HARMONIC INSTABILITY OF DIGITAL SOFT CLIPPING ALGORITHMS

Sean Enderby and Zlatko Baracskai

Department of Digital Media Technology
Birmingham City University
Birmingham, UK

ABSTRACT

In this paper several different digital soft clipping algorithms are described and analysed. It is discussed how the quality of each algorithm can be estimated. A testing methodology is devised to show the levels of nonlinearities produced as a function of the input signal amplitude. It is proposed that, while all soft clipping algorithms produce higher order nonlinearities, the instability of the produced harmonics plays a crucial role in the transparency of the effect. Existing and novel clipping algorithms are thus compared and classified based on their measured properties, including total harmonic distortion and inter-modulation distortion estimates. This paper proposes a conclusion related to the quality and properties of different algorithms.

1. INTRODUCTION

A clipping system has a limited output range, such that input values beyond a certain threshold will be capped at that threshold. Soft clipping does this in a manner in which the derivative of the characteristic curve will be continuous.

Soft clipping is used in digital audio processing to induce distortion and increase the perceived loudness of the signal. It compromises the dynamic range and introduces new spectral components. It is widely regarded as more musical than hard clipping [1]. This is due to the lower level of higher order harmonics [2]. This further decreases the chance of audible aliasing distortion which, due to its inharmonic properties greatly deteriorates the perceived sound quality. Further, lower levels of higher order harmonics supports perceptual fusion of the spectral components. If the harmonics are too loud they will be perceived as a distinct distortion whereas moderate distortion levels will merely change the timbre of the sound.

Musical distortion is considered to arise from even order harmonics that are produced by asymmetrical transfers typical of tube amplifiers [1]. Recently, extensive research has been conducted in digitally emulating analogue distortion circuits [3] as digital distortion became infamous for lacking body and definition. It is proposed that developing digital distortion based on existing core principles might yield distortion algorithms that would suit musical applications in their flexibility and improved perceptual qualities. In doing this, preliminary research into basic symmetrical transfers is conducted to shed light on the distinctions inherent in different soft clipping algorithms.

Many different solutions have been introduced in the literature [3] [4] [5], however analysis and comparison seems to be lacking. In this paper, algorithms are compared based on a novel testing methodology that provides insight into the instability of harmonics throughout the dynamic range of the clipping algorithm. Novel and existing algorithms are analysed and compared to demonstrate the effects produced. Further investigation into devising a 'knee' parameter is presented to propose the optimal choice of digital soft clipping algorithms for different requirements and purposes.

2. PERCEPTUAL CONSIDERATIONS

The perceptual features of distortion artefacts have been widely researched in striving for high fidelity reproduction as well as the intelligibility of speech in low bit-rate systems. From being a term describing unwanted deterioration of quality, distortion, mainly through its use in electric guitar playing, has become an essential ingredient in music performance and production. It has become apparent that low level, well designed distortion can enhance the perceived quality and presence of audio signals through the revealing of the core technology behind excitation effects [6].

Several psychoacoustic principles can be used to explain how distortion is perceived. In [7] the following are proposed:

- New frequency components created, which are lower in frequency than the original components, will be more perceptible.
- The higher the order of a nonlinearity the more perceptible it will be.
- Nonlinearities which affect signals at low levels have a more audible effect than those which only affect high level signals.

The perception of distortion also decreases for frequencies at either end of the audible spectrum [8].

Soft-clipping distortion is widely used for altering the timbre and the dynamics of digital audio. It would be challenging to try to devise an absolute quality measure as this is application specific. Instead, the proposed instability measure can serve as an aid in assessing the impact and the distinction among different algorithms. Along with the other presented measurements it can be concluded that different algorithms will have different advantages, but the final assessment has to be subjective and application specific.

3. ALGORITHMS

Several different clipping algorithms were tested. Each algorithm was optimised for comparison so that the clipping threshold was at $\frac{2}{3}$ of the maximum amplitude level. Equations 1 and 6 do not have thresholds but are asymptotic with the lines $y = 1$ and $y = -1$. For these equations the input was scaled so that at $\frac{2}{3}$ the output would be close to 1 but the curve would still cover most of the dynamic range below. The characteristic curves for the algorithms are shown in Figure 1.

