Minimum Feedback Vertex Set

Feedback Vertex Set

- A feedback vertex set of a graph is a set of vertices whose removal leaves a graph without cycles.

- Each feedback vertex set contains at least one vertex of any cycle in the graph.

- The feedback vertex set problem is an NP-complete problem in computational complexity theory

- Enumerate each simple cycle.

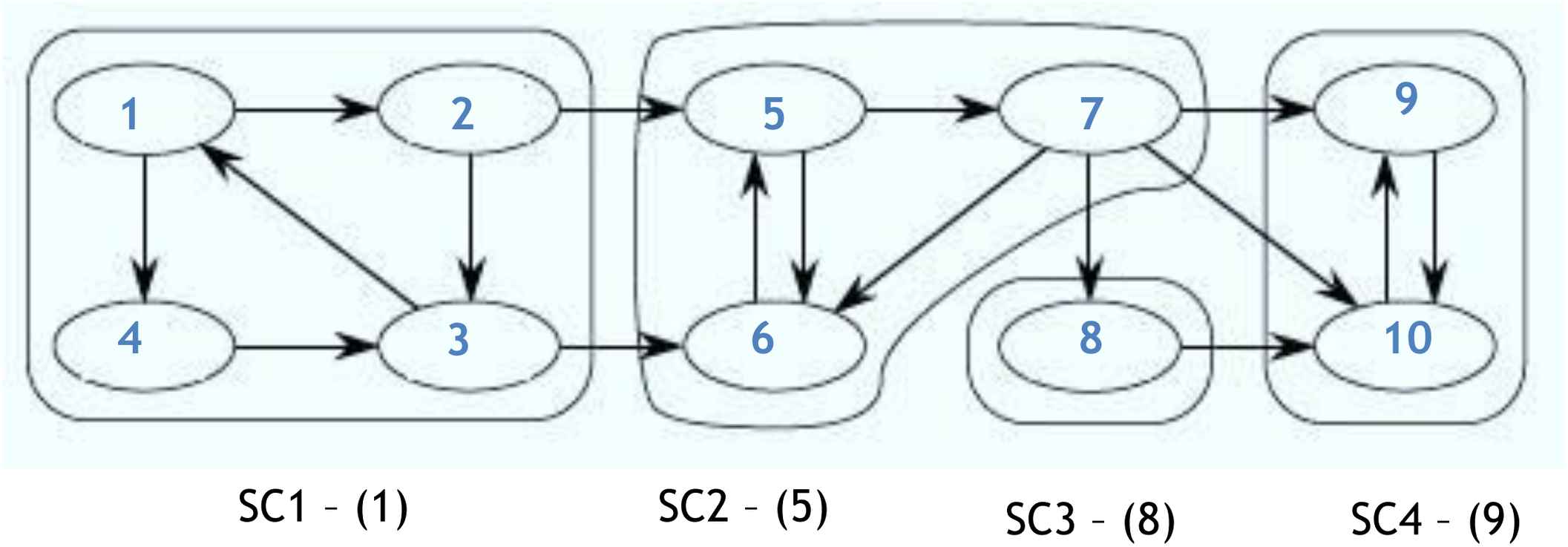
Formal Problem Statement

A feedback vertex set (FVS) in an undirected graph $G = (V, E)$ is a set $F \subseteq V$ of vertices such that every cycle in the graph G contains some vertex in F.

The minimum feedback vertex set problem is as follows:

Given an undirected graph $G = (V, E)$, with a non - negative cost function on its vertices, $c : V \rightarrow R$, find a minimum cost subset of vertices that is an FVS in G .

Strongly Connected Components



SC1 – (1) SC2 – (5) SC3 – (8) SC4 – (9)

Insight : Each strongly connected component can be considered in parallel since they do not share any cycle

Solution

Reduced Problem Statement:
Find a vertex which can eliminate most of the cycles in each component.

