Assignment 3 – Closed-Loop Dynamics

To be completed in GROUPS OF UP TO TWO (2)
Due: Mar 23, 2025 @ 11:59pm via the A2L Drop Box
Grading: 6% of course grade (76 Points Available)

Purpose

In this assignment, we will explore some controller design scenarios. We will investigate the impacts of controller parameter selection on closed-loop performance and stability in the presence of sensor delay, dead time, and other factors. We will also get some practice investigating stability via the Routh-Hurwitz Criterion.

Submission Instructions

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Please submit this assignment *electronically* before the due date. Late submissions will **not** be accepted. Submit via the A2L dropbox. Be sure that you have the names and student numbers of all students on the front page of your submission. Submit your solutions as a **.pdf** file including all relevant figures, tables, and math. You may embed code in your solutions, but you **must submit your coded solutions**.

ONE group member is to submit a single .zip file that includes the solutions and code using the naming convention (for both the .zip and .pdf contained in the .zip):

`4A03_AXX_MACID1_MACID2`

Where `AXX` is the assignment number, and `MACID#` is the McMaster ID (*e.g.* neasej, NOT your student number) of the submitting group member(s). Up to three McMaster IDs can be included on a single assignment.

Up to 10 points may be deducted from your submission for sloppy or otherwise unprofessional work. This is rare, but possible. The definition of unprofessional work may include:

- Low-resolution screenshots of figures and tables.
- Giving no context to an answer relating to the task (*i.e.,* "See code" or "113.289" with no units, context, or discussion whatsoever).
- Clear changes in author denoted by format changes, blatant writing style changes (including AI-generated discussion), or other factors that may deduct from the cohesiveness of the report.
- Failing to provide references for work that is obviously not yours (particularly bad cases will be considered as academic dishonesty).

Problem 1 – Design for First-Order Dynamics [46]

In this problem, we are going to get in the weeds around the design of a controller for a system with first-order dynamics. *Advice: we investigated first-order dynamics in the presence of P and PI control in lecture and came up with some handy formulas that will definitely be useful in this problem*.

In your solution, you are welcome to use *Simulink*, the built-in `MATLAB` functions like `tf()` and `step()`, analytical solutions, or any combination of these. In your report, make it clear what your strategy is to ensure you get as many points as possible.

Consider the following first-order transfer function experiencing relatively fast dynamics. Time units are in seconds, and gain units are linear cm of displacement from a rack/pinion assembly responding to angular changes in the pinion gear:

$$\frac{Y'(s)}{U'(s)} \triangleq G_P(s) = \frac{6}{s+2}$$

1.1. Simulate this system responding to an open-loop step change in $u(t)$ over a time span of 2 seconds. Clearly plot the results of the open-loop process and identify the gain and time constant for this system any way you choose. [2]

1.2. Consider a scenario where we place this system in closed-loop feedback control with a **proportional controller**. You may assume that we measure the output $y(t)$ directly and that there are no disturbances or noise signals. Draw a block diagram of this system if the setpoint/reference signal is depicted as $R'(s)$. [2]

1.3. Simulate this system for a time span of 2 seconds responding to a step change in $r(t)$ if a controller gain of $K_c = 4$ is used. Clearly plot the results. [2]

1.4. Determine the closed-loop transfer function for this system: $G(s) = \frac{Y'(s)}{R'(s)}$. Assume the same controller gain $K_c = 4$. Use the FVT to compute the offset of this system and compare to your simulation results. [4]

1.5. Simulate the closed-loop system responding to a step change in $r(t)$ for the set of controller gains $K_c = \{0.25, 1, 2, 4, 6, 10\}$ using a time span of 2 seconds. Plot all closed-loop responses for $y(t)$ on the same set of axes and comment on the results. [4]

1.6. Compute the controller gain K_c necessary to obtain a closed-loop offset of less than 0.08 in any fashion you choose. [2]

1.7. Consider now the use of a **proportional-integral controller** for this system. Using tuning parameters of $K_c = 3$ and $\tau_I = 0.05$, simulate the closed-loop system responding to a step change in $r(t)$ over a time of 2 seconds. Clearly plot the results. [2]

1.8. Using tuning parameters of $K_c = 3$ and $\tau_I = 0.05$, determine the closed-loop gain K, time constant τ, and damping factor ζ for this system. Comment on the results as they relate to your simulation results. [4]

1.9. Simulate the closed-loop system responding to a step change in $r(t)$ for a fixed controller gain $K_c = 3$ and a set of integral times $\tau_I = \{0.01, 0.05, 0.1, 0.4, 1, 2\}$ over 2 seconds. Plot all closed-loop responses for $y(t)$ on the same set of axes and comment on the results. [4]

1.10. Next, we would like to investigate the effects of tuning parameters on the closed-loop dynamic properties of the system. Create surface plots (you can use `surf()` in `MATLAB`) of the closed-loop time constant ζ and time constant τ (one on each set of axes) for tuning parameter ranges of $K_c = [0.05 \cdots 5]$ and $\tau_I = [0.01 \cdots 2]$. Comment briefly on the shape of the results and what you can learn from these plots. *Note – We explored a similar exercise in Workshop 6.5.* [4]

1.11. We know that "optimal" controller performance should include closed-loop results that are critically damped (or nearly critically damped). We therefore want to find the combination of tuning parameters K_c and τ_I that result in a critically damped closed-loop response.

