Assignment 3 – Closed-Loop Dynamics

To be completed in GROUPS OF UP TO TWO (2)
Due: Apr 08, 2025 @ 11:59pm via the A2L Drop Box
Grading: 6% of course grade (60 Points Available)

Purpose

In this assignment, we will explore how Routh-Hurwitz Stability can be used to help us establish feasible sets of controllers. Further, we will identify a system approximated with first-order dynamics and use a variety of tuning correlations to explore their results on closed-loop dynamics. Finally, we will design an "optimal" controller using a quantitative analysis strategy.

Submission Instructions

Please submit this assignment *electronically* before the due date. Late submissions will **not** be accepted. Submit via the A2L dropbox. Be sure that you have the names and student numbers of all students on the front page of your submission. Submit your solutions as a **.pdf** file including all relevant figures, tables, and math. You may embed code in your solutions, but you **must submit your coded solutions**.

ONE group member is to submit a single .zip file that includes the solutions and code using the naming convention (for both the .zip and .pdf contained in the .zip):

`4A03_AXX_MACID1_MACID2`

Where `AXX` is the assignment number, and `MACID#` is the McMaster ID (*e.g.* neasej, NOT your student number) of the submitting group member(s). Up to three McMaster IDs can be included on a single assignment.

Up to 10 points may be deducted from your submission for sloppy or otherwise unprofessional work. This is rare, but possible. The definition of unprofessional work may include:

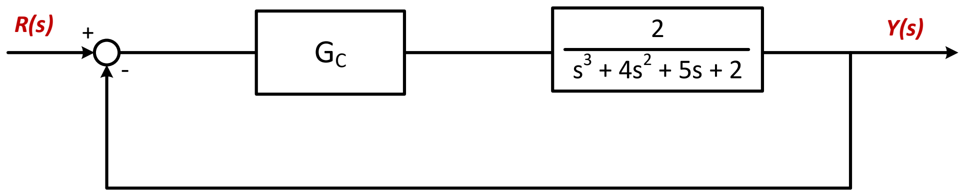
- Low-resolution screenshots of figures and tables.
- Giving no context to an answer relating to the task (*i.e.,* "See code" or "113.289" with no units, context, or discussion whatsoever).
- Clear changes in author denoted by format changes, blatant writing style changes (including AI-generated discussion), or other factors that may deduct from the cohesiveness of the report.
- Failing to provide references for work that is obviously not yours (particularly bad cases will be considered as academic dishonesty).

Problem 1 – Using RH to Establish Stability Criteria [23]

Consider the following simplified closed-loop block diagram of a third-order process being controlled with a controller $G_C(s)$. We would like to determine the range of values for a PI controller:

$$G_C(s) = K_C + \frac{1}{\tau_I s}$$

for which the closed-loop system $\frac{Y(s)}{R(s)}$ is closed-loop stable. Note that in this case we have a controller in **parallel form**, which is different from the **ideal form controller** presented in the course slides.



1.1. Use a Routh Array to determine the set of conditions for the above system under PI control that results in a stable closed-loop response. *Hint: for this problem I suggest making the substitution $T_I = \frac{1}{\tau_I}$ to make the algebra less cumbersome*. [5]

1.2. Plot the "feasible set" for this control design problem, which is a graphical representation of all combinations of K_C and τ_I (or T_I if you like) that results in a stable closed-loop response. [3]

1.3. Model this transfer function system in `Simulink` responding to a step in $R(s) = \frac{1}{s}$ and simulate it using three sets of controller parameters: One combination inside the feasible set, one combination outside the feasible set, and one combination that is perfectly on the "edge" of the feasible set. Plot the outputs $y(t)$ for all three scenarios and briefly comment on the results. [5]

Consider the same closed-loop system with the addition of a transport delay of 0.5 time units to transmit the feedback signal $Y(s)$ back to the controller. This scenario can be modeled as the same block diagram with measurement transfer function $G_M(s) = e^{-0.5s}$ inline with the feedback signal. The first-order Padé Approximant attempts to reproduce the curvature of a nonlinear function as a rational polynomial. Such an approximant centered at $s = 0$ for $e^{-\theta s}$ is:

$$e^{-\theta s} \approx \frac{1 - \theta/_2\, s}{1 + \theta/_2\, s}$$

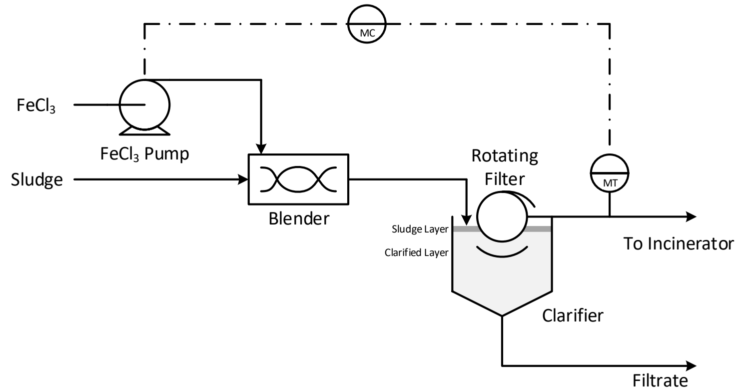
1.4. Complete a Routh Array for this system under closed-loop PI control using the Padé Approximant for $e^{-\theta s}$. Determine the feasible set for K_C and τ_I (or T_I) and plot it on the same axes as the feasible set for the system without transport delay. Comment on any differences. [6]

