IBEHS 4A03 Assignment #1

Baoze Lin, Hady Ibrahim

1. Question 1

   (1.1) yapyapyap

2. Question 2

(2.1) The ordinary differential equation (ODE) governing the temperature $T(t)$ is:

$$\frac{dT(t)}{dt} = \frac{Q_f(t) - UA(T(t) - T_a)}{\rho V c_p}$$

For this problem, the furnace is off, so $Q_f(t) = 0$. Substituting this into the equation:

$$\frac{dT(t)}{dt} = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

At steady state, the temperature $T(t)$ no longer changes with time, so:

$$\frac{dT(t)}{dt} = 0$$

Substitute this condition into the ODE:

$$0 = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

Solve:

$$T(t) - T_a = 0$$

Thus:

$$T(t) = T_a$$

(2.2) Using the dervied equation for $T(t)$, the behaviour of the furnace as defined below, and the given definition of one step of the univariate Euler's method, we can plot Figure 1.

$$Q_f(t) = \begin{cases} 0 & \text{When } T(t) > 23°\text{C}, \\ 1.5 \times 10^6 & \text{When } T(t) < 17°\text{C}, \\ \text{unchanged} & \text{For all } 17 \le T(t) \le 23°\text{C}. \end{cases}$$

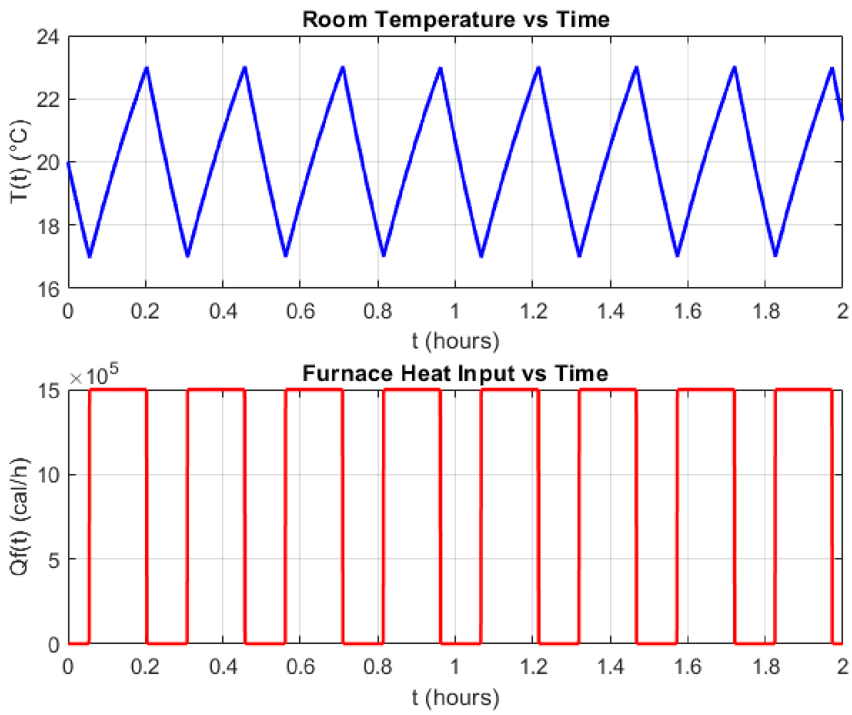

Figure 1: Temperature and Furnace Input Over Time

(2.3) To calculate the number of standard cubic meters of natural gas consumed, we follow the below equation:
Define $t_{on} \triangleq$ time the furnace was on in hours. The Matlab code says it was 1.182 hours
Define $Q_{f,on} \triangleq$ as the furnace heat input.
Define $\rho_e \triangleq$ as the energy density.
Define $e \triangleq$ as the efficiency.

$$\text{Volume of gas consumed} = \frac{t_{on}Q_{f,\text{on}}}{\rho_e e} = \frac{(1.182)(1.5 \times 10^6)}{(9 \times 10^6)(0.9)} = 0.2188888$$
$$\approx 0.219\,\text{standard cubic meters}$$

We also confirm with unit analysis that we get cubic meters.

$$\text{Volume of gas consumed} = \frac{t_{\text{on}} Q_{f,\text{on}}}{\rho_e e} = \frac{\text{hour} \cdot \frac{\text{cal}}{\text{hour}}}{\frac{\text{cal}}{\text{m}^3}} = \text{m}^3$$

3. Question 3

    (3.1) Part A
    SOLUTION

    (3.2) Part B
    SOLUTION

4. Question 4

   (4.1) Part A
   SOLUTION

   (4.2) Part B
   SOLUTION

5. Question 5

(5.1) Part A
SOLUTION

(5.2) Part B
SOLUTION