IMAGE PROCESSING

Chapter 5: Image Restoration

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CONTENTS

Image Degradation/Restoration Model

- Image degradation is modeled as an operator $\mathscr{H}$ together with an additive noise term for an input image **f(x,y)**, to generate the degraded image **g(x,y)**.
- Given **g(x,y)**, some knowledge about $\mathscr{H}$ and some knowledge about the additive noise term, the objective of restoration is to obtain an estimate of the original image.

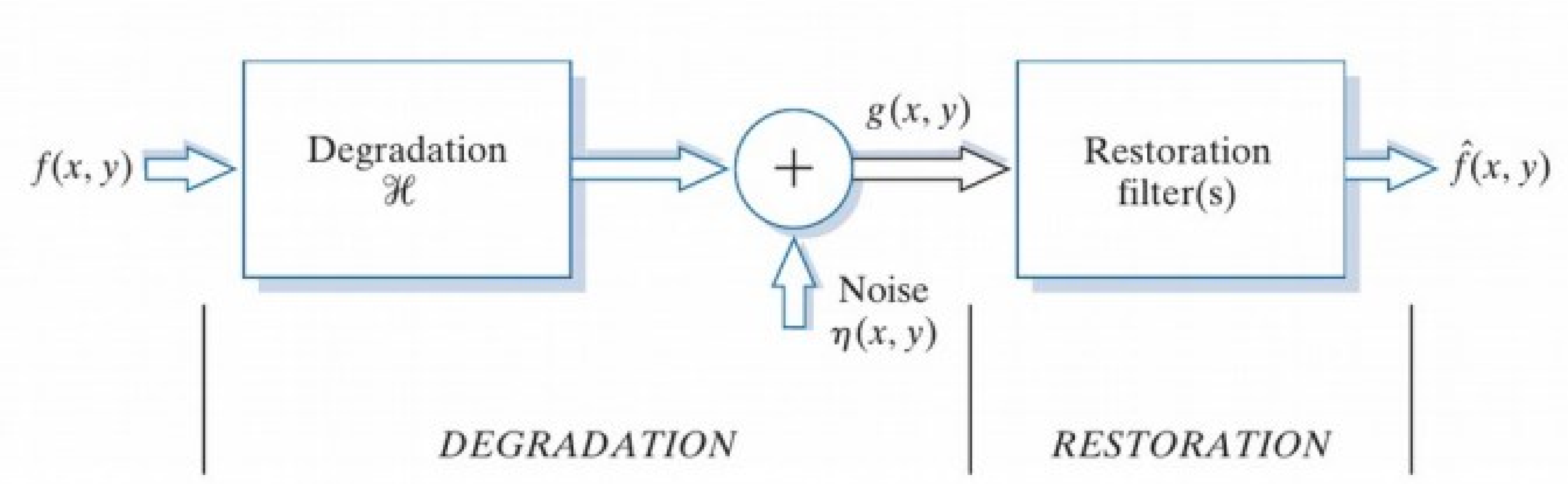


Image Degradation/Restoration Model

- We will show later that if the operator $\mathcal{H}$ is linear and position-invariant, then the degraded image is given in the spatial domain by:

$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + n(x,y)}$$

with h(x,y) as the spatial representation of the degradation function and n(x,y) as the additive noise term.

● Given the convolution theorem, the degraded image in the frequency domain is:

$$\boxed{G(u,v) = H(u,v)F(u,v) + N(u,v)}$$

with the terms in capital letters as the Fourier transform of the corresponding terms.

