HW ML1 Week1 NguyenNgocBangAnh 11200232

Nguyen Ngoc Bang Anh

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Problem 1:
a)

Marginal distributions of p(X) are:

$p(X = x1) = p(x1, y1) + p(x1, y2) + p(x1, y3) = 0.01 + 0.05 + 0.1 = 0.16$

$p(X = x2) = 0.02 + 0.1 + 0.05 = 0.17$

$p(X = x3) = 0.03 + 0.05 + 0.03 = 0.11$

$p(X = x4) = 0.1 + 0.07 + 0.05 = 0.22$

$p(X = x5) = 0.1 + 0.2 + 0.04 = 0.34$

Marginal distributions of p(y) are:

$p(Y = y1) = p(x1, y1) + p(x2, y1) + p(x3, y1) + p(x4, y1) + p(x5, y1) = 0.01 + 0.02 + 0.03 + 0.1 + 0.1 = 0.26$

$p(Y = y2) = 0.05 + 0.1 + 0.05 + 0.07 + 0.2 = 0.47$

$p(Y = y3) = 0.1 + 0.05 + 0.03 + 0.05 + 0.04 = 0.27$

b)

Formula for the conditional distribution:

$P(X|Y = y_i) = \frac{P(x,y_i)}{P(Y=y_i)}$

Conditional distributions of P(X—Y = y1) are:

$P(x_1|y_1) = \frac{P(x_1,y_1)}{P(Y=y_1)} = \frac{0.01}{0.26} = \frac{1}{26}$

$P(x_2|y_1) = \frac{0.02}{0.26} = \frac{1}{13}$

$P(x_3|y_1) = \frac{0.03}{0.26} = \frac{3}{26}$

$P(x_4|y_1) = \frac{0.1}{0.26} = \frac{5}{13}$

$P(x_5|y_1) = \frac{0.1}{0.26} = \frac{5}{13}$

Conditional distributions of P(X—Y = y3) are:

$P(x_1|y_3) = \frac{P(x_1,y_3)}{P(Y=y_3)} = \frac{0.01}{0.27} = \frac{1}{27}$

$P(x_2|y_3) = \frac{0.02}{0.27} = \frac{2}{27}$

$P(x_3|y_3) = \frac{0.03}{0.27} = \frac{1}{9}$

$P(x_4|y_3) = \frac{0.1}{0.27} = \frac{10}{27}$

$P(x_5|y_3) = \frac{0.1}{0.27} = \frac{10}{27}$
Problem 3:

P(X) = 0.207

P(Y) = 0.5

P(X—Y) = 0.365

a)

The probability that the respondent uses both X and Y is:

$P(XY) = P(X|Y) \cdot P(Y) = 0.365 \cdot 0.5 = 0.1825$

b)

Bayes's theory:

$P(X|Y) = \frac{P(Y|X) \cdot P(X)}{P(Y)}$

Probability that the respondent uses Y, knowing that he or she does not use X:

$P(Y|\bar{X}) = \frac{P(\bar{X}|Y) \cdot P(Y)}{P(\bar{X})}$

$= \frac{(1-P(X|Y)) \cdot P(Y)}{1-P(X)}$

$= \frac{(1-0.365)\cdot 0.5}{1-0.207}$

$= 0.4004$
Problem 4:

$Var(X) = E_X[(X - E_X[X])^2]$

$= E_X[X^2 - 2XE_X[X] + (E_X[X])^2]$

$= E_X[X^2] - 2E_X[XE_X[X]] + E_X[(E_X[X])^2](1)$

$While : E_X(X) = constant\ number\ and\ E_X[(E_X[X])^2] = E_X[X])^2$

$=> (1) = E_X[X^2] - 2E_X[X]E_X[X] + (E_X[X])^2$

$= E_X[X^2] - 2(E_X[X])^2 + (E_X[X])^2$

$= E_X[X^2] - (E_X[X])^2$