# SET THEORY

## CHAPTER I

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# OUTLINE

- Introduction
- Sets
- Universal Sets
- Empty Sets
- Set of Sets
- Set Operations
- Set cardinality
- Power Set
- Computer Representation of sets

# What is a set?

- A set is a group of "objects"
  - Students of CSE 4th semester: { Ram, Harish, Sita }
  - Subjects offered to CSE 4th semester: { BTCS 401, BTCS 402, BTCS 403}
  - Courses offered by GGI: { B. Tech, BBA, BCA, B Com, MBA }
  - Sets of even numbers: {2, 4, 6, 8……}
  - States of matter: { solid, liquid, gas, plasma }

Although a set can contain (almost) anything, we will most often use sets of numbers

# Definition

A set is an unordered collection of (unique) objects.

OR

A Set is a well-defined collection of distinct objects.

- Order does not matter
- Sets do not have duplicate elements

The objects in a set are called elements or members of a set.

Notation

Usually we denote sets with upper-case letters, elements with lower-case letters.

- $x \in A$ means that x is a member of the set A
- $x \notin A$ means that x is not a member of the set A.

# SET-BUILDER METHOD

$A = \{ x \mid x \in S, P(x) \}$ or $A = \{ x \in S \mid P(x) \}$

- A is the set of all elements x of S, such that x satisfies the property P
- Example:
  - If $X = \{2,4,6,8,10\}$, then in set-builder notation, X can be described as

$$X = \{n \in \mathbb{Z} \mid n \text{ is even and } 2 \leq n \leq 10\}$$

# Standard Symbols which denote sets of numbers

- N : The set of all natural numbers (i.e.,all positive integers)
- Z : The set of all integers
- Z+ : The set of all positive integers
- E : The set of all even integers
- Q : The set of all rational numbers
- Q+ : The set of all positive rational numbers
- R : The set of all real numbers
- R+ : The set of all positive real numbers
- C : The set of all complex numbers
- C* : The set of all nonzero complex numbers

# THE UNIVERSAL SET

- The universal set (***U***) – the set of all of elements from which given any set is drawn
  - For A= {-2, 0.4, 2}, ***U*** would be the real numbers
  - For A= {0, 1, 2}, ***U*** could be the Whole numbers, the integers, the rational numbers, or the real numbers, depending on the context
  - For the set of the students in this class, ***U*** would be all the students in the University (or perhaps all the people in the world)
  - For the set of the vowels of the alphabet, ***U*** would be all the letters of the alphabet

# The empty set

- Empty (or null) set has zero elements
  - Empty sets are represented by symbol $\varnothing$
  - Thus, $\varnothing = \{ \}$      **VERY IMPORTANT**

- Example????????

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## Sets of sets

- Sets can contain other sets
  - S = { {1}, {2}, {3} }
  - T = { {1}, {{2}}, {{{3}}} }
  - V = { {{1}, {{2}}}, {{{3}}}, { {1}, {{2}}, {{{3}}} } }
    - V has only 3 elements!
- Note that $1 \neq \{1\} \neq \{\{1\}\} \neq \{\{\{1\}\}\}$
  - They are all different
- Is $\varnothing = \{ \varnothing \}$?
  - The first is a set of zero elements
  - The second is a set of 1 element (that one element being the empty set)

# VENN DIAGRAMS

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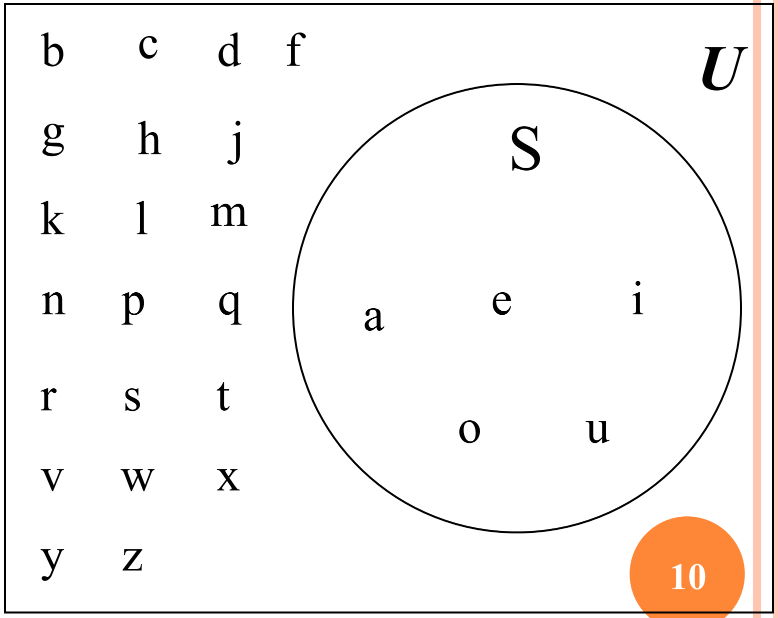
- A set can be represented graphically using a Venn Diagram
  - The box represents the universal set
  - Circles represent the set(s)
- Consider set S, which is the set of all vowels in the alphabet
- The individual elements are usually not written in a Venn diagram



