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Class Assignments - 7

(1) A factory has a machine that fills 80 ml of baby medicines in a bottle. An employee believes an average amount of baby medicine that is filled is not 80 ml. Using 40 samples he measures the average amount of medicine dispensed is around 78 ml with a standard deviation of 2.5

a) State the NULL and Alternate hypothesis
b) At 95% CI, is there enough evidence to support whether the Machine is working properly or not?

Ans:- Population average (μ) = 80

Sample average ($\bar{x}$) = 78

Sample SD (S) = 2.5

CI = 95% = 0.95

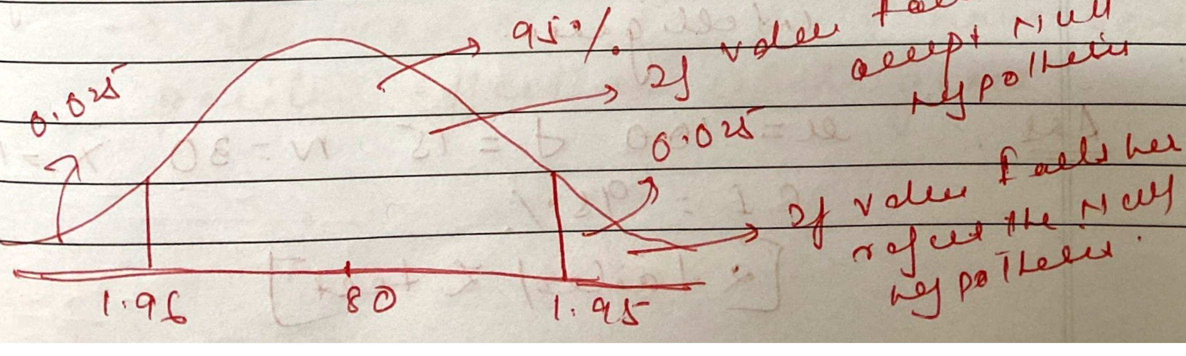
[Its a Two Tailed Z test (since n ≥ 30)]

(1) Null hypothesis $H_0: \mu = 80$

$H_1: \mu \neq 80$

(2) $\alpha = 0.05$

(3) Decision boundary



(4) Calculate test statistics

$z \cdot 0.025$
$= 1 - 0.025$
$= 0.975$
$= 1.96$

$$z = \frac{\bar{x} - \mu}{(s)/\sqrt{n}}$$

$$= \frac{78 - 80}{2.5 \times \sqrt{40}} = \frac{(-2)(\sqrt{40})}{2.5} = -5.05$$

(5) Conclusion :-

→ Since $-5.05 < -1.96$, we reject the Null hypothesis

→ This means, there is some fault in the machine and its not working properly

→ we accept the Alternate hypothesis

[2] In a population the average IQ is 100 with SD = 15. A team of scientists wants to test a new medicine to see if it has -ve or +ve effect or no effect at all. Sample of 30 participants who had taken the medicine have a mean of 140.

a) State the Null and alternate hypothesis
b) At 95% C.I did medicine effect intelligence.

Ans. $\mu = 100$ $\sigma = 15$ $n = 30$ $\bar{x} = 140$

CI = 95%

[2 tailed z test]