● Let's focus on **noise-only degradation**

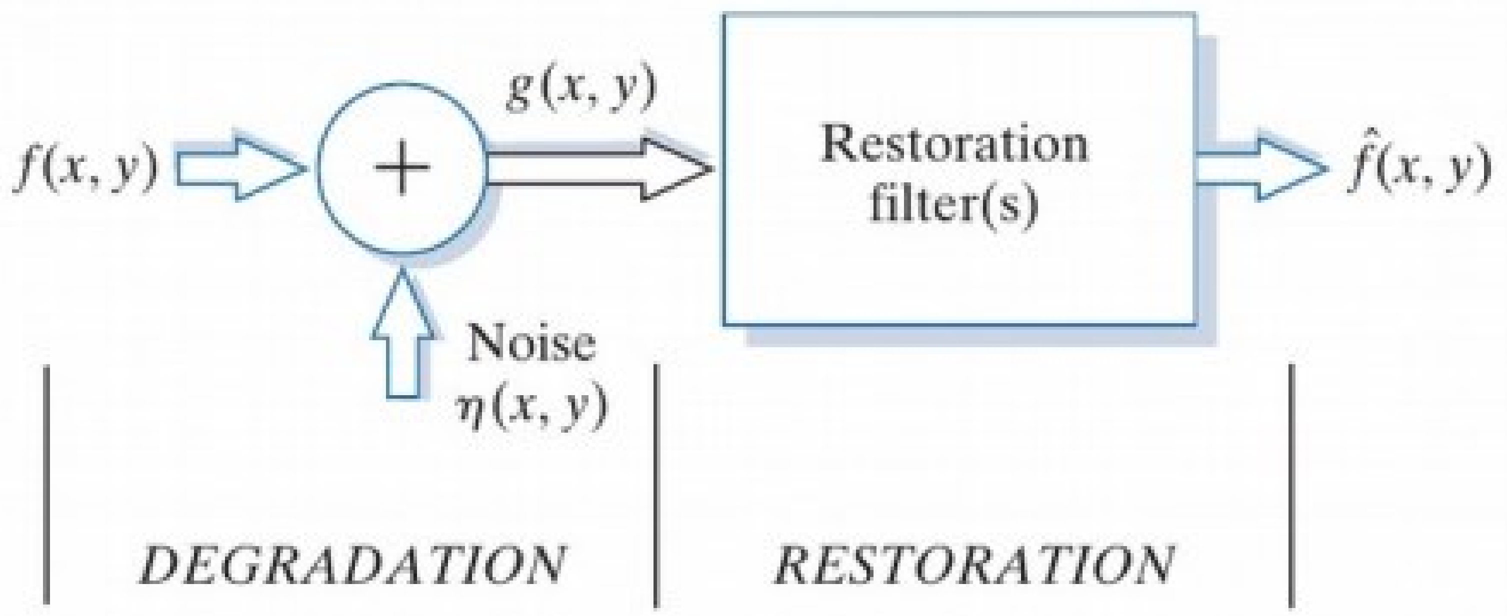
Performance

$$MSE = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

$$SNR = \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2}$$

Noise Models

- Sources of noise in digital images arise mainly during **image acquisition** and/or transmission.
- For example in CCD cameras, light levels and sensor temperature are major factors in the amount of noise.
- During transmission, images are affected by interference in the transmission channel.
- For example in wireless transmission, lightning or other atmospheric disturbances cause noise.



Restoration principle

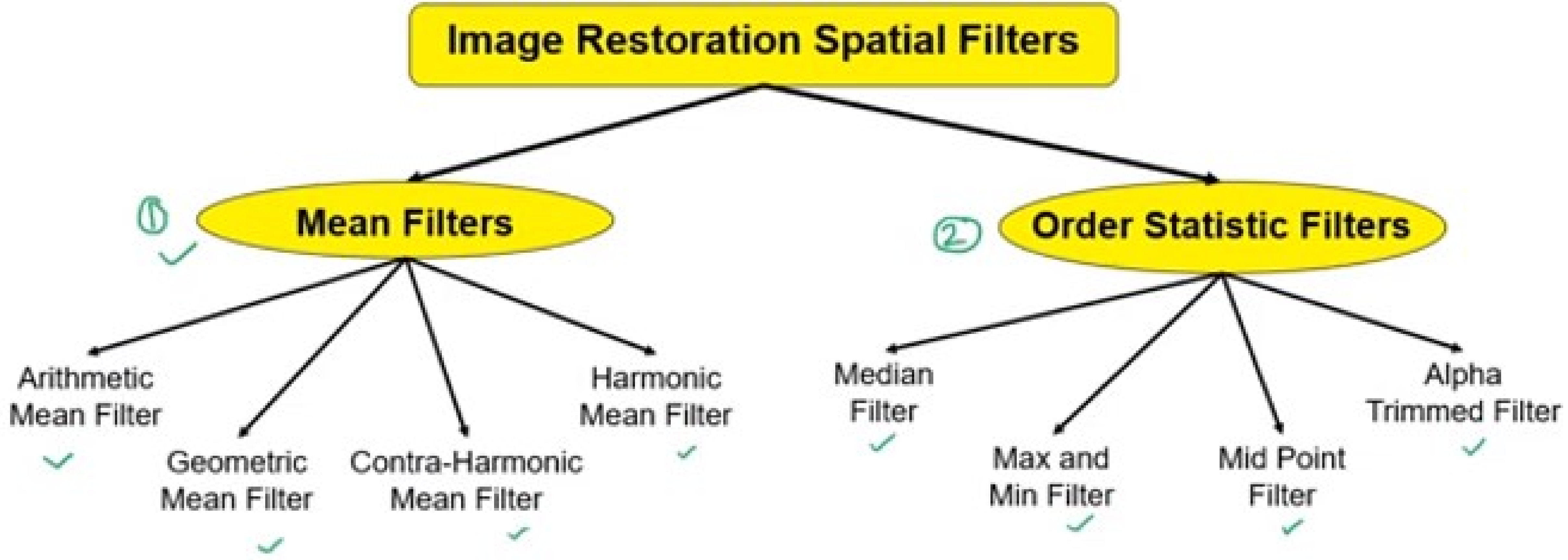
- A noise model is used to describe the characteristics of the noise in the image. Corresponding noise filters are then applied to restore the image
- The degree of image blur (an inverse filter is applied to deblur the image).