Using the equation for ζ for a first-order system under closed-loop PI control (see Workshop 6.5) and the nonlinear solver `fsolve()` in `MATLAB`, plot the curve representing the combination of K_c (x-axis) and τ_I (y-axis) corresponding to a closed-loop damping factor of $\zeta = 1$ on the range $K_c = [0.05 \cdots 5]$. That is, plot the curve that represents the intersection of the surface $\zeta = f(K_c, \tau_I)$ and plane $\zeta = 1$. [4].

1.12. In addition to having $\zeta = 1$, we also want the closed-loop time constant τ to be as small as possible to ensure a fast response. On the same axes as the plot of $\zeta = 1$ above, overlay a contour plot of τ as a function of K_c and τ_I. This should be the same results as your solution to problem (1.10). Based on the plot and assuming $K_c \leq 5$, suggest tuning parameters for K_c and τ_I that will result in optimal closed-loop performance of this system. [4]

1.13. Simulate the closed-loop system responding to a step change in $r(t)$ if the controller contains the optimal tuning parameters you have found over a time of 2 seconds. Plot the results $y(t)$ and comment on the results. [2]

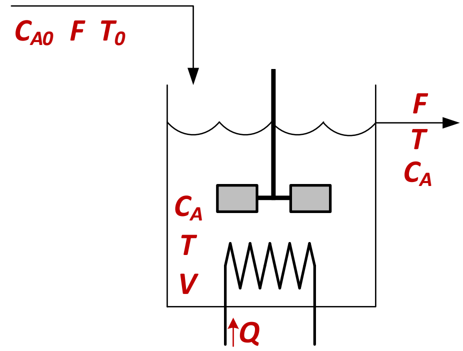
1.14. Finally (finally?), now that we have finely tuned this system in the simulation, consider a scenario when this controller is implemented for the real process represented by $G_P(s)$. However, in the real system there are (relatively short) **sensor dynamics** with the form:

$$\frac{Y_M'(s)}{Y'(s)} \triangleq G_M(s) = \frac{1}{0.05s + 1}$$

Where $Y_M'(s)$ is the measured output sent back to compute the feedback error $E'(s)$. Simulate your optimal controller with these new dynamics for a time span of 2 seconds and comment on the results. Then, re-tune the controller "by hand," by performing three tuning iterations. Record the parameters for each iteration and briefly explain how you decided on those parameters. Comment on the parameters you settle on that improve the closed-loop performance. [6]

Problem 2 – Design for Internally Interacting Systems [30]

Consider the non-isothermal stirred tank reactor depicted at right that was the focus for assignment 2. In this system, a reaction of the form $A \to B$ takes place that consumes component A. The reaction is **exothermic**, which means it releases energy in the form of heat as the reaction proceeds. The heat released by the reaction is ΔH [energy per mole reacted] and $\Delta H < 0$ for exothermic reactions. We may make all other assumptions that we have had for well mixed systems to date, such as constant ρ and constant c_p. Furthermore, the flow rate entering and leaving the system (F) is constant, and thus V is also constant.



The reaction follows first-order Arrhenius kinetics in the form $r_A = -k_0 e^{-\frac{E}{RT}} C_A$, where E, R, and k_0 are constants depending on the reaction (let the chemists worry about that, we do not care) and the reaction rate notably **depends on temperature** T.

In assignment 2, we linearized this system around a point for $\bar{C}_A$, $\bar{T}$, $\bar{C}_{A0}$, and $\bar{Q}$. Linearization around this point led to the following linearized open-loop dynamic model:

$$\frac{dC_A'(t)}{dt} = \left(-\frac{F}{V} - k_0 e^{-\frac{E}{R\bar{T}}}\right) C_A'(t) + \left(-\frac{E k_0 \bar{C}_A}{R\bar{T}^2} e^{-\frac{E}{R\bar{T}}}\right) T'(t) + \frac{F}{V} C_{A0}'(t) \quad (1)$$

$$\frac{dT'(t)}{dt} = \left(\frac{-\Delta H k_0}{\rho c_p} e^{-\frac{E}{R\bar{T}}}\right) C_A'(t) + \left(-\frac{F}{V} - \frac{k_0 \bar{C}_A \Delta H E}{\rho c_p R \bar{T}^2} e^{-\frac{E}{R\bar{T}}}\right) T'(t) + \left(\frac{1}{\rho c_p V}\right) Q'(t) \quad (2)$$

For this reactor, we have the following parameter data. The linearization point is also shown. We would like to implement a controller that uses the temperature state to control the rate of reaction while rejecting disturbances in the inlet concentration $C_{A0}(t)$. All of the analyses for this problem will be performed in deviation variables (away from the centering values provided).

