_z b_y), (i_z b_x - i_x b_z), (i_x b_y - i_y b_x) \quad \text{(III.13)}$$

$$(l_x, l_y, l_z) = (a_y i_z - a_z i_y), (a_z i_x - a_x i_z), (a_x i_y - a_y i_x) \quad \text{(III.14)}$$

(c) Numbers proportional to cosines of various angles.

$$m = g_x d_x + g_y d_y + g_z d_z \quad \text{(III.15)}$$

$$m_5 = g_{5x} d_x + g_{5y} d_y + g_{5z} d_z \quad \text{(III.16)}$$

$$n = b_x j_x + b_y j_y + b_z j_z \quad \text{(III.17)}$$

$$n_5 = b_x j_{5x} + b_y j_{5y} + b_z j_{5z} \quad \text{(III.18)}$$

$$p = i_x d_x + i_y d_y + i_z d_z \quad \text{(III.19)}$$

$$q = b_x g_x + b_y g_y + b_z g_z \quad \text{(III.20)}$$

$$g_5 = b_x g_{5x} + b_y g_{5y} + b_z g_{5z} \quad \text{(III.21)}$$

$$r = a_x b_x + a_y b_y + a_z b_z \quad \text{(III.22)}$$

$$s = a_x t_x + a_y t_y + a_z t_z \quad \text{(III.23)}$$

$$v = b_x t_x + b_y t_y + b_z t_z \quad \text{(III.24)}$$

$$w = i_x t_x + i_y t_y + i_z t_z \quad \text{(III.25)}$$

$$s_e = a_x e_x + a_y e_y + a_z e_z \quad \text{(III.26)}$$

$$v_e = b_x e_x + b_y e_y + b_z e_z \quad \text{(III.27)}$$

$$w_e = i_x e_x + i_y e_y + i_z e_z \quad \text{(III.28)}$$

$$s_5 = a_x f_{5x} + a_y f_{5y} + a_z f_{5z} \quad \text{(III.29)}$$

$$v_5 = b_x f_{5x} + b_y f_{5y} + b_z f_{5z} \quad \text{(III.30)}$$

$$w_5 = i_x f_{5x} + i_y f_{5y} + i_z f_{5z} \quad \text{(III.31)}$$

$$\lambda = i_x g_x + i_y g_y + i_z g_z \quad \text{(III.32)}$$

$$\lambda_5 = i_x g_{5x} + i_y g_{5y} + i_z g_{5z} \quad \text{(III.33)}$$

$$\varepsilon = f_x f_{5x} + f_y f_{5y} + f_z f_{5z} \quad \text{(III.34)}$$

(d) Miscellaneous factors.

$$R = \sqrt{1 - r^2} \quad \text{(III.35)}$$

$$= \frac{1}{R^2} \cdot \frac{nq}{|nq|} \quad \text{(III.36)}$$

$$\mu = \frac{1}{R^2} \cdot \frac{mq}{|mq|} \quad \text{(III.37)}$$

$$\upsilon = \frac{1}{R} \cdot \frac{p}{|p|} \quad \text{(III.38)}$$

$$G = g_x^2 + g_y^2 + g_z^2 \quad \text{(III.39)}$$

$$G_5 = g_{5x}^2 + g_{5y}^2 + g_{5z}^2 \quad \text{(III.40)}$$

$$M = (Gp^2 - 2mp\lambda + m^2R^2)^{1/2} \quad \text{(III.41)}$$

$$M_5 = (G_5p^2 - 2m_5p\lambda_5 + m_5^2R^2)^{1/2} \quad \text{(III.42)}$$

$$h = \frac{H1}{|g_z|} \quad \text{(III.43)}$$

$$h_5 = \frac{Mh - |p|L}{M_5} \quad \text{(III.44)}$$

$$B = [\tan^2\phi_1 + \tan^2\phi_2 - 2(\mu r/\rho) \times \tan\phi_1 \tan\phi_2]/R^2 \quad \text{(III.45)}$$

(e) Plunge and trend of line respectively of line of intersection of planes 1 and 2:

$$\psi_i = \arcsin(\nu i_z) \quad \text{(III.46)}$$

$$\alpha_i = \arctan\left(\frac{-\nu i_x}{-\nu i_y}\right) \quad \text{(III.47)}$$

The term $-\nu$ should not be cancelled out in equation (III.47) since this is required to determine the correct quadrant when calculating values for dip direction, α_i.

