

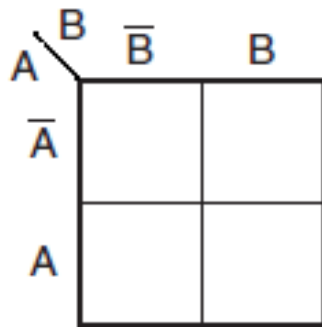
Karnaugh Maps(*K*- Map)

- A Karnaugh map is a graphical representation of the logic system.
- It can be drawn directly from either minterm (sum-of-products) or maxterm (product-of-sums) Boolean expressions
- It is desired to have a minimized sum-of-products or a minimized product-of-sums expression.
- **Types of *K*- Map**
- Two-variable Karnaugh map.

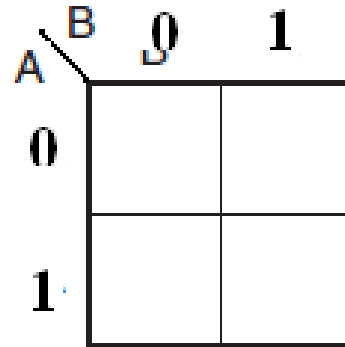
- *Construction of a Karnaugh Map(K- Map)*

- An n-variable Karnaugh map has 2^n squares
 - In the case of a **minterm** Karnaugh map,
 - '1' is placed in all those squares for which the output is '1',
 - '0' is placed in all those squares for which the output is '0'. 0s are omitted for simplicity.
 - An 'X' is placed in squares corresponding to 'don't care' conditions.
- In the case of a **maxterm** Karnaugh map,
 - '1' is placed in all those squares for which the output is '0',
 - '0' is placed for input entries corresponding to a '1' output. Again, 0s are omitted for simplicity,
 - an 'X' is placed in squares corresponding to 'don't care' conditions.

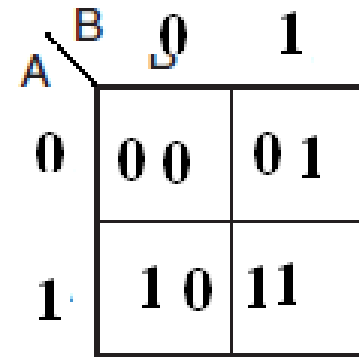
- Two-variable Karnaugh map.



(a)

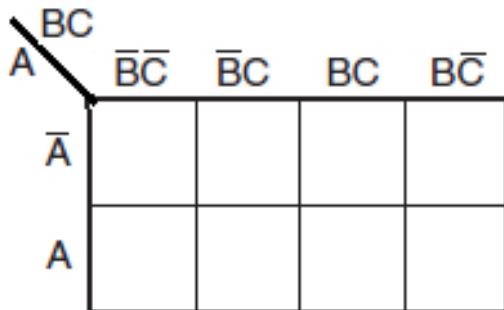


(a)

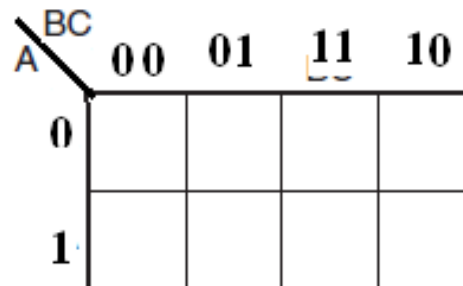


(a)

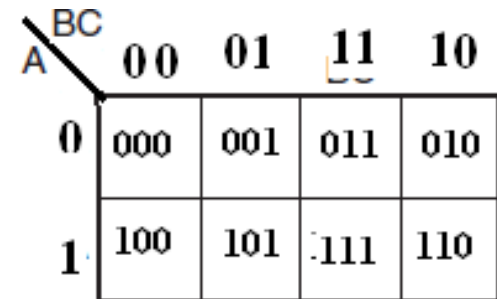
- Three-variable Karnaugh map.



(a)



(a)



(a)

- Four-variable K- map.

