(2) Group the minterm according to the number of 1's contained in them and separate by a horizontal line between each number of 1's category.

Min terms	Binary Equivalent					No. of 1's used
	A	B	C	D		
0	0	0	0	0	✓	0
2	0	0	1	0	✓	1
4	0	1	0	0	✓	
8	1	0	0	0	✓	
3	0	0	1	1	✓	2
6	0	1	1	0	✓	
10	1	0	1	0	✓	
12	1	1	0	0	✓	
7	0	1	1	1	✓	3
13	1	1	0	1	✓	

(3) The above table help to search the binary minterm, which are differ in one place only. After separate, each binary number is compared with every terms in next higher category. (4) If they differ by only one position, a check mark is placed beside each of the two terms, otherwise neglect that term and go for another term having difference of only one position.

(5) write the term in the second column with a "_" in the position that they differed

(6) This process of comparison continue for minterm until it complete

QM₂

Minterms (Part 2)	Result of first comparison A	B	C	D	
0, 2	0	0	—	0	✓
0, 4	0	—	0	0	✓
0, 8	—	0	0	0	✓
2, 3	0	0	1	—	✓
2, 6	0	—	1	0	✓
2, 10	—	0	1	0	✓
4, 6	0	1	—	0	✓
4, 12	—	1	0	0	✓
~~4, 10~~					
8, 10	1	0	—	0	✓
8, 12	1	—	0	0	✓
3, 7	0	—	1	1	✓
6, 7	0	1	1	—	✓
12, 13	1	1	0	—	*

0,
0 0 — 0
0 0 1 0

2 → 0 0 1 0
12 → 1 1 0 0
— — — 0

2,12 does not comes in the result of first comparison because 2,12 — on comparison gives the difference of three positions.

0,2 → 0 0 0 —
2,3 → 0 0 1 —
0 0 — — ×

0,2 → 0 0 — 0
4,6 → 0 1 — 0
0 — — 0 ✓ one position difference

0,2 → 0 0 — 0
8,10 → 1 0 — 0
— 0 — 0

Column 2

Minterms (quads)	Result of second comparison A	B	C	D	
0, 2, 4, 6	0	—	—	0	Same
0, 2, 8, 10	—	0	—	0	
0, 4, 2, 6	0	—	—	0	Same
0, 4, 8, 12	—	—	0	0	Same
0, 8, 2, 10	—	0	—	0	Same
0, 8, 4, 12	—	—	0	0	
2, 3, 6, 7	0	—	1	—	Same
2, 6, 3, 7	0	—	1	—	

If a particular minterm does not have check mark (✓) it show that this minterm cannot be combined with any other minterms and it is prime implicant. It is marked by [*]. Now list the prime implicant is given by

Prime implicants	Result of all comparison			
	A	B	C	D
12, 13, *	1	1	0	—
0, 2, 4, 6	0	—	—	0
0, 2, 8, 10	—	0	—	0
0, 4, 8, 12	—	—	0	0
2, 3, 6, 7	0	—	1	—

For selection of ~~minterms~~ minimum number of prime-implicant which must cover all the minterms.

[Prime implicant selection chart]

S No	Prime implicant	m_0	m_2	m_3	m_4	m_6	m_7	m_8	m_{10}	m_{12}	m_{13}
1	12, 13, * ✓									•	⊙
2	0, 2, 4, 6	•	•		•	•					
3	0, 2, 8, 10 ✓	•	•					•	⊙		
4	0, 4, 8, 12	•			•			•		•	
5	2, 3, 6, 7 ✓		•	⊙		•	⊙				
	Essential minterm			m_3			m_7		m_{10}		m_{13}

In the chart the Sr. No 2 and 4 have the Prime Implicant which are non-essential but it cover all the minterms, we have to select any one

Sr. No	
2	0, 2, 4, 6
Sr. No 4.	0, 4, 8, 12

⇒ 0, 2, minterms of serial number '2' are already covered by serial number '3'. Similarly 6th minterm of Sr. No '2' is already present in Sr. No. '5'. only 4th minterm is left.

→ So, if we select Sr. No '4' all the minterms including the previous one will be covered.

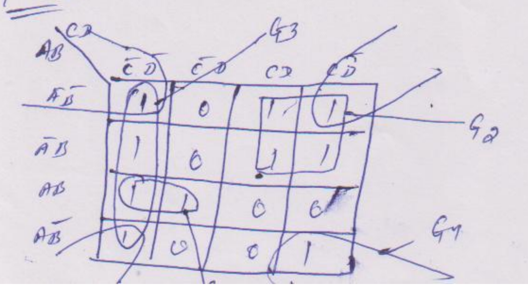
⇒ The minterms (0, 4, 8, 12) of serial No. 4 will also be include in Rest of prime implicants and serial No. '2' can be ignored.

⇒ Final Result for prime implicants will be :⇒

Prime Implicants	Result of comparison				O/P in ABCD form
	A	B	C	D	
12, 13	1	1	0	—	$AB\bar{C}$
0, 4, 8, 12	—	—	0	0	$\bar{C}\bar{D}$
0, 2, 8, 10	—	0	—	0	$\bar{B}\bar{D}$
2, 3, 6, 7	0	—	1	—	$\bar{A}C$

$$\therefore \boxed{f(A,B,C,D) = AB\bar{C} + \bar{C}\bar{D} + \bar{B}\bar{D} + \bar{A}C}$$

K-Map



$f(A,B,C,D) = G_1 + G_2 + G_3 + G_4$

$= AB\bar{C} + \bar{B}\bar{D} + \bar{C}\bar{D} + \bar{A}C$

Ex. 15 Obtain the set of prime implicants for the following function = Σm (0, 1, 6, 7, 8, 9, 13, 14, 15).

