

1. Question 1

(1.1) yapyapyap

## 2. Question 2

(2.1) The ordinary differential equation (ODE) governing the temperature  $T(t)$  is:

$$\frac{dT(t)}{dt} = \frac{Q_f(t) - UA(T(t) - T_a)}{\rho V c_p}$$

For this problem, the furnace is off, so  $Q_f(t) = 0$ . Substituting this into the equation:

$$\frac{dT(t)}{dt} = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

At steady state, the temperature  $T(t)$  no longer changes with time, so:

$$\frac{dT(t)}{dt} = 0$$

Substitute this condition into the ODE:

$$0 = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

Solve:

$$T(t) - T_a = 0$$

Thus:

$$T(t) = T_a$$

(2.2) Using the derived equation for  $T(t)$ , the behaviour of the furnace as defined below, and the given definition of one step of the univariate Euler's method, we can plot Figure 1.

$$Q_f(t) = \begin{cases} 0 & \text{When } T(t) > 23^\circ\text{C}, \\ 1.5 \times 10^6 & \text{When } T(t) < 17^\circ\text{C}, \\ \text{unchanged} & \text{For all } 17 \leq T(t) \leq 23^\circ\text{C}. \end{cases}$$

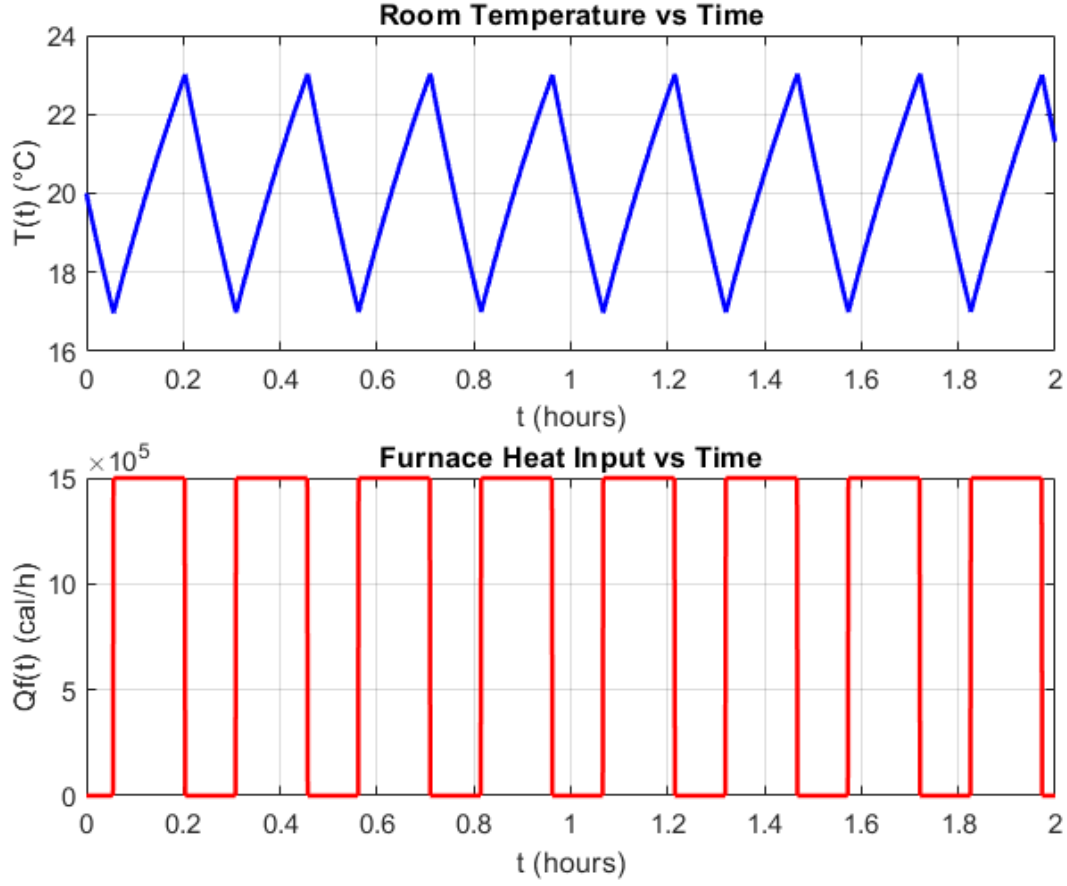


Figure 1: Temperature and Furnace Input Over Time

(2.3) To calculate the number of standard cubic meters of natural gas consumed, we follow the below equation:

Define  $t_{on} \triangleq$  time the furnace was on in hours. The Matlab code says it was 1.182 hours

Define  $Q_{f,on} \triangleq$  as the furnace heat input.