(f) Check on wedge geometry.

No wedge is formed, terminate computation
$$\begin{cases} \text{if } p\,i_z < 0, \text{ or} & \text{(III.48)} \\ \text{if } n\,q\,i_z < 0 & \text{(III.49)} \end{cases}$$

Tension crack invalid, terminate computation
$$\begin{cases} \text{if } \epsilon\,\eta\,q_5 i_z < 0, \text{or} & \text{(III.50)} \\ \text{if } h_5 < 0, \text{ or} & \text{(III.51)} \\ \text{if } \left[\left|\frac{m_5 h_5}{m\,h}\right|\right] > 1, \text{or} & \text{(III.52)} \\ \text{if } \left[\left|\frac{n\,q_5\,m_5\,h_5}{n_5\,q\,m\,h}\right|\right] > 1 & \text{(III.53)} \end{cases}$$

(g) Areas of faces and weight of wedge.

$$A_1 = \frac{|mq|h^2| - |m_5q_5|h_5^2}{2|p|} \quad \text{(III.54)}$$

$$A_2 = \frac{\left(|q|m^2h^2/|n| - |q_5|m_5^2h_5^2/|n_5|\right)}{|2p|} \quad \text{(III.55)}$$

$$A_5 = \frac{|m_5q_5|h_5^2}{2|n_5|} \quad \text{(III.56)}$$

$$W = \frac{\gamma\left(q^2m^2h^3/|n| - q_5^2m_5^2h_5^3/|n_5|\right)}{6|p|} \quad \text{(III.57)}$$

(h) Water pressure.

(i) With no tension crack

$$u_1 = u_2 = \frac{\gamma_w h|m_5|}{6|p|} \quad \text{(III.58)}$$

(ii) With tension crack

$$u_1 = u_2 = u_5 = \frac{\gamma_w h_5|m_5|}{3d_z} \quad \text{(III.59)}$$

$$V = u_5\,A_5\,\eta\left(\frac{\varepsilon}{|\varepsilon|}\right) \quad \text{(III.60)}$$

(i) Effective normal reactions on planes 1 and 2 assuming contact on both planes.

$$N_1 = \rho\{W\,k_z + T(r\,v - s) + E(r\,v_e - s_e) + V(r\,v_5 - s_5)\} - u_1\,A_1 \quad \text{(III.61)}$$

$$N_2 = \mu\{W\,l_z + T(r\,s - v) + E(r\,s_e - v_e) + V(r\,s_5 - v_5)\} - u_2\,A_2 \quad \text{(III.62)}$$

(j) Factor of safety when $N_1 < 0$ and $N_2 < 0$ (contact is lost on both planes).

$$FS = 0 \quad \text{(III.63)}$$

(k) If $N_1 > 0$ and $N_2 < 0$, contact is maintained on plane 1 only and the factor of safety is calculated as follows:

$$N_a = Wa_z - Ts - Es_e - Vs_5 - u_1 A_1 r \quad \text{(III.64)}$$

$$S_x = (Tt_x + Ee_x + N_a a_x + Vf_{5x} + u_1 A_1 b_x) \quad \text{(III.65)}$$

$$S_y = (Tt_y + Ee_y + N_a a_y + Vf_{5y} + u_1 A_1 b_y) \quad \text{(III.66)}$$

$$S_z = (Tt_z + Ee_z + N_a a_z + Vf_{5z} + u_1 A_1 b_z) + W \quad \text{(III.67)}$$

$$S_a = (S_x^2 + S_y^2 + S_z^2)^{1/2} \quad \text{(III.68)}$$

$$Q_a = (N_a - u_1 A_1)\tan\phi_1 + c_1 A_1 \quad \text{(III.69)}$$

$$\mathrm{FS}_1 = \left(\frac{Q_a}{S_a}\right) \quad \text{(III.70)}$$

(l) If $N_1 < 0$ and $N_2 > 0$, contact is maintained on plane 2 only and the factor of safety is calculated as follows:

