IBEHS 4A03 Assignment #1
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1. Question 1

   (1.1) yapyapyap

2. Question 2

(2.1) The ordinary differential equation (ODE) governing the temperature $T(t)$ is:

$$\frac{dT(t)}{dt} = \frac{Q_f(t) - UA(T(t) - T_a)}{\rho V c_p}$$

For this problem, the furnace is off, so $Q_f(t) = 0$. Substituting this into the equation:

$$\frac{dT(t)}{dt} = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

At steady state, the temperature $T(t)$ no longer changes with time, so:

$$\frac{dT(t)}{dt} = 0$$

Substitute this condition into the ODE:

$$0 = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

Solve:

$$T(t) - T_a = 0$$

Thus:

$$T(t) = T_a$$

(2.2) Using the derived equation for $T(t)$, the behaviour of the furnace as defined below, and the given definition of one step of the univariate Euler's method, we can plot Figure 1.

$$Q_f(t) = \begin{cases} 0 & \text{When } T(t) > 23°\text{C}, \\ 1.5 \times 10^6 & \text{When } T(t) < 17°\text{C}, \\ \text{unchanged} & \text{For all } 17 \leq T(t) \leq 23°\text{C}. \end{cases}$$

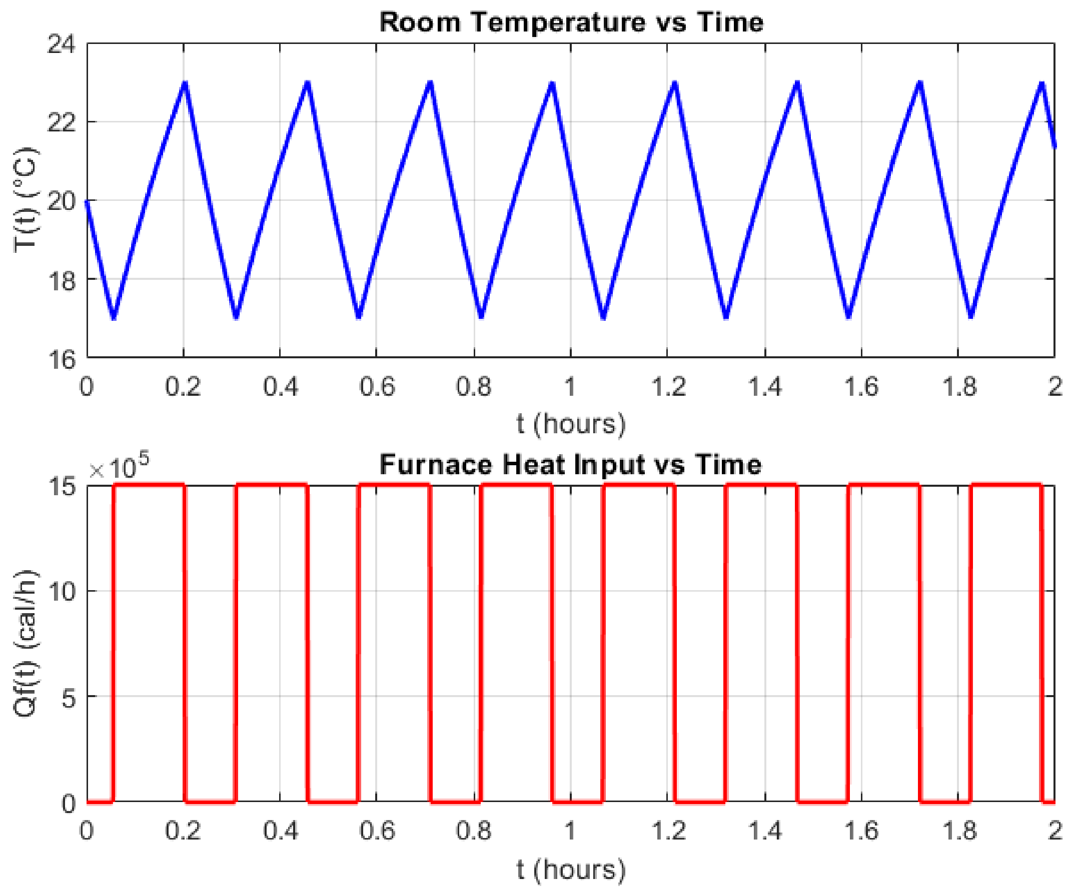


Figure 1: Temperature and Furnace Input Over Time

(2.3) To calculate the number of standard cubic meters of natural gas consumed, we follow the below equation:
Define $t_{on} \triangleq$ time the furnace was on in hours. The Matlab code says it was 1.182 hours
Define $Q_{f,on} \triangleq$ as the furnace heat input.
Define $\rho_e \triangleq$ as the energy density.
Define $e \triangleq$ as the efficiency.

$$\text{Volume of gas consumed} = \frac{t_{on} Q_{f,\text{on}}}{\rho_e e} = \frac{(1.182)(1.5 \times 10^6)}{(9 \times 10^6)(0.9)} = 0.2188888$$
$$\approx 0.219\,\text{standard cubic meters}$$

We also confirm with unit analysis that we get cubic meters.

$$\text{Volume of gas consumed} = \frac{t_{\text{on}} Q_{f,\text{on}}}{\rho_e e} = \frac{\text{hour} \cdot \frac{\text{cal}}{\text{hour}}}{\frac{\text{cal}}{\text{m}^3}} = \text{m}^3$$

(2.4) With a new oscillating defintion of Ta, we get Figure 2 below demonstrating the temperature and furnace input over time.