Imp

Solution :

Minterms	Binary Equivalent				No. of 1's
	A	B	C	D	
0	0	0	0	0	0
1 8	0 1	0 0	0 0	1 0	1
6 9	0 1	1 0	1 0	0 1	2
7 13 14	0 1 1	1 1 1	1 0 1	1 1 0	3
15	1	1	1	1	4

Result of 1st comparison, *i.e.*, Pairs

Solution :

Pairs	A	B	C	D	No. of 1's
0, 1 0, 8	0 –	0 0	0 0	– 0	0
1, 9 8, 9	– 1	0 0	0 0	1 –	1
6, 7 9, 13 6, 14	0 1 –	1 – 1	1 0 1	– 1 0	2
7, 15 13, 15 14, 15	– 1 1	1 1 1	1 – 1	1 1 –	3

Result of 2nd combination Quads

Quads	A	B	C	D	No. of 1's
0, 8, 1, 9 0, 8, 1, 9	– –	0 0	0 0	– –	0
6, 7, 14, 15	–	1	1	–	2

PRIME IMPLICANTS

Prime Implicants	0	1	6	7	8	9	13	14	15
0, 1, 8, 9	⊗	⊗			⊗	×			
6, 7, 14, 15			⊗	⊗				⊗	×
13, 15							×		×
9, 13						×	×		

∴ 0, 1, 8, 9 and 6, 7, 14, 15 have to be there.

Out of 13, 15 and 9, 13 one has to be selected.

We select 13, 15 as lesser number of NOT gates are used.

Essential	A	B	C	D	Output
0, 1, 8, 9	–	0	0	–	$\overline{B}\,\overline{C}$
6, 7, 14, 15	–	1	1	–	BC
13, 15	1	1	–	1	ABD

∴ Output, $Y = \overline{B}\,\overline{C} + BC + ABD$

Ex. 16 Obtain the set of prime-implicants for Σm (0, 1, 2, 4, 7, 8, 12, 15) using the binary designations of terms using Q-M method.

Solution :

Min-terms	Binary Equivalent			
	A	B	C	D
m_0	0	0	0	0
m_1	0	0	0	1
m_2	0	0	1	0
m_4	0	1	0	0
m_7	0	1	1	1
m_8	1	0	0	0
m_{12}	1	1	0	0
m_{15}	1	1	1	1

Min-terms	Binary Equivalent				No. of 1's
	A	B	C	D	
m_0	0	0	0	0	0
m_1	0	0	0	1	1
m_2	0	0	1	0	1
m_4	0	1	0	0	1
m_8	1	0	0	0	1
m_{12}	1	1	0	0	2
m_7	0	1	1	1	3
m_{15}	1	1	1	1	4

Pair of terms	Binary Equivalents				No. of 1's
	A	B	C	D	
0, 1	0	0	0	–	0
0, 2	0	0	–	0	0
0, 4	0	–	0	0	0
0, 8	–	0	0	0	0
4, 12	–	1	0	0	1
8, 12	1	–	0	0	1
7, 15	–	1	1	1	3

Quad of terms	Binary Equivalent				No. of 1's
0, 8, 4, 12	–	–	0	0	0

Prime Implicants	0	1	2	4	7	8	12	15
0, 8, 4, 12	×			⊗		⊗	⊗	
0, 1	×	⊗						
0, 2	×		⊗					
7, 15					⊗			⊗

$$\therefore \; Y = \overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{D} + BCD$$

REVIEW EXERCISE

Ex. 17 Minimize the logic function using Quine-McClusky method and verify it by using K-map :

$$f(A, B, C, D) = \Sigma m\ (1, 3, 5, 8, 9, 11, 15) \text{ and } d\ (2, 13).$$

Solution :

Decimal Equivalent	Binary Equivalent			
	A	B	C	D
1	0	0	0	1
2*	0	0	1	0
3	0	0	1	1
5	0	1	0	1
8	1	0	0	0
9	1	0	0	1
11	1	0	1	1
13*	1	1	0	1
15	1	1	1	1

Minterms	A	B	C	D	No. of 1's used
1	0	0	0	1	
2*	0	0	1	0	1
8	1	0	0	0	
3	0	0	1	1	
5	0	1	0	1	2
9	1	0	0	1	
11	1	0	1	1	3
13*	1	1	0	1	
15	1	1	1	1	4

Pairs	A	B	C	D	No. of 1's used
1, 3	0	0	–	1	
1, 5	0	–	0	1	
1, 9	–	0	0	1	1
2*, 3	0	0	1	–	
8, 9	1	0	0	–	
3, 11	–	0	1	1	
5, 13*	–	1	0	1	2
9, 13*	1	–	0	1	
11, 15	1	–	1	1	3
13*, 15	1	1	–	1	

Quads	A	B	C	D	No. of 1's used
1, 5, 9, 13*	–	–	0	1	
1, 9, 3, 11	–	0	–	1	1
1, 9, 5, 13*	–	–	0	1	
9, 13*, 11, 15	1	–	–	1	2

$m(\cancel{1}, \cancel{3}, \cancel{5}, 8, \cancel{9}, \cancel{11}, \cancel{15}) + d(12, \cancel{13})$

Prime Implicants	A	B	C	D	Output
✓1, 5, 9, 13*	–	–	0	1	$\overline{C}D$
✓1, 9, 3, 11	–	0	–	1	$\overline{B}D$
✓9, 13*, 11, 15	1	–	–	1	AD
2*, 3	0	0	1	–	$\overline{A}\,\overline{B}C$
✓8, 9	1	0	0	–	$A\overline{B}\,\overline{C}$