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
AB	$\bar{A}\bar{B}$				
	$\bar{A}B$				
	$A\bar{B}$				
	AB				

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
AB	$\bar{A}\bar{B}$	0000	0001	0011	0010
	$\bar{A}B$	0100	0101	0111	0110
	$A\bar{B}$	1100	1101	1111	1110
	AB	1000	1001	1011	1010

- five-variable K- map.

Five variable K-map is (i.e. A, B, C, D and E) as shown

		DE			
		00	01	11	10
BC	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10

A = 0

		DE			
		00	01	11	10
BC	00	16	17	19	18
	01	20	21	23	22
	11	28	29	31	30
	10	24	25	27	26

A = 1

- six-variable K- map.

A = 0		B = 0			
		CD \ EF			
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10
A = 1		B = 1			
		CD \ EF			
		32	33	35	34
		36	37	39	38
		44	45	47	46
		40	41	43	42
A = 0		B = 0			
		CD \ EF			
		16	17	19	18
		20	21	23	22
		28	29	31	30
		24	25	27	26
A = 1		B = 1			
		CD \ EF			
		48	49	51	50
		52	53	55	54
		60	61	63	62
		56	57	59	58

- For minterms (sum of product) table .

Decimal Number	Variables		Minterms	Minterm Designation
	A	B		
0	0	0	$\bar{A}\bar{B}$	m_0
1	0	1	$\bar{A}B$	m_1
2	1	0	$A\bar{B}$	m_2
3	1	1	AB	m_3

A \ B	0	1
0	00	01
1	10	11

A \ B	\bar{B}	B
\bar{A}	$\bar{A}\bar{B}$	$\bar{A}B$
A	$A\bar{B}$	AB

A \ B	0	1
0	0	1
1	2	3

A \ B	\bar{B}	B
\bar{A}	m_0	m_1
A	m_2	m_3

- For maxterms (product of sum) table .

Maxterms table is as shown (with corresponding minterms also)

Decimal Number	Variables		Minterms	Minterm Designation	Maxterms	Maxterm Designation
	A	B				
0	0	0	$\bar{A}\bar{B}$	m_0	$A + B$	M_0
1	0	1	$\bar{A}B$	m_1	$A + \bar{B}$	M_1
2	1	0	$A\bar{B}$	m_2	$\bar{A} + B$	M_2
3	1	1	AB	m_3	$\bar{A} + \bar{B}$	M_3

So the corresponding K-map will be

		B	
		0	1
A	0	$A+B$	$A+\bar{B}$
	1	$\bar{A}+B$	$\bar{A}+\bar{B}$

		B	
		0	1
A	0	M_0	M_1
	1	M_2	M_3

Three-variable K map.

Decimal Number	Variables			Minterms	Minterm Designation	Maxterms	Maxterm Designation
	A	B	C				
0	0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0	$A + B + C$	M_0
1	0	0	1	$\bar{A}\bar{B}C$	m_1	$A + B + \bar{C}$	M_1
2	0	1	0	$\bar{A}B\bar{C}$	m_2	$A + \bar{B} + C$	M_2
3	0	1	1	$\bar{A}BC$	m_3	$A + \bar{B} + \bar{C}$	M_3
4	1	0	0	$A\bar{B}\bar{C}$	m_4	$\bar{A} + B + C$	M_4
5	1	0	1	$A\bar{B}C$	m_5	$\bar{A} + B + \bar{C}$	M_5
6	1	1	0	$AB\bar{C}$	m_6	$\bar{A} + \bar{B} + C$	M_6
7	1	1	1	ABC	m_7	$\bar{A} + \bar{B} + \bar{C}$	M_7

For minterms (sum of product) table.

A	BC			
	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC
\bar{A}	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}B\bar{C}$	$\bar{A}BC$
A	$A\bar{B}\bar{C}$	$A\bar{B}C$	$AB\bar{C}$	ABC

or

A	BC			
	00	01	11	10
0	000	001	011	010
1	100	101	111	110

- Grouping of adjacent one pair.

