MATHEMATICS

SECTION A

January 27, 2024

1. If **A** is a square matrix satisfying $A'A = I$, write the value of $\left|A\right|$.
2. If $y = x\left|x\right|$, find $\dfrac{dy}{dx}$ for $x < 0$.
3. Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{d^2x} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

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4. Fid the direction cosines of a line which makes equal angles with the coordinate axes.
5. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.
6. Examine whether the operation $*$ defined on **R**, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.
7. Find:

$$\int \sqrt{3 - 2x - x^2}dx$$

8. Find:

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}dx$$

9. Find:

$$\int \frac{x - 3}{(x - 1)^3}e^x dx$$

10. If $\left|\overrightarrow{a}\right| = 2, \left|\overrightarrow{b}\right| = 7$ and $\overrightarrow{a} \times \overrightarrow{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.
11. Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$,$-5\hat{i} + 7hat\,j - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.
12. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.
13. Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$.
14. If $(\cos x)^y = (\sin y)^x$, find $\dfrac{dy}{dx}$.
15. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $\left[-1, 4\right]$.

16. Solve the differential equation :

$$\frac{dy}{dx} = \sqrt{\left(\frac{x + y\cos x}{1 + \sin x}\right)}$$

17. The scalar product of the vector $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{c} = \hat{\lambda} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\overrightarrow{b} + \overrightarrow{c}$.

18. If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find weather the lines are intersecting or not.

19. If $\mathbf{A} = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 1 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence solve the system of equations

$$\begin{aligned} x + 3y + 4z &= 8 \\ 2x + y + 2z &= 5 \\ \text{and } \ 5x + y + z &= 7 \end{aligned}$$

20. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

21. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

22. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹40 and that of type B ₹50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.