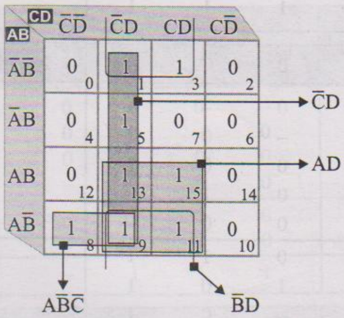
Minimum Selection of minterms

Prime Implicants	m_1	m_3	m_5	m_8	m_9	m_{11}	m_{15}
1, 5, 9, 13*	•		⊙✓		•		
1, 9, 3, 11	•	•			•	•	
9, 13*, 11, 15					•	•	⊙✓
2*, 3		•					
8, 9			single	⊙✓	•		

single single

∴ $f(A,B,C,D) = \overline{C}D + \overline{B}D + AD + A\overline{B}\,\overline{C}$

K-maps is as shown



$f(A,B,C,D) = \overline{C}D + \overline{B}D + AD + A\overline{B}\,\overline{C}$

∴ Output from Q-M method = Output from K-map.

Hence, verified.

Ex. 18 Solve the following function using Q-M method or Quine Mc-Clusky method or Tabular method :

F (A, B, C, D) = Σm (0, 5, 7, 8, 9, 10, 11, 14, 15).

Solution :

Minterms	Decimal Equivalent	Binary Equivalent				No. of 1's used
		A	B	C	D	
m_0	0	0	0	0	0	0
m_5	5	0	1	0	1	2
m_7	7	0	1	1	1	3
m_8	8	1	0	0	0	1
m_9	9	1	0	0	1	2
m_{10}	10	1	0	1	0	2
m_{11}	11	1	0	1	1	3
m_{14}	14	1	1	1	0	3
m_{15}	15	1	1	1	1	4

Minterms	Binary Equivalent				No. of 1's used
	A	B	C	D	
0	0	0	0	0	0
8	1	0	0	0	1
5	0	1	0	1	2
9	1	0	0	1	
10	1	0	1	0	
7	0	1	1	1	3
11	1	0	1	1	
14	1	1	1	0	
15	1	1	1	1	4

Result of 1st comparison

Minterms (Pairs)	Binary Equivalent				No. of 1's used
	A	B	C	D	
0, 8	–	0	0	0	0
8, 9	1	0	0	–	1
8, 10	1	0	–	0	
5, 7	0	1	–	1	2
9, 11	1	0	–	1	
10, 11	1	0	1	–	
10, 14	1	–	1	0	
7, 15	–	1	1	1	3
11, 15	1	–	1	1	
14, 15	1	1	1	–	

Result of 2nd comparison

Minterms	Binary Equivalent				No. of 1's used
	A	B	C	D	
8, 9, 10, 11	1	0	–	–	1
10, 11, 14, 15	1	–	1	–	2

Result of all comparisons

Prime Implicants	Binary Equivalent			
	A	B	C	D
5, 7	0	1	–	1
0, 8	–	0	0	0
7, 15	–	1	1	1
8, 9, 10, 11	1	0	–	–
10, 11, 14, 15	1	–	1	–

S. No.	Prime Implicants	m_0	m_5	m_7	m_8	m_9	m_{10}	m_{11}	m_{14}	m_{15}
1.	5, 7		⊗	×						
2.	0, 8	⊗			×					
3.	7, 15			×						×
4.	8, 9, 10, 11				×	⊗	×	×		
5.	10, 11, 14, 15						×	×	⊗	×

Essential Minterms : m_0, m_5, m_9, m_{14}

Prime Implicants	Binary Equivalent				Output in A B C D form
	A	B	C	D	
5, 7	0	1	–	1	$\bar{A}BD$
0, 8	–	0	0	0	$\bar{B}\bar{C}\bar{D}$
8, 9, 10, 11	1	0	–	–	$A\bar{B}$
10, 11, 14, 15	1	–	1	–	AC

$\therefore \quad f(A, B, C, D) = \bar{A}BD + \bar{B}\bar{C}\bar{D} + A\bar{B} + AC$

Ex. 19 $f(A, B, C, D) = \Sigma m\ (0, 1, 6, 7, 8, 11, 14, 15)$.

Solution :

Minterms	Decimal Equivalent	Binary Equivalent				No. of 1's used
		A	B	C	D	
m_0	0	0	0	0	0	0
m_1	1	0	0	0	1	1
m_6	6	0	1	1	0	2
m_7	7	0	1	1	1	3
m_8	8	1	0	0	0	1
m_{11}	11	1	0	1	1	3
m_{14}	14	1	1	1	0	3
m_{15}	15	1	1	1	1	4

Minterms	Binary Equivalent				No. of 1's used
	A	B	C	D	
0	0	0	0	0	0
1	0	0	0	1	1
8	1	0	0	0	
6	0	1	1	0	2
7	0	1	1	1	3
11	1	0	1	1	
14	1	1	1	0	
15	1	1	1	1	4

Result of 1st comparison

Minterm (Pairs)	Binary Equivalent				No. of 1's used
	A	B	C	D	
0, 1	0	0	0	–	0
0, 8	–	0	0	0	
6, 7	0	1	1	–	2
6, 14	–	1	1	0	
7, 15	–	1	1	1	3
11, 15	1	–	1	1	
14, 15	1	1	1	–	

Minterm (Quads)	Binary Representation			
	A	B	C	D
6, 7, 14, 15	–	1	1	–
6, 14, 7, 15	–	1	1	–