$$N_b = (Wb_z - Tv - Ev_e - Vv_5 - u_2 A_2 r) \quad \text{(III.71)}$$

$$S_x = (Tt_x + Ee_x + N_b b_x + Vf_{5x} + u_2 A_2 a_x) \quad \text{(III.72)}$$

$$S_y = (Tt_y + Ee_y + N_b b_y + Vf_{5y} + u_2 A_2 a_y) \quad \text{(III.73)}$$

$$S_z = (Tt_z + Ee_z + N_b b_z + Vf_{5z} + u_2 A_2 a_z) + W \quad \text{(III.74)}$$

$$S_b = (S_x^2 + S_y^2 + S_z^2)^{1/2} \quad \text{(III.75)}$$

$$Q_b = (N_b - u_2 A_2)\tan\phi_2 + c_2 A_2 \quad \text{(III.76)}$$

$$\mathrm{FS}_2 = \left(\frac{Q_b}{S_b}\right) \quad \text{(III.77)}$$

(m) If $N_1 > 0$ and $N_2 > 0$, contact is maintained on both planes and the factor of safety is calculated as follows:

$$S = \nu(Wi_z - Tw - Ew_e - Vw_5) \quad \text{(III.78)}$$

$$Q = N_1 \tan\phi_1 + N_2 \tan\phi_2 + c_1 A_1 + c_2 A_2 \quad \text{(III.79)}$$

$$\mathrm{FS}_3 = \left(\frac{Q}{S}\right) \quad \text{(III.80)}$$

2 *Minimum factor of safety produced when load E of given magnitude is applied in the worst direction.*

(a) Evaluate $N_1'', N_2'', S'', Q'', FS_3''$ by use of equations (III.61), (III.62), (III.78), (III.79) and (III.80) with $E = 0$.

(b) If $N_1'' < 0$ and $N_2'' < 0$, even before E is applied. Then FS = 0, terminate computation.

(c) $$D = \{(N_1'')^2 + (N_2'')^2 + 2\left(\frac{mn}{|mn|}\right) N_1'' N_2'' r\}^{1/2} \quad \text{(III.81)}$$

$$\psi_e = \arcsin\left\{\left(-\frac{1}{G}\left(\frac{m}{|m|}\right) \cdot N_1'' a_z + \frac{n}{|n|} \cdot N_2'' b_z\right)\right\} \quad \text{(III.82)}$$

$$\alpha_e = \arctan\left\{\frac{\frac{m}{|m|}.N_1'' a_x + \frac{n}{|n|}.N_2'' b_x}{\frac{m}{|m|}.N_1'' a_y + \frac{n}{|n|}.N_2'' b_y}\right\} \quad \text{(III.83)}$$

If $E > D$, and E is applied in the direction ψ_e, α_e, or within a certain range encompassing this direction, then contact is lost on both planes and FS = 0. Terminate calculation.

(d) If $N_1'' > 0$ and $N_2'' < 0$, assume contact on plane 1 only after application of E. Determine $S_x'', S_y'', S_z'', S_a'', Q_a'', \mathrm{FS}_1''$ from equations (III.65) to (III.70) with $E = 0$. If $\mathrm{FS}_1'' < 1$, terminate computation. If $\mathrm{FS}_1'' > 1$:

$$\mathrm{FS}_1 = \frac{S_a'' Q_a'' - E\{(Q_a'')^2 + ((S_a'')^2 - E^2)\tan^2\phi_1\}^{1/2}}{(S_a'')^2 - E^2} \quad \text{(III.84)}$$

$$\psi_{e1} = \arcsin\left(\frac{S_z''}{S_a''}\right) - \arctan\left(\frac{\tan\phi_1}{(\mathrm{FS}_1)}\right) \quad \text{(III.85)}$$

$$\alpha_{e1} = \arctan\left(\frac{S_x''}{S_a''}\right) + 180° \quad \text{(III.86)}$$

(e) If $N_1'' < 0$ and $N_2'' > 0$, assume contact on plane 2 only after application of E. Determine $S_x'', S_y'', S_z'', S_b'', Q_b'', \mathrm{FS}_2''$ from equations (III.72) to (III.77) with $E = 0$. If $\mathrm{FS}_2'' < 1$, terminate computation. If $\mathrm{FS}_2'' > 1$:

$$\mathrm{FS}_2 = \frac{S_b'' Q_b'' - E\{(Q_b'')^2 + ((S_b'')^2 - E^2)\tan^2\phi_2\}^{1/2}}{(S_b'')^2 - E^2} \quad \text{(III.87)}$$

$$\psi_{e2} = \arcsin\left(\frac{S_z''}{S_b''}\right) - \arctan\left(\frac{\tan\phi_2}{(\mathrm{FS}_2)}\right) \quad \text{(III.88)}$$

$$\alpha_{e2} = \arctan\left(\frac{S_x''}{S_y''}\right) + 180° \quad \text{(III.89)}$$