## Set equality

- Two sets are said to be equal if they have the same elements
  - $\{1, 2, 3, 4, 5\} = \{5, 4, 3, 2, 1\}$
    - Remember that order does not matter!

## SUBSETS

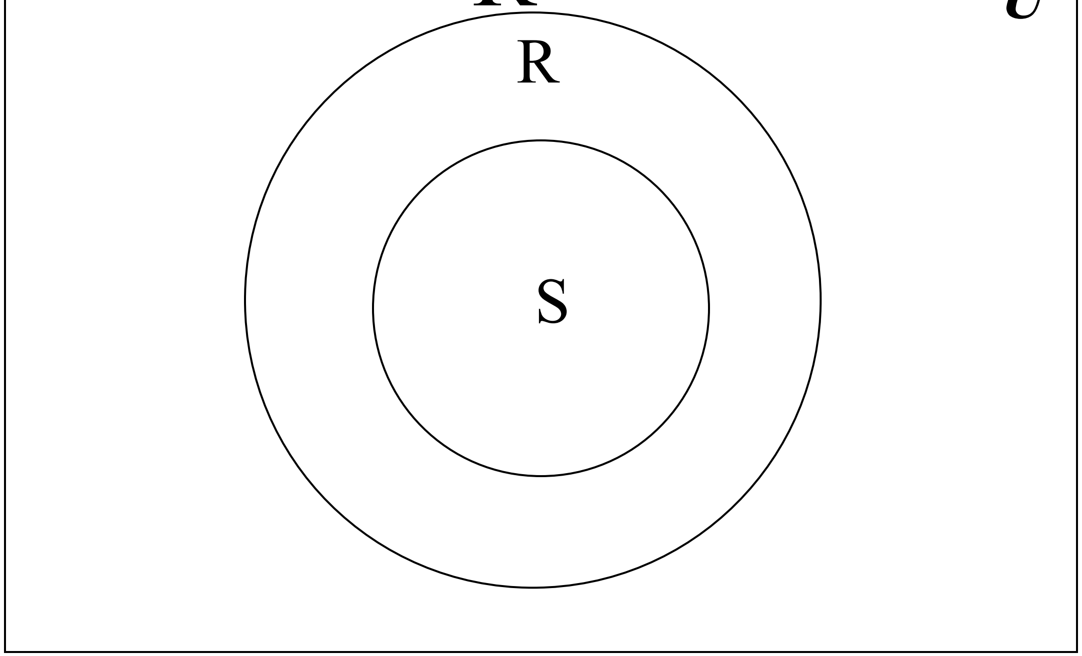
- Set S is said to be subset of T ($S \subseteq T$), If all the elements of a set S are also elements of a set T
  - For example, $S = \{2, 4, 6\}$, $T = \{1, 2, 3, 4, 5, 6, 7\}$, then $S \subseteq T$
- Note that
  - Every set is a subset of itself
  - The empty set is a subset of *all* sets
  - All sets are subsets of the universal set

# Proper Subsets

- If S is a subset of T, and S is not equal to T, then S is a proper subset of T
  - Let T = {0, 1, 2, 3, 4, 5}
  - If S = {1, 2, 3}, S is not equal to T, and S is a subset of T
  - A proper subset is written as $S \subset T$
  - Let R = {0, 1, 2, 3, 4, 5}. R is equal to T, and thus is a subset (but not a proper subset) or T
    - Can be written as: $R \subseteq T$ and $R \not\subset T$ (or just R = T)
  - Let Q = {4, 5, 6}. Q is neither a subset or T nor a proper subset of T

