Karnaugh Maps*(K*- Map*)*

- A Karnaugh map is a graphical representation of the logic system.
- It can be drawn directly from either minterm (sum-of-products) or maxterm (product-of-sums) Boolean expressions
- It is desired to have a minimized sum-of-products or a minimized product-of-sums expression.

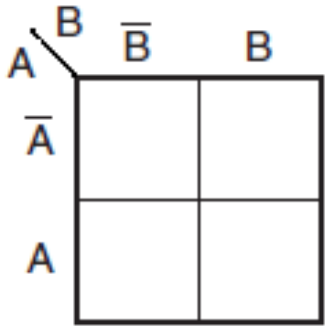
- **<u>Types of *K*- Map</u>**
- Two-variable Karnaugh map.

- *Construction of a Karnaugh Map(K-* Map*)*

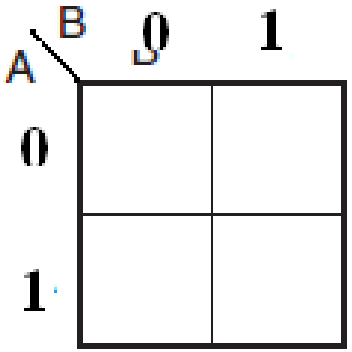
- An n-variable Karnaugh map has 2n squares
  - In the case of a **minterm** Karnaugh map,
  - '1' is placed in all those squares for which the output is '1',
  - '0' is placed in all those squares for which the output is '0'. 0s are omitted for simplicity.
  - An 'X' is placed in squares corresponding to 'don't care' conditions.

- In the case of a **maxterm** Karnaugh map,
  - '1' is placed in all those squares for which the output is '0',
  - '0' is placed for input entries corresponding to a '1' output. Again, 0s are omitted for simplicity,
  - an 'X' is placed in squares corresponding to 'don't care' conditions.

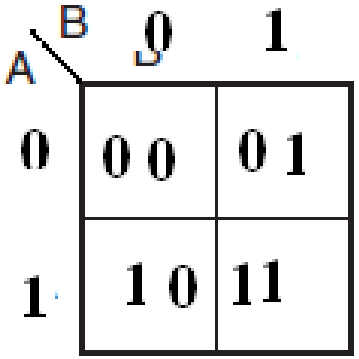
- Two-variable Karnaugh map.



(a)

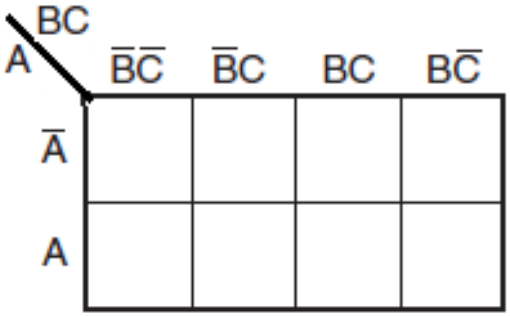


(a)

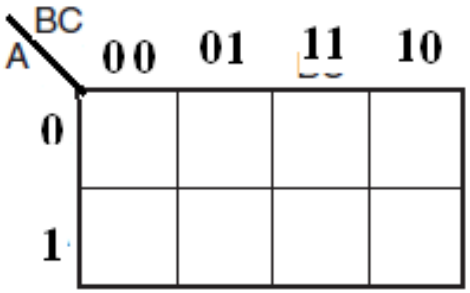


(a)

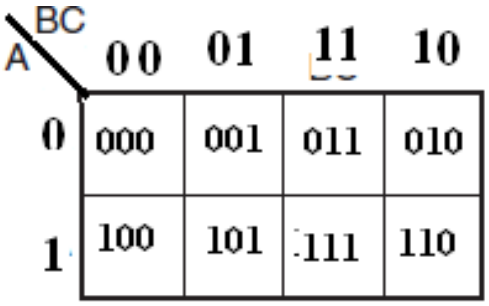
- Three-variable Karnaugh map.



(a)

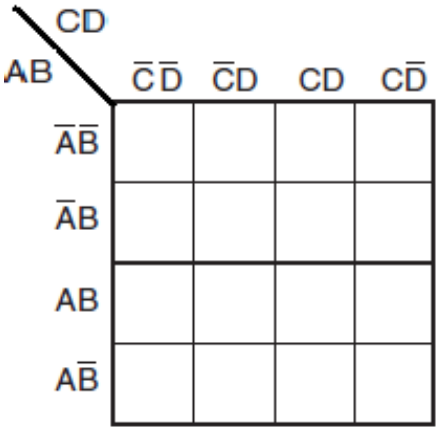


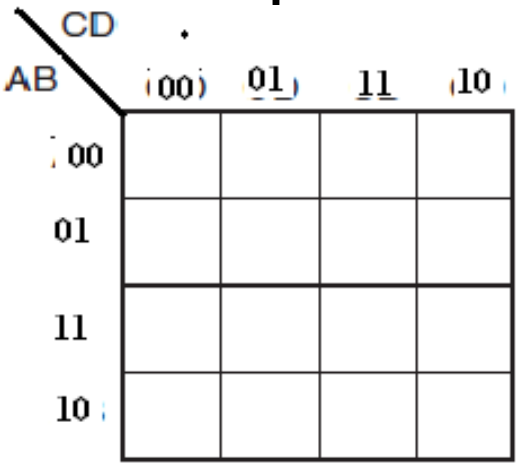
(a)



(a)

- Four-variable K- map.