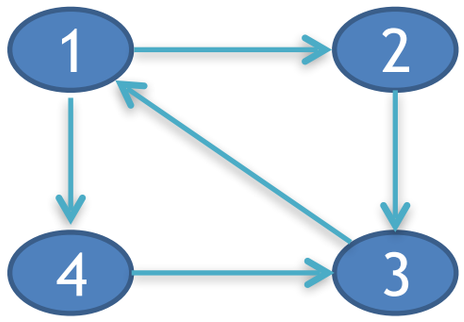
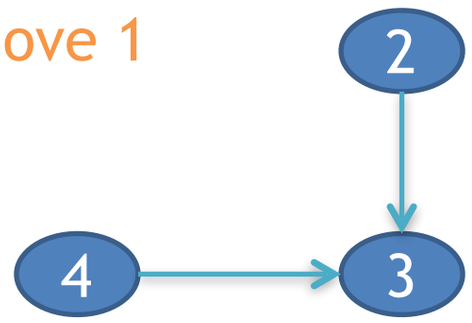
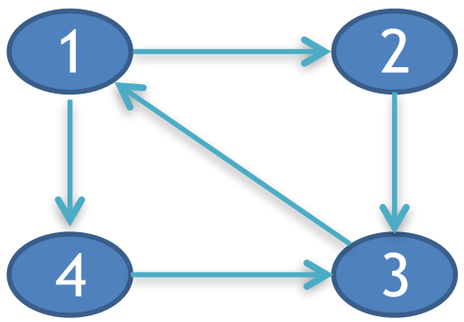
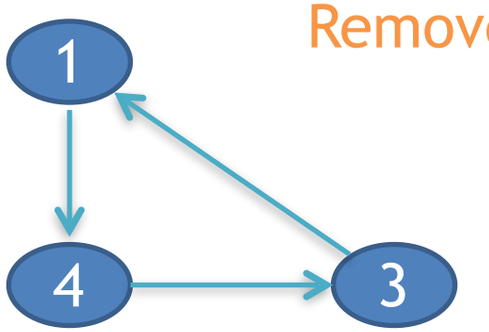
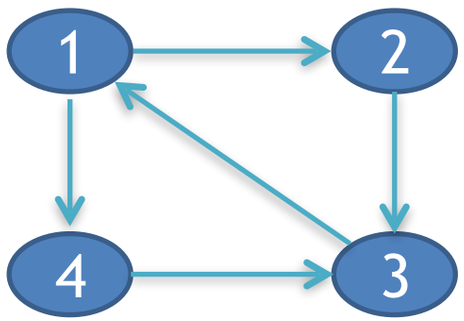
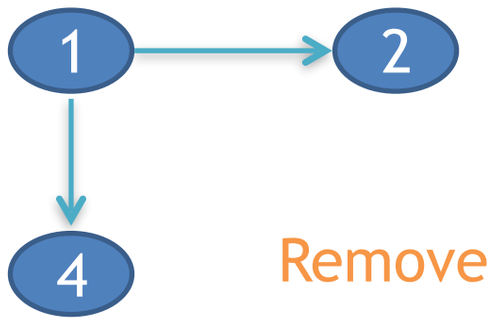
Solution Heuristics :
Vertex, removing which then create highest number of SCC in that component. Apply this recursively.

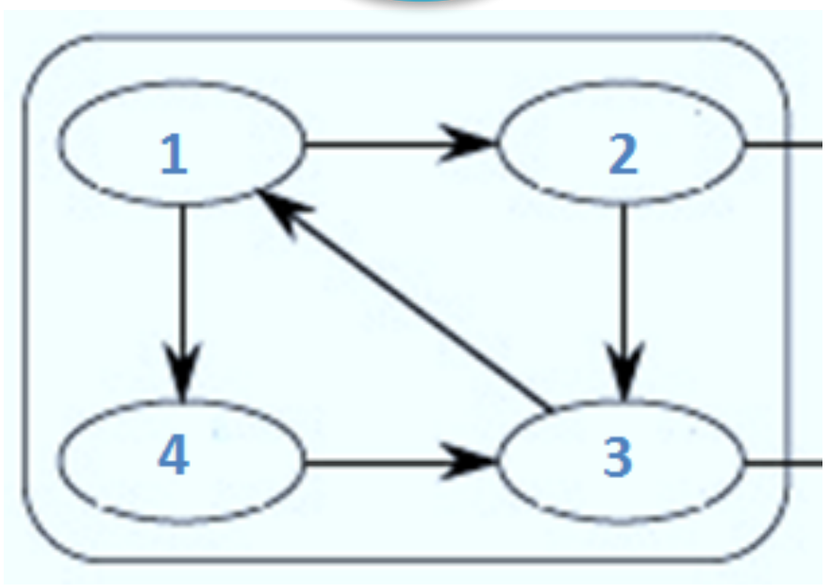
Fitness Function :
maximum no of strongly connected component.