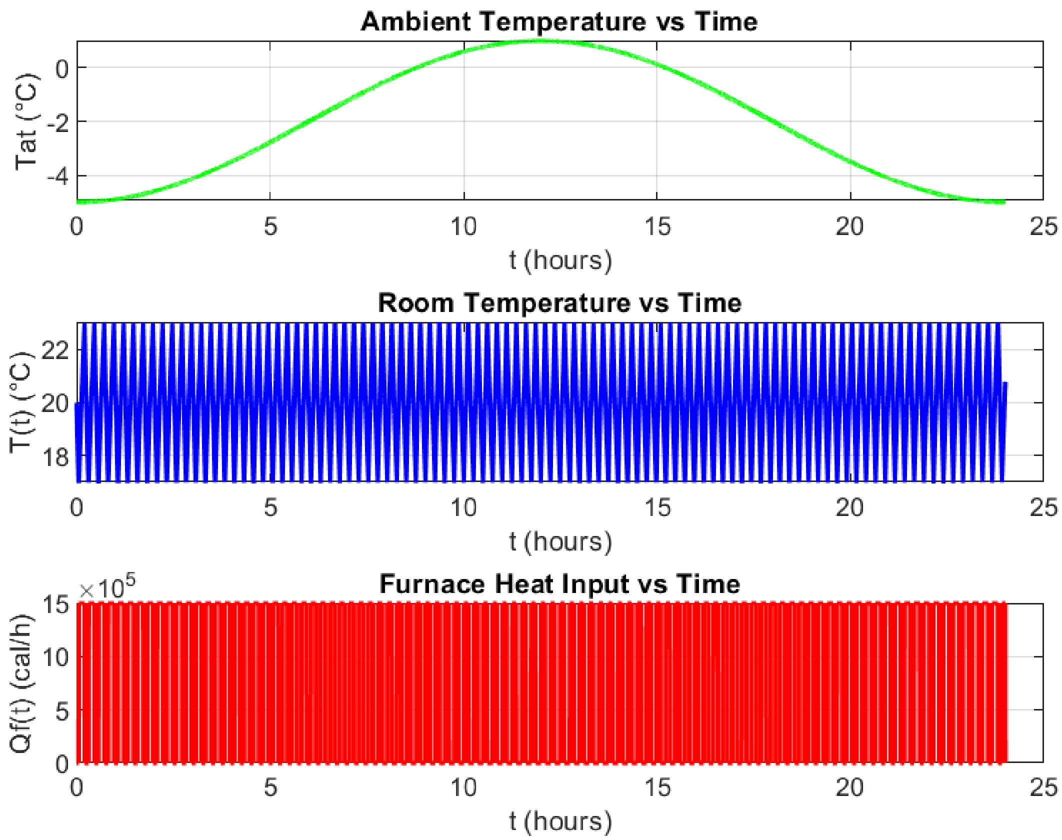


Figure 2: Temperature and Furnace Input Over Time

The simulation results align with the expected behavior of the system. The room temperature $T(t)$ is maintained within the desired range of 17°C to 23°C as the furnace responds to changes in the ambient temperature $T_a(t)$. When $T_a(t)$ is lower, the furnace operates more frequently to offset increased heat loss, while at higher $T_a(t)$, the furnace operates less often due to reduced heat loss. This behavior reflects the thermal dynamics of the system and the influence of the bang-bang control logic.

3. Question 3

(3.1) We are given the following ODE describing the current going thorugh the system.

$$V_s(t) = Ri(t) + L\frac{di(t)}{dt}$$

During steady state, the current $i(t)$ is constant, so the derivative of $i(t)$ with respect to time is zero. Substituting this into the ODE:

$$V_s(t) = Ri(t) + L(0)$$

Rearranging, we get the following equation for $i(t)$:

$$i(t) = \frac{V_s(t)}{R}$$

This is useful when designing a cirtcuit with a target current i(t) because you can control the voltage source to achieve the desired current while the resistance will always be a constant property of the circuit's hardware.

(3.2) The results of the Matlab code was plotted below in Figure 3. You can see that it plateaus to a value of 0.1. We know $V_s(t)$ was defined as a constant $5V$ and R as $50\Omega$. We know the steady state is $\frac{V_s(t)}{R}$ which in this case would give us $0.1A$, so we can see that the plot shows our steady state expression holds.

