Assignment 1

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May 1, 2025

Question 1

Proof that $\mathcal{R} = \{(A, B) \mid A \supseteq B\} \subseteq \mathcal{P}(S) \times \mathcal{P}(S)$ is a partial order

We prove that $\mathcal{R}$ is a partial order by verifying that it satisfies reflexivity, antisymmetry, and transitivity.

1. reflexivity

For any $A \in \mathcal{P}(S)$, clearly $A \supseteq A$ since every set contains itself. Therefore, $(A, A) \in \mathcal{R}$, and so $\mathcal{R}$ is reflexive.

2. antisymmetry

Suppose $(A, B) \in \mathcal{R}$ and $(B, A) \in \mathcal{R}$. This means $A \supseteq B$ and $B \supseteq A$. This means they are equal. Therefore, $A = B$, and $\mathcal{R}$ is antisymmetric.

3. Transitivity

Suppose $(A, B) \in \mathcal{R}$ and $(B, C) \in \mathcal{R}$. Then $A \supseteq B$ and $B \supseteq C$. By transitivity of the superset relation, $A \supseteq C$, so $(A, C) \in \mathcal{R}$. Therefore, $\mathcal{R}$ is transitive.

Conclusion

Since $\mathcal{R}$ is reflexive, antisymmetric, and transitive, it follows that $\mathcal{R}$ is a **partial order** on $\mathcal{P}(S)$. ■

Question 2

To run the program implemented in Python, follow the steps below.

Prerequisites

You must have Python 3 installed on your system.

Steps

```
git clone https://github.com/BangruiW/transaction_systems_ss25.git
cd transaction_systems_ss25/assignment_1
python3 serializability_checker.py
```

The input should be a space-separated list of operations, for example:

```
w 1 x r 2 x w 2 y r 3 y w 3 z r 1 z
```

Question 3

Our algorithm constructs a **precedence graph** based on conflicting operations between transactions. According to serializability theory, a schedule is **conflict-serializable** if and only if this graph has **no cycles**. We apply the algorithm to a theoretical instance:

```
w 1 x   r 2 x   w 2 y   r 3 y   w 3 z   r 1 z
```

The algorithm detects WR conflicts: $1 \to 2 \to 3 \to 1$, forming a cycle. Therefore, the algorithm returns `false`, providing that the history is not conflict-serializable. This matches the theoretical prediction and confirms the algorithm's correctness for this instance.