(f) If $N_1'' > 0$ and $N_2'' > 0$, assume contact on both planes after application of E. If $\mathrm{FS}_3'' < 1$, terminate computation. If $\mathrm{FS}_3'' > 1$:

$$\mathrm{FS}_3 = \frac{S'' Q'' - E\{(Q'')^2 + B((S'')^2 - E^2)\}^{1/2}}{(S'')^2 - E^2} \quad \text{(III.90)}$$

$$\chi = \sqrt{B + (\mathrm{FS}_3)^2} \quad \text{(III.91)}$$

$$e_x = -\frac{((\mathrm{FS}_3)\nu i_x - \rho k_x \tan\phi_1 - \mu l_x \tan\phi_2)}{\chi} \quad \text{(III.92)}$$

$$e_y = -\frac{((\mathrm{FS}_3)\nu i_y - \rho k_y \tan\phi_1 - \mu l_y \tan\phi_2)}{\chi} \quad \text{(III.93)}$$

$$e_z = -\frac{((\mathrm{FS}_3)\nu i_z - \rho k_z \tan\phi_1 - \mu l_z \tan\phi_2)}{\chi} \quad \text{(III.94)}$$

$$\psi_{e3} = \arcsin(-e_z) \quad \text{(III.95)}$$

$$\alpha_{e3} = \arctan\left(\frac{e_x}{e_y}\right) \quad \text{(III.96)}$$

Compute s_e and v_e using equations (III.26) and (III.27)

$$N_1 = N_1'' + E\rho(r\, v_e - s_e) \quad \text{(III.97)}$$

$$N_2 = N_2'' + E\mu(r\, s_e - v_e) \quad \text{(III.98)}$$

Check that $N_1 \geqq 0$ and $N_2 \geqq 0$

3 *Minimum cable or bolt tension $T_{\min}$ required to raise the factor of safety to some specified value* FS.

(a) Evaluate N_1', N_2', S', Q' by means of equations (III.61), (III.62), (III.78), (III.79) with $T = 0$.

(b) If $N_2' < 0$, contact is lost on plane 2 when $T = 0$. Assume contact on plane 1 only, after application on T. Evaluate S_x', S_y', S_z', S_a' and Q_a' using equations (III.65) to (III.69) with $T = 0$.

$$T_1 = \frac{((\mathrm{FS})S_a' - Q_a')}{\sqrt{(\mathrm{FS})^2 + \tan^2\phi_1}} \quad \text{(III.99)}$$

$$\psi_{t1} = \arctan\left(\frac{\tan\phi_1}{(\text{FS})}\right) - \arcsin\left(\frac{S'_z}{S'_a}\right) \tag{III.100}$$

$$\alpha_{t1} = \arctan\left(\frac{S'_x}{S'_y}\right) \tag{III.101}$$

(a) If $N'_1 < 0$, contact is lost on plane 1 when $T = 0$. Assume contact on plane 2 only, after application of T. Evaluate S'_x, S'_y, S'_z, S'_b and Q'_b using equations (III.72) to (III.76) with $T = 0$.

$$T_2 = \frac{((\text{FS})S'_b - Q'_b)}{\sqrt{(\text{FS})^2 + \tan^2\phi_2}} \tag{III.102}$$

$$\psi_{t2} = \arctan\left(\frac{\tan\phi_2}{(\text{FS})}\right) - \arcsin\left(\frac{S'_z}{S'_b}\right) \tag{III.103}$$

$$\alpha_{t2} = \arctan\left(\frac{S'_x}{S'_y}\right) \tag{III.104}$$

(a) All cases. No restrictions on values of N'_1 and N'_2. Assume contact on both planes after application of T.