S. No.	Prime Implicants	m_0	m_1	m_6	m_7	m_8	m_{11}	m_{14}	m_{15}
1.	6, 7, 14, 15			⊗	⊗			⊗	×
2.	0, 1	×	⊗						
3.	0, 8	×				⊗			
4.	11, 15						⊗		×

Essential Minterms : m_1, m_6, m_7, m_8, m_{11}, m_{14}

Prime Implicants	Binary Equivalent Result of all comparisons				Output in A B C D
	A	B	C	D	
6, 7, 14, 15	–	1	1	–	B C
0, 1	0	0	0	–	$\overline{A}\,\overline{B}\,\overline{C}$
0, 8	–	0	0	0	$\overline{B}\,\overline{C}\,\overline{D}$
11, 15	1	–	1	1	A C D

$f(A, B, C, D) = BC + \overline{A}\,\overline{B}\,\overline{C} + \overline{B}\,\overline{C}\,\overline{D} + ACD$

Ex. 20 Solve the function using Q-M method : f (A, B, C, D) = Σm (0, 2, 3, 4, 6, 7, 8, 10, 12, 13).

Solution :

Decimal Equivalent	Binary Equivalent				No. of 1's
	A	B	C	D	
0	0	0	0	0	0
2	0	0	1	0	
4	0	1	0	0	1
8	1	0	0	0	
3	0	0	1	1	
6	0	1	1	0	2
10	1	0	1	0	
12	1	1	0	0	
7	0	1	1	1	3
13	1	1	0	1	

Pairs	A	B	C	D	No. of 1's
0, 2	0	0	–	0	
0, 4	0	–	0	0	0
0, 8	–	0	0	0	
2, 3	0	0	1	–	
2, 6	0	–	1	0	
2, 10	–	0	1	0	1
4, 6	0	1	–	0	
4, 12	–	1	0	0	
8, 12	1	0	–	0	
3, 7	0	–	1	1	
6, 7	0	1	1	–	2
12, 13	1	1	0	–	

Quads	A	B	C	D	No. of 1's
0–2–4–6	0	–	–	0	
0–4–2–6	0	–	–	0	0
0–8–4–12	–	–	0	0	
0–8–2–10	–	0	–	0	
2–3–6–7	0	–	1	–	1

Prime Implicants	Result of all Comparisons									
	m_0	m_2	m_3	m_4	m_6	m_7	m_8	m_{10}	m_{12}	m_{13}
2–3–6–7		×	⊗		⊗	⊗				
0–8–4–12	×			⊗			×		×	
0–8–2–10	×	×					×	⊗		
0–4–2–6		×	×					×		×
0–2–8–12										
12–13									×	⊗

Prime Implicants	Result of all Comparisons				Output
12–13	1	1	0	–	→ $AB\overline{C}$
0–2–8–10	–	–	0	0	→ $\overline{C}\,\overline{D}$
0–4–8–12	–	0	–	0	→ $\overline{B}\,\overline{D}$
2–3–6–7	0	–	1	–	→ $\overline{A}C$

$\therefore \quad Y = AB\overline{C}+\overline{C}\,\overline{D}+\overline{B}\,\overline{D}+\overline{A}C$

Ex. 21 Solve the following using Q-Method : F (A, B, C, D) = Σm (0, 2, 5, 8, 12, 13, 15).

Solution :

Minterms	Binary Equivalent					No. of 1's
	A	B	C	D		
0	0	0	0	0	√	0
2	0	0	1	0	√	1
8	1	0	0	0	√	
5	0	1	0	1	√	2
12	1	1	0	0		
13	1	1	0	1	√	3
15	1	1	1	1	√	4

Minterm (Pairs)	Binary Equivalent					No. of 1's
	A	B	C	D		
0, 2	0	0	–	0	*	0
0, 8	–	0	0	0	*	
8, 12	1	–	0	0	*	1
5, 13	–	1	0	1	*	2
12, 13	1	1	0	–	*	
13, 15	1	1	–	1	*	3

Prime Implicants table :

Prime Implicants	0	2	5	8	12	13	15
0, 2	•	⊙					
0, 8	•			•			
8, 12				•	•		
5, 13			⊙			•	
12, 13					•	•	
13, 15						•	⊙
Essential minterms		m_2	m_5				m_{15}

Prime Implicants	Final output
(0, 2)	$\overline{A}\,\overline{B}\,\overline{D}$
(5, 13)	$B\overline{C}D$
(13, 15)	ABD
(8, 12)	$A\overline{C}\,\overline{D}$

$\therefore \quad Y = \overline{A}\,\overline{B}\,\overline{D}+B\overline{C}D+ABD+A\overline{C}\,\overline{D}$

Ex. 22 Minimize using Q-M method : (A, B, C, D) = Σm (0, 2, 3, 5, 6, 7, 8, 9) + d (10, 11, 12, 13, 14, 15).