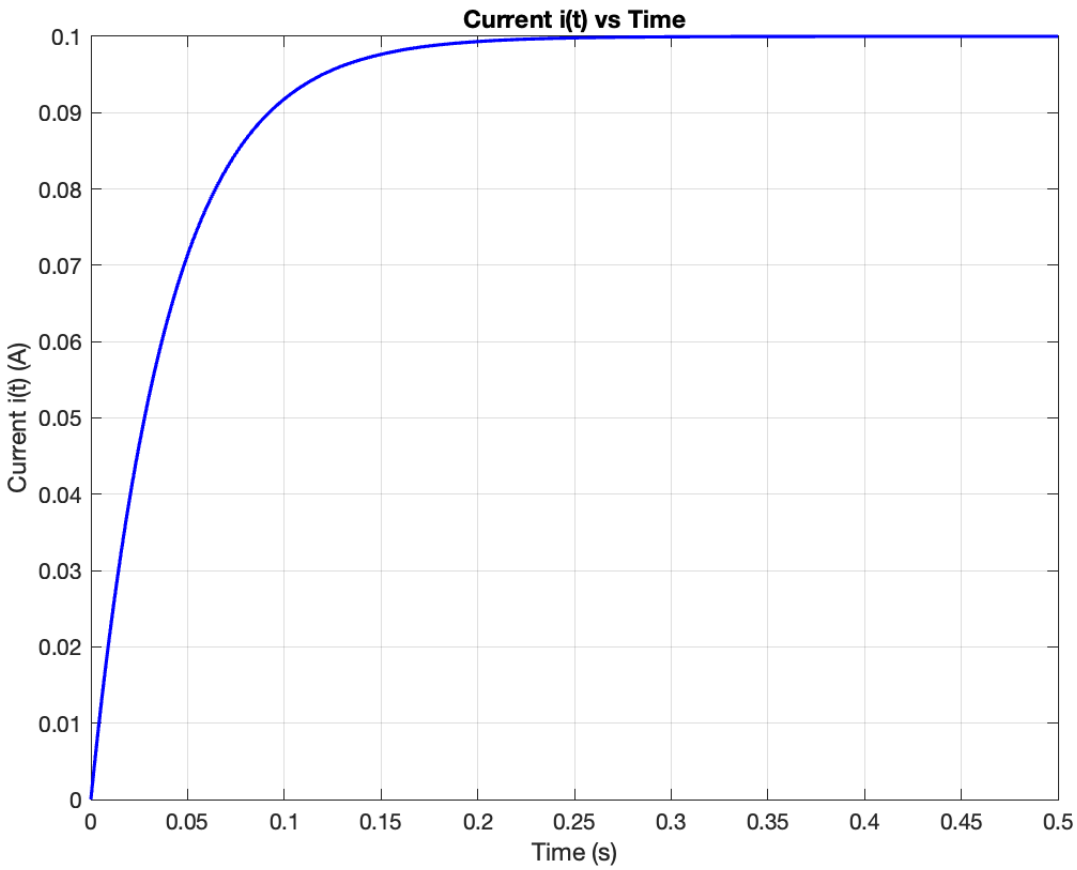


Figure 3: Current Over Time

(3.3)

$$V_s(t) = Ri(t) + L\frac{di(t)}{dt}$$

Based on the ODE above, inductance $L$ should have no effect on the steady state current $i(t)$ because the derivative of $i(t)$ with respect to time is zero, and inductance is a coeffecient on the derivative. This means that the inductance $L$ term in the ODE will not affect the current $i(t)$ when the system reaches steady state, and the current $i(t)$ will be determined solely by the voltage source $V_s(t)$ and the resistance $R$ in the circuit. This can be seen below in Figure 4, where changing R causes the steady state current $i(t)$ to change, while changing L has no effect on the steady state current $i(t)$.

