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1. Question 1

   (1.1) yapyapyap

2. Question 2

(2.1) The ordinary differential equation (ODE) governing the temperature $T(t)$ is:

$$\frac{dT(t)}{dt} = \frac{Q_f(t) - UA(T(t) - T_a)}{\rho V c_p}$$

For this problem, the furnace is off, so $Q_f(t) = 0$. Substituting this into the equation:

$$\frac{dT(t)}{dt} = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

At steady state, the temperature $T(t)$ no longer changes with time, so:

$$\frac{dT(t)}{dt} = 0$$

Substitute this condition into the ODE:

$$0 = \frac{-UA(T(t) - T_a)}{\rho V c_p}$$

Solve:

$$T(t) - T_a = 0$$

Thus:

$$T(t) = T_a$$

(2.2) Using the dervied equation for $T(t)$, the behaviour of the furnace as defined below, and the given definition of one step of the univariate Euler's method, we can plot Figure 1.

$$Q_f(t) = \begin{cases} 0 & \text{When } T(t) > 23°\text{C}, \\ 1.5 \times 10^6 & \text{When } T(t) < 17°\text{C}, \\ \text{unchanged} & \text{For all } 17 \leq T(t) \leq 23°\text{C}. \end{cases}$$

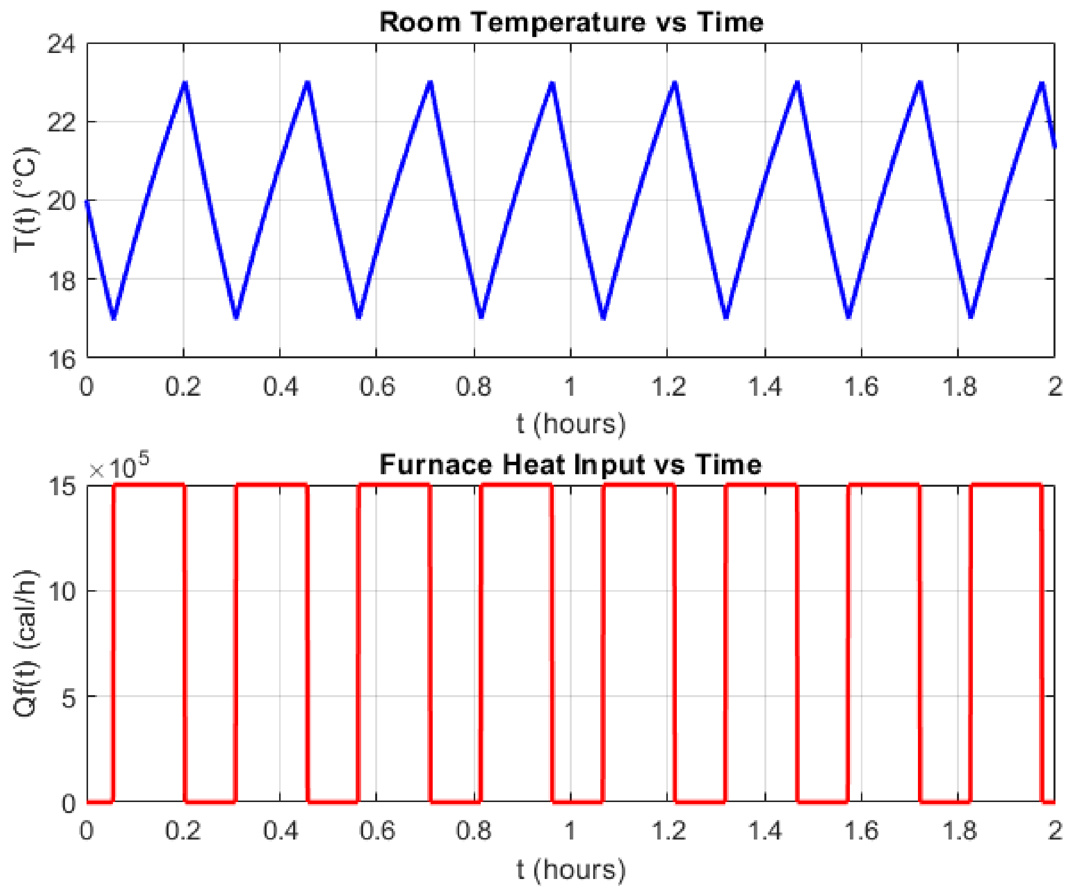

Figure 1: Temperature and Furnace Input Over Time

(2.3) To calculate the number of standard cubic meters of natural gas consumed, we follow the below equation:

Define $t_{on} \triangleq$ time the furnace was on in hours. The Matlab code says it was 1.182 hours

Define $Q_{f,on} \triangleq$ as the furnace heat input.

Define $\rho_e \triangleq$ as the energy density.