Solution :

Decimal	Binary Representation					No. of 1's
	A	B	C	D		
0	0	0	0	0	√	0
2	0	0	1	0	√	1
8	1	0	0	0	√	
3	0	0	1	1	√	2
5	0	1	0	1	√	
6	0	1	1	0	√	
9	1	0	0	1	√	
10	1	0	1	0	√	
12	1	1	0	0	√	
7	0	1	1	1	√	3
11	1	0	1	1	√	
13	1	1	0	1	√	
14	1	1	1	0	√	
15	1	1	1	1	√	4

Pairs	A	B	C	D		No. of 1's
0–2	0	0	–	0	√	0
0–8	–	0	0	0	√	
2–3	0	0	1	–	√	1
2–6	0	–	1	0	√	
2–10	–	0	1	0	√	
8–9	1	0	0	–	√	
8–10	1	0	–	0	√	
8–12	1	–	0	0		
3–7	0	–	1	1	√	2
3–11	–	0	1	1	√	
5–7	0	1	–	1	√	
5–13	–	1	0	1	√	
6–7	0	1	1	–	√	
6–14	–	1	1	0	√	
9–11	1	0	–	1	√	
9–13	1	–	0	1	√	
10–11	1	0	1	–	√	
10–14	1	–	1	0	√	
12–13	1	1	0	–	√	
12–14	1	1	–	0	√	
7–15	–	1	1	1	√	3
11–15	1	–	1	1	√	
13–15	1	1	–	1	√	
14–15	1	1	1	–	√	

Quads	A	B	C	D		No. of 1's
0–2–8–10	–	0	–	0	*	0
2–3–6–7	0	–	1	–	√	1
2–3–10–11	–	0	1	–	√	
2–6–10–14	–	–	1	0	√	
8–9–10–11	1	0	–	–	*	
8–9–12–13	1	–	0	–	√	
8–10–12–14	1	–	–	0	√	
3–7–11–15	–	–	1	1	√	2
5–7–13–15	–	1	–	1	*	
6–7–14–15	–	1	1	–	√	
9–11–13–15	1	–	–	1	√	
10–11–14–15	1	–	1	–	√	

Octets	A	B	C	D	No. of 1's
2–3–6–7–10–11–14–15	–	–	1	–	1
8–9–12–13–10–11–14–15	1	–	–	–	

Table of Prime Implicants

Sr. No.	Prime Implicants	A	B	C	D	No. of 1's
1.	8–9–10–11	1	0	–	–	$A\overline{B}$
2.	5–7–13–15	–	1	–	1	BD
3.	2–3–6–7–10–11–14–15	–	–	1	–	C
4.	0–2–8–10	–	0	–	0	$\overline{B}\,\overline{D}$
5.	8–9–12–13–10–11–14–15	1	–	–	–	A

Sr. No.	Prime Implicants	m_0	m_2	m_3	m_5	m_6	m_7	m_8	m_9
1.	8–9–10–11							•	•
2.	5–7–13–15 √				⊙		•		
3.	2–3–6–7–10–11–14–15 √		⊙	⊙		⊙	•		
4.	8–9–12–13–10–11–14–15 √							•	•
5.	0–2–8–10 √	⊙	•					•	

Output :

$$F = BD + \overline{B}\,\overline{D} + A + C$$

REVIEW EXERCISE

Ex. 24 Solve the following using Q-M method : $f(a, b, c, d, e) = \Sigma m$ (0, 1, 3, 8, 9, 13, 14, 15, 16, 17, 19, 25, [illegible]

Solution :

Minterms	Binary Equivalent					No. of 1's used
	a	*b*	*c*	*d*	*e*	
0	0	0	0	0	0	0
1	0	0	0	0	1	1
8	0	1	0	0	0	
16	1	0	0	0	0	
3	0	0	0	1	1	2
9	0	1	0	0	1	
17	1	0	0	0	1	
13	0	1	1	0	1	3
14	0	1	1	1	0	
19	1	0	0	1	1	
15	0	1	1	1	1	4
25	1	0	1	1	1	
27	1	1	0	1	1	
31	1	1	1	1	1	5

Result of 1st comparison

Pair of Minterms	Binary Equivalent					No. of 1's used
	a	*b*	*c*	*d*	*e*	
0–1	0	0	0	0	–	0
0–8	0	–	0	0	0	
0–16	–	0	0	0	0	
1–3	0	0	0	–	1	1
1–9	0	–	0	0	1	
1–17	–	0	0	0	1	
8–9	0	1	0	0	–	
16–17	1	0	0	0	–	
3–19	–	0	0	1	1	2
9–13	0	1	–	0	1	
17–19	1	0	0	–	1	
13–15	0	1	1	–	1	3
14–15	0	1	1	1	–	
19–25	1	–	0	1	1	
19–27	1	–	0	1	1	
15–31	–	1	1	1	1	4
25–31	1	–	1	1	1	
27–31	1	1	–	1	1	

Result of 2nd comparison

Minterms of Quads	Binary Equivalent					No. of 1's used
	a	b	c	d	e	
0–1–8–9	0	–	0	0	–	
0–1–16–17	–	0	0	0	–	0
0–8–1–9	0	–	0	0	–	
0–16–1–17	–	0	0	0	–	
1–3–17–19	–	0	0	–	1	1
1–17–3–19	–	0	0	–	1	
19–25–25–31	1	–	–	1	1	3

S. No.	Prime Implicants	m_0	m_1	m_3	m_8	m_9	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{19}	m_{25}	m_{27}	m_{31}
1.	0–1–8–9	×	×		⊗	⊗									
2.	0–1–16–17	×	×												
3.	1–3–17–19		×	⊗						⊗	×				
4.	19–25–25–31										×	×			
5.	13–15											×	⊗		⊗
6.	14–15						⊗		×						
7.	19–27							⊗	×						
												×		⊗	

Essential Minterms : $m_3, m_8, m_9, m_{13}, m_{14}, m_{16}, m_{25}, m_{27}, m_{31}$.

