

1. Question 1

(1.1) yapyapyap

## 2. Question 2

(2.1) The ordinary differential equation (ODE) governing the temperature  $T(t)$  is:

$$\frac{dT(t)}{dt} = \frac{Q_f(t) - UA(T(t) - T_a)}{\rho V c_p}$$

For this problem, the furnace is off, so  $Q_f(t) = 0$ . Substituting this into the equation:

$$\frac{dT(t)}{dt} = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

At steady state, the temperature  $T(t)$  no longer changes with time, so:

$$\frac{dT(t)}{dt} = 0$$

Substitute this condition into the ODE:

$$0 = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

Solve:

$$T(t) - T_a = 0$$

Thus:

$$T(t) = T_a$$

(2.2) Using the derived equation for  $T(t)$ , the behaviour of the furnace as defined below, and the given definition of one step of the univariate Euler's method, we can plot Figure 1.

$$Q_f(t) = \begin{cases} 0 & \text{When } T(t) > 23^\circ\text{C}, \\ 1.5 \times 10^6 & \text{When } T(t) < 17^\circ\text{C}, \\ \text{unchanged} & \text{For all } 17 \leq T(t) \leq 23^\circ\text{C}. \end{cases}$$

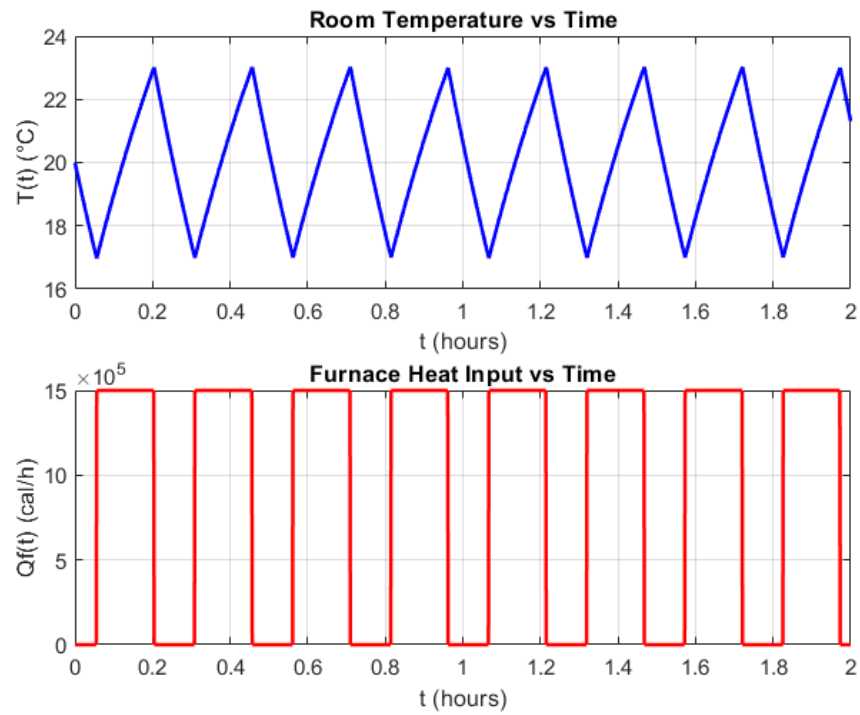


Figure 1: Temperature and Furnace Input Over Time

### 3. Question 3

(3.1) Part A

SOLUTION

(3.2) Part B

SOLUTION

4. Question 4

(4.1) Part A

SOLUTION

(4.2) Part B

SOLUTION

5. Question 5

(5.1) Part A

SOLUTION

(5.2) Part B

SOLUTION