Arithmetic Circuits – Basic Building Blocks

- ***Half-Adder:***
  - A half-adder is an arithmetic circuit block that can be used to add two bits
    - Such a circuit thus has two inputs that represent the two bits to be added and two outputs, with
    - one producing the **SUM** output and the other producing the **CARRY.**

- The Boolean expressions for the SUM and CAR… n by the equations

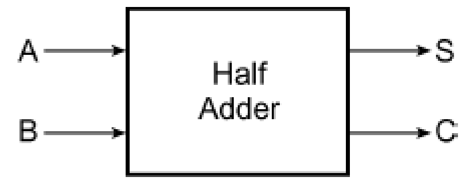
$$\text{SUM } S = A.\overline{B} + \overline{A}.B$$

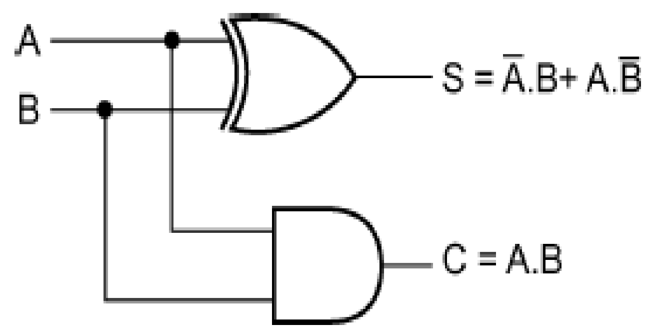
$$\text{CARRY } C = A.B$$

$$\text{SUM } S = A \oplus B$$

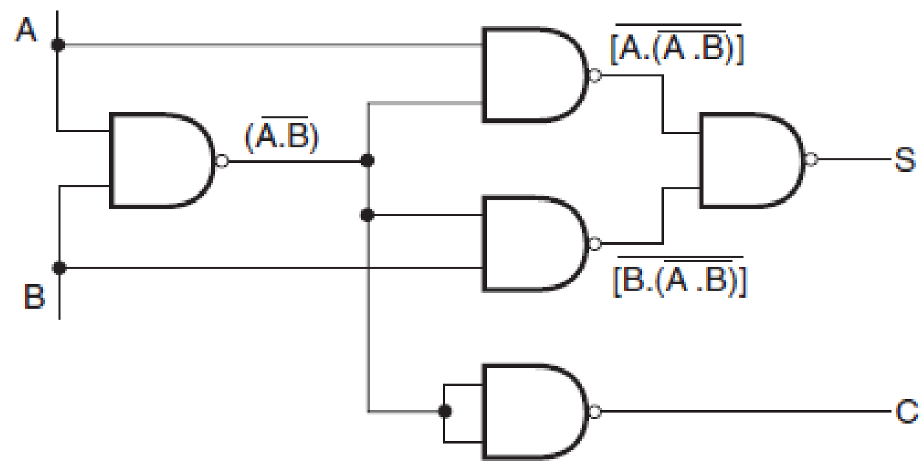
$$\text{CARRY } C = A.B$$

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1





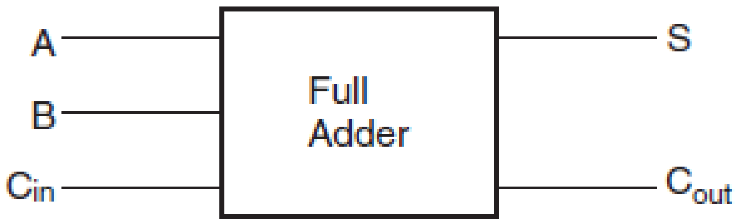
Half-adder implementation using NAND gates