Prime Implicants	Binary Representation					Output
	a	b	c	d	e	
0–1–8–9	0	–	0	0	–	$\bar{a}\bar{c}\bar{d}$
0–1–16–17	–	0	0	0	–	$\bar{b}\bar{c}\bar{d}$
1–3–17–19	–	0	0	–	1	$\bar{b}\bar{c}e$
19–25–25–31	1	–	–	1	1	$a d e$
13–15	0	1	1	–	1	$\bar{a}bce$
14–15	0	1	1	1	–	$\bar{a}bcd$
19–27	1	–	0	1	1	$a\bar{c}de$

$$f(a,b,c,d,e) = \bar{a}\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d} + \bar{b}\bar{c}e + ade + \bar{a}bce + \bar{a}bcd + a\bar{c}de$$

Ex. 25 Solve the following using Q-M method or Tabular method :

F (A, B, C, D, E, F) = (1, 3, 8, 7, 15, 18, 23, 26, 29, 35, 43, 57, 62).

Solution :

Minterms	Binary Equivalent						No. of 1's
	A	B	C	D	E	F	
1	0	0	0	0	0	1 √	1
8	0	0	1	0	0	0	
3	0	0	0	0	1	1 √	2
18	0	1	0	0	1	0 √	
7	0	0	0	1	1	1 √	
26	0	1	1	0	1	0 √	3
35	1	0	0	0	1	1 √	

15	0	0	1	1	1	1	√	
23	0	1	0	1	1	1	√	
29	0	1	1	1	0	1		4
43	1	0	1	0	1	1	√	
57	1	1	1	0	0	1		
62	1	1	1	1	1	0		5

Minterm (Pairs)	Binary Equivalent						No. of 1's used
	A	B	C	D	E	F	
(1, 3)	0	0	0	0	–	1	1
(3, 7)	0	0	0	–	1	1	
(3, 35)	–	0	0	0	1	1	2
(18, 26)	0	1	–	0	1	0	
(7, 15)	0	0	–	1	1	1	3
(7, 23)	0	–	0	1	1	1	
(35, 43)	1	0	–	0	1	1	

S. No.	Prime Implicants	m_1	m_3	m_8	m_7	m_{15}	m_{18}	m_{23}	m_{26}	m_{29}	m_{35}	m_{43}	m_{57}	m_{62}
1.	8			⊙										
2.	29									⊙				
3.	57												⊙	
4.	62													⊙
5.	(1, 3)	⊙	•											
6.	(3, 7)		•		•									
7.	(3, 35)		•								•			
8.	(18, 26)						⊙		⊙					
9.	(7, 15)				•	⊙								
10.	(7, 23)				•			⊙						
11.	(35, 43)										•	⊙		
	Essential Minterms	m_1		m_8		m_{15}	m_{18}	m_{23}	m_{26}	m_{29}		m_{43}	m_{57}	m_{62}

Prime Implicants	A	B	C	D	E	F	Outputs
(1, 3)	0	0	0	0	–	1	$\bar{A}\bar{B}\bar{C}\bar{D}F$
8	0	0	1	0	0	0	$\bar{A}\bar{B}C\bar{D}\bar{E}\bar{F}$
(7, 15)	0	0	–	1	1	1	$\bar{A}\bar{B}DEF$
(18, 26)	0	1	–	0	1	0	$\bar{A}B\bar{D}E\bar{F}$
(7, 23)	0	–	0	1	1	1	$\bar{A}\bar{C}DEF$
29	0	1	1	1	0	1	$\bar{A}BCD\bar{E}F$
(35, 43)	1	0	–	0	1	1	$A\bar{B}\bar{D}EF$
57	1	1	1	0	0	1	$ABC\bar{D}\bar{E}F$
62	1	1	1	1	1	0	$ABCDEF$

$$\therefore\ Y = \bar{A}\bar{B}\bar{C}\bar{D}F + \bar{A}\bar{B}C\bar{D}\bar{E}\bar{F} + \bar{A}\bar{B}DEF + \bar{A}B\bar{D}\bar{E}F$$
$$+ \bar{A}\bar{C}DEF + \bar{A}BCD\bar{E}F + A\bar{B}\bar{D}EF + ABC\bar{D}\bar{E}F + \bar{A}\bar{B}\bar{C}\bar{D}\bar{E}F$$

REVIEW EXERCISE

Ex. 26 **Obtain the set of prime implicants for Σm (0, 2, 4, 7, 8, 16, 24, 32, 36, 40, 48) using the binary designations of min-terms using Q-M method.**

Solution :

Minterms	Binary Equivalent					
	A	B	C	D	E	F
m_0	0	0	0	0	0	0
m_2	0	0	0	0	1	0
m_4	0	0	0	1	0	0
m_7	0	0	0	1	1	1
m_8	0	0	1	0	0	0
m_{16}	0	1	0	0	0	0
m_{24}	0	1	1	0	0	0
m_{32}	1	0	0	0	0	0
m_{36}	1	0	0	1	0	0
m_{40}	1	0	1	0	0	0
m_{48}	1	1	0	0	0	0

Minterms	Binary Equivalent						No. of 1's used
	A	B	C	D	E	F	
m_0	0	0	0	0	0	0	0
m_2	0	0	0	0	1	0	1
m_4	0	0	0	1	0	0	1
m_8	0	0	1	0	0	0	1
m_{16}	0	1	0	0	0	0	1
m_{32}	1	0	0	0	0	0	1
m_{24}	0	1	1	0	0	0	2
m_{36}	1	0	0	1	0	0	2
m_{40}	1	0	1	0	0	0	2
m_{48}	1	1	0	0	0	0	2
m_7	0	0	0	1	1	1	3

