IBEHS 4A03 Assignment #3

Baoze Lin, Hady Ibrahim

---

1. Question 1

   (1.1) The open-loop step response of the system is shown in the Simulink model (Figure 1) below with the response shown in Figure 2.

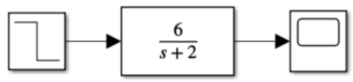

Figure 1: Simulink model of the open-loop step response

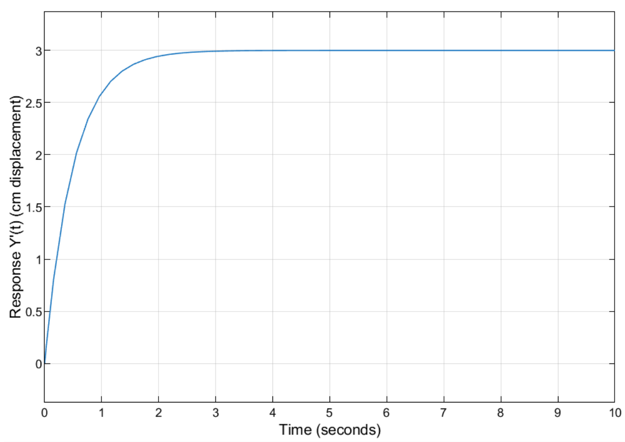

Figure 2: Open-loop step response

To identify the gain and time constant for this system, we can rearrange the transfer function to isolate the gain K and time constant τ:

The given transfer function is:

$$G_p(s) = \frac{6}{s+2}$$

To express this in the standard first-order form, which is:

$$G(s) = \frac{K}{\tau s + 1}$$

where K is the gain and τ is the time constant.
We rearrange $G_p(s)$ as follows:

$$G_p(s) = \frac{6}{2(\frac{1}{2}s + 1)} = \frac{3}{\frac{1}{2}s + 1}$$

Thus, we identify the gain K and the time constant τ as:

$$K = 3, \quad \tau = \frac{1}{2}$$

(1.2) The closed-loop feedback control with a proportional controller is shown below in Figure 3. The transfer function $G_c(s)$ can also be represented by a proportional gain K_c.

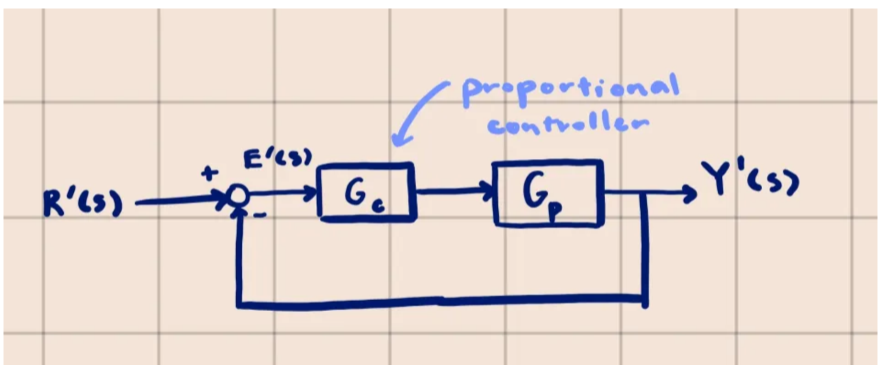

Figure 3: Simulink model of the closed-loop feedback control with a proportional controller

(1.3) The Simulink model is shown below in Figure 4. The response of the system is shown in Figure 5.

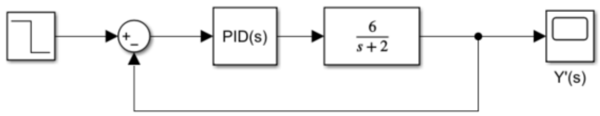

Figure 4: Simulink model of the closed-loop feedback control with a proportional controller

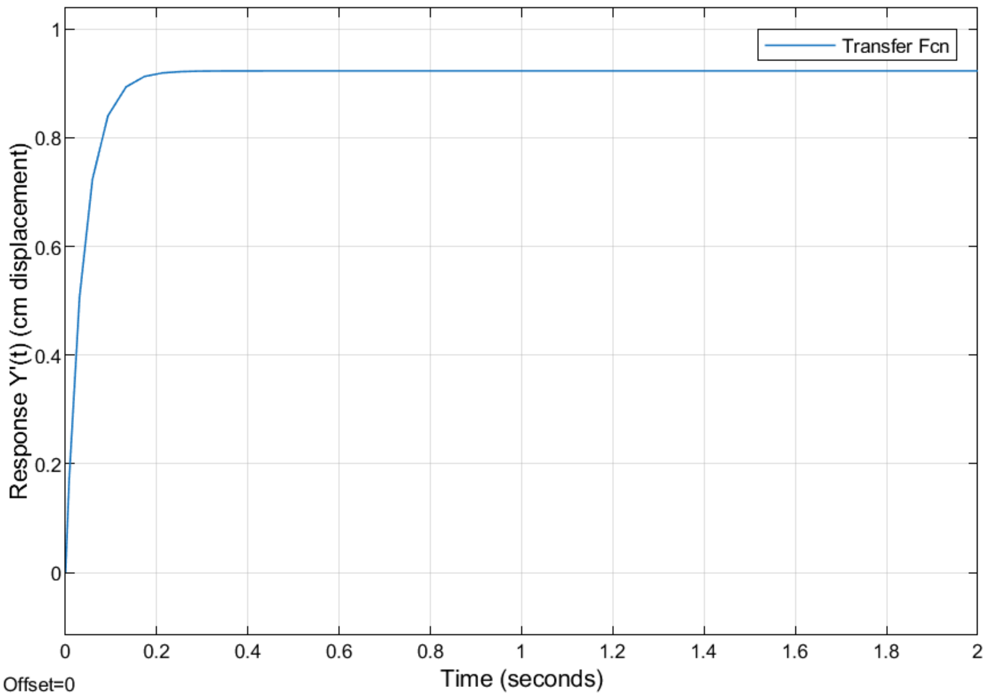

Figure 5: Closed-loop step response

(1.4) The closed-loop transfer function $T(s)$ is given by:

$$T(s) = \frac{Y'(s)}{R'(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}$$

