Appendix

6.1 Noise analysis in current mirror

In this section, we will step by step analyse the noise contribution in a current mirror architechture

show them the image

In this schematics, we place noise source i_{nref}, i_{n1} and i_{n2} to represent the current noise of these npn transistor.

Our purpose for the current mirror is to dominate by the fundamental noise (in this case is short noise). In this case we've chosen to optimise the circuit in order to minimize the size and negligible also the contribution of thermal noise and flicker noise.

6.2 Derive the noise contribution in current mirror

6.2.1 Assumption

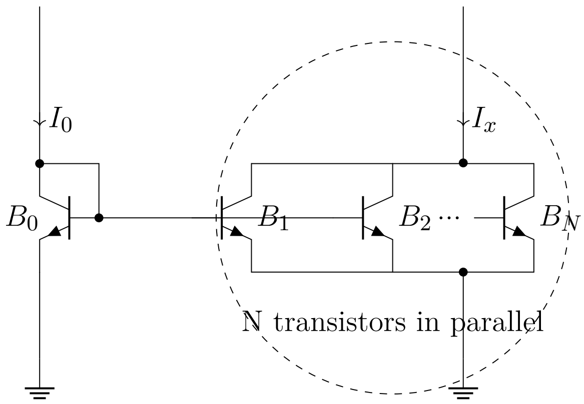

Figure 6.1: Simple N-multiplier current mirror model

In this section, we are gonna use the simple model that is N-multiplier current mirror. In this model, we assume using one primodial transistor at the first branch, then a large transistor composed by N - transistors in parallel in the second branch. Now we are going to discuss about different parameter of this large transistor