	B	\bar{B}	B
\bar{A}	0	1	
A	0	1	

	B	\bar{B}	B
\bar{A}	$\bar{A}\bar{B}$	$\bar{A}B$	
A	$A\bar{B}$	AB	

– If we consider the minterm of K map. Then

- Output of the K map is $Y = B$

$$\begin{aligned}
 Y &= \bar{A}B + AB \\
 &= B(\bar{A} + A) \\
 &= B.1 = B
 \end{aligned}$$

- Three variable K map

B is common

BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
\overline{A}	0	0	1	1
A	0	0	0	0

- Output of the K map is $Y = \overline{A}B$

BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
\overline{A}	0	0	0	1
A	0	0	0	1

- Output of the K map is $Y = B\overline{C}$

BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
\overline{A}	1	0	0	1
A	0	0	0	0

- Output of the K map is $Y = \overline{A} \cdot \overline{C}$

C is common

BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
\overline{A}	0	1	1	0
A	0	0	1	0

- Output of the K map is $Y = \overline{A}C + BC$

		$\overline{B}\overline{C}$		$\overline{B}C$		BC		$B\overline{C}$	
A	\overline{A}	1	1			0	0		
	A	1	1			0	0		

\overline{B} is the common term

- Output of the K map is $Y = \overline{B}$

		BC		$\overline{B}\overline{C}$		BC	
A	BC	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$		
	\overline{A}	0	1	1	0		
A	\overline{A}	0	1	1	0		

C is the common term

- Output of the K map is $Y = C$

		BC		BC		$B\overline{C}$	
A	BC	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$		
	\overline{A}	0	0	1	1		
A	\overline{A}	0	0	1	1		

B is the common term

- Output of the K map is $Y = B$

		BC		$\overline{B}\overline{C}$		$B\overline{C}$	
A	BC	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$		
	\overline{A}	1	0	0	1		
A	\overline{A}	1	0	0	1		

\overline{C} is common term

- Output of the K map is $Y = \overline{C}$

Four-variable K- map.

		CD			
		00	01	11	10
AB	00				
	01				
	11				
	10				

		CD			
		00	01	11	10
AB	00	0000	0001	0011	0010
	01	0100	0101	0111	0110
	11	1100	1101	1111	1110
	10	1000	1001	1011	1010

- Example:

Minimize the following function using K – map. Right truth table and Draw the logic ckt diagram,

$$f(A,B,C,D) = \sum m(0,1,2,3,6,8,9,10,11,12,13)$$

	A	B	C	D	F
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}$	1 ₀	1 ₁	1 ₃	1 ₂
	$\bar{A}B$	0 ₄	0 ₅	0 ₇	1 ₆
	$A\bar{B}$	1 ₁₂	1 ₁₃	0 ₁₅	0 ₁₄
	AB	1 ₈	1 ₉	1 ₁₁	1 ₁₀

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}$	1 ₀	1 ₁	1 ₃	1 ₂
	$\bar{A}B$	0 ₄	0 ₅	0 ₇	1 ₆
	$A\bar{B}$	1 ₁₂	1 ₁₃	0 ₁₅	0 ₁₄
	AB	1 ₈	1 ₉	1 ₁₁	1 ₁₀

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}$	1 ₀	1 ₁	1 ₃	1 ₂
	$\bar{A}B$	0 ₄	0 ₅	0 ₇	1 ₆
	$A\bar{B}$	1 ₁₂	1 ₁₃	0 ₁₅	0 ₁₄
	AB	1 ₈	1 ₉	1 ₁₁	1 ₁₀

Group 1: $\bar{C}\bar{D}$, $\bar{C}D$, CD , $C\bar{D}$ (top row)
Group 2: $\bar{A}\bar{B}$, $\bar{A}B$, $A\bar{B}$, AB (left column)
Group 3: $\bar{C}\bar{D}$, $\bar{C}D$, CD , $C\bar{D}$ (bottom row)

		\bar{C}			
		CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD
A	AB	1	1	0	0
	$\bar{A}B$	1	1	0	0
	$\bar{A}\bar{B}$	0	0	0	0
	AB	0	0	0	0

$Y = \bar{A}.\bar{C}$

		D			
		CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD
A	AB	0	1	1	0
	$\bar{A}B$	0	1	1	0
	$\bar{A}\bar{B}$	0	0	0	0
	AB	0	0	0	0