# Proper subsets: Venn diagram

$S \subset R$



# Sets Operations

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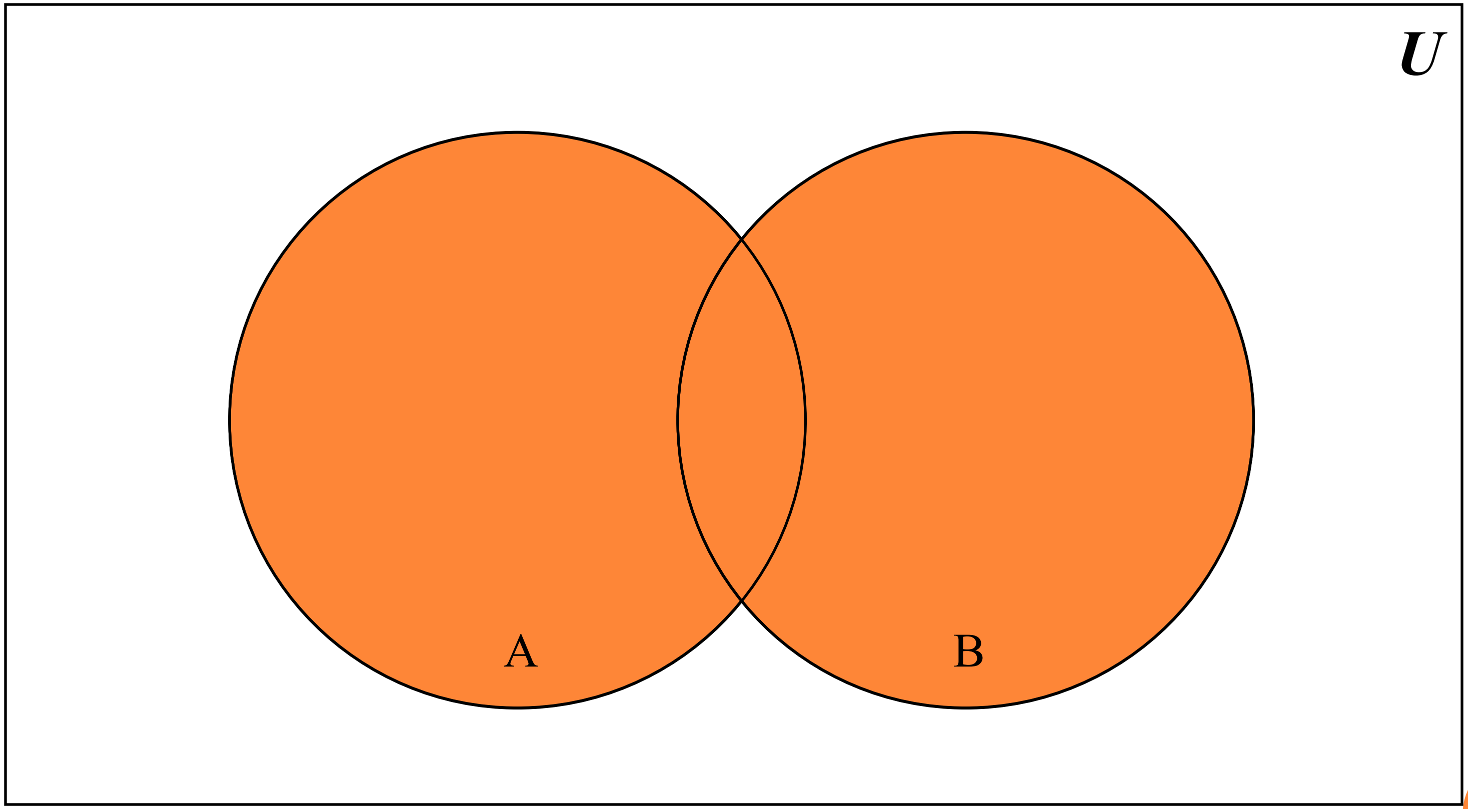
## SET OPERATIONS: UNION

