Data Structure Final Exam

2019.1.8

Closed-book exam. Lecturer: Chih-Ya Shen (沈之涯)

Please answer the following questions in the answer sheet
Total points: 105 (maximum points given: 100)

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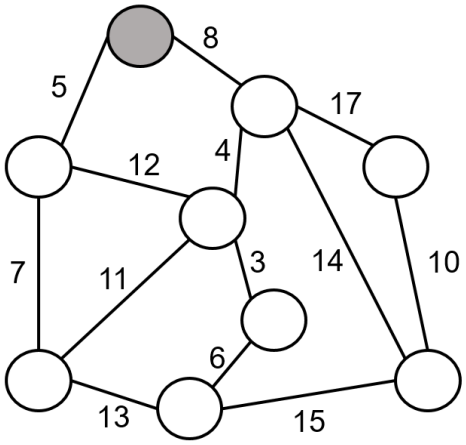
1. **(10%, 2% each)** For each of the following algorithms, what is the **tightest** asymptotic upper bound for its time complexity? Please use the following table of (A)~(E) choices to answer.

(A)	(B)	(C)	(D)	(E)
O(1)	O(n)	O(lgn)	O(nlgn)	$O(n^2)$

(i). Best-case time complexity of insertion sort for **n** numbers
(ii). Worst-case time complexity of merge sort for **n** numbers
(iii). Search an element in an AVL tree
(iv). Search an element in a hash table with **n** static keys and perfect hashing
(v). Best-case time complexity of merge sort for **n** numbers

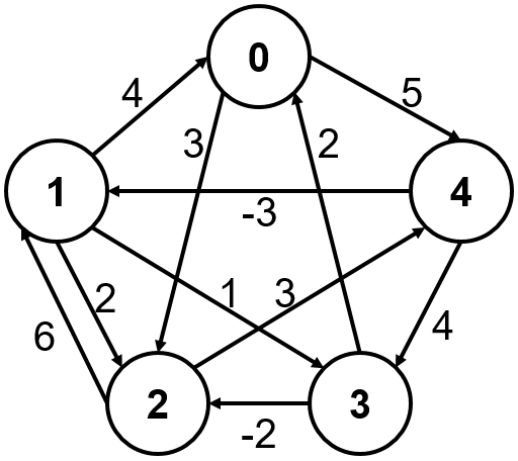
2. **(12%)** Consider the graph below.
   A. Please use the Kruskal's algorithm to find the minimum-spanning tree **step-by-step**. (6%)
   B. Let the gray node be the starting node. Please use Prim's algorithm to find the minimum-spanning tree **step-by-step**. (6%)



3. **(10%)** Given the following sequence of keys [79,68,106,39,118,99], please answer the following questions if the hash function **h(x) = (x mod 7)** is used. Assume each bucket has only one slot and the bucket numbers start from **0**.
   A. Hash table with ***linear probing***. What is the number in bucket 4? (5%)
   B. Hash table with **rehashing** where the i^{th} ($i \geq 1$) hash function is defined as:
   $$h_i(x) = (x+3*i^2) \bmod 7.$$
   What is the number in bucket 3? (5%)

4. **(18%)** Suppose we want to sort the following sequence of decimal numbers [57, 120, 361, 232, 447, 236, 126, 82] in increasing order.
   - **A.** Please write down the result for each pass of using the **radix sort** with radix **r=6** to sort the sequence in LSD order. (6%)
   - **B.** Please write down the result for each pass of using the **<u>iterative</u> merge sort**. (6%)
   - **C.** Please write down the result for each pass of using the **insertion sort**. (6%)

5. **(8%)** Consider the graph below. Please write down the result of each step of using the Bellman-Ford algorithm to find the shortest path starting from vertex 0 to all other vertices. If there exists a negative cycle, please identify it with the Bellman-Ford algorithm.