Pairs of minterms	Binary Equivalent						No. of 1's used
	A	B	C	D	E	F	
0, 2	0	0	0	0	–	0	0
0, 4	0	0	0	–	0	0	0
0, 8	0	0	–	0	0	0	0
0, 16	0	–	0	0	0	0	0
0, 32	–	0	0	0	0	0	0
4, 36	–	0	0	1	0	0	1
8, 24	0	–	1	0	0	0	1
8, 40	–	0	1	0	0	0	1
16, 24	0	1	–	0	0	0	1
16, 48	–	1	0	0	0	0	1
32, 36	1	0	0	–	0	0	1
32, 40	1	0	–	0	0	0	1
32, 48	1	–	0	0	0	0	1

Pairs of terms	Binary Equivalent						No. of 1's used
	A	B	C	D	E	F	
0, 4, 32, 36	–	0	0	–	0	0	0
0, 8, 16, 24	0	–	–	0	0	0	0
0, 8, 32, 40	–	0	–	0	0	0	0
0, 16, 8, 24	0	–	–	0	0	0	0
0, 16, 32, 48	–	–	0	0	0	0	0
0, 32, 4, 36	–	0	0	–	0	0	0
0, 32, 8, 40	–	0	–	0	0	0	0
0, 32, 16, 48	–	–	0	0	0	0	0

Prime Implicants	0	2	4	7	8	16	24	32	36	40	48
0, 4, 32, 36	×		⊗					×	⊗		
0, 8, 16, 24	×				×	×	⊗				
0, 8, 32, 40	×				×			×		⊗	
0, 16, 32, 48	×					×		×			⊗
0, 2	×	⊗									
7			⊗								

$$\therefore\ Y = \overline{B}\,\overline{C}\,\overline{E}\,\overline{F} + \overline{A}\,\overline{D}\,\overline{E}\,\overline{F} + \overline{B}\,\overline{D}\,\overline{E}\,\overline{F} + \overline{C}\,\overline{D}\,\overline{E}\,\overline{F} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\,\overline{F} + \overline{A}\,\overline{B}\,\overline{C}\,DEF$$

Ex. 27 Solve the following expression using Q-M method

$$F = \Sigma m\ (0, 2, 6, 7, 8, 10, 12, 14, 15, 52).$$

Solution :

No. of 1's	Binary Equivalent						Decimal Equivalent	
	A	B	C	D	E	F		
0	0	0	0	0	0	0	0	√
1	0	0	0	0	1	0	2	√
	0	0	1	0	0	0	8	√
	0	0	0	1	1	0	6	√
2	0	0	1	0	1	0	10	√
	0	0	1	1	0	0	12	√
	0	0	0	1	1	1	7	√
3	0	0	1	1	1	0	14	√
	1	1	0	1	0	0	52	P–1
4	0	0	1	1	1	1	15	√

REVIEW EXERCISE

Two Term Combinations (Pairs)

Pairs	Binary Equivalent						No. of 1's used
	A	B	C	D	E	F	
0, 2	0	0	0	0	—	0	0
0, 8	0	0	—	0	0	0	
2, 6	0	0	0	—	1	0	1
2, 10	0	0	—	0	1	0	
8, 10	0	0	1	0	—	0	
8, 12	0	0	1	—	0	0	
6, 7	0	0	0	1	1	—	2
6, 14	0	0	—	1	1	0	
10, 14	0	0	1	—	1	0	
12, 14	0	0	1	1	—	0	
7, 15	0	0	—	1	1	1	3
14, 15	0	0	1	1	1	—	

Four Term Combinations (Quads)

Quads	Binary Equivalent						No. of 1's used
	A	B	C	D	E	F	
0, 2, 8, 10	0	0	—	0	—	0	0
2, 6, 10, 14	0	0	—	—	1	0	1
8, 10, 12, 14	0	0	1	—	—	0	
6, 7, 14, 15	0	0	—	1	1	—	2

PRIME IMPLICANTS TABLE

Prime Implicants	0	2	6	7	8	10	12	14	15	52
52										⊗
0, 2, 8, 10	⊗	×			×	×				
2, 6, 10, 14		×	×		×		×			
8, 10, 12, 14					×	×	⊗	×		
6, 7, 14, 15			×	⊗				×	⊗	

Therefore, no extra terms are needed as it will cover all the given minterms.

Essential Minterms	Binary Equivalent						Outputs
	A	B	C	D	E	F	
52	1	1	0	1	0	0	$AB\overline{C}D\overline{E}\overline{F}$
0, 2, 8, 10	0	0	–	0	–	0	$\overline{A}\overline{B}\overline{D}\overline{F}$
8, 10, 12, 14	0	0	1	–	–	0	$\overline{A}\overline{B}C\overline{F}$
6, 7, 14, 15	0	0	–	1	1	–	$\overline{A}\overline{B}DE$

$\therefore \quad F = AB\overline{C}D\overline{E}\overline{F} + \overline{A}\overline{B}\overline{D}\overline{F} + \overline{A}\overline{B}C\overline{F} + \overline{A}\overline{B}DE$

Ex. 28 Minimize the following function using Q-M method.

Y (A, B, C, D, E, F) = Σ*m* (0, 1, 2, 5, 7, 9, 10, 13, 14, 17, 19, 21, 23, 29, 31, 35, 39, 43, 45, 49, 52, 55) + Σ*d* (3, 11, 18, 56, 60).

Ex. 29 Minimize the following function using Q-M method

$f(A, B, C, D, E, F) = \Sigma m\,(6, 9, 13, 18, 19, 25, 27, 29, 41, 45, 57, 61)$.