- Formal definition for the union of two sets:
  A U B = { $x$ | $x \in$ A **or** $x \in$ B }
- Further examples
  - {1, 2, 3} U {3, 4, 5} = {1, 2, 3, 4, 5}
  - {New York, Washington} U {3, 4} = {New York, Washington, 3, 4}
  - {1, 2} U ∅ = {1, 2}

# Set operations: Union 2

A U B



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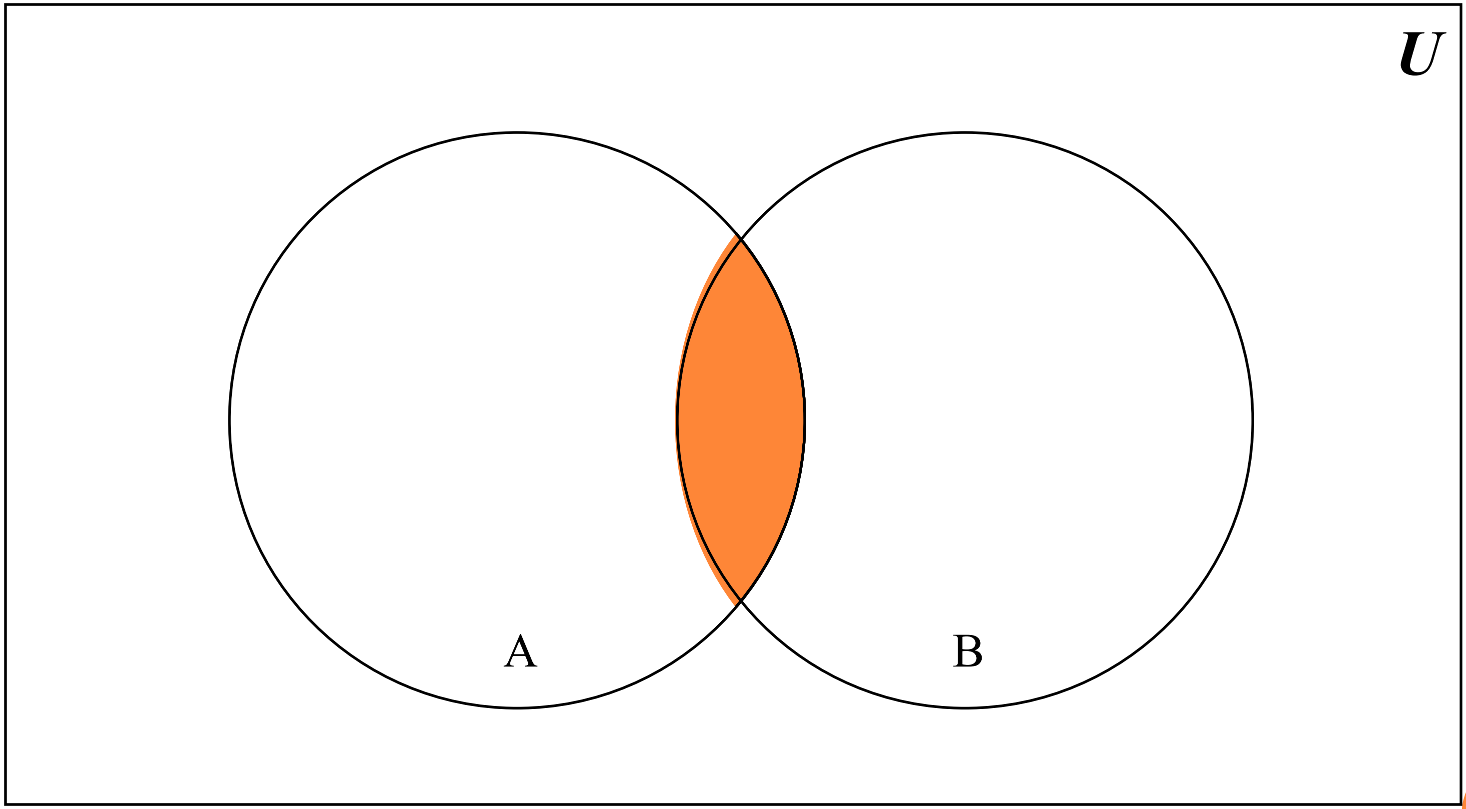
- Properties of the union operation
  - A U ∅ = A Identity law
  - A U ***U*** = ***U*** Domination law
  - A U A = A Idempotent law
  - A U B = B U A Commutative law
  - A U (B U C) = (A U B) U C Associative law