CD

AB	00	01	11	10
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010

- five-variable K- map.

Five variable K-map is (*i.e.* A, B, C, D and E) as shown

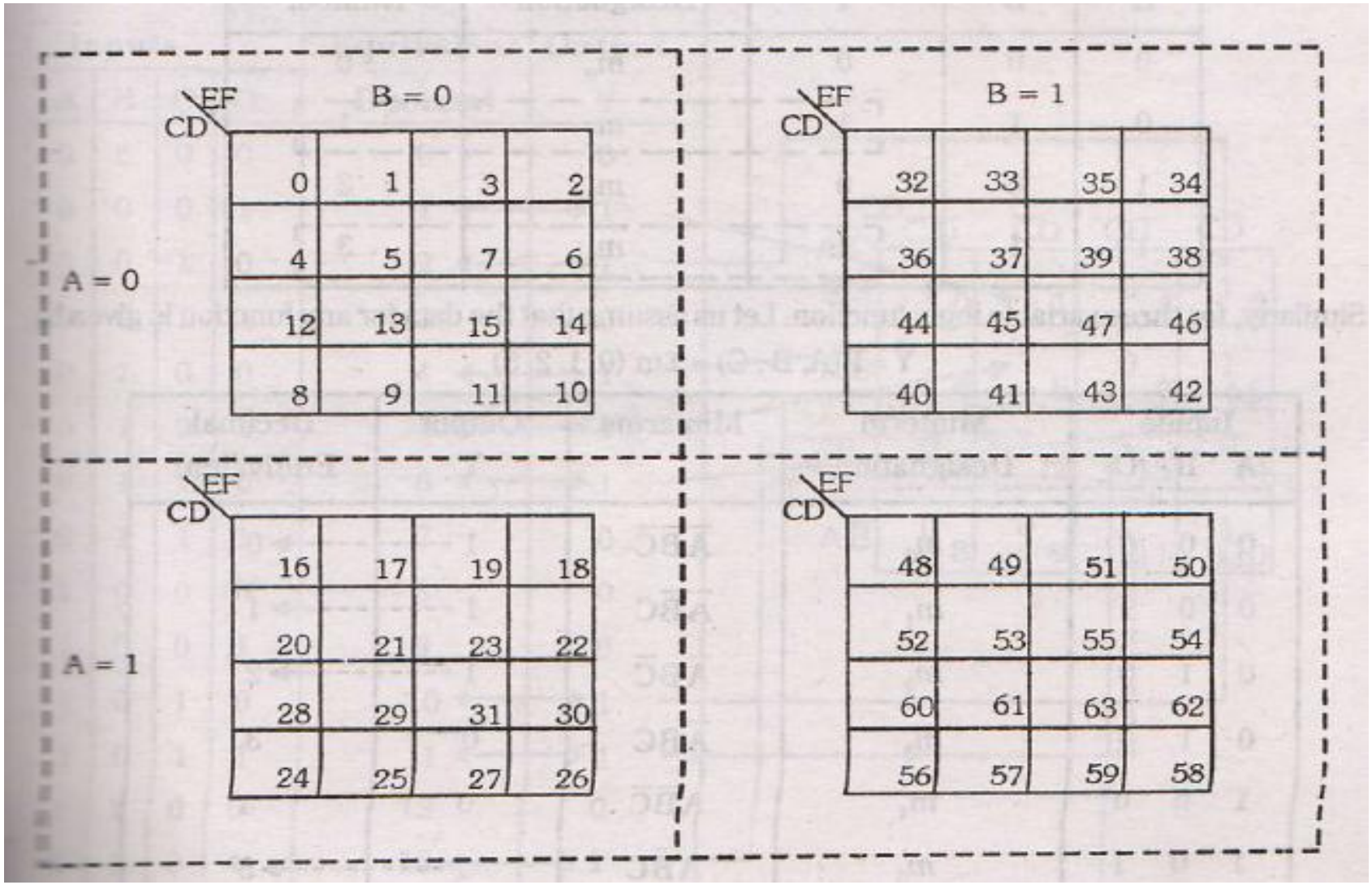
BC \ DE	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

A = 0

BC \ DE	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

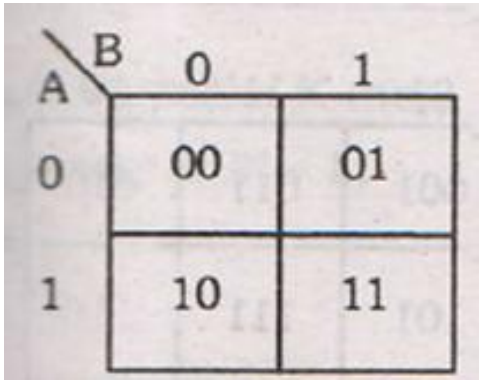
A = 1

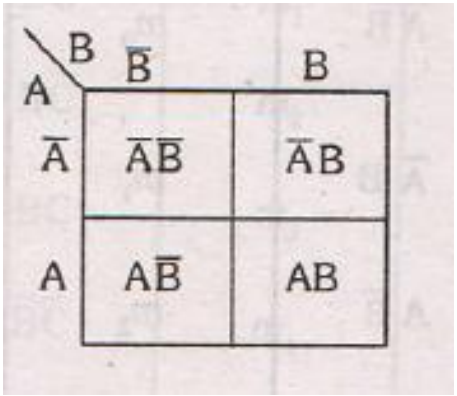
- six-variable K- map.