Hyperbolic Tangent Soft Clipping (Tanh)

As suggested in [3]

$$y = \tanh(5x) \tag{1}$$

Sinusoidal Soft Clipping

$$y = \begin{cases} \operatorname{sgn}(x) & \text{for} \quad |x| > \frac{2}{3} \\ \sin\left(\frac{3\pi x}{4}\right) & \text{for} \quad -\frac{2}{3} \leq x \leq \frac{2}{3} \end{cases} \tag{2}$$

Exponential Soft Clipping

As this algorithm has a parameter other than clipping threshold it was tested twice with exponent E taking values of 2 (Exp2) and 5 (Exp5) respectively.

$$y = \begin{cases} \operatorname{sgn}(x) & \text{for} \quad |x| > \frac{2}{3} \\ \operatorname{sgn}(x)\left(1 - \left|\frac{3x}{2} - \operatorname{sgn}(x)\right|^E\right) & \text{for} \quad -\frac{2}{3} \leq x \leq \frac{2}{3} \end{cases} \tag{3}$$

Two Stage Quadratic Soft Clipping (TSQ)

As proposed in [4].

$$y = \begin{cases} \operatorname{sgn}(x) & \text{for} \quad |x| > \frac{2}{3} \\ \operatorname{sgn}(x)\frac{3-(2-|3x|)^2}{3} & \text{for} \quad \frac{1}{3} \leq |x| \leq \frac{2}{3} \\ 2x & \text{for} \quad -\frac{1}{3} < x < \frac{1}{3} \end{cases} \tag{4}$$

Cubic Soft Clipping

Modified from an equation in [5].

$$y = \begin{cases} \operatorname{sgn}(x) & \text{for} \quad |x| > \frac{2}{3} \\ \frac{9x}{4} - \frac{27x^3}{16} & \text{for} \quad -\frac{2}{3} \leq x \leq \frac{2}{3} \end{cases} \tag{5}$$

Reciprocal Soft Clipping

$$y = \operatorname{sgn}(x)\left(1 - \frac{1}{|30x|+1}\right) \tag{6}$$

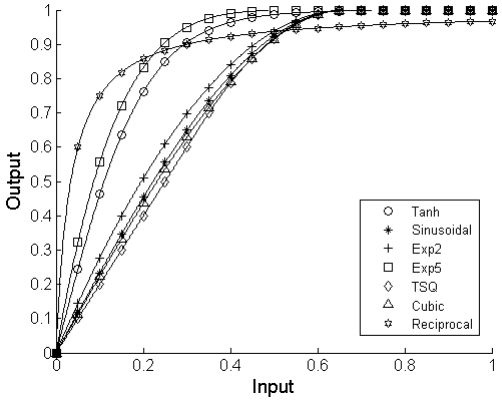

Figure 1: *Algorithm Characteristic Curves*

4. MEASUREMENTS

For each of the algorithms a graph is plotted that represents the distortion levels as a function of the input amplitude. The plots display odd order harmonics from the 3^{rd} to 9^{th} order to make the differences visually apparent. In analysing the total harmonic distortion, harmonics up to the 23^{rd} order were summed. The excitation of harmonic spectral components is inherent to soft clipping and desired in the context of producing a distortion effect. Aliasing and quantisation distortions produce inharmonic partials and are considered artefacts typical of digital distortion that is widely considered less musical. Intermodulation distortion is measured to estimate the presence of inharmonic artefacts due to nonlinearities. Aliasing and quantisation distortions are not examined as oversampling and bit-rate improvement can successfully mitigate these anomalies.