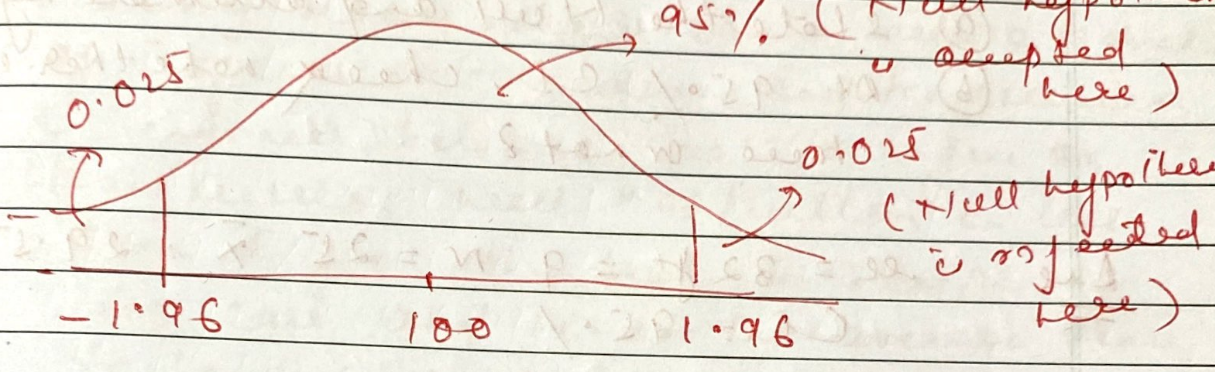
1. Null hypothesis

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$

2. $\alpha = 0.05$

3. Decision boundary



4. Z test statistics —

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{140 - 100}{15/\sqrt{30}} = \frac{\cancel{40}^{8} \times \sqrt{30}}{\cancel{15}_{3}}$$

$$= \frac{8 \times 5.47}{3}$$

$$= 14.60$$

5. Conclusion —

Since $14.6 > 1.96$

→ Reject the Null Hypothesis

→ Accept the Alternate hypothesis

→ The medicine definitely have some positive effect on intelligence.

3 A complaint was registered that the boys in a Govt school were underfed. Average weight of the boys of age 10 is 32 kg with SD = 9 kg. From the Govt school the average weight was found to be 29.5 kg for a sample of 25 selected boys.

(a) State the Null and alternate hypotheses

(b) At 95% CI, check whether it is true or not?

Ans $\mu = 32$, $\sigma = 9$, $N = 25$, $\bar{x} = 29.5$

CI p = 95%

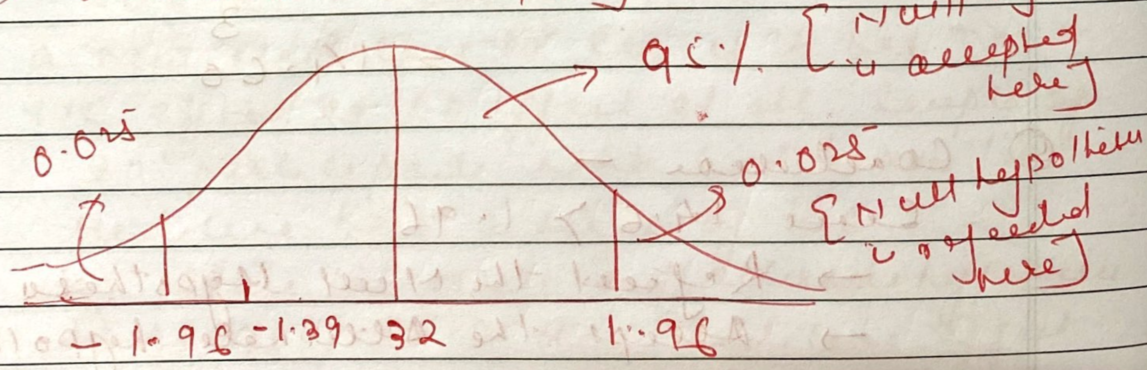
[2 tailed z test]

(1) $H_0 = \mu = 32$ kg

$H_1 = \mu \neq 32$ kg

(2) $\alpha = 0.05$

(3) Decision boundary



(4) Z test statistics

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{29.5 - 32}{9/\sqrt{25}} = \frac{-2.5 \times 5}{9}$$

$$= -\frac{12.5}{9} = -1.39$$

(5) Conclusion –

~~Score~~ $-1.39 > -1.96$

So we accept the null hypothesis

→ we reject the alternate hypothesis

→ The boys are not underfed.

[4] A factory manufacturer comes with a warranty of 5 years on the engine and transmission. An engineer believes that the engine or transmission will malfunction in less than 5 yrs. He tests a sample of 40 cars and finds the average time to be 4.8 years with an SD of 0.50.