Define $e \triangleq$ as the efficiency.

$$\text{Volume of gas consumed} = \frac{t_{on}Q_{f,\text{on}}}{\rho_e e} = \frac{(1.182)(1.5 \times 10^6)}{(9 \times 10^6)(0.9)} = 0.2188888$$
$$\approx 0.219\,\text{standard cubic meters}$$

We also confirm with unit analysis that we get cubic meters.

$$\text{Volume of gas consumed} = \frac{t_{\text{on}}Q_{f,\text{on}}}{\rho_e e} = \frac{\text{hour} \cdot \frac{\text{cal}}{\text{hour}}}{\frac{\text{cal}}{\text{m}^3}} = \text{m}^3$$

(2.4) With a new oscillating defintion of Ta, we get Figure 2 below demonstrating the temperature and furnace input over time.

The simulation results align with the expected behavior of the system. The room temperature $T(t)$ is maintained within the desired range of 17°C to 23°C as the furnace responds to changes in the ambient temperature $T_a(t)$. When $T_a(t)$ is lower, the furnace operates more frequently to offset increased heat loss, while at higher $T_a(t)$, the furnace operates less often due to reduced heat loss. This behavior reflects the thermal dynamics of the system and the influence of the bang-bang control logic.

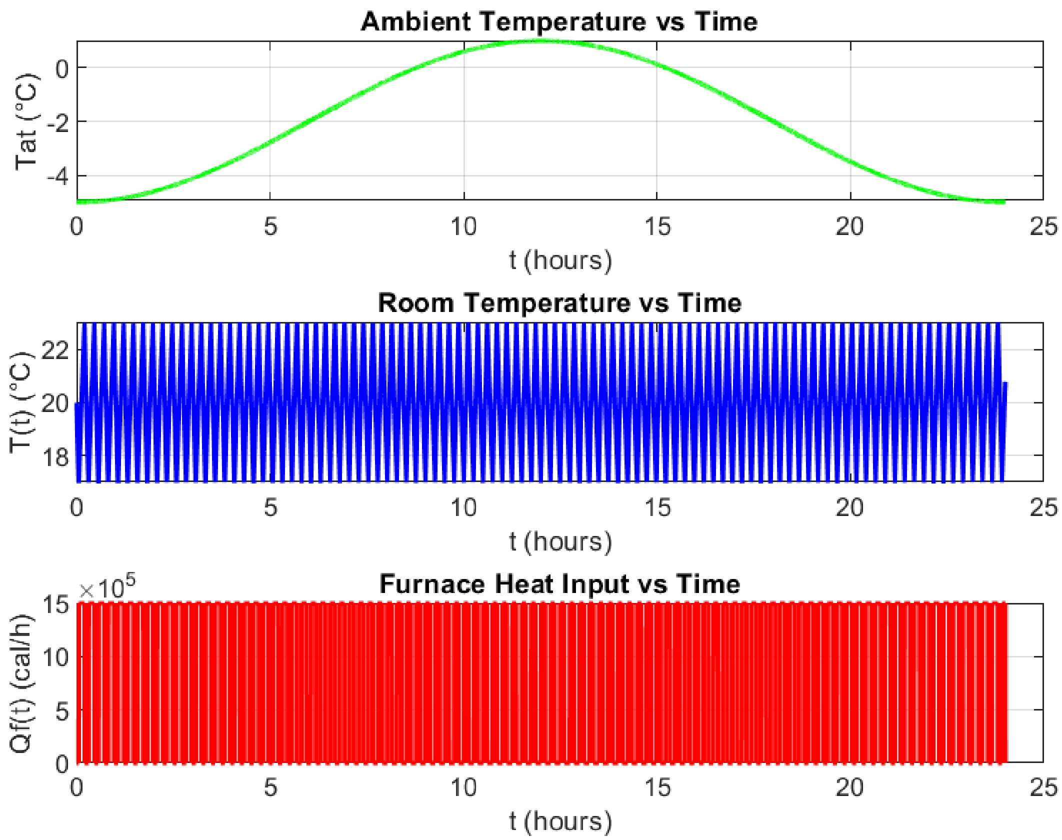

Figure 2: Temperature and Furnace Input Over Time

3. Question 3

(3.1) We are given the following ODE describing the current going thorugh the system.

$$V_s(t) = Ri(t) + L\frac{di(t)}{dt}$$

During steady state, the current $i(t)$ is constant, so the derivative of $i(t)$ with respect to time is zero. Substituting this into the ODE:

$$V_s(t) = Ri(t) + L(0)$$

Rearranging, we get the following equation for $i(t)$:

$$i(t) = \frac{V_s(t)}{R}$$

This is useful when designing a cirtcuit with a target current i(t) because you can control the voltage source to achieve the desired current while the resistance will always be a constant property of the circuit's hardware.

(3.2) Part B

SOLUTION

4. Question 4

    (4.1) Part A
    SOLUTION

    (4.2) Part B
    SOLUTION

5. Question 5

   (5.1) Part A
   SOLUTION

   (5.2) Part B
   SOLUTION