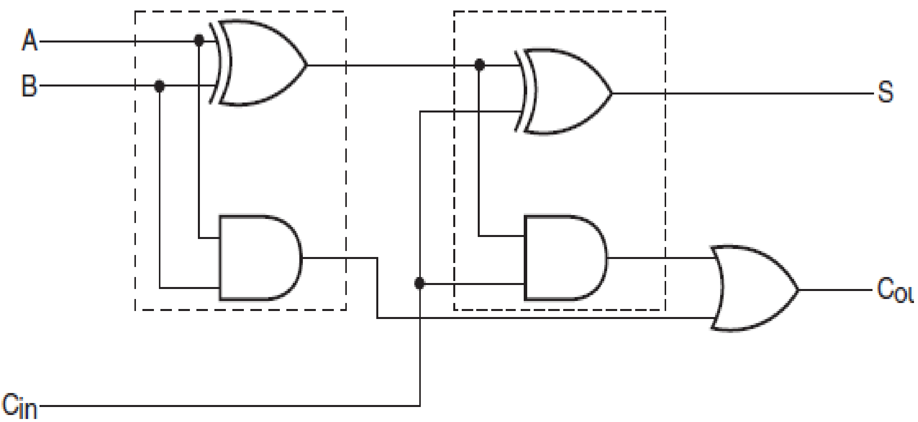
- Half-adder implementation using NOR gates

Full Adder

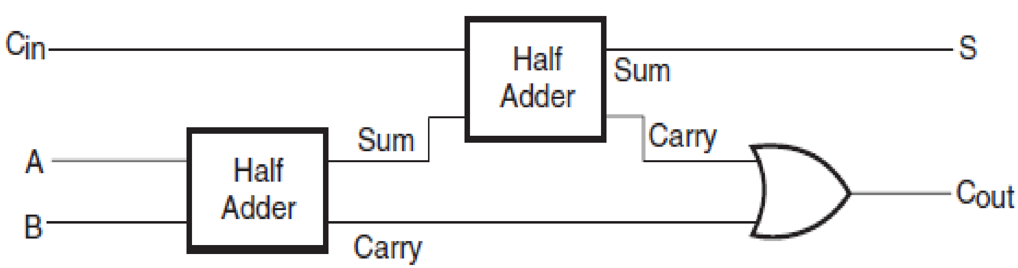
- A full adder circuit is an arithmetic circuit block that can be used to add three bits to produce a SUM and a CARRY output.



A	B	C_{in}	SUM (S)	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



$$S = \overline{A}.\overline{B}.C_{\text{in}} + \overline{A}.B.\overline{C}_{\text{in}} + A.\overline{B}.\overline{C}_{\text{in}} + A.B.C_{\text{in}}$$



$$C_{\text{out}} = \overline{A}.B.C_{\text{in}} + A.\overline{B}.C_{\text{in}} + A.B.\overline{C}_{\text{in}} + A.B.C_{\text{in}}$$

$$S = \overline{A}.\overline{B}.C_{in} + \overline{A}.B.\overline{C}_{in} + A.\overline{B}.\overline{C}_{in} + A.B.C_{in}$$

$$X = \overline{A}.B + A.\overline{B} = A \oplus B$$

$$S = \overline{C}_{in}.(\overline{A}.B + A.\overline{B}) + C_{in}.(A.B + \overline{A}.\overline{B})$$

$$A.B + \overline{A}.\overline{B} = \overline{A \oplus B} = \overline{X}$$

$$S = \overline{C}_{in}.(A \oplus B) + C_{in}.(\overline{A \oplus B})$$

$$S = \overline{C}_{in}.X + C_{in}\overline{X}$$

$$S = X \oplus C_{in}$$

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = B.C_{in}.(A + \overline{A}) + A.B + A.C_{in}.(B + \overline{B})$$

$$= A.B + A.B.C_{in} + \overline{A}.B.C_{in} + A.B.C_{in} + A.\overline{B}.C_{in}$$

$$= A.B + A.B.C_{in} + \overline{A}.B.C_{in} + A.\overline{B}.C_{in}$$

$$= A.B.(1 + C_{in}) + C_{in}.(\overline{A}.B + A.\overline{B})$$

$$C_{out} = A.B + C_{in}.(\overline{A}.B + A.\overline{B})$$

$$= A.B + C_{in}.(A \oplus B)$$

Four-bit binary adder