4.1. Harmonic and Intermodulation Distortion

The signal used to test harmonic distortion was a 2kHz sine wave. For intermodulation distortion the sum of a 2kHz and a 5kHz sine wave was used. The sampling frequency of these signals was 96kHz, to allow measuring up to the 23^{rd} harmonic overtone. To measure the distortion a large window FFT spectrum of the processed signal was calculated. This was used to separate out harmonic and intermodulation components based on the resulting magnitude spectrum.

Harmonic distortion was calculated using Equation 7 as recommended in [9].

$$D_h = 20\log_{10}\left(\frac{A_n}{\sqrt{A_1^2 + A_s^2}}\right) \tag{7}$$

Where A_1 is the RMS amplitude of the fundamental, A_s is the combined RMS amplitude of all harmonics and A_n is the RMS amplitude of a specific harmonic.

Intermodulation distortion was calculated using Equation 8:

$$D_i = 20\log_{10}\left(\frac{A_i}{A_T}\right) \tag{8}$$

Where A_i is the combined RMS amplitude for the intermodulation frequencies not harmonically related to the fundamentals. And A_T is the RMS amplitude of the processed signal.

In both measurements any amplitudes which were too low to be represented in 16 bit audio were ignored.

4.2. Amplitude Dependence

For each algorithm a plot of amplitude dependence for the 3^{rd} through 9^{th} odd order harmonics is shown. These graphs are shown in Figures 2 - 8. The legend for these graphs is shown in Table 1.

Symbol	Order of Harmonic
✳	Third Order
□	Fifth Order
△	Seventh Order
◆	Ninth Order