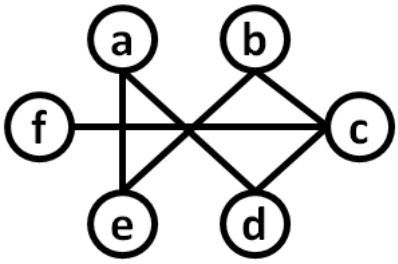
6. **(10%)** Suppose all vertices in an undirected graph G have **odd degree k**. Please show that the total number of edges in G is a multiple of k.

7. **(7%)** Given a graph G=(V,E), please design an algorithm with time complexity O(|V|+|E|) that answers whether the given graph contains cycle(s) or not.

8. **(10%)** Show how to sort **n** integers in the range [0,n^4-1] in **<u>linear</u>** time (assume we have infinite memory).

9. **(10%)** Some sorting algorithms are not stable, for example, Quick Sort. Please show that we can make every sorting algorithm stable by pre-processing the input data and slightly adjusting the algorithm. Please note that the new algorithm should have the same time complexity.

10. **(10%)** Given a graph $G = (V, E)$, a subset of vertices $S \subseteq V$ is said to be a k-plex if the minimum degree of the subgraph induced by S on G is at least $|S| - k$. For example, $S^* = \{a, b, c, d, e\}$ is a 3-plex with |S|=5. In contrast, $S^* = \{a, b, c, d, e, f\}$ is not a 3-plex.



Given a k-plex S and any subset $S' \subseteq S$, is S' a k-plex as well? If yes, please provide the proof. If no, please provide a counterexample.

The definition of an **induced subgraph** is as follows. Let G = (V, E) be a graph, and let $S \subseteq V$ be a subset of vertices of G. The induced subgraph G[S] is the graph whose vertex set is S, and whose edge set consists of all edges in E that have both endpoints in S. For example, given the graph in Figure (a), the subgraph induced by S = {1,2,3} is shown in Figure (b). The induced subgraph of S = {1,2,3,5} is shown in Figure (c).

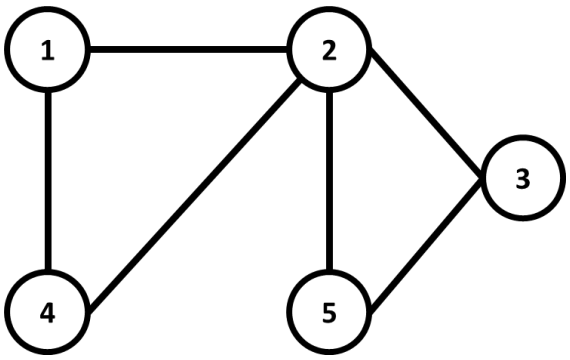


Figure (a)

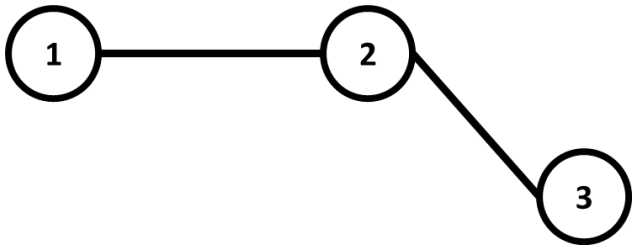


Figure (b)

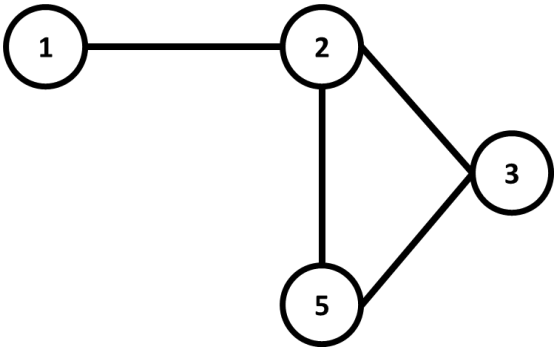


Figure (c)