Define  $\rho_e \triangleq$  as the energy density.

Define  $e \triangleq$  as the efficiency.

$$\begin{aligned}\text{Volume of gas consumed} &= \frac{t_{\text{on}} Q_{f,\text{on}}}{\rho_e e} = \frac{(1.182)(1.5 \times 10^6)}{(9 \times 10^6)(0.9)} = 0.2188888 \\ &\approx 0.219 \text{ standard cubic meters}\end{aligned}$$

We also confirm with unit analysis that we get cubic meters.

$$\text{Volume of gas consumed} = \frac{t_{\text{on}} Q_{f,\text{on}}}{\rho_e e} = \frac{\text{hour} \cdot \frac{\text{cal}}{\text{hour}}}{\frac{\text{cal}}{\text{m}^3}} = \text{m}^3$$

- (2.4) With a new oscillating definition of  $T_a$ , we get Figure 2 below demonstrating the temperature and furnace input over time.

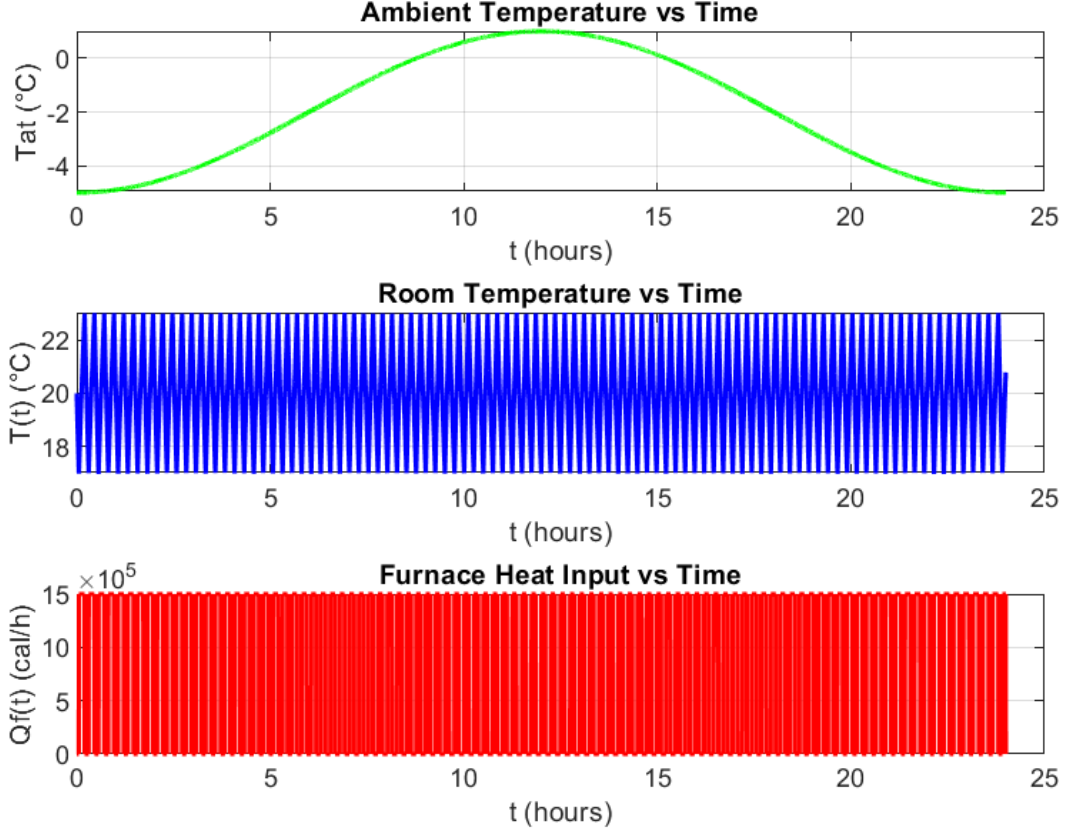


Figure 2: Temperature and Furnace Input Over Time

The simulation results align with the expected behavior of the system. The room temperature  $T(t)$  is maintained within the desired range of  $17^{\circ}\text{C}$  to  $23^{\circ}\text{C}$  as the furnace responds to changes in the ambient temperature  $T_a(t)$ . When  $T_a(t)$  is lower, the furnace operates more frequently to offset increased heat loss, while at higher  $T_a(t)$ , the furnace operates less often due to reduced heat loss. This behavior reflects the thermal dynamics of the system and the influence of the bang-bang control logic.

### 3. Question 3

(3.1) We are given the following ODE describing the current going through the system.

$$V_s(t) = Ri(t) + L \frac{di(t)}{dt}$$

During steady state, the current  $i(t)$  is constant, so the derivative of  $i(t)$  with respect to time is zero. Substituting this into the ODE:

$$V_s(t) = Ri(t) + L(0)$$

Rearranging, we get the following equation for  $i(t)$ :

$$i(t) = \frac{V_s(t)}{R}$$

This is useful when designing a circuit with a target current  $i(t)$  because you can control the voltage source to achieve the desired current while the resistance will always be a constant property of the circuit's hardware.

(3.2) I don't remember how to use ode45, so I referenced [1] to get started. The results of the Matlab code was plotted below in Figure 3.