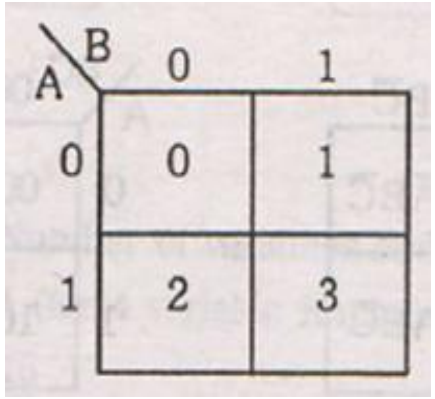


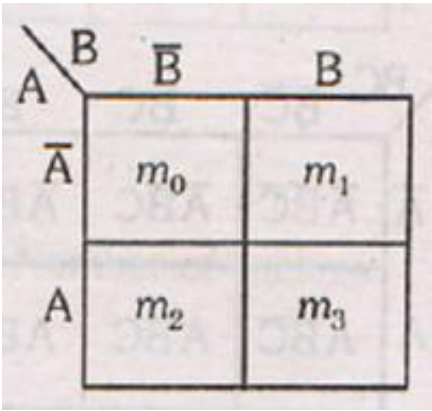
- For minterms (sum of product)table .

Decimal Number	Variables A	Variables B	Minterms	Minterm Designation
0	0	0	$\overline{A}\,\overline{B}$	m_0
1	0	1	$\overline{A}B$	m_1
2	1	0	$A\overline{B}$	m_2
3	1	1	AB	m_3







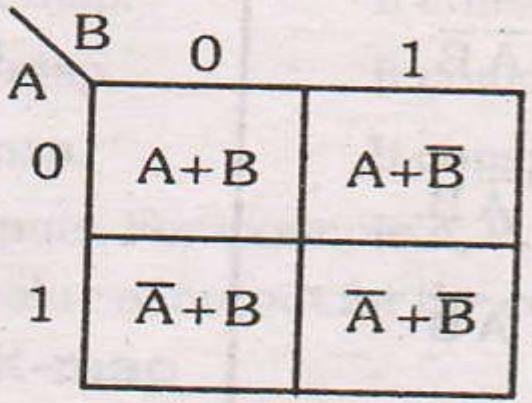


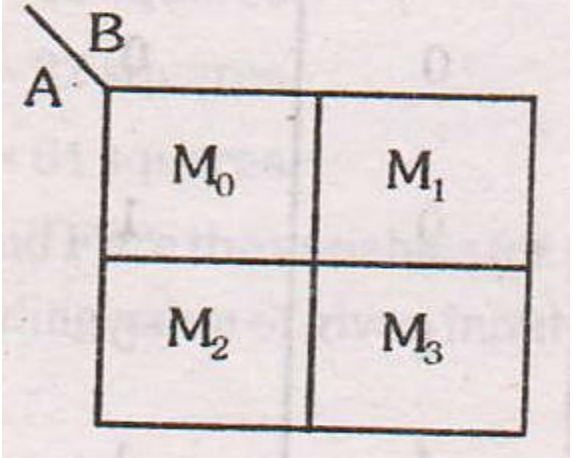
- For maxterms (product of sum)table .

Maxterms table is as shown (with corresponding minterms also)

Decimal Number	Variables A	Variables B	Minterms	Minterm Designation	Maxterms	Maxterm Designation
0	0	0	$\overline{A}\,\overline{B}$	m_0	$A + B$	M_0
1	0	1	$\overline{A}\,B$	m_1	$A + \overline{B}$	M_1
2	1	0	$A\,\overline{B}$	m_2	$\overline{A} + B$	M_2
3	1	1	AB	m_3	$\overline{A} + \overline{B}$	M_3

So the corresponding K-map will be





Three-variable K map.

Decimal Number	Variables A	B	C	Minterms	Minterm Designation	Maxterms	Maxterm Designation
0	0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0	$A+B+C$	M_0
1	0	0	1	$\bar{A}\bar{B}C$	m_1	$A+B+\bar{C}$	M_1
2	0	1	0	$\bar{A}B\bar{C}$	m_2	$A+\bar{B}+C$	M_2
3	0	1	1	$\bar{A}BC$	m_3	$A+\bar{B}+\bar{C}$	M_3
4	1	0	0	$A\bar{B}\bar{C}$	m_4	$\bar{A}+B+C$	M_4
5	1	0	1	$A\bar{B}C$	m_5	$\bar{A}+B+\bar{C}$	M_5
6	1	1	0	$AB\bar{C}$	m_6	$\bar{A}+\bar{B}+C$	M_6
7	1	1	1	ABC	m_7	$\bar{A}+\bar{B}+\bar{C}$	M_7

From above table the K-map is given by

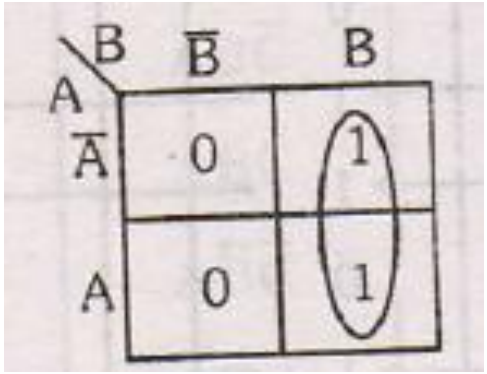
For minterms (sum of product)table.