(a) State the null and alternate hypothesis

(b) At 2% significance level is there enough evidence to support the idea that the warranty should be reversed?

Ans $\mu = 5 \quad \bar{x} = 4.8 \quad s = 0.5 \quad n = 40$

$\alpha = 0.02 \quad CL = 98\%$

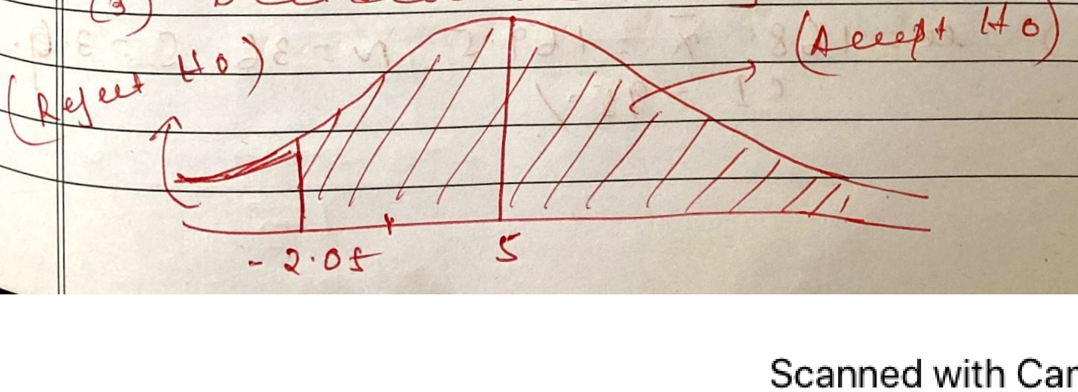
(1) $H_0 : \mu \geq 5$

$H_1 : \mu < 5$ [1 tailed test]

(2) $\alpha = 0.02$

$Z_{0.02} = -2.05$

(3) Decision boundary



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(4) Test statistics

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{4.8 - 5}{0.5/\sqrt{40}}$$

$$= \frac{-.2}{0.5} \times 6.32$$

$$= \frac{-1.264}{0.5} = \frac{-1.264}{0.5}$$

$$= \frac{-12.64}{5} = -2.52$$

(5) Conclusion –

Since $-2.52 < -2.05$

→ Reject the Null hypothesis

→ Accept the Null hypothesis

⇒ The warranty needs to be revered

$\boxed{5}$ The average weight of all residents in XYZ town is 168 pounds. A nutritionist believes that the true mean is different. She measured the weight of 36 individuals and found the mean to be 169.5 pounds with SD = 3.9

(a) State the Null and alternate hypotheses

(b) At 95% CI state is there enough evidence to discard Null hypothesis?

(1) $\mu = 168$ $\bar{x} = 169.5$ $n = 36$ $s = 3.9$

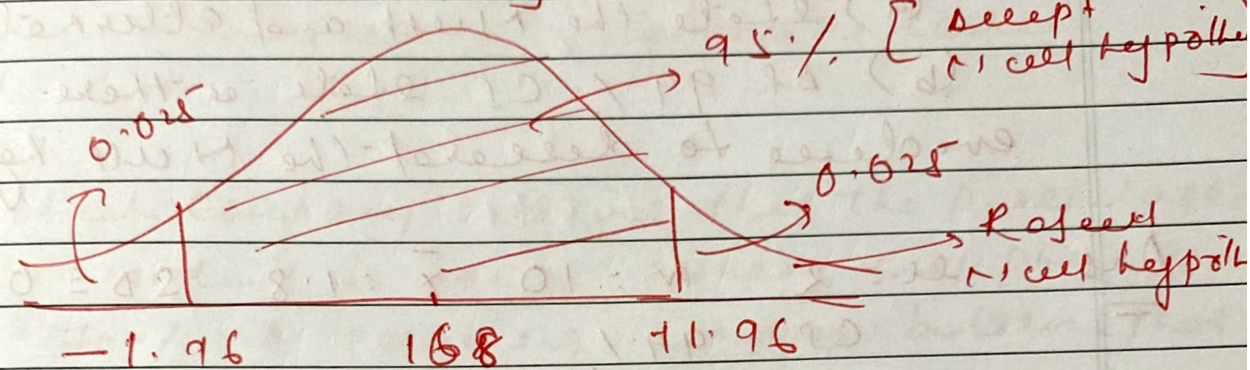
CI = 95%

$H_0 = \mu = 168$ pounds

~~$H_1 = \mu \neq 168$ pounds~~

(2) $\alpha = 0.05$ [2 tailed Z test]