$$\chi = \sqrt{((\text{FS})^2 + B)} \tag{III.105}$$

$$T_3 = \frac{((\text{FS})S' - Q')}{\chi} \tag{III.106}$$

$$t_x = \frac{((\text{FS})\upsilon i_x - \rho k_x \tan\phi_1 - \mu l_x \tan\phi_2)}{\chi} \tag{III.107}$$

$$t_y = \frac{((\text{FS})\upsilon i_y - \rho k_y \tan\phi_1 - \mu l_y \tan\phi_2)}{\chi} \tag{III.108}$$

$$t_z = \frac{((\text{FS})\upsilon i_z - \rho k_z \tan\phi_1 - \mu l_z \tan\phi_2)}{\chi} \tag{III.109}$$

$$\psi_{t3} = \arcsin(-t_z) \tag{III.110}$$

$$\alpha_{t3} = \arctan\left(\frac{t_x}{t_y}\right) \tag{III.111}$$

Compute s and v using equations (III.23) and (III.24).

$$N_1 = N'_1 + T_3\rho(rv - s) \tag{III.112}$$

$$N_2 = N'_2 + T_3\mu(rs - v) \tag{III.113}$$

If $N_1 < 0$ or $N_2 < 0$, ignore the results of this section.
If $N'_1 > 0$ and $N'_2 > 0$, $T_{\min} = T_3$
If $N'_1 > 0$ and $N'_2 < 0$, $T_{\min}$ = smallest of T_1, T_3
If $N'_1 < 0$ and $N'_2 > 0$, $T_{\min}$ = smallest of T_2, T_3
If $N'_1 < 0$ and $N'_2 < 0$, $T_{\min}$ = smallest of T_1, T_2, T_3

Example *Calculate the factor of safety for the following wedge:*

Plane	1	2	3	4	5
ψ	45	70	12	65	70
α	105	235	195	185	165

$\eta = +1$
$H1 = 100$ ft, $L = 40$ ft, $c_1 = 500\ \text{lb/ft}^2$, $c_2 = 1000\ \text{lb/ft}^2$
$\phi_1 = 20°, \phi_2 = 30°, \gamma = 160\ \text{lb/ft}^3$.

(1a) $T = 0, E = 0, u_1 = u_2 = u_5$

u_5 calculated from equation (III.59).

$(a_x, a_y, a_z) = (0.68301, -0.18301, 0.70711)$
$(b_x, b_y, b_z) = (-0.76975, -0.53899, 0.34202)$
$(d_x, d_y, d_z) = (-0.05381, -0.20083, 0.97815)$
$(f_x, f_y, f_z) = (-0.07899, -0.90286, 0.42262)$
$(f_{5x}, f_{5y}, f_{5z}) = (0.24321, -0.90767, 0.34202)$
$(g_x, g_y, g_z) = (-0.56107, 0.34451, 0.63112)$
$(g_{5x}, g_{5y}, g_{5z}) = (-0.57923, 0.061627, 0.57544)$
$(i_x, i_y, i_z) = (-0.31853, 0.77790, 0.50901)$
$(j_x, j_y, j_z) = (-0.79826, 0.05452, -0.03272)$
$(j_{5x}, j_{5y}, j_{5z}) = (-0.81915, -0.25630, -0.09769)$
$(k_x, k_y, k_z) = (0.54041, -0.28287, 0.77047)$
$(l_x, l_y, l_z) = (-0.64321, -0.57289, 0.47302)$

$m = 0.57833$

$m_5 = 0.58166$

$n = 0.57388$

$n_5 = 0.73527$

$p = 0.35880$

$q = 0.46206$

$q_5 = 0.60945$

$r = -0.18526$

$s_5 = 0.57407$

$v_5 = 0.41899$

$w_5 = -0.60945$

$\lambda = 0.76796$

$\lambda_5 = 0.52535$

$\varepsilon = 0.94483$

$R = 0.98269$

$\rho = 1.03554$

$\mu = 1.03554$

$\nu = 1.01762$

$G = 0.83180$

$G_5 = 0.67044$

$M = 0.33371$

$M_5 = 0.44017$

$h = 158.45$

$h_5 = 87.521$

$B = 0.56299$

$\psi_i = 31.20°$

$\alpha_i = 157.73°$

$$\left.\begin{array}{l} pi_z > 0 \\ nqi_z > 0 \end{array}\right\} \text{Wedge is formed}$$

$$\left.\begin{array}{l} \varepsilon\eta q_5 i_z > 0 \\ h_5 > 0 \\ \frac{|m_5h_5|}{|mh|} = 0.5554 < 1 \\ \frac{|nq_5m_5h_5|}{|n_5qmh|} = 0.57191 < 1 \end{array}\right\} \text{Tension crack valid}$$