Where $G_c(s) = K_c = 4$ (proportional controller) and $G_p(s) = \frac{6}{s+2}$ (plant transfer function). Substituting the transfer functions:

$$T(s) = \frac{4 \cdot \frac{6}{s+2}}{1 + 4 \cdot \frac{6}{s+2}}$$

Simplifying:

$$T(s) = \frac{\frac{24}{s+2}}{1 + \frac{24}{s+2}} = \frac{24}{s+2+24} = \frac{24}{s+26}$$

Thus, the closed-loop transfer function is:

$$T(s) = \frac{24}{s+26}$$

We then apply the Final Value Theorem, which states that:

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$

For a unit step input $R'(s) = \frac{1}{s}$, the Laplace transform of the output is:

$$Y'(s) = T(s)R'(s) = \frac{24}{s+26} \cdot \frac{1}{s}$$

Applying the FVT:

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot \frac{24}{(s+26)s} = \lim_{s \to 0} \frac{24}{s+26} = \frac{24}{26} = \frac{12}{13} \approx 0.923$$

The steady-state value of the output is approximately 0.923. Since the system is operating on displacement from a rock assembly with a step input, the set point is 1. Thus, the offset of this system is approximately 0.923. Referring to the step response in Figure 5, we can see the offset is approximately $1 - 0.923 = 0.077$.

(1.5) The closed-loop system for controller gains $K_c = 0.25, 1, 2, 4, 10$ are shown in Figure 6. Observing the results, we can see that the offset of the system decreases as the proportional gain increases.

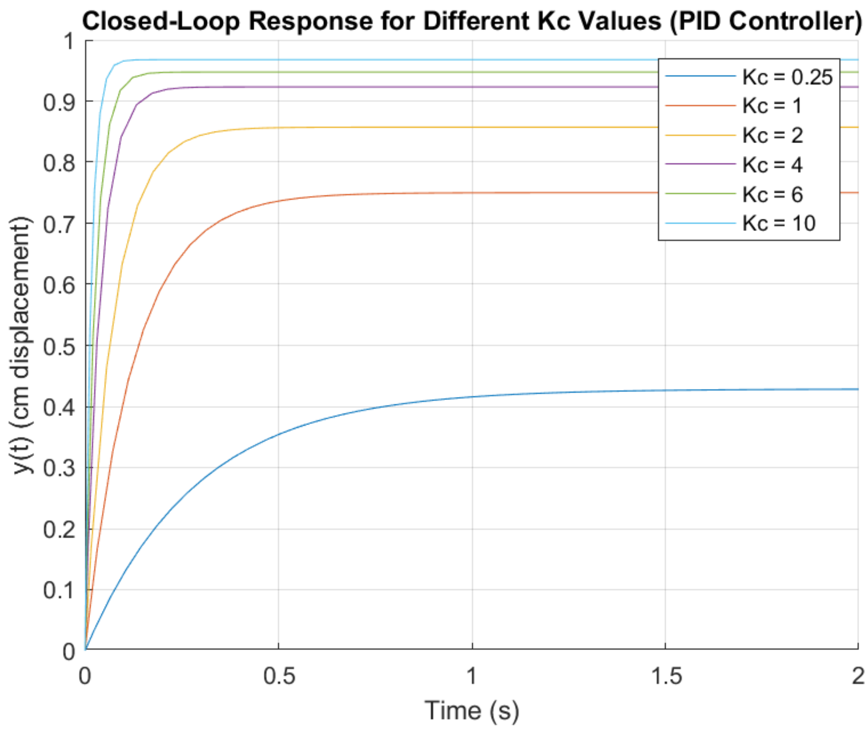

Figure 6: Closed-loop step response for different controller gains

(1.6) We want to find the controller gain K_c such that the offset is less than 0.08. The offset is given by:

$$\text{Offset} = \lim_{t\to\infty}(r(t) - y(t))$$

Using the Final Value Theorem:

$$\text{Offset} = \lim_{s\to 0}\left(1 - \frac{K_c \cdot G_p}{1 + K_c \cdot G_p}\right)$$

With $G_p = \frac{6}{s+2}$, we evaluate at $s = 0$:

$$\text{Offset} = 1 - \frac{K_c \cdot \frac{6}{0+2}}{1 + K_c \cdot \frac{6}{0+2}} = 1 - \frac{3K_c}{1 + 3K_c}$$

We require that the offset be less than 0.08:

$$0.08 > 1 - \frac{3K_c}{1 + 3K_c}$$

Rearranging the inequality:

$$\frac{3K_c}{1 + 3K_c} > 1 - 0.08 = 0.92$$

$$3K_c > 0.92(1 + 3K_c)$$

$$3K_c > 0.92 + 2.76K_c$$

$$(3 - 2.76)K_c > 0.92$$

$$0.24K_c > 0.92$$

$$K_c > \frac{0.92}{0.24}$$

$$K_c > 3.8333...$$

Therefore, the controller gain K_c must be greater than approximately 3.83 to achieve an offset less than 0.08.

2. Question 2

   (2.1) placeholder