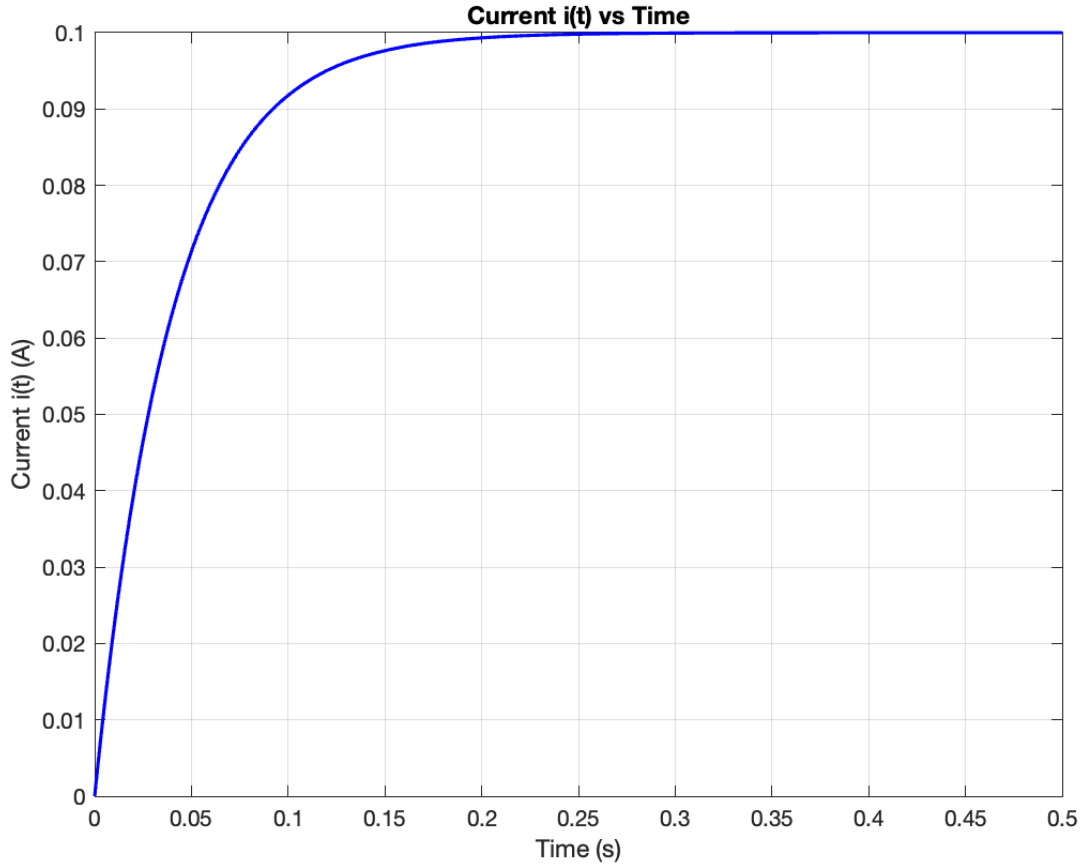


Figure 3: Current Over Time

(3.3)

$$V_s(t) = Ri(t) + L \frac{di(t)}{dt}$$

Based on the ODE above, inductance  $L$  should have no effect on the steady state current  $i(t)$  because the derivative of  $i(t)$  with respect to time is zero, and inductance is a coefficient on the derivative. This means that the inductance  $L$  term in the ODE will not affect the current  $i(t)$  when the system reaches steady state, and the current  $i(t)$  will be determined solely by the voltage source  $V_s(t)$  and the resistance  $R$  in the circuit. This can be seen below in Figure 4, where

changing  $R$  causes the steady state current  $i(t)$  to change, while changing  $L$  has no effect on the steady state current  $i(t)$ .