# Set operations: Intersection

- Formal definition for the intersection of two sets: $A \cap B = \{ x \mid x \in A \textbf{ and } x \in B \}$
- Further examples
  - $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$
  - $\{\text{New York, Washington}\} \cap \{3, 4\} = \varnothing$
    - No elements in common
  - $\{1, 2\} \cap \varnothing = \varnothing$
    - **Any set intersection with the empty set yields the empty set**

# Set operations: Intersection

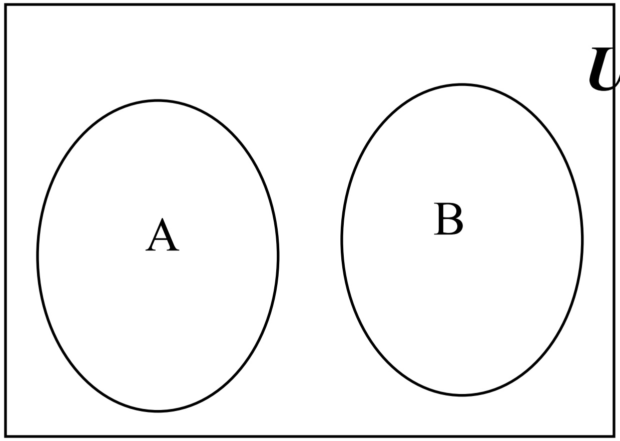
$A \cap B$



- Properties of the intersection operation
  - $A \cap \boldsymbol{U} = A$ Identity law
  - $A \cap \varnothing = \varnothing$ Domination law
  - $A \cap A = A$ Idempotent law
  - $A \cap B = B \cap A$ Commutative law
  - $A \cap (B \cap C) = (A \cap B) \cap C$ Associative law

# DISJOINT SETS

- Two sets are disjoint if their intersection is the empty set ($A \cap B = \varnothing$)
- Examples
  - {1, 2, 3} and {3, 4, 5} are not disjoint
  - {New York, Washington} and {3, 4} are disjoint
  - {1, 2} and $\varnothing$ are disjoint
    - Their intersection is the empty set
  - $\varnothing$ and $\varnothing$ are disjoint!
    - Their intersection is the empty set