Table 1: *Legend for Subsequent Graphs*

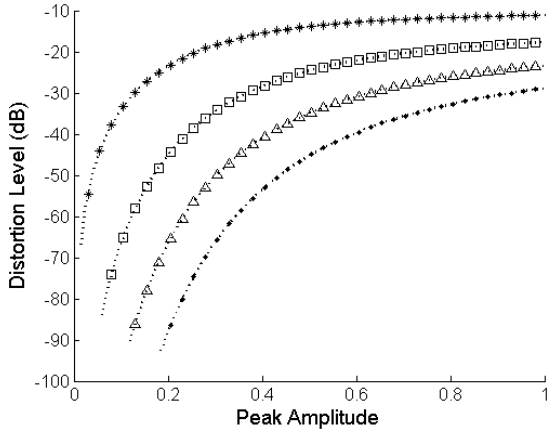

Figure 2: *Hyperbolic Tangent Soft Clipping*

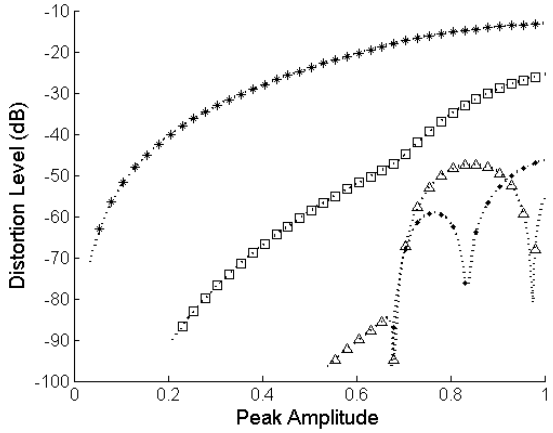

Figure 3: *Sinusoidal Soft Clipping*

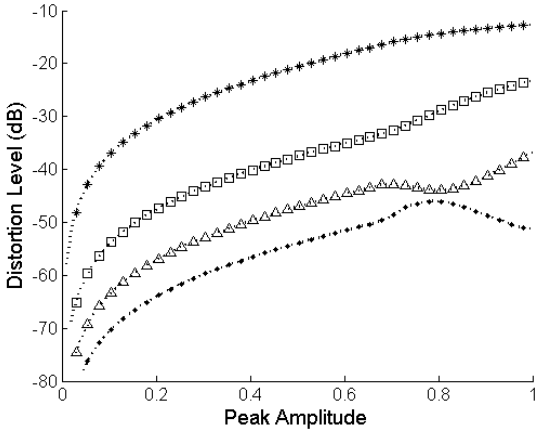

Figure 4: *Exponential Soft Clipping with an Exponent of 2*

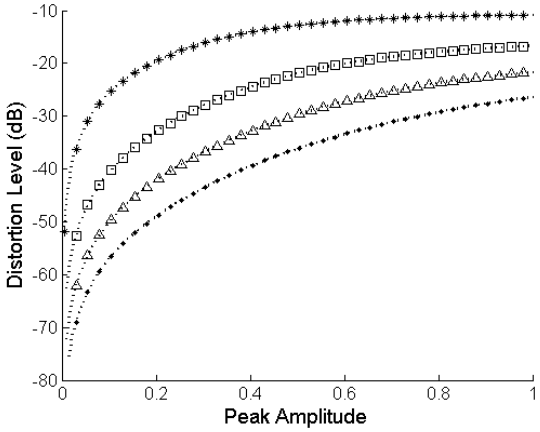

Figure 5: *Exponential Soft Clipping with an Exponent of 5*

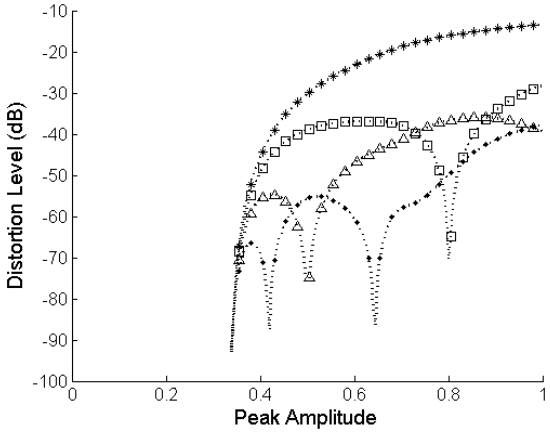

Figure 6: *Two Stage Quadratic Soft Clipping*

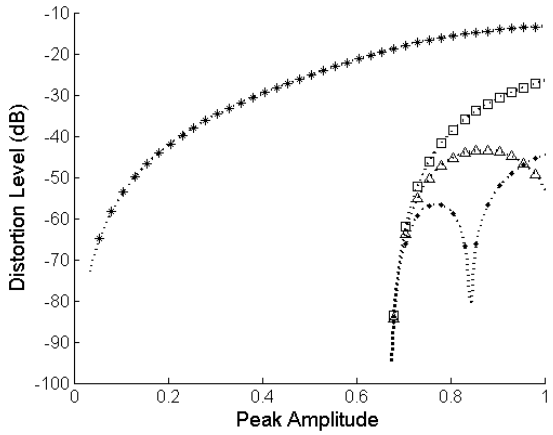

Figure 7: *Cubic Soft Clipping*

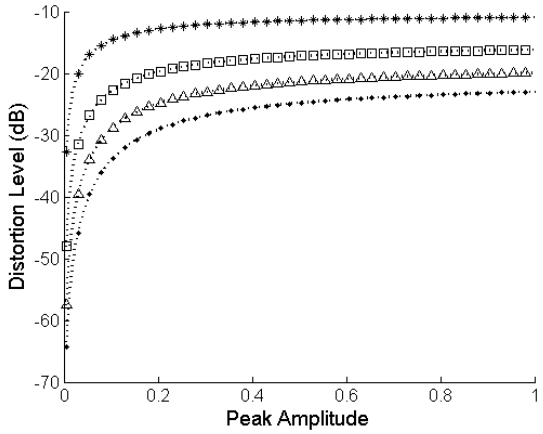

Figure 8: *Reciprocal Soft Clipping*

4.3. Instability

Figures 2 - 8 show that there are vast differences in dynamic response for each algorithm. It was proposed that taking the standard deviation of the aplitude dependence curve would give a basic measurement of the instability of the algorithms dynamic response. The level of instability is measured with reference to the level of the third order harmonic. This is because the behaviour of the third order harmonic was found to be similar across every algorithm. It is introduced at the first point the curve becomes nonlinear. It then follows a logarithmic curve which finishes at approximately -15dB at full input amplitude. As the third order harmonic is the most stable in each algorithm, it was used as a benchmark against which the instability of other order harmonics was judged. Hereby the proposed instability measure is considered to be a measure of the internal instability among the harmonics across the full dynamic range.