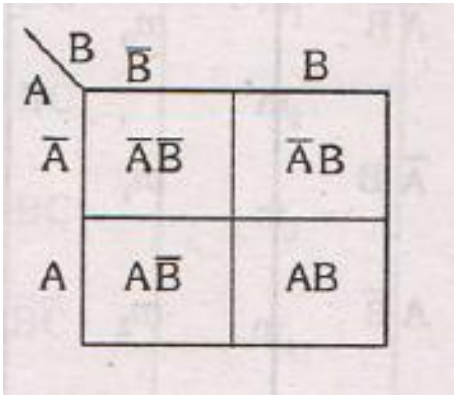
A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
A	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

or

A \ BC	00	01	11	10
0	000	001	011	010
1	100	101	111	110

- Grouping of adjacent one pair.



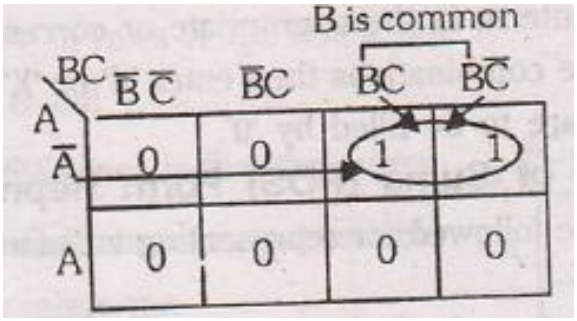


  - If we consider the minterm of K map. Then

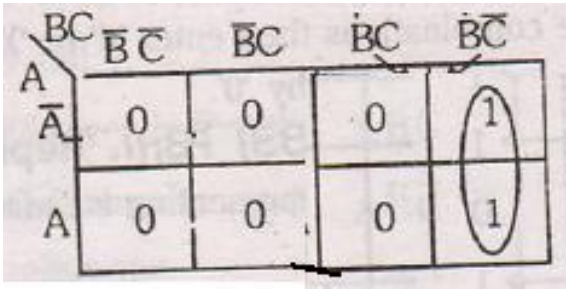
$$Y = \overline{A}B + AB$$
$$= B(\overline{A} + A)$$
$$= B.1 = B$$

  - Output of the K map is Y = B

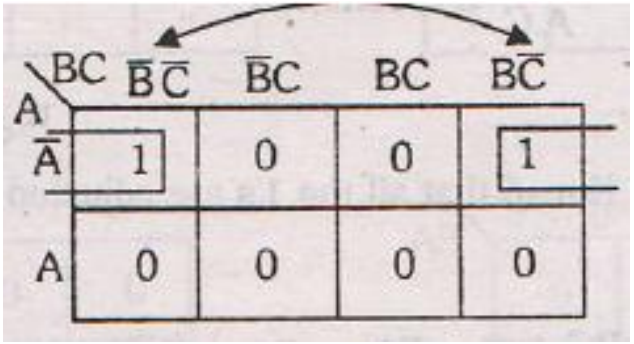
- Three variable K map



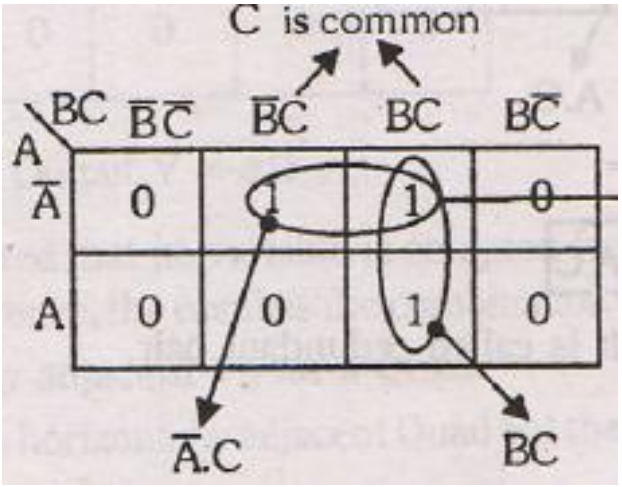
- Output of the K map is Y = $\bar{A}B$



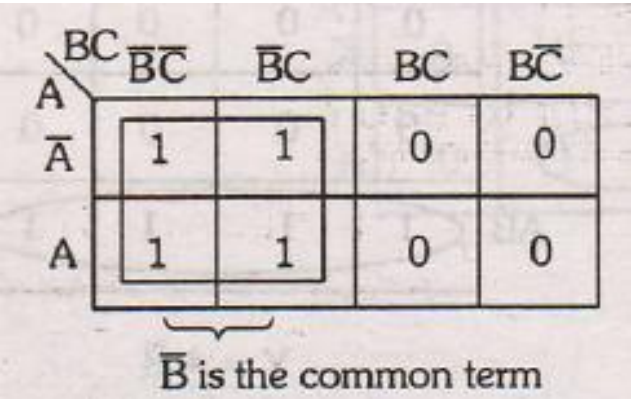
- Output of the K map is Y = $B\bar{C}$



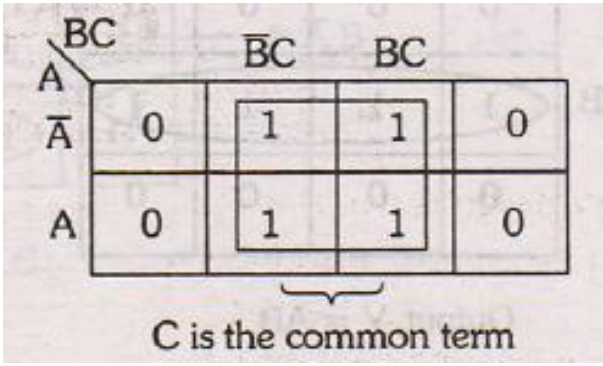
- Output of the K map is Y = $\bar{A}.\bar{C}$