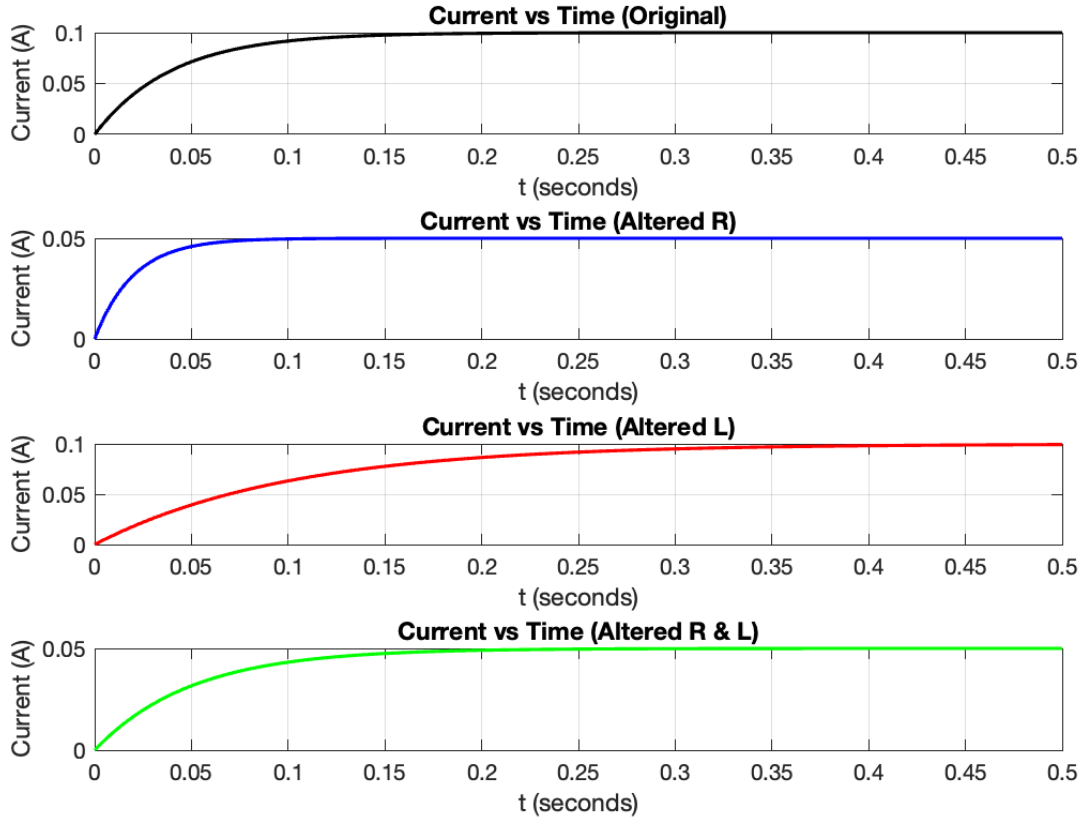


Figure 4: Current Over Time

- (3.4) Figure 5 and 6 below demonstrates the LTI nature of the system. An LTI system is defined as a system that is both Linear and Time Invariant. A linear system is one that follows the condition that if  $x(t) = \alpha x_1(t) + \beta x_2(t)$ , then  $y(t) = \alpha y_1(t) + \beta y_2(t)$ . That is, the output of a system is a linear combination of the inputs. We can observe in Figure 5 that the combination of the inputs  $V_{s1} = 5V$  and  $V_{s2} = 10V$  results in the outputs  $i_1(t)$  and  $i_2(t)$ , which is equal to  $i_3(t)$ .  $i_3(t)$  is