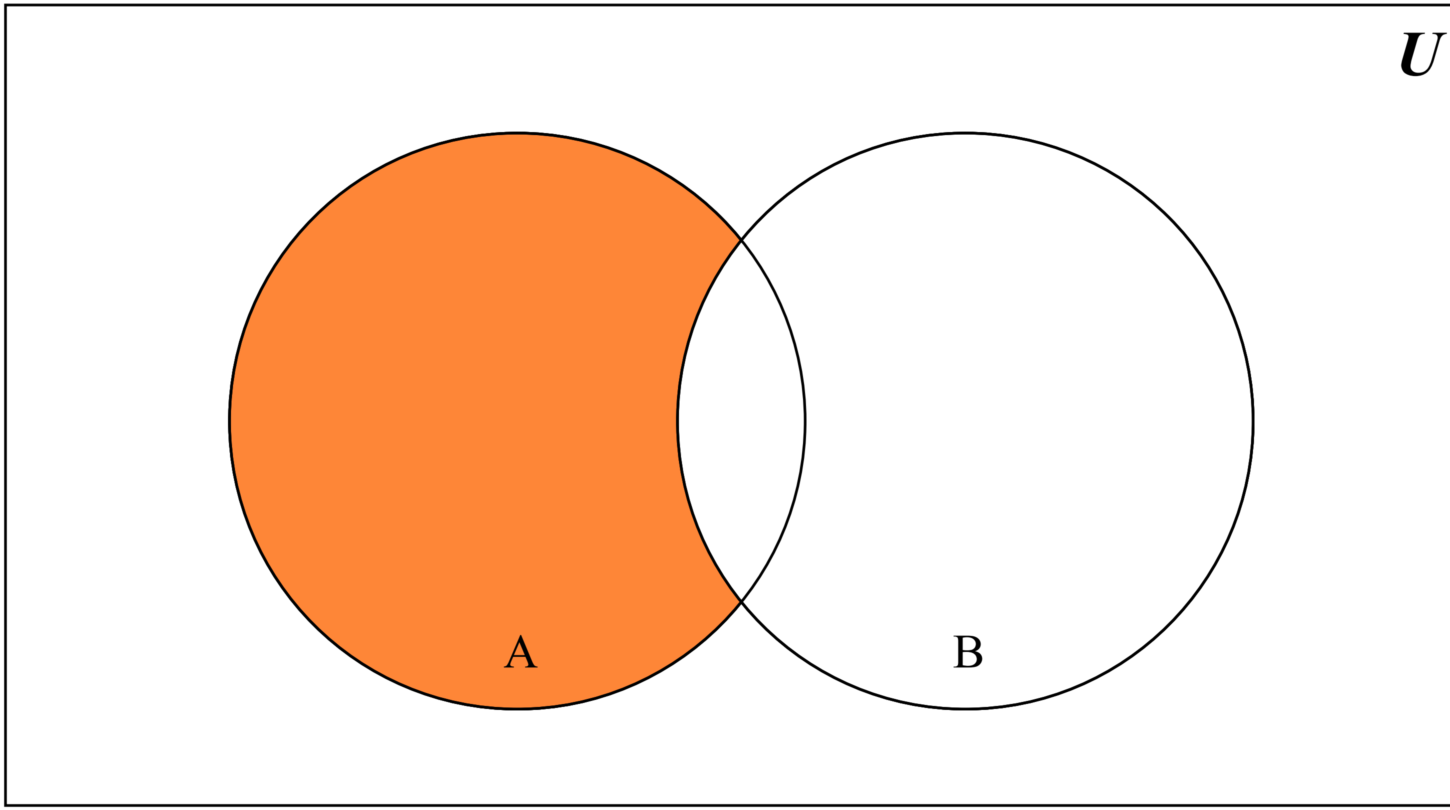


# Set operations: Difference

- Difference of two sets:
  $A - B = \{ x \mid x \in A \textbf{ and } x \notin B \}$
  Elements of Set A which are not elements of B
- Examples
  - $\{1, 2, 3\} - \{3, 4, 5\} = \{1, 2\}$
  - {New York, Washington} - $\{3, 4\}$ = {New York, Washington}
  - $\{1, 2\} - \varnothing = \{1, 2\}$
    - The difference of any set S with the empty set will be the set S

# SET OPERATIONS: DIFFERENCE

**A - B**



# Set operations: Symmetric Difference



- The symmetric difference of two sets:

  $A \oplus B = \{ x \mid (x \in A \text{ or } x \in B) \text{ and } x \notin A \cap B\}$