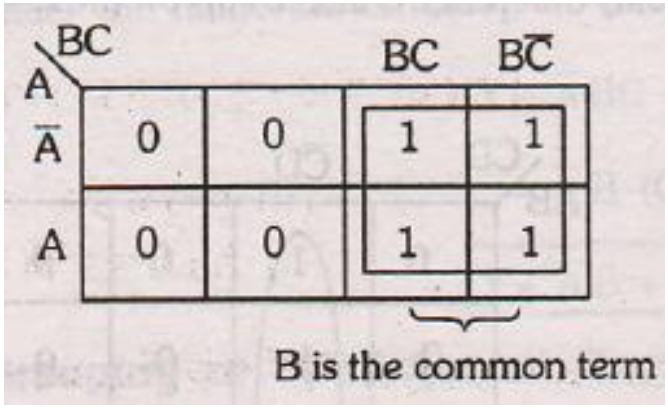
- Output of the K map is Y = $\bar{A}C + BC.$



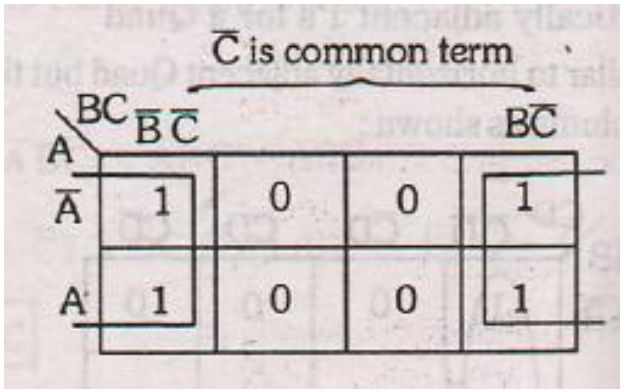
- Output of the K map is Y = $\overline{B}$



- Output of the K map is Y = C



- Output of the K map is Y = B



- Output of the K map is Y = $\overline{C}$

Four-variable K- map.

CD / AB	00	01	11	10
00				
01				
11				
10				

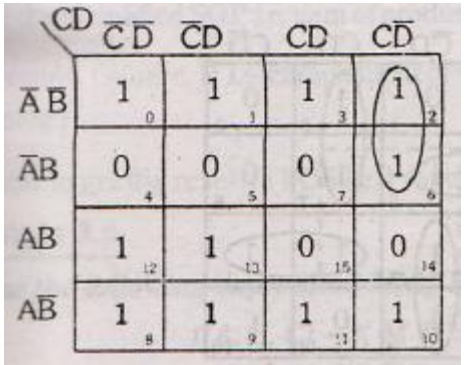
CD / AB	00	01	11	10
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010

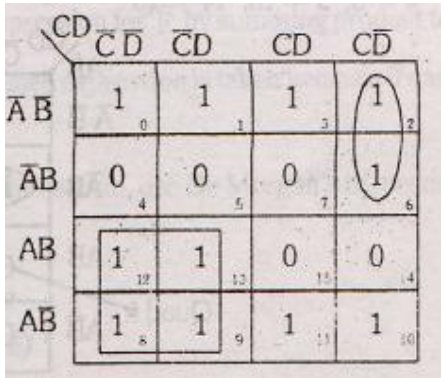
		A	B	C	D	F
0		0	0	0	0	
1		0	0	0	1	
2		0	0	1	0	
3		0	0	1	1	
4		0	1	0	0	
5		0	1	0	1	
6		0	1	1	0	
7		0	1	1	1	
8		1	0	0	0	
9		1	0	0	1	
10		1	0	1	0	
11		1	0	1	1	
12		1	1	0	0	
13		1	1	0	1	
14		1	1	1	0	
15		1	1	1	1	

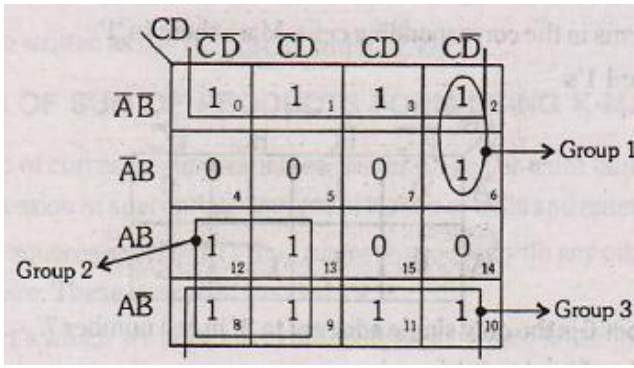
- Example:

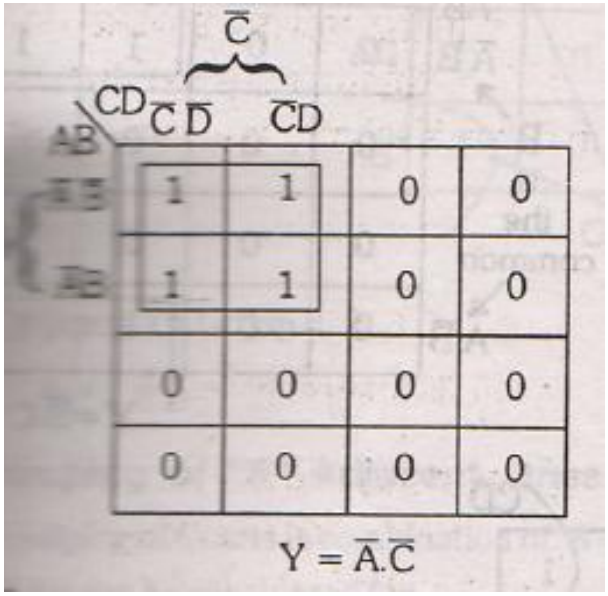
Minimize the flowwing function using K – map. Right truth table and Drow the logic ckt diagram,

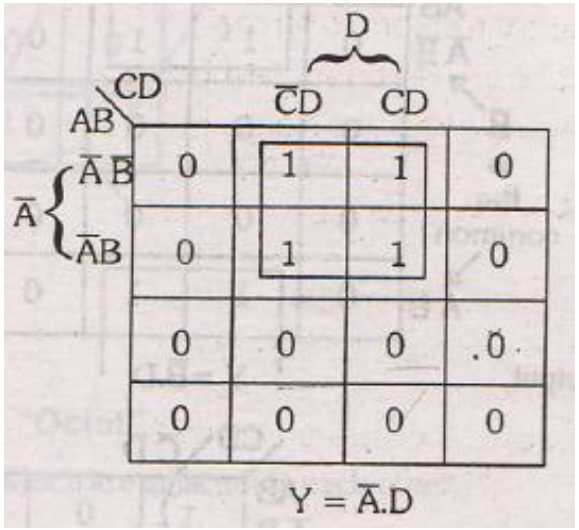
$$f(A,B,C,D) = \sum m(0,1,2,3,6,8,9,10,11,12,13)$$