(3) Decision boundary



(4) Test statistics

$$Z \text{ score} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{169.5 - 168}{3.9/\sqrt{36}}$$

$$= \frac{1.5}{3.9} \times 6 = \frac{9}{3.9} = 2.31$$

(5) Conclusion →

Since $2.31 > 1.96$

→ Reject the Null hypothesis

→ Accept the alternate hypothesis

→ The average weight of the the residents $\neq$ 169.5 pounds.

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6. A company manufacturers bike batteries with an average life span of 2 years or more. An engineer believes these values to be less. Using 10 samples he measures the average life span to be 1.8 years with a SD = 0.15

(a) State the null and alternate hypotheses

(b) At 99% CI state is there enough evidence to discard the null hypothesis.

Ans $\mu = 2$ $n = 10$ $\bar{x} = 1.8$ $SD = 0.15$

CI = 99%

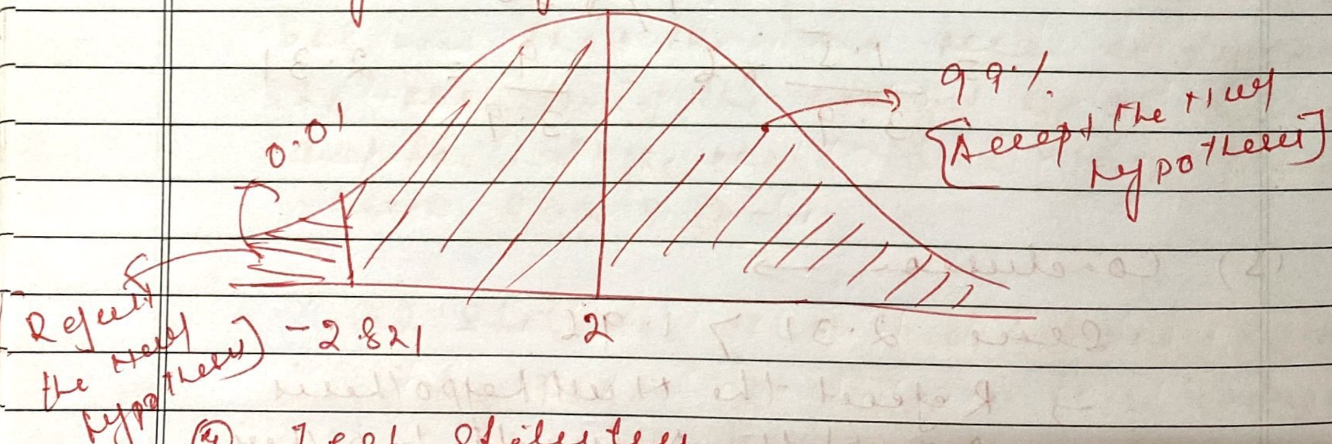
[1 tail T test]

(1) H_0 : ~~$\mu = 2$ years~~ $\mu \geq 2$ years

~~$\mu \neq 2$ years~~ $\mu < 2$ years

(2) $\alpha = 0.01$

(3) Degrees of freedom $= n - 1 = 9$



(4) Test statistics

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1.8 - 2}{0.15/\sqrt{10}} = \frac{-0.2 \times 100}{15/3.16}$$

$$= \frac{-0.2 \times 100 \times 3.16}{15}$$

$$= -4.21$$

(5) Conclusion —

Since $-4.21 < -2.821$

→ Reject the Null hypothesis

→ Accept the Null hypothesis

→ The average lifespan is less than 2 years. Engineer is correct.