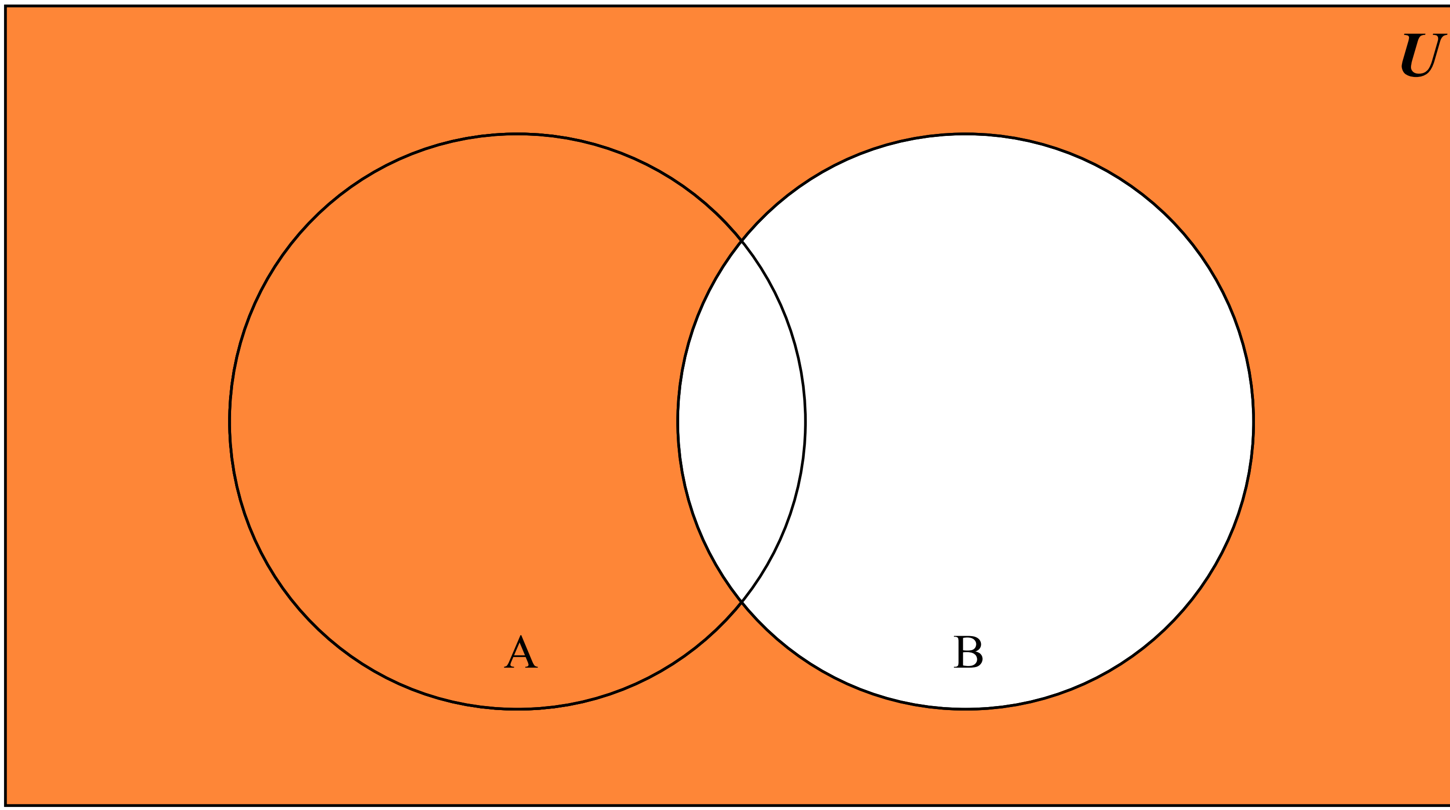
  $A \oplus B = (A \cup B) - (A \cap B)$ Important!

  $A \oplus B = (A - B) \cup (B - A)$

- Examples
  - $\{1, 2, 3\} \oplus \{3, 4, 5\} = \{1, 2, 4, 5\}$
  - {New York, Washington} $\oplus$ {3, 4} = {New York, Washington, 3, 4}
  - $\{1, 2\} \oplus \varnothing = \{1, 2\}$
    - The symmetric difference of any set S with the empty set will be the set S