The instability of harmonics with respect to input amplitude is calculated using Equation 9:

$$\sigma_i = 20\log_{10}\sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\frac{A_k}{T_k}-\mu\right)^2} \tag{9}$$

Where A_k is the level of a specific order harmonic at input amplitude k. T_k is the level of the third order harmonic at input amplitude k and μ is the mean level of the harmonic given by Equation 10:

$$\mu = \frac{1}{N}\sum_{k=1}^{N}\frac{A_k}{T_k} \tag{10}$$

The instability is calculated for each order harmonic, present in the signal, and then the mean of these is taken. This gives one value for the instability of the harmonics.

For each effect the instability for three separate dynamic bands was calculated. These bands split the dynamic range into three equal sections (0 to $\frac{1}{3}$, $\frac{1}{3}$ to $\frac{2}{3}$ and $\frac{2}{3}$ to 1). This was done so that the instability above and below the clipping thresholds could be measured separately. The various instability values are shown in Figure 9.

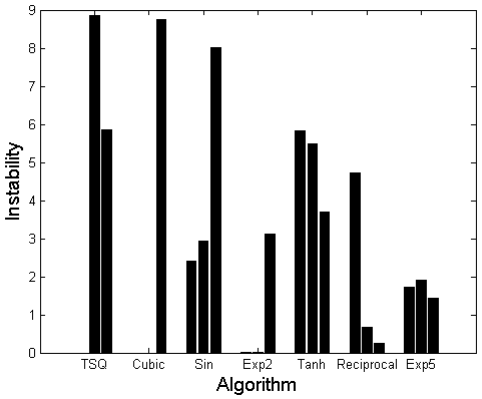

Figure 9: *Instability Levels for Three Dynamic Bands*

The mean THD for each dynamic band was also measured, the results are shown in Figure 10.

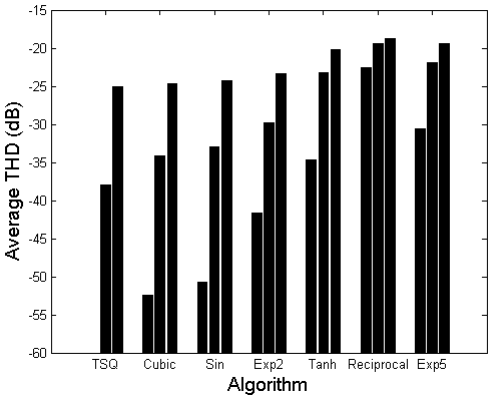

Figure 10: *Average THD for Three Dynamic Bands*

4.4. Knee

For each algorithm the knee parameter was measured as the value of the second derivative of the characteristic curve at the clipping threshold, where $y = f(x)$, Knee $= f''(\frac{2}{3})$. For algorithms with more than one threshold, such as Equation 4, the values of the second derivative for each threshold are summed. The second derivative of the characteristic curve has been used before in the measurement of distortion. In [7] it is used to bias a metric towards higher order nonlinearities. In this paper it is used to check for discontinuity, in change of gradient, between sections of the characteristic curve.

The knee parameter for each algorithm is shown in Table 2 as well as values of THD and IMD for full amplitude signals.

Algorithm	Knee	THD (dB)	IMD (dB)
TSQ	-5.00	-13.32	-15.80
Cubic	-3.38	-13.10	-15.37
Sinusoidal	-2.78	-12.85	-15.03
Exp2	-2.25	-12.27	-14.17
Tanh	-0.25	-9.99	-11.27
Reciprocal	-0.19	-9.01	-9.56
Exp5	0.00	-9.42	-10.55

Table 2: *Comparison Table of Analysis Data.*

5. COMPARISON

As all the effects are symmetrical for both negative and positive portions of a signal, only odd order harmonics are being introduced [10]. In each of the effects tested the higher the order of the nonlinearity the lower its average level.