Solution :

Decimal Equivalent	Binary Equivalent					
	A	B	C	D	E	F
6	0	0	0	1	1	0
9	0	0	1	0	0	1
13	0	0	1	1	0	1
18	0	1	0	0	1	0
19	0	1	0	0	1	1
25	0	1	1	0	0	1
27	0	1	1	0	1	1
29	0	1	1	1	0	1
41	1	0	1	0	0	1
45	1	0	1	1	0	1
57	1	1	1	0	0	1
61	1	1	1	1	0	1

Minterms	A	B	C	D	E	F	No. of 1's used
6	0	0	0	1	1	0	
9	0	0	1	0	0	1	2
18	0	1	0	0	1	0	
13	0	0	1	1	0	1	
19	0	1	0	0	1	1	3
41	1	0	1	0	0	1	
25	0	1	1	0	0	1	
27	0	1	1	0	1	1	
29	0	1	1	1	0	1	4
45	1	0	1	1	0	1	
57	1	1	1	0	0	1	
61	1	1	1	1	0	1	5

Pairs	A	B	C	D	E	F	No. of 1's used
9, 13	0	0	1	–	0	1	
9, 41	–	0	1	0	0	1	2
9, 25	0	–	1	0	0	1	
18, 19	0	1	0	0	1	–	
13, 29	0	–	1	1	0	1	
25, 27	0	1	1	0	–	1	
13, 45	–	0	1	1	0	1	
25, 29	0	1	1	–	0	1	3
19, 27	0	1	–	0	1	1	
25, 57	–	1	1	0	0	1	
41, 45	1	0	1	–	0	1	
41, 57	1	–	1	0	0	1	
29, 61	–	1	1	1	0	1	
45, 61	1	–	1	1	0	1	4
57, 61	1	1	1	–	0	1	

REVIEW EXERCISE

Quads	A	B	C	D	E	F	No. of 1's used
9, 13, 41, 45 Same	–	0	1	–	0	1	2
9, 41, 13, 45 Same							
9, 25, 13, 29 Same	–	–	1	0	0	1	
9, 25, 41, 57	0	–	1	–	0	1	
9, 13, 25, 29							
9, 41, 25, 57							
13, 45, 29, 61	–	–	1	1	0	1	3
25, 57, 29, 61	–	1	1	–	0	1	
13, 29, 45, 61							
41, 57, 45, 61	1	1	1	–	0	1	4
25, 29, 57, 61							
41, 45, 57, 61							

Octet	A	B	C	D	E	F	Output
9, 13, 41, 45 25, 57, 29, 61	–	–	1	–	0	1	$C\bar{E}F$

$\Sigma m(\cancel{6}, \cancel{9}, \cancel{13}, \cancel{18}, \cancel{19}, \cancel{25}, \cancel{27}, \cancel{29}, \cancel{41}, \cancel{45}, \cancel{57}, \cancel{61})$.

Thus all minterms are cover in the prime implicant table shown below :

Prime Implicants	m_6	m_9	m_{13}	m_{18}	m_{19}	m_{25}	m_{27}	m_{29}	m_{41}	m_{45}	m_{57}	m_{61}
9, 13, 41, 45, 25, 57, 29, 61		⊙	⊙			⊙		⊙	•	⊙	•	•
18, 19				⊙	⊙							
41, 57									•		•	
57, 61											•	•
6	⊙											

$\therefore \quad f(A, B, C, D, E, F) = C\bar{E}F + \bar{A}\bar{B}\bar{C}DE\bar{F} + \bar{A}B\bar{C}\bar{D}E$

Ex. 30 Solve the following using Q-M method

F (A, B, C, D, E, F) = Σm (0, 2, 4, 8, 16, 24, 32, 36, 40, 48) + *d* (18, 22, 54, 56).

Solution :

Minterms	Binary Representation						No. of 1's used
	A	B	C	D	E	F	
0	0	0	0	0	0	0	0
2	0	0	0	0	1	0	1
4	0	0	0	1	0	0	
8	0	0	1	0	0	0	
16	0	1	0	0	0	0	
32	1	0	0	0	0	0	
18	0	1	0	0	1	0	
24	0	1	1	0	0	0	2
36	1	0	0	1	0	0	
40	1	0	1	0	0	0	
48	1	1	0	0	0	0	
22	0	1	0	1	1	0	3
25	0	1	1	0	0	1	
56	1	1	1	0	0	0	
54	1	1	0	1	1	0	4

Q-M Method OR Tabular Method (QM-1st)

→ In case the number of variable increases, 7, 8, 9 or even 10 variables. It is difficult to use K-Map bcoz it becomes very complex. To overcome this problem W.V Quine and E.J. Mc-cluskey developed a tabular method that is also known as Q-M Method, used for minimization of large number of variables.

→ It make the use of same technique as of K-Map, but in this minterms are written in binary form 1st and binary equivalent which are differ only in one place can be combined for reduction of minterm.

Q- Simplify and minimize the following function using Q-M Method

$$\boxed{f(A,B,C,D) = \sum m\,(0, 2, 3, 4, 6, 7, 8, 10, 12, 13)}$$

Sol. Step 1st: write all the given minterms in the binary form

Min terms	Decimal Equivalent	Binary equivalent A 8	B 4	C 2	D 1	Number of 1's used in minterm
m_0	0	0	0	0	0	0
m_2	2	0	0	1	0	1
m_3	3	0	0	1	1	2
m_4	4	0	1	0	0	1
m_6	6	0	1	1	0	2
m_7	7	0	1	1	1	3
m_8	8	1	0	0	0	1
m_{10}	10	1	0	1	0	2
m_{12}	12	1	1	0	0	2
m_{13}	13	1	1	0	1	3