# Complement sets 2

$\overline{B}$



# Complement sets

A complement of a set is all the elements that are NOT in the set

$$\overline{A} = \{ x \mid x \notin A \}$$

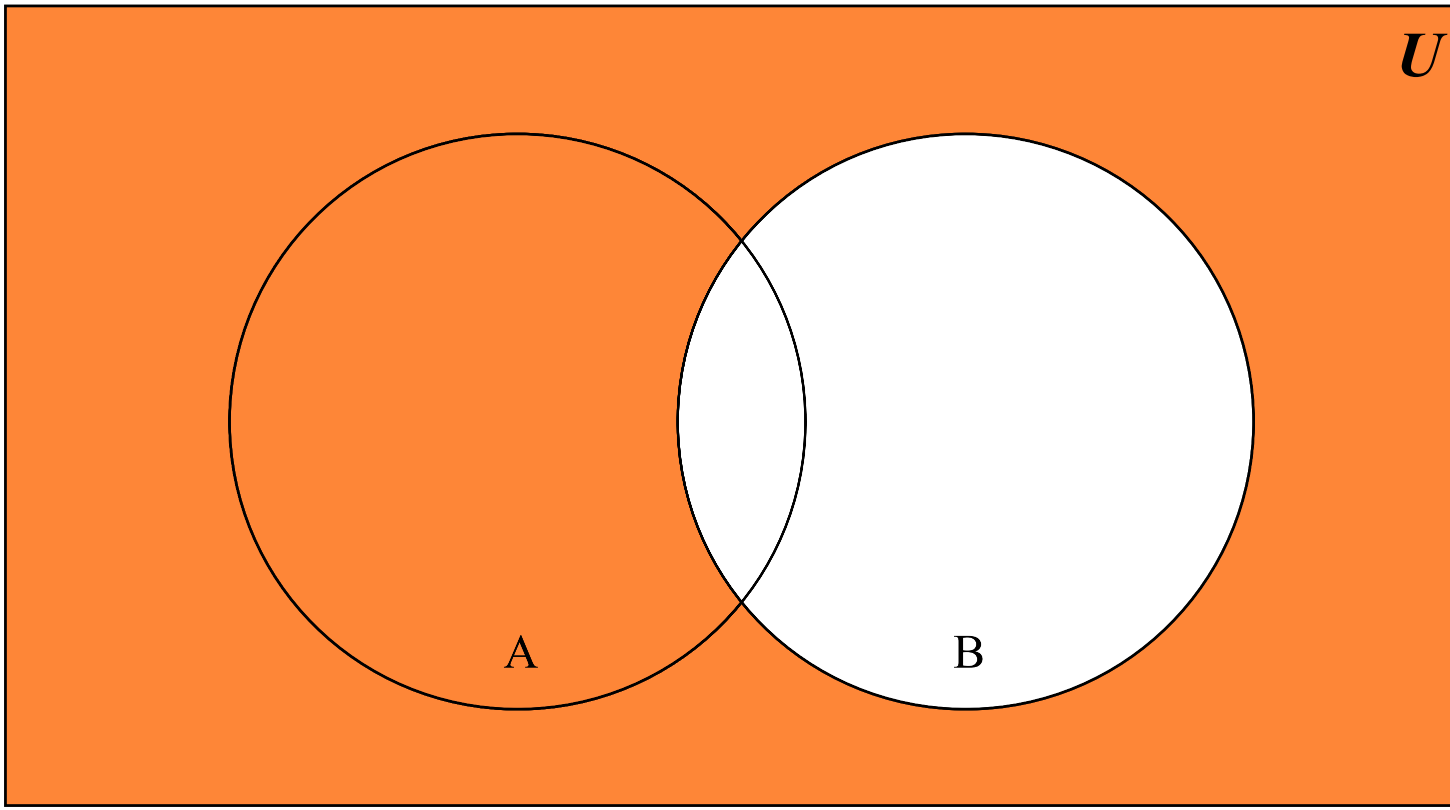
$$\overline{A} = U - A$$

Examples (assuming $\boldsymbol{U} = \mathbf{Z}$)

$\{1, 2, 3\} = \{ \ldots, -2, -1, 0, 4, 5, 6, \ldots \}$

# Complement sets 2

$\overline{B}$



# Complement sets 4

- Properties of complement sets

  - $\bar{\bar{A}} = A$ Complementation law
  - $A \cup \bar{A} = \boldsymbol{U}$ Complement law
  - $A \cap \bar{A} = \varnothing$ Complement law

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## SET CARDINALITY

- The cardinality of a set ($|A|$) is the number of elements in a set
- Examples
  - Let $R = \{1, 2, 3, 4, 5\}$. Then $|R| = 5$
  - $|\varnothing| = 0$
  - Let $S = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. Then $|S| = 4$

# Power sets

- The power set of S (P(S)) is the set of all the subsets of S
- Given the set S = {0, 1}. What are all the possible subsets of S?
  - They are: ∅ (as it is a subset of all sets), {0}, {1}, and {0, 1}
  - P(S) = { ∅, {0}, {1}, {0,1} }
    - Note that |S| = 2 and |P(S)| = 4
- If a set has n elements, then the power set will have 2n elements

# Computer representation of sets

- Assume that ***U*** is finite (and reasonable!)
  - Let ***U*** be the alphabet

- Each bit represents whether the element in ***U*** is in the set

- The vowels in the alphabet:
  abcdefghijklmnopqrstuvwxyz
  10001000100000100000100000

- The consonants in the alphabet:
  abcdefghijklmnopqrstuvwxyz
  01110111011111011111011111

- Consider the union of these two sets:

$$\begin{array}{r} 1000100010000001000000100000 \\ \vee\ \underline{0111011101111110111111011111} \\ 1111111111111111111111111111 \end{array}$$

- Consider the intersection of these two sets:

$$\begin{array}{r} 1000100010000001000000100000 \\ \wedge\ \underline{0111011101111110111111011111} \\ 0000000000000000000000000000 \end{array}$$

The cost of being wrong is less than the cost of doing nothing


THANKS !