B_0	B_{tot}
I_C	$N \cdot I_C$
$R_{BB'}$	$R_{BB'}/N$
β	β
g_m	$N \cdot g_m$

Table 6.1: Caption

Remember to discuss different params

6.2.2 Noise derivation

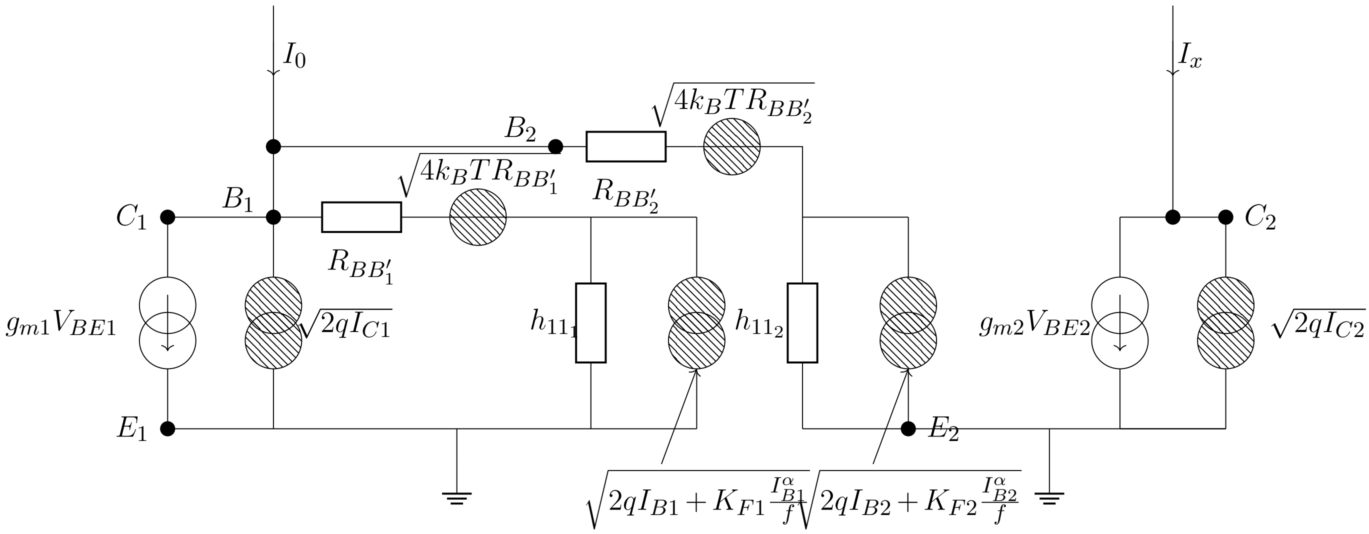

Figure 6.2: Circuit uses to analyse the noise

From this one, we are interested to find the current density S_{Ix}

$$S_{Ix} = g_{m2}^2 S_{V_{BE2}} + 2qI_{C2} = \frac{\beta_2^2}{h_{11_2}^2} S_{V_{BE2}} + 2qI_{C2} \tag{6.1}$$

Now we have to identify $S_{V_{BE2}}$:

$$
\begin{aligned}
S_{V_{BE2}} = &(\sqrt{4k_BTR_{BB'_2}} \cdot \frac{h_{11_2}}{R_{BB'_1}+R_{BB'_2}+h_{11_1}+h_{11_2}})^2 \\
&+(\sqrt{2qI_{B2}+K_{F2}\frac{I_{B2}^{\alpha}}{f}} \cdot \frac{h_{11_2}\cdot(R_{BB'_1}+R_{BB'_2}+h_{11_1})}{h_{11_2}+R_{BB'_1}+R_{BB'_2}+h_{11_1}})^2 \\
&+(\sqrt{2qI_{C1}} \cdot \frac{h_{11_1}+R_{BB'_1}}{h_{11_1}+h_{11_2}+R_{BB'_1}+R_{BB'_2}} \cdot h_{11_2})^2 \\
&+(\sqrt{2qI_{B1}+K_{F1}\frac{I_{B1}^{\alpha}}{f}} \cdot \frac{h_{11_1}}{R_{BB'_1}+R_{BB'_2}+h_{11_2}} \cdot h_{11_2})^2 \\
&+(\sqrt{4k_BTR_{BB'_1}} \cdot \frac{h_{11_2}}{R_{BB'_1}+R_{BB'_2}+h_{11_1}+h_{11_2}})^2
\end{aligned}
$$

While the current

For $h_{11_2}=h_{11_1}/N=h_{11}/N$, $\beta_1=\beta_2=\beta$, $I_{C2}=NI_{C1}=NI_C$, $I_{B2}=NI_{B1}=NI_B$, $R_{BB'_2}=R_{BB'_1}/N=R_{BB'}/N$, we can simplify the V_{BE_2}