1.5. Finally, simulate the system in Simulink responding to a step change in $R(s) = \frac{1}{s}$ for a stable combination of K_C and τ_I with and without the transport delay (use the same tuning parameters for both). Plot both outputs on the same set of axes and comment on any differences. [4]

Problem 2 – Design for Internally Interacting Systems [37]

Consider the figure below, which shows a filtration system for a biological waste treatment plant. The plant accepts activated sludge from the digesters and is intended to filter out the sludge before sending the wastewater to tertiary treatment. This can be achieved using a vacuum filtration system that extracts the solid waste (sludge) from the water phase as efficiently as possible.

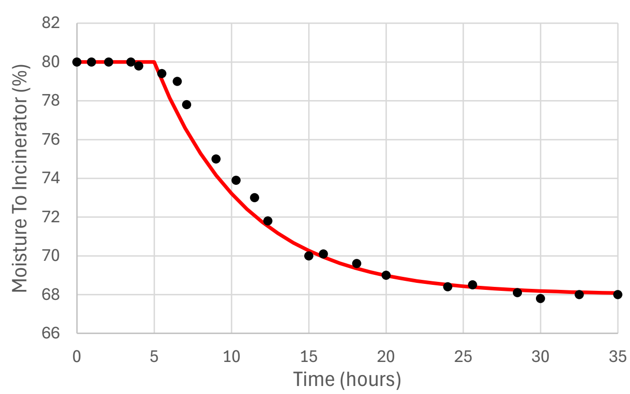
To reduce the environmental impact of the downstream sludge incinerator, we would like to remove as much water as possible during the filtration stage. One such method is to use ferric chloride ($FeCl_3$) as a "dewatering" agent, the addition of which before the filter causes the outgoing sludge to contain less moisture and thus burn more readily in the incinerator.



The sensor (MT) transmits the moisture content of the sludge destined for the incinerator to the controller (MC), which adjusts the pump output of $FeCl_3$ to the blender. **Currently, the pump is operating at 50% capacity**. For the last few years, the control loop has been disabled and the flow of $FeCl_3$ has been adjusted manually from time-to-time, leading to inconsistent performance in the incinerator. Your boss wants you to design an efficient controller to not waste $FeCl_3$ (it is expensive!) but still maintain an expected moisture content in the sludge stream.

To determine the transfer function model of the process above, you apply a step change of +10% in the flow rate of $FeCl_3$ (input). The data below was obtained by taking samples from the sludge stream and measuring the moisture content at various times.

Time (hr)	Moisture (%)	Time (hr)	Moisture (%)
0	80	12.35	71.8
0.95	80	15	70
2.05	80	15.95	70.1
3.5	80	18.1	69.6
4	79.8	20	69
5.5	79.4	24	68.4
6.5	79	25.6	68.5
7.1	77.8	28.5	68.1
9	75	30	67.8
10.3	73.9	32.5	68
11.5	73	35	68



Based on the data shown in the table and figure, you determine that this system seems to exhibit second-order dynamics but can likely be approximated by the following first-order plus dead-time model:

$$\frac{Y(s)}{U(s)} = \frac{-1.2e^{-5s}}{6s+1}$$

2.1. Assuming no sensor or actuation dynamics, design a PID controller for this system for set-point tracking using the following design heuristics: [6]

a. Cohen-Coon
b. Ciancone
c. ITAE

2.2. Create a Bode plot for this system. Using the Bode plot, determine the critical frequency, ultimate gain, and ultimate period for this system. Using the Bode plot, design a PID controller for this process using the Ziegler-Nichols design heuristic. [6]

2.3. Simulate this system under closed-loop control for all four controller designs. Plot the results for each system responding to a desired step change of -5% moisture in the incinerator stream and comment on the different performances for the controllers. [5]

2.4. Now that we have a general idea of the *neighbourhood* of acceptable controller tuning parameters, we would like to *optimize* the controller using a quantitative metric. Assume that the K_D (derivative gain) for this controller is fixed at the value found using the ITAE correlation. For the other tuning parameters (τ_I and K_C), you are to **determine the combination that results in the lowest possible Integral Squared Error (ISE)**.

To do this, simulate the system responding to a step change of -5% moisture in the incinerator stream for spectra of K_C and τ_I around the neighbourhood of the guesses provided by the tuning correlations. You do not need to change K_D. Explore whatever range of values for K_C and τ_I that you deem appropriate. Make a contour plot of ISE (level sets) versus K_C (x-axis) and τ_I (y-axis) to visualize how the tuning parameters affect controller performance. Select the combination of K_C and τ_I that result in optimal performance for this scenario and plot the results compared to the other controllers. Comment on the results. Your grade will be based on how thoroughly you explore the search space and how good your solution is according to ISE. [20]

This process is basically identical to the ISE optimization exercise in tutorial exercise 06, which you are encouraged to attempt and check the solutions for when they are available. Note that we are not asking you to perform a Routh Array for this problem.

Congratulations for making it through all the assignments and all the best for the exam (and beyond!).