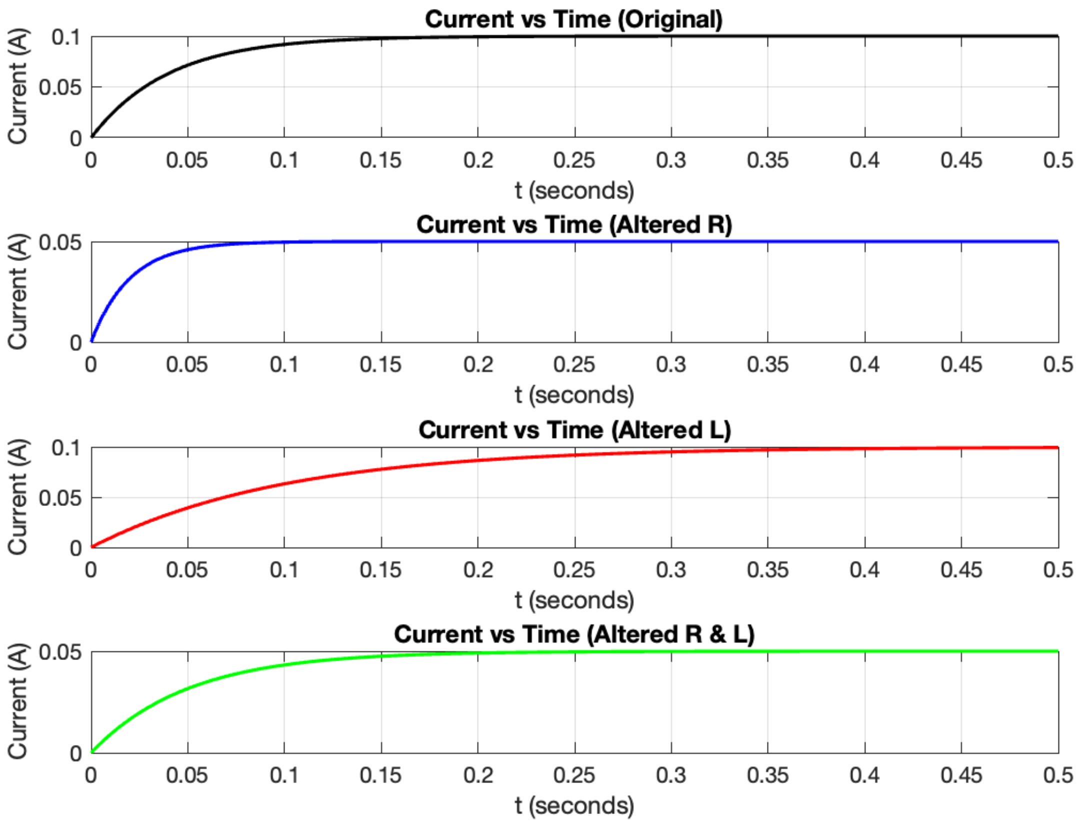


Figure 4: Current Over Time

(3.4) Figure 5 and 6 below demonstrates the LTI nature of the system. An LTI system is defined as a system that is both Linear and Time Invariant. A linear system is one that follows the condition that if $x(t) = \alpha x_1(t) + \beta x_2(t)$, then $y(t) = \alpha y_1(t) + \beta y_2(t)$. That is, the output of a system is a linear combination of the inputs. We can observe in Figure 5 that the combination of the inputs $V_s1 = 5V$ and $V_s2 = 10V$ results in the outputs $i_1(t)$ and $i_2(t)$, which is equal to $i_3(t).i_3(t)$ is the resultant output from the combination input of $V_s3 = V_s1 + V_s2 = 15V$. This is exhibits the linear property. We can also observe the system is time-invariant by looking at the equation — the equation has no coeffecients that change over time. We can also see in Figure 6 that when taking the input and adding time delay, it is equivalent to taking the original output and time shifting that. This proves the equation is time-invariant.

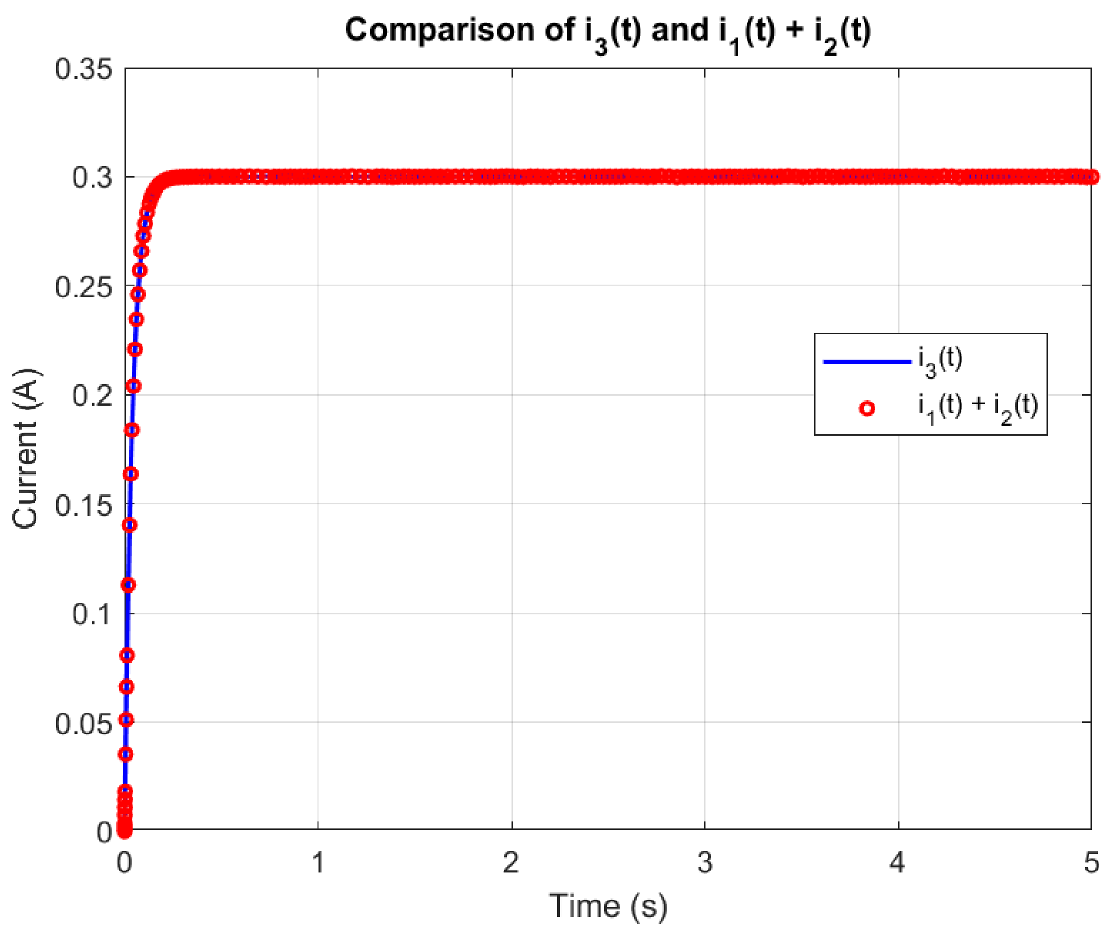


Figure 5: Comparing Linear Combination of Inputs to an Equivalent Output Over Time

