Sharpening Minds, Brightening Tomorrow.
Gulzar Group of Institutes

Basic counting techniques
pigeon-hole principle
chapter III

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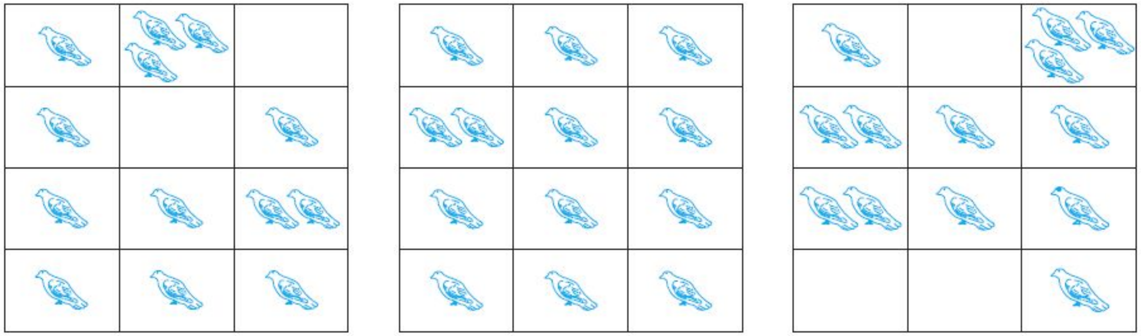
PIGEON-HOLE PRINCIPLE

- If there are more pigeons than pigeonholes, then there must be at least one pigeonhole with at least two pigeons in it
- If k is a positive integer and $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.



(a) (b) (c)

FIGURE 1 There Are More Pigeons Than Pigeonholes.

A function f from a set with $k + 1$ or more elements to a set with k elements is not one-to-one.

Proof: Suppose that for each element y in the co-domain of f we have a box that contains all elements x of the domain of f such that $f(x) = y$.

Because the domain contains $k + 1$ or more elements and the codomain contains only k elements, the pigeonhole principle tells us that one of these boxes contains two or more elements x of the domain. This means that f cannot be one-to-one.

PROBLEMS

EXAMPLE 1 Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

EXAMPLE 2 In any group of 27 English words, there must be at least two that begin with the same letter, because there are 26 letters in the English alphabet.

EXAMPLE 3 How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

Solution: There are 101 possible scores on the final. The pigeonhole principle shows that among any 102 students there must be at least 2 students with the same score.

THE GENERALIZED PIGEONHOLE PRINCIPLE

- If *N* objects are placed into *k* boxes, then there is at least one box containing at least ⌈N/k⌉ (quotient when N is divide by k) objects.
- Among 100 people there are at least ⌈ 100/12 ⌉ = 9 who were born in the same month.

- What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

Solution: The minimum number of students needed to ensure that at least six students receive the same grade is the smallest integer N such that $\lceil N/5 \rceil = 6$. The smallest such integer is $N = 5 \cdot 5 + 1 = 26$. If you have only 25 students, it is possible for there to be five who have received each grade so that no six students have received the same grade. Thus, 26 is the minimum number of students needed to ensure that at least six students will receive the same grade.