Above algorithm is a genetic algorithm.

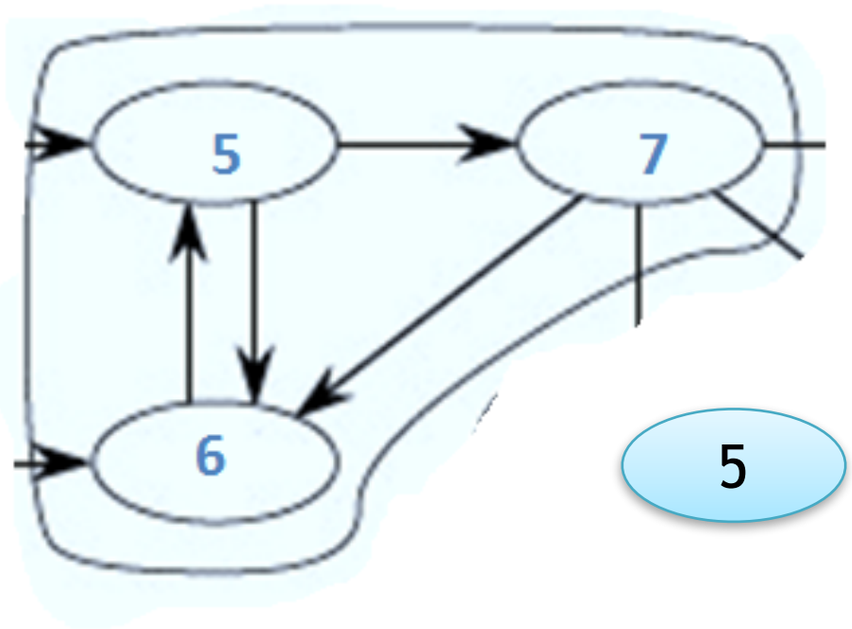
Limitation : All the cycles in the graphs should have more than two vertices.

Graph should not have self loop and parallel loop.
parallel loop exist when there are only two vertices in it.

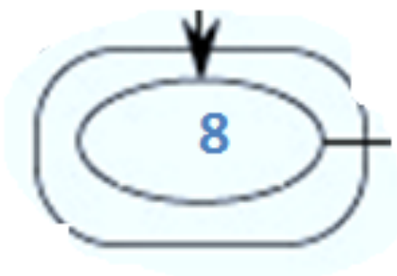
Graph	Iteration	SCC count
	Remove 1 	3
	Remove 2 	1
	 Remove 3	3



1



5



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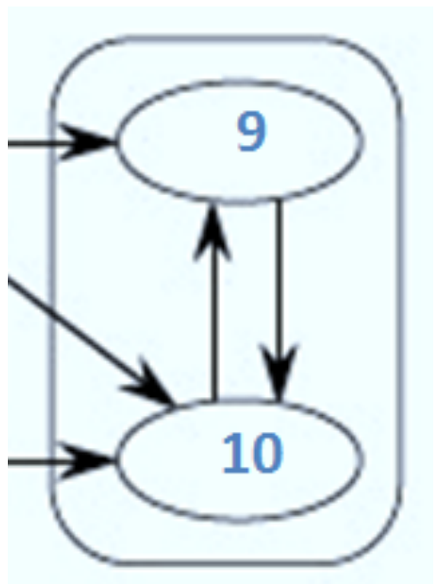

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Feedback Vertex Set

{ 1 5 8 9 }

Here we had no recursion, since removing 1, 5, 8, 9 removed all the cycles.

Implementation

Algorithm has been implemented in following three languages:

Ocaml (Functional)
Java (Imperative)

Since the nature of algorithm is distributed, It has also been implemented in Apache Spark (Scala)

Findings

Algorithm needs to be tested on multiple types of complex and large graphs to show performance metrics.

This is work in progress!

Application

Deadlock prevention [1]

Program verification [2]

Bayesian Inference [3, 4]

Genome Sequence Assembly [5]

Large scale biological networks [7]

Can there be a application in blockchain? There are few blockchains which are DAGs.

References

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