Figure 6 shows a lack of distortion components in the linear section of the characteristic curve. Producing no distortion for low level signals, this will theoretically reduce the audibility of these distortions [7] [8].

In Figures 2 through 5 distortion components are present for all parts of the dynamic range. It can be noticed that for the trigonometric algorithms (Figures 2 and 3) higher order nonlinearities are introduced later in the dynamic range. Whereas with exponential soft clipping all orders are present at the lowest input level. This would suggest that the trigonometric algorithms are less perceptible than the exponential ones as the distortion present at low levels is of a lower order.

In cubic soft clipping, shown in Figure 7, only the third order nonlinearity is present before the threshold. This is due to the fact that before the threshold a cubic function is used. Raising a signal to a power n will only produce nonlinearities of the same parity as n up to the n^{th} order [11].

The results in Table 2 and Figure 10 show a correlation between the value of the knee and the levels of distortion components. While decreasing the knee value does produce higher levels of distortion, it can be seen in Figure 9 that this can sometimes improve the dynamic resonse of the algorithm.

Of all the algorithms exponential soft clipping is the most stable. In Figure 10 it can be seen that an increase in exponent (exp5 rather than exp2) will increase the levels of distortion components. This increase in exponent also gives lower dynamic instability above the clipping threshold. This is at the expense of the stability of the lower end of the dynamic range.

Reciprocal Clipping is very stable in the higher end of the dynamic range. It does however produce the highest levels of distortion and is dynamically unstable with low input amplitudes.

6. CONCLUSION

Soft clipping algorithms can be analysed in several ways. The knee parameter of an algorithm can give an approximation of its distortion levels and dynamic stability. However, this conclusion is based on a limited set of measurements and further investigation is needed. Exponential soft clipping gives the most stable dynamic response but at the expense of distortion levels. Cubic and Two Stage Quadratic clipping sit at the other end of the scale, with lower distortion levels but a high degree of dynamic instability. Trigonometric algorithms pose a middle ground with slightly more stable dynamic response and slightly greater levels of distortion components.

7. REFERENCES

[1] E. Barbour, "The cool sound of tubes [vacuum tube musical applications]," *Spectrum, IEEE*, vol. 35, no. 8, pp. 24 –35, Aug 1998.

[2] P. H. Kraght, "Aliasing in digital clippers and compressors," *J. Audio Eng. Soc*, vol. 48, no. 11, pp. 1060–1065, 2000.

[3] D. T. Yeh, *Digital Implementations of Musical Distortion Circuits by Analysis and Simulation*, Ph.D. thesis, Stanford University, 2009.

[4] U. Zölzer, *Digital Audio Effects*, Wiley, 2002.

[5] C. R. Sullivan, "Extending the karplus-strong algorithm to synthesize electric guitar timbres with distortion and feedback," *Computer Music Journal*, vol. 14, no. 3, pp. 26–37, 1990.

[6] J. Chalupper, "Aural exciter and loudness maximizer: What's psychoacoustic about -psychoacoustic processors?-," in *Audio Engineering Society Convention 109*, Sep 2000.

[7] E. R. Geddes and L. W. Lee, "Auditory perception of nonlinear distortion - theory," in *Audio Engineering Society Convention 115*, Oct 2003.

[8] A. Voishvillo, "Assessment of nonlinearity in transducers and sound systems: From thd to perceptual models," in *Audio Engineering Society Convention 121*, Oct 2006.

[9] IEC 60268-5, "Sound system equipment - part 5: Loudspeakers," .

[10] C. Dodge and T. A. Jerse, *Computer Music: Synthesis, Composition and Performance*, Schirmer: Thomson Learning, 2^{nd} edition, 1997.

[11] M. Le Brun, "Digital waveshaping synthesis," *J. Audio Eng. Soc*, vol. 27, no. 4, pp. 250–266, 1979.