$$
\begin{aligned}
S_{V_{BE2}} = &(4k_BT\frac{R_{BB'}}{N})\left(\frac{\frac{h_{11}}{N}}{R_{BB'}+\frac{R_{BB'}}{N}+h_{11}+\frac{h_{11}}{N}}\right)^2 \\
&+(2qNI_B+K_{F2}\frac{N^{\alpha}I_B^{\alpha}}{f})\left(\frac{\frac{h_{11}}{N}\cdot(R_{BB'}+\frac{R_{BB'}}{N}+h_{11})}{\frac{h_{11}}{N}+R_{BB'}+\frac{R_{BB'}}{N}+h_{11}}\right)^2 \\
&+2qI_C\left(\frac{h_{11}+R_{BB'}}{h_{11}+\frac{h_{11}}{N}+R_{BB'}+\frac{R_{BB'}}{N}}\cdot\frac{h_{11}}{N}\right)^2 \\
&+(2qI_B+K_{F1}\frac{I_B^{\alpha}}{f})\left(\frac{h_{11}}{R_{BB'}+\frac{R_{BB'}}{N}+\frac{h_{11}}{N}}\cdot\frac{h_{11}}{N}\right)^2 \\
&+(4k_BTR_{BB'})\left(\frac{\frac{h_{11}}{N}}{R_{BB'}+\frac{R_{BB'}}{N}+h_{11}+\frac{h_{11}}{N}}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
S_{V_{BE2}} &= (4k_BT\frac{R_{BB'}}{N})\left(\frac{h_{11}}{(N+1)(R_{BB'}+h_{11})}\right)^2 \\
&+ (2qNI_B + K_{F2}\frac{N^\alpha I_B^\alpha}{f})\left(\frac{h_{11}((N+1)R_{BB'}+Nh_{11})}{N(N+1)(R_{BB'}+h_{11})}\right)^2 \\
&+ 2qI_C\left(\frac{h_{11}(R_{BB'}+h_{11})}{(N+1)(R_{BB'}+h_{11})}\right)^2 \\
&+ (2qI_B + K_{F1}\frac{I_B^\alpha}{f})\left(\frac{h_{11}^2}{(N+1)R_{BB'}+h_{11}}\right)^2 \\
&+ (4k_BTR_{BB'})\left(\frac{h_{11}}{(N+1)(R_{BB'}+h_{11})}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
S_{V_{BE2}} &= (4k_BTR_{BB'})\left(\frac{h_{11}^2}{N(N+1)(R_{BB'}+h_{11})^2}\right) \\
&+ (2qNI_B + K_{F2}\frac{N^\alpha I_B^\alpha}{f})\left(\frac{h_{11}[(N+1)R_{BB'}+Nh_{11}]}{N(N+1)(R_{BB'}+h_{11})}\right)^2 \\
&+ 2qI_C\left(\frac{h_{11}}{N+1}\right)^2 \\
&+ (2qI_B + K_{F1}\frac{I_B^\alpha}{f})\left(\frac{h_{11}^2}{(N+1)R_{BB'}+h_{11}}\right)^2
\end{aligned}
$$

Substitute $I_C = \beta I_B$, also $K_{F1} = K_{F2} = K_F$ we have

In this simple assumption, we consider K_F not change for the same technology (but after this, please consider changing it

$$
\begin{aligned}
S_{V_{BE2}} &= (4k_BTR_{BB'})\left(\frac{h_{11}^2}{N(N+1)(R_{BB'}+h_{11})^2}\right) \\
&+ (2qN\frac{I_C}{\beta} + K_F\frac{N^\alpha I_C^\alpha}{f\beta^\alpha})\left(\frac{h_{11}[(N+1)R_{BB'}+Nh_{11}]}{N(N+1)(R_{BB'}+h_{11})}\right)^2 \\
&+ 2qI_C\left(\frac{h_{11}}{N+1}\right)^2 \\
&+ (2q\frac{I_C}{\beta} + K_F\frac{I_C^\alpha}{f\beta^\alpha})\left(\frac{h_{11}^2}{(N+1)R_{BB'}+h_{11}}\right)^2
\end{aligned}
$$

Assuming condition on transistor, we have equation 6.1 become:

$$S_{Ix} = \frac{N^2\beta^2}{h_{11}^2}S_{V_{BE2}} + 2NqI_C \tag{6.2}$$

Substituting $S_{V_{BE2}}$ to 6.2, we have:

$$\begin{aligned}
S_{V_{BE2}} = &(4k_BTR_{BB'})\frac{N\beta^2}{(N+1)(R_{BB'}+h_{11})^2}\\
&+(2qNI_C\beta+K_F\frac{N^{\alpha}I_C^{\alpha}}{f\beta^{\alpha-2}})\left(\frac{(N+1)R_{BB'}+Nh_{11}}{(N+1)(R_{BB'}+h_{11})}\right)^2\\
&+2qI_C\frac{N^2\beta^2}{(N+1)^2}\\
&+(2qI_C\beta+K_F\frac{I_C^{\alpha}}{f\beta^{\alpha-2}})\left(\frac{Nh_{11}}{(N+1)R_{BB'}+h_{11}}\right)^2\\
&+2qNI_C
\end{aligned}$$