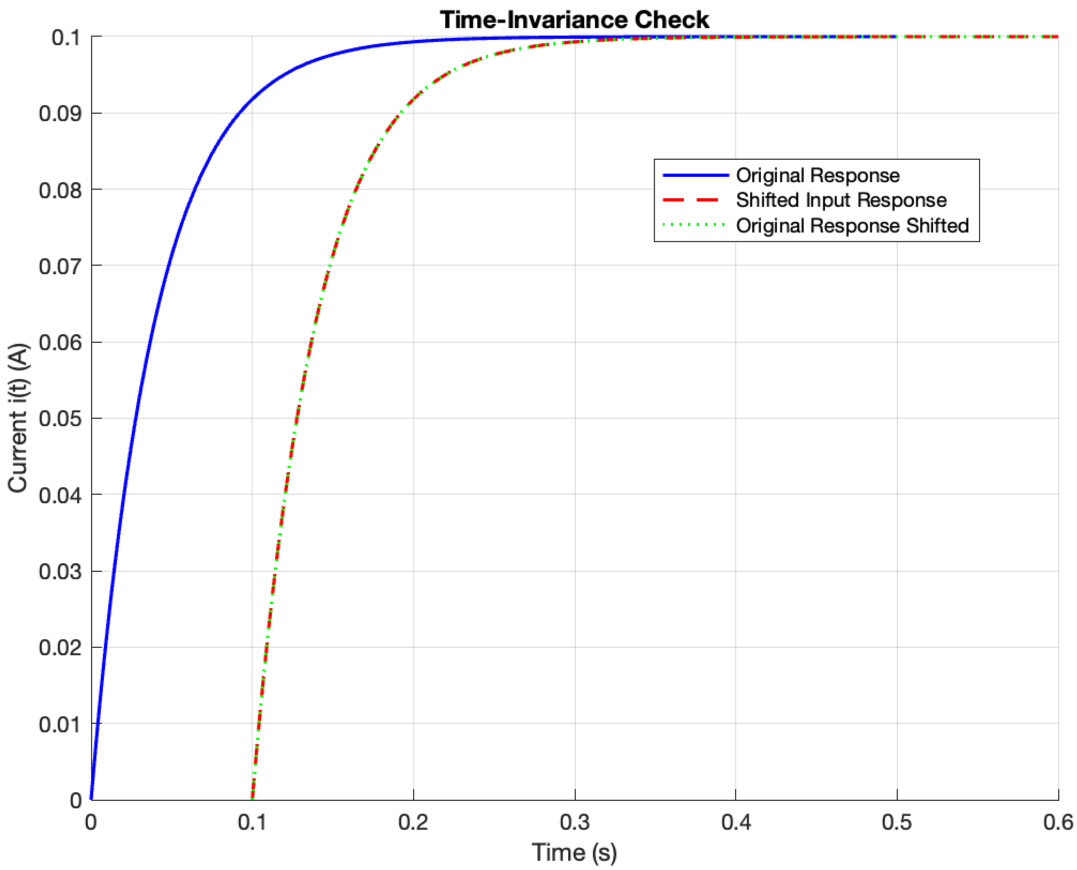


Figure 6: Comparing a Time-Shifted Input with the Time-Shifted Output of the Original Signal

4. Question 4
   (4.1) Part A
   SOLUTION
   (4.2) Part B
   SOLUTION

5. Question 5

   (5.1) Part A
   SOLUTION

   (5.2) Part B
   SOLUTION

# References

[1] MATLAB Documentation: `ode45`. Available at: `https://www.mathworks.com/help/matlab/ref/ode45.html`.