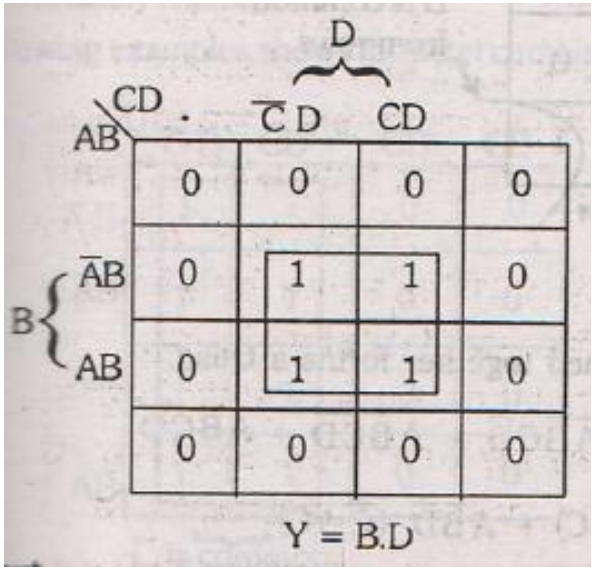


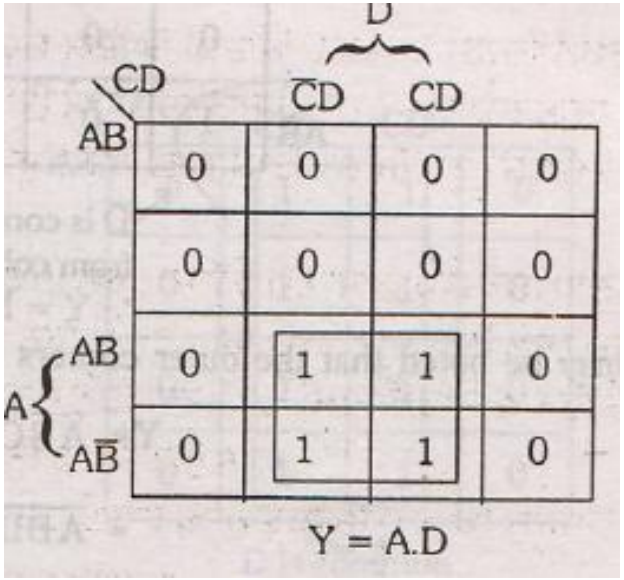


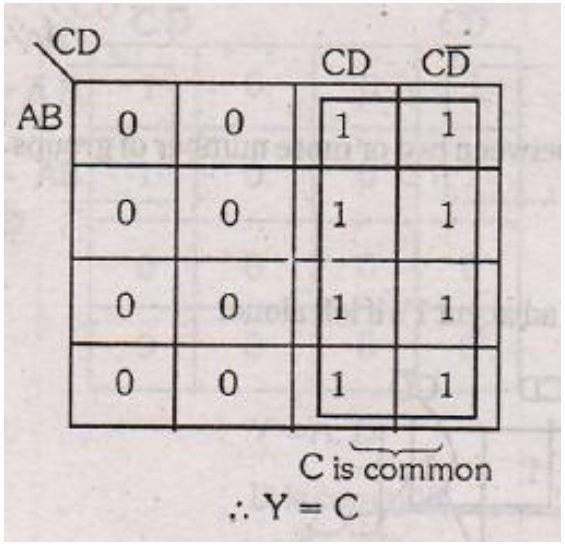


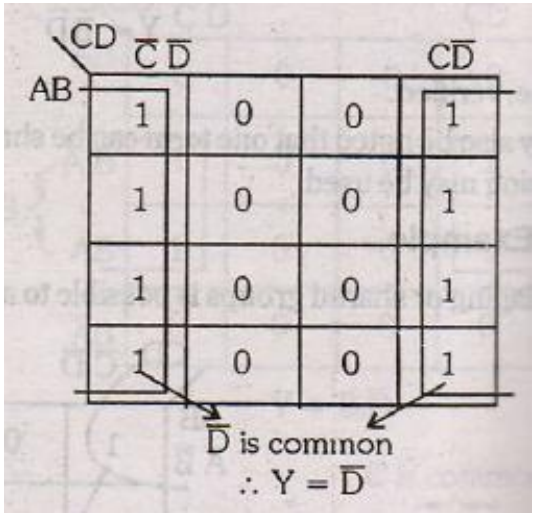

C̄
CD
C̄D̄
C̄D
AB
1 1 0 0
1 1 0 0
0 0 0 0
0 0 0 0
Y = Ā.C̄

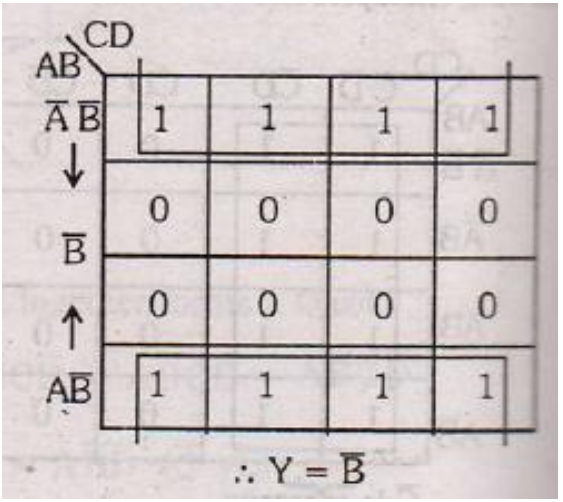

D
CD
C̄D
CD
AB
ĀB̄
ĀB
Ā
0 1 1 0
0 1 1 0
0 0 0 0
0 0 0 0
Y = Ā.D

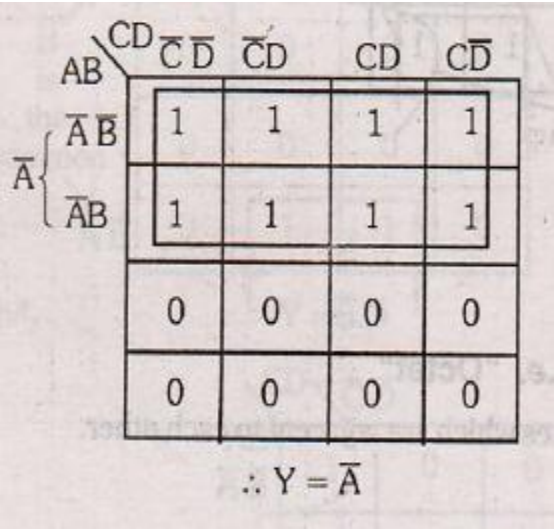

D
CD
C̄D
CD
AB
B̄A
BA
B
0 0 0 0
0 1 1 0
0 1 1 0
0 0 0 0
Y = B.D

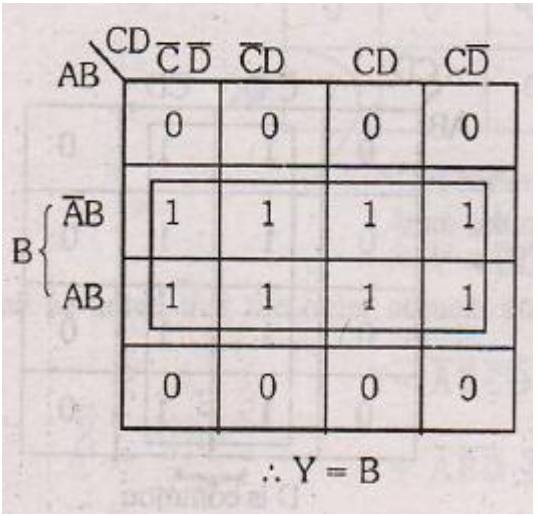

D
CD
C̄D
CD
AB
AB
AB̄
A
0 0 0 0
0 0 0 0
0 1 1 0
0 1 1 0
Y = A.D

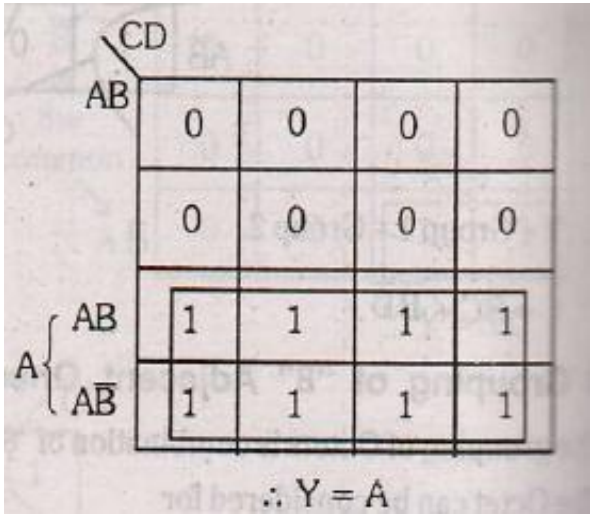

CD
CD
CD
AB
0 0 1 1
0 0 1 1
0 0 1 1
0 0 1 1
C is common
∴ Y = C


CD
C D
CD
AB
1 0 0 1
1 0 0 1
1 0 0 1
1 0 0 1
D is common
∴ Y = D


CD
AB
A B
1 1 1 1
0 0 0 0
B
0 0 0 0
AB
1 1 1 1
∴ Y = B


CD
C D
C D