$A_1 = 5565.01\text{ft}^2$

$A_2 = 6428.1\text{ft}^2$

$A_5 = 1846.6\text{ft}^2$

$W = 2.8272 \times 10^7$ lb

$u_1 = u_2 = u_5 = 1084.3\,\text{lb/ft}^2$;

$V = 2.0023 \times 10^6$ lb

$$\left.\begin{array}{l} N_1 = 1.5171 \times 10^7 \text{ lb} \\ N_2 = 5.7892 \times 10^6 \text{ lb} \end{array}\right\} \text{Both positive therefore contact on planes 1 and 2.}$$

$S = 1.5886 \times 10^7$ lb

$Q = 1.8075 \times 10^7$ lb

FS = 1.1378—*Factor of Safety*

(1b) $T = 0, E = 0$, dry slope, $u_1 = u_2 = u_5 = 0$.

As in (1a) except as follows:

$$\left.\begin{array}{l} V = 0 \\ N_1 = 2.2565 \times 10^7 \text{lb} \\ N_2 = 1.3853 \times 10^7 \text{ lb} \end{array}\right\} \text{Both positive, therefore contact on both planes 1 and 2.}$$

$S = 1.4644 \times 10^7$ lb

$Q = 2.5422 \times 10^7$ lb

$FS_3 = 1.7360$—*Factor of Safety*

(2) As in (1b), except $E = 8 \times 10^6$ lb. Find the value of FS_{min}.

Values of $N_1'', N_2'', S'', Q'', FS_3''$ as given in (1b).

$N_1'' > 0, N_2'' > 0, FS_3'' > 1$, continue calculation.

$B = 0.56299$

$FS_3 = 1.04$—FS_{min} *(minimum factor of safety)*

$\chi = 1.2798$

$e_x = 0.12128$

$e_y = -0.99226$

$e_z = 0.028243$

$\psi_{e3} = -1.62°$—*plunge of force (upwards)*

$\alpha_{e3} = 173.03°$—*trend of force*

$$\left.\begin{array}{l} N_1 = 1.9517 \times 10^7 \text{ lb} \\ N_2 = 9.6793 \times 10^6 \text{ lb} \end{array}\right\} \text{Both positive therefore contact maintained on both planes.}$$

(3) As in (1a) except that the minimum cable tension T_{min} required to increase the factor of safety to 1.5 is to be determined

N_1', N_2', S' and Q'—as given in (1a)

$\chi = 1.6772$

$T_3 = 3.4307 \times 10^6$ lb—T_{min} *(minimum cable tension)*

$t_x = -0.18205$

$t_y = 0.97574$

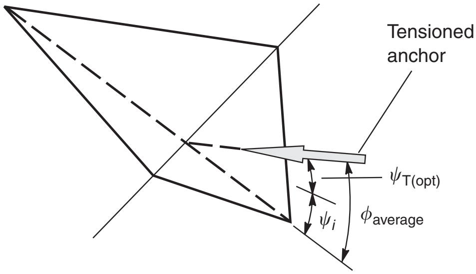

Figure III.2 Optimum anchor orientation for reinforcement of a wedge.

$t_z = 0.12148$

$\psi_{t3} = -6.98°$—*plunge of cable (upwards)*

$\alpha_{t3} = 349.43°$—*trend of cable*

Note that the optimum plunge and trend of the anchor are approximately:

$$\psi_{t3} = \frac{1}{2}(\phi_1 + \phi_2) - \psi_i$$

$$\approx 25 - 31.2$$

$$\approx -6.2° \text{ (upwards)}$$

$$\alpha_{t3} \approx \alpha_i \pm 180°$$

$$\approx 157.73 + 180$$

$$\approx 337.73°$$

That is, the best direction in which to install an anchor to reinforce a wedge is

The anchor should be aligned with the line of intersection of the two planes, viewed from the bottom of the slope, and it should be inclined at the average friction angle to the line of intersection (Figure III.2).