[7] A tech company believes that the percentage of residents in town owning cell phone is 70%. A Marketing manager believes that value to be different. He conducts a survey of 200 individuals and found that 130 responded yes to owning a cell phone.

(a) State the Null and alternate hypothesis

(b) At 95% CI state is there enough evidence to discard the null hypothesis

Ans $\mu = 70$ $N = 200$ $x = 130$ $CI = 95\%$

[Z test with proportion]

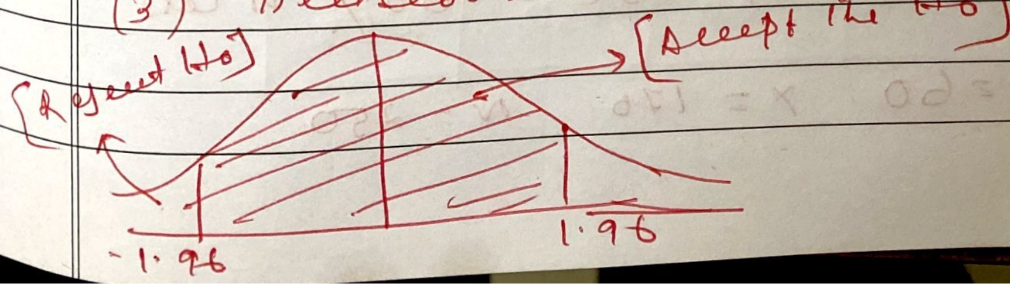
(1) Null hypothesis $H_0 = P_0 = 70\%$

$H_1 = P_0 \neq 70\%$

(2) $\alpha = 0.05$ $\hat{P} = \frac{x}{N} = \frac{130}{200} = 0.65$

$q_0 = 1 - P_0 = 1 - 0.7 = 0.3$

(3) Decision boundary



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(4) Test statistic

z test with proportion

$$= Z_{test} = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{N}}}$$

$$= \frac{0.65 - 0.70}{\sqrt{\frac{0.7 \times 0.3}{200}}}$$

$$= -1.571$$

(5) Conclusion –

Since $-1.571 > -1.96$

→ Accept the Null hypothesis

→ Reject the alternate hypothesis

→ The tech company is right.

[8] A car company believes that the percentage of residents in city ABC that owns a vehicle is 60%. A sales manager disagrees with this and conducts a hypothesis testing for 250 residents and found that only 170 responded Yes to owning a vehicle.

(a) State the Null and alternate hypothesis

(b) At 10% significance level state is there enough evidence to support the idea that vehicle ownership in city ABC is 60% or less

Ans $\mu = 60$ $x = 170$ $N = 250$

[Z test with proportion]

(1) Null hypothesis = $H_0 = 60\%$

$H_1 < 60\%$

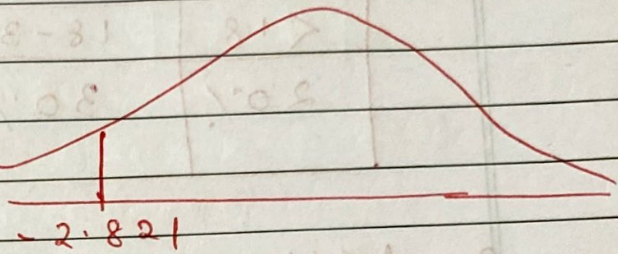
(2) $\alpha = 0.01$

$P_0 = 0.6$

$\hat{P} = \frac{x}{n} = \frac{170}{250} = 0.68$

$q_0 = 1 - P_0$

$= 0.4$



(3) ~~Test statistics~~

$$Z_{test} = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}}$$

$$= \frac{0.68 - 0.6}{\sqrt{\frac{0.6 \times 0.4}{250}}}$$

$$= \frac{0.08}{\sqrt{0.24}} \times 15.811$$

$$= \frac{1.264}{0.48} = 2.63$$

(4) Conclusion

Since $2.63 > -2.821$

→ ~~Null~~ Alternate hypothesis is rejec

→ Vehicle ownership in city ABC is not less than 60%