$Y = \bar{A}.D$

		D			
		CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD
B	AB	0	0	0	0
	$\bar{A}B$	0	1	1	0
	$\bar{A}\bar{B}$	0	1	1	0
	AB	0	0	0	0

$Y = B.D$

		D			
		CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD
A	AB	0	0	0	0
	$\bar{A}B$	0	0	0	0
	$\bar{A}\bar{B}$	0	1	1	0
	AB	0	1	1	0

$Y = A.D$

		CD		$C\bar{D}$	
AB	CD	0	0	1	1
	$\bar{C}\bar{D}$	0	0	1	1
	$\bar{C}D$	0	0	1	1
	$C\bar{D}$	0	0	1	1

C is common
 $\therefore Y = C$

		$\bar{C}\bar{D}$		$C\bar{D}$	
AB	CD	1	0	0	1
	$\bar{C}\bar{D}$	1	0	0	1
	$\bar{C}D$	1	0	0	1
	$C\bar{D}$	1	0	0	1

\bar{D} is common
 $\therefore Y = \bar{D}$

		CD			
AB	$\bar{A}\bar{B}$	1	1	1	1
	$\bar{A}B$	0	0	0	0
	$A\bar{B}$	0	0	0	0
	AB	1	1	1	1

$\therefore Y = \bar{B}$

		$\bar{C}\bar{D}$		$\bar{C}D$		$C\bar{D}$	
\bar{A}	$\bar{A}\bar{B}$	1	1	1	1		
	$\bar{A}B$	1	1	1	1		
	$A\bar{B}$	0	0	0	0		
	AB	0	0	0	0		

$\therefore Y = \bar{A}$

		$\bar{C}\bar{D}$		$\bar{C}D$		$C\bar{D}$	
B	$\bar{A}\bar{B}$	0	0	0	0		
	$\bar{A}B$	1	1	1	1		
	$A\bar{B}$	1	1	1	1		
	AB	0	0	0	0		

$\therefore Y = B$

		CD			
A	$\bar{A}\bar{B}$	0	0	0	0
	$\bar{A}B$	0	0	0	0
	$A\bar{B}$	1	1	1	1
	AB	1	1	1	1

$\therefore Y = A$

- Example:

Minimize the following function using K – map. Right truth table and Draw the logic ckt diagram,

$$f(A, B, C, D) = \sum m(3, 4, 5, 7, 9, 13, 14, 15)$$

Five-variable K- map.

A=0 i.e. \bar{A}					A=1 i.e. A				
BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE	BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE
$\bar{B}\bar{C}$	0	1	3	2	$\bar{B}\bar{C}$	16	17	19	18
$\bar{B}C$	4	5	7	6	$\bar{B}C$	20	21	23	22
BC	12	13	15	14	BC	28	29	31	30
BC	8	9	11	10	BC	24	25	27	26

- Example:

Minimize the following function using K – map. Right truth table and Drawn the logic ckt diagram,

$$f(A, B, C, D, E) = \sum m(3, 5, 6, 8, 9, 12, 13, 14, 19, 22, 23, 24, 25, 30)$$

A = 0 or \bar{A}					A = 1 or A				
BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE	BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	$D\bar{E}$	DE
$\bar{B}\bar{C}$	0	0	1	0	$\bar{B}\bar{C}$	0	0	1	0
$\bar{B}C$	0	1	0	1	$\bar{B}C$	0	0	0	1
BC	1	1	0	1	BC	0	0	0	1
BC	1	1	0	0	BC	1	1	0	0

G_5 ← (Group 1: $\bar{B}\bar{C}, D\bar{E}$)
 G_4 ← (Group 2: $\bar{B}C, \bar{D}E$)
 G_3 ← (Group 3: $BC, \bar{D}E$)

- Example:

Minimize the following function using K – map. Right truth table and Draw the logic ckt diagram,

$$f(A, B, C, D, E) = \sum m(0, 5, 6, 8, 9, 10, 11, 16, 20, 24, 25, 26, 27)$$