the resultant output from the combination input of  $V_s3 = V_s1 + V_s2 = 15V$ . This exhibits the linear property. We can also observe the system is time-invariant by looking at the equation — the equation has no coefficients that change over time. We can also see in Figure 6 that when taking the input and adding time delay, it is equivalent to taking the original output and time shifting that. This proves the equation is time-invariant.

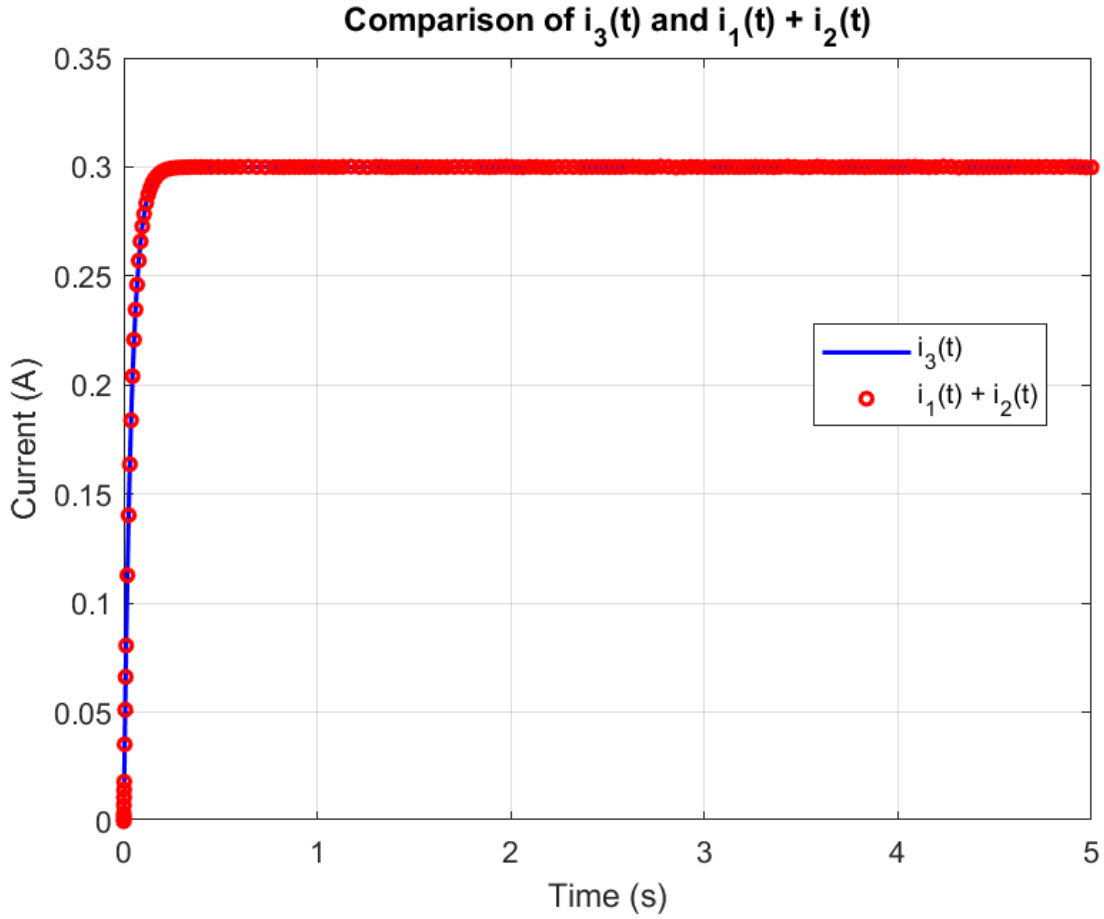


Figure 5: Comparing Linear Combination of Inputs to an Equivalent Output Over Time