V = 0.5 [m^3]	R = 8.314 [kJ·kmol^{-1}·K^{-1}]	T_0 = 352.6634 [K]
ΔH = -4.78×10^4 [kJ·kmol^{-1}]	k_0 = 65×10^9 [min]	E = 8.314×10^4 [kJ·kmol^{-1}]
c_p = 0.329 [kJ·kg^{-1}·K]	ρ = 1000 [kg·m^3]	F = 0.2 [m^3·min^{-1}]
$\bar{C}_{A0}$ = 0.6767 [kmol·m^{-3}]	$\bar{Q}$ = 0 [kJ·min^{-1}]	$\bar{C}_A$ = 0.0199 [kmol·m^{-3}]
$\bar{T}$ = 448.09 [K]		

2.1. If $C_{A0}(t)$ and $Q(t)$ are the open-loop inputs for this scenario, make an argument that any change in either input will result in an unstable **open-loop** system. [2]

2.2. Since $C_A(t)$ and $T(t)$ are dependent, it is unrealistic to dictate both of their values at the same time (in fact, it is impossible unless both setpoints are in equilibria with each other). Instead, we would be better served to exploit temperature control as an ad-hoc way of controlling the reaction extension. We showed this was possible in the previous assignment by computing the explicit transfer function $\frac{C_A'(s)}{Q'(s)}$.

Draw a block diagram of this process where $C_A'(s)$ is the controlled variable, and $C_{A0}'(s)$ is a **disturbance**. You may define the transfer functions by inputting the data above or algebraically (the suggestion is to make it as simple as possible). State $T'(s)$ does not need to exit the system and can be considered an intermediate state. The state $C_A'(s)$ should be fed back to a comparator (sum) block with $R'(s)$, the setpoint for $C_A'(s)$. The error signal resulting from this feedback signal should enter a to-be-determined controller $G_C(s)$ that **selects a value for $\boldsymbol{Q'(s)}$**, the heat input to the reactor system. At the end of it all you should be able to show $C_{A0}'(s)$ and $R'(s)$ are the only inputs to the system, and $C_A'(s)$ is the only output. [4]

2.3. Using your block diagram or solution to assignment 2, show that the expected gain in the transfer function $\frac{C_A'(s)}{Q'(s)}$ is **negative**. [2]

2.4. Make an argument (using this process or any other simple process as an example) that the sign of K_C for a feedback controller in ideal form needs to be the same sign as the process gain that it is controlling, or the control loop will be unstable. [2]

2.5. Your task for this problem is to design a PI controller that rejects disturbances in $C_{A0}'(t)$ to keep the outlet concentration $C_A'(t)$ as close to zero as possible. You are to do this by designing block $G_C(s)$, selecting the "best" values of parameters K_C and τ_I in any manner you choose. Your controller should be designed to reject a **step disturbance** in $C_{A0}'(t)$ of **magnitude +0.1** with the following information and constraints: [20]

- Simulate the system for a time span of $t = [0, 0.01, \cdots 40]$. Do not exceed 40 minutes as the simulation may become numerically unstable (not to be confused with "control loop unstable").
- The measurement device for $C_A'(t)$ can provide you with the state directly, but it has its own dynamics with a time constant of 12 seconds (and a gain of 1.0).
- When tuning the controller, it is important that the value for $C_A'(t)$ does not exceed ± 0.01 of its desired value (0 in deviation terms).
- When tuning the controller, it is important that the reactor temperature does not increase by more than 15°C at any time due to safety constraints.
- The maximum cooling to the reactor at any one time is $Q'(t) = -2000$ kJ·min^{-1}.
- Any sustained disturbance in $C_{A0}'(t)$ should ideally be rejected in 40 minutes (aka the simulation time) or less.

Describe your design procedure, including at least three sets of tuning parameters and why you tried that combination. For each set of tuning parameters **including** the final values:

- Plot $C_A'(t)$, $Q'(t)$, and $T'(t)$.
- Discuss what the results inform you should try next (if appropriate).
- Whether the set of parameters accomplishes the design objectives.