CD
CD
AB
A B
1 1 1 1
A
AB
1 1 1 1
0 0 0 0
0 0 0 0
∴ Y = A


CD
C D
C D
CD
CD
AB
0 0 0 0
AB
B
AB
1 1 1 1
AB
1 1 1 1
0 0 0 0
∴ Y = B

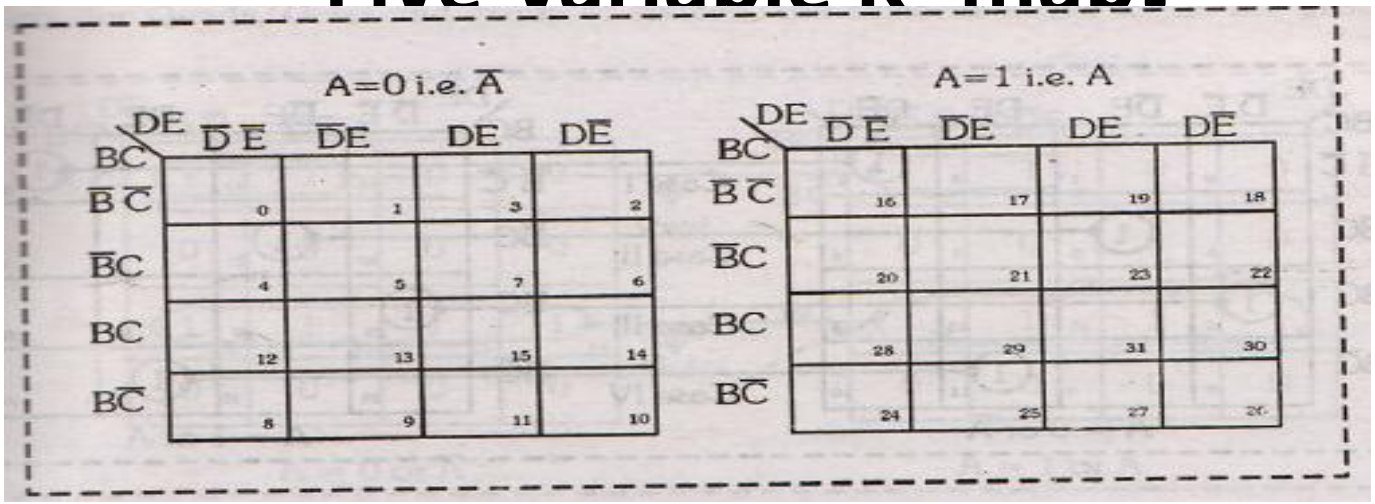

CD
AB
0 0 0 0
0 0 0 0
AB
A
AB
1 1 1 1
1 1 1 1
∴ Y = A

- Example:

  Minimize the flowing function using K – map. Right truth table and Drown the logic ckt diagram,

$$f(A,B,C,D) = \sum m(3,4,5,7,9,13,14,15)$$

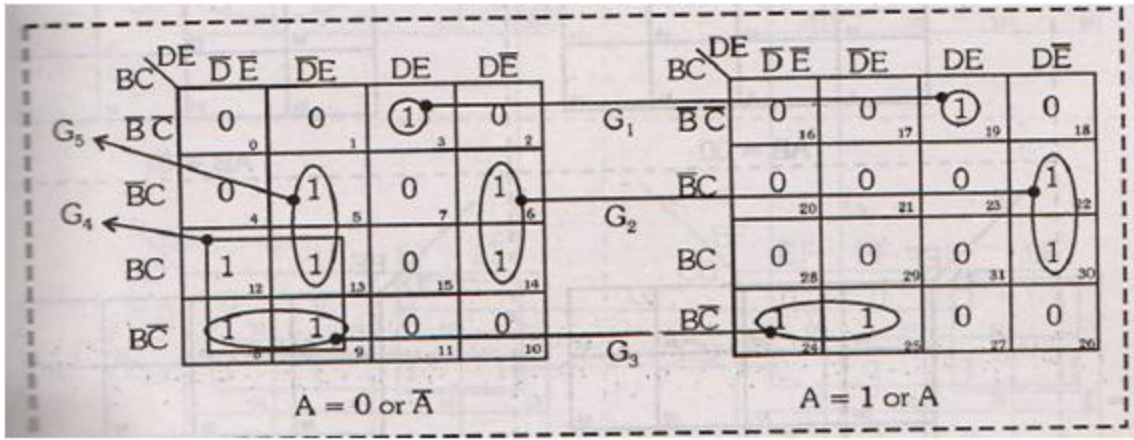
Five-variable K- map.



- Example:

Minimize the flowing function using K – map. Right truth table and Drown the logic ckt diagram,

$$f(A,B,C,D,E) = \sum m(3,5,6,8,9,12,13,14,19,22,23,24,25,30)$$



- Example:

Minimize the flowing function using K – map. Right truth table and Drown the logic ckt diagram,

$$f(A,B,C,D,E) = \sum m(0,5,6,8,9,10,11,16,20,24,25,26,27)$$