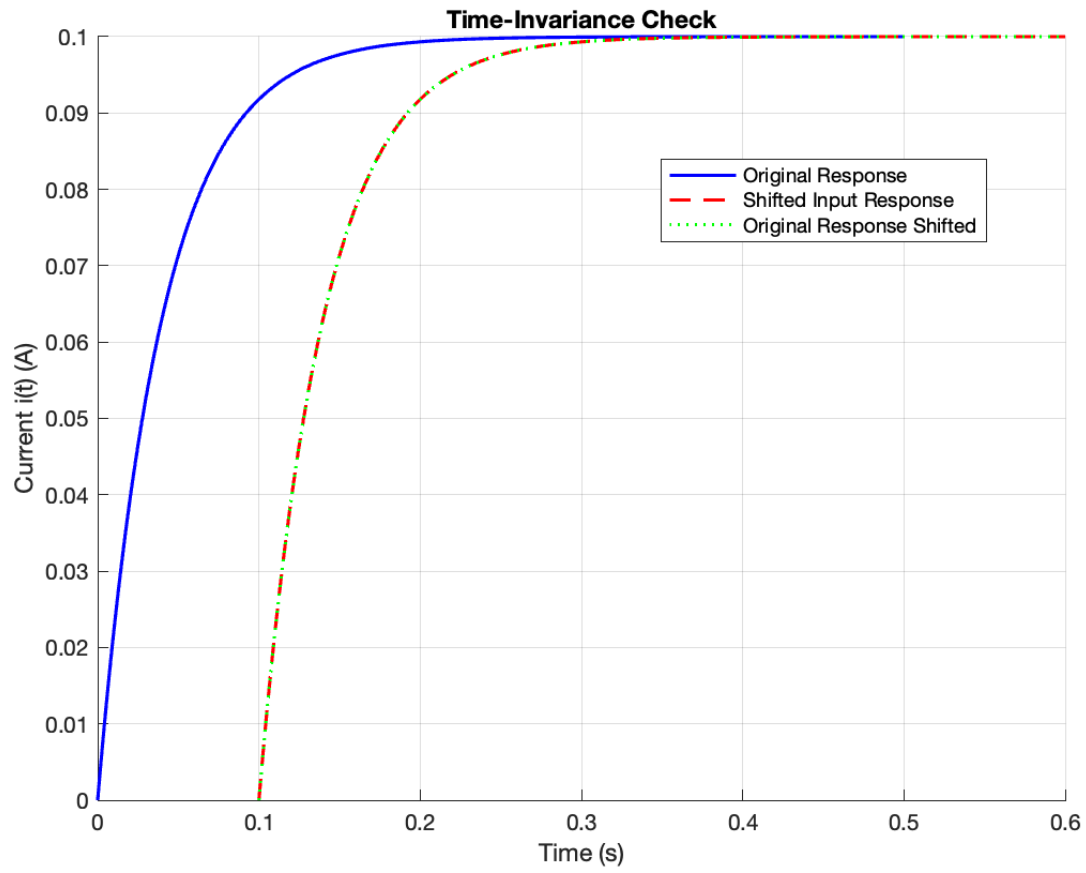


Figure 6: Comparing a Time-Shifted Input with the Time-Shifted Output of the Original Signal

4. Question 4

(4.1) Part A

SOLUTION

(4.2) Part B

SOLUTION

5. Question 5

(5.1) Part A

SOLUTION

(5.2) Part B

SOLUTION

## References

- [1] MATLAB Documentation: `ode45`. Available at: <https://www.mathworks.com/help/matlab/ref/ode45.html>.