Since we are using SiGe HBT bipolar transistor, α can be chosen at $\alpha=2$ [4], the equation above become:

provide some more paper, or even better if s.o provide the experiment for this

$$\begin{aligned}
S_{V_{BE2}} = &(4k_BTR_{BB'})\frac{N\beta^2}{(N+1)(R_{BB'}+h_{11})^2}\\
&+(2qNI_C\beta+K_F\frac{N^2I_C^2}{f})\left(\frac{(N+1)R_{BB'}+Nh_{11}}{(N+1)(R_{BB'}+h_{11})}\right)^2\\
&+2qI_C\frac{N^2\beta^2}{(N+1)^2}\\
&+(2qI_C\beta+K_F\frac{I_C^2}{f})\left(\frac{Nh_{11}}{(N+1)R_{BB'}+h_{11}}\right)^2\\
&+2qNI_C
\end{aligned}$$

$$
\begin{aligned}
S_{V_{BE2}} = & (4k_BTR_{BB'})\frac{N\beta^2}{(N+1)(R_{BB'}+h_{11})^2} \\
& +(2qNI_C\beta+K_F\frac{N^2I_C^2}{f})\left(\frac{-h_{11}}{(N+1)(R_{BB'}+h_{11})}\right)^2 \\
& +2qI_C\frac{N^2\beta^2}{(N+1)^2} \\
& +(2qI_C\beta+K_F\frac{I_C^2}{f})\left(\frac{Nh_{11}}{(N+1)R_{BB'}+h_{11}}\right)^2 \\
& +2qNI_C
\end{aligned}
$$

For $h_{11} = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C}$, we have

$$
\begin{aligned}
S_{V_{BE2}} = & (4k_BTR_{BB'})\frac{N\beta^2}{(N+1)(R_{BB'}+\frac{\beta V_T}{I_C})^2} \\
& +(2qNI_C\beta+K_F\frac{N^2I_C^2}{f})\left(\frac{-\frac{\beta V_T}{I_C}}{(N+1)(R_{BB'}+\frac{\beta V_T}{I_C})}\right)^2 \\
& +2qI_C\frac{N^2\beta^2}{(N+1)^2} \\
& +(2qI_C\beta+K_F\frac{I_C^2}{f})\left(\frac{N\frac{\beta V_T}{I_C}}{(N+1)R_{BB'}+\frac{\beta V_T}{I_C}}\right)^2 \\
& +2qNI_C
\end{aligned}
$$

$$
\begin{aligned}
S_{V_{BE2}} = & (4k_BTR_{BB'})\frac{N\beta^2I_C^2}{(N+1)(I_CR_{BB'}+\beta V_T)^2} \\
& +(2qNI_C\beta+K_F\frac{N^2I_C^2}{f})\left(\frac{-\beta V_T}{(N+1)(I_CR_{BB'}+\beta V_T)}\right)^2 \\
& +2qI_C\frac{N^2\beta^2}{(N+1)^2} \\
& +(2qI_C\beta+K_F\frac{I_C^2}{f})\left(\frac{N\beta V_T}{(N+1)I_CR_{BB'}+\beta V_T}\right)^2 \\
& +2qNI_C
\end{aligned}
$$

Distinguish different types of noise

$$S_{V_{BE2}} = (4k_BTR_{BB'})\frac{N\beta^2 I_C^2}{(N+1)(I_C R_{BB'}+\beta V_T)^2} + K_F\frac{I_C^2}{f}\left(\left(\frac{-N\beta V_T}{(N+1)(I_C R_{BB'}+\beta V_T)}\right)^2 + \left(\frac{N\beta V_T}{(N+1)I_C R_{BB'}+\beta V_T}\right)^2\right) + 2qI_C\left(\frac{\beta^3 V_T^2}{(N+1)^2(I_C R_{BB'}+\beta V_T)^2} + \frac{N^2\beta^2}{(N+1)^2} + \frac{N^2\beta^3 V_T^2}{((N+1)I_C R_{BB'}+\beta V_T)^2} + N\right)$$

lots of work to do

6.3 Mathematical background

6.3.1 Fourrier Transform

6.3.2 Wiener-Khinchin theory

6.3.3 Langevin equation