Noise models

- Salt and pepper:
- Gaussian
- Speckle:
- Impulse-nhiễu xung:

Image Restoration in presence of Noise Only

Spatial Filter, which are used in image smoothing and sharpening, are also useful for removing noise. Classification is shown as:



Mean Filters

1. Arithmetic Mean Filter

- This filter removes local variations within the image
- It is similar to low pass filter
- It is useful in removing Gaussian Noise and Uniform Noise

$$\boxed{\hat{f}(x,y) = \frac{1}{mn}\sum_{(s,t)\in S_{x,y}} g(s,t)}$$

2. Geometric Mean Filter

- This filter eliminates Gaussian Noise
- It is ineffective for Pepper type of Noise
- Geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process

$$\boxed{\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{x,y}} g(s,t)\right]^{\frac{1}{mn}}}$$

Mean Filters

3. Harmonic Mean Filter

- It works well for salt noise, but fails for pepper noise.
- It does well also with other types of noise like Gaussian noise

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

4. Contra-harmonic Mean Filter

- It is well suited for reducing the effects of salt-and-pepper noise
- Q>0 for elimination of pepper noise and Q<0 for elimination of salt noise
- Q is the order of the filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{x,y}} \left[g(s,t)\right]^{Q+1}}{\sum_{(s,t)\in S_{x,y}} \left[g(s,t)\right]^{Q}}$$

Image Restoration in presence of Noise Only

Order Statistic Filters

- Order statistic filters are also known as rank, rank order or order filters
- These filters are not based on convolution
- These filters are differentiated based on how they choose the values in the sorted list. The position indicates the rank.

1. Median Filter

- It is an example of non linear filter
- It simply sorts the list and finds the median
- Centre pixel is replaced by median value

$$f(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

2. Maximum Filter

- This filter selects the largest value in the sorted list
- It is used for removing pepper type of noises

$$f(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

Image Restoration in presence of Noise Only

Order Statistic Filters

3. Minimum Filter

- This filter selects the smallest value
- It is very effective in elimination of salt type of noises

$$f(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

4. Midpoint Filter

- This filter selects the midpoint
- Midpoint is the average of minimum and maximum values
- It is very effective in removing Gaussian noise and Uniform noise

$$f(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\}\right]$$

Alpha-trimmed Mean Filter

Minimum Mean Square Error (Wiener) Filtering

Restoration of blur

The noise and image are assumed uncorrelated, with either one having a zero mean, and that the intensity levels in the estimate are a linear function of the levels in the degraded image.

- Considering all these assumptions, the restored image is represented as:

$$
\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2+S_n(u,v)}\right]G(u,v) \\
&= \left[\frac{H^*(u,v)}{|H(u,v)|^2+S_n(u,v)/S_f(u,v)}\right]G(u,v) \\
&= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+S_n(u,v)/S_f(u,v)}\right]G(u,v)
\end{aligned}
$$

- In this formulation:

$$
\begin{aligned}
H^*(u,v) &: \text{ complex conjugate of } H(u,v) \\
|H(u,v)|^2 &= H^*(u,v)H(u,v) \\
S_n(u,v) &= |N(u,v)|^2 \text{ power spectrum of the noise} \\
S_f(u,v) &= |F(u,v)|^2 \text{ power spectrum of the undegraded image}
\end{aligned}
$$

- The derivation is out of the scope of the course.

Restoration of blur

Minimum Mean Square Error (Wiener) Filtering

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}\right] G(u,v)$$

- If the noise is zero, the Wiener filter reduces to simple inverse filter.
- The signal-to-noise ratio measures the level of the original, undegraded image power to the level of noise power:

$$\boxed{SNR = \frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |\hat{F}(u,v)|^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |N(u,v)|^2} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} |\hat{f}(x,y)|^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - \hat{f}(x,y)]^2}}$$

Geometric Mean Filter

- The Wiener filter can be generalized in the form of the geometric mean filter:

$$\hat{F}(u,v)=\left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha}\left[\frac{H^*(u,v)}{|H(u,v)|^2+\beta\frac{S_n(u,v)}{S_f(u,v)}}\right]^{1-\alpha}G(u,v)$$

  with **α** and **β** as non-negative real constants.

- When **α=1**, the filter reduces to the ***inverse filter***.
- When **α=0**, the filter becomes a ***parametric Wiener filter***, which reduce to standard ***Wiener filter*** when **β=1**.
- When **α=1/2** and **β=1** the filter is called a ***spectrum equalization filter***.
- The geometric mean filter is useful in image restoration, since it represents a family of filters as a single expression.

```
function ImageRestoration(image)
img = imread(image);
figure;
subplot(1,3,1);
imshow(img);
title('Ảnh gốc');
noisy_img = imnoise(img,'salt & pepper',0.01);%imnoise(img,'gaussian', 0, 0.02);
subplot(1,3,2);
imshow(noisy_img);
title('Ảnh bị nhiễu Gaussian');
restored_img = medfilt2(noisy_img, [3 3]);
subplot(1,3,3);
imshow(restored_img);
title('Ảnh sau khi khử nhiễu');
end
```

Homework

Create a function to compare different filtering algorithms by calculating their Mean Squared Error on a given image