CSCI 3320 Homework 1.

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1. (a). $\mu = \frac{1}{5}(2+3+6+8+11) = 6.$

(b). $\sigma^2 = \frac{1}{5}[(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2] = 10.8.$ $\sigma = \sqrt{\sigma^2} = 3.29$

(c). There $\binom{5}{1}=25$ kinds of sample, (2,2) . (2,3). (2,6). (2,8) (2,11)

(3,2). (3,3). (3,6). (3,8) (3,11). (6,2), (6,3), (6,6), (6,8), (6,11).

(8,2). (8,3). (8,6). (8,8) (8,11). (11,2), (11,3) (11,6) (11,8) (11,11).

the means of these samplings are 2, 2.5, 4, 5. 6.5, 2.5, 3, 4.5,

5.5, 7, 4, 4.5, 6, 7, 8.5, 5, 5.5, 7, 8, 9.5, 6.5, 7,

8.5, 9.5, 11.

The mean of the means of sampling $\mu_{\bar{x}} = \frac{150}{25} = 6.$

(d). $\sigma_{\bar{x}}^2 = \frac{1}{25}[(2-6)^2+(2.5-6)^2+\cdots+(11-6)^2] = 5.4$ $\sigma_{\bar{x}} = \sqrt{\sigma_{\bar{x}}^2} = 2.32.$

2. (a). $\mu_{\bar{x}} = \mu = 68.0$ inches.

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = 0.6$ inches.

(b). $\mu_{\bar{x}} = \mu = 68.0$ inches

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{3}{\sqrt{25}}\sqrt{\frac{3000-25}{3000-1}} = 0.5976.$

3. According to Central Limit Theorem.

$z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-68}{0.6}$ follows normal distribution $N(0,1)$.

$\frac{66.8-68.0}{0.6} = -2.$ $\frac{68.3-68}{0.6} = 0.5.$ $P(-2 < z < 0.5) = 0.6687.$

From the table, the expect number of samples is

$80 \times 0.6687 = 53.496 \approx 53.$

(b). $\frac{66.4-68}{0.6} = -2.67.$ $P(z < -2.67) = 0.0038.$

From the table, the expect number of samples is

$80 \times 0.0038 = 0.304 \approx 0.$

4. (a). Flipping a coin follows Binomial distribution, with mean $\mu = 0.5$,

standard deviation $= \sqrt{\frac{pq}{N}} = \sqrt{\frac{0.5 \times 0.5}{120}} = 0.0456.$

According to Central Limit Theorem.

$z = \frac{\bar{x} - \mu}{\sigma}$ follows normal distribution $N(0,1)$.

Since the proportion of heads is discrete, calculate the proportion of heads between $0.4 - \frac{1}{2 \times 120}$ and $0.6 + \frac{1}{2 \times 120}$, which is $(0.3958, 0.6042)$.

$P\left(\frac{0.3958 - 0.5}{0.0456} \leq z \leq \frac{0.6042 - 0.5}{0.0456} \right) = P(-2.28 \leq z \leq 2.28)$

$= 0.0113 \times 2 = 0.0226.$

Thus, the probability of less than 40% or more than 60% heads is 0.0226.

(b). From (a), calculat the proportion of heads more than $\frac{5}{8} - \frac{1}{2 \times 240} = 0.621$

$P\left(z \geq \frac{0.621 - 0.5}{0.0456} \right) = 0.004.$

5. (a). $\bar{x} = \frac{1}{n} \sum x = \frac{1}{5}(6.33 + 6.37 + 6.36 + 6.32 + 6.37) = 6.35$ cm.

(b). $s^2 = \frac{n}{n-1} s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{5-1} [(6.33 - 6.35)^2 + (6.37 - 6.35)^2 + \cdots + (6.37 - 6.35)^2]$

$= 0.00055$ cm^2.

6. (a). For 95% confidence.

$\frac{1.96 \times 0.05}{\sqrt{N}} \leq 0.01 \Rightarrow N \geq 96.04 \approx 97.$

(b). For 99% confidence.

$\frac{2.58 \times 0.05}{\sqrt{N}} \leq 0.01 \Rightarrow N \geq 166.41 \approx 167.$

7. (a). For 95% confidence. $z_c = 1.96$.

the confidence interval is $(75 - 1.96\sigma_{\bar{x}}, 75 + 1.96\sigma_{\bar{x}})$.

$= \left(75 - 1.96 \times \frac{10}{\sqrt{50}} \sqrt{\frac{200 - 50}{200 - 1}},\ 75 + 1.96 \times \frac{10}{\sqrt{50}} \sqrt{\frac{200 - 50}{200 - 1}}\right) = (72.5935, 77.4065).$

(b). $z_c \times \frac{10}{\sqrt{50}} \sqrt{\frac{200-50}{200-1}} = 1 \Rightarrow z_c = 0.8145$ · the corresponding probability is 79.1%.

According to the table, the degree of confidence is $1-2(1-79.1\%) = 58.2\%$.

8. (a). $0.674\,\sigma_{\bar{x}} = 0.674 \frac{\sigma}{\sqrt{n}} = 0.674 \times \frac{0.5}{\sqrt{50}} = 0.0477$.

(b). With 50% confidence limits. $z_c = 0.67$

The confidence interval is $(18.2 - 0.67 \times \frac{0.5}{\sqrt{50}}, 18.2 + 0.67 \times \frac{0.5}{\sqrt{50}})$

$= (18.153, 18.247)$.

9. a). Let X be the number of heads out of 100 tosses of a fair coin.

X follows binomial distribution with mean $\mu = 100 \times 0.5 = 50$.

variance $\sigma^2 = 100 \times 0.5 \times 0.5 = 25$.

$z = \frac{X - \mu}{\sigma}$ follows normal distribution.

Calculate the probability $39.5 < X < 60.5$.

which is $P(\frac{39.5-50}{5} < z < \frac{60.5-50}{5}) = P(-2.1 < z < 2.1)$.

$= 1 - 2 \times (1 - 0.9821) = 0.9642$

(b). The probability of rejecting the hypothesis when it is actually correct

is $1 - 0.9642 = 0.0358$.

(c). Decision rule:

H_0: the coin is fair. H_1: the coin is not fair.

1. Reject H_0 if the z score of the number of heads is outside the range -2.1 to 2.1
2. Accept H_0 otherwise.

(d). Based on (c). H_0 is accepted in both cases with 0.9642 confidence level, as they both lie in range 40 to 60.

(e). Yes, which is Type II error, we accept H_0 whereas we should actually reject it.